1990

Numerical approaches to flood routing in rivers

Dedi Budiawan

University of Wollongong

NUMERICAL APPROACHES TO FLOOD ROUTING IN RIVERS

A thesis submitted in fulfilment of the requirements

for the award of the degree of

MASTER OF ENGINEERING (HONOURS)

from

THE UNIVERSITY OF WOLLONGONG

by

Dedi Budiawan, Ir.

DEPARTMENT OF CIVIL AND MINING ENGINEERING

1990

To my parents:

Mr. and Mrs. Budiawan

Acknowledgements

The author wishes to thank the chairman and staff of the Department of Civil and Mining Engineering at the University of Wollongong for their assistance, and for the study and research facilities provided during the research and computer work related to this thesis. In particular, the author wishes to express his great gratitude and appreciation to his supervisor, Dr. M.J. Boyd, for his advice, patience and guidance throughout the period of this project.

The author also wishes to thank the staff of the computer laboratory of the Department of Civil and Mining Engineering at the University of Wollongong for their assistance during the computer programming and analysis work.

Finally, but certainly not in the least, the author wishes to express his special gratitude to his parents and family for the on-going support and encouragement which led to the completion of this thesis.

Abstract

Flood routing is commonly used to calculate the shape of the flood hydrograph at the downstream end of a reservoir or a river reach, if the flood hydrograph at the upstream end of the reach is known. The flood routing procedure also enables prediction of the time at which the flood will occur at the downstream station.

One of the methods of flood routing which has been widely applied in engineering practice because of its simplicity and accuracy is the Muskingum method. This method is based on the assumption of a linear algebraic relationship between inflow I, outflow Q and storage S in a reach. The equation used is basically and numerically derived from the differential equation of continuity or conservation of mass.

As mentioned above, flood routing normally involves the use of an upstream hydrograph to estimate a downstream hydrograph, an example is estimating the flood hydrograph at the downstream end of a river reach. An estimate of the upstream hydrograph from the recorded flood hydrograph at the downstream end is sometimes required. This case is less common, but still significant. For example, it can be needed to fill in missing records using those at a downstream station.

This reverse routing equation, mathematically, can be deduced easily from the conventional Muskingum equation, i.e.: re-arranging the Muskingum equation to solve for inflow I given outflow Q. Difficulties often arise, since the process is numerically unstable. This numerical instability can cause the process to diverge from the true solution or oscillations to occur in the calculated upstream hydrograph. In practice, satisfactory upstream hydrographs cannot be obtained.

This project is intended to investigate that problem, to determine the cause of the numerical instability and to develop some alternative approaches which can overcome the problem.

Several methods of solution were investigated, including an iterative approach combined with a smoothing and averaging algorithms. Results using this method show that the numerical instability can be overcome by selecting an appropriate time step (routing period), which has been shown to depend on the values of the Muskingum model parameters. The solution converges rapidly because of the use of the averaging algorithm, and accurate estimates of the upstream hydrograph are obtained. It can be said that this method has the same order of accuracy as the conventional downstream routing using the Muskingum method.

Table of Contents

Chapter		Page
	Title page	i
	Declaration	ii
	Acknowledgements	iii
	Abstract	iv
	Table of Contents	vi
	List of Figures	xi
	List of Tables	xiv
	List of Notation	xvi
1	**INTRODUCTION**	**1-1**
	1.0 INTRODUCTORY REMARKS	1-1
	1.1 THE AIM AND THE SCOPE OF THIS PROJECT	1-3
	1.2 DESCRIPTION OF DATA USED	1-5
2	**LITERATURE SURVEY**	**2-1**
	2.0 INTRODUCTION	2-1
	2.1 THE MUSKINGUM METHOD	2-2
	2.2 PARAMETER EVALUATION	2-4
	2.2.1 Graphical and Trial-and-Error Methods	2-4
	2.2.2 Least-Squares Method	2-7
	2.2.3 Direct Optimization	2-9
	2.3 HYDRODYNAMIC APPROACH	2-10
	2.3.1 Convection-Diffusion and Kinematic Wave Equation	2-10
	2.3.2 The Analogy between the Muskingum and the	

	Kinematic Wave Equation	2-12
	2.4 ALLOWABLE VALUES OF PARAMETERS K AND x AND CHOICE OF Δt	2-14
	2.5 SUMMARY	2-20
3	**SOME ASPECTS OF DOWNSTREAM ROUTING USING MUSKINGUM METHOD**	**3-1**
	3.0 INTRODUCTION	3-1
	3.1 EFFECTS OF MODEL PARAMETERS ON DOWNSTREAM HYDROGRAPH	3-2
	3.1.1 Effect of Varying Time Step Δt	3-2
	3.1.2 Effect of Varying K Value	3-4
	3.1.3 Effect of Varying Parameter x Value	3-5
	3.2 NEGATIVE OR REDUCED INITIAL DOWNSTREAM DISCHARGES	3-6
	3.3 CASE OF PURE TRANSLATION	3-11
	3.4 NASH COEFFICIENTS	3-15
	3.5 SUMMARY	3-20
4	**UPSTREAM ROUTING USING CONVENTIONAL MUSKINGUM EQUATION**	**4-1**
	4.0 INTRODUCTION	4-1
	4.1 UPSTREAM ROUTING DERIVED FROM CONVENTIONAL DOWNSTREAM ROUTING	4-1
	4.2 UPSTREAM ROUTING COMPUTATIONS USING EQUATION (4.1.1)	4-2
	4.2.1 Further Computations Using Parameter x=0.0	

	with Various Δt	4-11
	4.2.2 Further Computations Using Parameter x=0.45 with Various Δt	4-29
	4.2.3 Further Computations with Various K Values	4-50
	4.3 INVESTIGATION OF THE CAUSE OF THE INSTABILITY	4-56
	4.4 PROOF OF THE INSTABILITY	4-61
	4.4.1 Muskingum Coefficients	4-62
	4.4.2 Nash Coefficients	4-63
	4.5 SUMMARY	4-67
5	**ALTERNATIVE APPROACHES TO UPSTREAM ROUTING**	**5-1**
	5.0 INTRODUCTION	5-1
	5.1 ITERATIVE METHOD	5-2
	5.1.1 Criterion to Terminate the Iteration	5-4
	5.1.2 Condition to Converge	5-5
	5.1.3 Weighting Factor (α)	5-8
	5.1.4 Summary of the Computation Procedure	5-9
	5.1.5 Tests of Computations	5-11
	5.2 ITERATIVE METHOD WITH BACKWARD DIFFERENCE AT THE END OF HYDROGRAPH	5-29
	5.2.1 Computation using Backward Difference Based on the Second Derivative at the End of Hydrograph	5-33
	5.2.2 Computation using Backward Difference Based on the First Derivative at the End of Hydrograph	5-40

5.3 ITERATIVE METHOD WITH NEWTON BACKWARD FORMULA AT THE END OF HYDROGRAPH 5-45

5.3.1 The Theory of Newton Backward Formula 5-45

5.3.2 The Application of Newton Backward Formula 5-48

5.4 UPSTREAM ROUTING MOVING BACKWARD IN TIME 5-63

5.5 THE CUBIC SPLINE AND RUNGE-KUTTA METHODS 5-64

5.5.1 The Cubic Spline 5-66

5.5.2 The Runge-Kutta Method 5-69

5.5.3 The Application of the Cubic Spline and Runge-Kutta Methods 5-71

5.6 SUMMARY 5-73

6 DOWNSTREAM ROUTING USING ITERATIVE METHOD 6-1

6.0 INTRODUCTION 6-1

6.1 COMPUTATION PROCEDURE 6-2

6.2 CONDITION TO CONVERGE 6-7

6.3 WEIGHTING FACTOR (α) 6-9

6.4 TESTS OF COMPUTATIONS 6-10

6.5 SUMMARY 6-11

7 CONCLUSIONS 7-1

REFERENCES R-1

APPENDIX :

A EXAMPLES OF RUNNING PROGRAMS App.A-1

A.0 INTRODUCTION App.A-1

A.1 HOW TO RUN THE PROGRAM App.A-2

A.2 WORKED EXAMPLES App.A-4

A.2.1 Downstream Routing App.A-4

A.2.1.1 Downstream Routing Using Conventional Muskingum Method App.A-4

A.2.1.2 Downstream Routing Using Iterative Method App.A-10

A.2.2 Upstream Routing App.A-12

A.2.2.1 Upstream Routing Moving Backward in Time App.A-13

A.2.2.2 Upstream Routing Using Iterative Method App.A-16

List of Figures

		Page
Figure 1.0.1	Attenuation and Lag	1-2
Figure 2.1.1	Illustration of Storage in a River Reach (Raudkivi, 1979)	2-3
Figure 2.2.1	Determination of the (a) Parameter x, and (b) Parameter K (Raudkivi, 1979)	2-5
Figure 2.2.2	Description of Storage S (Raudkivi, 1979)	2-6
Figure2.2.3	Determination of Parameters x and K for the Muskigum Method (Raudkivi, 1979)	2-6
Figure 2.3.1	Difference Scheme of the Muskingum Method in s-t plane	2-12
Figure 2.4.1	Variation of Muskingum Coefficient C_0 as a Function of $\Delta t/K$ and Parameter x (Ponce, 1978)	2-16
Figure 2.4.2	Critical Value of $1/(\omega r)$ Plotted Against x for Different Values of λ (Jones, 1981)	2-17
Figure 3.1.1	Calculated Downstream Hydrograph with Various Time Steps	3-3
Figure 3.1.2	Downstream Hydrograph with Various K Values	3-5
Figure 3.1.3	Downstream Hydrograph with Various x Values	3-6
Figure 3.2.1	Routing Through Storage with x=0.5 (Nash, 1959)	3-8
Figure 3.3.1	Routing Through Storage with x=0.5, K=66 Hours and Δt=24 Hours and 66 Hours	3-14
Figure 4.3.1	Oscillations in Upstream Hydrograph Using Parameter x=0.0	4-59
Figure 4.4.1	Graphic f(a) and g(a) Vs. a	4-66
Figure 5.1.1	River Reach Routing Using Instantaneous Discharges	5-4
Figure 5.1.2	Graphic x/2 Vs. Min. Time Step/K	5-7

Figure 5.1.3	Graphic α Vs. Number of Iterations for x=0.2	5-9
Figure 5.1.4	Flow Chart of the Computation	5-10
Figures 5.1.5	Graphics of Samples of Computations	5-20
Figure 5.1.6a	Upstream Routing to Obtain Upstream Hydrograph Using Iterative Method	5-24
Figure 5.1.6b	Downstream Routing to Obtain Downstream Hydrograph Using Standard Muskingum Equation	5-25
Figures 5.1.6c	Upstream Routing Using Observed Downstream Hydrograph	5-26
Figures 5.2.1	Upstream Routing with Backward Difference Based on the Second Derivative at the End of Hydrograph Using Observed Downstream Hydrograph	5-35
Figures 5.2.2	Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Backward Difference Based on the Second Derivative at the End of Hydrograph	5-38
Figures 5.2.3	Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Basic Method (Eq. 5.1.5).	5-39
Figures 5.2.4	Upstream Routing with Backward Difference Based on the First Derivative at the End of Hydrograph Using Observed Downstream Hydrograph	5-41
Figures 5.2.5	Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Backward Difference Based on the First Derivative at the End of Hydrograph	5-44
Figures 5.3.1	Upstream Routing Using Observed Downstream Hydrograph with Newton Backward Formula at the End of Hydrograph, n=2	5-50
Figures 5.3.2	Upstream Routing Using Observed Downstream	

	Hydrograph with Newton Backward Formula at the End of Hydrograph, n=3	5-53
Figure 5.4.1	Upstream Routing Moving Backward in Time	5-64
Figure 5.5.1	Upstream Routing with Spline Function	5-72
Figure 6.1.1	Flow Chart of the Computation	6-4
Figure 6.2.1	Graphic (1-x)/2 Vs. Min. Time Step/K	6-8
Figure 6.3.1	Graphic α Vs. Number of Iterations for Parameter x=0.3	6-9
Figure 6.4.1	Downstream Routing to Obtain Downstream Hydrograph Using the Standard Muskingum and Iterative Methods	6-11

List of Tables

		Page
Table I.2.1	Storage Analysis of Flood of September-October 1960 in the Reach of Murray River	1-6
Table III.1.1	The Values of C_0, C_1 and C_2 for K=66 Hours, x=0.45 and Δt=24, 48 and 72 Hours	3-2
Table III.1.2	The Values of C_0, C_1 and C_2 for x=0.45 and Δt=24 Hours and Various Parameter K Values	3-4
Table III.1.3	The Values of C_0, C_1 and C_2 for K=66 Hours, Δt=24 Hours and Various Parameter x Values	3-6
Table III.3.1	Result of Computation Using x=0.5, K=66 Hours and Δt=K	3-15
Table III.4.1	Sample of Computation Using Muskingum and Nash Coefficients	3-19
Tables IV.2.1	Results of Computations Using Various Parameter x Values, K=66 Hours and Δt=24 Hours	4-4
Tables IV.2.2	Results of Computations Using Parameter x=0, Various Time Step Δt and K=66 Hours	4-12
Tables IV.2.3	Results of Computations Using Parameter x=0.45, Various Time Step Δt and K=66 Hours	4-30
Tables IV.2.4	Results of Computations Using Parameter x=0.45, Various K Values and Δt=24 Hours	4-51
Table IV.3.1	Values of Muskingum and Nash Coefficients with K=66 Hours, Δt=24 Hours and Various Parameter x Values	4-57
Table IV.3.2	Values of $-C_1/C_0$ with Parameter x=0.45, K=66 Hours	

	and Various Δt	4-60
Table IV.3.3	Values of $-C_1/C_0$ with Parameter $x=0.45$, $\Delta t=24$ Hours and Various K Values	4-61
Tables V.1.1	Samples of Computations	5-13
Table V.3.1	Backward Difference	5-46
Table V.3.2	Samples of Backward Difference	5-47
Tables V.3.3	Upstream Routing Calculation with Newton Backward Formula, n=2 and Finite Difference Based on the Second Derivative at the End of Hydrograph	5-56

List of Notation

A	wetted cross-sectional area of channel
B	mean channel width
C	Courant number
C_0	Muskingum coefficient, Nash coefficient
C_1	Muskingum coefficient, Nash coefficient
C_2	Muskingum coefficient, Nash coefficient
D	reciprocal of cell Reynolds number, differential operator d/dt
d	relative difference in each value of I or Q from one iteration to the next
E	error function
g	acceleration due to gravity
I	inflow or upstream discharge into a reach
I^*	inflow or upstream discharge assumed for first trial or obtained from previous iteration
i	increment counter
j	increment counter
K	Muskingum method parameter
k	iteration number
M	conveyance
m	slope of inflow curve [Nash (1959)]
N	time interval at which last hydrograph ordinate was observed or is calculated
n	Manning coefficient
Q	outflow or downstream discharge, discharge
Q^*	outflow or downstream discharge assumed for first trial or obtained from previous iteration
q	lateral inflow per unit length

q_o	reference discharge per unit width
R	hydraulic radius, cell Reynolds number
r	ratio of time increment to space increment
S	volume of temporary or channel storage
S_e	estimated storage
S_f	friction slope
S_o	observed storage, bed slope
s	downstream distance
T_R	time of rise of inflow hydrograph
t	time
V_q	downstream component of velocity of lateral inflow
x	Muskingum method parameter
y	water depth
z	weighting factor
ΔS	storage increment
Δs	space increment (reach length)
Δt	time increment or time step or routing period
α	weighting factor
λ	ratio of time base to time increment
μ	diffusion parameter
σ	difference between absolute and relative storage
τ	time taken by the flood wave to reach the downstream end of the river reach as defined by Gill (1979a)
ω	kinematic wave speed
∇	backward difference operator

Chapter One

Introduction

1.0 INTRODUCTORY REMARKS

In hydrologic practice, the need to determine a flood hydrograph at a certain site when the flood hydrograph at an upstream site on a river channel or reservoir system is known, is a common problem. For example, a major flood hydrograph may be known at a certain site on a river and it is required to calculate the corresponding flood hydrograph at a downstream station, in order that ample flood protection can be provided. As another example, assume that a major flood hydrograph has been recorded by the stream gauging station at a certain site of a river just upstream from a proposed reservoir site. The corresponding outflow hydrograph is required for the proposed reservoir as a test of the sufficiency of the proposed outlet works.

Both of the examples above show the need for flood routing. The first is concerned with river routing and the second with reservoir routing. In scientific terms, flood routing is a technique used to compute the effect of system storage on the shape and movement of a flood wave. Storage, in this context, is the volume of water temporarily stored within the reach at any given time and which is in transit to the outlet or downstream site. It does not include water which is retained permanently. Because this storage is temporary, the total volume of inflow must be equal to the total volume of outflow.

The hydrograph for the downstream site differs from the one for the upstream site. It has a different pattern in which peak is lower and base is broader. The peak itself occurs at a later time. The effect of the system which leads to a lower peak is called **attenuation** and a delay between the peaks of the downstream and the upstream hydrographs is called **lag-time**.

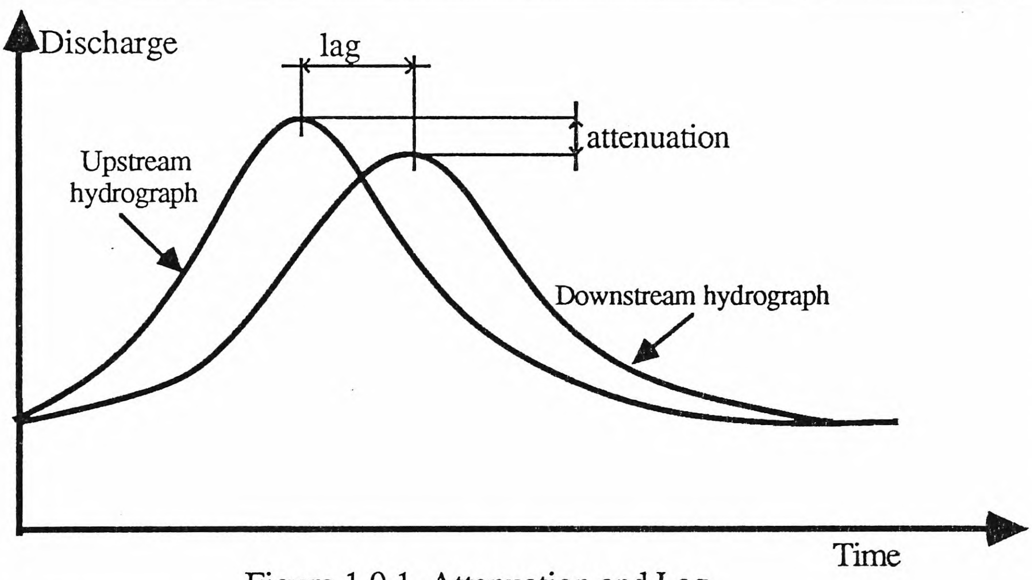


Figure 1.0.1 Attenuation and Lag

The lower peak indicates the degree of peak flow reduction resulting from passage through the reservoir or the reach of river. The change in time tells whether the peak of the outflow hydrograph occurs at time when the outlet at the downstream site can pass the flood without any trouble or whether it occurs at a time when the outlet is being flooded with water from any other tributaries. If this happens, some measures have to be taken which will change the time adequately to avoid the occurrence of flood peaks from other tributaries at the same time.

The Muskingum method of flood routing in rivers is one of the methods which has been widely applied in engineering practice. This method is based on the assumption of a linear algebraic relationship between inflow I, outflow Q and storage S in the reach. The Muskingum equation is numerically derived from the differential equation of conservation of mass.

It is sometimes necessary to estimate the hydrograph at the upstream end of a reach from a known hydrograph at the downstream end. This reverse routing or upstream routing process can be deduced from the conventional downstream routing procedure. The problem is that difficulties often arise in this upstream routing since the process is computationally unstable and unrealistic fluctuations may occur in the calculated hydrograph at the upstream end of the reservoir or river reach.

1.1 THE AIM AND THE SCOPE OF THIS PROJECT

The aim of this project is to investigate the problem mentioned above, to determine the cause of the computational instability and eventually to develop some alternative approaches which can overcome the problem. The methods of solution retain a numerical method which is based on a finite difference approximation.

The scope of this project is restricted to the problem of upstream routing in a river using the Muskingum assumption for the storage system.

In investigating the problem, several computer programs have been written. They are also provided with a graphic program in order that the analyses can be displayed clearly and quickly. This program consists of several subprograms which were taken from 'Turbo Graphix Toolbox' by Borland International (1985). However, some modifications to those subprograms were made to suit the needs of the numerical analyses. A diskette containing the computer programs which allow normal downstream routing calculations, downstream and upstream routing calculations using iterative method and upstream routing moving backward in time is enclosed. All of the programs were written in Turbo Pascal language. Examples of running these programs are given in appendix A.

In order to avoid ambiguities of symbols and definitions used in this thesis, the following terms are used:

- 'Upstream discharge' and 'downstream discharge' have the same meaning as 'inflow' and 'outflow' respectively which are often used in text books. Similarly, 'upstream hydrograph' is the same as 'inflow hydrograph' and 'downstream hydrograph' is the same as 'outflow hydrograph'.
- 'Time step' with the symbol Δt is the 'routing period', which in some text books is symbolized by 'T'. The symbol 'T' is also used in this thesis in the results of computer computations in the form of the graphics and tables to represent Δt, because of the difficulty in writing 'Δ' in the computer graphics.

This thesis is divided into seven chapters. Chapter two consists of the theoretical background and literature survey. Chapter three discusses some specific aspects of downstream routing using the Muskingum equation. This can be looked upon as a further investigation of the literature described in chapter two. Chapter four presents the analyses of the problem of upstream routing for which an equation is derived from the equation for conventional downstream routing. This chapter contains a great number of pages presenting computer outputs in the form of tables of computations. These are **deliberately not** placed in the appendix for the ease of the reader to follow the discussion. Chapter five introduces some alternative approaches for upstream routing. The Runge-Kutta method combined with a cubic spline fitting method which is used in the graphic program (Turbo Graphix Toolbox) are also discussed here as one of the methods. Chapter six presents an iterative method for downstream routing. This applies the methods of chapter five to conventional downstream routing. Finally, conclusions are highlighted in chapter seven.

This thesis forms part of a study into upstream routing in rivers and reservoirs, as reported by Boyd et.al. (1989). It considers in greater detail the problem of upstream routing in rivers.

1.2 DESCRIPTION OF DATA USED

This investigation is concerned with numerical approaches to Muskingum flood routing rather than the analysis of floods in actual rivers. Therefore, only one flood event was used in the example calculations. The methods developed in this thesis however are generally applicable to a wide range of flood events.

The data used in this project are of the September-October 1960 flood in the reach of the Murray River from Doctors Point at Albury (National Station No. 409017) to Corowa (409002). The respective catchment areas are 16800 and 18800 km^2, and no major tributaries enter the reach between the stations. These data were taken from "Australian Rainfall and Runoff - A Guide to Flood Estimation" Vol.1, chapter 7, Table 7.1, page 134 [Pilgrim, I.E., Australia, 1987] referred to herein as ARR87. The data are given in Table I.2.1.

The storage at instant i in column (4) of Table I.2.1 was obtained by cumulating the storage increments before instant i (see Fig. 2.2.2 in chapter 2). The storage increments were obtained by multiplying the average values of the differences between the inflow and outflow discharges over each 24-hour period with the number of seconds in the period.

The parameter x = 0.45 in column (5) was obtained by applying the trial-end-error method discussed in chapter 2 section 2.2.1. The parameter K value obtained from this method is K = 66 hours. These parameter x and K values are consistently used in this project.

Table I.2.1 Storage Analysis of Flood of September-October 1960 in the reach of Murray River

9am, Date (1)	Doctors Pt. Inflow I m^3/sec (2)	Adjusted Corowa Ouflow Q m^3/sec (3)	Storage S $m^3 \times 10^6$ (4)	[x.I + (1-x).Q] for x = 0.45 m^3/sec (5)
Sept. 15	274	274	0	274
16	314	298	0.7	305
17	355	320	2.9	336
18	404	361	6.3	380
19	495	383	13.0	433
20	566	405	24.8	477
21	586	446	37.8	509
22	572	502	46.8	534
23	575	543	51.2	557
24	572	593	51.8	584
25	571	593	49.9	583
26	676	593	52.4	630
27	1026	614	73.9	799
28	1156	686	112.0	898
29	1081	899	140.1	981
30	1001	1100	143.7	1055
Oct. 1	816	1061	128.8	951
2	681	972	105.7	841
3	568	884	79.4	742
4	538	817	53.7	691
5	534	678	35.4	613
6	535	606	26.2	574
7	551	558	22.8	555
8	555	539	23.2	546
9	549	534	24.5	541
10	544	529	25.8	536
11	493	524	25.1	510
12	428	517	20.0	477
13	376	476	11.8	431
14	357	413	5.0	388
15	301	301	2.6	301
16	274	295	1.7	286
17	271	290	0	281

Total inflow volume = total outflow volume = $1.583 \cdot 10^9$ m^3.

Chapter Two

Literature Survey

2.0 INTRODUCTION

Since its development in 1930's, the Muskingum method of flood routing in rivers has been the subject of many investigations. Several useful papers dealing with various aspects of the method have been published.

The aim of this chapter is to describe not only the basic theory of the Muskingum method but also those aspects which contribute to its use in flood routing. For example, Gill (1978) proposed a least-squares method to replace the trial-and-error procedure for obtaining the Muskingum parameters x and K of a river reach, Cunge (1969) developed the Muskingum method using a hydrodynamic approach and Jones (1981) discussed the choice of the space and time steps Δs and Δt in terms of the parameters of the convection-diffusion equation.

The sources of this chapter were taken from the text books or papers written by Cunge (1969), Dooge (1973), Price (1973a), Gill (1978), Ponce et.al.(1978), Raudkivi (1979), Strupczewski and Kundzewicz (1980a), Singh and McCann (1980), Jones (1981), Linsley et.al.(1982), and Pilgrim (I.E. Australia, 1987).

2.1 THE MUSKINGUM METHOD

In routing floods through a river, the river is divided into convenient segments called 'reaches'. In this project, only the reach which has no accretion from precipitation, ground water, or tributaries is taken into account. All flow is looked upon as entering the reach at its upstream limit, then progressing to the downstream end of the reach, and it is considered to be unaffected by backwater from lower reaches.

The Muskingum method, originated by Mc Carthy (1938), is the most widely used method of flood routing in rivers. The method is based on a linear algebraic relationship between storage S and both inflow I and outflow Q, along with parameters x and K. Parameter x, the value of which lies between 0 and 0.5, is a weighting factor which expresses the relative influence of the inflow I and the outflow Q. K is a storage parameter which has a time dimension and expresses the average storage to discharge ratio for the river reach. The K value is approximately equal to the average travel time through the reach. It measures the delay between the center of gravity of the input wave and the center of gravity of the output wave.

The basic continuity or storage equation is

$$\frac{dS}{dt} = I - Q \qquad (2.1.1)$$

This is also often called 'the equation of conservation of mass'. With reference to Figure 2.1.1, the total storage is expressed:

$$S = K.Q + K.x.(I - Q) = K.[x.I + (1 - x).Q] \qquad (2.1.2a)$$

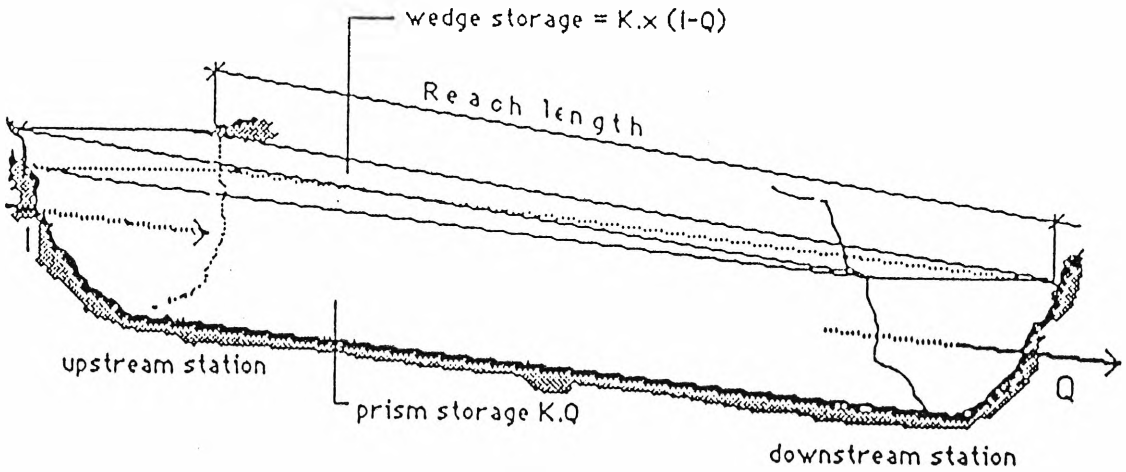


Figure 2.1.1 Illustration of Storage in a River Reach (Raudkivi, 1979)

or $$\Delta S = S_2 - S_1 = K.[x.(I_2 - I_1) + (1 - x).(Q_2 - Q_1)] \qquad (2.1.2b)$$

Solution of eqs.(2.1.1) and (2.1.2a) can be obtained algebraically if I can be expressed as a mathematical function. Such a solution is presented by Kulandaiswamy (1966) and also Diskin (1967). Normally, the inflow data I are available only at a certain time step, or in other words the inflow I is available only in discrete form. Therefore, a solution is obtained using finite difference method instead of the differential equation in eq. (2.1.1). Equation (2.1.1) can thus be expressed in finite difference terms as

$$\frac{1}{2}.\Delta t.(I_1 + I_2) - \frac{1}{2}.\Delta t.(Q_1 + Q_2) = S_2 - S_1$$

Substituting eq.(2.1.2b) into this equation yields

$$Q_2 = C_0.I_2 + C_1.I_1 + C_2.Q_1$$

or in common numerical expression

$$Q_{i+1} = C_0.I_{i+1} + C_1.I_i + C_2.Q_i \qquad (2.1.3)$$

where

$$C_0 = \frac{\Delta t - 2.K.x}{2.K.(1 - x) + \Delta t} \qquad (2.1.4a)$$

$$C_1 = \frac{\Delta t + 2.K.x}{2.K.(1 - x) + \Delta t} \qquad (2.1.4b)$$

$$C_2 = \frac{2.K.(1 - x) - \Delta t}{2.K.(1 - x) + \Delta t} \qquad (2.1.4c)$$

2.2 PARAMETER EVALUATION

2.2.1 Graphical and Trial-and-Error Methods

If the inflow and the outflow hydrographs for the reach are available, the value of x can be determined from the observation that the storage is maximum at the time when the inflow and the outflow hydrographs intersect, Fig. 2.2.1a. At this point dS/dt = 0. Differentiating eq. (2.1.2a) and setting dS/dt equal to zero yields:

$$x.\left(\frac{dI}{dt}\right)_c = -(1-x).\left(\frac{dQ}{dt}\right)_c \qquad (2.2.1)$$

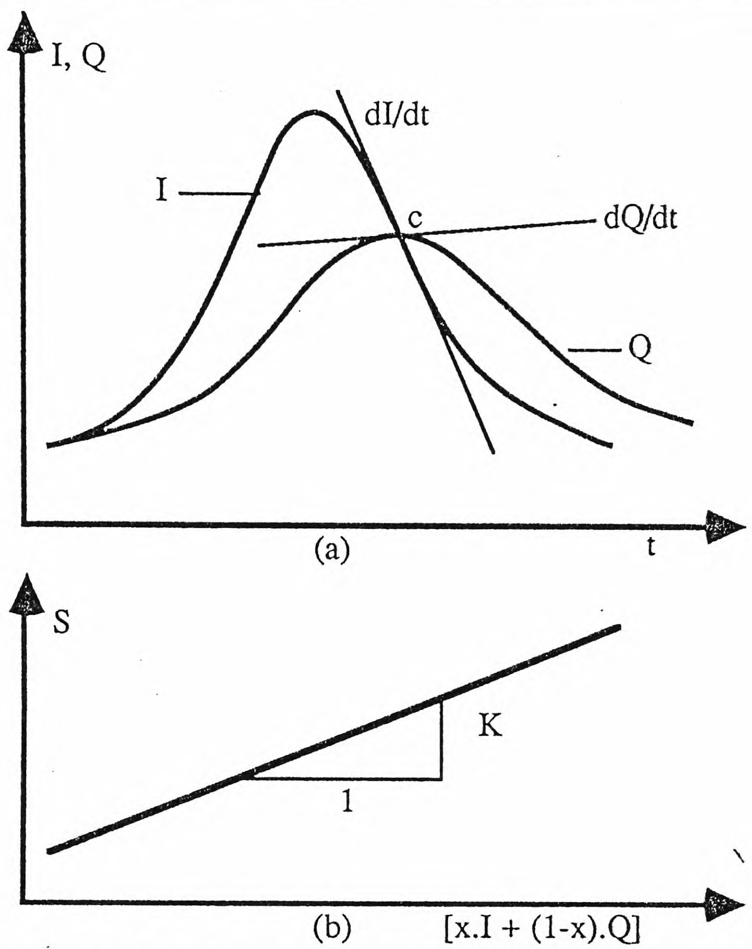


Figure 2.2.1 Determination of the (a) Parameter x, and (b) Parameter K (Raudkivi, 1979)

in which x is the only unknown. With x determined in this way, the value of K can be determined by plotting S versus [x.I + (1-x).Q], Fig. 2.2.1b. The slope of this line is the storage coefficient [Raudkivi (1979)].

Another method of determining values of K and x is to plot values of S against the weighted discharges at successive times t. The volumes of storage in the river reach S_i at instants t_i, i = 0, 1, 2, ... are represented by the area between the inflow and outflow hydrographs (usually the area under inflow or outflow hydrograph is obtained by adding up the area of trapezoidal elements) as can be seen in Fig. 2.2.2. These values plotted against [x.I + (1-x).Q] for arbitrary values of x, give K as the slope. The value of x which yields a loop closest to a single line, Fig. 2.2.3, is taken to be the correct value.

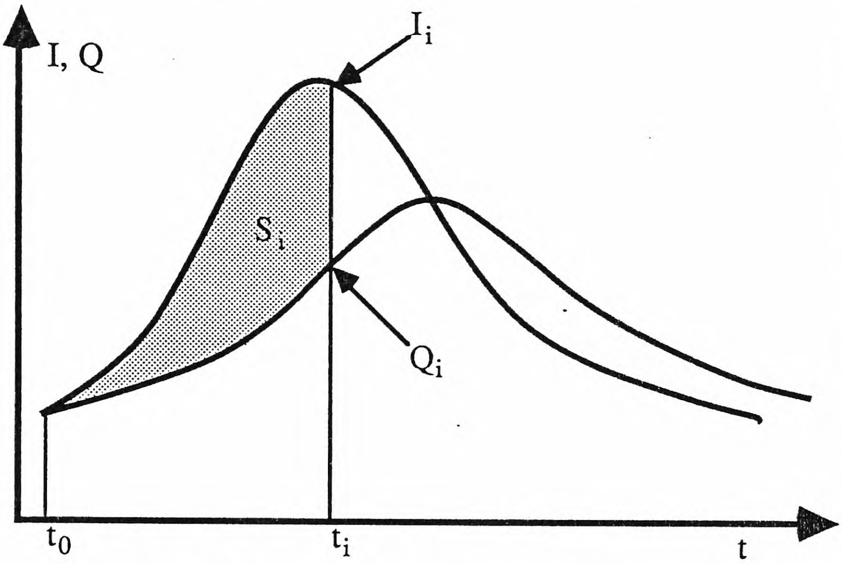


Figure 2.2.2 Description of Storage S (Raudkivi, 1979)

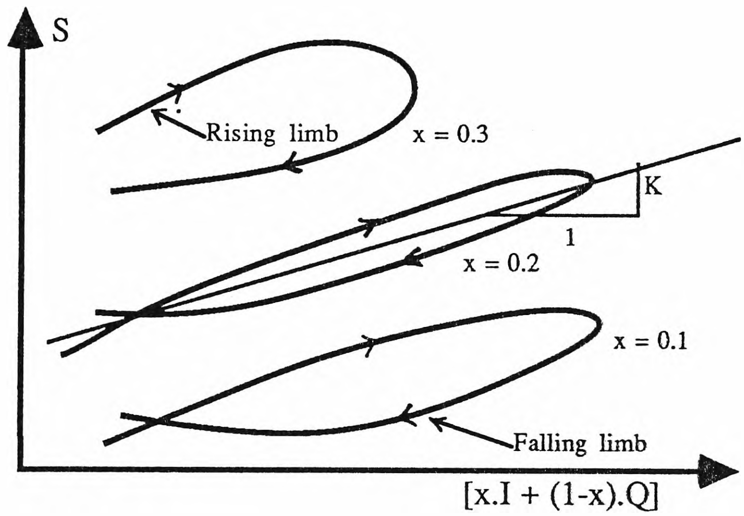


Figure 2.2.3 Determination of Parameters x and K for the Muskingum Method (Raudkivi, 1979)

This trial-and-error procedure can be replaced by other methods. Gill (1978) proposed the least-squares method and Stephenson (1979) proposed a direct optimization method for parameter estimation. These methods are briefly discussed herein. The description is based on the appendix of the paper by Singh and McCann (1980).

2.2.2 Least-squares Method

The storage S that is normally available is the relative storage (the storage volume in excess of the base value of storage which existed at the start of the flood) unless the initial flow in the river reach is zero. The storage equation, i.e.: eq.(2.1.2a) refers to the absolute value. Therefore, it is necessary to modify eq.(2.1.2a), if the initial storage is significant or the difference between relative and absolute storage is significant. Equation (2.1.2a) is modified into

$$S = K.[x.I + (1 - x).Q] + \sigma \qquad (2.2.2)$$

where σ is the difference between absolute and relative storages.

The method is based on minimizing the squares of deviations between the estimated and the observed values of S. The error function which represents this condition can be expressed as

$$E = \sum_{j=0}^{N}[S_o(j) - S_e(j)]^2 \qquad (2.2.3)$$

where $S_o(j)$ is the observed storage at the time interval j, $S_e(j)$ is the estimated storage at the time interval j and N is the time interval at which last hydrograph ordinate was observed or is estimated. The error E has to be minimized. There are two cases which have to be considered.

Case 1 : $\sigma \neq 0$

Firstly, assume A = K.x and B = K.(1-x). By dropping j for brevity, eq. (2.2.3) can be written as

$$E = \sum_{j=0}^{N}[S_o - K.x.I - K.(1-x).Q - \sigma]^2 \tag{2.2.4}$$

This error E has to be minimized. Using the usual procedure, the following normal equations are obtained.

$$\sum_0^N S_o - A.\sum_0^N I - B.\sum_0^N Q - N.\sigma = 0 \tag{2.2.5}$$

$$\sum_0^N S_o.I - A.\sum_0^N I^2 - B.\sum_0^N I.Q - \sigma.\sum_0^N I = 0 \tag{2.2.6}$$

$$\sum_0^N Q.S_o - A.\sum_0^N I.Q - B.\sum_0^N Q^2 - \sigma.\sum_0^N Q = 0 \tag{2.2.7}$$

The values of A, B and σ can be obtained from these equations.

$$B = (y_1.z_2 - z_1.y_2)/(z_2.y_3 - y_2.z_3) \tag{2.2.8}$$

$$A = y_1/y_2 - B.(y_3/y_2) \tag{2.2.9}$$

$$\sigma = (\sum S_o - A.\sum I - B.\sum Q)/N \tag{2.2.10}$$

where

$y_1 = \sum S_o.I - (\sum S_o.\sum I)/N$; $y_2 = \sum I^2 - (\sum I)^2/N$

$y_3 = \sum Q.I - \sum Q.\sum I/N$; $z_1 = \sum S_o.Q - (\sum S_o.\sum Q)/N$

$z_2 = \sum I.Q - (\sum I.\sum Q)/N$; $z_3 = \sum Q^2 - (\sum Q.\sum Q)/N$

Then

$$K = A + B \text{ and } x = A/(A+B) \tag{2.2.11}$$

Case 2 : $\sigma = 0$

Solving for A and B as before:

$$A = (\sum S_o.I.\sum Q^2 - \sum S_o.Q.\sum I.Q)/D \tag{2.2.12}$$

$$B = (\sum S_o.Q.\sum I^2 - \sum S_o.I.\sum I.Q)/D \tag{2.2.13}$$

$$D = \sum I^2.\sum Q^2 - (\sum I.Q)^2 \tag{2.2.14}$$

K and x can be obtained using eq.(2.2.11).

2.2.3 Direct Optimization

This is a direct method of deriving the routing coefficients C_0, C_1 and C_2 without performing the intermediate step of obtaining K and x. This involves minimizing the difference between the observed hydrograph and computed hydrograph. The difference can be expressed by the error defined in a least-squares function. Therefore, this method is none other than a least-squares optimization method which is similar to the one discussed previously.

There are only two unknowns in this method since the third can be determined from $C_0 + C_1 + C_2 = 1$. If C_1 and C_2 are the two unknowns then

$$x = \frac{C_1 + 0.5.C_2 - 0.5}{C_1 + C_2} \quad (2.2.15)$$

$$K = \frac{\Delta t.(C_1 + C_2)}{1 - C_2} \quad (2.2.16)$$

By re-arranging, eq.(2.1.3) becomes

$$C_1.(I_{i+1} - I_i) + C_2.(I_{i+1} - Q_i) = I_{i+1} - Q_{i+1} \quad (2.2.17)$$

if

$$R_{i+1} = I_{i+1} - Q_{i+1}\,; \qquad F_{i+1} = I_{i+1} - I_i\,; \qquad G_{i+1} = I_{i+1} - Q_i$$

then

$$R_{i+1} = C_1.F_{i+1} + C_2.G_{i+1} \quad (2.2.18)$$

By dropping the subscript i+1 for brevity, the error function follows:

$$E = \sum(R_o - R_e)^2 \quad (2.2.19)$$

where subscripts o and e refer to observed and estimated R, respectively.

Following the usual procedure,

$$\sum R_o.F = C_1.\sum F^2 + C_2.\sum F.G \quad (2.2.20a)$$

and

$$\sum R_o.G = C_1.\sum F.G + C_2.\sum G^2 \quad (2.2.20b)$$

Then C_1 and C_2 can be obtained.

$$C_1 = (\sum R_o.F.\sum G^2 - \sum R_o.G.\sum F.G)/DET \quad (2.2.21a)$$

$$C_2 = (\sum R_o.G.\sum F^2 - \sum R_o.F.\sum F.G)/DET \quad (2.2.21b)$$

where

$$DET = \sum G^2.\sum F^2 - (\sum F.G)^2$$

Eventually, after knowing the values of C_1 and C_2, x and K can be determined by using eqs.(2.2.15) and (2.2.16) respectively.

2.3 HYDRODYNAMIC APPROACH

2.3.1 Convection-Diffusion and Kinematic Wave Equation

Basically, flood routing methods are based on the St. Venant equations which describe the conservation of volume and momentum in a channel.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial s} = q \quad (2.3.1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial s}\left(\frac{Q^2}{A}\right) = A.g.(S_o - \frac{\partial y}{\partial s} - S_f) + q.V_q \quad (2.3.2)$$

Flood routing methods can be classified into three groups [see Jones (1981)]

a. those methods based on a numerical solution of the St. Venant equations without simplication

b. methods based on momentum governed by bed, friction and surface slopes only, which yields diffusion analogy models

c. methods based on momentum governed by bed and friction slopes only, which yields kinematic models.

Since the slope terms have much greater effect on the momentum if compared to the other terms, eq.(2.3.2) can be approximated as

$$S_o - \frac{\partial y}{\partial s} - S_f = 0$$

or

$$S_o = \frac{\partial y}{\partial s} + S_f \tag{2.3.3}$$

Equation (2.3.3) can be combined with eq.(2.3.1) to yield *convection-diffusion* equation using

$$S_f = Q^2/M^2 \tag{2.3.4}$$

where M is the conveyance which is assumed to be a function of depth and channel parameters. The convection-diffusion equation is then expressed as

$$\frac{\partial Q}{\partial t} + \omega.\frac{\partial Q}{\partial s} = \mu.\frac{\partial^2 Q}{\partial s^2} + \omega.q \tag{2.3.5}$$

where

$$\omega = \frac{Q.(dM/dy)}{B.M} \tag{2.3.6}$$

$$\mu = M^2/(2.B.Q) \tag{2.3.7}$$

$$M = (A.R^{2/3})/n \tag{2.3.8}$$

B is mean channel width,

A is wetted cross-sectional area of channel,

n is Manning coefficient,

R is hydraulic radius.

Since the water surface slope has only a secondary effect on momentum, the momentum equation, eq.(2.3.3), can be further approximated to

$$S_o = S_f \tag{2.3.9}$$

Combining this equation with eq.(2.3.1) yields

$$\frac{\partial Q}{\partial t} + \omega.\frac{\partial Q}{\partial s} = \omega.q \tag{2.3.10}$$

This equation is called *kinematic wave equation*, where ω is the kinematic wave speed which may depend on Q [Jones (1981)].

2.3.2 The Analogy between the Muskingum and the Kinematic Wave Equation

If there is no lateral inflow, eq.(2.3.10) can be written as

$$\frac{\partial Q}{\partial t} + \omega.\frac{\partial Q}{\partial s} = 0 \qquad (2.3.11)$$

As mentioned previously, ω is a function of Q, therefore eq.(2.3.11) is a *quasi linear equation.* For certain applications, however, ω is considered a constant and eq.(2.3.11) reduces to a linear form.

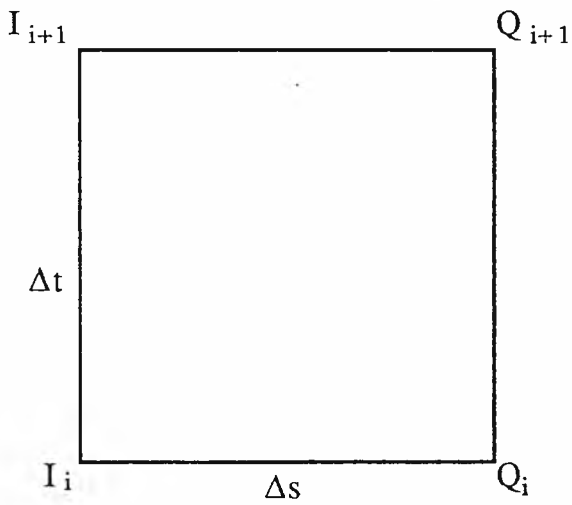


Figure 2.3.1 Difference scheme of the Muskingum Method in s-t Plane

Assuming a difference scheme in the s-t plane (Fig. 2.3.1), eq. (2.3.11) is discretized to yield (the discharges on the left-hand side of Fig.2.3.1 are symbolized by I not Q to refer to upstream discharge) :

$$\left[\frac{x.(I_{i+1} - I_i) + (1\text{-}x).(Q_{i+1} - Q_i)}{\Delta t}\right] + \omega.\left[\frac{z.(Q_i - I_i) + (1\text{-}z).(Q_{i+1} - I_{i+1})}{\Delta s}\right] = 0 \qquad (2.3.12)$$

where x and z are weighting factors. By taking z = 0.5, eq.(2.3.12) reduces to

$$\Delta t.\left[\left(\frac{I_i + I_{i+1}}{2}\right) - \left(\frac{Q_i + Q_{i+1}}{2}\right)\right] = \frac{\Delta s}{\omega}.\left[x.(I_{i+1} - I_i) + (1-x).(Q_{i+1} - Q_i)\right]$$

(2.3.13)

if $\Delta s/\omega = K$, eq.(2.3.13) becomes

$$Q_{i+1} = \frac{\Delta t - 2.K.x}{2.K.(1-x) + \Delta t}.I_{i+1} + \frac{\Delta t + 2.K.x}{2.K.(1-x) + \Delta t}.I_i + \frac{2.K.(1-x) - \Delta t}{2.K.(1-x) + \Delta t}.Q_i$$

which is the Muskingum formula.

The convection-diffusion equation [eq.(2.3.5)] with no lateral inflow is

$$\frac{\partial Q}{\partial t} + \omega.\frac{\partial Q}{\partial s} = \mu.\frac{\partial^2 Q}{\partial s^2} \quad (2.3.14)$$

Cunge (1969) noted that the solution of the finite difference forms of equations (2.1.1) and (2.1.2a), by means of a Taylor series expansion, can be shown to approximate eq.(2.3.14) with an error of order $(\Delta s)^2$ provided that

$$K = \Delta s/\omega \quad (2.3.15)$$

and

$$x = \frac{1}{2} - \frac{\mu}{\omega.\Delta s} \quad (2.3.16)$$

It can be noticed from eq.(2.3.16), once the parameter values of μ and ω for a reach are known, the determination of parameter x is equivalent to the determination of Δs, that is to say the value of x should depend on the reach length adopted.

Price (1973a) in his paper mentioned that the parameters μ and ω can vary significantly with the magnitude of the flood. This is the reason why there is a disadvantage with the approach of using the values of μ and ω resulting from calibration to route other floods of significantly different magnitude in the same river. It should be noted that calibration is a process for determining a certain parameter value through comparing the predicted result with a recorded result.

This is done repeatedly using various trial values of that parameter until the parameter value which yields the most accurate result is obtained.

Price suggested that curves for μ and ω, where possible, be defined. This can be done by correlating values of μ and ω calculated for a number of recorded floods with the **average peak discharge** along the reach in each case. Thus, the functions $\mu(Q)$ and $\omega(Q)$ can be obtained in the form of curves which are drawn through the resulting points. However, the use of μ and ω values from the curves to route a future flood has to be performed with caution, since the curves may not be smooth, or in other words there may be some scatter about the curves due to observational error and also to the dependence of the calculated values of μ and ω on the shapes of the discharge hydrographs. To overcome this difficulty, Price developed the variable parameter diffusion method [Price (1973b)].

2.4 ALLOWABLE VALUES OF PARAMETERS K AND x AND CHOICE OF Δt

Since K is the parameter which has time dimension, its value must be greater than zero. It can be seen from eq.(2.3.15) that its value depends on the length of the reach and the wave speed.

The range of parameter x value, in practice, is [0,0.5]. However, Dooge (1973) and Strupczewski and Kundzewicz (1980a) in their paper asserted that the parameter x value can be negative. This principle was proved by the formulae obtained from matching the moments of the impulse response of the Muskingum model with those of a linear dynamic model. The negative x value is needed in the case when the river reach is short. In general, the parameter x value can theoretically lie in the range $(-\infty,0.5]$.

According to Ponce et.al. (1978), the range of parameter x value is [0,0.5]. Further, Ponce mentioned that values of $x \geq 0.5$ cause numerical instability and values of $x < 0$ will be associated with very small values of Δs (river reach length) which lead to inefficient computation. They also presented a graphic which correlates the Muskingum coefficient C_0 to parameter x, time step Δt and parameter K (Figure 2.4.1). Before presenting the graphic, it is necessary to explain briefly the derivation of the parameter used. Equation (2.3.16) can be written as

$$\mu = (1/2 - x).\omega.\Delta s \qquad (2.4.1)$$

This is the numerical diffusion coefficient of a second order approximation of the finite difference equation. The physical diffusion coefficient is $\mu = q_o/(2.S_o)$, where q_o is a reference discharge per unit width and S_o is the channel bed slope. The parameter x can be obtained by matching the physical diffusion coefficient with eq.(2.4.1).

$$x = \frac{1}{2}.(1 - \frac{q_o}{S_o.\omega.\Delta s}) \qquad (2.4.2)$$

or

$$x = \frac{1}{2}.(1 - D)$$

where D is the reciprocal of a cell Reynolds number R.

$$D = 1/R \qquad (2.4.3)$$

$$R = \frac{\omega.\Delta s}{q_o/S_o} \qquad (2.4.4)$$

Defining the Courant number $C = \Delta t/K$ to be used as a substitution in equations (2.1.4a,b,c), those equations become

$$C_0 = \frac{-1 + C + D}{1 + C + D} \qquad (2.4.5a)$$

$$C_1 = \frac{1 + C - D}{1 + C + D} \qquad (2.4.5b)$$

$$C_2 = \frac{1 - C + D}{1 + C + D} \qquad (2.4.5c)$$

Figure 2.4.1 shows the values of C_0 bounded between +1 and -1 to be a function of C and D. The shaded area satisfies the condition $2.x \leq \Delta t/K \leq 2.(1-x)$ and $0 \leq x < 0.5$. The condition represented by the shaded area in Fig.2.4.1 can be regarded as the criterion for choosing the time step.

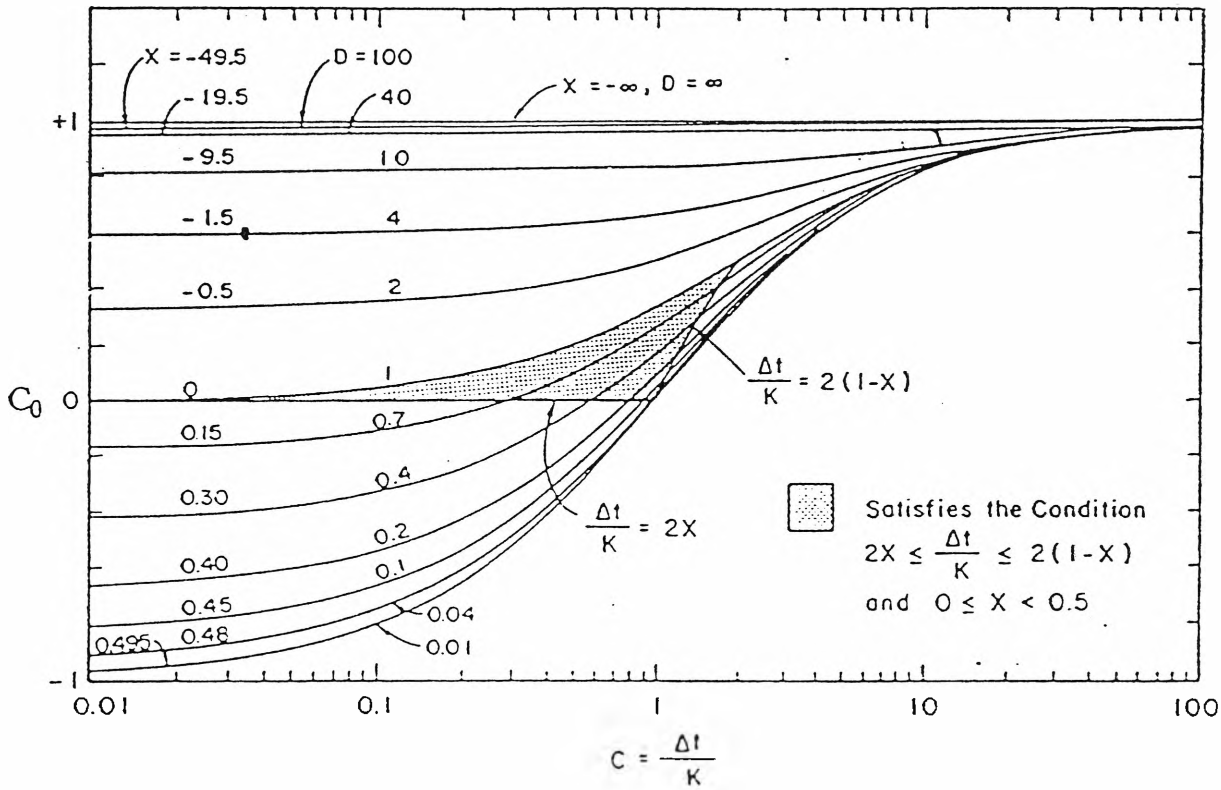


Figure 2.4.1 Variation of Muskingum Coefficient C_0 as a Function of $\Delta t/K$ and Parameter x [Ponce (1978)]

A Similar but more restrictive criterion of choosing the time step is described by Pilgrim (I.E. Australia, 1987). The time step Δt chosen should generally conform with the following conditions:

$$\Delta t \leq 0.25\ T_R \tag{2.4.6}$$

where T_R is the time of rise of the inflow hydrograph, and

$$\Delta t \leq K \qquad (2.4.7)$$

$$\Delta t \geq 2.K.x \qquad (2.4.8)$$

In some cases it is not possible to satisfy all of these conditions, therefore a compromise value may have to be taken. Inability to satisfy the condition may result in some practical problems. One of these problems is the occurrence of an unexpected decreasing value of calculated discharge represented by the dip near the start of the hydrograph. This problem is further discussed in chapter 3.

The criterion of choosing the time step Δt and space step Δs for the Muskingum-Cunge method was discussed by Jones (1981). In order to apply the Muskingum-Cunge method, the parameters μ and ω of the convection-diffusion eq.(2.3.14) which are assumed to be constant, must be determined. Substituting eq.(2.4.1) into eq.(2.3.14) yields

$$\frac{\partial Q}{\partial t} + \omega.\frac{\partial Q}{\partial s} = (\frac{1}{2} - x).\frac{\omega}{r}.\Delta t.\frac{\partial^2 Q}{\partial s^2} \qquad (2.4.9)$$

where $$r = \Delta t/\Delta s \qquad (2.4.10)$$

Jones presented the true solution to the convection-diffusion equation and some related graphics. One of them is the graphic which permits the choice of Δt and Δs for wave forms of a number of time steps λ (λ is the ratio of time base to time step). However, in application the time base of the inflow hydrograph, and hence the value of λ, may not be known in advance, so the model should be chosen to be applicable and accurate in as many cases as possible. Figure 2.4.2 shows the graphic $1/(\omega r)$ vs. x. The Muskingum-Cunge parameters K and x and the space and time steps Δs and Δt may be found using equations (2.3.15) and (2.3.16) together with Fig.2.4.2. It can be seen in Fig 2.4.2 that the behaviour of the model for $0.3 \leq x \leq 0.5$ is similar for a wide range of values of λ. The value of $\lambda = 10$

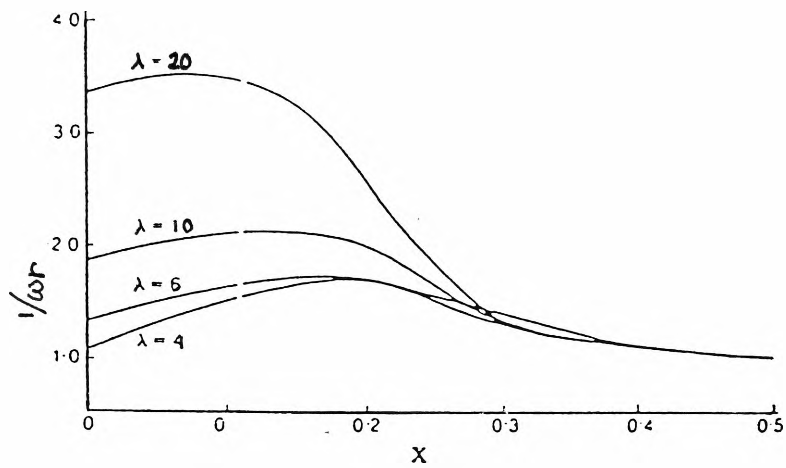


Figure 2.4.2 Critical Value of 1/(ωr) Plotted against x for Different Values of λ [Jones (1981)]

can be taken as a representative choice for that range of x value. That is, the time step Δt can be taken as one tenth of the hydrograph time base.

There are three approaches to the choice of the values of Δs and Δt which depend on whether or not one of them is more clearly defined by the physical model. They are:

a. space step Δs fixed,

b. time step Δt fixed,

c. checks when both space step and time step are fixed.

The third approach is used when the space and time steps Δs and Δt are specified from physical conditions. This is usually found in the case of pipe routing. It is not discussed herein, for further detail see Jones (1981).

a. Space step Δs fixed

If the space step Δs is obviously suggested by the physical model, but the time step Δt is not, the parameter x can be determined using eq.(2.3.16) with known values of parameters μ and ω. An increased value of Δs should be considered if the parameter x value is less than 0.3 since the corresponding choice of the time step Δt will depend on λ. To obtain the calculated outflow hydrograph at the end of the true reach at a distance Δs, interpolation on the final solution has to be performed.

Using the parameter x value calculated from eq.(2.3.16) and $\lambda = 10$ curve in Fig.2.4.2, the value of $1/(r.\omega)$ is obtained, and hence the time step Δt using $r = \Delta t/\Delta s$. Another alternative to obtain the value of $1/(r.\omega)$ is using eq.(2.4.11) which is a good fit to the $\lambda = 10$ curve in the region $0.3 \leq x \leq 0.5$.

$$\frac{1}{\omega.r} = 1.0 - 0.0939.(\tfrac{1}{2} - x) + 9.015.(\tfrac{1}{2} - x)^2 \qquad (2.4.11)$$

<u>b.Time step Δt fixed</u>

If time step Δt is determined in advance but space step Δs is not, what has to be performed first is checking the time step Δt, whether it is at most a fifth of the rise time of the inflow hydrograph T_R (to give $\lambda > 10$).

Substituting for 1/2 - x from eq.(2.3.16) into eq.(2.4.11) yields

$$\frac{\Delta s}{\omega.\Delta t} = 1.0 - 0.0939.\left(\frac{\mu}{\omega.\Delta s}\right) + 9.015.\left(\frac{\mu}{\omega.\Delta s}\right)^2 \qquad (2.4.12)$$

This is a cubic equation in Δs which may be cumbersome to solve for each reach. For convenience, a simpler approximation is used :

$$\frac{1}{\omega.r} = 1.0 + 0.767.\left(\frac{\mu}{\omega.\Delta s}\right) \qquad (2.4.13)$$

or

$$\Delta s^2 - \omega.\Delta t.\Delta s - 0.767.\mu.\Delta t = 0 \qquad (2.4.14)$$

Solving this equation yields

$$\Delta s = \tfrac{1}{2}.\omega.\Delta t.\left[1 + \sqrt{\left(1 + \frac{3.068.\mu}{\omega^2.\Delta t}\right)}\right] \qquad (2.4.15)$$

If x is in the range $0.3 \leq x \leq 0.5$, equation (2.3.16) gives

$$\Delta s > 5.\mu/\omega \qquad (2.4.16)$$

and from eq.(2.4.15) this leads to the requirement that

$$\Delta t > 4.335.\mu/\omega^2 \qquad (2.4.17)$$

If this condition is not satisfied, a larger time step Δt should be chosen.

2.5 SUMMARY

The parameters x and K can be obtained by a trial-and-error procedure. The value of x which results in a loop closest to a single line in a graphic of S vs. [x.I + (1-x).Q] using historical data is adopted, while the value of K is obtained as the slope of the straight line. Alternatively, the value of x can be first found by calculating dI/dt and dQ/dt at the intersection of the inflow and outflow hydrographs. This value of x can then be used to plot S versus [x.I + (1-x).Q] and the value of K determined from the slope of the resulting straight line. The trial-and-error procedure can be replaced either by the least-squares method proposed by Gill (1978) or the direct optimization method for parameter estimation proposed by Stephenson (1979).

The conventional Muskingum equation has an analogy with the kinematic wave equation, where $K = \Delta s/\omega$ and ω is the kinematic wave speed.

The parameter x value can theoretically lie in the range $(-\infty, 0.5]$. The negative x value is needed in the case when the river reach is short. However, in practice, the range of parameter x is [0, 0.5].

The criterion of choosing the time step Δt for the conventional Muskingum method in terms of T_R, x and K was presented by Pilgrim (I.E. Australia, 1987). The time step Δt chosen should generally conform with the three stated conditions. But in some cases, it is not possible to satisfy all of those conditions, therefore a compromise value may have to be taken. Inability to satisfy the condition may result in some practical problems, such as the occurrence of an unexpected decreasing value of calculated discharge represented by the dip near the start of the hydrograph (further discussed in chapter 3). The criterion of choosing the time step Δt and space step Δs was presented by Jones (1981) in terms of parameters ω and μ (the kinematic wave speed and the diffusion parameter).

Chapter Three

Some Aspects of Downstream Routing Using Muskingum Method

3.0 INTRODUCTION

The aim of this chapter is to consider some specific aspects which are significant in the conventional Muskingum downstream routing equation. One of them involves an explanation for the failure of the Muskingum method when $\Delta t/K$ is not small. This is demonstrated by the widely accepted belief that Muskingum routing with parameter $x = 0.5$ operates as a pure delay when the time step Δt equals K. Another aspect considered is the reduced or sometimes negative outflows which occur near the start of the hydrograph. Finally, an alternative way of calculating Muskingum coefficients, Nash coefficients, which are potentially more accurate than the Muskingum coefficients is considered.

Some of the sources of this chapter were taken from the papers written by Nash (1959), Kulandaiswamy (1966), Gill (1979a,b), Singh and McCann (1980), Strupczewski and Kundzewicz (1980b) and Pilgrim (I.E.Australia, 1987).

3.1 EFFECTS OF MODEL PARAMETERS ON DOWNSTREAM HYDROGRAPH

In order to describe more clearly the effects of model parameters on the calculated downstream hydrograph, results of computations using the observed upstream hydrograph taken from ARR87 page 134 table 7.1 with various values of model parameters are presented. The computations encompassed the effect of varying time step Δt, varying parameter K and varying parameter x values.

3.1.1 Effect of Varying Time Step Δt

The computations used time steps Δt : 24, 48 and 72 hours, parameter K = 66 hours and parameter x = 0.45. Figure 3.1.1 shows the result. It can be noticed from the figure that unexpected decreasing values occur for Δt = 24 and 48 hours. They are shown by the dips at time t = 288 hours. The unexpected decreasing value is due to the negative value of C_0 in the Muskingum equation (see Table III.1.1) and the high value of I_2 (I_{i+1}) for that period. This negative value of C_0 also results in fluctuations which are evident in the calculated hydrograph beyond this time. Using a longer time step, i.e.: Δt = 72 hours, the dip is eliminated.

Table III.1.1 The values of C_0, C_1 and C_2 for K = 66 Hours, x = 0.45 and Δt = 24, 48 and 72 Hours

Δt (hours)	C_0	C_1	C_2
24	-0.366	0.863	0.503
48	-0.095	0.891	0.204
72	0.087	0.909	0.004

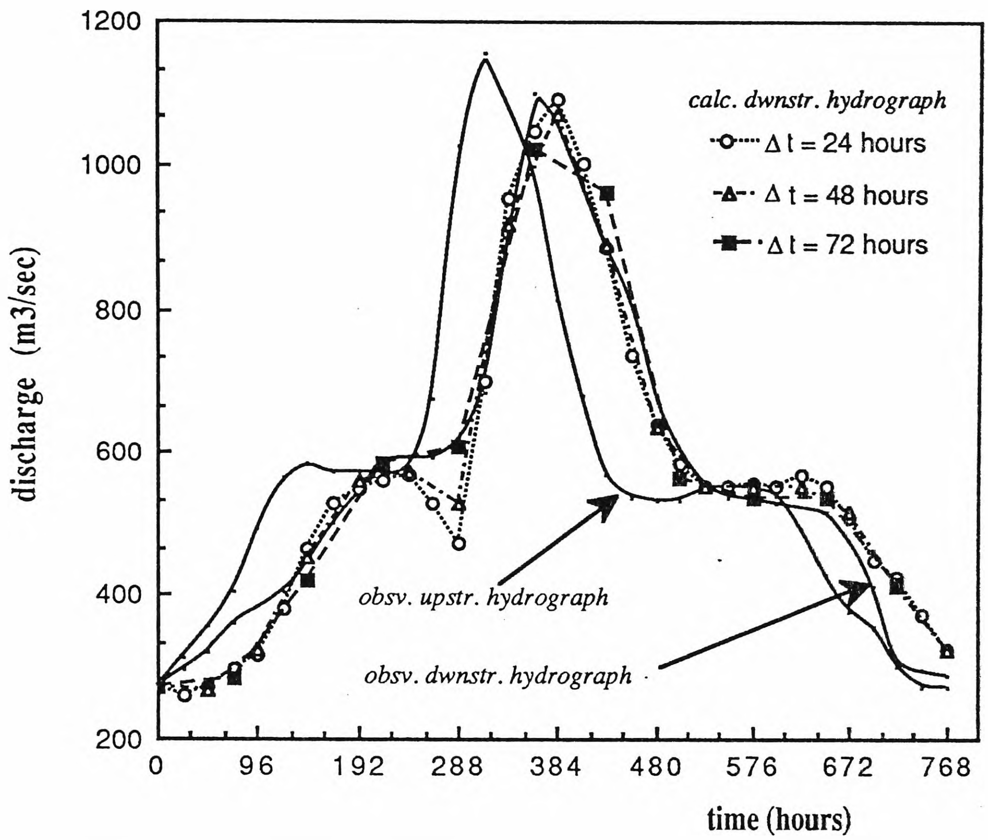


Figure 3.1.1 Calculated Downstream Hydrograph with Various Time Steps

The time steps used in this computation do not satisfy the three conditions discussed in section 7.4 of chapter 7 by Pilgrim (I.E.Aust., 1987). The conditions are:

* $\Delta t \leq 0.25\ T_R$

  $\leq 0.25 \times 72$ hours ≤ 18 hours

  where T_R is the time of rise of the major peak of the inflow hydrograph,

* $\Delta t \leq K$

  ≤ 66 hours,

* $\Delta t \geq 2.K.x$

  $\geq 2 \times 66 \times 0.45 \geq 59.4$ hours.

Since the three conditions cannot be satisfied by any one value of Δt, a compromise is necessary. In spite of the dip, time step $\Delta t = 24$ hours provides a result which agrees with the observed downstream hydrograph reasonably well.

Using a longer time step, i.e.: $\Delta t = 72$ hours, the dip is eliminated, but the spacing of the computed points is so great that the shape of hydrograph and particularly the peak, is not adequately defined.

The other criterion for choosing time step Δt presented by Jones (1981) as discussed in chapter 2 cannot be applied in this case since the parameters μ and ω are not known.

It can be concluded that since the time step Δt has a significant effect on the calculated downstream hydrograph, it must be chosen with care.

3.1.2 Effect of Varying K Value

The computations used K values: 6, 12, 24, 33 and 66 hours with parameter $x = 0.45$ and $\Delta t = 24$ hours. Figure 3.1.2 shows the result. It can be noticed from the figure that the larger the K value, the longer the time lag is and the more the peak is reduced (attenuation). In addition, the dip at time 288 hours is more pronounced with the larger K value, since it makes the value of C_0 decrease to become negative (see Table III.1.2).

Table III.1.2 The values of C_0, C_1 and C_2 for $x = 0.45$ and $\Delta t = 24$ Hours and Various Parameter K Values

K (hours)	C_0	C_1	C_2
6	0.608	0.961	-0.569
12	0.355	0.935	-0.290
24	0.048	0.905	0.048
33	-0.095	0.891	0.204
66	-0.366	0.863	0.503

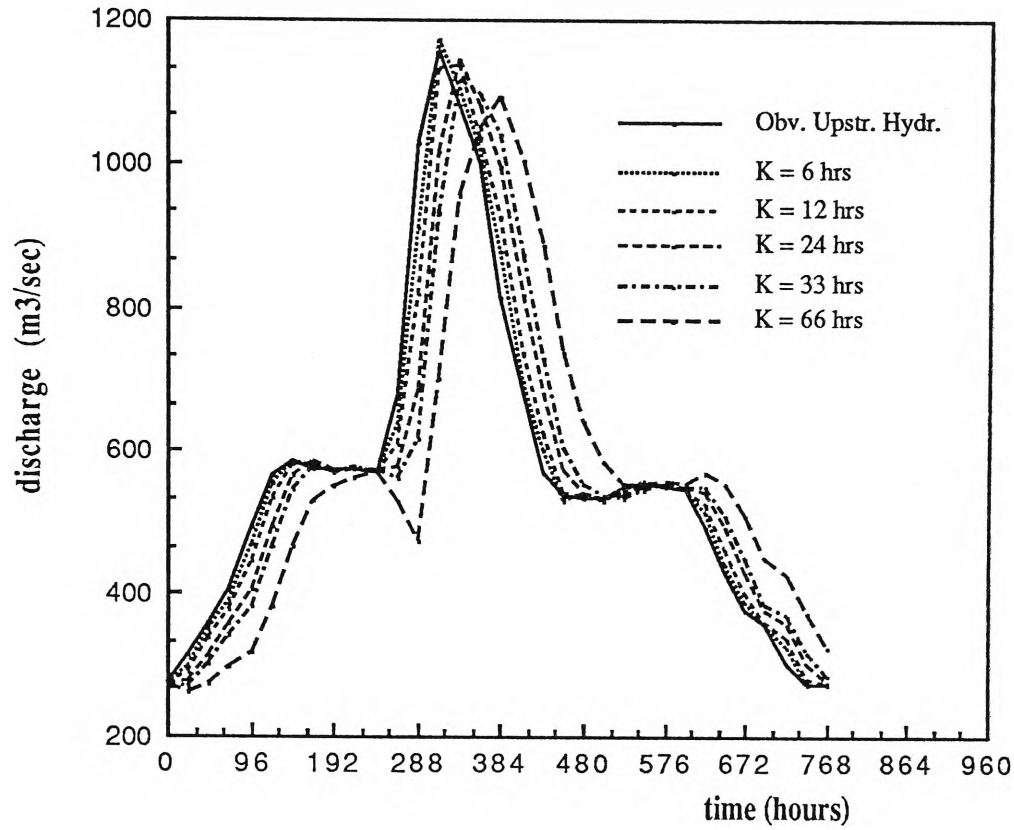


Figure 3.1.2 Downstream Hydrograph with Various K Values

3.1.3 Effect of Varying Parameter x Value

The computations used parameter x values: 0, 0.1, 0.2, 0.3, 0.4, 0.45 and 0.5, parameter K = 66 hours and time step Δt = 24 hours. It can be noticed from Figure 3.1.3 that as parameter x decreases to zero, attenuation is greater so that the peak discharge decreases. Also, as x decreases, the dip in the outflow hydrograph becomes less pronounced. As mentioned previously, the dip results from the negative value of C_0 and the high value of I_2 (I_{i+1}) for the corresponding period. The more negative the value of C_0 is, the more pronounced the dip becomes. As can be seen in Table III.1.3, the most negative value of C_0 is given by x = 0.5. This parameter x value results in the most pronounced dip (Fig. 3.1.3).

Table III.1.3 The values of C_0, C_1 and C_2 for K = 66 hours, Δt = 24 Hours and Various Parameter x Values

x	C_0	C_1	C_2
0	0.154	0.154	0.692
0.1	0.076	0.261	0.664
0.2	-0.019	0.389	0.630
0.3	-0.134	0.546	0.588
0.4	-0.279	0.744	0.535
0.45	-0.366	0.863	0.503
0.5	-0.467	1	0.467

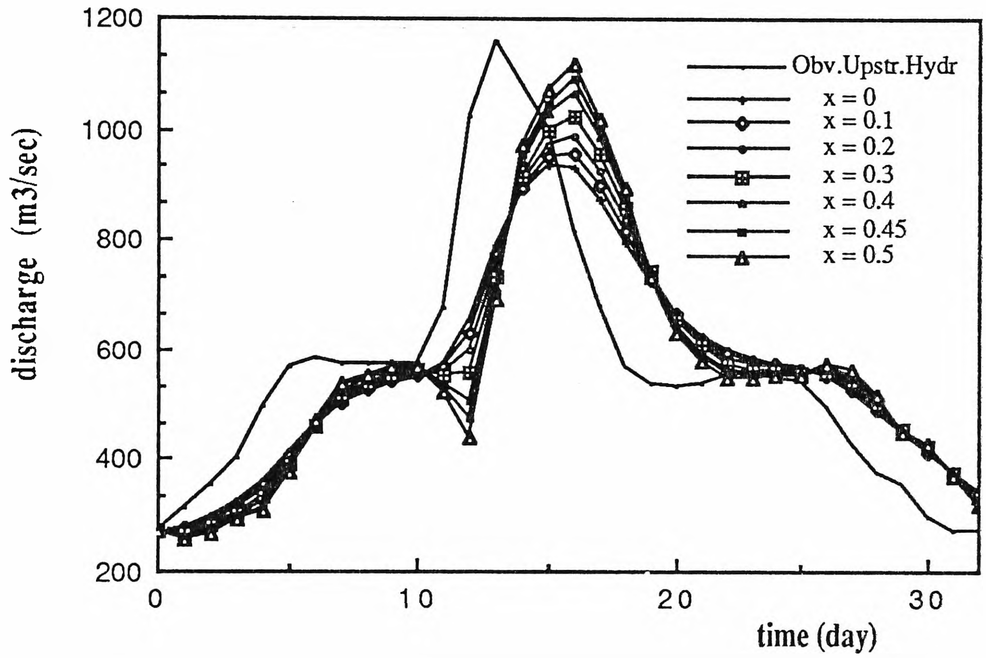


Figure 3.1.3 Downstream Hydrograph with Various x Values

3.2 NEGATIVE OR REDUCED INITIAL DOWNSTREAM DISCHARGES

Reduced or sometimes negative initial downstream discharges may occur at the start of the computation, as can be seen in Figure 3.1.1, for time near to zero.

This is investigated to reveal whether this phenomenon, which results from using a finite difference method of solution, is in accordance with the analytical solution of the Muskingum equation.

The description presented by Nash (1959) is given below.

The fundamental equations are (as previously mentioned):

$$I = Q + \frac{dS}{dt} \tag{3.2.1}$$

$$S = K\,(x.I + (1-x).Q) \tag{3.2.2}$$

from which

$$I - x.K.\frac{dI}{dt} = Q + (1-x).K.\frac{dQ}{dt} \tag{3.2.3a}$$

This can be re-arranged in terms of I

$$Q(t) = \frac{1 - x.K.D}{1 + (1-x).K.D}\, I(t) \tag{3.2.3b}$$

where D = the differential operator d/dt.

When x = 0, the linear reservoir case is obtained.

$$Q(t) = \frac{1}{1 + K.D}\, I(t) \tag{3.2.4}$$

which has the solution

$$Q = \frac{1}{K}.e^{-t/K} \int e^{t/K}.I\,dt \tag{3.2.5}$$

Now eq.(3.2.3b) may be looked upon as the result of operating on I(t) successively with 1 - x.K.D and 1/[1 + (1-x).K.D]. The operation 1 - x.K.D merely involves differentiation of the inflow (upstream discharge), and the operation 1/[1 + (1-x).K.D] represents reservoir routing with S = (1-x).K.Q. Therefore eq. (3.2.3b) is equivalent to subtracting x.K times the first derivative of I from I and routing the remainder through reservoir storage with S = (1-x).K.Q. From eq. (3.2.3b), another significant point can be obtained by defining

$$I'(t) = (1 - x.K.D).I(t) \quad (3.2.6a)$$

or

$$I'(t) = I(t) - x.K.D.I(t) \quad (3.2.6b)$$

By comparing with eq.(3.2.1), this means that I' is the result of routing I backwards through linear reservoir storage S = -x.K.I. The effect of the negative x.K is achieved by taking the routing procedure from right to left; that is, in the negative direction of time (Fig.3.2.1).

When time t_1 at which I' becomes zero is reached, I would fall off logarithmically and never actually reach zero unless I' took negative values. This means that when I starts from zero and rises at a finite rate, I' must always take negative values initially.

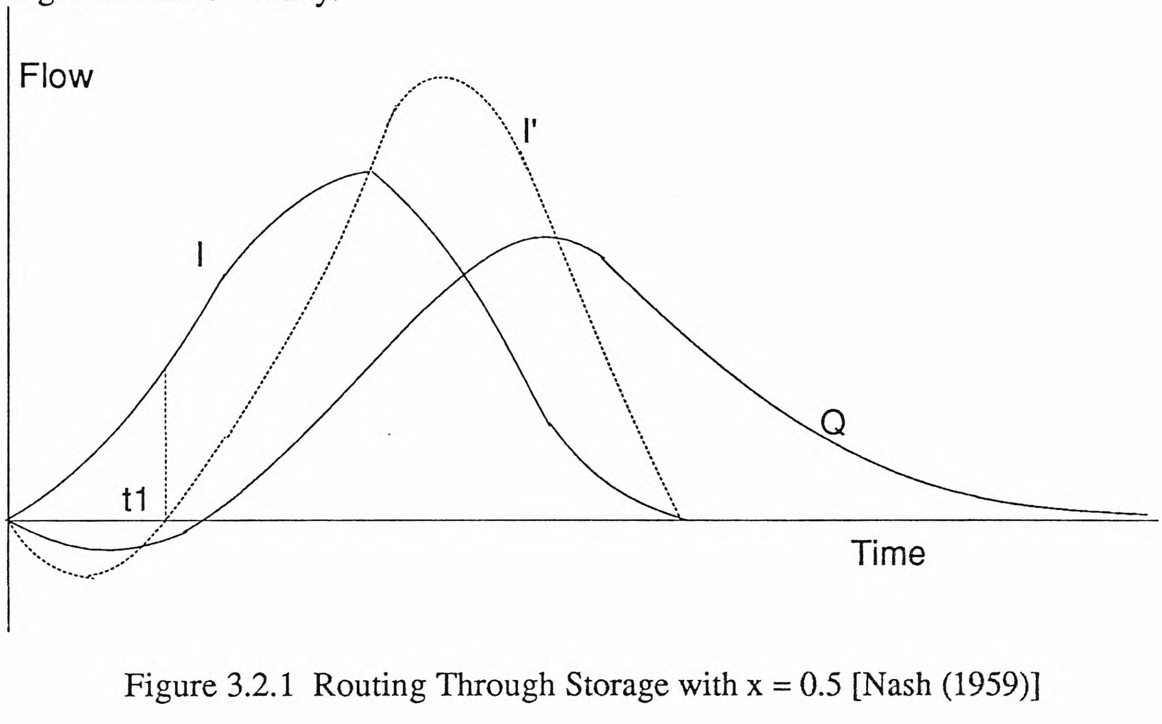


Figure 3.2.1 Routing Through Storage with x = 0.5 [Nash (1959)]

It is clear that the interval between the centres of area of I'(t) and I(t) is x.K. Further routing moving forwards through S = (1-x).K.Q should be carried out to obtain Q (Fig. 3.2.1). Clearly this involves a further shift of the centre of area by (1-x).K so that the total shift is K. It is shown in Fig 3.2.1 that I and Q are not identical when parameter x = 0.5 or any other value, so that pure translation cannot occur. This circumstance is further discussed later in section

3.3. It should be noted that the negative initial values of I' result in negative initial values of Q.

Gill in his paper (1979b) asserted that the reduced or sometimes negative initial downstream discharges are the result of using a wrong initial condition rather than due to any inherent defect of the Muskingum method. That the negative outflows are due to a wrong initial condition is demonstrated by considering specific examples (not discussed herein). Further, Gill proposed an initial condition:

$$I(0) = Q(\tau), \quad \tau > 0 \qquad (3.2.7)$$

and emphasized that the use of this condition would prevent the occurrence of negative outflow in the Muskingum method.

Singh and McCann (1980) criticized this assertion and stated that the condition is incompatible with the formulation of the Muskingum method and, therefore, cannot be used. Below is their explanation.

If eq.(3.2.3a) is solved using the initial condition proposed by Gill [eq.(3.2.7)] then the solution is, for $\tau \leq t$:

$$Q(t) = -\frac{x}{1-x}.I(t) + [Q(\tau) + \frac{x}{1-x}.I(\tau)].e^{-(t-\tau)/[K(1-x)]} + \frac{1}{K(1-x)^2}\int_{\tau}^{t} e^{-(t-s)/[K(1-x)]}.I(s)\,ds \qquad (3.2.8)$$

This solution was obtained by Singh and McCann (1979) with the explicit statement of eq.(3.2.7).

To show that the initial condition in eq.(3.2.7) proposed by Gill (1979a,b) is incompatible with eq. (3.2.3a), an inflow represented by a finite-duration rectangular pulse is considered:

$$I(t) = A \quad \text{for } 0 \leq t < T$$

$$I(t) = 0 \quad \text{for } t \geq T \qquad (3.2.9)$$

where T is the duration of inflow, and A some constant > 0. In order to obtain Q(t) from eq. (3.2.8), using eq. (3.2.9) two cases must be distinguished.

a). $T < \tau$

b). $T \geq \tau$

In case (a) for $t \leq T < \tau$, Q(t) cannot be obtained since eq. (3.2.8) is valid only for $t \geq \tau$. Further, Q(t) cannot be obtained either for $t \geq T$ since Q(t) is not known for $t \leq T$. Therefore, this condition is incompatible for I(t), $t \leq \tau$.

In case (b) for $\tau \leq t \leq T$, Q(t) is obtained from eq. (3.2.8):

$$Q(t) = A, \qquad \tau \leq t \leq T \tag{3.2.10a}$$

and

$$Q(t) = A.e^{-(t-T)/[K(1-x)]}, \quad t \geq T \tag{3.2.10b}$$

From these equations, it can be seen immediately that eq. (3.2.1) for conservation of mass cannot be satisfied. To illustrate, the inflow volume applied is A.T. The total outflow produced, if Q(t) is assumed to be zero during $0 \leq t \leq \tau$ is:

$$A(T-\tau) + \int_T^{\infty} A.e^{-(t-T)/[K(1-x)]}\, dt = A(T-\tau) + A.K(1-x)$$

It is obvious that the total volume of inflow does not equal the total volume of outflow produced.

Furthermore, if the outflow during $0 \leq t \leq \tau$ is assumed as Q(t) = A, then the total volume of outflow becomes

$$A.T + A.K(1-x)$$

which again violates eq. (3.2.1). Therefore, the conclusion that can be deduced is that eq. (3.2.7) is not consistent with the Muskingum hypothesis. Gill (1979a,b) is mistaken to assert the adequacy of this condition in the Muskingum flood routing method. Another inflow which can be considered is

$$I(t) = \sin(t.\pi/\tau), \qquad \text{for } 0 \leq t \leq \tau$$

$$I(t) = 0, \quad \text{for } \tau \leq t \tag{3.2.11}$$

Then $Q(\tau) = I(0) = 0$ and $I(0) = 0$ for $t \geq \tau$. For this choice of I eq. (3.2.8) becomes $Q(t) = 0$ for $t \geq \tau$. This is obviously incorrect since there is no outflow before $t \leq \tau$. Apparently, any lag time τ to be imposed on Muskingum method should be imposed through the basic equations, not through the initial condition.

Strupczewski and Kundzewicz in their paper (1980b) alleged that: Gill's idea of shifting the initial conditions on outflow is only 'skipping' the problem of negative outflows. Outflows are simply not calculated in the periods when they should be negative. Again, this opposes Gill's opinion.

As has been explained by Nash above, it is clear that the reduced or sometimes negative initial downstream discharges which may occur when the inflow rises steeply, is associated with the storage assumption and not with any particular method of solution. Apparently, based on the analyses above, Gill's idea of shifting the initial condition on the outflow is physically incorrect.

3.3 CASE OF PURE TRANSLATION

A curious feature of the Muskingum method which directly leads to the consideration of translatory waves is the special case in which $x = 0.5$ and time step $\Delta t = K$. Substituting these values into Muskingum coefficients yields C_0 and C_2 being equal to zero and $C_1 = 1$. From eq. (2.1.3), it is seen that

$$Q_{i+1} = I_i \tag{3.3.1}$$

Equation (3.3.1) indicates that, the downstream discharge at any time i+1 is equal to the upstream discharge at time i. In other words, the flood wave is merely translated with a time lag of $\Delta t = K$. Whether this circumstance is correct is rather doubtful. It may be considered to happen because of adopting a large value for the time step Δt and making it equal to K. If the time step $\Delta t \neq K$, the coefficients C_0 and C_2 are not zero and with parameter $x = 0.5$, it can be seen that

$$Q_{i+1} \neq I_i \tag{3.3.2}$$

Kulandaiswamy in his paper (1966) alleged that the value of time step Δt that may be adopted for solving numerically the differential equation in eq.(2.1.1) is purely arbitrary and the value adopted for Δt should not change the basic nature of the result. But actual routing has also shown that with parameter x = 0.5 the downstream hydrograph is more or less the same as upstream hydrograph translated over a certain period even without time step Δt being equal to K. With respect to this, Kulandaiswamy investigated whether $Q_{i+1} = I_i$ when parameter $x = 0.5$, or Q_{i+1} is at least very nearly equal to I_i if $Q_{i+1} \neq I_i$. The investigation is described as follows:

The differential equation in eq.(2.1.1) can be written in operator form as

$$Q(t) = \frac{1 - K.x.D}{1 + K.(1-x).D} \cdot I(t) \tag{3.3.3}$$

where D = d/dt, when x = 0.5

$$Q(t) = \frac{1 - \frac{1}{2}.K.D}{1 + \frac{1}{2}.K.D} \cdot I(t) \tag{3.3.4}$$

The term 1/(1 + 1/2.K.D) can be expanded into series and eq.(3.3.4) becomes

$$Q(t) = (1 - \tfrac{1}{2}.K.D + \tfrac{1}{4}.K^2.D^2 - \tfrac{1}{8}.K^3.D^3 \ldots)(1 - \tfrac{1}{2}.K.D).I(t) \tag{3.3.5}$$

Since the operator can be treated as an algebraic quantity, the multiplication can be performed and

$$Q(t) = (1 - K.D + \tfrac{1}{2}.K^2.D^2 - \tfrac{1}{4}.K^3.D^3 \ldots)\, I(t) \tag{3.3.6}$$

Then, the upstream hydrograph I(t) which is merely translated by a time lag K is considered. The expression I(t-K) is now regarded as the resulting downstream hydrograph which can be expanded in Taylor series:

$$\begin{aligned} I(t-K) &= I(t) - K.I'(t) + \frac{K^2}{2!}.I''(t) - \frac{K^3}{3!}.I'''(t) \ldots \\ &= (1 - K.D + \frac{K^2}{2!}.D^2 - \frac{K^3}{3!}.D^3 \ldots)\, I(t) \end{aligned} \tag{3.3.7}$$

If eq.(3.3.6) and (3.3.7) are compared, it can be seen that the first three terms on the right hand side of those equations are identical. The difference starts from the fourth term. However, this difference is very small. If I(t) is such that the third and higher order derivatives are not significant, the following equation can be written:

$$Q(t) = I(t-K) \tag{3.3.8}$$

The conclusion which can be extracted from the discussion above is that when x = 0.5, the value of Q_{i+1} is not identically equal to I_i. It is approximately equal provided that the third and higher order derivatives of I(t) are very small and can be ignored. Equation (2.1.3) shows that $Q_{i+1} = I_i$ when x = 0.5 and Δt = K. This is purely due to the approximation inherent in the numerical procedure used for the solution of the continuity differential equation.

Gill in his paper (1979a) alleged that Kulandaiswamy's conclusions were rather vague because he could not reduce eq.(3.3.8) to eq.(3.3.1) for any value of Δt not necessarily equal to K. This was mentioned with respect to Kulandaiswamy's statement that the actual routing has also shown that with parameter x = 0.5 the downstream hydrograph is more or less the same as the upstream hydrograph translated over a certain period, even without time step Δt being equal to K. Furthermore, Gill proposed that for a translatory wave, a general condition which is required to be satisfied is Q(t) = I(t-T), where T is the time lag which in some cases may be different from K. It really depends on the form of the inflow function. Gill's conclusion was based on his example in terms of sinusoidal flood without any further explanation regarding the proof.

Figure 3.3.1 shows an example using an observed upstream hydrograph taken from ARR87 page 134 table 7.1 with Δt = 24 and 66 hours, K = 66 hours and parameter x = 0.5. The result of using time step Δt = K = 66 hours is a translatory wave. This can be seen in Table III.3.1. The observed upstream discharges which have time interval 24 hours had been interpolated using $\Delta t = 66$ hours, before the computation was carried out. Figure 3.3.1 cannot show the translatory wave properly for $\Delta t = K = 66$ hours, because the observed upstream discharges were plotted using a time interval of 24 hours while the calculated downstream discharges were plotted using a time interval of 66 hours, so that the shape and the peak of the calculated downstream hydrograph cannot be adequately defined. The result of using time step $\Delta t = 24$ hours (regardless of the dip occurring at time $t = 288$ hours) seems to give a translatory wave. But, careful examination shows that it does not. The peak of the hydrograph is slightly attenuated and in addition, a reduced initial outflow occurs as has been discussed by Nash (1959), and given in section 3.2 of this thesis.

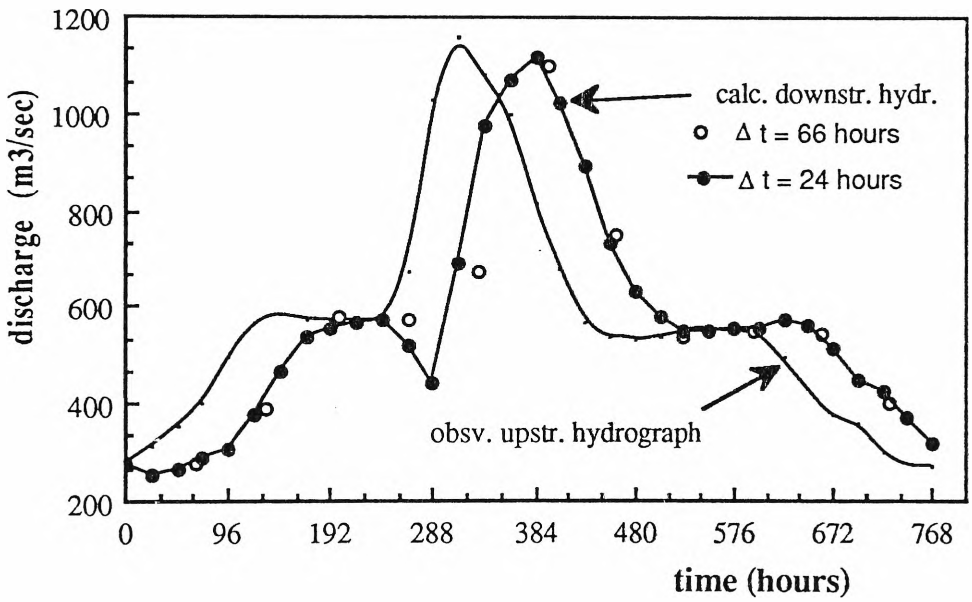


Figure 3.3.1 Routing Through Storage with x = 0.5, K = 66 Hours and Δt = 24 and 66 Hours

Table III.3.1 Result of Computation Using
$x = 0.5$, $K = 66$ Hours and $\Delta t = K$

PERIOD (x 66 hrs)	INFLOW (m^3/sec)	OUTFLOW (m^3/sec)
0	274.000	274.000
1	391.750	274.000
2	576.000	391.750
3	574.250	576.000
4	676.000	574.250
5	1099.750	676.000
6	748.500	1099.750
7	537.000	748.500
8	551.000	537.000
9	545.250	551.000
10	402.000	545.250
11	294.250	402.000

The translatory wave occurs in Table III.3.1 because a large value of time step $\Delta t = 66$ hours is used and this is made equal to K. This leads coefficients C_0 and C_2 to being equal to zero and C_1 being to 1 in the numerical approximation for the solution of the differential equation. If the time step Δt is not equal to K, a translatory wave does not occur as can be seen on Figure 3.3.1. This is in accordance with what has been discussed by Nash (1959), see section 3.2. Kulandaiswamy's approach (i.e.: the existence of translatory wave, even though Δt is not equal to K) prevails, if the third and higher order derivatives are small enough to be ignored. According to Singh and McCann (1980), the appearance of the upstream hydrograph frequently encountered in nature resembles a gamma or log-normal distribution. Obviously their third-and higher-order derivatives do not vanish in this case and therefore pure translation is only approximated.

3.4 NASH COEFFICIENTS

The derivation of the coefficients below are cited from Nash (1959).

Equation (3.2.3b) can be divided into two parts, i.e.:

$$Q = \frac{1}{1 + (1\text{-}x).K.D}\frac{I}{1\text{-}x} - \frac{x.I}{1\text{-}x} \tag{3.4.1}$$

This equation will be used to obtain the expression for the C's of the conventional downstream routing equation, i.e:

$$Q_1 = C_0.I_1 + C_1.I_0 + C_2.Q_0 \tag{3.4.2}$$

In expressing Q as a function of I_0, I_1 and Q_0 only, second and higher order derivatives of I must be neglected; that is, I must be assumed to consist of straight line segments. If the second or higher order derivatives are required, three or more values of I in eq.(3.4.2) must be used. However, by choosing time intervals which are sufficiently short, the calculation using only I_0, I_1 and Q_0 can be made as precise as is desired. The only difference between the present calculation and the usual development of the Muskingum coefficient equation is that the values of the time interval are not limited to the small values compared with K.

The solution of eq.(3.4.1) when I is a series of straight segments is obtained as follows. Let $m = (I_1 - I_0)/\Delta t$ be the slope of a segment.

Let

$$q(t) = \frac{1}{1 + (1\text{-}x).K.D} I(t)$$

then

$$Q = \frac{q}{1\text{-}x} - \frac{x.I}{1\text{-}x} \tag{3.4.3}$$

Let $k = (1\text{-}x).K$ and $c = e^{-\Delta t/[K(1\text{-}x)]}$ to simplify the notation. From eq.(3.2.5)

$$q = (1/k).e^{-t/k}\int (I_0 + m.t).e^{t/k}\,dt$$

$$q = (1/k).e^{-t/k}[k.I_0.e^{t/k} + m.k^2.e^{t/k}(t/k - 1) + A]$$

$$q = I_0 + m.k.(t/k - 1) + A/k.e^{-t/k} \tag{3.4.4}$$

The constant value of A can be defined by letting $q = q_0$ at time $t = 0$. Equation (3.4.4) becomes:

$$q_0 = I_0 - m.k + A/k$$

or

$$A = k.(q_0 - I_0 + m.k)$$

By substituting this A value, equation (3.4.4) becomes

$$q = I_0 + m.k.(t/k - 1) + (q_0 - I_0 + m.k).e^{-t/k}$$

By substituting $(I_1 - I_0)/\Delta t$ for m and letting $t = \Delta t$, eq. (3.4.5) is obtained.

$$q_1 = I_0 + k/\Delta t.(\Delta t/k - 1)(I_1 - I_0) + [q_0 - I_0 + k/\Delta t.(I_1 - I_0)].c$$

$$q_1 = I_0\,[k/\Delta t.(1 - c) - c] + I_1\,[-k/\Delta t.(1 - c) + 1] + q_0.c \qquad (3.4.5)$$

whence by eq.(3.4.3)

$$Q_1 = I_0.\left[\frac{k}{\Delta t}.\frac{1-c}{1-x} - \frac{c}{1-x}\right] + I_1.\left[-\frac{k}{\Delta t}.\frac{1-c}{1-x} + \frac{1}{1-x} - \frac{x}{1-x}\right] + q_0.\frac{c}{1-x} \qquad (3.4.6)$$

From eq.(3.4.3)

$$\frac{q_0}{1-x} = Q_0 + \frac{x.I_0}{1-x}$$

which when substituted in eq.(3.4.6) with $k = K.(1-x)$, gives

$$Q_1 = I_0.\left[\frac{K}{\Delta t}.(1 - c) - c\right] + I_1.\left[-\frac{K}{\Delta t}.(1 - c) + 1\right] + Q_0.c \qquad (3.4.7)$$

or it can be written in numerical expression as

$$Q_{i+1} = C_0.I_{i+1} + C_1.I_i + C_2.Q_i \qquad (3.4.8)$$

where:

$$C_0 = -\frac{K}{\Delta t}.(1-c) + 1 \qquad (3.4.9a)$$

$$C_1 = \frac{K}{\Delta t}.(1-c) - c \qquad (3.4.9b)$$

$$C_2 = c \qquad (3.4.9c)$$

$$c = e^{-\Delta t/[K.(1-x)]} \qquad (3.4.9d)$$

These Nash coefficients are more accurate than conventional Muskingum coefficients of equations (2.1.4a) to (2.1.4c). However, the differences are not great in many cases as can be seen in Table III.4.1 which is a sample of computation using the observed upstream hydrograph taken from ARR87 page 134 Table 7.1 with parameter $x = 0.45$, $K = 66$ hours and $\Delta t = 24$ hours.

Guidelines for choice of the form of the coefficients are given below [Pilgrim (I.E.Aust, 1987)]. Assuming the Nash coefficients give the more accurate answer:

a). If the maximum acceptable difference in the calculated hydrograph peak using the conventional Muskingum is set at 5%,

for $0 \leq x \leq 0.35$, both methods are satisfactory,

for $0.35 < x \leq 0.5$, use Nash with $\Delta t = K$, as long as $\Delta t \leq 0.25\ T_R$. Otherwise, a compromise value must be used (T_R is the time of rise of the upstream hydrograph).

b). If the maximum acceptable difference in the calculated hydrograph peak using the conventional Muskingum coefficients is set at 2%,

for $0 \leq x \leq 0.15$, both methods are satisfactory.

$0.15 < x \leq 0.4$, use Nash if $\Delta t > 0.1\ T_R$, but both methods are satisfactory if $\Delta t \leq 0.1\ T_R$.

$0.4 < x \leq 0.5$, use Nash with $\Delta t = K$, as long as $\Delta t \leq 0.25\ T_R$. Otherwise a compromise value must be used.

The conventional Muskingum coefficients generally overestimate the peak flow. The above criteria apply most critically to narrow, sharp-peaked hydrographs. For flatter hydrographs, the criteria are rather too severe, and the Muskingum coefficients will give answers within the indicated accuracies for a wider range of values of x than indicated above.

Nuaber of data = 33
K = 66.0 hours
T = 24.0 hours
x = 0.45000

PERIOD (day)	INFLOW (observed) (m3/sec)	OUTFLOW (Muskingum) (m3/sec)	OUTFLOW (N a s h) (m3/sec)
0	274.000	274.000	274.000
1	314.000	259.342	260.788
2	355.000	271.476	272.987
3	404.000	295.022	296.475
4	495.000	315.825	318.433
5	566.000	378.837	380.395
6	586.000	464.508	463.575
7	572.000	530.007	527.422
8	575.000	549.774	547.995
9	572.000	563.409	562.050
10	571.000	568.044	567.193
11	676.000	531.034	534.353
12	1026.000	474.806	487.268
13	1156.000	701.052	704.939
14	1081.000	954.597	947.911
15	1001.000	1046.723	1038.716
16	816.000	1091.798	1081.577
17	681.000	1004.228	997.696
18	569.000	885.028	881.820
19	538.000	738.492	739.920
20	534.000	640.335	643.563
21	535.000	587.131	590.232
22	551.000	555.364	558.229
23	555.000	551.730	553.411
24	549.000	555.553	556.161
25	544.000	554.129	554.349
26	493.000	567.786	566.188
27	428.000	554.445	552.253
28	376.000	510.671	509.322
29	357.000	450.716	451.104
30	301.000	424.671	424.078
31	274.000	373.114	373.458
32	271.000	324.964	326.336

Table III.4.1 Sample of Computation Using Muskingum and Nash Coefficients

Values of coefficients:
Muskingum : C_0 = -0.366, C_1 = 0.863 and C_2 = 0.503
Nash : C_0 = -0.330, C_1 = 0.814 and C_2 = 0.516

3.5 SUMMARY

Based on the description by Nash (1959), the reduced or sometimes negative initial outflow which sometimes occurs when the inflow rises steeply, is associated with the storage assumption in the Muskingum method and not with any particular method of solution.

The Muskingum method of flood routing is not a translatory solution. The translatory wave obtained when time step $\Delta t = K$ with parameter $x = 0.5$ is due to the approximation inherent in the numerical procedure. When time step $\Delta t \neq K$, a translatory wave does not occur, even though the calculated wave seemingly resembles a wave translated over a certain time period.

Sometimes an unexpected decreasing value shown by the existence of a dip in the calculated hydrograph occurs (as is shown on Fig. 3.1.1). This is caused by a negative value of the coefficient C_0, due either to the value of time step Δt being too small, K being too large, or x being too large. If values of K and x are given, then the dip can be avoided by using a larger time step Δt. But if the time step Δt used is too large, the shape of the hydrograph and particularly the peak is not adequately defined.

The coefficients derived by Nash (1959) yield very similar results to those resulting from the standard Muskingum coefficients. For smaller values of the time step Δt, the Nash and Muskingum coefficients become almost identical. For larger time steps, and for larger values of x, the Nash coefficients should give more accurate results.

Chapter four

Upstream Routing Using Conventional Muskingum Equation

4.0 INTRODUCTION

This chapter is intended to give a description of the problems which are associated with upstream routing. Upstream routing is deduced mathematically from the conventional downstream routing procedures. Samples of the computations showing the problem are given.

4.1 UPSTREAM ROUTING DERIVED FROM CONVENTIONAL DOWNSTREAM ROUTING

Mathematically, upstream routing can be deduced easily from the Muskingum operating equation (Eq. 2.1.3), which is obtained by combining the equation of linear relationship between I, Q and S and the equation of conservation of mass in terms of finite differences. That equation can be expressed as:

$$I_{i+1} = \frac{Q_{i+1}}{C_0}. - \frac{C_1}{C_0}.I_i - \frac{C_2}{C_0}.Q_i \qquad (4.1.1)$$

where:

$$\frac{1}{C_0} = -\frac{K - K.x + 0.5\Delta t}{K.x - 0.5\Delta t} \qquad (4.1.2)$$

$$\frac{C_1}{C_0} = -\frac{K.x + 0.5\Delta t}{K.x - 0.5\Delta t} \qquad (4.1.3)$$

$$\frac{C_2}{C_0} = -\frac{K - K.x - 0.5\Delta t}{K.x - 0.5\Delta t} \qquad (4.1.4)$$

and

$$\frac{1}{C_0} - \frac{C_1}{C_0} - \frac{C_2}{C_0} = 1 \qquad (4.1.5)$$

The method of solving this equation is similar to that used to solve conventional downstream routing. If the routing coefficients $1/C_0, C_1/C_0$ and C_2/C_0 are evaluated, routing is carried out by solving equation (4.1.1) consecutively for I_{i+1} period by period throughout the flood. In each routing period, Q_i and Q_{i+1} are known from the observed hydrograph at the downstream station, while I_i is set equal to the value of I_{i+1} calculated for the previous routing period.

The routing coefficients in terms of K, x and Δt are the same as the ones described in chapter 2, namely Muskingum coefficients. The coefficients derived by Nash (1959) which are more accurate if applied in conventional routing, still can be used. However, the computations using both Muskingum and Nash coefficients show that unexpected results arise.

4.2 UPSTREAM ROUTING COMPUTATIONS USING EQUATION (4.1.1)

The values of the Muskingum parameters on which the Muskingum coefficients depend were adopted as: average travel time K = 66 hours, time step Δt = 24 hours and parameter x = 0.0, 0.1, 0.2, 0.3, 0.4, 0.45 and 0.5.

Firstly, downstream hydrograph ordinates were calculated from the given observed upstream hydrograph, applying conventional downstream routing [eq.(2.1.3)] with each set of parameter values. Secondly, each calculated downstream hydrograph was used to calculate back the upstream hydrograph ordinates, applying upstream routing (eq. 4.1.1). The results of the computations are given in Tables IV.2.1.

It can be noticed from the results in Tables IV.2.1 that the only x value which makes the calculated upstream discharges agree exactly with the observed ones is $x = 0$. It might be expected that the other x values should give similar results, since the conventional downstream and the upstream routings are basically derived from the same equation. However, this does not occur, and all other values of x give unsatisfactory results. A value of $x=0.2$ gives the worst result, and $x=0.5$ seems to give a satisfactory result, but as a matter of fact it does not, since the last few calculated upstream discharges do not match the observed ones.

In addition, fluctuations are likely to occur in the calculated hydrograph. This circumstance can be noticed most clearly from the computation with $x=0.1$. These problems are due to the computational instability of the process. It should be noted that they result not only from using Muskingum coefficients but also with Nash coefficients.

Tables IV.2.1

Results of Computations Using Various Parameter x Values, K = 66 Hours and Δt = 24 Hours

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.00000
```

PERIOD (x 24.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	280.154	314.000
2	355.000	296.876	355.000
3	404.000	322.299	404.000
4	495.000	361.437	495.000
5	566.000	413.457	566.000
6	586.000	463.470	586.000
7	572.000	499.018	572.000
8	575.000	521.935	575.000
9	572.000	537.801	572.000
10	571.000	548.170	571.000
11	676.000	571.349	676.000
12	1026.000	657.395	1026.000
13	1156.000	790.812	1156.000
14	1081.000	891.639	1081.000
15	1001.000	937.596	1001.000
16	816.000	928.644	816.000
17	681.000	873.215	681.000
18	568.000	796.687	568.000
19	538.000	721.707	538.000
20	534.000	664.566	534.000
21	535.000	624.546	535.000
22	551.000	599.455	551.000
23	555.000	585.161	555.000
24	549.000	574.958	549.000
25	544.000	566.201	544.000
26	493.000	551.524	493.000
27	428.000	523.517	428.000
28	376.000	486.127	376.000
29	357.000	449.319	357.000
30	301.000	412.298	301.000
31	274.000	373.898	274.000
32	271.000	342.699	271.000

Notes : (i) Values of coefficients:

$$1/C_0 = 6.50, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -4.50$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.10000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	277.025	314.000
2	355.000	292.555	355.000
3	404.000	317.251	404.000
4	495.000	353.292	495.000
5	566.000	406.295	566.000
6	586.000	461.490	586.000
7	572.000	502.283	572.000
8	575.000	525.944	575.000
9	572.000	542.207	572.000
10	571.000	552.146	571.000
11	676.000	566.424	676.000
12	1026.000	629.727	1026.000
13	1156.000	772.760	1155.999
14	1081.000	895.908	1081.003
15	1001.000	952.073	1000.989
16	816.000	954.528	816.039
17	681.000	897.754	680.867
18	568.000	816.349	568.459
19	538.000	730.601	536.420
20	534.000	665.559	539.442
21	535.000	621.413	516.256
22	551.000	593.577	615.564
23	555.000	579.568	332.612
24	549.000	570.856	1315.004
25	544.000	563.131	-2094.459
26	493.000	552.843	9581.027
27	428.000	527.812	-30875.205
28	376.000	490.329	108198.149
29	357.000	450.462	-371030.403
30	301.000	414.811	1279524.278
31	274.000	374.513	-4405939.512
32	271.000	340.500	15177228.653

Notes : (i) Values of coefficients:

$$1/C_0 = 13.22, \quad -(C_1/C_0) = -3.44 \quad \text{and} \quad -(C_2/C_0) = -8.78$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.20000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	273.259	314.000
2	355.000	287.589	355.000
3	404.000	311.649	404.000
4	495.000	344.168	495.000
5	566.000	398.717	566.003
6	586.000	460.303	586.064
7	572.000	507.117	573.345
8	575.000	531.092	603.238
9	572.000	547.410	1164.996
10	571.000	556.536	13023.918
11	676.000	559.948	262187.288
12	1026.000	596.449	5492763.039
13	1156.000	753.135	115327633.830
14	1081.000	903.733	2421857115.500
15	1001.000	970.869	50858977725.000
16	816.000	985.4541068038512000.000	
17	681.000	925.19422428808737000.000	
18	568.000	836.844471004983460000.000	
19	538.000	737.8289891104652900000.000	
20	534.000	663.89220771319771000000.000	
21	535.000	615.765436197715200000000.000	
22	551.000	585.5569160152019400000000.000	
23	555.000	572.68319236319241000000000.000	
24	549.000	566.245403962704070000000000.000	
25	544.000	559.9518483216785500000000000.000	
26	493.000	554.98717814755250000000000000.000	
27	428.000	533.233374109860250000000000000.000	
28	376.000	495.2217856307065400000000000000.000	
29	357.000	451.41716498244837000000000000000.000	
30	301.000	417.485346463141590000000000000000.000	
31	274.000	374.8427275725973500000000000000000.000	
32	271.000	337.54915279024545000000000000000000.000	

Notes : (i) Values of coefficients:

$$1/C_0 = -54.00, \quad -(C_1/C_0) = 21.00 \quad \text{and} \quad -(C_2/C_0) = 34.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.30000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	268.639	314.000
2	355.000	281.850	355.000
3	404.000	305.448	404.000
4	495.000	333.892	495.000
5	566.000	390.813	566.000
6	586.000	460.375	586.000
7	572.000	514.055	572.000
8	575.000	537.548	575.000
9	572.000	553.394	572.000
10	571.000	561.201	571.000
11	676.000	551.169	675.998
12	1026.000	555.739	1025.992
13	1156.000	732.238	1155.967
14	1081.000	917.037	1080.865
15	1001.000	995.372	1000.449
16	816.000	1022.487	813.753
17	681.000	955.430	671.841
18	568.000	857.408	530.659
19	538.000	742.065	385.766
20	534.000	658.462	-86.648
21	535.000	607.004	-1995.335
22	551.000	575.167	-9764.982
23	555.000	564.665	-41502.464
24	549.000	561.484	-170916.046
25	544.000	557.006	-698505.804
26	493.000	558.478	-2849479.276
27	428.000	540.188	-11618689.741
28	376.000	500.894	-47369873.254
29	357.000	451.938	-193124505.340
30	301.000	420.293	-787354907.010
31	274.000	374.719	-3209986343.300
32	271.000	333.587	-13086868246.000

Notes : (i) Values of coefficients:

$$1/C_0 = -7.46, \quad -(C_1/C_0) = 4.08 \quad \text{and} \quad -(C_2/C_0) = 4.38$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

Number of data = 33
K = 66.00 hours
T = 24.00 hours
x = 0.40000

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	262.837	314.000
2	355.000	275.192	355.000
3	404.000	298.638	404.000
4	495.000	322.248	495.000
5	566.000	382.784	566.000
6	586.000	462.419	586.000
7	572.000	523.806	572.000
8	575.000	545.384	575.000
9	572.000	559.996	572.000
10	571.000	565.858	571.000
11	676.000	538.948	676.000
12	1026.000	505.018	1026.000
13	1156.000	711.056	1156.000
14	1081.000	938.937	1081.000
15	1001.000	1027.338	1001.000
16	816.000	1066.716	816.000
17	681.000	987.778	680.999
18	568.000	876.626	567.998
19	538.000	741.451	537.995
20	534.000	647.939	533.987
21	535.000	594.665	534.966
22	551.000	562.449	550.909
23	555.000	556.007	554.758
24	549.000	557.213	548.356
25	544.000	554.789	542.281
26	493.000	564.003	488.417
27	428.000	549.118	415.779
28	376.000	507.296	343.410
29	357.000	451.530	270.093
30	301.000	423.191	69.248
31	274.000	373.893	-344.005
32	271.000	328.268	-1377.013

Notes : (i) Values of coefficients:

$$1/C_0 = -3.58,\ -(C_1/C_0) = 2.67 \quad \text{and} \quad -(C_2/C_0) = 1.92$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.45000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	259.342	314.000
2	355.000	271.476	355.000
3	404.000	295.022	404.000
4	495.000	315.825	495.000
5	566.000	378.837	566.000
6	586.000	464.508	586.000
7	572.000	530.007	572.000
8	575.000	549.774	575.000
9	572.000	563.408	572.000
10	571.000	568.044	571.000
11	676.000	531.034	676.000
12	1026.000	474.806	1026.000
13	1156.000	701.052	1156.000
14	1081.000	954.597	1081.000
15	1001.000	1046.723	1001.000
16	816.000	1091.798	816.000
17	681.000	1004.228	681.001
18	568.000	885.028	568.002
19	538.000	738.492	538.006
20	534.000	640.335	534.013
21	535.000	587.131	535.031
22	551.000	555.364	551.074
23	555.000	551.730	555.174
24	549.000	555.553	549.410
25	544.000	554.129	544.967
26	493.000	567.786	495.278
27	428.000	554.445	433.367
28	376.000	510.671	388.645
29	357.000	450.716	386.790
30	301.000	424.671	371.184
31	274.000	373.114	439.348
32	271.000	324.964	660.548

Notes : (i) Values of coefficients:

$$1/C_0 = -2.73, \quad -(C_1/C_0) = 2.36 \quad \text{and} \quad -(C_2/C_0) = 1.37$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.50000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	255.333	314.000
2	355.000	267.489	355.000
3	404.000	291.295	404.000
4	495.000	308.938	495.000
5	566.000	375.038	566.000
6	586.000	467.551	586.000
7	572.000	537.257	572.000
8	575.000	554.387	575.000
9	572.000	566.780	572.000
10	571.000	570.031	571.000
11	676.000	521.548	676.000
12	1026.000	440.589	1026.000
13	1156.000	692.142	1156.000
14	1081.000	974.533	1081.000
15	1001.000	1068.649	1001.000
16	816.000	1118.903	816.000
17	681.000	1020.355	681.000
18	568.000	892.099	567.999
19	538.000	733.246	537.999
20	534.000	630.982	533.997
21	535.000	578.791	534.994
22	551.000	547.969	550.988
23	555.000	547.719	554.974
24	549.000	554.402	548.944
25	544.000	553.854	543.880
26	493.000	572.399	492.743
27	428.000	560.386	427.449
28	376.000	514.047	374.820
29	357.000	449.289	354.471
30	301.000	426.201	295.581
31	274.000	372.027	262.387
32	271.000	321.146	246.116

Notes : (i) Values of coefficients:

$$1/C_0 = -2.14, \quad -(C_1/C_0) = 2.14 \quad \text{and} \quad -(C_2/C_0) = 1.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

4.2.1 Further Computations Using Parameter x = 0.0 with Various Δt

Tables IV.2.1 show that only x = 0.0 gives a satisfactory result, where all of the calculated upstream discharges agree exactly with the observed ones. The computation used only one set of parameter values, i.e.: K = 66 hours and Δt = 24 hours. In order to be able to verify whether or not the parameter x = 0.0 always gives satisfactory results, it is necessary that further computations be carried out. For this purpose, various time steps Δt are used besides Δt = 24 hours. They are respectively 3, 6, 12, 24, 36 and 48 hours.

Firstly, the observed upstream hydrograph ordinates were interpolated using linear interpolation according to the time step Δt used. Secondly, conventional downstream routing was applied to obtain calculated downstream hydrograph ordinates. These hydrographs were then used to compute back the upstream ones applying upstream routing [eq.(4.1.1)].

Tables IV.2.2 show the results. It can be noticed that all of the calculated upstream discharges agree exactly with the observed ones, no matter what the time step Δt is used.

Tables IV.2.2

Results of Computations Using Parameter x = 0, Various Time Step Δt and K = 66 hours

Number of data = 25:
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	279.000	274.111	279.000
2	284.000	274.440	284.000
3	289.000	274.976	289.000
4	294.000	275.710	294.000
5	299.000	276.634	299.000
6	304.000	277.739	304.000
7	309.000	279.017	309.000
8	314.000	280.461	314.000
9	319.125	282.066	319.125
10	324.250	283.827	324.250
11	329.375	285.737	329.375
12	334.500	287.790	334.500
13	339.625	289.980	339.625
14	344.750	292.301	344.750
15	349.875	294.746	349.875
16	355.000	297.310	355.000
17	361.125	300.010	361.125
18	367.250	302.862	367.250
19	373.375	305.860	373.375
20	379.500	308.997	379.500
21	385.625	312.266	385.625
22	391.750	315.663	391.750
23	397.875	319.181	397.875
24	404.000	322.814	404.000
25	415.375	326.675	415.375
26	426.750	330.870	426.750
27	438.125	335.384	438.125
28	449.500	340.203	449.500
29	460.875	345.314	460.875
30	472.250	350.703	472.250
31	483.625	356.357	483.625
32	495.000	362.267	495.000
33	503.875	368.363	503.875

Notes : (i) Values of coefficients:

$$1/C_0 = 45.00, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -43.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 257
K               = 66.00 hours
T               = 3.00 hours
x               = 0.00000
```

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
34	512.750	374.583	512.750
35	521.625	380.921	521.625
36	530.500	387.372	530.500
37	539.375	393.930	539.375
38	548.250	400.592	548.250
39	557.125	407.351	557.125
40	566.000	414.205	566.000
41	568.500	421.007	568.500
42	571.000	427.618	571.000
43	573.500	434.046	573.500
44	576.000	440.300	576.000
45	578.500	446.386	578.500
46	581.000	452.314	581.000
47	583.500	458.089	583.500
48	586.000	463.718	586.000
49	584.250	469.114	584.250
50	582.500	474.192	582.500
51	580.750	478.967	580.750
52	579.000	483.452	579.000
53	577.250	487.659	577.250
54	575.500	491.602	575.500
55	573.750	495.292	573.750
56	572.000	498.740	572.000
57	572.375	502.005	572.375
58	572.750	505.141	572.750
59	573.125	508.154	573.125
60	573.500	511.050	573.500
61	573.875	513.834	573.875
62	574.250	516.510	574.250
63	574.625	519.085	574.625
64	575.000	521.562	575.000
65	574.625	523.928	574.625
66	574.250	526.173	574.250
67	573.875	528.302	573.875

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
68	573.500	530.319	573.500
69	573.125	532.230	573.125
70	572.750	534.039	572.750
71	572.375	535.751	572.375
72	572.000	537.370	572.000
73	571.875	538.907	571.875
74	571.750	540.369	571.750
75	571.625	541.761	571.625
76	571.500	543.086	571.500
77	571.375	544.346	571.375
78	571.250	545.544	571.250
79	571.125	546.684	571.125
80	571.000	547.767	571.000
81	584.125	549.092	584.125
82	597.250	550.940	597.250
83	610.375	553.290	610.375
84	623.500	556.119	623.500
85	636.625	559.405	636.625
86	649.750	563.129	649.750
87	662.875	567.271	662.875
88	676.000	571.811	676.000
89	719.750	577.414	719.750
90	763.500	584.712	763.500
91	807.250	593.631	807.250
92	851.000	604.097	851.000
93	894.750	616.043	894.750
94	938.500	629.402	938.500
95	982.250	644.112	982.250
96	1026.000	660.113	1026.000
97	1042.250	676.735	1042.250
98	1058.500	693.342	1058.500
99	1074.750	709.932	1074.750
100	1091.000	726.507	1091.000
101	1107.250	743.068	1107.250

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
102	1123.500	759.615	1123.500
103	1139.750	776.149	1139.750
104	1156.000	792.670	1156.000
105	1146.625	808.610	1146.625
106	1137.250	823.424	1137.250
107	1127.875	837.164	1127.875
108	1118.500	849.876	1118.500
109	1109.125	861.606	1109.125
110	1099.750	872.399	1099.750
111	1090.375	882.295	1090.375
112	1081.000	891.335	1081.000
113	1071.000	899.542	1071.000
114	1061.000	906.940	1061.000
115	1051.000	913.565	1051.000
116	1041.000	919.451	1041.000
117	1031.000	924.631	1031.000
118	1021.000	929.136	1021.000
119	1011.000	932.997	1011.000
120	1001.000	936.241	1001.000
121	977.875	938.606	977.875
122	954.750	939.837	954.750
123	931.625	939.986	931.625
124	908.500	939.101	908.500
125	885.375	937.227	885.375
126	862.250	934.408	862.250
127	839.125	930.687	839.125
128	816.000	926.104	816.000
129	799.125	920.835	799.125
130	782.250	915.051	782.250
131	765.375	908.774	765.375
132	748.500	902.026	748.500
133	731.625	894.827	731.625
134	714.750	887.199	714.750
135	697.875	879.159	697.875

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
136	681.000	870.727	681.000
137	666.875	861.981	666.875
138	652.750	852.996	652.750
139	638.625	843.782	638.625
140	624.500	834.350	624.500
141	610.375	824.710	610.375
142	596.250	814.870	596.250
143	582.125	804.839	582.125
144	568.000	794.627	568.000
145	564.250	784.471	564.250
146	560.500	774.600	560.500
147	556.750	765.002	556.750
148	553.000	755.663	553.000
149	549.250	746.572	549.250
150	545.500	737.719	545.500
151	541.750	729.092	541.750
152	538.000	720.683	538.000
153	537.500	712.552	537.500
154	537.000	704.761	537.000
155	536.500	697.294	536.500
156	536.000	690.137	536.000
157	535.500	683.275	535.500
158	535.000	676.696	535.000
159	534.500	670.387	534.500
160	534.000	664.337	534.000
161	534.125	658.547	534.125
162	534.250	653.020	534.250
163	534.375	647.744	534.375
164	534.500	642.708	534.500
165	534.625	637.902	534.625
166	534.750	633.314	534.750
167	534.875	628.936	534.875
168	535.000	624.759	535.000
169	537.000	620.814	537.000

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
170	539.000	617.133	539.000
171	541.000	613.705	541.000
172	543.000	610.518	543.000
173	545.000	607.562	545.000
174	547.000	604.826	547.000
175	549.000	602.300	549.000
176	551.000	599.976	551.000
177	551.500	597.810	551.500
178	552.000	595.763	552.000
179	552.500	593.829	552.500
180	553.000	592.003	553.000
181	553.500	590.281	553.500
182	554.000	588.657	554.000
183	554.500	587.128	554.500
184	555.000	585.689	555.000
185	554.250	584.309	554.250
186	553.500	582.956	553.500
187	552.750	581.630	552.750
188	552.000	580.330	552.000
189	551.250	579.054	551.250
190	550.500	577.802	550.500
191	549.750	576.572	549.750
192	549.000	575.363	549.000
193	548.375	574.177	548.375
194	547.750	573.017	547.750
195	547.125	571.880	547.125
196	546.500	570.766	546.500
197	545.875	569.673	545.875
198	545.250	568.602	545.250
199	544.625	567.550	544.625
200	544.000	566.517	544.000
201	537.625	565.375	537.625
202	531.250	564.000	531.250
203	524.875	562.403	524.875

```
Number of data   = 257
K                = 66.00 hours
T                = 3.00 hours
x                = 0.00000
```

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
204	518.500	560.593	518.500
205	512.125	558.581	512.125
206	505.750	556.374	505.750
207	499.375	553.983	499.375
208	493.000	551.414	493.000
209	484.875	548.637	484.875
210	476.750	545.623	476.750
211	468.625	542.381	468.625
212	460.500	538.923	460.500
213	452.375	535.257	452.375
214	444.250	531.392	444.250
215	436.125	527.339	436.125
216	428.000	523.104	428.000
217	421.500	518.733	421.500
218	415.000	514.267	415.000
219	408.500	509.711	408.500
220	402.000	505.068	402.000
221	395.500	500.343	395.500
222	389.000	495.539	389.000
223	382.500	490.659	382.500
224	376.000	485.708	376.000
225	373.625	480.779	373.625
226	371.250	475.964	371.250
227	368.875	471.257	368.875
228	366.500	466.654	366.500
229	364.125	462.150	364.125
230	361.750	457.741	361.750
231	359.375	453.421	359.375
232	357.000	449.189	357.000
233	350.000	444.936	350.000
234	343.000	440.561	343.000
235	336.000	436.069	336.000
236	329.000	431.466	329.000
237	322.000	426.757	322.000

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.00000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
238	315.000	421.945	315.000
239	308.000	417.037	308.000
240	301.000	412.035	301.000
241	297.625	407.025	297.625
242	294.250	402.088	294.250
243	290.875	397.220	290.875
244	287.500	392.419	287.500
245	284.125	387.681	284.125
246	280.750	383.003	280.750
247	277.375	378.384	277.375
248	274.000	373.819	274.000
249	273.625	369.375	273.625
250	273.250	365.111	273.250
251	272.875	361.020	272.875
252	272.500	357.094	272.500
253	272.125	353.326	272.125
254	271.750	349.708	271.750
255	271.375	346.235	271.375
256	271.000	342.900	271.000

```
Number of data   = 129
K                = 66.00 hours
T                = 6.00 hours
x                = 0.00000
```

PERIOD (x 6.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	284.000	274.435	284.000
2	294.000	275.701	294.000
3	304.000	277.727	304.000
4	314.000	280.447	314.000
5	324.250	283.810	324.250
6	334.500	287.772	334.500
7	344.750	292.281	344.750
8	355.000	297.289	355.000
9	367.250	302.840	367.250
10	379.500	308.974	379.500
11	391.750	315.639	391.750
12	404.000	322.790	404.000
13	426.750	330.841	426.750
14	449.500	340.170	449.500
15	472.250	350.666	472.250
16	495.000	362.228	495.000
17	512.750	374.545	512.750
18	530.500	387.334	530.500
19	548.250	400.555	548.250
20	566.000	414.170	566.000
21	571.000	427.590	571.000
22	576.000	440.278	576.000
23	581.000	452.297	581.000
24	586.000	463.706	586.000
25	582.500	474.188	582.500
26	579.000	483.454	579.000
27	575.500	491.611	575.500
28	572.000	498.753	572.000
29	572.750	505.155	572.750
30	573.500	511.065	573.500
31	574.250	516.527	574.250
32	575.000	521.579	575.000
33	574.250	526.192	574.250

Notes : (i) Values of coefficients:

$$1/C_0 = 23.00, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -21.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data   = 129
K                = 66.00 hours
T                = 6.00 hours
x                = 0.00000
```

PERIOD (x 6.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
34	573.500	530.338	573.500
35	572.750	534.059	572.750
36	572.000	537.391	572.000
37	571.750	540.389	571.750
38	571.500	543.105	571.500
39	571.250	545.564	571.250
40	571.000	547.786	571.000
41	597.250	550.946	597.250
42	623.500	556.114	623.500
43	649.750	563.115	649.750
44	676.000	571.790	676.000
45	763.500	584.656	763.500
46	851.000	604.012	851.000
47	938.500	629.293	938.500
48	1026.000	659.985	1026.000
49	1058.500	693.226	1058.500
50	1091.000	726.402	1091.000
51	1123.500	759.519	1123.500
52	1156.000	792.583	1156.000
53	1137.250	823.369	1137.250
54	1118.500	849.848	1118.500
55	1099.750	872.393	1099.750
56	1081.000	891.348	1081.000
57	1061.000	906.970	1061.000
58	1041.000	919.495	1041.000
59	1021.000	929.191	1021.000
60	1001.000	936.305	1001.000
61	954.750	939.919	954.750
62	908.500	939.198	908.500
63	862.250	934.518	862.250
64	816.000	926.223	816.000
65	782.250	915.171	782.250
66	748.500	902.145	748.500
67	714.750	887.317	714.750

Number of data = 129
K = 66.00 hours
T = 6.00 hours
x = 0.00000

PERIOD (x 6.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
68	681.000	870.844	681.000
69	652.750	853.108	652.750
70	624.500	834.457	624.500
71	596.250	814.972	596.250
72	568.000	794.724	568.000
73	560.500	774.683	560.500
74	553.000	755.732	553.000
75	545.500	737.777	545.500
76	538.000	720.731	538.000
77	537.000	704.798	537.000
78	536.000	690.164	536.000
79	535.000	676.715	535.000
80	534.000	664.348	534.000
81	534.250	653.024	534.250
82	534.500	642.707	534.500
83	534.750	633.309	534.750
84	535.000	624.749	535.000
85	539.000	617.119	539.000
86	543.000	610.500	543.000
87	547.000	604.804	547.000
88	551.000	599.952	551.000
89	552.000	595.738	552.000
90	553.000	591.979	553.000
91	554.000	588.633	554.000
92	555.000	585.665	555.000
93	553.500	582.933	553.500
94	552.000	580.308	552.000
95	550.500	577.781	550.500
96	549.000	575.344	549.000
97	547.750	572.999	547.750
98	546.500	570.749	546.500
99	545.250	568.586	545.250
100	544.000	566.502	544.000
101	531.250	563.991	531.250

```
Number of data   = 129
K                = 66.00 hours
T                = 6.00 hours
x                = 0.00000
```

PERIOD (x 6.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
102	518.500	560.590	518.500
103	505.750	556.376	505.750
104	493.000	551.419	493.000
105	476.750	545.633	476.750
106	460.500	538.936	460.500
107	444.250	531.409	444.250
108	428.000	523.124	428.000
109	415.000	514.287	415.000
110	402.000	505.088	402.000
111	389.000	495.559	389.000
112	376.000	485.727	376.000
113	371.250	475.979	371.250
114	366.500	466.666	366.500
115	361.750	457.749	361.750
116	357.000	449.195	357.000
117	343.000	440.569	343.000
118	329.000	431.476	329.000
119	315.000	421.957	315.000
120	301.000	412.047	301.000
121	294.250	402.098	294.250
122	287.500	392.426	287.500
123	280.750	383.009	280.750
124	274.000	373.823	274.000
125	273.250	365.110	273.250
126	272.500	357.090	272.500
127	271.750	349.702	271.750
128	271.000	342.891	271.000

```
Number of data  =  65
K               = 66.00 hours
T               = 12.00 hours
x               = 0.00000
```

PERIOD (x 12.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	294.000	275.667	294.000
2	314.000	280.389	314.000
3	334.500	287.699	334.500
4	355.000	297.208	355.000
5	379.500	308.881	379.500
6	404.000	322.693	404.000
7	449.500	340.036	449.500
8	495.000	362.071	495.000
9	530.500	387.184	530.500
10	566.000	414.029	566.000
11	576.000	440.191	576.000
12	586.000	463.659	586.000
13	579.000	483.466	579.000
14	572.000	498.805	572.000
15	573.500	511.129	573.500
16	575.000	521.649	575.000
17	573.500	530.416	573.500
18	572.000	537.472	572.000
19	571.500	543.185	571.500
20	571.000	547.862	571.000
21	623.500	556.094	623.500
22	676.000	571.703	676.000
23	851.000	603.669	851.000
24	1026.000	659.474	1026.000
25	1091.000	725.979	1091.000
26	1156.000	792.232	1156.000
27	1118.500	849.735	1118.500
28	1081.000	891.404	1081.000
29	1041.000	919.670	1041.000
30	1001.000	936.559	1001.000
31	908.500	939.590	908.500
32	816.000	926.700	816.000
33	748.500	902.625	748.500

Notes : (i) Values of coefficients:

$$1/C_0 = 12.00, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -10.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

Number of data = 65
K = 66.00 hours
T = 12.00 hours
x = 0.00000

PERIOD (x 12.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
34	681.000	871.313	681.000
35	624.500	834.886	624.500
36	568.000	795.113	568.000
37	553.000	756.011	553.000
38	538.000	720.926	538.000
39	536.000	690.271	536.000
40	534.000	664.393	534.000
41	534.500	642.702	534.500
42	535.000	624.710	535.000
43	543.000	610.425	543.000
44	551.000	599.854	551.000
45	553.000	591.879	553.000
46	555.000	585.566	555.000
47	552.000	580.221	552.000
48	549.000	575.268	549.000
49	546.500	570.681	546.500
50	544.000	566.443	544.000
51	518.500	560.577	518.500
52	493.000	551.439	493.000
53	460.500	538.991	460.500
54	428.000	523.201	428.000
55	402.000	505.168	402.000
56	376.000	485.806	376.000
57	366.500	466.714	366.500
58	357.000	449.220	357.000
59	329.000	431.516	329.000
60	301.000	412.097	301.000
61	287.500	392.456	287.500
62	274.000	373.838	274.000
63	272.500	357.073	272.500
64	271.000	342.853	271.000

```
Number of data  =  33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.00000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	280.154	314.000
2	355.000	296.876	355.000
3	404.000	322.299	404.000
4	495.000	361.437	495.000
5	566.000	413.457	566.000
6	586.000	463.470	586.000
7	572.000	499.018	572.000
8	575.000	521.935	575.000
9	572.000	537.801	572.000
10	571.000	548.170	571.000
11	676.000	571.349	676.000
12	1026.000	657.395	1026.000
13	1156.000	790.812	1156.000
14	1081.000	891.639	1081.000
15	1001.000	937.596	1001.000
16	816.000	928.644	816.000
17	681.000	873.215	681.000
18	568.000	796.687	568.000
19	538.000	721.707	538.000
20	534.000	664.566	534.000
21	535.000	624.546	535.000
22	551.000	599.455	551.000
23	555.000	585.161	555.000
24	549.000	574.958	549.000
25	544.000	566.201	544.000
26	493.000	551.524	493.000
27	428.000	523.517	428.000
28	376.000	486.127	376.000
29	357.000	449.319	357.000
30	301.000	412.298	301.000
31	274.000	373.898	274.000
32	271.000	342.699	271.000

Notes : (i) Values of coefficients:

$$1/C_0 = 6.50, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -4.50$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data   =  22
K                = 66.00 hours
T                = 36.00 hours
x                = 0.00000
```

PERIOD (x 36.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	334.500	286.964	334.500
2	404.000	322.230	404.000
3	530.500	384.381	530.500
4	586.000	458.896	586.000
5	573.500	510.691	573.500
6	572.000	537.288	572.000
7	623.500	563.200	623.500
8	1026.000	675.293	1026.000
9	1118.500	845.417	1118.500
10	1001.000	937.274	1001.000
11	748.500	910.478	748.500
12	568.000	802.380	568.000
13	536.000	695.074	536.000
14	535.000	626.685	535.000
15	553.000	591.249	553.000
16	549.000	573.999	549.000
17	518.500	556.750	518.500
18	428.000	520.964	428.000
19	366.500	467.944	366.500
20	301.000	410.432	301.000
21	272.500	357.426	272.500

Notes : (i) Values of coefficients:

$$1/C_0 = 4.67, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -2.67$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 17
K               = 66.00 hours
T               = 48.00 hours
x               = 0.00000
```

PERIOD (x 48.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	355.000	295.600	355.000
2	495.000	364.613	495.000
3	586.000	458.420	586.000
4	575.000	523.529	575.000
5	571.000	549.914	571.000
6	1026.000	682.493	1026.000
7	1081.000	880.363	1081.000
8	816.000	916.703	816.000
9	568.000	796.861	568.000
10	534.000	665.735	534.000
11	551.000	600.010	551.000
12	549.000	573.338	549.000
13	493.000	545.424	493.000
14	376.000	486.265	376.000
15	301.000	407.457	301.000
16	271.000	342.680	271.000

Notes : (i) Values of coefficients:

$$1/C_0 = 3.75, \quad -(C_1/C_0) = -1.00 \quad \text{and} \quad -(C_2/C_0) = -1.75$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

4.2.2 Further Computations Using Parameter x = 0.45 with Various Δt

Since the parameter x value derived from the September-October 1960 flood in the reach of the Murray River from which the data were taken is 0.45, this value was adopted to investigate the effect of Δt in the numerical computation. The time steps Δt used were chosen to cover a wide range. They are, respectively, 3, 6, 12, 24, 36, 48, 60, 72 and 96 hours while the parameter K value is 66 hours. It should be noted that these time steps Δt of which some of them are larger than the K value are only for the use of numerical investigation. In practice, the time step Δt used would always be less than or equal to the K value.

The method of computation is the same as that discussed in the previous section. After interpolating the data according to the time step Δt, conventional downstream routing was applied to obtain calculated downstream hydrograph ordinates. Afterwards, this result was used to compute back the upstream hydrograph ordinates.

Tables IV.2.3 show the results. It can be noticed from these tables that the only time step Δt giving calculated upstream discharges which agree precisely with the observed ones is $\Delta t = 96$ hours. With this large time step, the number of data points becomes very few. The time step $\Delta t = 12$ hours apparently gives a satisfactory result, but actually it does not since the last few calculated upstream discharges do not match the observed ones. The time step $\Delta t = 72$ hours gives fairly good result, but again the last few calculated upstream discharges do not match precisely the observed ones. If the number of data points were more than that shown in the table, the differences would propagate and magnify. The worst result is given by time step $\Delta t = 60$ hours, even though the number of data is very few, the computation diverges very rapidly. The cause of these results will be discussed later in this chapter.

Tables IV.2.3

Results of Computations Using Parameter x = 0.45, Various Time Step Δt and K = 66 hours

```
Number of data  = 257
K               = 66.00 hours
T               = 3.00 hours
x               = 0.45000
```

PERIOD (x 3.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	279.000	270.270	279.000
2	284.000	267.233	284.000
3	289.000	264.833	289.000
4	294.000	263.021	294.000
5	299.000	261.749	299.000
6	304.000	260.976	304.000
7	309.000	260.660	309.000
8	314.000	260.767	314.000
9	319.125	261.168	319.125
10	324.250	261.944	324.250
11	329.375	263.066	329.375
12	334.500	264.505	334.500
13	339.625	266.237	339.625
14	344.750	268.238	344.750
15	349.875	270.487	349.875
16	355.000	272.964	355.000
17	361.125	274.905	361.125
18	367.250	277.179	367.250
19	373.375	279.758	373.375
20	379.500	282.618	379.500
21	385.625	285.738	385.625
22	391.750	289.096	391.750
23	397.875	292.674	397.875
24	404.000	296.454	404.000
25	415.375	296.503	415.375
26	426.750	297.451	426.750
27	438.125	299.227	438.125
28	449.500	301.764	449.500
29	460.875	305.003	460.875
30	472.250	308.888	472.250
31	483.625	313.367	483.625
32	495.000	318.393	495.000
33	503.875	325.789	503.875

Notes : (i) Values of coefficients:

$$1/C_0 = -1.34, \quad -(C_1/C_0) = 1.11 \quad \text{and} \quad -(C_2/C_0) = 1.23$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data   = 257
K                = 66.00 hours
T                = 3.00 hours
x                = 0.45000
```

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
34	512.750	333.302	512.750
35	521.625	340.923	521.625
36	530.500	348.643	530.500
37	539.375	356.455	539.375
38	548.250	364.351	548.250
39	557.125	372.326	557.125
40	566.000	380.371	566.000
41	568.500	393.239	568.500
42	571.000	405.283	571.000
43	573.500	416.570	573.500
44	576.000	427.160	576.000
45	578.500	437.107	578.500
46	581.000	446.464	581.000
47	583.500	455.276	583.500
48	586.000	463.588	586.000
49	584.250	474.609	584.250
50	582.500	484.616	582.500
51	580.750	493.690	580.750
52	579.000	501.905	579.000
53	577.250	509.329	577.250
54	575.500	516.025	575.500
55	573.750	522.051	573.750
56	572.000	527.460	572.000
57	572.375	530.715	572.375
58	572.750	533.742	572.750
59	573.125	536.558	573.125
60	573.500	539.180	573.500
61	573.875	541.624	573.875
62	574.250	543.904	574.250
63	574.625	546.033	574.625
64	575.000	548.022	575.000
65	574.625	550.443	574.625
66	574.250	552.642	574.250
67	573.875	554.637	573.875

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.45000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
68	573.500	556.443	573.500
69	573.125	558.077	573.125
70	572.750	559.551	572.750
71	572.375	560.878	572.375
72	572.000	562.070	572.000
73	571.875	562.952	571.875
74	571.750	563.753	571.750
75	571.625	564.481	571.625
76	571.500	565.141	571.500
77	571.375	565.739	571.375
78	571.250	566.280	571.250
79	571.125	566.767	571.125
80	571.000	567.207	571.000
81	584.125	557.716	584.125
82	597.250	550.020	597.250
83	610.375	543.977	610.375
84	623.500	539.455	623.500
85	636.625	536.334	636.625
86	649.750	534.502	649.750
87	662.875	533.857	662.875
88	676.000	534.304	676.000
89	719.750	512.911	719.750
90	763.500	496.688	763.500
91	807.250	485.225	807.250
92	851.000	478.143	851.000
93	894.750	475.096	894.750
94	938.500	475.763	938.500
95	982.250	479.850	982.250
96	1026.000	487.084	1026.000
97	1042.250	517.732	1042.250
98	1058.500	547.237	1058.500
99	1074.750	575.691	1074.750
100	1091.000	603.176	1091.000
101	1107.250	629.769	1107.250

```
Number of data  = 257
K               = 66.00 hours
T               = 3.00 hours
x               = 0.45000
```

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
102	1123.500	655.541	1123.500
103	1139.750	680.558	1139.750
104	1156.000	704.878	1156.000
105	1146.625	747.676	1146.625
106	1137.250	786.332	1137.250
107	1127.875	821.177	1127.875
108	1118.500	852.512	1118.500
109	1109.125	880.616	1109.125
110	1099.750	905.746	1099.750
111	1090.375	928.137	1090.375
112	1081.000	948.007	1081.000
113	1071.000	966.023	1071.000
114	1061.000	981.814	1061.000
115	1051.000	995.559	1051.000
116	1041.000	1007.420	1041.000
117	1031.000	1017.545	1031.000
118	1021.000	1026.073	1021.000
119	1011.000	1033.131	1011.000
120	1001.000	1038.835	1001.000
121	977.875	1053.084	977.875
122	954.750	1064.367	954.750
123	931.625	1072.919	931.624
124	908.500	1078.957	908.499
125	885.375	1082.681	885.374
126	862.250	1084.274	862.249
127	839.125	1083.905	839.124
128	816.000	1081.730	815.999
129	799.125	1073.230	799.124
130	782.250	1064.064	782.249
131	765.375	1054.288	765.374
132	748.500	1043.947	748.499
133	731.625	1033.088	731.624
134	714.750	1021.752	714.748
135	697.875	1009.976	697.873

```
Number of data   = 257
K                = 66.00 hours
T                = 3.00 hours
x                = 0.45000
```

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
136	681.000	997.795	680.998
137	666.875	983.191	666.873
138	652.750	968.624	652.749
139	638.625	954.092	638.622
140	624.500	939.593	624.497
141	610.375	925.123	610.372
142	596.250	910.681	596.247
143	582.125	896.264	582.121
144	568.000	881.870	567.996
145	564.250	859.757	564.245
146	560.500	839.102	560.495
147	556.750	819.788	556.744
148	553.000	801.710	552.994
149	549.250	784.768	549.243
150	545.500	768.874	545.492
151	541.750	753.944	541.741
152	538.000	739.901	537.990
153	537.500	724.250	537.489
154	537.000	709.801	536.988
155	536.500	696.460	536.487
156	536.000	684.138	535.986
157	535.500	672.754	535.484
158	535.000	662.234	534.982
159	534.500	652.509	534.481
160	534.000	643.516	533.979
161	534.125	634.731	534.101
162	534.250	626.653	534.224
163	534.375	619.226	534.346
164	534.500	612.399	534.468
165	534.625	606.123	534.589
166	534.750	600.355	534.711
167	534.875	595.055	534.831
168	535.000	590.186	534.952
169	537.000	584.314	536.947

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.45000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
170	539.000	579.067	538.941
171	541.000	574.395	540.935
172	543.000	570.252	542.928
173	545.000	566.597	544.920
174	547.000	563.391	546.912
175	549.000	560.598	548.902
176	551.000	558.186	550.892
177	551.500	557.242	551.380
178	552.000	556.414	551.868
179	552.500	555.690	552.354
180	553.000	555.064	552.838
181	553.500	554.527	553.321
182	554.000	554.073	553.802
183	554.500	553.694	554.281
184	555.000	553.385	554.757
185	554.250	554.073	553.981
186	553.500	554.646	553.203
187	552.750	555.115	552.421
188	552.000	555.487	551.636
189	551.250	555.769	550.848
190	550.500	555.970	550.055
191	549.750	556.096	549.257
192	549.000	556.152	548.455
193	548.375	556.050	547.772
194	547.750	555.907	547.083
195	547.125	555.726	546.387
196	546.500	555.510	545.684
197	545.875	555.261	544.972
198	545.250	554.982	544.251
199	544.625	554.676	543.519
200	544.000	554.345	542.777
201	537.625	558.280	536.271
202	531.250	561.396	529.752
203	524.875	563.760	523.218

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.45000

PERIOD (x 3.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
204	518.500	565.430	516.667
205	512.125	566.461	510.097
206	505.750	566.905	503.506
207	499.375	566.807	496.892
208	493.000	566.211	490.253
209	484.875	566.462	481.836
210	476.750	566.049	473.388
211	468.625	565.023	464.905
212	460.500	563.434	456.384
213	452.375	561.326	447.822
214	444.250	558.741	439.212
215	436.125	555.716	430.551
216	428.000	552.286	421.833
217	421.500	547.271	414.677
218	415.000	542.138	407.451
219	408.500	536.897	400.148
220	402.000	531.556	392.760
221	395.500	526.123	385.277
222	389.000	520.605	377.689
223	382.500	515.010	369.986
224	376.000	509.342	362.155
225	373.625	500.531	358.307
226	371.250	492.231	354.302
227	368.875	484.401	350.124
228	366.500	477.005	345.755
229	364.125	470.006	341.173
230	361.750	463.375	336.356
231	359.375	457.081	331.280
232	357.000	451.098	325.916
233	350.000	448.853	315.609
234	343.000	446.229	304.950
235	336.000	443.259	293.902
236	329.000	439.968	282.424
237	322.000	436.384	270.469

Number of data = 257
K = 66.00 hours
T = 3.00 hours
x = 0.45000

PERIOD (x 3.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
238	315.000	432.528	257.987
239	308.000	428.422	244.922
240	301.000	424.087	231.211
241	297.625	416.836	220.412
242	294.250	409.893	208.823
243	290.875	403.233	196.360
244	287.500	396.833	182.930
245	284.125	390.674	168.430
246	280.750	384.736	152.747
247	277.375	379.001	135.755
248	274.000	373.453	117.314
249	273.625	365.840	100.270
250	273.250	358.801	81.453
251	272.875	352.291	60.674
252	272.500	346.268	37.725
253	272.125	340.693	12.374
254	271.750	335.531	-15.634
255	271.375	330.749	-46.582
256	271.000	326.316	-80.782

```
Number of data   = 129
K                = 66.00 hours
T                = 6.00 hours
x                = 0.45000
```

PERIOD (x 6.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	284.000	267.206	284.000
2	294.000	262.976	294.000
3	304.000	260.919	304.000
4	314.000	260.702	314.000
5	324.250	261.875	324.250
6	334.500	264.435	334.500
7	344.750	268.168	344.750
8	355.000	272.896	355.000
9	367.250	277.108	367.250
10	379.500	282.548	379.500
11	391.750	289.027	391.750
12	404.000	296.388	404.000
13	426.750	297.361	426.750
14	449.500	301.659	449.500
15	472.250	308.774	472.250
16	495.000	318.276	495.000
17	512.750	333.198	512.750
18	530.500	348.551	530.500
19	548.250	364.270	548.250
20	566.000	380.300	566.000
21	571.000	405.254	571.000
22	576.000	427.162	576.000
23	581.000	446.488	581.000
24	586.000	463.627	586.000
25	582.500	484.688	582.500
26	579.000	501.999	579.000
27	575.500	516.133	575.500
28	572.000	527.574	572.000
29	572.750	533.847	572.750
30	573.500	539.277	573.500
31	574.250	543.992	574.250
32	575.000	548.102	575.000
33	574.250	552.718	574.250

Notes : (i) Values of coefficients:

$$1/C_0 = -1.47, \quad -(C_1/C_0) = 1.22 \quad \text{and} \quad -(C_2/C_0) = 1.25$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

Number of data = 129
K = 66.00 hours
T = 6.00 hours
x = 0.45000

PERIOD (x 6.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
34	573.500	556.515	573.500
35	572.750	559.618	572.750
36	572.000	562.132	572.000
37	571.750	563.809	571.750
38	571.500	565.191	571.500
39	571.250	566.324	571.250
40	571.000	567.246	571.000
41	597.250	549.985	597.250
42	623.500	539.367	623.500
43	649.750	534.378	649.750
44	676.000	534.158	676.000
45	763.500	496.367	763.500
46	851.000	477.704	851.000
47	938.500	475.249	938.500
48	1026.000	486.528	1026.000
49	1058.500	546.810	1058.500
50	1091.000	602.850	1091.000
51	1123.500	655.297	1123.500
52	1156.000	704.698	1156.000
53	1137.250	786.338	1137.250
54	1118.500	852.651	1118.500
55	1099.750	905.977	1099.750
56	1081.000	948.299	1081.000
57	1061.000	982.147	1061.000
58	1041.000	1007.773	1041.000
59	1021.000	1026.434	1021.000
60	1001.000	1039.192	1001.000
61	954.750	1064.783	954.750
62	908.500	1079.406	908.500
63	862.250	1084.735	862.250
64	816.000	1082.190	816.000
65	782.250	1064.479	782.250
66	748.500	1044.320	748.500
67	714.750	1022.086	714.750

Number of data = 129
K = 66.00 hours
T = 6.00 hours
x = 0.45000

PERIOD (x 6.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
68	681.000	998.094	681.000
69	652.750	968.876	652.751
70	624.500	939.805	624.501
71	596.250	910.859	596.251
72	568.000	882.020	568.001
73	560.500	839.174	560.501
74	553.000	801.723	553.002
75	545.500	768.846	545.502
76	538.000	739.843	538.002
77	537.000	709.706	537.003
78	536.000	684.018	536.004
79	535.000	662.100	535.004
80	534.000	643.374	534.005
81	534.250	626.506	534.257
82	534.500	612.251	534.508
83	534.750	600.211	534.760
84	535.000	590.047	535.012
85	539.000	578.926	539.015
86	543.000	570.112	543.018
87	547.000	563.256	547.023
88	551.000	558.056	551.028
89	552.000	556.300	552.034
90	553.000	554.964	553.041
91	554.000	553.985	554.051
92	555.000	553.308	555.062
93	553.500	554.585	553.576
94	552.000	555.438	552.093
95	550.500	555.933	550.614
96	549.000	556.122	549.140
97	547.750	555.884	547.921
98	546.500	555.492	546.710
99	545.250	554.968	545.507
100	544.000	554.334	544.315
101	531.250	561.418	531.635

```
Number of data   = 129
K                = 66.00 hours
T                = 6.00 hours
x                = 0.45000
```

PERIOD (x 6.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
102	518.500	565.475	518.972
103	505.750	566.965	506.328
104	493.000	566.281	493.708
105	476.750	566.134	477.617
106	460.500	563.527	461.562
107	444.250	558.838	445.550
108	428.000	552.384	429.593
109	415.000	542.226	416.951
110	402.000	531.634	404.389
111	389.000	520.675	391.926
112	376.000	509.404	379.583
113	371.250	492.264	375.639
114	366.500	477.016	371.875
115	361.750	463.370	368.333
116	357.000	451.083	365.062
117	343.000	446.230	352.874
118	329.000	439.981	341.092
119	315.000	432.549	329.810
120	301.000	424.114	319.138
121	294.250	409.904	316.464
122	287.500	396.833	314.705
123	280.750	384.727	314.069
124	274.000	373.438	314.806
125	273.250	358.766	323.226
126	272.500	346.220	333.707
127	271.750	335.475	346.711
128	271.000	326.255	362.806

Number of data = 65
K = 66.00 hours
T = 12.00 hours
x = 0.45000

PERIOD (x 12.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	294.000	262.794	294.000
2	314.000	260.441	314.000
3	334.500	264.149	334.500
4	355.000	272.621	355.000
5	379.500	282.264	379.500
6	404.000	296.122	404.000
7	449.500	301.233	449.500
8	495.000	317.801	495.000
9	530.500	348.180	530.500
10	566.000	380.012	566.000
11	576.000	427.172	576.000
12	586.000	463.790	586.000
13	579.000	502.381	579.000
14	572.000	528.039	572.000
15	573.500	539.670	573.500
16	575.000	548.427	575.000
17	573.500	556.806	573.500
18	572.000	562.382	572.000
19	571.500	565.391	571.500
20	571.000	567.404	571.000
21	623.500	539.009	623.500
22	676.000	533.563	676.000
23	851.000	475.921	851.000
24	1026.000	484.277	1026.000
25	1091.000	601.539	1091.000
26	1156.000	703.975	1156.000
27	1118.500	853.219	1118.500
28	1081.000	949.487	1081.000
29	1041.000	1009.207	1041.000
30	1001.000	1040.638	1001.000
31	908.500	1081.219	908.500
32	816.000	1084.047	816.000
33	748.500	1045.824	748.500

Notes : (i) Values of coefficients:

$$1/C_0 = -1.78,\ -(C_1/C_0) = 1.51 \quad \text{and} \quad -(C_2/C_0) = 1.28$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data   =  65
K                = 66.00 hours
T                = 12.00 hours
x                = 0.45000
```

PERIOD (x 12.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
34	681.000	999.296	681.000
35	624.500	940.655	624.500
36	568.000	882.622	568.000
37	553.000	801.772	553.000
38	538.000	739.602	538.001
39	536.000	683.531	536.001
40	534.000	642.799	534.001
41	534.500	611.654	534.502
42	535.000	589.486	535.003
43	543.000	569.547	543.004
44	551.000	557.533	551.006
45	553.000	554.559	553.009
46	555.000	552.996	555.013
47	552.000	555.246	552.020
48	549.000	556.006	549.030
49	546.500	555.419	546.546
50	544.000	554.290	544.069
51	518.500	565.658	518.604
52	493.000	566.567	493.156
53	460.500	563.906	460.735
54	428.000	552.780	428.355
55	402.000	531.949	402.534
56	376.000	509.651	376.805
57	366.500	477.059	367.712
58	357.000	451.017	358.826
59	329.000	440.034	331.750
60	301.000	424.223	305.142
61	287.500	396.830	293.740
62	274.000	373.378	283.399
63	272.500	346.026	286.658
64	271.000	326.008	292.327

Number of data = 33
K = 66.00 hours
T = 24.00 hours
x = 0.45000

PERIOD (x 24.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	259.342	314.000
2	355.000	271.476	355.000
3	404.000	295.022	404.000
4	495.000	315.825	495.000
5	566.000	378.837	566.000
6	586.000	464.508	586.000
7	572.000	530.007	572.000
8	575.000	549.774	575.000
9	572.000	563.408	572.000
10	571.000	568.044	571.000
11	676.000	531.034	676.000
12	1026.000	474.806	1026.000
13	1156.000	701.052	1156.000
14	1081.000	954.597	1081.000
15	1001.000	1046.723	1001.000
16	816.000	1091.798	816.000
17	681.000	1004.228	681.001
18	568.000	885.028	568.002
19	538.000	738.492	538.006
20	534.000	640.335	534.013
21	535.000	587.131	535.031
22	551.000	555.364	551.074
23	555.000	551.730	555.174
24	549.000	555.553	549.410
25	544.000	554.129	544.967
26	493.000	567.786	495.278
27	428.000	554.445	433.367
28	376.000	510.671	388.645
29	357.000	450.716	386.790
30	301.000	424.671	371.184
31	274.000	373.114	439.348
32	271.000	324.964	660.548

Notes : (i) Values of coefficients:

$$1/C_0 = -2.73, \quad -(C_1/C_0) = 2.36 \quad \text{and} \quad -(C_2/C_0) = 1.37$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 22
K               = 66.00 hours
T               = 36.00 hours
x               = 0.45000
```

PERIOD (x 36.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	334.500	260.964	334.500
2	404.000	294.742	404.000
3	530.500	339.921	530.500
4	586.000	454.313	586.000
5	573.500	544.313	573.500
6	572.000	563.987	572.000
7	623.500	558.203	623.500
8	1026.000	514.767	1026.000
9	1118.500	833.775	1118.500
10	1001.000	1047.861	1001.000
11	748.500	1071.199	748.498
12	568.000	896.147	567.992
13	536.000	685.486	535.968
14	535.000	586.595	534.870
15	553.000	548.510	552.470
16	549.000	552.349	546.839
17	518.500	556.700	509.690
18	428.000	550.874	392.083
19	366.500	482.662	220.069
20	301.000	419.762	-295.987
21	272.500	347.166	-2161.369

Notes : (i) Values of coefficients:

$$1/C_0 = -4.64, \quad -(C_1/C_0) = 4.08 \quad \text{and} \quad -(C_2/C_0) = 1.56$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  17
K               = 66.00 hours
T               = 48.00 hours
x               = 0.45000
```

PERIOD (x 48.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	355.000	266.343	355.000
2	495.000	323.682	495.000
3	586.000	451.453	586.000
4	575.000	559.595	575.000
5	571.000	572.236	571.000
6	1026.000	528.242	1026.000
7	1081.000	919.268	1081.001
8	816.000	1073.060	816.010
9	568.000	891.878	568.094
10	534.000	637.279	534.883
11	551.000	553.460	559.315
12	549.000	551.691	627.340
13	493.000	554.842	1231.046
14	376.000	516.674	7329.166
15	301.000	411.784	65807.144
16	271.000	326.434	617407.827

Notes : (i) Values of coefficients:

$$1/C_0 = -10.58, \quad -(C_1/C_0) = 9.42 \quad \text{and} \quad -(C_2/C_0) = 2.16$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  13
K               = 66.00 hours
T               = 60.00 hours
x               = 0.45000
```

PERIOD (x 60.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	379.500	379.500	23589.500
2	566.000	566.000	-4577193.999
3	573.500	573.500	910976463.340
4	571.000	571.000	-181284202050.000
5	1091.000	1091.000	36075556434000.000
6	1001.000	1001.000	-7179035729800000.000
7	624.500	624.500	1428628110100000000.000
8	534.000	534.000	-284296993900000000000.000
9	553.000	553.000	56575101783000000000000.000
10	544.000	544.000	-11258445254000000000000000.000
11	402.000	402.000	2240430605400000000000000000.000
12	301.000	301.000	-445845690450000000000000000000.000

Notes : (i) Values of coefficients:

$$1/C_0 = 221.00, \quad -(C_1/C_0) = -199.00 \quad \text{and} \quad -(C_2/C_0) = -21.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 11
K               = 66.00 hours
T               = 72.00 hours
x               = 0.45000
```

PERIOD (x 72.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	404.000	285.328	404.000
2	586.000	419.367	586.000
3	572.000	584.089	572.000
4	1026.000	611.610	1026.000
5	1001.000	1022.102	1001.000
6	568.000	963.357	568.000
7	535.000	566.765	534.999
8	549.000	536.352	549.011
9	428.000	538.404	427.882
10	301.000	417.392	302.232

Notes : (i) Values of coefficients:

$$1/C_0 = 11.48, \quad -(C_1/C_0) = -10.43 \quad \text{and} \quad -(C_2/C_0) = -0.05$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

Number of data = 9
K = 66.00 hours
T = 96.00 hours
x = 0.45000

PERIOD (x 96.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	495.000	321.975	495.000
2	575.000	536.381	575.000
3	1026.000	678.264	1026.000
4	816.000	1028.675	816.000
5	534.000	725.266	534.000
6	549.000	510.710	549.000
7	376.000	516.759	376.000
8	271.000	333.670	271.000

Notes : (i) Values of coefficients:

$1/C_0 = 4.61$, $-(C_1/C_0) = -4.25$ and $-(C_2/C_0) = 0.64$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

4.2.3 Further Computations with Various K Values

Computations using various parameter x values and time steps Δt have been implemented. In order that the problem of upstream routing discussed herein can be analysed more thoroughly, it is essential that the computation using various average travel time K values be considered as well.

The K values used in the computations are respectively 6, 12, 24, 33 and 66 hours with time step $\Delta t = 24$ hours and parameter $x = 0.45$.

Tables IV.2.4 show that all of the K values yield unsatisfactory results, except K = 6 hours. This K value almost gives a perfect result. However, differences still occur in the last few calculated values even though they are very small. These results therefore are consistent with the previous sections, where Δt and x were varied. The overall result is that upstream routing using equation (4.1.1) gives unsatisfactory results in almost all cases.

It should be noted that in the real case, the value of time step Δt used in the computation should be made less than or equal to the K value. In this section, the value of time step Δt used remains unchanged, i.e.: $\Delta t = 24$ hours, no matter what the K value is. This is for the use of numerical investigation only.

Tables IV.2.4

Results of Computations Using Parameter x = 0.45, Various K Values and Δt = 24 hours

Number of data = 33
K = 6.00 hours
T = 24.00 hours
x = 0.45000

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	298.314	314.000
2	355.000	347.841	355.000
3	404.000	388.855	404.000
4	495.000	467.926	495.000
5	566.000	553.552	566.000
6	586.000	585.235	586.000
7	572.000	577.925	572.000
8	575.000	570.454	575.000
9	572.000	575.761	572.000
10	571.000	569.253	571.000
11	676.000	635.817	676.000
12	1026.000	911.594	1026.000
13	1156.000	1170.074	1156.000
14	1081.000	1102.409	1081.000
15	1001.000	1020.199	1001.000
16	816.000	877.632	816.000
17	681.000	698.895	681.000
18	568.000	602.138	568.000
19	538.000	530.353	538.000
20	534.000	539.917	534.000
21	535.000	531.243	535.000
22	551.000	546.862	551.000
23	555.000	555.785	555.000
24	549.000	550.907	549.000
25	544.000	544.877	544.000
26	493.000	512.502	493.000
27	428.000	442.401	428.000
28	376.000	388.203	376.000
29	357.000	357.512	357.000
30	301.000	322.670	301.000
31	274.000	272.266	274.001
32	271.000	273.162	270.999

Notes : (i) Values of coefficients:

$$1/C_0 = 1.65, \quad -(C_1/C_0) = -1.58 \quad \text{and} \quad -(C_2/C_0) = 0.94$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               = 12.00 hours
T               = 24.00 hours
x               = 0.45000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	288.194	314.000
2	355.000	336.041	355.000
3	404.000	377.891	404.000
4	495.000	443.870	495.000
5	566.000	535.038	566.000
6	586.000	582.086	586.000
7	572.000	582.169	572.000
8	575.000	570.112	575.000
9	572.000	575.354	572.000
10	571.000	570.671	571.000
11	676.000	608.353	676.000
12	1026.000	819.833	1026.000
13	1156.000	1131.984	1156.000
14	1081.000	1136.359	1081.000
15	1001.000	1036.541	1001.000
16	816.000	925.037	816.000
17	681.000	736.441	681.000
18	568.000	624.807	568.001
19	538.000	540.862	537.998
20	534.000	535.750	534.005
21	535.000	533.847	534.986
22	551.000	541.012	551.038
23	555.000	555.319	554.900
24	549.000	552.778	549.265
25	544.000	546.129	543.302
26	493.000	525.285	494.841
27	428.000	460.562	423.146
28	376.000	400.095	388.797
29	357.000	362.263	323.263
30	301.000	335.601	389.943
31	274.000	281.374	39.514
32	271.000	270.795	889.190

Notes : (i) Values of coefficients:

$$1/C_0 = 2.82, \quad -(C_1/C_0) = -2.64 \quad \text{and} \quad -(C_2/C_0) = 0.82$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

Number of data = 33
K = 24.00 hours
T = 24.00 hours
x = 0.45000

PERIOD (x 24.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	275.905	314.000
2	355.000	314.138	355.000
3	404.000	355.388	404.000
4	495.000	406.018	495.000
5	566.000	494.144	566.000
6	586.000	563.531	586.000
7	572.000	584.263	572.002
8	575.000	572.727	574.960
9	572.000	574.749	572.759
10	571.000	572.083	556.585
11	676.000	576.052	949.884
12	1026.000	687.907	-4177.805
13	1156.000	1016.091	100028.292
14	1081.000	1145.766	-1877492.540
15	1001.000	1080.275	35693898.262
16	816.000	995.965	-678164231.970
17	681.000	818.141	12885136592.000
18	568.000	682.150	-244817581740.000
19	538.000	572.007	4651534064400.000
20	534.000	539.429	-88379147212000.000
21	535.000	534.306	1679203797000000.000
22	551.000	535.729	-31904872143000000.000
23	555.000	550.463	606192570710000000.000
24	549.000	554.498	-11517658843000000000.000
25	544.000	549.024	218835518020000000000.000
26	493.000	541.811	-4157874842400000000000.000
27	428.000	492.229	78999622004000000000000.000
28	376.000	428.582	-1500992818100000000000000.000
29	357.000	377.599	28518863543000000000000000.000
30	301.000	355.314	-541858407310000000000000000.000
31	274.000	302.301	10295309739000000000000000000.000
32	271.000	275.205	-195610885030000000000000000000.000

Notes : (i) Values of coefficients:

$$1/C_0 = 21.00, \quad -(C_1/C_0) = -19.00 \quad \text{and} \quad -(C_2/C_0) = -1.00$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  =  33
K               =  33.00 hours
T               =  24.00 hours
x               =  0.45000
```

PERIOD (x 24.00 hours) (1)	INFLOW (observed) (m3/sec) (2)	OUTFLOW (calculated) (m3/sec) (3)	INFLOW (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	270.219	314.000
2	355.000	301.194	355.000
3	404.000	339.393	404.000
4	495.000	382.219	495.000
5	566.000	465.284	566.000
6	586.000	543.565	586.000
7	572.000	578.668	571.998
8	575.000	573.076	574.985
9	572.000	574.891	571.857
10	571.000	572.684	569.651
11	676.000	561.418	663.294
12	1026.000	619.543	906.292
13	1156.000	930.802	28.220
14	1081.000	1117.154	-9543.873
15	1001.000	1095.937	-99096.492
16	816.000	1037.853	-942207.744
17	681.000	874.015	-8883595.328
18	568.000	731.053	-83698666.882
19	538.000	604.095	-788534359.050
20	534.000	551.860	-7428828232.900
21	535.000	537.549	-69987386269.000
22	551.000	534.007	-659354854080.000
23	555.000	547.156	-6211816787800.000
24	549.000	553.967	-58521852900000.000
25	544.000	550.486	-551337456280000.000
26	493.000	550.144	-5194179193300000.000
27	428.000	510.800	-48934635558000000.000
28	376.000	449.805	-461015777100000000.000
29	357.000	392.851	-4343253900100000000.000
30	301.000	369.606	-40918023585000000000.000
31	274.000	317.547	-385490853770000000000.000
32	271.000	283.166	-3631729622400000000000.000

Notes : (i) Values of coefficients:

$$1/C_0 = -10.58, \quad -(C_1/C_0) = 9.42 \quad \text{and} \quad -(C_2/C_0) = 2.16$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

```
Number of data  = 33
K               = 66.00 hours
T               = 24.00 hours
x               = 0.45000
```

PERIOD (x 24.00 hours) (1)	I N F L O W (observed) (m3/sec) (2)	O U T F L O W (calculated) (m3/sec) (3)	I N F L O W (calculated) (m3/sec) (4)
0	274.000	274.000	274.000
1	314.000	259.342	314.000
2	355.000	271.476	355.000
3	404.000	295.022	404.000
4	495.000	315.825	495.000
5	566.000	378.837	566.000
6	586.000	464.508	586.000
7	572.000	530.007	572.000
8	575.000	549.774	575.000
9	572.000	563.408	572.000
10	571.000	568.044	571.000
11	676.000	531.034	676.000
12	1026.000	474.806	1026.000
13	1156.000	701.052	1156.000
14	1081.000	954.597	1081.000
15	1001.000	1046.723	1001.000
16	816.000	1091.798	816.000
17	681.000	1004.228	681.001
18	568.000	885.028	568.002
19	538.000	738.492	538.006
20	534.000	640.335	534.013
21	535.000	587.131	535.031
22	551.000	555.364	551.074
23	555.000	551.730	555.174
24	549.000	555.553	549.410
25	544.000	554.129	544.967
26	493.000	567.786	495.278
27	428.000	554.445	433.367
28	376.000	510.671	388.645
29	357.000	450.716	386.790
30	301.000	424.671	371.184
31	274.000	373.114	439.348
32	271.000	324.964	660.548

Notes : (i) Values of coefficients:

$$1/C_0 = -2.73, \quad -(C_1/C_0) = 2.36 \quad \text{and} \quad -(C_2/C_0) = 1.37$$

(ii) Column (3) is calculated from column (2) using eq. (2.1.3)

(iii) Column (4) is calculated from column (3) using eq. (4.1.1)

4.3 INVESTIGATION OF THE CAUSE OF THE INSTABILITY

The existence of error in numerical processes is inevitable. The important thing is to attempt to lessen the error as much as possible.

There are three types of error in numerical processes performed by digital computer. These are:

a. *Round-off error.* This is machine made and is caused by the limitations of the particular computer.

b. *Truncation error.* With the truncation of series after only a few terms, a generally known error is committed. This error is not machine-caused but it is due to the method used in the numerical process.

c. *Propagation or inherited error.* This is caused by the use of values previously calculated by the computer, which already are erroneous owing to either (a) or (b) or both errors above, to calculate values at the next time step. Since they are already off the correct solution, any new computed points cannot be expected to have the correct solution [Grove (1966)].

The method used to solve eq. (4.1.1) is repetitive. Subscript i refers to the values which are obtained from the previous routing period. The downstream discharges Q are known from the given data and these are obviously fixed, therefore they do not have any inherited errors. The upstream discharge variables I_i and I_{i+1} however are obtained from calculation and are therefore susceptible to error. If the previously calculated value of upstream discharge I_i which already contains an error is used to calculate the next one (I_{i+1}), the error is inherited. Since the routing is carried out by successively solving equation (4.1.1) for I_{i+1} period by period throughout the flood, errors tend to accumulate or magnify. It was suspected that the coefficient $-C_1/C_0$ which multiplies I_i is the cause of error propagation. Table IV.3.1 gives values of Muskingum and Nash coefficients (for the example calculations given in Table IV.2.1).

Table IV.3.1 Values of Muskingum and Nash Coefficients with K = 66 Hours, Δt = 24 Hours and Various Parameter x Values

	MUSKINGUM				NASH			
x	C_0	C_1	C_2	$-C_1/C_0$	C_0	C_1	C_2	$-C_1/C_0$
0.0	0.154	0.154	0.692	-1	0.162	0.143	0.695	-0.886
0.1	0.076	0.261	0.664	-3.444	0.086	0.246	0.668	-2.867
0.2	-0.019	0.389	0.630	21.000	-0.004	0.370	0.635	82.626
0.3	-0.134	0.546	0.588	4.077	-0.114	0.519	0.595	4.547
0.4	-0.279	0.744	0.535	2.667	-0.250	0.704	0.545	2.819
0.45	-0.366	0.863	0.503	2.356	-0.330	0.814	0.516	2.465
0.5	-0.467	1	0.467	2.143	-0.421	0.938	0.483	2.227

It can be noticed from Tables IV.2.1 that the calculated hydrograph is worst for x=0.2 and best for x = 0.0. If the parameter x values are set in order according to the accuracy of the calculated upstream hydrograph, starting from the worst to the best, the order will be: 0.2, 0.3, 0.1, 0.4, 0.45, 0.5 and 0.0. There is a correlation between this order and the values of $-C_1/C_0$ given in Table IV.3.1. If the parameter x values in that table are also set in order according to the value of $|-C_1/C_0|$, starting from the largest to the least, the order will be precisely the same as that above. The largest value of $|-C_1/C_0|$ given by x=0.2, gives the worst calculated hydrograph. Conversely, the least value of $|-C_1/C_0|$ given by x=0.0, gives the best calculated hydrograph.

The error propagation in the computation can be described as follows:

If parameter x=0.0 then

$$I_{i+1} = \frac{Q_{i+1}}{C_0} \cdot - \frac{C_1}{C_0} . I_i - \frac{C_2}{C_0} . Q_i$$

$$I_{i+1} = 6.5 \, . Q_{i+1} \; - 1. \, I_i \; - \; 4.5. \, Q_i$$

at instant t, if error in I is ΔI,

at $t+\Delta t$, the error becomes $\Delta I.(-1) = -\Delta I$

at $t+2.\Delta t$, the error becomes $\Delta I.(-1)^2 = \Delta I$

at $t+3.\Delta t$, the error becomes $\Delta I.(-1)^3 = -\Delta I$

at $t+n.\Delta t$, the error becomes $\Delta I.(-1)^n$

In this case, the absolute value of error remains unchanged. It does not magnify but the sign of the error changes repeatedly. This circumstance may cause oscillations but, since the error is very small and does not magnify, these oscillations do not affect the computation. However in some cases, i.e. if an observed downstream hydrograph which is used in the computation to obtain calculated upstream hydrograph has even slight oscillations, the circumstance above may allow these oscillations to amplify.

<u>If parameter x=0.2 then</u>

$$I_{i+1} = \frac{Q_{i+1}}{C_0} - \frac{C_1}{C_0}.I_i - \frac{C_2}{C_0}.Q_i$$

$$I_{i+1} = -54 .Q_{i+1} + 21. I_i + 34. Q_i$$

at instant t, if error in I is ΔI,

at $t+\Delta t$, the error becomes $\Delta I.(21)$

at $t+2.\Delta t$, the error becomes $\Delta I.(21)^2$

at $t+3.\Delta t$, the error becomes $\Delta I.(21)^3$

at $t+n.\Delta t$, the error becomes $\Delta I.(21)^n$,

in this case, any error in I will magnify dramatically.

It is clear that the coefficients relating to I_i affect the instability of the process. Parameter x = 0.2 gives the worst result since any error entering to I_i can magnify very rapidly due to the very large value of $|-C_1/C_0|$. Parameter x = 0.0 gives the best result since the value of $|-C_1/C_0|$ is equal to 1 therefore any error entering to I_i does not magnify. This does not imply that parameter x = 0.0 always yields satisfactory results, as it may result in oscillations in the computation since the value of $-C_1/C_0$ which multiplies I_i is negative. Figure 4.3.1 shows an example. The downstream hydrograph ordinates used in the computation are observed data taken from ARR87 table 7.1 page 134, rather than values calculated from the observed upstream hydrograph.

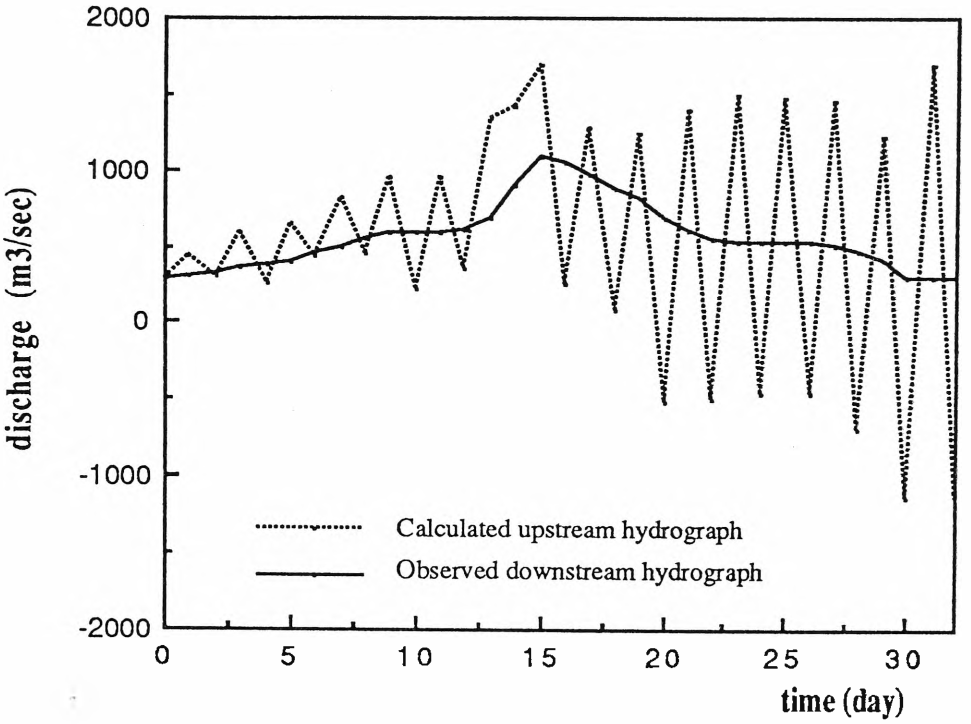


Figure 4.3.1 Oscillations in Upstream Hydrograph Using Parameter x = 0.0

The reason why the computations using parameter x = 0.0 in Tables IV.2.2 do not result in oscillations may be explained by noting that the downstream hydrograph ordinates used in the computation are those which were calculated using the equation for conventional downstream routing. In that equation, the multiplying factor of $Q_{i,}$ namely C_2, is **positive** and **less than 1.0**. Therefore no oscillations occurred and any error entering to Q_i diminished towards zero. Since there was no oscillation in the calculated downstream hydrograph and this result was used to calculate back the upstream hydrograph ordinates using eq.(4.1.1), the coefficient $-C_1/C_0 = -1$ did not affect the computation and therefore the calculated upstream hydrograph ordinates agree exactly with the observed ones. The result in Figure 4.3.1 was obtained using observed downstream hydrograph ordinates. They have slight oscillations (which cannot be detected, since the scale in that figure is too small), and these are amplified by the coefficient $-C_1/C_0 = -1$

From Table IV.3.1, it can be seen that the coefficient $-C_1/C_0$ for parameter x = 0.1 is -3.444. This value is negative and the absolute value $|-C_1/C_0|$ is larger than 1.0. Any error entering to I_i will therefore magnify, with a sign change at each time step. This is the the reason why oscillations and divergence occurred in the computation (see Tables IV.2.1 for parameter x = 0.1).

The results in Tables IV.2.3 can be explained in conjunction with the coefficient $-C_1/C_0$. Table IV.3.2 shows the values of that coefficient corresponding to the time steps used.

Table IV.3.2 Values of $-C_1/C_0$ with Parameter x = 0.45, K = 66 Hours and Various Δt

Δt (hours)	3	6	12	24	36	48	60	72	96
$-C_1/C_0$	1.106	1.225	1.506	2.356	4.077	9.421	-199	-10.429	-4.246

According to the value of $-C_1/C_0$ for Δt = 60 hours, oscillations and very rapid divergence will occur. It is the worst case since the value $|-C_1/C_0|$ is the largest one. Result in Tables IV.2.3 for Δt = 60 shows that the computation oscillates and diverges very rapidly. The only calculated upstream discharges which agree precisely with the observed ones are given by Δt = 96 hours. Seemingly, this is a contradiction since the value $|-C_1/C_0|$ is larger than 1.0 and it therefore should have given a bad result. The reason why it gives a satisfactory result is that the number of data points becomes very few (from 33 to 9) since interpolation was used, so that any error entering to I_i does not have the opportunity to magnify. The time step Δt = 72 hours also gives quite satisfactory result even though the value $|-C_1/C_0|$ is larger than 1.0. However, there are some small differences in the last few calculated upstream discharges if compared to the observed ones. If the number of data were more than that shown in the table, the differences would propagate and magnify. In practice, a time step Δt which is larger than K would not be used since it is too coarse and good definition of the hydrograph is not possible. The shape of the calculated hydrograph, and especially the peak, are not adequately defined.

Table IV.3.3 shows the values of the coefficient $-C_1/C_0$ for various K used in the computation of which results are presented in Table IV.2.4.

Table IV.3.3 Values of $-C_1/C_0$ with Parameter x = 0.45, Δt = 24 Hours and Various Parameter K values

K (hours)	6	12	24	33	66
$-C_1/C_0$	-1.581	-2.636	-19	9.421	2.356

According to the value of $-C_1/C_0$ for K = 24 hours, the result should be the worst. Oscillations and rapid divergence will occur since the value of $-C_1/C_0$ is negative and its absolute value is the largest. Result in Table IV.2.4 for K = 24 hours precisely show that condition. The only K value which gives adequately satisfactory result is K = 6 hours, but as a matter of fact, the calculated upstream discharges do not agree exactly with the observed ones since there are some differences in the last few discharges. In other words, the error started magnifying at almost the end of the computation.

4.4 PROOF OF THE INSTABILITY

Stability (convergence) of the numerical process can only be achieved by selecting a time step Δt relative to the K value so as to make $|-C_1/C_0| \leq 1$. However, as shown in Tables IV.2.3 (for x = 0.45), this cannot be done, since no time step Δt can make the process converge for any parameter x values other than x=0.0. This result can be proved mathematically, either using Muskingum or Nash coefficients, as described below.

4.4.1 Muskingum Coefficients

From equation (2.1.4) :

$$\left|\frac{-C_1}{C_0}\right| = \left|\frac{K.x + 0.5.\Delta t}{K.x - 0.5.\Delta t}\right| \leq 1 \tag{4.4.1}$$

where : K and T are positive,

and $0 \leq x \leq 0.5$

Condition (4.4.1) can be written as:

$$-1 \leq \frac{K.x + 0.5.\Delta t}{K.x - 0.5.\Delta t} \leq 1 \tag{4.4.2}$$

To solve this condition, it is necessary to assume the value of the denominator whether >0 or <0, since it affects the mathematical operators.

a. Suppose $K.x - 0.5.\Delta t > 0$ or $x > 0.5.\Delta t/K$ then

condition (4.4.2) can be written as:

$-K.x + 0.5.\Delta t \leq K.x + 0.5.\Delta t \leq K.x - 0.5.\Delta t$

The first condition: $-K.x + 0.5.\Delta t \leq K.x + 0.5.\Delta t$

$$-K.x \leq K.x$$

$$x \geq 0$$

The second condition: $K.x + 0.5.\Delta t \leq K.x - 0.5.\Delta t$

since K and Δt are positive, there is no solution for it.

From these conditions, it can be concluded that there is no solution for this case.

b. Suppose $K.x - 0.5.\Delta t < 0$ or $x < 0.5.\Delta t/K$ then

condition (4.4.2) can be written as:

$-K.x + 0.5.\Delta t \geq K.x + 0.5.\Delta t \geq K.x - 0.5.\Delta t$

The first condition: $-K.x + 0.5.\Delta t \geq K.x + 0.5.\Delta t$

$$-K.x \geq K.x$$

$$x \leq 0$$

The second condition: $K.x + 0.5.\Delta t \geq K.x - 0.5.\Delta t$

$$K.x \geq K.x - \Delta t$$

since K and Δt are positive, any x will satisfy this condition.

From those conditions above, $x < 0.5.\Delta t/K$ and $x \leq 0$, it can be concluded that only $x \leq 0$ will satisfy condition (4.4.1). This is the reason why only x=0.0 gives a satisfactory result in the samples of the computations (Table IV.2.1).

4.4.2 Nash Coefficients

From equation (3.4.9) :

$$\left|\frac{-C_1}{C_0}\right| = \left|\frac{c.\Delta t - K(1-c)}{\Delta t - K(1-c)}\right| \leq 1 \qquad (4.4.3)$$

where : K and Δt are positive,

$\Delta t \leq K$,

$0 \leq x \leq 0.5$, and

$c = e^{\frac{-\Delta t}{K(1-x)}}$ is always positive

Condition (4.4.3) can be written as:

$$-1 \leq \frac{c.\Delta t - K.(1-c)}{\Delta t - K(1-c)} \leq 1 \qquad (4.4.4)$$

Again, to solve this equation, it is necessary to assume the value of the denominator whether >0 or <0 since it affects the mathematical operators.

a. Suppose $\Delta t - K(1-c) < 0$ then condition (4.4.4) can be written as:

$-\Delta t + K - K.c \geq c.\Delta t - K + K.c \geq \Delta t - K + K.c$

The second condition: $c.\Delta t - K + K.c \geq \Delta t - K + K.c$

$$c.\Delta t - \Delta t \geq 0$$

$$c \geq 1$$

$$e^{\frac{-\Delta t}{K(1-x)}} \geq 1$$

$$\frac{-\Delta t}{K(1-x)} \geq 0$$

since x lies in [0,0.5], no x will satisfy that condition, therefore it is not necessary to consider the first condition.

b. Suppose $\Delta t - K(1-c) > 0$ then $c > 1 - (\Delta t / K)$

If $\Delta t / K = 1$ then $c > 0$

$$e^{\frac{-\Delta t}{K(1-x)}} > 0$$

$$e^{\frac{-1}{1-x}} > 0$$

any x in [0, 0.5] will satisfy that condition, therefore that condition can be ignored.

If $\Delta t / K < 1$ then

$$e^{\frac{-\Delta t}{K(1-x)}} > 1 - \frac{\Delta t}{K}$$

$$\frac{-\Delta t}{K(1-x)} > \ln(1 - \frac{\Delta t}{K})$$

since : $0 \leq x \leq 0.5$, (1-x) is always positive and

$\Delta t / K < 1$, $\ln(1-\Delta t / K)$ is negative then

$$1 - x > \frac{-\Delta t}{K.\ln(1 - \frac{\Delta t}{K})}$$

$$x < 1 + \frac{\Delta t}{K.\ln(1 - \frac{\Delta t}{K})}$$

let $a = \Delta t / K$:

$$x < 1 + \frac{a}{\ln(1-a)} \tag{4.4.5}$$

Since $\Delta t - K(1-c) > 0$, condition (4.4.4) can be written as:

$-\Delta t + K - K.c \leq c.\Delta t - K + K.c \leq \Delta t - K + K.c$

The second condition: $c.\Delta t - K + K.c \leq \Delta t - K + K.c$

$$c.\Delta t - \Delta t \leq 0$$

$$c \leq 1$$

$$e^{\frac{-\Delta t}{K(1-x)}} \leq 1$$

$$\frac{-\Delta t}{K(1-x)} \leq 0$$

any $x < 1$ will satisfy that condition.

The first condition: $-\Delta t + K - K.c \leq c.\Delta t - K + K.c$

$$-\Delta t + 2K \leq c.\Delta t + 2K.c$$

$$\frac{2K - \Delta t}{\Delta t + 2K} \leq c$$

$$\frac{2K - \Delta t}{\Delta t + 2K} \leq e^{\frac{-\Delta t}{K(1-x)}}$$

$$\ln\left(\frac{2K - \Delta t}{2K + \Delta t}\right) \leq \frac{-\Delta t}{K(1 - x)}$$

since: * K and Δt are positive and $\Delta t \leq K$, $\ln((2K-\Delta t)/(2K+\Delta t))$ is negative,

* x lies in [0, 0.5], (1-x) is always positive then

$$1 - x \geq \frac{-\Delta t}{K.\ln\left(\frac{2K - \Delta t}{2K + \Delta t}\right)}$$

$$x \leq 1 + \frac{\Delta t}{K.\ln\left(\frac{2K - \Delta t}{2K + \Delta t}\right)} \tag{4.4.6}$$

From the first and the second conditions above, condition (4.4.6) should be chosen. Let $a = \Delta t/K$, condition (4.4.6) becomes:

$$x \le 1 + \frac{a}{\ln(\frac{2 - a}{2 + a})} \qquad (4.4.7)$$

Both conditions (4.4.5) and (4.4.7) should be taken into account. In order to know which condition will entirely satisfy condition (4.4.3), they are illustrated in graphic (see Fig. 4.4.1).

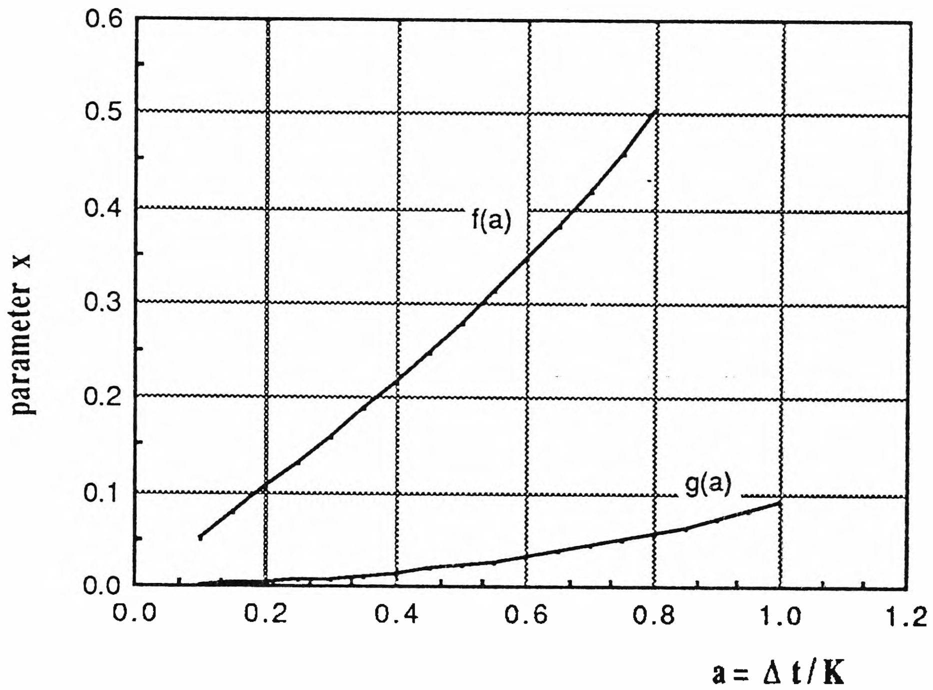


Figure 4.4.1 Graphic f(a) and g(a) vs. a, where :

$x = f(a) = 1 + \frac{a}{\ln(1 - a)}$ and $x = g(a) = 1 + \frac{a}{\ln(\frac{2 - a}{2 + a})}$

It can be noticed from Fig. 4.4.1, condition (4.4.7) satisfies condition (4.4.3). It also can be noticed that the value of parameter x which can satisfy condition (4.4.3) is $x \le 0.0898$ (0.0898 is obtained by substituting $a = \Delta t/K = 1$ into condition (4.4.7)). This is the reason why only x=0.0 gives a satisfactory result, if upstream routing is computed by using eq. (4.1.1).

4.5 SUMMARY

Upstream routing using a re-arranged form of the conventional downstream routing equation is numerically unstable. The only parameter x value which reproduces precisely the observed upstream hydrograph is x = 0.0. This result for x = 0.0 occurs only when the downstream hydrograph has been calculated from a given upstream hydrograph using normal Muskingum routing procedures, i.e. the downstream hydrograph contains 'perfect' data. In practical problems, where a recorded downstream hydrograph, which is not error free, must be used, satisfactory upstream hydrographs cannot be obtained, since very rapid and great oscillations will occur, as shown in Figure 4.3.1.

It has been found that the coefficient multiplying I_i is the cause of the instability. If its value is negative and its absolute value is much larger than 1.0, oscillations and very rapid divergence will occur. If its value is positive and greater than 1.0 then monotonic divergence, either increasing or decreasing, will occur.

Satisfactory results can only be obtained by making the absolute value of the coefficient relating to I_i equal to or less than 1.0 with the appropriately chosen time step Δt. However, as has been proved, no time step Δt and parameter x value can satisfy that condition, except x = 0.0. In view of this, other techniques for upstream routing must be developed and these are covered in the following chapter.

Chapter Five

Alternative Approaches to Upstream Routing

5.0 INTRODUCTION

It is clear that the upstream routing derived from standard Muskingum routing equation gives unsatisfactory results as described in chapter 4. In view of this point, this chapter is intended to investigate some alternative approaches to upstream routing.

An iterative solution which is based on finite differences is introduced as one of the methods. Both first order and second order finite difference formulations are applied in conjunction with this method. The method of cubic spline fitting combined with the Runge-Kutta method and an alternative approach, in which the upstream hydrograph is calculated moving backward in time are also investigated.

5.1 ITERATIVE METHOD

As described in chapter 4, Equation (4.1.1) cannot be used due to its instability. An alternative solution has been developed which is still based on the equation of conservation of mass [eq.(2.1.1)] but using a different approach. Equation (2.1.1) is re-arranged into

$$I_i = Q_i + dS/dt|_i \qquad (5.1.1)$$

This follows a procedure used by Pilgrim and Watson (1967) for a similar problem in estimating the input from a recorded output, for a radiation ratemeter involving an electrical storage system.

The derivative in equation (5.1.1) is expressed in central finite difference form as discussed for example in Salvadori and Baron (1964). The simplest two point scheme is used [eq.(5.1.2)].

$$dS/dt|_i = (S_{i+1} - S_{i-1})/(2.\Delta t) \qquad (5.1.2)$$

The storage S at any specified discharge is expressed by the linear relationship between upstream discharge I, downstream discharge Q and storage S, i.e.:

$$S = K[x.I + (1-x).Q] \qquad (5.1.3)$$

as mentioned in chapter 2.

Since eq.(5.1.1) is a differential equation, it is necessary to assume the initial value of I (at time i = 0). Therefore, equation (5.1.2) is not used to obtain $dSdt|_0$. This assumption of initial value of I is actually dependent on the judgement of the hydrologist. Normally, the hydrologist takes equation (5.1.4) into account, although the initial discharges do not have to be equal.

$$I_0 = Q_0 \qquad (5.1.4)$$

Difficulty arises in calculating the derivative at the end of the hydrograph (at time i = N, the time at which the last downstream discharge was observed) using eq.(5.1.2) since the observed downstream discharge at time i = N+1 is not known. Assumptions or methods for obtaining that derivative are discussed in the latter part of this chapter. In this section, it is first assumed that $S_{N+1} = S_N$, therefore the derivative becomes

$$dS/dt|_N = (S_N - S_{N-1})/(2.\Delta t) \qquad (5.1.5)$$

If equations (5.1.1), (5.1.2) and (5.1.3) are combined, they will yield an implicit equation, since the storage S is expressed in terms of the upstream discharge I, the value of which itself is being sought.

An iterative solution using instantaneous discharges is required. The method of solution used is to adopt the downstream hydrograph ordinates Q as the first estimate of the upstream hydrograph ordinates I, give an initial value at time i = 0 to I_0 which remains unchanged throughout the iterative process, use equation (5.1.3) to calculate the values of storage S, use equations (5.1.2) and equation (5.1.5) to determine the derivative dS/dt, then use equation (5.1.1) to make an improved estimate of I. These steps are repeated until successive calculated upstream hydrographs converge. A detailed explanation of the procedures with help of a flowchart is given in Section 5.1.4.

In the first application of this method, the results of computations were unsatisfactory. Oscillations occurred in the estimated upstream hydrograph. These oscillations became greater with each iteration. The reason why this occurred was that the first derivative dS/dt estimated using eq.(5.1.2) possessed slight oscillations. Much more satisfactory results were obtained when those oscillations were eradicated by the smoothing algorithm [eq.(5.1.6)].

$$\frac{dS}{dt}\Big|_i^* = \left(\frac{dS}{dt}\Big|_{i-1}^* + 2.\frac{dS}{dt}\Big|_i + \frac{dS}{dt}\Big|_{i+1}\right)/4 \qquad (5.1.6)$$

Superscript * refers to the value which has been or is being smoothed.

The smoothing algorithm is carried out from the derivative at time i = 1 up to time i = N-1. Since the value of I at time i = 0 (I_0) is assumed, $dS/dt|_0$ is not calculated using eq.(5.1.2) but using

$$\frac{dS^*}{dt}\Big|_0 = \frac{dS}{dt}\Big|_0 = I_0 - Q_0 \tag{5.1.7}$$

The expression of equation (2.1.1) in the finite difference form of equations (5.1.1), (5.1.2) and (5.1.5) provides a satisfactory procedure for upstream routing in river reaches, and gives much better results than the reverse application of normal routing procedures as expressed in equation (2.1.3). Figure (5.1.1) shows results of the iterative method including this smoothing algorithm.

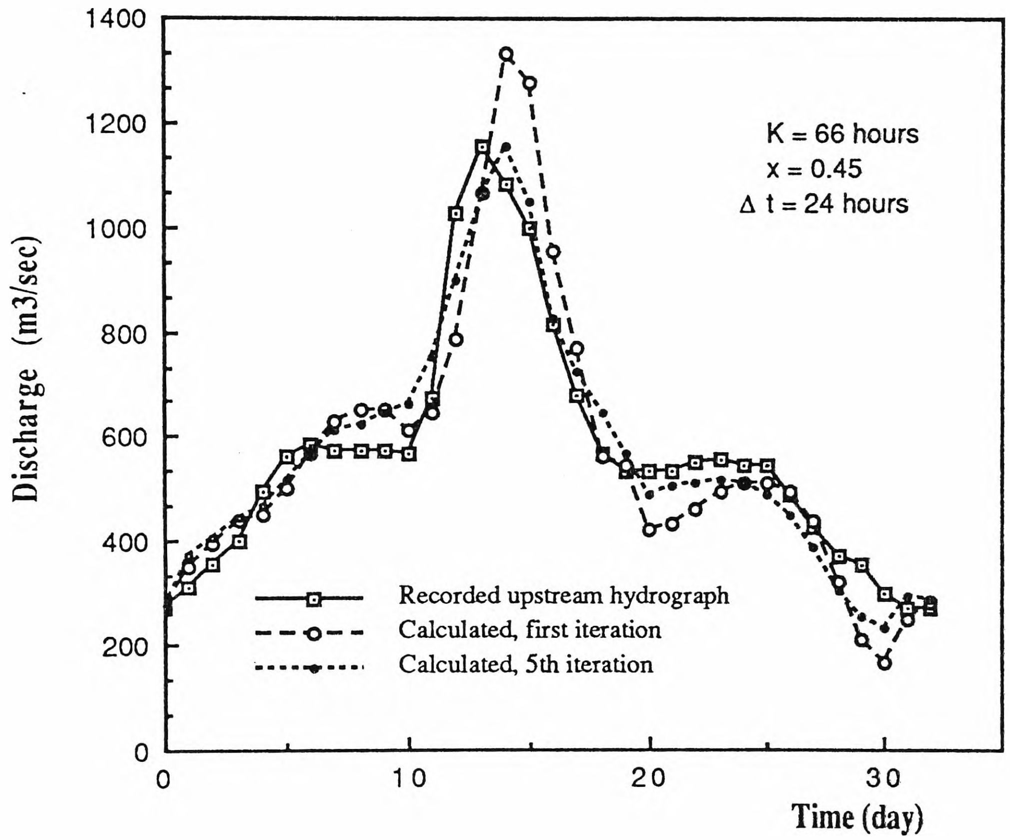


Figure 5.1.1 River Reach Routing Using Instantaneous Discharges

5.1.1 Criterion to Terminate the Iteration

As mentioned above, the downstream hydrograph ordinates Q are adopted as the first estimate of the upstream hydrograph ordinates I. These results are used to estimate the ordinates I at the next iteration. The iteration is repeated until successive calculated upstream hydrographs converge. The question of when to terminate the iteration arises, and this depends on the convergence criterion adopted.

The vector of estimates after k th iteration is denoted as I_1^k, I_2^k,..., I_{N-1}^k, I_N^k while the (k-1) th iteration results in the vector of estimates denoted as I_1^{k-1}, I_2^{k-1},..., I_{N-1}^{k-1}, I_N^{k-1}. Convergence is most easily measured in terms of the relative change in each value of I from one iteration to the next. If the quantities:

$$d_i = \left| \frac{I_i^k - I_i^{k-1}}{I_i^k} \right| \qquad i = 1, 2, \ldots, N \tag{5.1.8}$$

are computed for each value of i, then convergence can be said to have been reached when each d_i is equal to or less than some specified small quantity [de Vahl Davis (1986)]. In this project, the criterion of convergence is taken as

$$d_i \le 0.001 \tag{5.1.9}$$

The small quantity in (5.1.9) is selected depending on the precision of the computation required by the hydrologist. However, the value of d_i affects the total number of iterations, the smaller that quantity is, the more iterations are required.

5.1.2 Condition to Converge

Equations (5.1.1), (5.1.2) and (5.1.3) can be combined and yield:

$$I_i = Q_i + \frac{K}{2.\Delta t}\left[x.I_{i+1}^* + (1-x).Q_{i+1} - x.I_{i-1}^* - (1-x).Q_{i-1}\right] \tag{5.1.10}$$

Superscript * refers to the values which are assumed for the first trial or obtained from the previous iteration.

It is clear that equation (5.1.10) is implicit since the variable being solved also appears on the right hand side of the equation. Therefore, it is necessary to use an iterative solution in which values of the variable I calculated from a previous trial are used in the computation.

It has been mentioned in chapter 4 that the multiplying factor related to the unknown variable I affects convergence. Convergence can be reached as long as the absolute value of the multiplying factor is less than 1.0. This condition is expressed from equation (5.1.10) as

$$(K.x)/(2.\Delta t) < 1, \quad \text{or}$$

$$\Delta t > (K.x)/2. \qquad (5.1.11)$$

If this condition is fulfilled, the process of computation should converge. In addition, the time step Δt should be well taken into account. If it is too large, not all points on the hydrograph are considered and the peak may be missed. However, the larger the time step Δt , the fewer the number of iterations required, in accordance with eq.(5.1.11).

In practice, the limiting time step Δt required to converge is somewhat larger than that given by condition (5.1.11). This can be noticed more clearly from Figure 5.1.2.

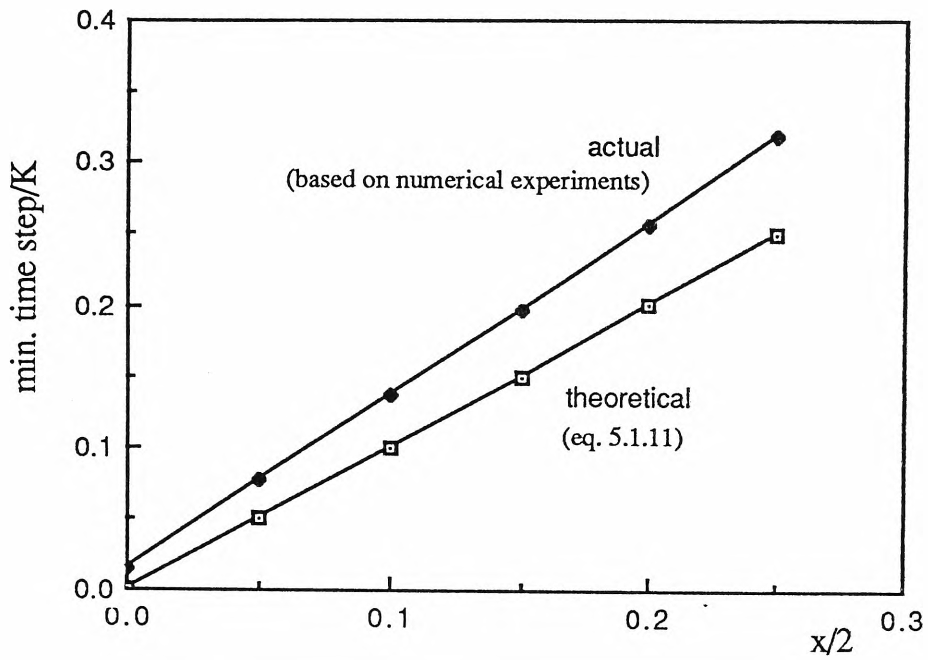


Figure 5.1.2 Graphic x/2 Vs. Min. Time Step/K
data were obtained from upstream routing calculation

The values in the actual line (Fig.5.1.2) were obtained by trial and error computations using the data taken from ARR87 Table 7.1 page 134, and it can be seen that the time step required for convergence is somewhat greater than indicated by eq. (5.1.11). However, these values can be reduced down to those in theoretical line, if a weighting factor is applied, as discussed in the section 5.1.3.

In the particular case when parameter x = 0.0, equation (5.1.10) becomes

$$I_i = Q_i + \frac{K}{2.\Delta t}\left[Q_{i+1} - Q_{i-1} \right] \tag{5.1.12}$$

This equation becomes explicit and it can be solved without using an iterative solution. There is no error which will magnify, since the variable involved on the right hand side of the equation (Q) is fixed once the given outflow hydrograph is adopted. Therefore, the condition for choosing time step Δt in order to converge is no longer necessary for this case.

5.1.3 Weighting Factor (α)

If the upstream hydrograph ordinates at iteration k are combined with those at iteration k-1 as a weighted average to make a new estimate of upstream hydrograph ordinates (I_a), before commencing iteration k+1, results can be dramatically improved, with fewer iterations required. This condition is expressed as follows:

$$I_{a_i} = I_i^{k-1} + (I_i^k - I_i^{k-1}).\alpha \tag{5.1.13}$$

where i = 0,1,2,...,N and $0 < \alpha < 1$. It was found by numerical experiments that the effective α lies between 0.1 and 0.7.

The other advantage of applying a weighting factor α in the iterations is that, as mentioned in section 5.1.2, the actual limiting time steps Δt can be reduced down to those in the theoretical line given by condition (5.1.11) or even to values of Δt which are less than those in the theoretical line if the appropriate weighting factor α is used. The particular values of Δt that can be reached should be determined by numerical experiments. For example if parameter x = 0.5 and K = 66 hours, then using condition (5.1.11), Δt > 16.5 hours. In practice, the minimum Δt which still can make the process converge without weighting (i.e. $\alpha = 1$) is 21 hours. If a weighting factor $\alpha = 0.4$ is applied, Δt can be reduced down to 12 hours which is less than that given by condition (5.1.11).

The question which arises is: what is the optimum α to be chosen? In this context, 'optimum' implies the value of α which requires the fewest number of iterations. It should be determined by numerical experiments. According to the experiments using various values of parameter x, with K = 66 hours and Δt = K.x/2 and condition (5.1.9) for terminating the iterations, the optimum α is close to 0.4 (see Figure 5.1.3).

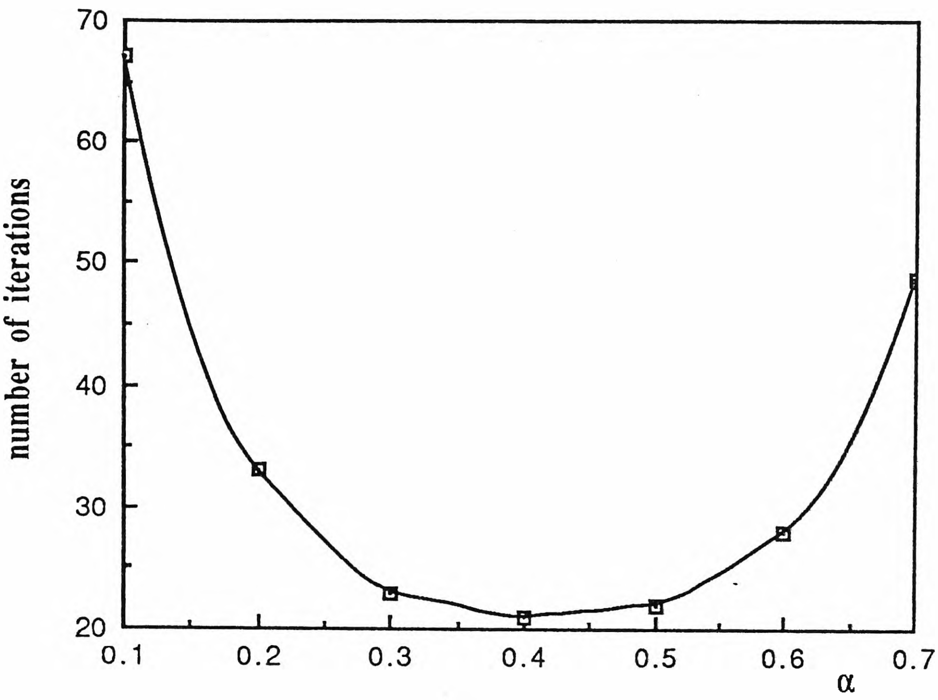


Figure 5.1.3 Graphic α Vs. Number of Iterations for x = 0.2

Other values of parameter x result in similar graphics to that in Figure 5.1.3 with approximately 20 being the minimum number of iterations. If the computation is carried out without weighting (i.e. $\alpha = 1$), the minimum time step used in order to converge is somewhat greater than that given by condition $\Delta t = K.x/2$ as shown in Fig. 5.1.2, and the number of iterations also becomes greater.

5.1.4 Summary of the Computation Procedure

All of the steps discussed in the previous sections of this chapter can be summarized with the help of a flow chart as described in figure 5.1.4.

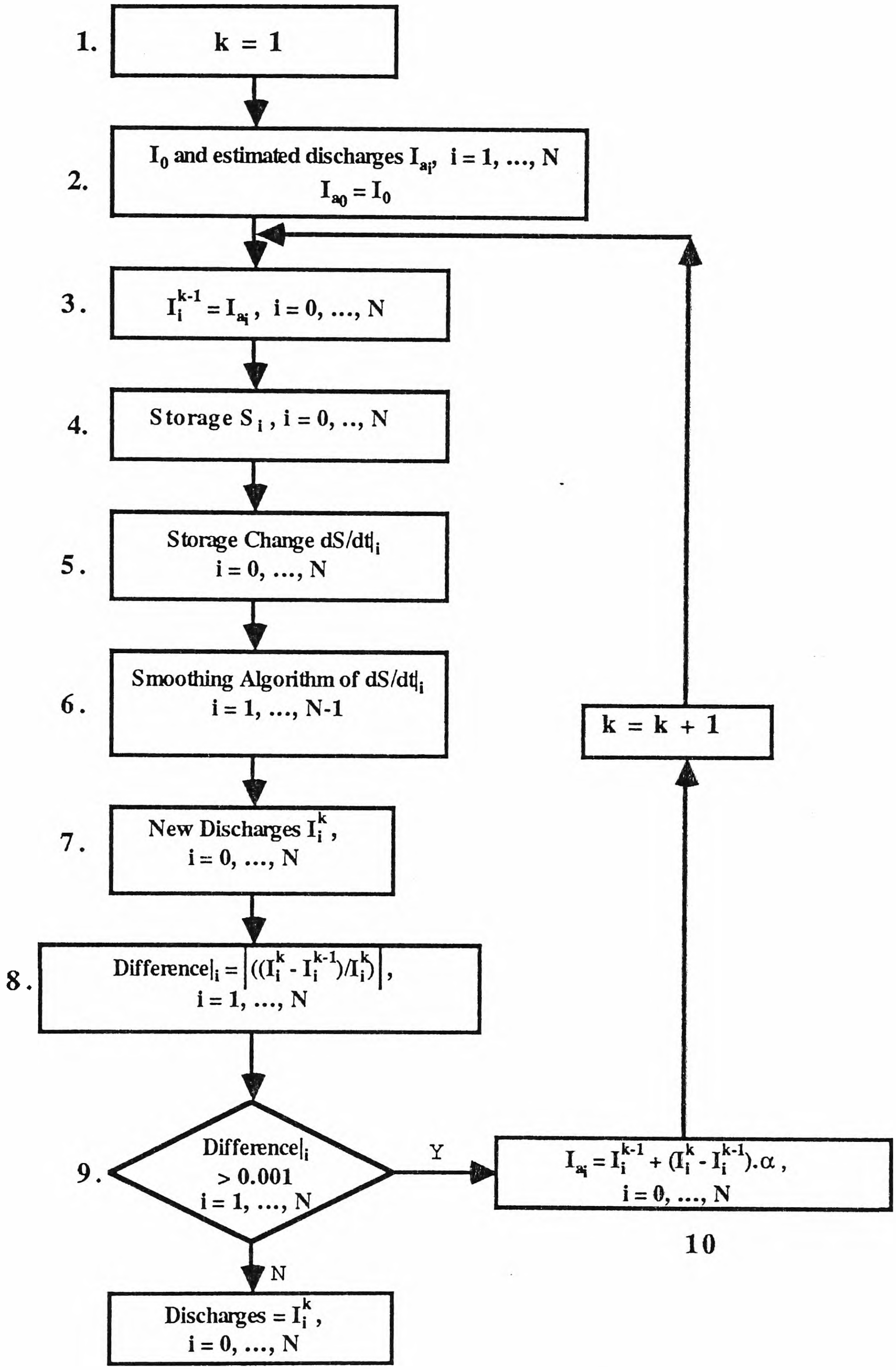


Figure 5.1.4 Flow Chart of the Computation

<u>Explanation of the steps of the computation</u>

Step 1 : initialize iteration $k = 1$

Step 2 : give an initial value at time $i = 0$ to I (I_0)which remains unchanged throughout the required number of iterations to converge [eq.(5.1.4)] and adopt the downstream hydrograph ordinates Q as the first estimate of the upstream hydrograph ordinates (I_a)

Step 3 : equate I^{k-1} (upstream hydrograph ordinates at iteration k-1) with I_a

Step 4 : calculate storage S for all ordinates throughout the flood according to the given data Q and the values of I obtained in step 3 using equation (5.1.3).

Step 5 : calculate storage change dS/dt using eq.(5.1.2) for all ordinates, except for the first and the last ordinates. The value of dS/dt at the last ordinate is calculated using eq.(5.1.5) and dS/dt at the first ordinate is calculated using eq.(5.1.7).

Step 6 : apply smoothing algorithm using eq.(5.1.6).

Step 7 : calculate new upstream hydrograph ordinates I^k using eq.(5.1.1).

Step 8 : calculate the relative change in each value of I from the previous value to the new one using eq.(5.1.8).

Step 9 : check the results of step 8, if they are all equal to or less than 0.001, the upstream hydrograph ordinates I are set equal to the I^k ordinates and the process is finished. If not, continue to step 10.

Step 10 : use eq.(5.1.13) to make a new estimate of upstream hydrograph ordinates I and return to step 3 to get into the next iteration (k+1).

5.1.5 Tests of Computations

Firstly, tests were carried out using samples of computations to check their stability and convergence. The upstream hydrograph from Australian Rainfall and

Run-off (ARR87) Table 7.1 page 134 (Pilgrim, I.E. Aust. 1987) was used to obtain a downstream hydrograph by applying an iterative method to downstream routing (to be discussed in chapter 6). These results were then used to calculate the upstream hydrograph using the method outlined in the preceding sections of this chapter. Tables V.1.1 and Figures 5.1.5 show results for K = 66 hours, $\Delta t = 24$ hours, $\alpha = 0.4$ and x = 0, 0.1, 0.2, 0.3, 0.4, 0.45 and 0.5. 'Total iterations' in those tables indicates the number of iterations which are required for upstream routing to converge. It can be noticed from these results that upstream routing reproduces upstream discharges which are almost the same as those observed.

Tables V.1.1

Samples of Computations

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.00000
alfa             = 0.40000
Total iterations = 13
```

PERIOD (x 24.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	281.679	313.839
2	355.000	303.587	354.572
3	404.000	316.062	403.821
4	495.000	359.749	495.436
5	566.000	421.956	566.675
6	586.000	478.863	586.198
7	572.000	506.048	571.509
8	575.000	517.677	574.344
9	572.000	540.789	571.836
10	571.000	565.437	571.427
11	676.000	494.374	676.532
12	1026.000	653.114	1026.127
13	1156.000	844.577	1155.680
14	1081.000	905.357	1080.610
15	1001.000	973.837	1000.901
16	816.000	930.406	816.215
17	681.000	875.011	681.269
18	568.000	776.182	568.078
19	538.000	702.896	537.868
20	534.000	655.923	533.825
21	535.000	614.680	534.941
22	551.000	597.178	551.074
23	555.000	589.002	555.106
24	549.000	572.407	549.040
25	544.000	573.467	543.963
26	493.000	560.154	492.943
27	428.000	529.137	427.976
28	376.000	474.748	376.014
29	357.000	454.544	357.024
30	301.000	410.279	301.012
31	274.000	371.594	273.999
32	271.000	329.236	270.995

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.10000
alfa             = 0.40000
Total iterations = 14
```

PERIOD (x 24.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	279.629	314.027
2	355.000	300.453	354.724
3	404.000	310.063	403.569
4	495.000	351.753	494.903
5	566.000	416.600	566.418
6	586.000	479.425	586.553
7	572.000	508.797	572.150
8	575.000	520.281	574.601
9	572.000	548.051	571.434
10	571.000	566.791	570.800
11	676.000	475.995	676.334
12	1026.000	631.104	1026.513
13	1156.000	837.065	1156.203
14	1081.000	912.332	1080.784
15	1001.000	991.704	1000.674
16	816.000	953.559	815.892
17	681.000	896.455	681.151
18	568.000	790.375	568.203
19	538.000	709.134	538.060
20	534.000	656.532	533.905
21	535.000	611.400	534.873
22	551.000	592.706	550.946
23	555.000	584.761	555.029
24	549.000	568.564	549.063
25	544.000	572.308	544.049
26	493.000	563.106	493.013
27	428.000	532.969	427.983
28	376.000	476.849	375.973
29	357.000	456.684	356.980
30	301.000	412.313	300.991
31	274.000	370.891	273.997
32	271.000	326.434	271.001

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.20000
alfa             = 0.40000
Total iterations = 14
```

PERIOD (x 24.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	277.011	313.919
2	355.000	296.405	355.182
3	404.000	303.288	404.359
4	495.000	343.752	495.131
5	566.000	412.278	565.721
6	586.000	480.983	585.604
7	572.000	511.419	571.878
8	575.000	523.305	575.205
9	572.000	556.808	572.285
10	571.000	567.283	571.140
11	676.000	454.111	675.961
12	1026.000	606.869	1025.882
13	1156.000	830.276	1155.955
14	1081.000	922.288	1081.122
15	1001.000	1013.780	1001.172
16	816.000	980.568	816.022
17	681.000	919.787	680.843
18	568.000	804.079	567.816
19	538.000	713.434	537.934
20	534.000	654.811	534.060
21	535.000	606.143	535.103
22	551.000	586.971	551.072
23	555.000	579.729	555.022
24	549.000	564.339	548.988
25	544.000	571.529	543.972
26	493.000	566.874	492.966
27	428.000	537.231	427.972
28	376.000	479.060	375.985
29	357.000	459.027	356.994
30	301.000	414.250	300.996
31	274.000	369.615	274.000
32	271.000	323.049	271.002

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.30000
alfa             = 0.40000
Total iterations = 14
```

PERIOD (x 24.00 hours)	INFLOW (observed) (m3/sec)	OUTFLOW (calculated) (m3/sec)	INFLOW (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	274.520	314.096
2	355.000	292.143	354.905
3	404.000	294.902	403.681
4	495.000	333.975	494.917
5	566.000	409.347	566.250
6	586.000	484.681	586.144
7	572.000	515.834	571.767
8	575.000	527.965	574.719
9	572.000	566.149	572.148
10	571.000	564.262	571.575
11	676.000	426.992	676.523
12	1026.000	581.305	1026.051
13	1156.000	826.669	1155.616
14	1081.000	937.171	1080.513
15	1001.000	1040.388	1000.692
16	816.000	1010.461	816.013
17	681.000	943.862	681.281
18	568.000	816.749	568.289
19	538.000	715.706	538.081
20	534.000	650.614	533.911
21	535.000	598.597	534.907
22	551.000	579.656	550.965
23	555.000	573.733	554.981
24	549.000	559.878	548.977
25	544.000	571.579	543.989
26	493.000	571.868	493.006
27	428.000	542.095	428.011
28	376.000	481.451	375.997
29	357.000	461.558	356.979
30	301.000	415.904	300.978
31	274.000	367.539	273.994
32	271.000	318.970	271.007

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.40000
alfa             = 0.40000
Total iterations = 16
```

PERIOD (x 24.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	271.150	314.003
2	355.000	286.557	355.045
3	404.000	285.704	403.945
4	495.000	324.699	494.822
5	566.000	405.836	565.991
6	586.000	469.201	586.376
7	572.000	520.695	572.434
8	575.000	534.868	575.042
9	572.000	577.075	571.656
10	571.000	557.234	570.634
11	676.000	593.455	675.895
12	1026.000	553.925	1026.161
13	1156.000	826.742	1156.229
14	1081.000	958.380	1081.084
15	1001.000	1073.352	1000.887
16	816.000	1044.327	815.816
17	681.000	968.059	680.904
18	568.000	826.401	568.067
19	538.000	713.968	538.169
20	534.000	643.017	534.118
21	535.000	588.888	534.995
22	551.000	571.306	550.936
23	555.000	567.347	554.941
24	549.000	555.811	548.940
25	544.000	573.026	543.939
26	493.000	578.259	492.970
27	428.000	547.377	428.005
28	376.000	483.816	376.026
29	357.000	464.094	357.042
30	301.000	417.048	301.045
31	274.000	364.507	274.027
32	271.000	314.181	271.003

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.45000
alfa             = 0.40000
Total iterations = 17
```

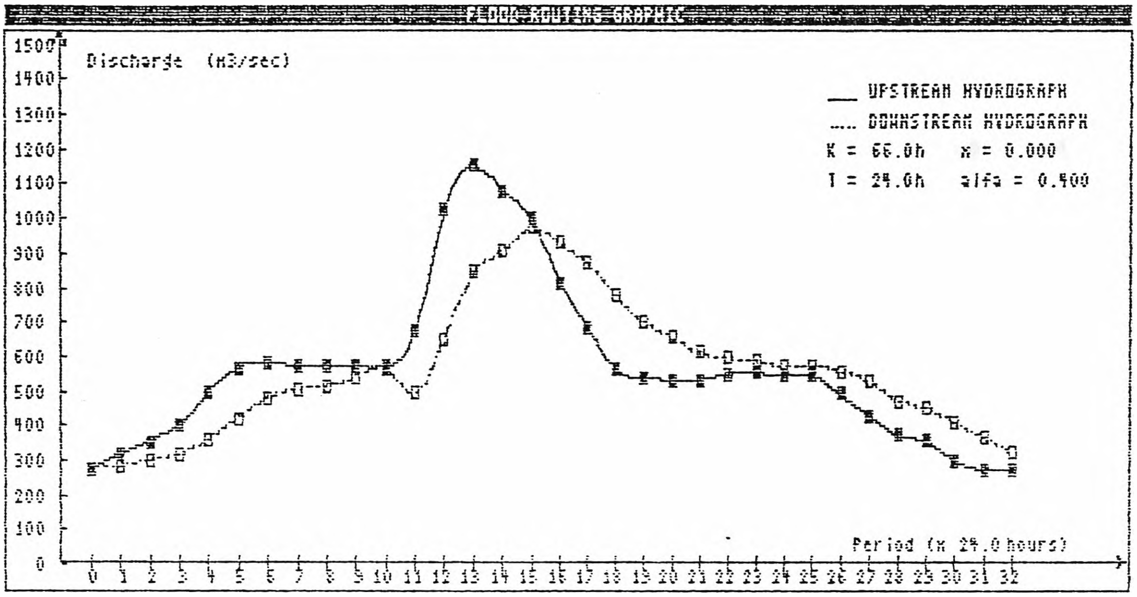
PERIOD (x 24.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	269.450	313.997
2	355.000	283.490	354.958
3	404.000	280.261	403.922
4	495.000	319.690	495.090
5	566.000	405.558	566.321
6	586.000	492.670	586.227
7	572.000	523.492	571.842
8	575.000	538.461	574.669
9	572.000	582.129	571.863
10	571.000	551.749	571.104
11	676.000	374.840	676.149
12	1026.000	540.651	1026.037
13	1156.000	828.887	1155.898
14	1081.000	971.354	1080.848
15	1001.000	1091.695	1000.935
16	816.000	1062.150	816.093
17	681.000	979.958	681.188
18	568.000	830.254	568.151
19	538.000	711.748	538.011
20	534.000	637.809	533.882
21	535.000	582.993	534.866
22	551.000	566.637	550.925
23	555.000	563.999	554.960
24	549.000	554.001	548.977
25	544.000	574.440	544.022
26	493.000	582.130	493.060
27	428.000	550.260	428.057
28	376.000	485.060	376.043
29	357.000	465.388	357.031
30	301.000	417.344	301.006
31	274.000	362.516	273.982
32	271.000	311.456	270.978

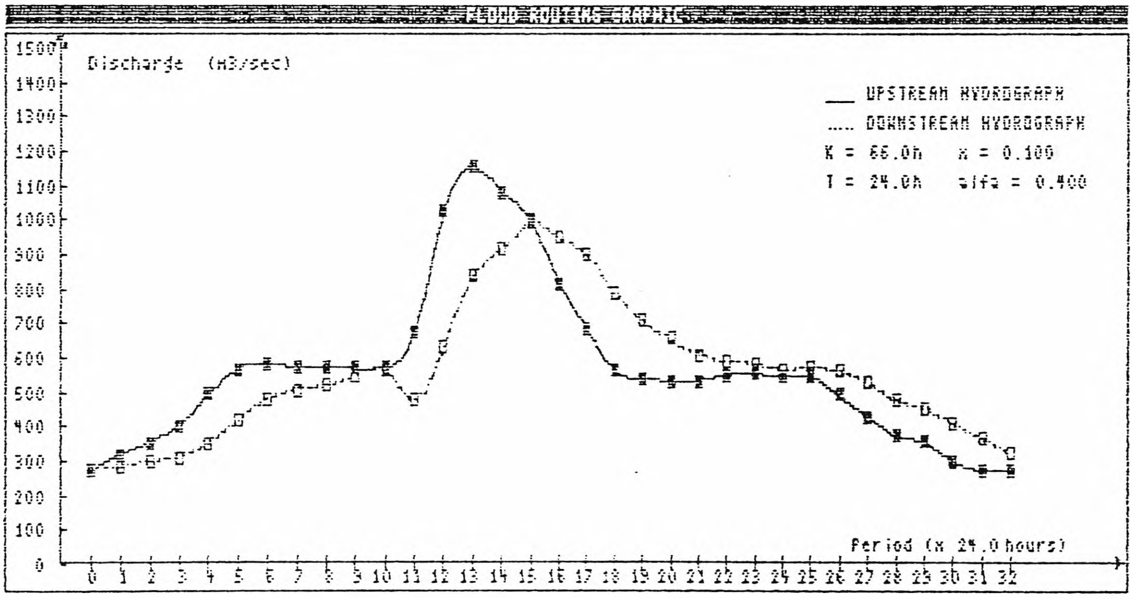
```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.50000
alfa             = 0.40000
Total iterations = 18
```

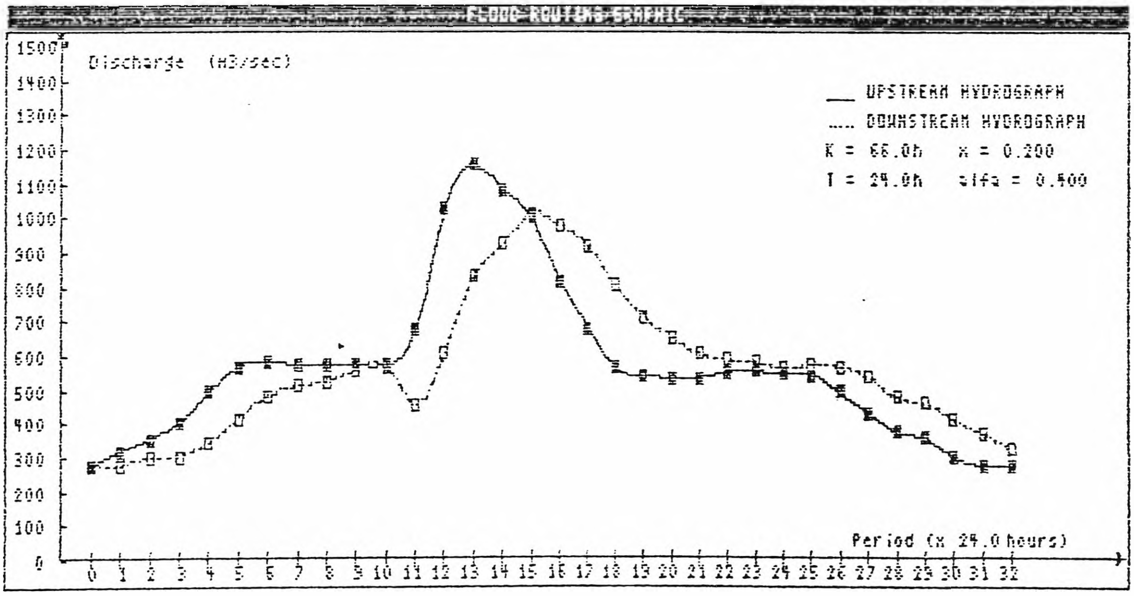
PERIOD (x 24.00 hours)	I N F L O W (observed) (m3/sec)	O U T F L O W (calculated) (m3/sec)	I N F L O W (calculated) (m3/sec)
0	274.000	274.000	274.000
1	314.000	267.481	313.988
2	355.000	279.901	355.037
3	404.000	274.544	404.103
4	495.000	315.060	495.188
5	566.000	405.784	566.152
6	586.000	496.190	585.916
7	572.000	526.573	571.697
8	575.000	543.134	574.760
9	572.000	587.394	572.004
10	571.000	544.242	571.165
11	676.000	553.969	676.150
12	1026.000	527.305	1026.029
13	1156.000	832.946	1155.923
14	1081.000	986.869	1080.920
15	1001.000	1112.087	1001.008
16	816.000	1080.735	816.096
17	681.000	991.147	681.081
18	568.000	832.501	568.010
19	538.000	707.923	537.931
20	534.000	631.574	533.902
21	535.000	576.720	534.943
22	551.000	561.955	550.994
23	555.000	560.805	555.011
24	549.000	552.637	549.020
25	544.000	576.523	544.039
26	493.000	586.419	493.043
27	428.000	553.138	428.028
28	376.000	486.191	376.014
29	357.000	466.554	357.000
30	301.000	417.339	300.982
31	274.000	360.173	273.977
32	271.000	308.545	270.986

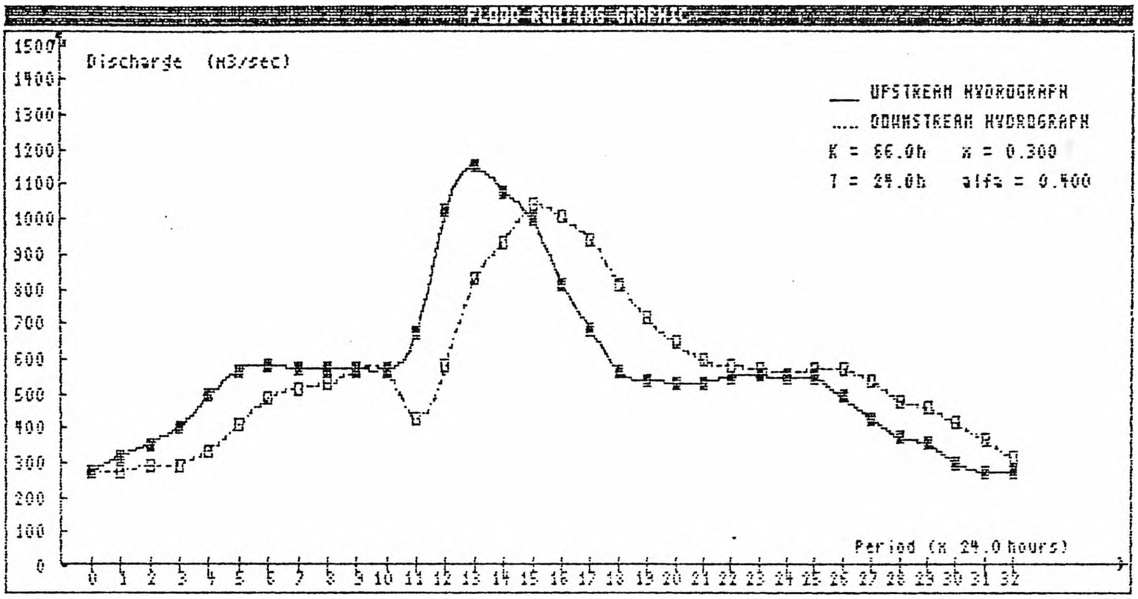
Figures 5.1.5

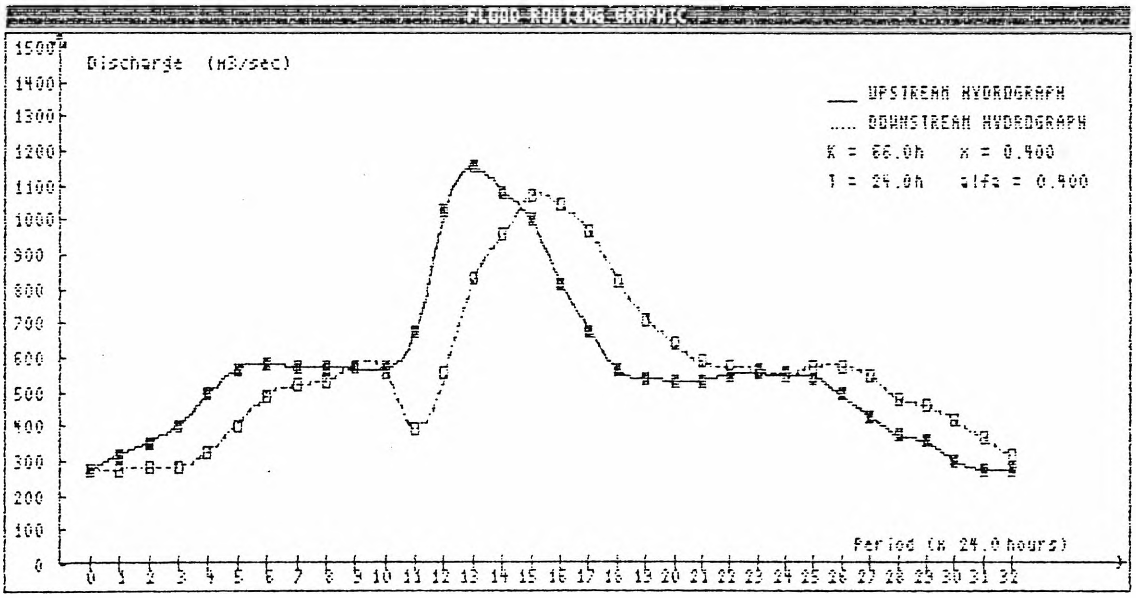
Graphics of Samples of Computations

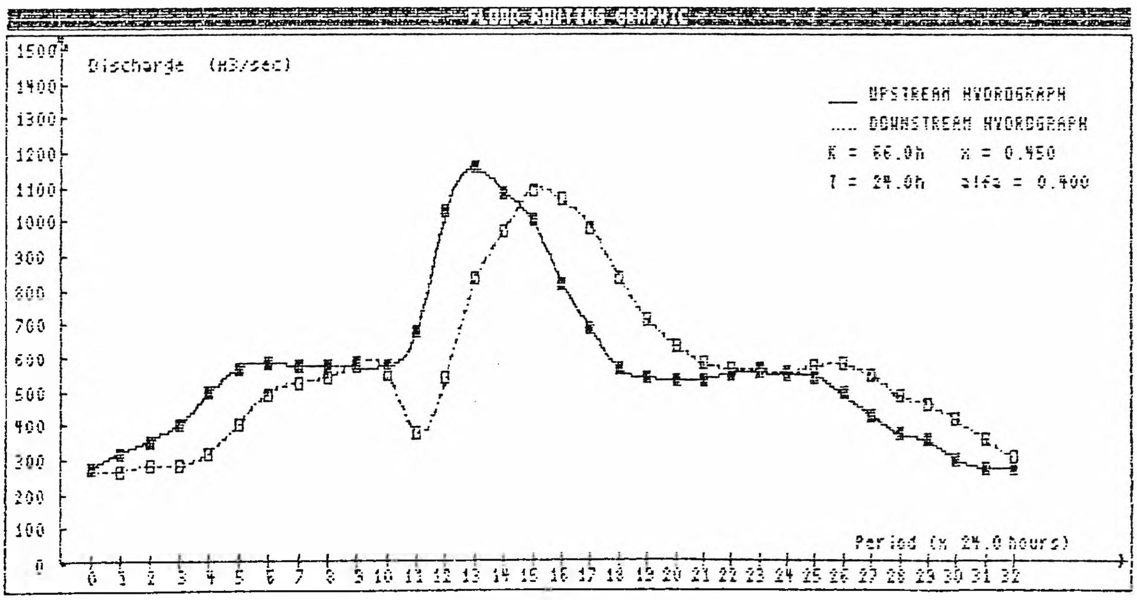

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

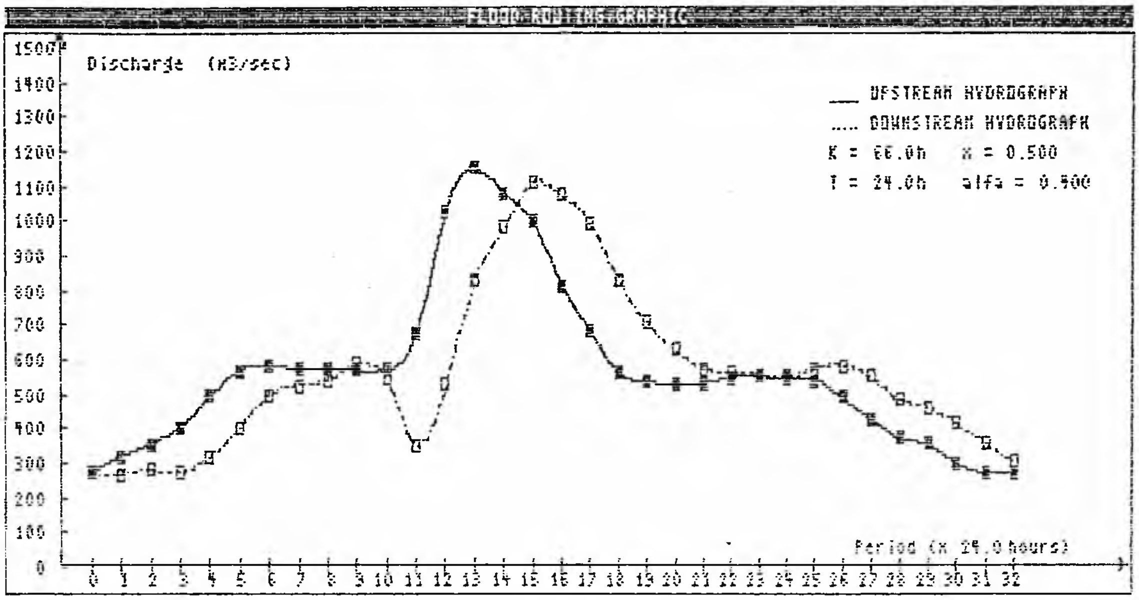

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.900
Period (x 24.0 hours)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Secondly, the iterative method for upstream routing was applied to recorded upstream and downstream hydrographs, to examine the case which occurs in practice where the known downstream hydrograph is used to estimate an unknown upstream hydrograph. In this test, the upstream plus the downstream hydrograph from ARR 87 Table 7.1, page 134 were used. Figure 5.1.6a shows results for K = 66 hours, Δt = 24 hours and x = 0.45. As for conventional Muskingum routing from upstream to downstream, the estimated and recorded hydrographs cannot be expected to agree exactly because the movement of flood waves in river reaches does not exactly conform with the behaviour assumed in the linear Muskingum equation. Note however that the method gives calculated hydrographs that agree reasonably well with the recorded upstream hydrograph. Figure 5.1.6b shows conventional Muskingum downstream routing to estimate the downstream hydrograph from a recorded upstream hydrograph. Comparison of Figures 5.1.6a and 5.1.6b shows that the upstream routing method developed in this study has the same order of accuracy as conventional Muskingum downstream routing.

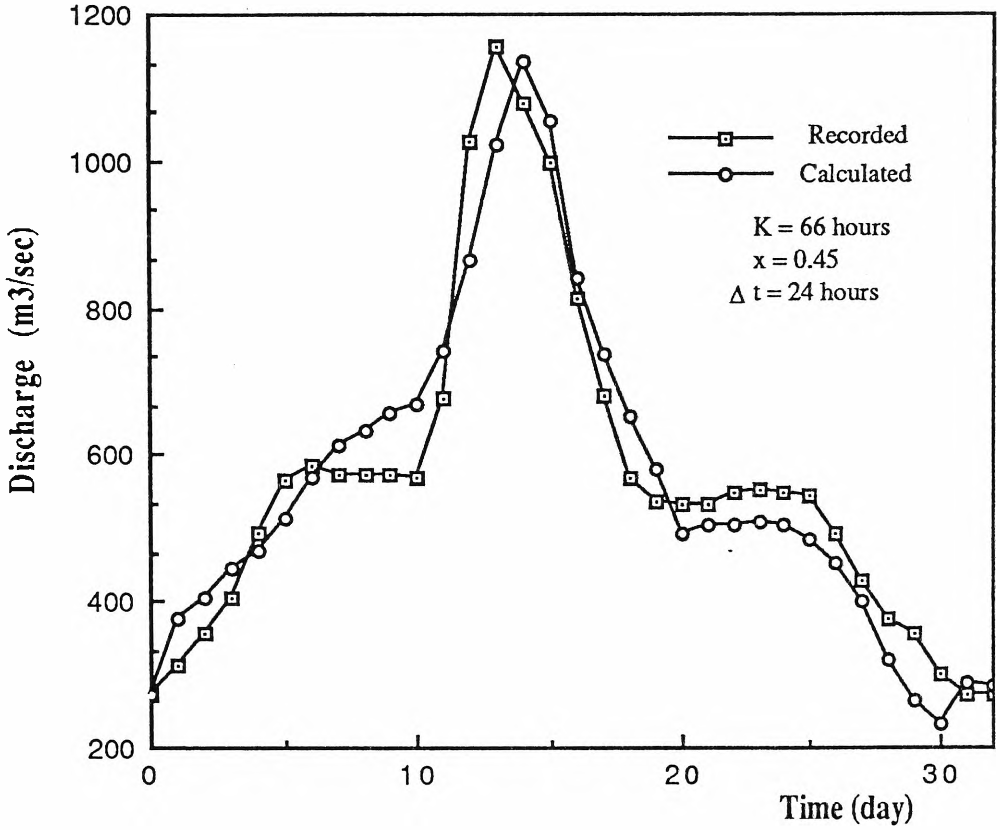


Figure 5.1.6a Upstream Routing to Obtain Upstream Hydrograph Using Iterative Method

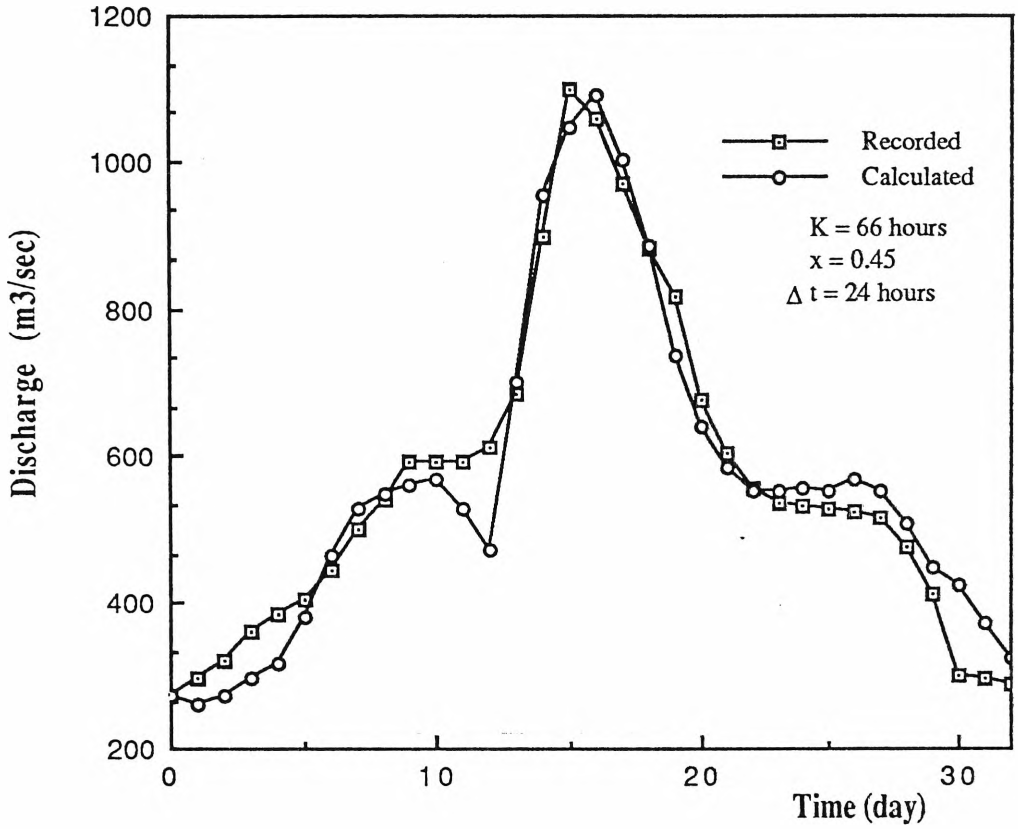
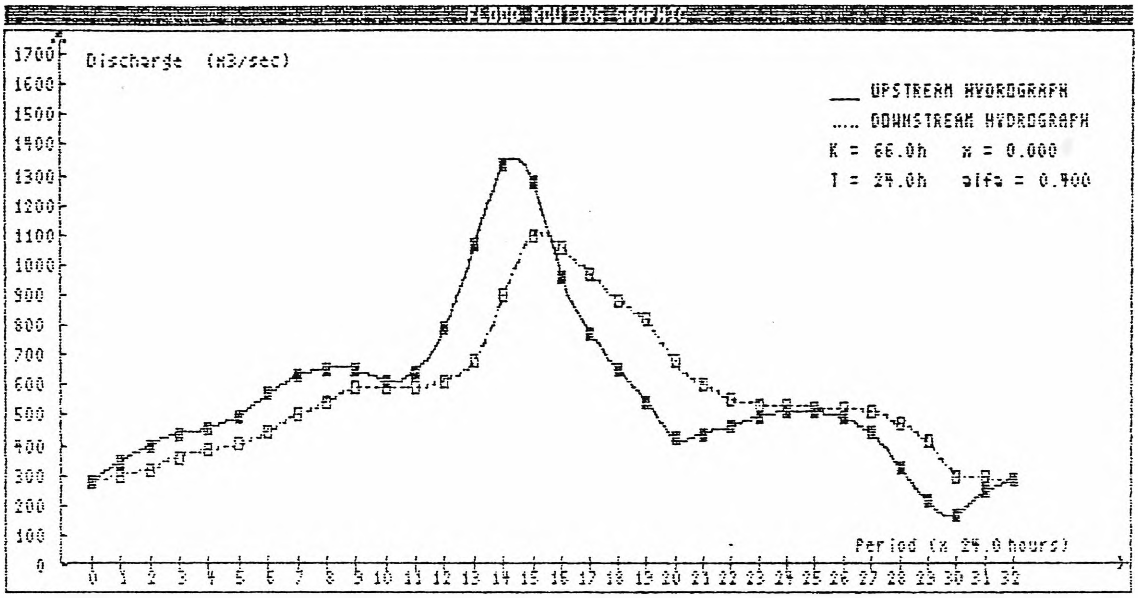


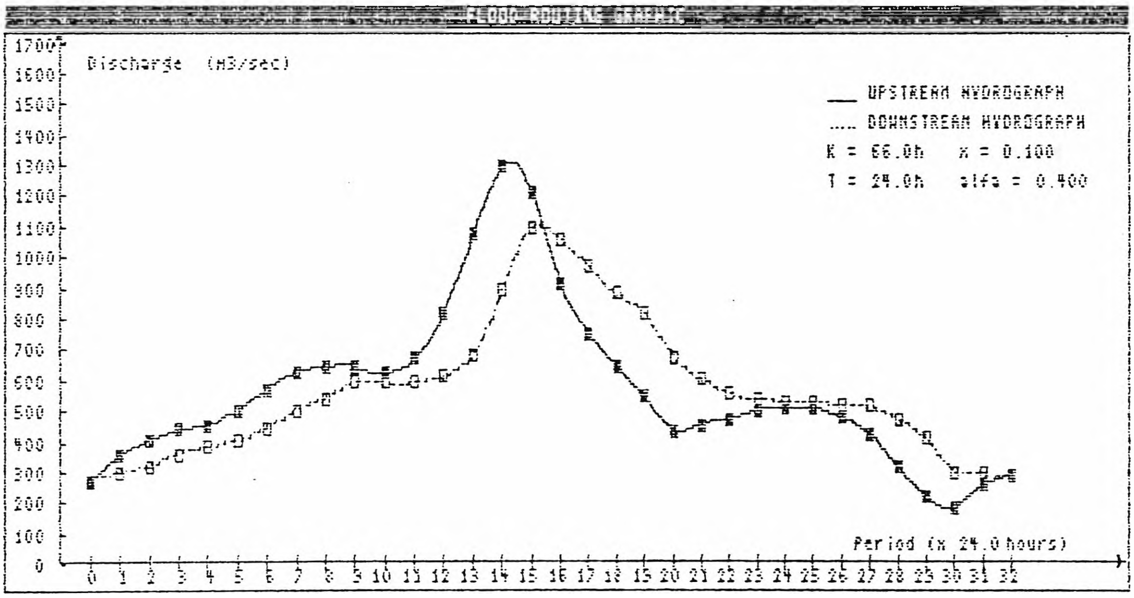
Figure 5.1.6b Downstream Routing to Obtain Downstream Hydrograph Using Standard Muskingum Equation

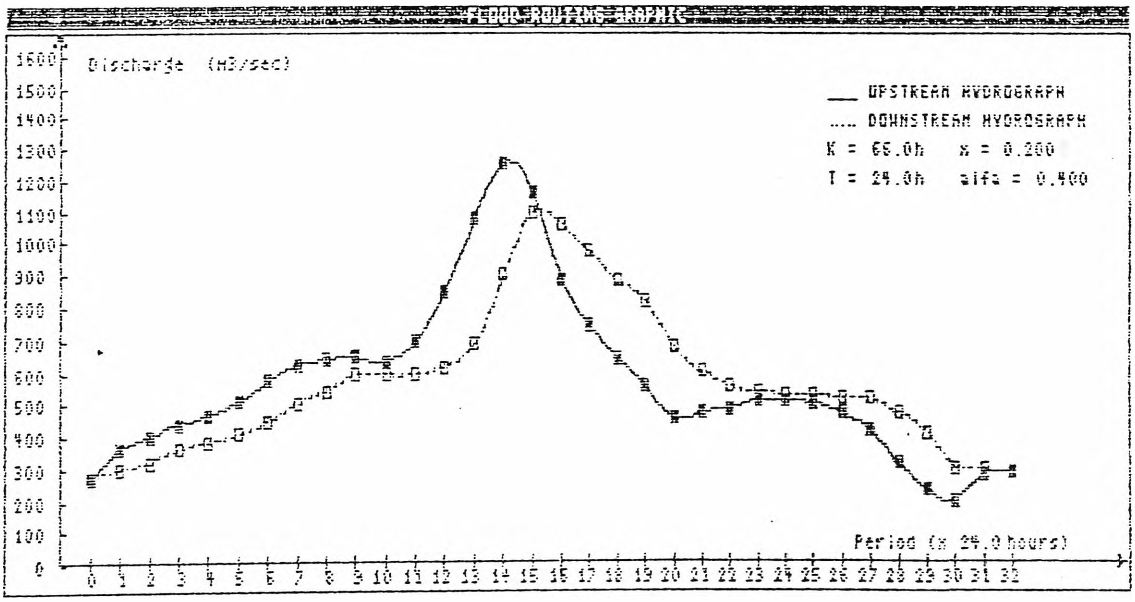
Results of computations using the same observed downstream hydrograph, K = 66 hours and Δt = 24 hours but different parameter x values are shown in Figs. 5.1.6c. In each case the iterative method works well, with approximately 15 iterations required to produce an upstream hydrograph.

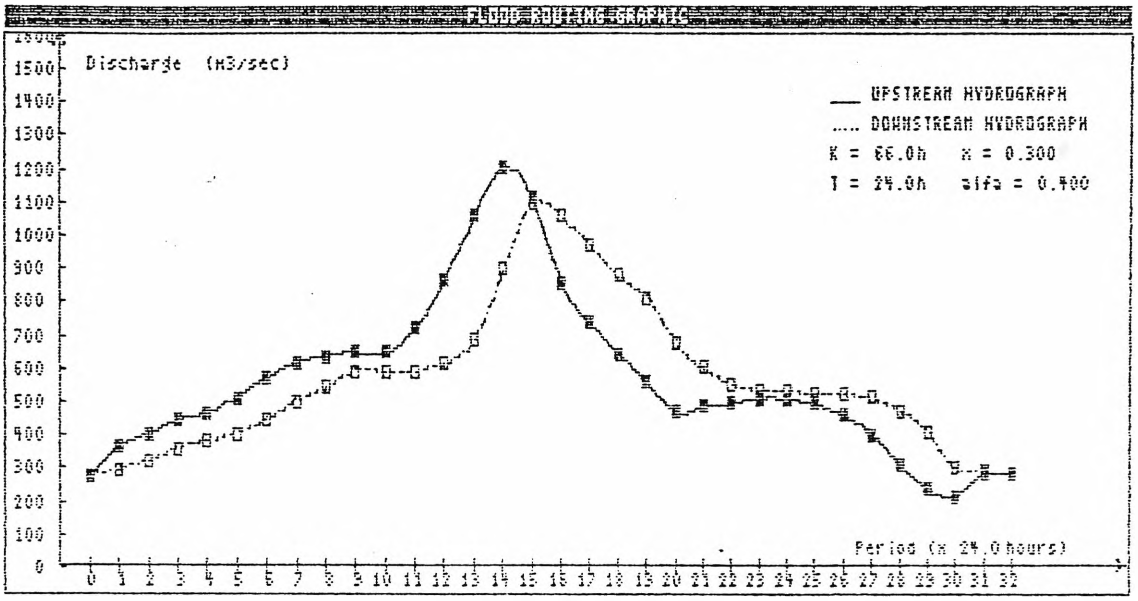
Figures 5.1.6c

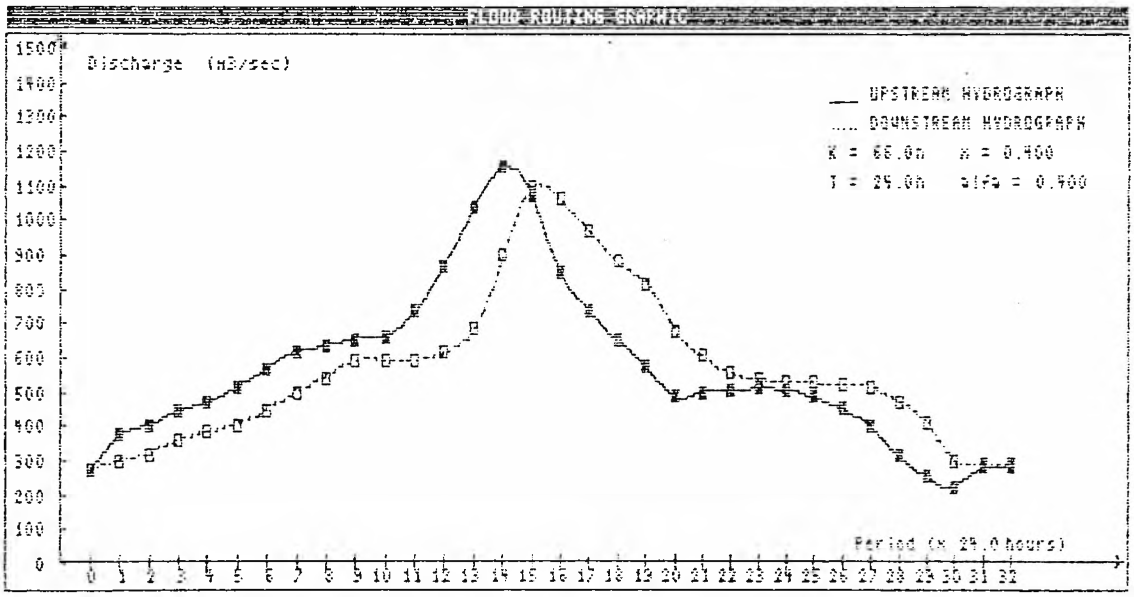
Upstream Routing Using Observed Downstream Hydrograph

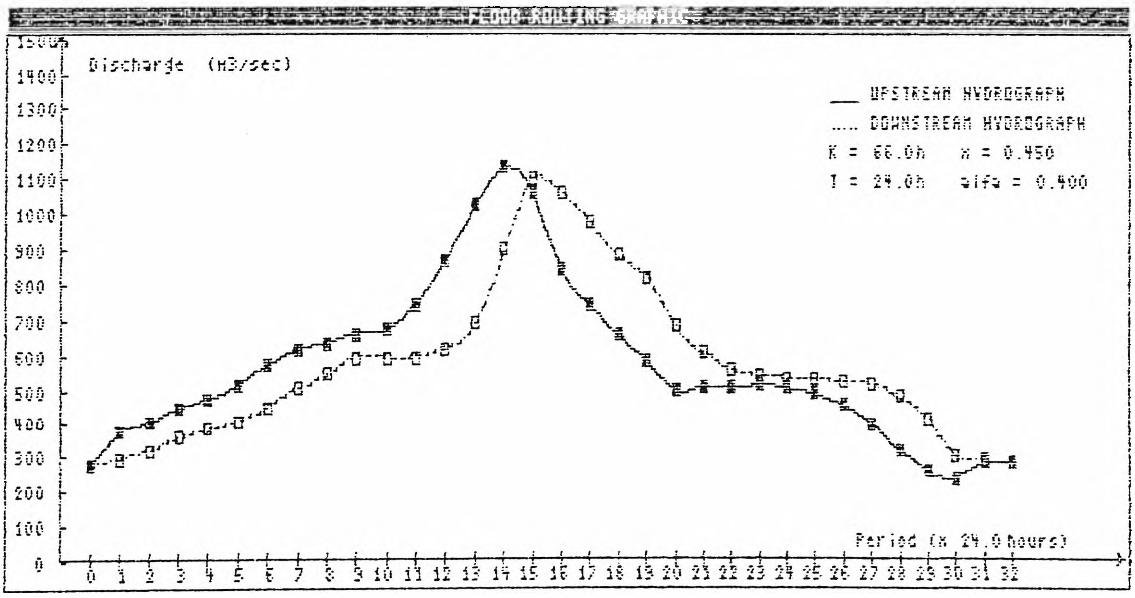

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

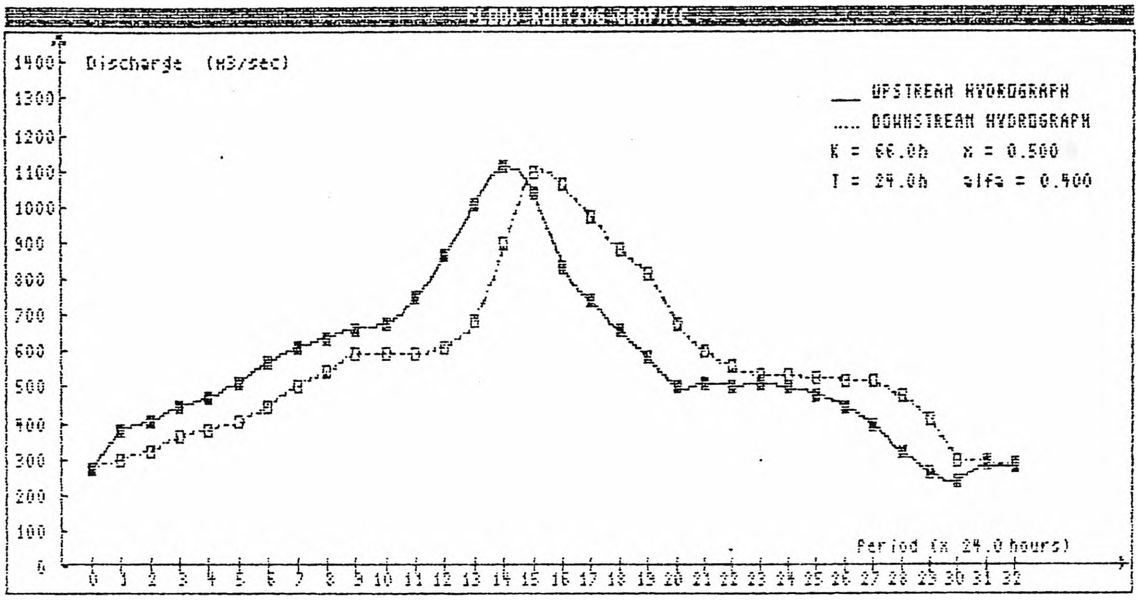

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

5.2 ITERATIVE METHOD WITH BACKWARD DIFFERENCE AT THE END OF THE HYDROGRAPH

As mentioned in section 5.1, a difficulty arises in calculating the derivative at the end of the hydrograph (at time $i = N$). Equation (5.1.2) cannot be properly used to calculate $dS/dt|_N$, since S_{N+1} is not known. Therefore, an assumption must be made. Equation (5.1.5) is one of the assumptions which can be taken into account. The other possible assumption is applying a backward finite difference. There are two types which have been investigated, backward finite difference based on the first derivative and backward finite difference based on the second derivative. The procedure of computation is entirely the same as that discussed in section 5.1.

First of all, these backward differences are derived prior to their application in the computation. The derivation is taken from Salvadori and Baron (1964). Given the values

$$y_0, y_1, y_2, \ldots, y_{ll}, y_l, y_i, y_r, y_{rr}, \ldots, y_{n-2}, y_{n-1}, y_n$$

of a function y(x) at the pivotal points of its interval of definition, evenly spaced by h, the first backward difference of y at i is

$$\nabla y_i = y_i - y_l \qquad (5.2.1)$$

The second backward difference of y at i is defined as the difference of the first difference and is therefore given by

$$\nabla(\nabla y_i) \equiv \nabla^2 y_i = (y_i - y_l) - (y_l - y_{ll})$$
$$= y_i - 2.y_l + y_{ll} \qquad (5.2.2)$$

Similarly, the n th backward difference is the difference of (n-1) th difference:

$$\nabla^n y_i \equiv \nabla(\nabla^{n-1} y_i)$$

It is well known that the differential operator $D \equiv d/dx$ can be used symbolically as if it were a number, in as much as it satisfies formally the fundamental laws of algebra. The difference operator ∇ may also be used symbolically as a number (or variable), since it satisfies formally the laws of algebra, as shown by the following identities:

$$\nabla(y_i + y_j) = \nabla y_i + \nabla y_j = \nabla y_j + \nabla y_i;$$
$$\nabla(c.y_i) = c\nabla y_i;$$
$$\nabla^m(\nabla^n y_i) = \nabla^{m+n} y_i;$$

Making use of these properties, it is possible to express the differences of a function y in terms of its successive derivatives and, conversely, its derivatives in terms of its successive differences. The derivation of these expressions by symbolical methods is by far the most efficient.

Consider for this purpose the Taylor expansion of y(x+h) about x:

$$y(x+h) = y(x) + \frac{h}{1!}y'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + \ldots, \quad \text{(a)}$$

which, using the powers of symbol D to indicate the derivatives of y, becomes

$$y(x+h) = y(x) + \frac{h}{1!}Dy(x) + \frac{h^2}{2!}D^2y(x) + \frac{h^3}{3!}D^3y(x) + \ldots$$
$$= (1 + \frac{h}{1!}D + \frac{h^2}{2!}D^2 + \frac{h^3}{3!}D^3 + \ldots)\, y(x) \quad \text{(b)}$$

By means of the series expansion for $e^{\pm x}$,

$$e^{\pm x} = 1 \pm \frac{x}{1!} + \frac{x^2}{2!} \pm \frac{x^3}{3!} + \ldots,$$

the differential operator on the right-hand side of eq.(b) may be written symbolically as

$$1 + \frac{hD}{1!} + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \ldots = e^{hD}, \quad (5.2.3)$$

and hence y(x+h) may also be written symbolically as

$$y(x+h) = e^{hD}.y(x) \quad (5.2.4)$$

Setting $x = x_i$ and indicating as before $y(x_i+h)$ by y_r and $y(x_i)$ by y_i, eq.(5.2.4) becomes

$$y_r = e^{hD}.y_i \quad (5.2.5)$$

Similarly, changing h into -h in eq.(5.2.4),

$$y(x-h) = e^{-hD}.y(x) \tag{5.2.6}$$

and letting, as before, $y(x) = y_i$, and $y_l = y(x_i - h)$, eq.(5.2.6) becomes

$$y_l = e^{-hD}.y_i \tag{5.2.7}$$

The first backward difference ∇y_i [eq.(5.2.1)] may now be written by means of eq.(5.2.7) as

$$\nabla y_i = y_i - y_l = [1 - e^{-hD}].y_i \tag{5.2.8}$$

or, by eq.(5.2.3), as

$$\begin{aligned}\nabla y_i &= \left(\frac{hD}{1!} - \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} - \frac{h^4D^4}{4!} + \dots\right) y_i \\ &= \left(1 - \frac{hD}{2} + \frac{h^2D^2}{6!} - \frac{h^3D^3}{24} + \dots\right)hDy_i\end{aligned} \tag{5.2.9}$$

Equation (5.2.9) gives the expression of ∇y_i into an infinite series of all the derivatives of y at i.

If eq.(5.2.8) is written in purely operational form, by dropping y_i on both sides of the equation,

$$\nabla = 1 - e^{-hD}, \tag{5.2.10}$$

its 'powers' may be used to evaluate the series expansions for the successive differences of a function. Thus, squaring eq.(5.2.10), and making use of eq.(5.2.3), the expansion for the second difference ∇^2 can be obtained in the form

$$\begin{aligned}\nabla^2 = (1 - e^{-hD})^2 &= 1 + e^{-2hD} - 2.e^{-hD} \\ &= 1 + \left(1 - \frac{2hD}{1!} + \frac{4h^2D^2}{2!} - \frac{8h^3D^3}{3!} + \frac{16h^4D^4}{4!} - \dots\right) \\ &\quad - 2\left(1 - \frac{hD}{1!} + \frac{h^2D^2}{2!} - \frac{h^3D^3}{3!} + \frac{h^4D^4}{4!} - \dots\right),\end{aligned}$$

or

$$\nabla^2 = h^2.D^2 - h^3.D^3 + \frac{7}{12}h^4.D^4 - \dots \tag{5.2.11}$$

Conversely, to obtain expressions for the derivatives of y in terms of its difference, solve eq.(5.2.10) for e^{-hD} :

$$e^{-hD} = 1 - \nabla \tag{5.2.12}$$

and take the natural logarithms of both sides of this equation, obtaining

$$\ln e^{-hD} = -hD = \ln (1 - \nabla) \qquad (5.2.13)$$

The series expansion of ln (1 ± x) equals

$$\ln (1 \pm x) = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} - \frac{x^4}{4} \pm \frac{x^5}{5} - \dots$$

therefore eq.(5.2.13) can be written as

$$\ln e^{-hD} = - hD = \ln (1 - \nabla) = - (\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \dots)$$

the expansion of the first derivative D into an infinite series of differences becomes

$$hD = \nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \dots \qquad (5.2.14)$$

The difference expansions (5.2.9), (5.2.10), (5.2.11) and (5.2.14) allow the simple derivation of *unilateral differentiation formula* and of their errors.

For example, solving eqs. (5.2.9) and (5.2.11) for D and D^2, respectively, eqs.(5.2.15) can be obtained.

$$D = \frac{\nabla}{h} + \frac{hD^2}{2} - \frac{h^2D^3}{6} + \frac{h^3D^4}{24} - \dots ,$$

$$D^2 = \frac{\nabla^2}{h^2} + hD^3 - \frac{7h^2D^4}{12} + \dots , \qquad (5.2.15)$$

from which, taking into account the first term of the series only,

$$Dy_i = \frac{1}{h} (y_i - y_l) + O(h) , \qquad (5.2.16a)$$

$$D^2y_i = \frac{1}{h^2} (y_i - 2.y_l + y_{ll}) + O(h) , \qquad (5.2.16b)$$

Where the symbol O(h) stands for an error 'of the order of h' and is the sum of the terms neglected in eqs.(5.2.15).

It can similarly be proved that the approximation of the n th derivative by the first term of its backward difference expansion has an error of the order of h.

To obtain formulas with errors of order h^2, the first two terms of the derivative expansions into differences must be taken into account.

Thus, eliminating h^2D^2 between eqs.(5.2.9) and (5.2.11) results in

$$\nabla + \frac{\nabla^2}{2} = hD - \frac{1}{3}h^3D^3 + \ldots$$

or, by eqs.(5.2.1) and (5.2.2),

$$Dy_i = \frac{1}{2h}(3.y_i - 4.y_1 + y_1) + O(h^2) \qquad (5.2.17)$$

In general, if the first m terms of the derivative expansions into backward differences are taken into account, the corresponding formulas have errors of order h^m.

5.2.1 Computation Using Backward Difference Based on the Second Derivative at the End of Hydrograph

Neglecting the error term, equation (5.2.17) can be adopted to calculate the derivative S at the end of hydrograph (at time i = N). That equation becomes

$$\frac{dS}{dt}\Big|_N = \frac{1}{2.\Delta t}(3.S_N - 4.S_{N-1} + S_{N-2}) \qquad (5.2.18)$$

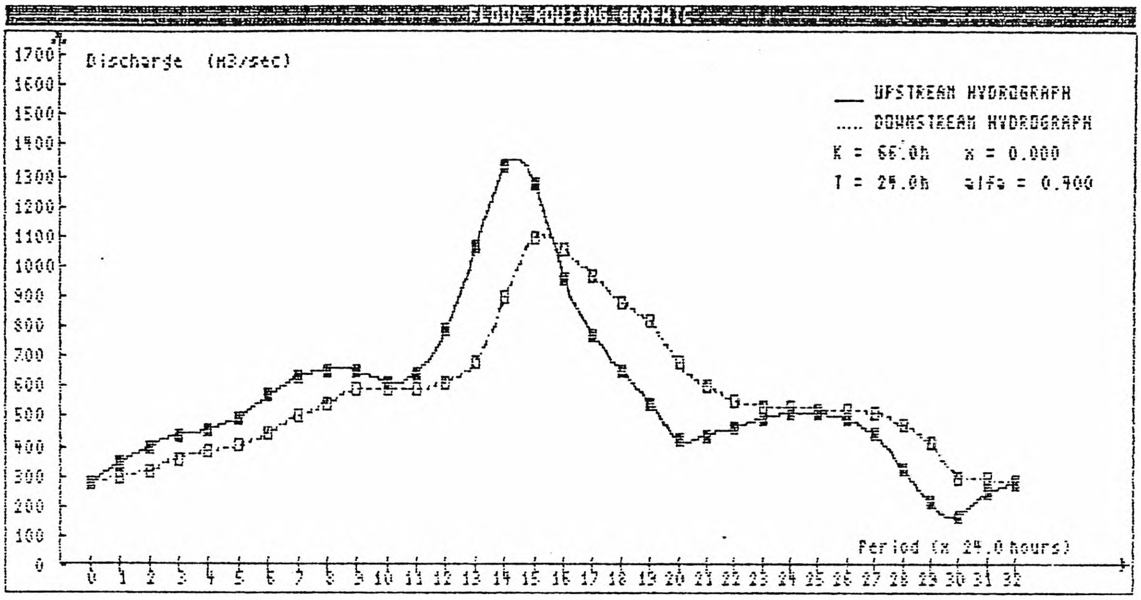
Test results indicate that this scheme is less satisfactory than that of eq.(5.1.5). The results are very similar to those obtained from section 5.1, except for the tails of the hydrograph. The discharges in the tail tend to become smaller as the parameter x value increases. Figures 5.2.1 show results using the observed downstream hydrograph ordinates taken from ARR87 Table 7.1 page 134.

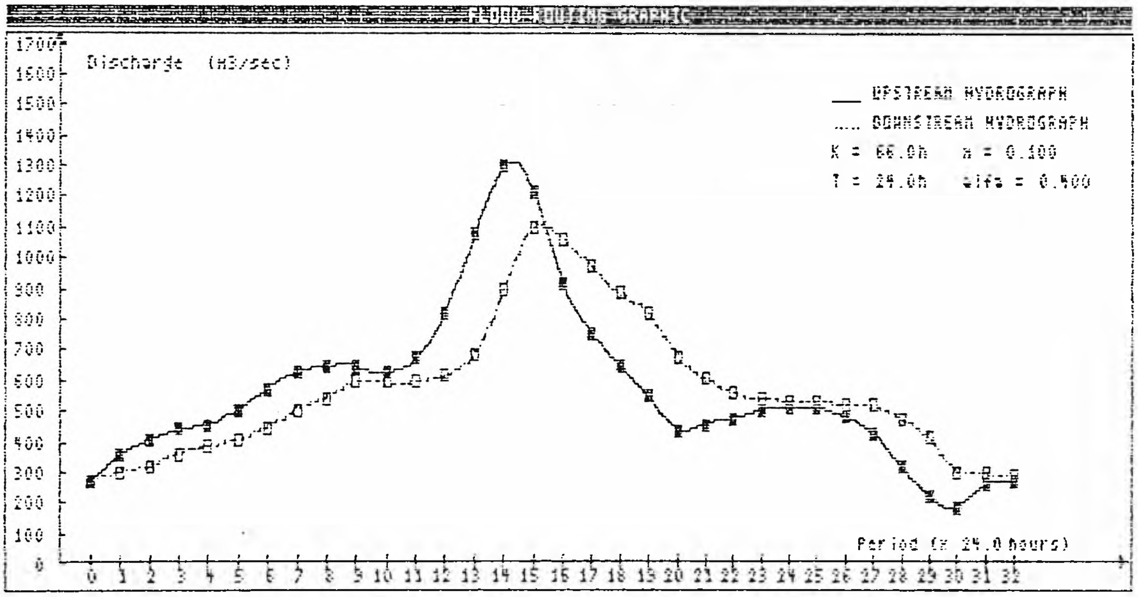
The other problem which arises is that if various time steps Δt are used, the tails of the hydrographs are inconsistent. For example using the same data above with Δt = 22, 24 and 26 hours, K = 66 hours and parameter x = 0.1, the results are presented in Figures 5.2.2.

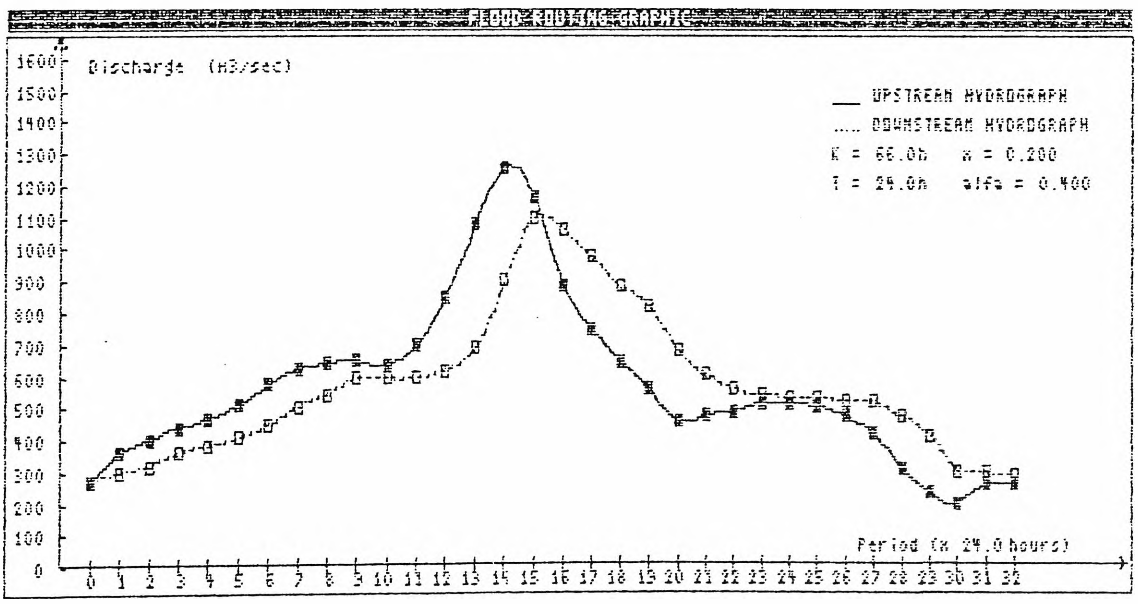
It can be noticed from Figures 5.2.2 that each tail of the upstream hydrograph is not consistent with the others. The hydrograph tails do not vary consistently as the time step Δt changes. This circumstance does not occur if the same data are evaluated using the scheme discussed in section 5.1, i.e.: eq.(5.1.5). Small differences among the tails of the hydrographs occur in this case, but these occur only because of the linear interpolation needed to estimate discharges for other than 24 hours. The results are shown in Figures 5.2.3.

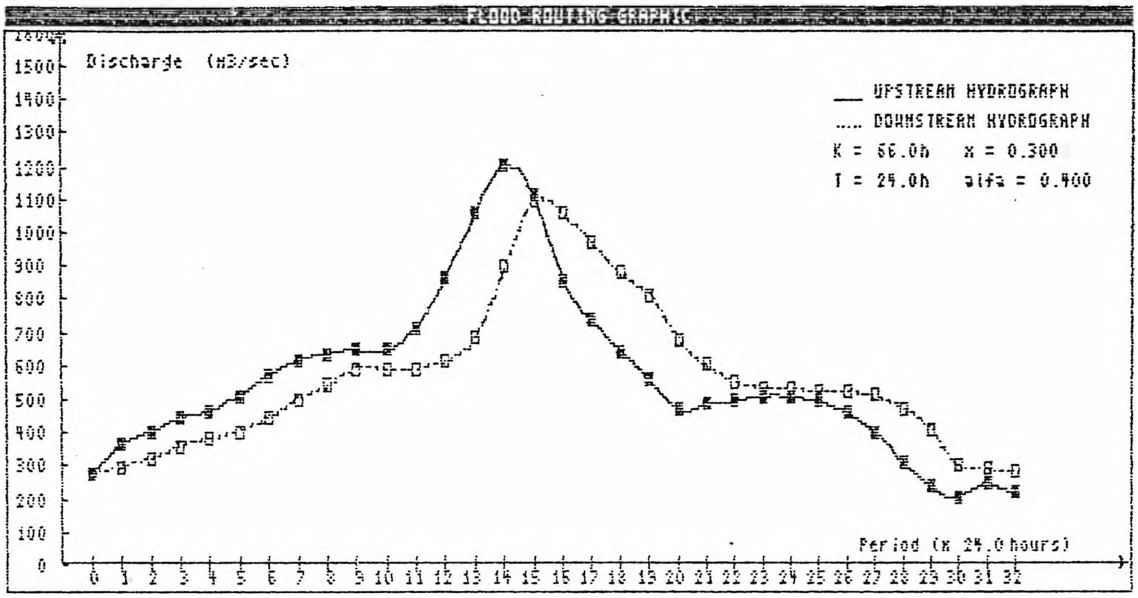
Figures 5.2.1

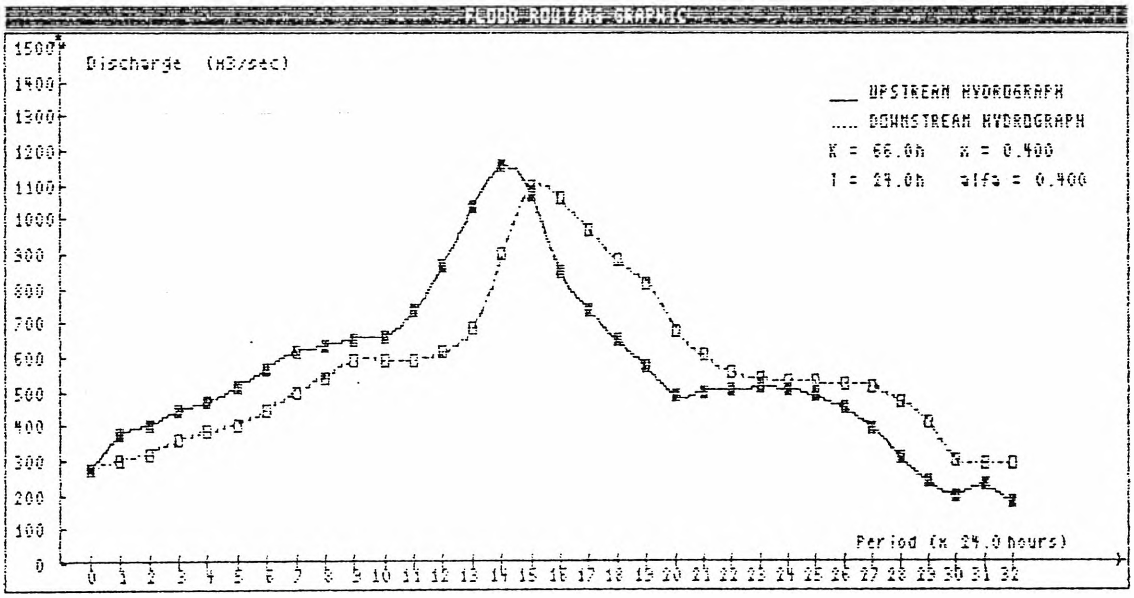
Upstream Routing with Backward Difference Based on the Second Derivative at the End of Hydrograph Using Observed Downstream Hydrograph

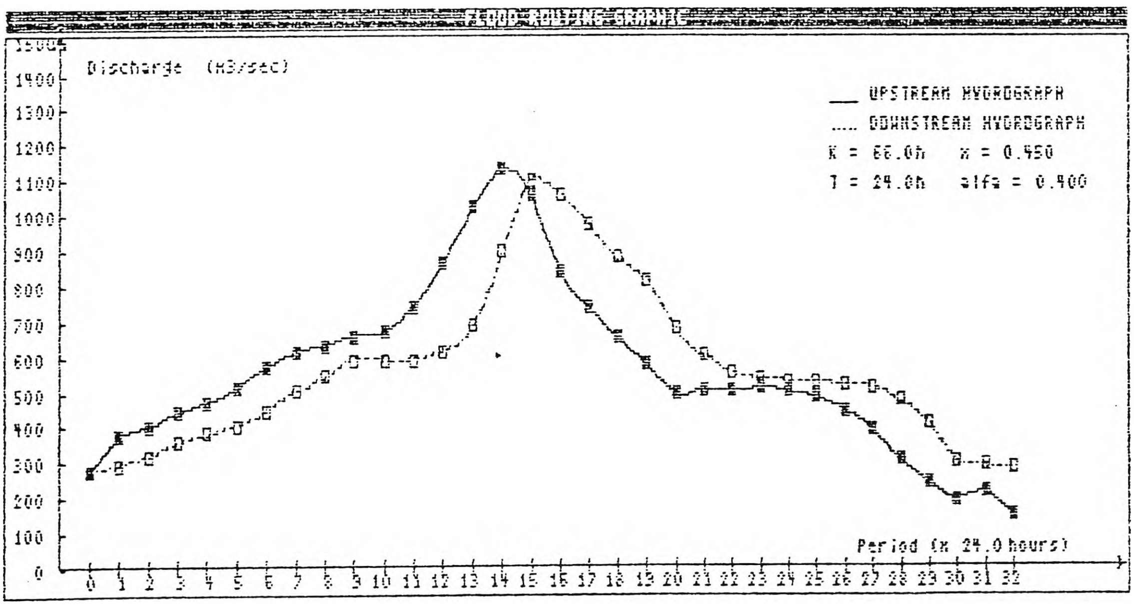

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

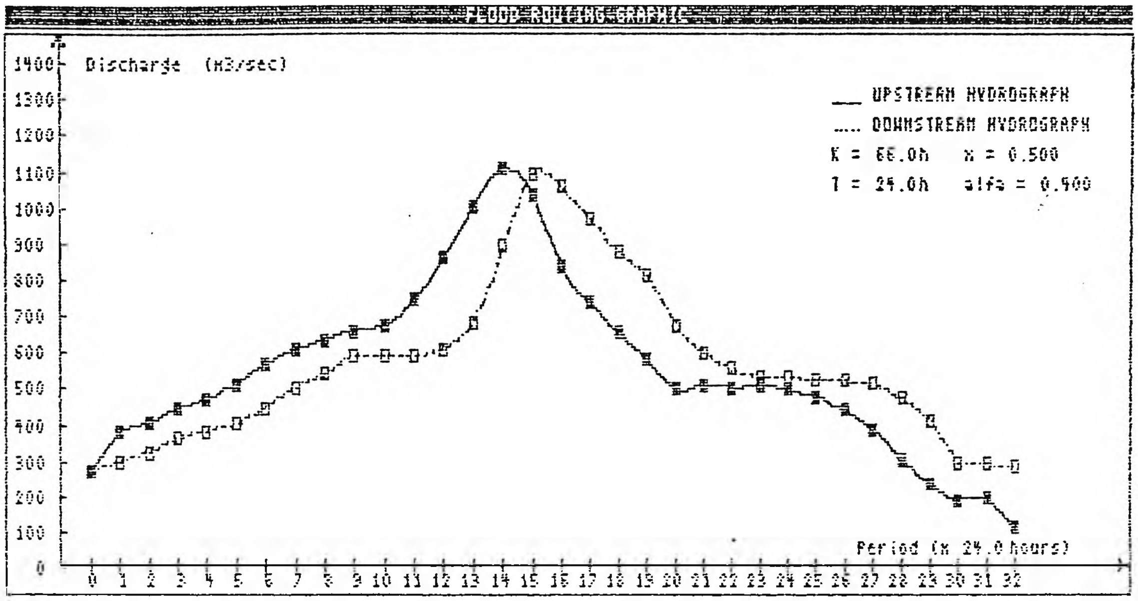

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

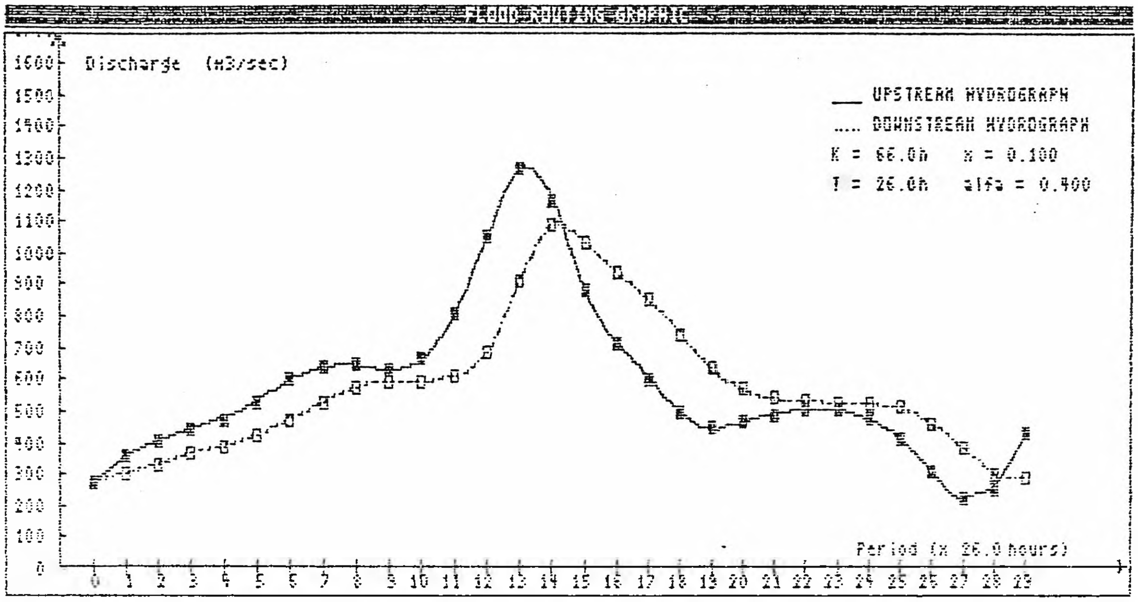

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

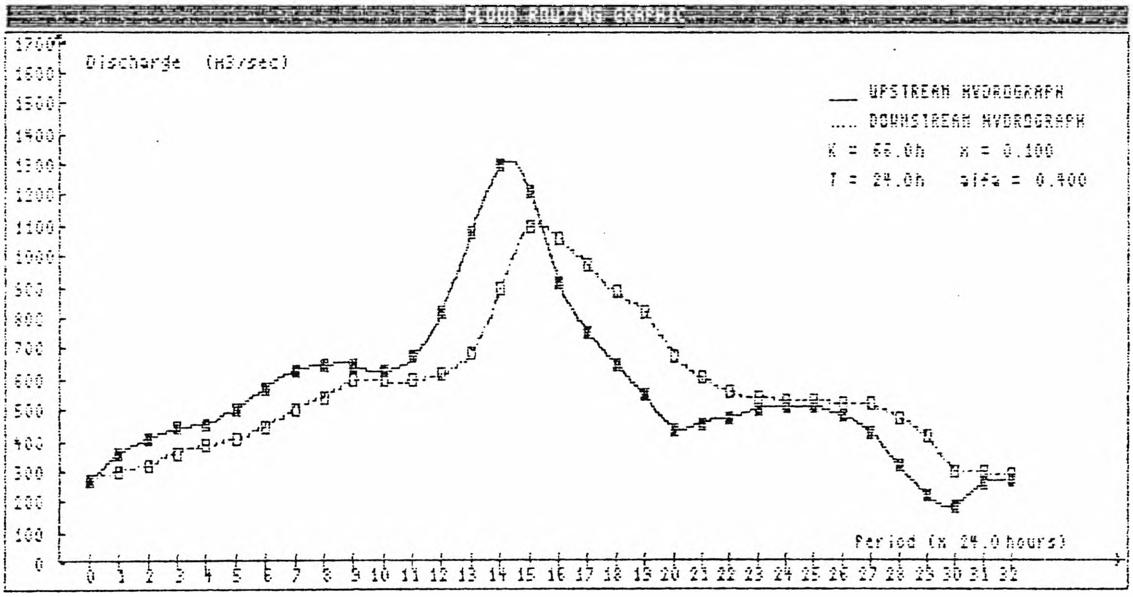

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

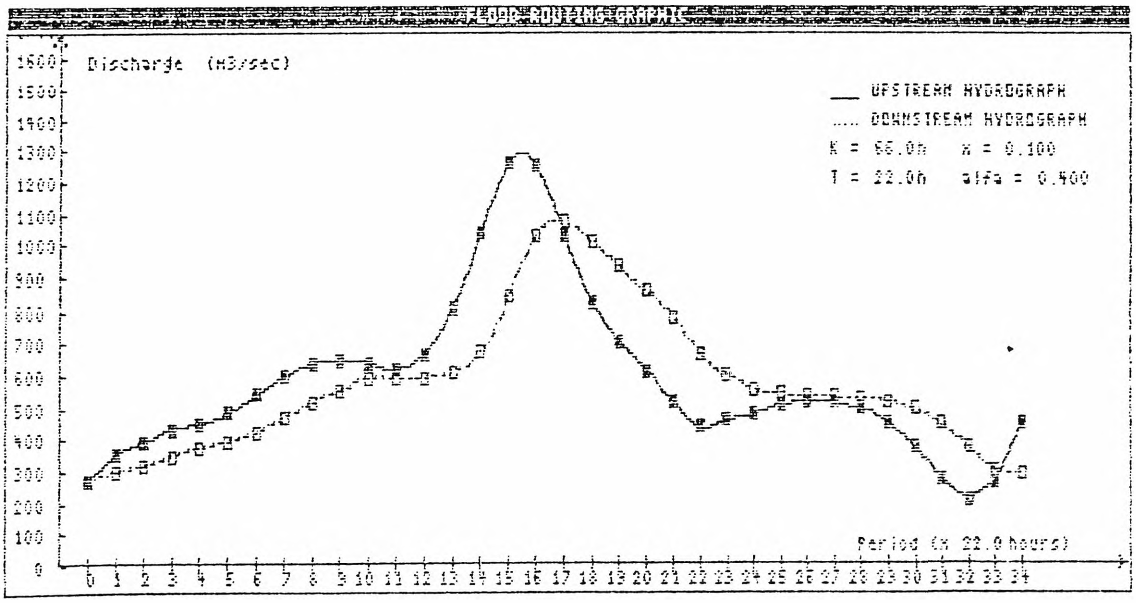

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.900
Period (x 24.0 hours)
0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Figures 5.2.2

Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Backward Difference Based on the Second Derivative at the End of Hydrograph

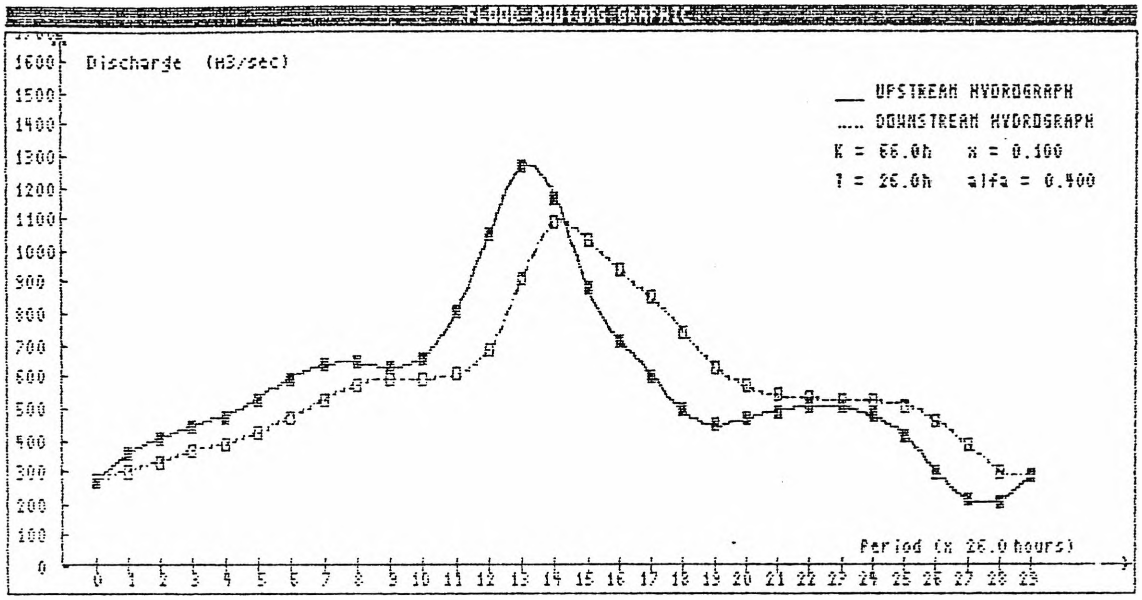

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 26.0h alfa = 0.400
Period (x 26.0 hours)

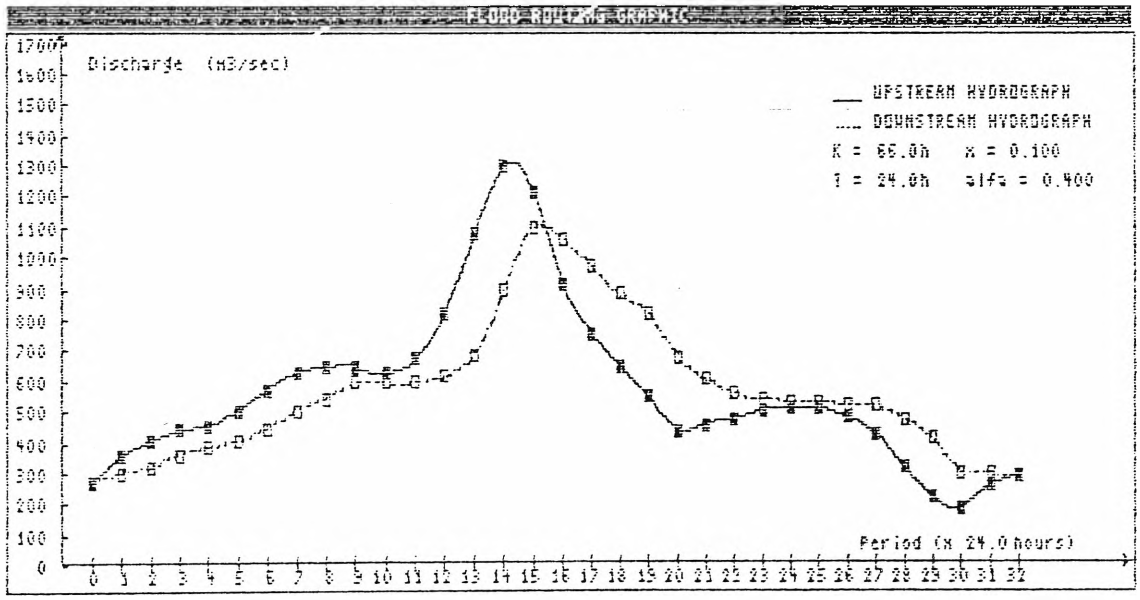

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

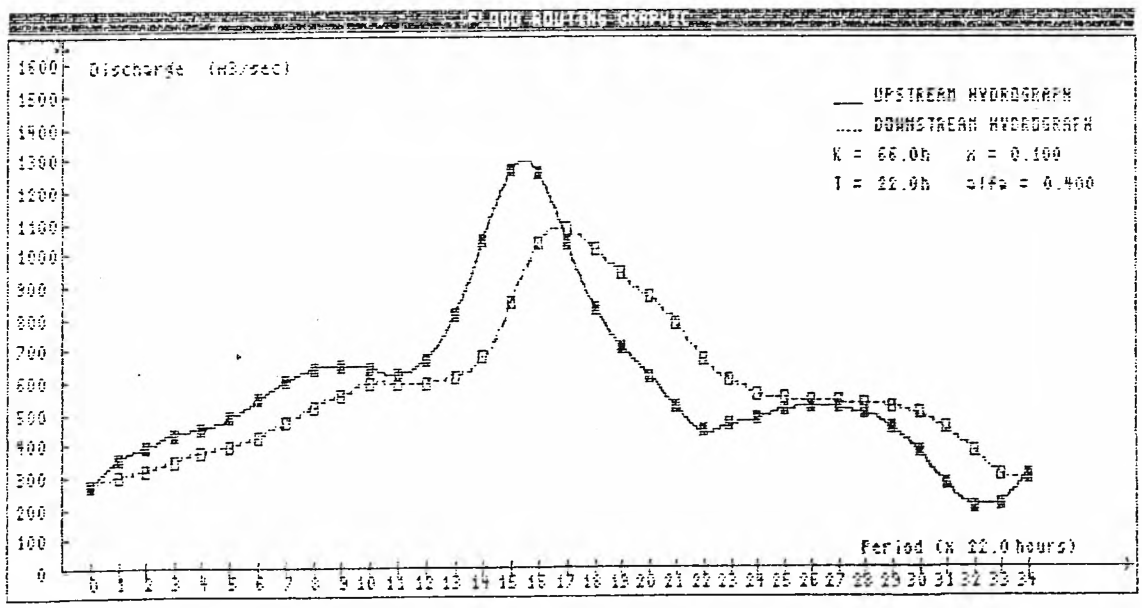

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 22.0h alfa = 0.400
Period (x 22.0 hours)

Figures 5.2.3

Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Basic Method (Eq. 5.1.5)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 26.0h alfa = 0.400
Period (x 26.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 22.0h alfa = 0.400
Period (x 22.0 hours)

5.2.2 Computation Using Backward Difference Based on the First Derivative at the End of Hydrograph

Another backward difference which can be taken into account for calculating the derivative S at the end of hydrograph (at time i = N) is equation (5.2.16a). By neglecting the error term and converting the variables used, the equation can be written as

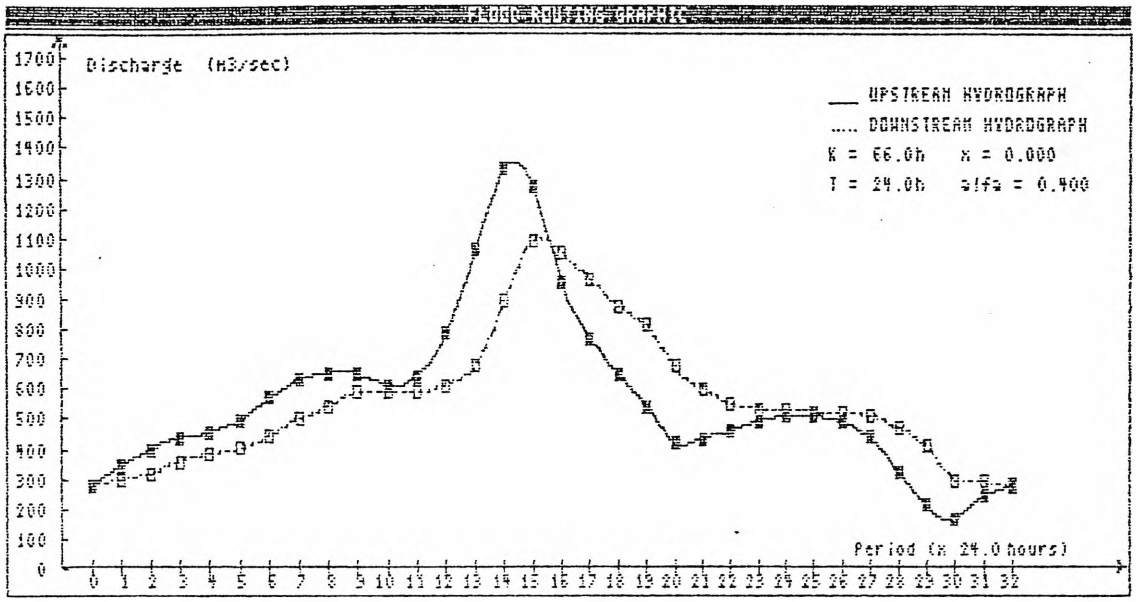
$$\frac{dS}{dt}\Big|_N = (S_N - S_{N-1})/\Delta t \qquad (5.2.19)$$

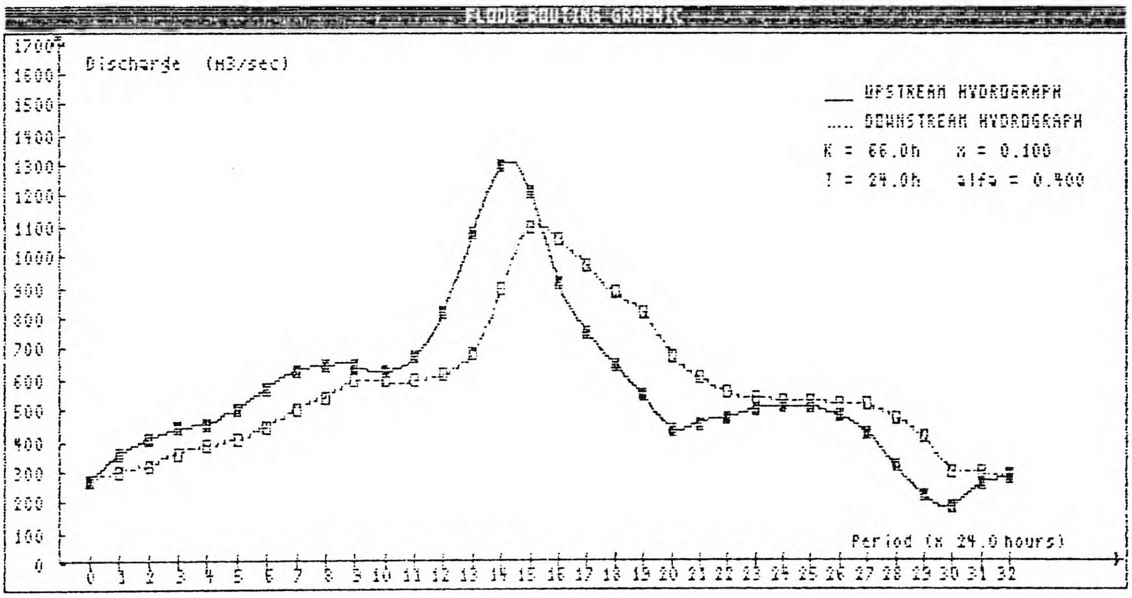
Results of the test of computation show that this scheme gives very similar results to those obtained by using eq.(5.1.5). The slight difference is only at the tail of the hydrograph. It can be said that the rest of the ordinates are precisely the same. This result is reasonable since the method used is iterative, the last calculated ordinate (at time i = N) affects the other ordinates which are relatively close to it in time, or in other words, it propagates up to a certain ordinate. Figures 5.2.4 show results of computations using the observed downstream hydrograph ordinates taken from ARR87 Table 7.1 page 134 for K = 66 hours, Δt = 24 hours and various parameter x values.

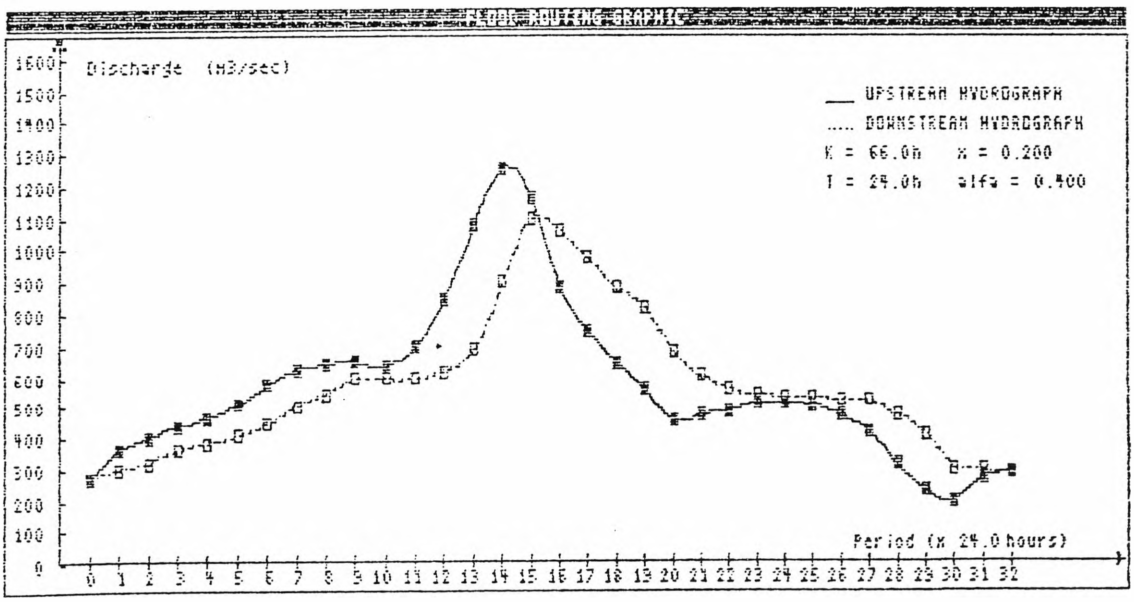
This scheme encounters the same problem as that discussed in section 5.2.1 does, if various time steps Δt are applied for a certain parameter x value. Each tail of the hydrograph is not consistent with the others. However, the deviation of each tail is not as much as that in Figs. 5.2.2. Figures 5.2.5 show the results using K = 66 hours, parameter x = 0.1 and Δt = 22, 24 and 26 hours.

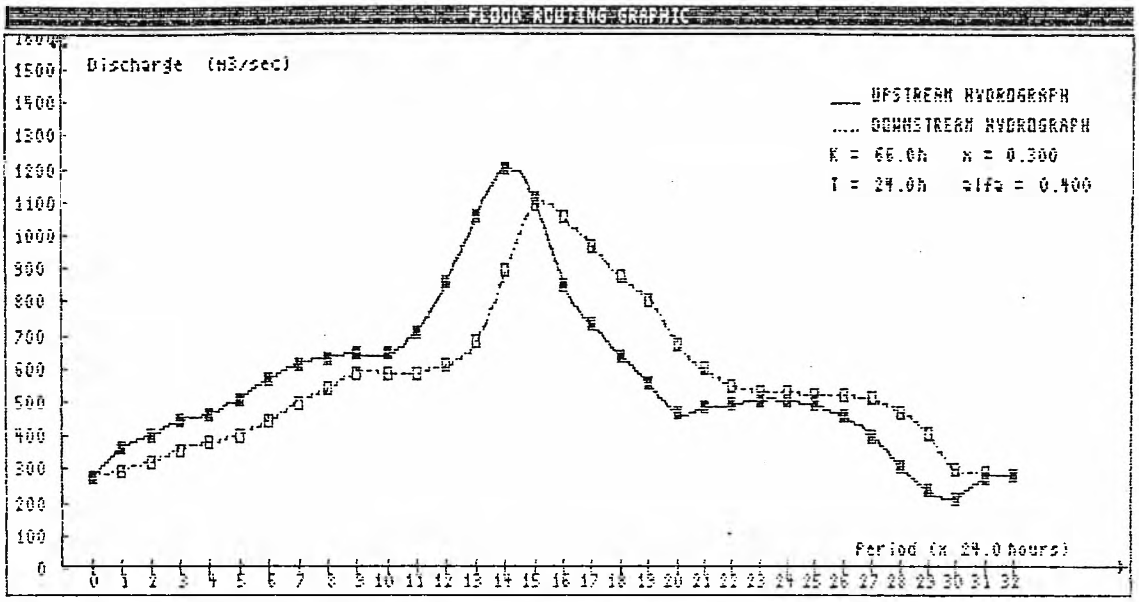
Figures 5.2.4

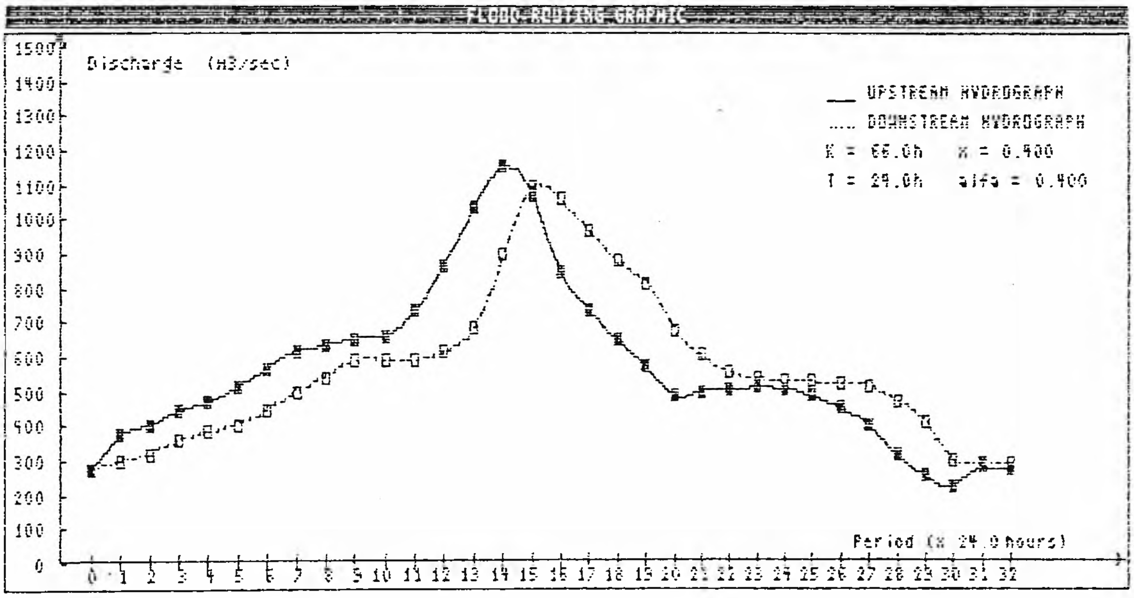
Upstream Routing with Backward Difference Based on the First Derivative at the End of Hydrograph Using Observed Downstream Hydrograph

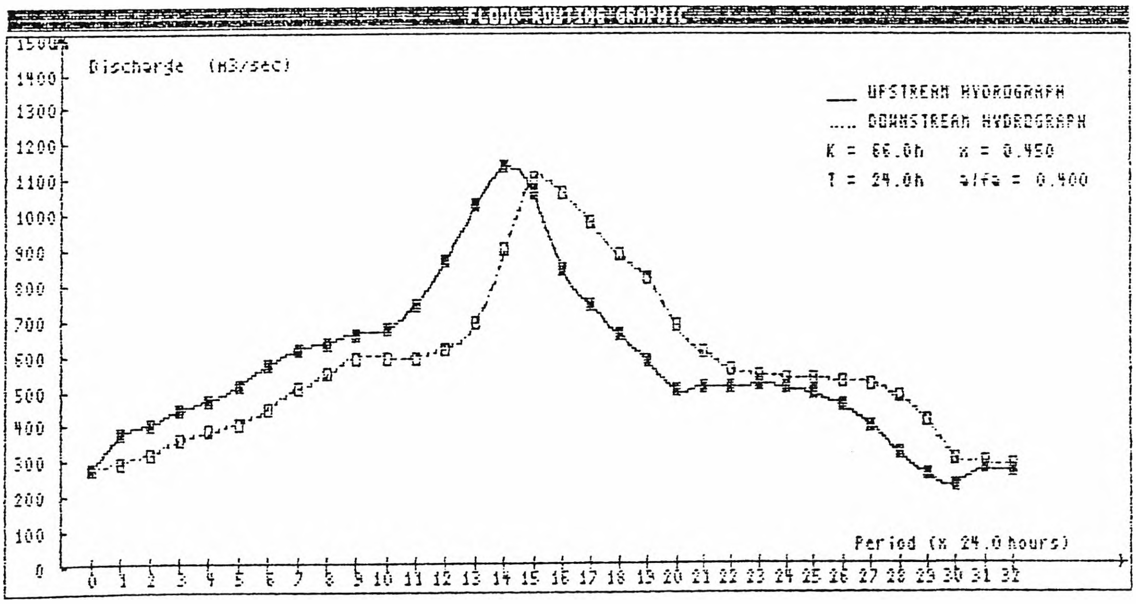

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

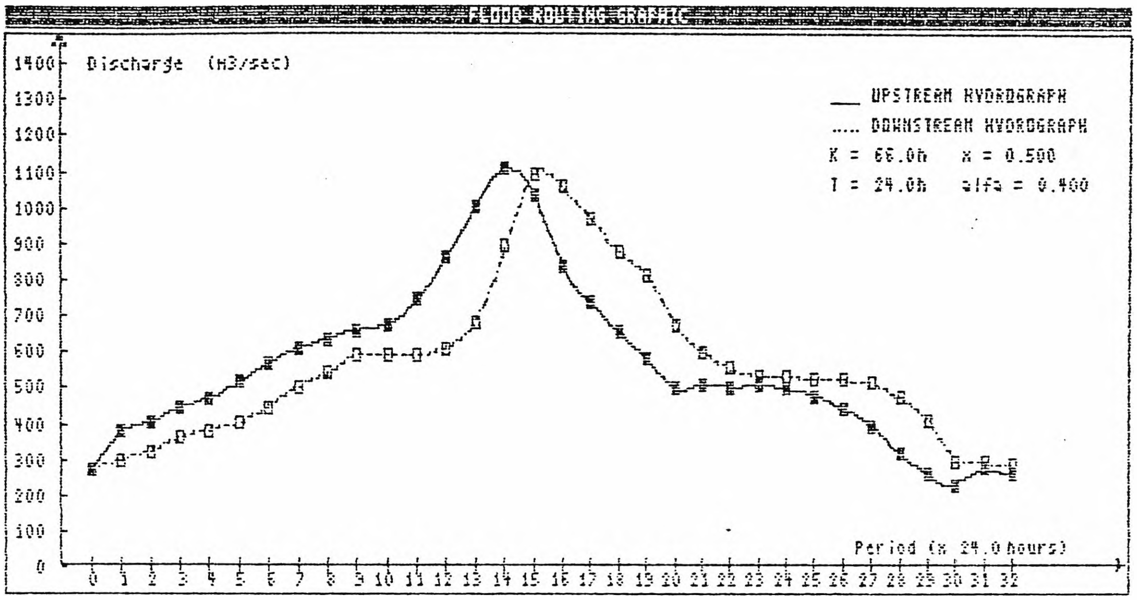

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

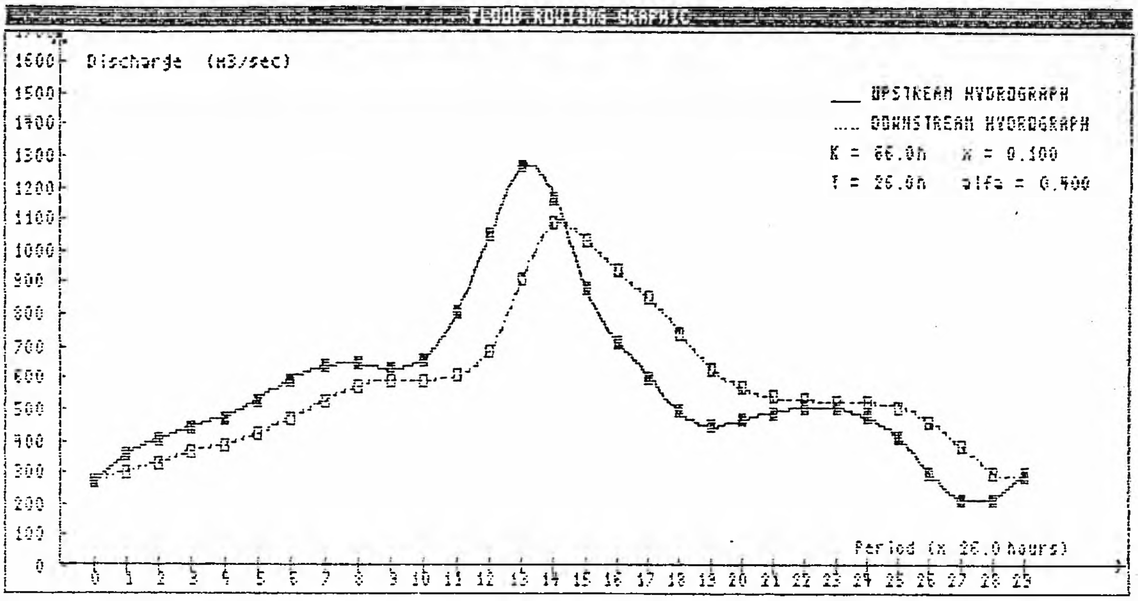

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

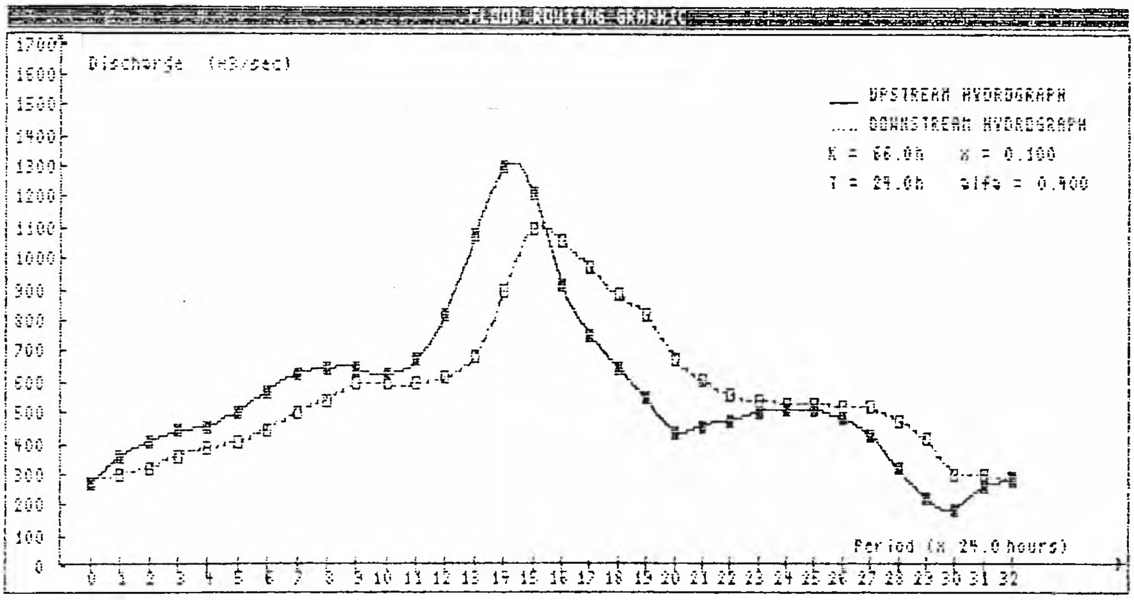

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

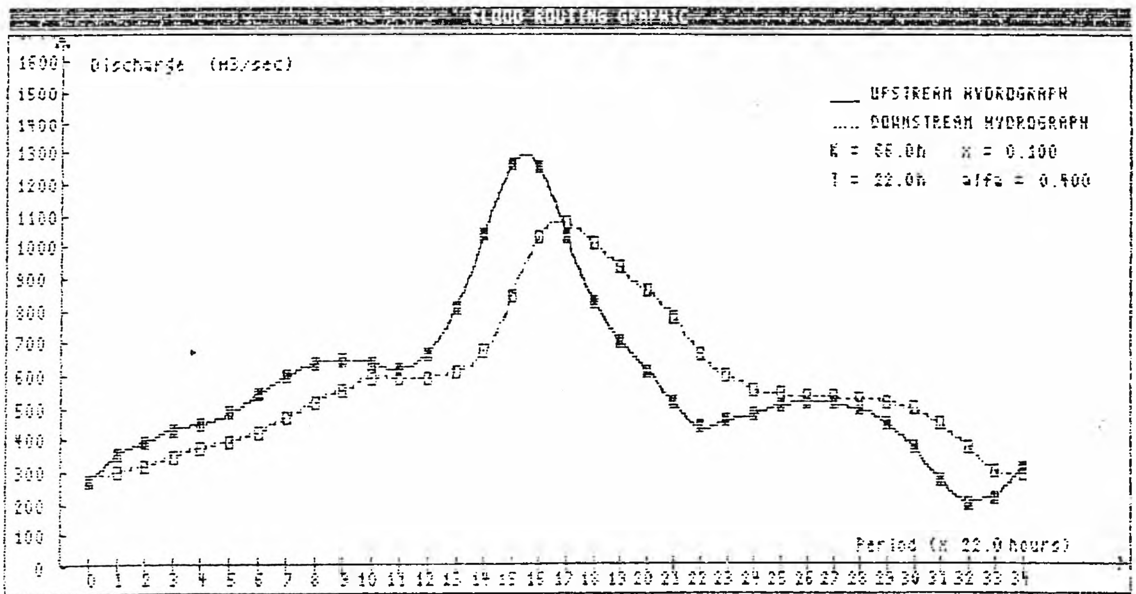

FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Figures 5.2.5

Calculated Upstream Hydrographs with Δt = 22, 24 and 26 Hours Using Backward Difference Based on the First Derivative at the End of Hydrograph


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 26.0h alfa = 0.400
Period (x 26.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 22.0h alfa = 0.400
Period (x 22.0 hours)

5.3 ITERATIVE METHOD WITH NEWTON BACKWARD FORMULA AT THE END OF HYDROGRAPH

Equation (5.1.2) can be used to calculate $dS/dt|_N$ as long as the value S_{N+1} is known. It may be obtained using the assumption made in section 5.1, that S_{N+1} is assumed to be equal to S_N. Therefore equation (5.1.2) becomes equation (5.1.5).

There is another way to obtain the value S_{N+1}, namely by applying the Newton backward formula. Its role is to predict (extrapolate) the value outside the data interval. Firstly, before the iterative computation begins, it is necessary to obtain the value Q_{N+1} (downstream discharge at time i = N+1) applying Newton backward formula. Since downstream hydrograph ordinates are adopted as the first estimate of the upstream hydrograph ordinates I, I_{N+1} is equal to Q_{N+1}. Thus, the storage S at time i = N+1 can be calculated using eq.(5.1.3) and the derivative of S can be calculated using eq.(5.1.2). Secondly, in the iteration process, the Newton backward difference formula is applied to obtain the value I_{N+1} (upstream discharge at time i = N+1) based on the calculated upstream hydrograph ordinates (I_N, I_{N-1}, ...). The derivative of S at time i = N can be continuously calculated using eq.(5.1.2) after storage S has been calculated using eq.(5.1.3).

5.3.1 The Theory of Newton Backward Formula

The Newton backward formula is derived below, prior to the test of computation. This is taken from Scheid (1968).

Given a discrete function, that is, a finite set of arguments $x_k - x_{k-1} = h$, the backward differences of the y_k values are denoted

$$\nabla y_k = y_k - y_{k-1}$$

and called first differences. The differences of these first differences are denoted

$$\nabla^2 y_k = \nabla(\nabla y_k) = \nabla y_k - \nabla y_{k-1} = y_k - 2.y_{k-1} + y_{k-2}$$

and called second differences. In general

$$\nabla^{n} y_k = \nabla^{n-1} y_k - \nabla^{n-1} y_{k-1}$$

defines the n th differences.

Backward differences are normally applied only at the bottom of a table, using negative arguments as shown in table V.3.1.

Table V.3.1

	x	y				
-4	x_{-4}	y_{-4}				
			∇y_{-3}			
-3	x_{-3}	y_{-3}		$\nabla^2 y_{-2}$		
			∇y_{-2}		$\nabla^3 y_{-1}$	
-2	x_{-2}	y_{-2}		$\nabla^2 y_{-1}$		$\nabla^4 y_0$
			∇y_{-1}		$\nabla^3 y_0$	
-1	x_{-1}	y_{-1}		$\nabla^2 y_0$		
			∇y_0			
0	x_0	y_0				

Each difference proves to be a combination of the y values in column three. A simple example is

$$\nabla^{3} y_0 = y_0 - 3.y_{-1} + 3.y_{-2} - y_{-3}\ .$$

The general result is

$$\nabla^{k} y_0 = \sum_{i=0}^{k} (-1)^{i+k} . \binom{k}{i} . y_{i-k} \qquad (5.3.1)$$

where

$$\binom{k}{i} = \frac{k!}{i!\ (k-i)!}$$

The Newton backward formula , in terms of k, is expressed as

$$P(x_k) = y_0 + k\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \ldots + \frac{k\ldots(k+n-1)}{n!}\nabla^n y_0 \qquad (5.3.2a)$$

or

$$P(x_k) = y_0 + \sum_{i=1}^{n}\frac{k(k+1)\ldots(k+i-1)}{i!}\nabla^i y_0 \qquad (5.3.2b)$$

where

$$k = \frac{x_k - x_0}{h} \qquad (5.3.3)$$

h = the increment of x values in the data.

Example.

* Apply Newton backward formula to the prediction of $\sqrt{1.35}$ in table V.3.2.

Table V.3.2

	x	$y(x) = \sqrt{x}$	∇	∇^2	∇^3	∇^4	∇^5	∇^6
-6	1.00	1.00000						
			2470					
-5	1.05	1.02470		-59				
			2411		5			
-4	1.10	1.04881		-54		-1		
			2357		4		-1	
-3	1.15	1.07238		-50		-2		4
			2307		2		3	
-2	1.20	1.09544		-48		1		
			2259		3			
-1	1.25	1.11803		-45				
			2214					
0	1.30	1.14017						

k (in eq. 5.3.2a,b) can be found using equation (5.3.3)

$$k = (1.35 - 1.30)/0.05 = 1.$$

By choosing n = 3 and substituting into eq.(5.3.2a), the result will be

P(1.35) = 1.14017 + 1.(0.02214) + 1.(-0.00045) + 1.(0.00003) = 1.16189,

while the exact solution of $\sqrt{1.35}$ is 1.161895.

5.3.2 The Application of Newton Backward Formula

It is clear that the purpose of applying Newton backward formula is to predict (extrapolate) the values of Q_{N+1} and I_{N+1} in order to able to obtain the derivative of S at time i = N using equation (5.1.2).

When applied to this problem, equation (5.3.2a) becomes

$$Q_{N+1} = Q_N + k\nabla Q_N + \frac{k(k+1)}{2!}\nabla^2 Q_N + \ldots + \frac{k \ldots (k+n-1)}{n!}\nabla^n Q_N$$

and

$$I_{N+1} = I_N + k\nabla I_N + \frac{k(k+1)}{2!}\nabla^2 I_N + \ldots + \frac{k \ldots (k+n-1)}{n!}\nabla^n I_N$$

where:

$$k = \frac{x_k - x_0}{h} = \frac{i_{N+1} - i_N}{\Delta t} = \frac{\Delta t}{\Delta t} = 1$$

The differences ∇ are calculated using eq. (5.3.1), where

$$y_0 \equiv Q_N \text{ or } I_N$$
$$y_{-1} \equiv Q_{N-1} \text{ or } I_{N-1}$$
$$y_{-2} \equiv Q_{N-2} \text{ or } I_{N-2}$$
$$\vdots$$
$$y_{-n} \equiv Q_{N-n} \text{ or } I_{N-n}$$

The number of ordinates n which are considered to be involved in the equation is dependent on the hydrologist's judgement. According to numerical experiments carried out here, computation using the larger n value gives less satisfactory results at the tail of hydrograph. This is explained by noting that as more ordinates are taken into account, the more uncertain the interpolation is, since the hydrograph ordinates do not follow any function which can be expressed precisely as a mathematical equation as with the example in section 5.3.1.

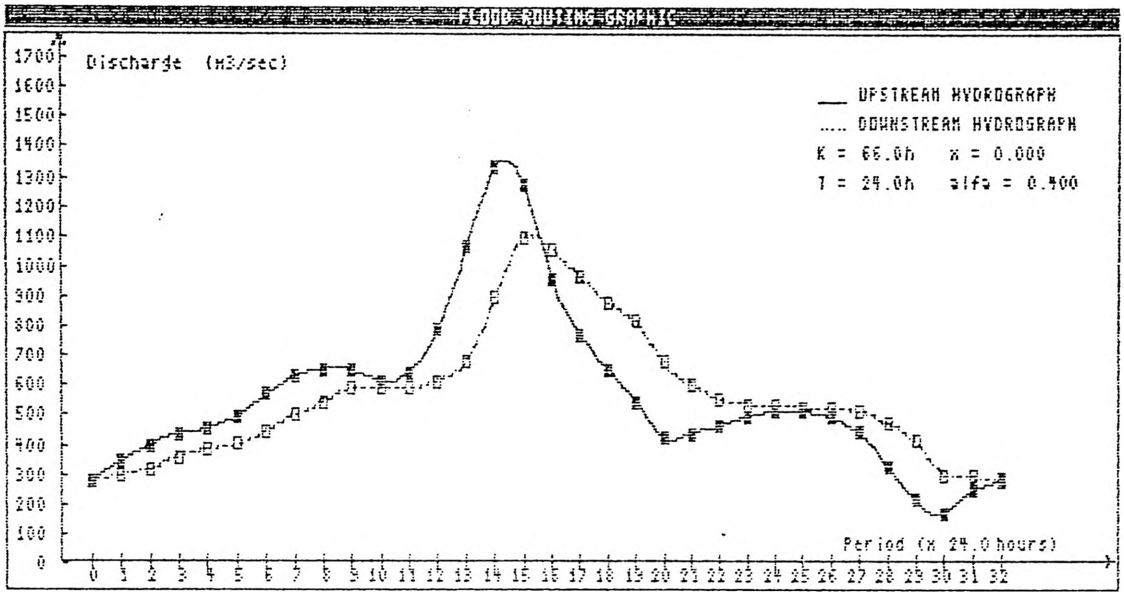
Results of computations using n=2 and n=3 are presented in Figures 5.3.1 and 5.3.2 respectively. The downstream hydrograph ordinates are also taken from ARR87 Table 7.1 page 134. The computations used K = 66 hours, Δt = 24 hours and weighting factor $\alpha = 0.4$ for various parameter x values. It can be noticed from these figures that n = 3 gives poorer results. Negative discharges come out at the tail of hydrograph. The larger the parameter x value, the more negative the tail is.

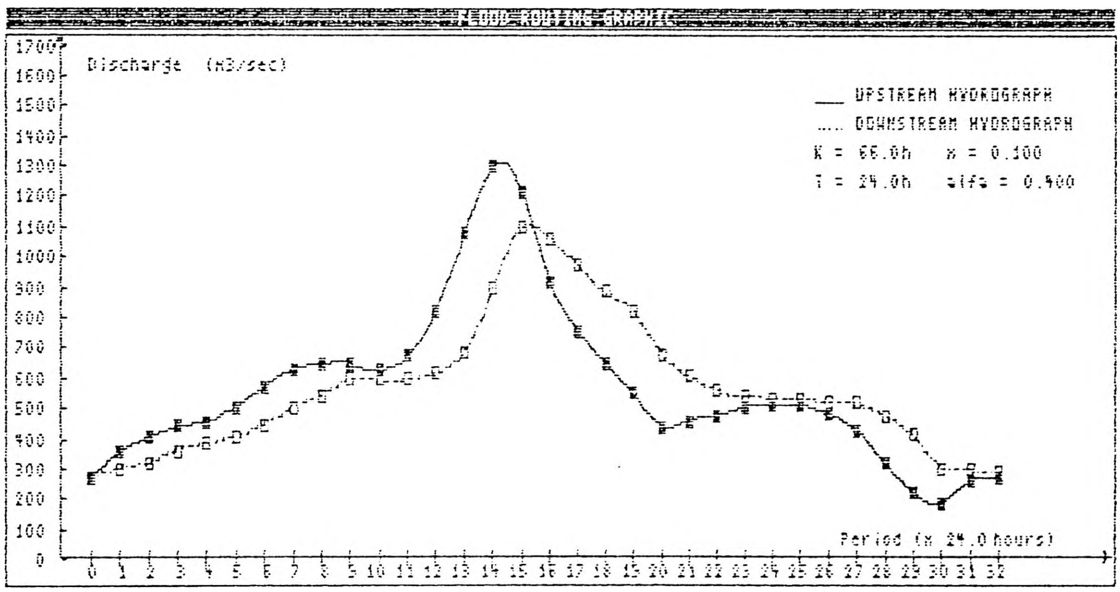
The iterative method using the Newton backward formula with n = 2 and the iterative method using backward difference based on the second derivative at the tail of hydrograph were found to give identical results. To demonstrate this, the results of these two methods are shown in Tables V.3.3.

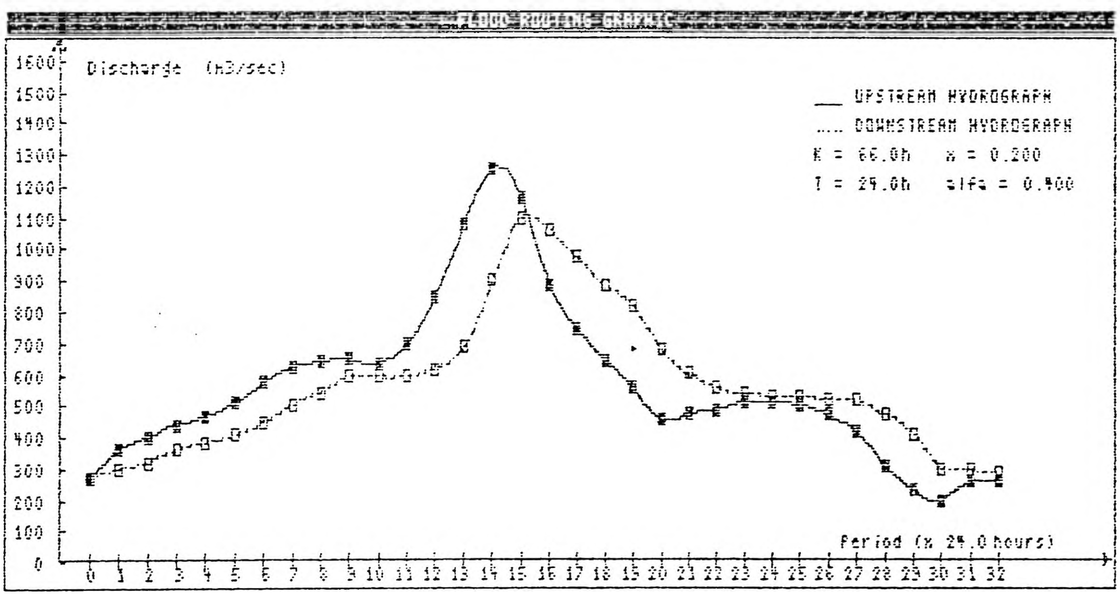
The problem encountered by Newton backward formula with n=2 and n=3 and various time steps Δt for a certain parameter x value is the same as that encountered by backward difference based on the second derivative. Each tail of the upstream hydrograph is not consistent with the others.

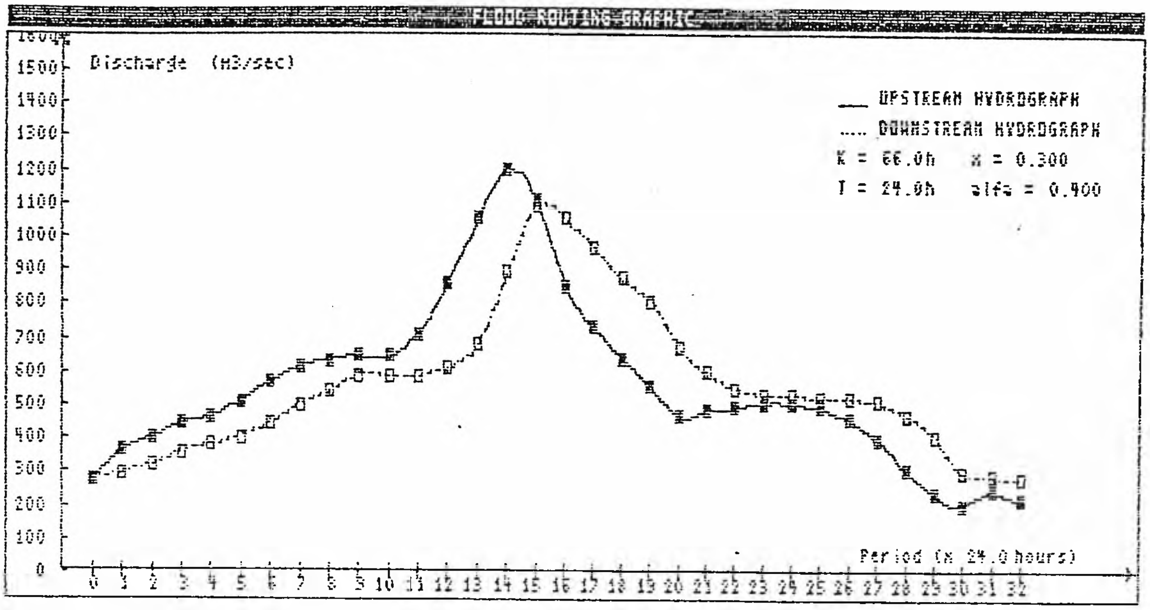
Figures 5.3.1

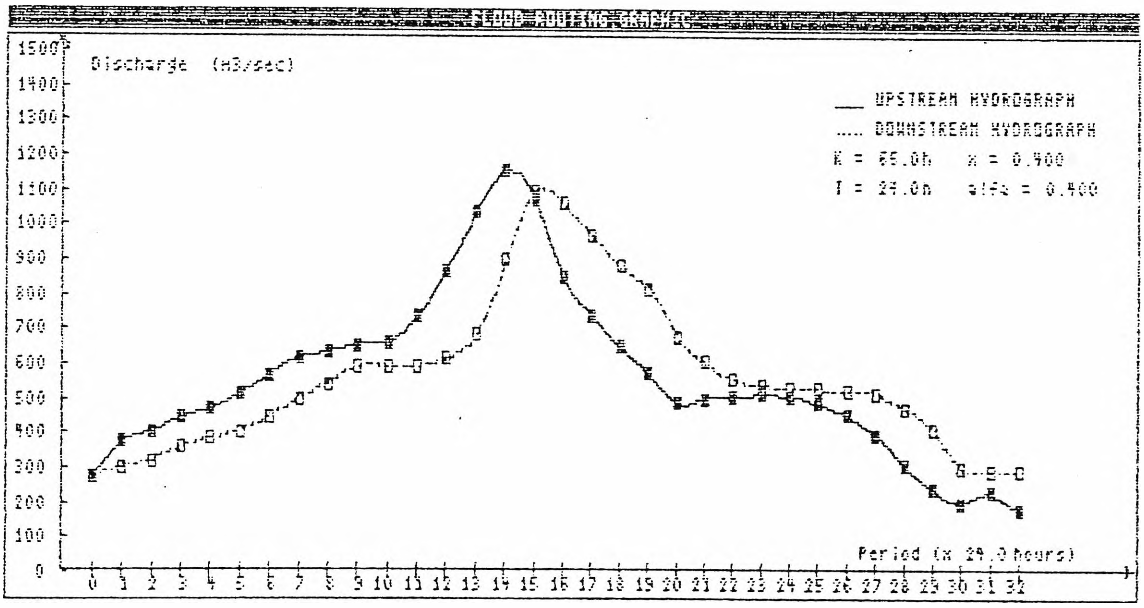
Upstream Routing Using Observed Downstream Hydrograph with Newton Backward Formula at the End of Hydrograph, n=2

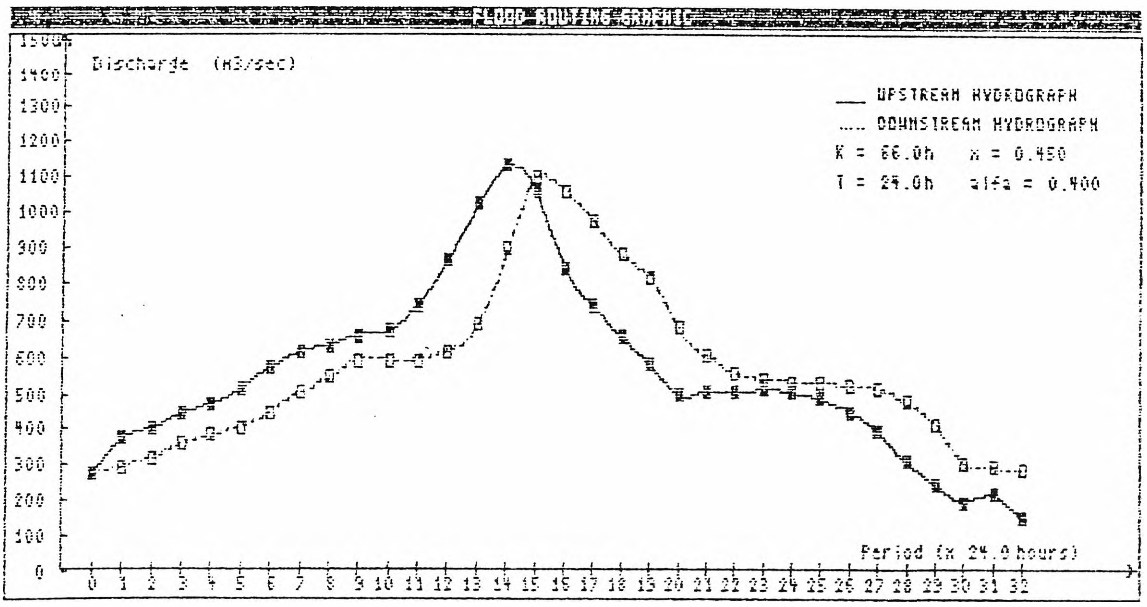

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

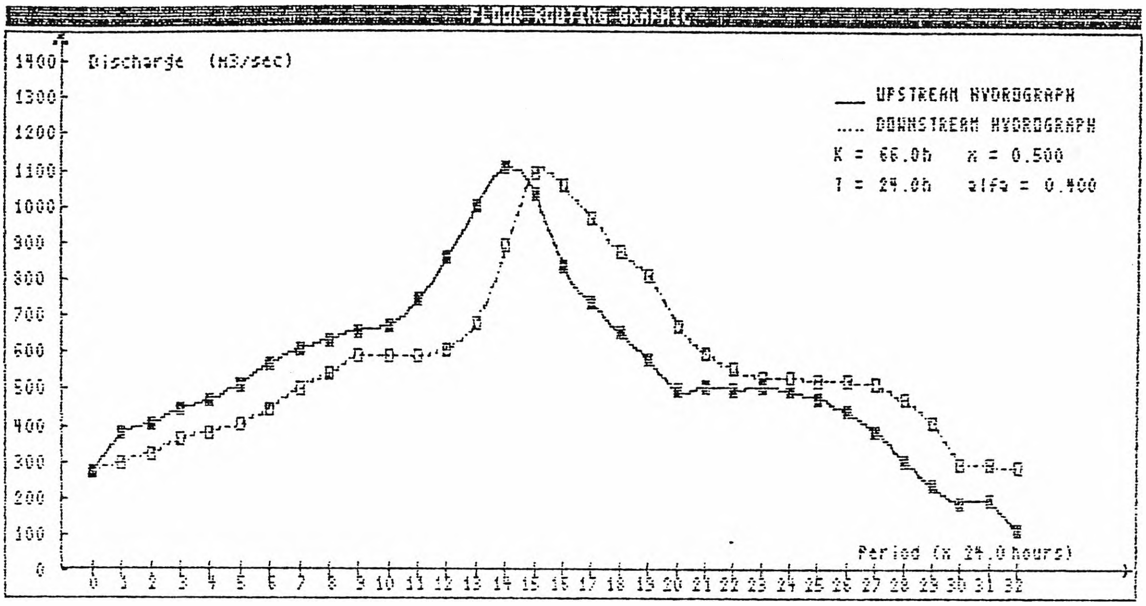

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

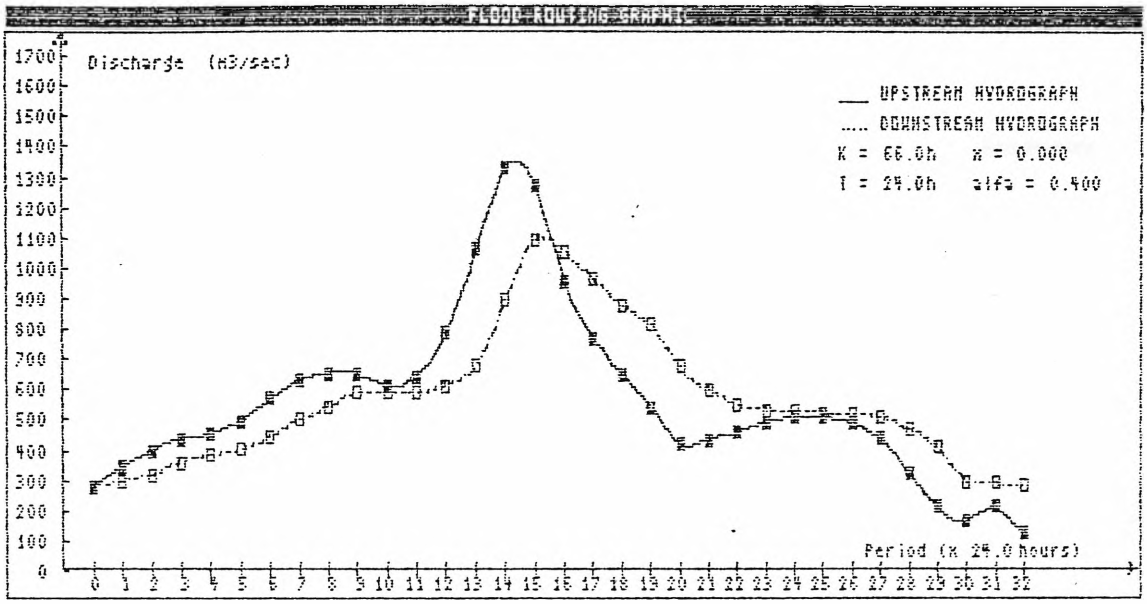

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

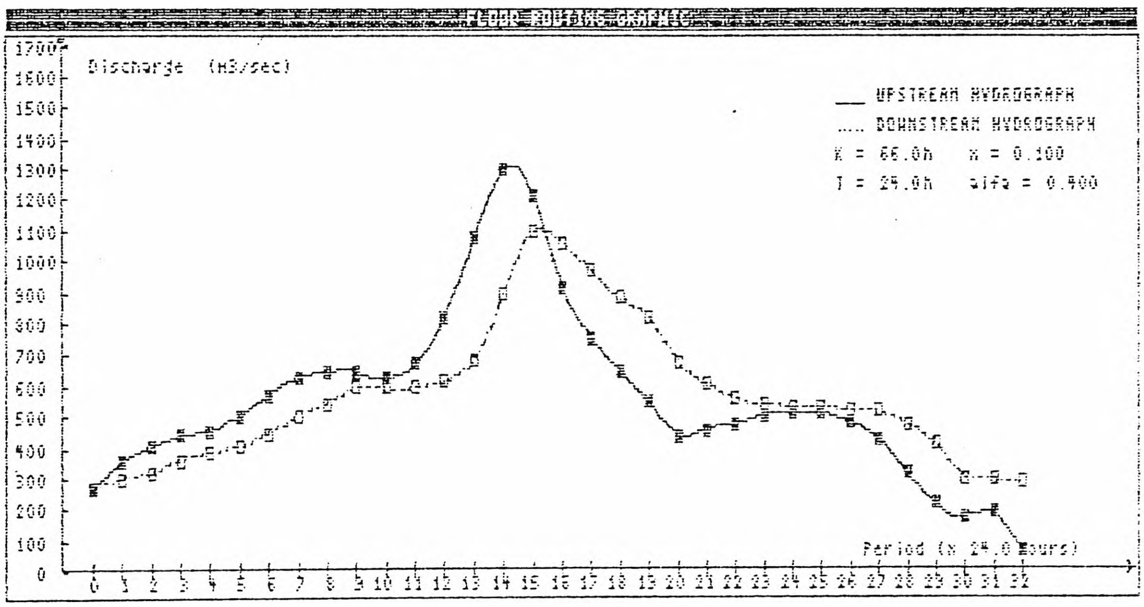

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
___ UPSTREAM HYDROGRAPH
..... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

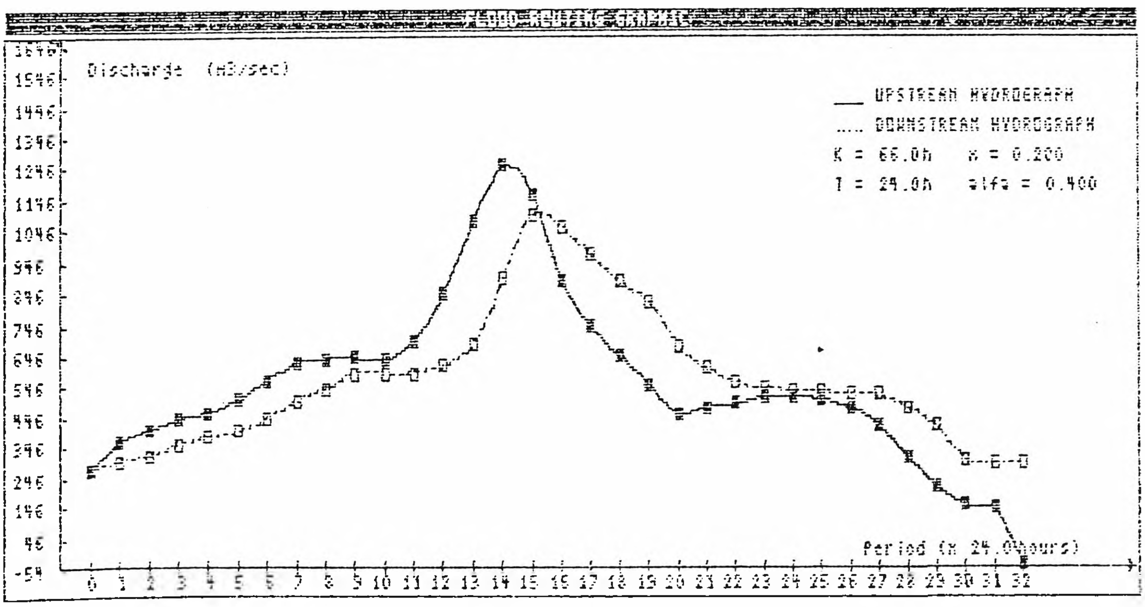

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.400
Period (x 24.0 hours)
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

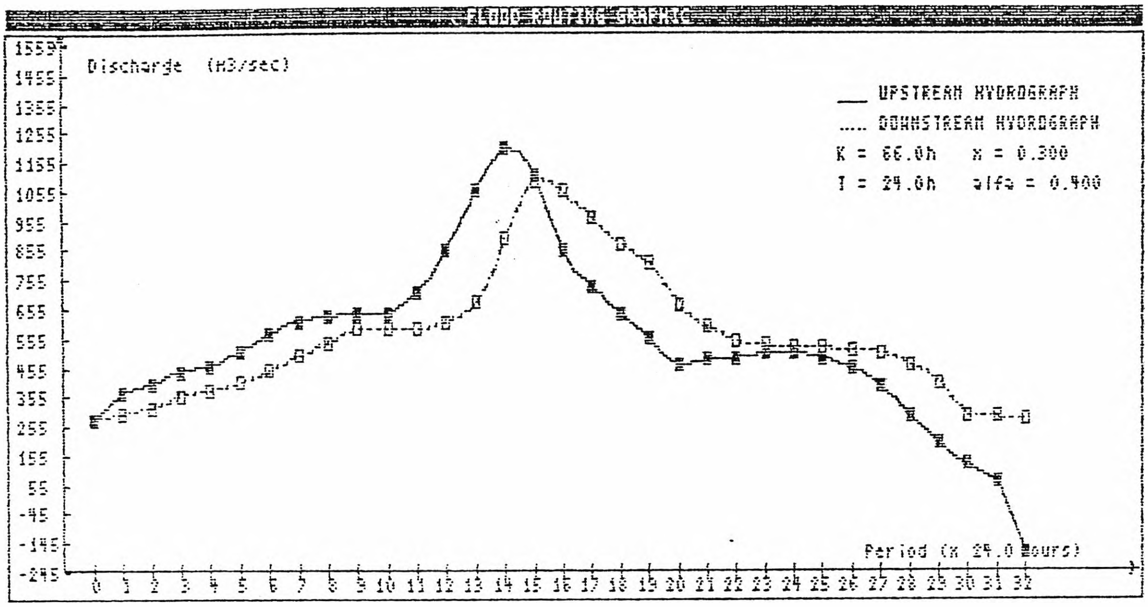
Figures 5.3.2

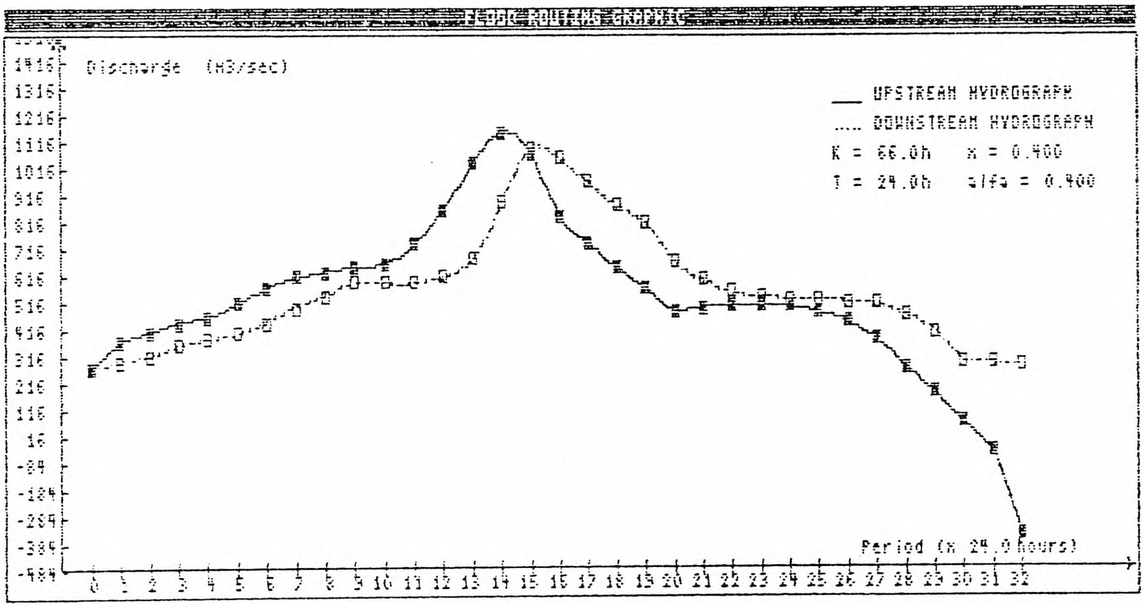
Upstream Routing Using Observed Downstream Hydrograph with Newton Backward Formula at the End of Hydrograph, n=3

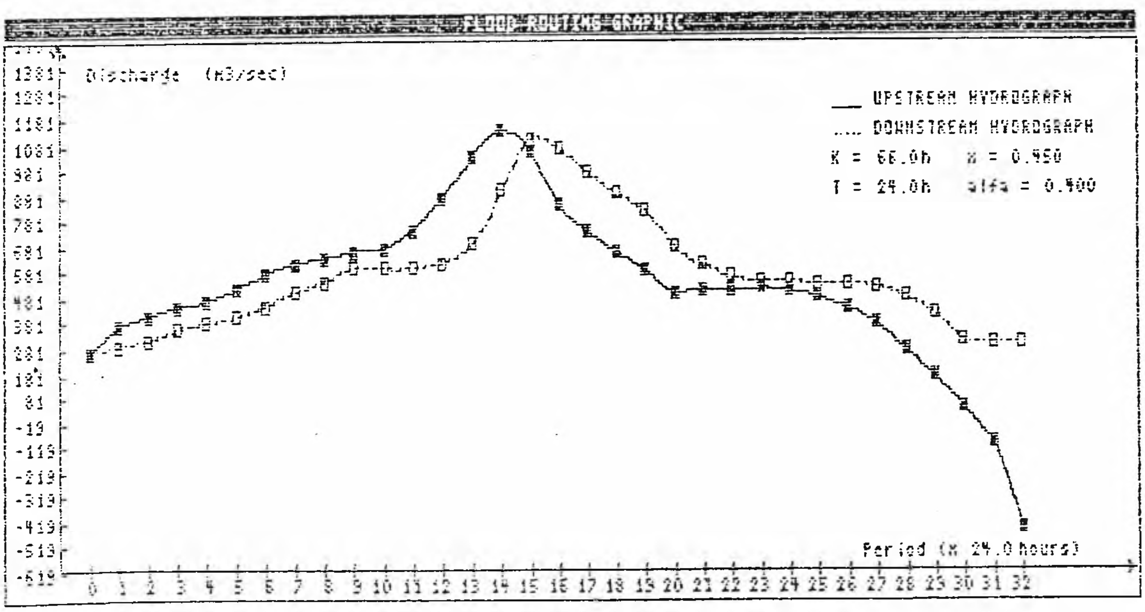

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
.... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.000
T = 24.0h alfa = 0.400
Period (x 24.0 hours)

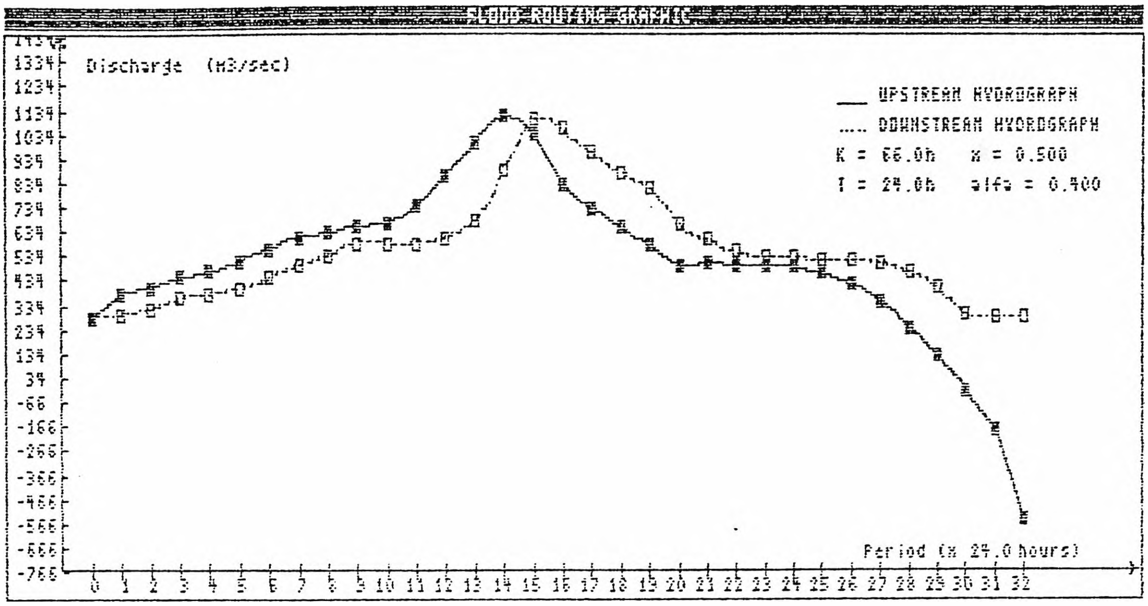

FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
.... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.100
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
.... DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.200
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.300
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.400
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (M3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.450
T = 24.0h alfa = 0.400
Period (x 24.0 hours)


FLOOD ROUTING GRAPHIC
Discharge (m3/sec)
UPSTREAM HYDROGRAPH
DOWNSTREAM HYDROGRAPH
K = 66.0h x = 0.500
T = 24.0h alfa = 0.400
1334
1234
1134
1034
934
834
734
634
534
434
334
234
134
34
-66
-166
-266
-366
-466
-566
-666
-766
Period (x 24.0 hours)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Tables V.3.3

Upstream Routing Calculation with Newton Backward Formula, n = 2 and Finite Difference Based on the Second Derivative at the End of Hydrograph

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.00000
alfa             = 0.40000
Total iterations = 15
```

PERIOD (x 24.00 hours)	OUTFLOW (observed) (m3/sec)	INFLOW (calculated-1) (m3/sec)	INFLOW (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	351.281	351.281
2	320.000	398.289	398.289
3	361.000	439.010	439.010
4	383.000	454.409	454.409
5	405.000	499.508	499.508
6	446.000	569.658	569.658
7	502.000	630.883	630.883
8	543.000	654.971	654.971
9	593.000	655.368	655.368
10	593.000	615.811	615.811
11	593.000	645.109	645.109
12	614.000	788.933	788.933
13	686.000	1067.983	1067.983
14	899.000	1334.808	1334.808
15	1100.000	1276.327	1276.327
16	1061.000	956.238	956.238
17	972.000	770.841	770.841
18	884.000	656.335	656.335
19	817.000	545.928	545.928
20	678.000	423.919	423.919
21	606.000	436.949	436.949
22	558.000	461.425	461.425
23	539.000	494.919	494.919
24	534.000	512.667	512.667
25	529.000	512.667	512.667
26	524.000	495.167	495.167
27	517.000	441.042	441.042
28	476.000	325.354	325.354
29	413.000	214.464	214.464
30	301.000	166.460	166.460
31	295.000	250.709	250.709
32	290.000	277.625	277.625

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference based on the second derivative

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.10000
alfa             = 0.40000
Total iterations = 15
```

PERIOD (x 24.00 hours)	O U T F L O W (observed) (m3/sec)	I N F L O W (calculated-1) (m3/sec)	I N F L O W (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	357.513	357.513
2	320.000	400.974	400.974
3	361.000	439.972	439.972
4	383.000	457.835	457.835
5	405.000	504.769	504.769
6	446.000	571.986	571.986
7	502.000	627.546	627.546
8	543.000	646.768	646.768
9	593.000	648.877	648.877
10	593.000	621.833	621.833
11	593.000	669.943	669.943
12	614.000	823.565	823.565
13	686.000	1080.368	1080.368
14	899.000	1300.182	1300.182
15	1100.000	1218.486	1218.486
16	1061.000	915.447	915.447
17	972.000	752.710	752.710
18	884.000	648.208	648.208
19	817.000	547.668	547.668
20	678.000	437.935	437.935
21	606.000	455.282	455.282
22	558.000	476.099	476.099
23	539.000	503.483	503.483
24	534.000	515.233	515.233
25	529.000	509.516	509.516
26	524.000	485.352	485.352
27	517.000	426.738	426.738
28	476.000	316.152	316.152
29	413.000	220.426	220.426
30	301.000	182.342	182.342
31	295.000	260.653	260.653
32	290.000	273.404	273.404

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference based on the second derivative

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.20000
alfa             = 0.40000
Total iterations = 19
```

PERIOD (x 24.00 hours)	OUTFLOW (observed) (m3/sec)	INFLOW (calculated-1) (m3/sec)	INFLOW (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	363.702	363.702
2	320.000	403.078	403.078
3	361.000	441.217	441.217
4	383.000	461.522	461.522
5	405.000	509.023	509.023
6	446.000	572.339	572.339
7	502.000	622.929	622.929
8	543.000	639.963	639.963
9	593.000	647.276	647.276
10	593.000	633.911	633.911
11	593.000	696.213	696.213
12	614.000	849.606	849.606
13	686.000	1077.172	1077.172
14	899.000	1255.259	1255.259
15	1100.000	1162.896	1162.896
16	1061.000	883.427	883.427
17	972.000	742.021	742.021
18	884.000	645.628	645.628
19	817.000	554.188	554.188
20	678.000	454.371	454.371
21	606.000	472.782	472.782
22	558.000	488.045	488.045
23	539.000	508.877	508.877
24	534.000	514.737	514.737
25	529.000	503.708	503.708
26	524.000	474.425	474.425
27	517.000	414.520	414.520
28	476.000	311.685	311.685
29	413.000	229.612	229.612
30	301.000	195.999	195.999
31	295.000	262.805	262.805
32	290.000	256.872	256.872

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference based on the second derivative

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.30000
alfa             = 0.40000
Total iterations = 21
```

PERIOD (x 24.00 hours)	O U T F L O W (observed) (m3/sec)	I N F L O W (calculated-1) (m3/sec)	I N F L O W (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	369.769	369.769
2	320.000	404.681	404.681
3	361.000	442.679	442.679
4	383.000	465.033	465.033
5	405.000	512.075	512.075
6	446.000	571.384	571.384
7	502.000	618.479	618.479
8	543.000	635.815	635.815
9	593.000	650.088	650.088
10	593.000	648.841	648.841
11	593.000	719.229	719.229
12	614.000	864.664	864.664
13	686.000	1061.311	1061.311
14	899.000	1206.250	1206.250
15	1100.000	1113.884	1113.884
16	1061.000	860.648	860.648
17	972.000	737.508	737.508
18	884.000	647.246	647.246
19	817.000	563.810	563.810
20	678.000	471.305	471.305
21	606.000	488.113	488.113
22	558.000	496.842	496.842
23	539.000	511.317	511.317
24	534.000	511.851	511.851
25	529.000	496.451	496.451
26	524.000	463.881	463.881
27	517.000	405.053	405.053
28	476.000	310.489	310.489
29	413.000	238.533	238.533
30	301.000	203.438	203.438
31	295.000	253.881	253.881
32	290.000	225.779	225.779

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference based on the second derivative

Number of data = 33
K = 66.00 hours
T = 24.00 hours
x = 0.40000
alfa = 0.40000
Total iterations = 23

PERIOD (x 24.00 hours)	OUTFLOW (observed) (m3/sec)	INFLOW (calculated-1) (m3/sec)	INFLOW (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	375.658	375.658
2	320.000	405.836	405.836
3	361.000	444.273	444.273
4	383.000	468.169	468.169
5	405.000	514.140	514.140
6	446.000	569.902	569.902
7	502.000	615.033	615.033
8	543.000	634.276	634.276
9	593.000	655.594	655.594
10	593.000	663.515	663.515
11	593.000	736.434	736.434
12	614.000	869.382	869.382
13	686.000	1037.126	1037.126
14	899.000	1157.986	1157.986
15	1100.000	1073.202	1073.202
16	1061.000	845.960	845.960
17	972.000	737.492	737.492
18	884.000	651.679	651.679
19	817.000	575.025	575.025
20	678.000	487.383	487.383
21	606.000	500.702	500.702
22	558.000	502.671	502.671
23	539.000	511.368	511.368
24	534.000	507.340	507.340
25	529.000	488.590	488.590
26	524.000	454.156	454.156
27	517.000	397.452	397.452
28	476.000	309.873	309.873
29	413.000	243.497	243.497
30	301.000	201.384	201.384
31	295.000	231.711	231.711
32	290.000	179.098	179.098

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference based on the second derivative

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.45000
alfa             = 0.40000
Total iterations = 24
```

PERIOD (x 24.00 hours)	O U T F L O W (observed) (m3/sec)	I N F L O W (calculated-1) (m3/sec)	I N F L O W (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	378.522	378.522
2	320.000	406.258	406.258
3	361.000	445.095	445.095
4	383.000	469.572	469.572
5	405.000	514.900	514.900
6	446.000	569.145	569.145
7	502.000	613.767	613.767
8	543.000	634.259	634.259
9	593.000	658.766	658.766
10	593.000	670.054	670.054
11	593.000	742.600	742.600
12	614.000	868.513	868.513
13	686.000	1023.226	1023.226
14	899.000	1135.141	1135.141
15	1100.000	1056.032	1056.032
16	1061.000	841.107	841.107
17	972.000	738.673	738.673
18	884.000	654.575	654.575
19	817.000	580.849	580.849
20	678.000	494.827	494.827
21	606.000	505.926	505.926
22	558.000	504.590	504.590
23	539.000	510.659	510.659
24	534.000	504.628	504.628
25	529.000	484.513	484.513
26	524.000	449.456	449.456
27	517.000	393.803	393.803
28	476.000	308.846	308.846
29	413.000	243.486	243.486
30	301.000	196.067	196.067
31	295.000	215.298	215.298
32	290.000	149.887	149.887

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference
based on the second derivative

```
Number of data   =  33
K                = 66.00 hours
T                = 24.00 hours
x                = 0.50000
alfa             = 0.40000
Total iterations = 25
```

PERIOD (x 24.00 hours)	O U T F L O W (observed) (m3/sec)	I N F L O W (calculated-1) (m3/sec)	I N F L O W (calculated-2) (m3/sec)
0	274.000	274.000	274.000
1	298.000	381.322	381.322
2	320.000	406.577	406.577
3	361.000	445.928	445.928
4	383.000	470.876	470.876
5	405.000	515.539	515.539
6	446.000	568.437	568.437
7	502.000	612.776	612.776
8	543.000	634.566	634.566
9	593.000	661.929	661.929
10	593.000	675.832	675.832
11	593.000	747.204	747.204
12	614.000	865.937	865.937
13	686.000	1008.716	1008.716
14	899.000	1113.467	1113.467
15	1100.000	1040.844	1040.844
16	1061.000	837.607	837.607
17	972.000	740.417	740.417
18	884.000	657.757	657.757
19	817.000	586.654	586.654
20	678.000	501.780	501.780
21	606.000	510.448	510.448
22	558.000	505.909	505.909
23	539.000	509.521	509.521
24	534.000	501.641	501.641
25	529.000	480.296	480.296
26	524.000	444.687	444.687
27	517.000	389.895	389.895
28	476.000	306.855	306.855
29	413.000	241.372	241.372
30	301.000	187.628	187.628
31	295.000	195.258	195.258
32	290.000	116.843	116.843

Note :
calculated-1 is obtained using Newton Backward Formula, n=2
calculated-2 is obtained using Backward finite difference
based on the second derivative

5.4 UPSTREAM ROUTING MOVING BACKWARD IN TIME

In chapter 4, it has been shown that the cause of the instability of equation (4.1.1) is the coefficient in terms of C_1 and C_0. If the value of $|-C_1/C_0|$ is larger than 1.0, the computation diverges. It has also been proved that only x = 0.0 will give satisfactory results.

If equation (4.1.1) is re-arranged to

$$I_i = \frac{1}{C_1}.Q_{i+1} - \frac{C_2}{C_1}.Q_i - \frac{C_0}{C_1}.I_{i+1} \qquad (5.4.1)$$

then

$$\left|-\frac{C_0}{C_1}\right| = \left|-\frac{(-K.x + \Delta t/2)}{(K.x + \Delta t/2)}\right|$$

is always less than 1.0 and any error entering into the calculated I_{i+1} value is carried forward into the calculation but diminishes towards zero. Therefore, equation (5.4.1) is numerically stable.

To solve eq.(5.4.1), it is required that the calculation be carried out backward in time, starting from the tail of the hydrograph (the ordinate at time i = N) and moving backward to the start of rise of the hydrograph. Seemingly, this step is rather unusual, but as long as it can be computed mathematically and the concept is valid, it is still acceptable.

A problem which arises with this approach is that the starting discharge (at the tail of the hydrograph) may not be known. However, any uncertainty in this discharge diminishes rapidly towards zero, since the value of $|-C_0/C_1|$ is always less than 1.0.

For a sample of computation, firstly the downstream hydrograph is calculated from the observed upstream hydrograph taken from ARR87 Table 7.1 page 134 using conventional downstream routing [equation (2.1.3)] with K = 66 hours, Δt = 24 hours and parameter x = 0.45. Secondly, this result is used to calculate back the upstream hydrograph using eq.(5.4.1). Figure 5.4.1 shows the resulting upstream hydrograph calculated from various assumed starting discharges. Convergence is reached rapidly and good upstream hydrograph reproduction is obtained.

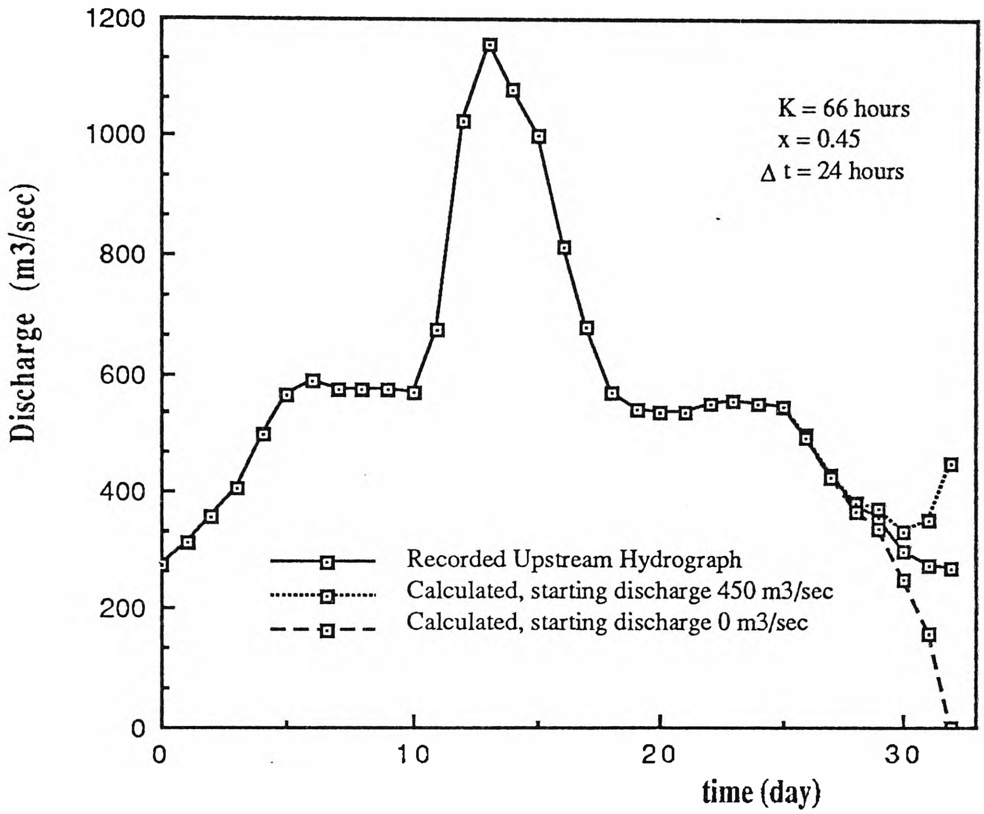


Figure 5.4.1 Upstream Routing Moving Backward in Time

This method works very well since there is no error propagation. It can be said that satisfactory results can always be obtained.

5.5 THE CUBIC SPLINE AND RUNGE-KUTTA METHODS

If the derivative term in the equation of conservation of mass [eq.(2.1.1)] is based on the storage S equation [eq.(5.1.3)], that derivative can be written as follows:

$$\frac{dS}{dt} = K.x.\frac{dI}{dt} + K.(1-x).\frac{dQ}{dt} \tag{5.5.1}$$

Thus, the equation of conservation of mass:

$$I - Q = \frac{dS}{dt}$$

becomes

$$I - Q = K.x.\frac{dI}{dt} + K.(1\text{-}x).\frac{dQ}{dt}$$

$$I - K.x.\frac{dI}{dt} = Q + K.(1\text{-}x).\frac{dQ}{dt} \tag{5.5.2}$$

On the right-hand side of eq.(5.5.2) are the known variables obtained from observed data, while on the left-hand side are the variables for which the solution is sought. This ordinary differential equation may be solved numerically by the *Runge-Kutta.* method. Difficulty arises in applying this method, since it requires that the right hand side Q variable in eq.(5.5.2) be available as a function, not as a set of data points, in order to be able to obtain the Q values and their derivatives at any time required in the method of solution. However, this problem can be overcome by fitting a *Cubic Spline* through the ordinates of the downstream hydrograph.

In the particular case where parameter x = 0.0, equation (5.5.2) becomes

$$I = Q + K.\frac{dQ}{dt} \tag{5.5.3}$$

thus, it is not necessary to apply Runge-Kutta to solve. It can be solved for I straight-forwardly with help of cubic spline to obtain the derivative of Q. If parameter x is not equal to 0.0, eq.(5.5.2) can be written as

$$\frac{1}{K.x}.I - \frac{dI}{dt} = \frac{1}{K.x}.Q + \frac{K(1\text{-}x)}{K.x}.\frac{dQ}{dt} \tag{5.5.4}$$

To solve eq.(5.5.4), both the cubic spline and Runge-Kutta methods have to be applied.

5.5.1 The Cubic Spline

The theory described below is derived from Young (1972).

Suppose that one is interested in determining a function F(x) which approximates a given function f(x) in an interval I = [a,b]. One method would be to subdivide the interval into N subintervals $I_1 = [x_0,x_1]$; $I_2 = [x_1,x_2]$; ...; $I_N = [x_{N-1},x_N]$ where $a = x_0$, $b = x_N$, and $x_0 < x_1 < ... < x_N$. One could then determine by *Lagrangian* interpolation, or if the intervals are of equal length, by *Gregory-Newton* interpolation, a polynomial F(x) of degree N or less such that $F(x_i) = f(x_i)$, i = 0, 1, ..., N. However, for certain functions the approximate representation of f(x) by a single polynomial throughout the interval is not satisfactory.

It is possible to use a cubic polynomial in each subinterval to obtain a function S(x) which interpolates to f(x) at the $\{x_i\}$ in the entire interval. Such a function is known as a cubic spline function.

In using cubic spline interpolation, F'(x) and f'(x) are not required to agree at the points of interpolation. A function $F_k(x)$ in the interval I_k has to be determined such that

$$F_k(x_i) = f(x_i), \qquad i = k-1, k.$$

For k = 1, 2, ..., N-1, it is also required that

$$F'_k(x_k-) = F'_{k+1}(x_k+)$$

$$F''_k(x_k-) = F''_{k+1}(x_k+)$$

The procedure involves determining M_k, where

$$M_k = F''_k(x_k-) = F''_{k+1}(x_k+)$$

Since $F_k(x)$ is a cubic polynomial, $F''_k(x)$ is a linear function of x in I_k, i.e.:

$$F''_k(x) = M_{k-1}.\frac{x_k - x}{x_k - x_{k-1}} + M_k.\frac{x - x_{k-1}}{x_k - x_{k-1}} \qquad (5.5.5)$$

By integrating eq.(5.5.5), it becomes

$$F'_k(x) = -M_{k-1}.\frac{(x_k - x)^2}{2h_k} + M_k.\frac{(x - x_{k-1})^2}{2h_k} + c_1$$

where

$$h_k = x_k - x_{k-1} \tag{5.5.6}$$

and where c_1 is a constant of integration to be determined. By integrating again, it becomes

$$F_k(x) = M_{k-1}.\frac{(x_k - x)^3}{6h_k} + M_k.\frac{(x - x_{k-1})^3}{6h_k} + c_1.x + c_2 \tag{5.5.7}$$

By letting $y_k = f(x_k)$, eqs.(5.5.8) is obtained.

$$y_{k-1} = M_{k-1}.\frac{h_k^2}{6} + c_1.x_{k-1} + c_2$$

$$y_k = M_k.\frac{h_k^2}{6} + c_1.x_k + c_2 \tag{5.5.8}$$

hence,

$$c_1 = \frac{(y_k - y_{k-1}) - (M_k - M_{k-1})(h_k^2/6)}{h_k}$$

$$c_2 = \frac{(x_k.y_{k-1} - x_{k-1}.y_k) - (x_k.M_{k-1} - x_{k-1}.M_k)(h_k^2/6)}{h_k}$$

by substituting in eq.(5.5.7), eq.(5.5.9) is obtained.

$$F_k(x) = M_{k-1}.\left(\frac{(x_k - x)((x_k - x)^2 - h_k^2)}{6.h_k}\right) + M_k.\left(\frac{(x - x_{k-1})((x - x_{k-1})^2 - h_k^2)}{6.h_k}\right) + \frac{1}{h_k}.y_{k-1}(x_k - x) + \frac{1}{h_k}.y_k(x - x_{k-1}) \tag{5.5.9}$$

By differentiating, eq.(5.5.10) is obtained.

$$F'_k(x) = M_{k-1}.\left(\frac{h_k^2 - 3(x_k - x)^2}{6.h_k}\right) + M_k.\left(\frac{3(x - x_{k-1})^2 - h_k^2}{6.h_k}\right) + \frac{1}{h_k}(y_k - y_{k-1}) \tag{5.5.10}$$

If $F'_k(x_k-) = F'_{k+1}(x_k+)$ then

$$\frac{h_k}{6}.M_{k-1} + \frac{h_k}{3}.M_k + \frac{1}{h_k}(y_k - y_{k-1}) = -\frac{h_{k+1}}{3}.M_k - \frac{h_{k+1}}{6}.M_{k+1} + \frac{1}{h_{k+1}}(y_{k+1} - y_k)$$

(5.5.11)

or

$$\frac{h_k}{6}.M_{k-1} + \frac{h_k + h_{k+1}}{3}.M_k + \frac{h_{k+1}}{6}.M_{k+1} = \left[\frac{1}{h_{k+1}}(y_{k+1} - y_k) - \frac{1}{h_k}(y_k - y_{k-1})\right]$$

$(k = 1,2, ..., N-1)$ (5.5.12)

This is a system of N-1 linear algebraic equations with N+1 unknowns, i.e.: M_0, M_1, ..., M_N. Two more equations are needed to solve that system. Arbitrarily, M_0 and M_1 can be assumed by taking:

$$F_1'(x_0) = 0$$
$$F_N'(x_N) = 0$$

(5.5.13)

(Equations (5.5.13) imply that the slopes of downstream Q hydrograph at time i = 0 and i = N are assumed to be equal to zero.)

Using eqs. (5.5.13), eq.(5.5.10) gives

$$\frac{h_1}{3}.M_0 + \frac{h_1}{6}.M_1 = \frac{1}{h_1}(y_1 - y_0)$$
$$\frac{h_N}{6}.M_{N-1} + \frac{h_N}{3}.M_N = -\frac{1}{h_N}(y_N - y_{N-1})$$

(5.5.14)

Eventually, the system has N+1 linear algebraic equations with N+1 unknowns. The values of M_0, M_1, ..., M_N can uniquely determined. This follows since the determinant of matrix A of the system, does not vanish.

$$[A].[M] = [D]$$

$$A = \begin{bmatrix} \frac{h_1}{3} & \frac{h_1}{6} & . & . \cdots & . & . & . \\ \frac{h_1}{6} & \frac{h_1+h_2}{3} & \frac{h_2}{6} & . \cdots & . & . & . \\ . & \frac{h_2}{6} & \frac{h_2+h_3}{3} & \frac{h_3}{6} \cdots & . & . & . \\ . & . & . & . \cdots & . & . & . \\ . & . & . & . \cdots & . & . & . \\ . & . & . & . \cdots & \frac{h_{N-1}}{6} & \frac{h_{N-1}+h_N}{3} & \frac{h_N}{6} \\ . & . & . & . \cdots & . & \frac{h_N}{6} & \frac{h_N}{3} \end{bmatrix}$$

$$M = \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ \vdots \\ M_{N-1} \\ M_N \end{bmatrix} \qquad D = \begin{bmatrix} \frac{1}{h_1}(y_1 - y_0) \\ \frac{1}{h_2}(y_2 - y_1) - \frac{1}{h_1}(y_1 - y_0) \\ \frac{1}{h_3}(y_3 - y_2) - \frac{1}{h_2}(y_2 - y_1) \\ \vdots \\ \frac{1}{h_N}(y_N - y_{N-1}) - \frac{1}{h_{N-1}}(y_{N-1} - y_{N-2}) \\ -\frac{1}{h_N}(y_N - y_{N-1}) \end{bmatrix}$$

The solution of a system of linear algebraic equations with a tri-diagonal matrix can easily be carried out. In this project, that system is solved by using *Gauss elimination methods*. This method is common, hence it is not discussed herein.

5.5.2 The Runge-Kutta Method

The theory described below is all derived from Grove (1966).

This version of the Runge-Kutta method uses terms through the fourth derivative. The equations are given below:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2.k_2 + 2.k_3 + k_4) \tag{5.5.15}$$

where

$$k_1 = h.f(x_n, y_n) \tag{5.5.16}$$

$$k_2 = h.f(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_1) \tag{5.5.17}$$

$$k_3 = h.f(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_2) \tag{5.5.18}$$

$$k_4 = h.f(x_n + h,\ y_n + k_3) \tag{5.5.19}$$

The differential equation must, of course, be written as $y' = f(x,y)$ with an initial condition that $x = x_0$ when $y = y_0$.

Use of the Runge-Kutta method is as follows: compute the four k values from (5.5.16), (5.5.17), (5.5.18) and (5.5.19) and substitute into (5.5.15). This yields a new point (x_{n+1}, y_{n+1}), which is then re-used as the initial point. The process is repeated across the interval of the desired solution.

<u>Example</u> : Solve $y' = x - y$ for the initial condition $x = 0$, $y = 2$ with $h = 0.1$.

Solution:

$x_0 = 0,\ y_0 = 2,\ h = 0.1,\ y' = x - y$

$k_1 = 0.1(0 - 2) = -0.2$

$k_2 = 0.1\{0.05 - [2 + 1/2(-0.2)]\} = 0.1[0.05 - 1.9] = -0.185$

$k_3 = 0.1\{0.05 - [2 + 1/2(-0.185)]\} = 0.1[0.05 - 1.9075] = -0.18575$

$k_4 = 0.1\{0.1 - (2 - 0.18575)\} = 0.1[0.1 - 1.81425] = -0.171425$

$y_1 = 2 + 1/6[-0.2 + 2(-0.185) + 2(-0.18575) - 0.171425] = 1.8145125$

Now, using $x = 0.1$ and $y = 1.8145125$ as the initial point,

$k_1 = -0.17145125$

$k_2 = -1.5787869$

$k_3 = -1.5855732$

$k_4 = 1.4559552$

$$y_2 = 1.6561927$$

Now

$$x_2 = 0.2, \quad y_2 = 1.6561927$$

$$x_3 = 0.3, \quad y_3 = 1.5224553$$

$$x_4 = 0.4, \quad y_4 = 1.4109609$$

$$x_5 = 0.5, \quad y_5 = 1.3195929$$

$$x_6 = 0.6, \quad y_6 = 1.2464359.$$

5.5.3 The Application of the Cubic Spline and Runge-Kutta Methods

Equation (5.5.4) can be written as

$$\frac{dI}{dt} = -\frac{1}{K.x}.Q - \frac{K(1-x)}{K.x}.\frac{dQ}{dt} + \frac{1}{K.x}.I$$

or $$I'(t) = D(t) + c.I \tag{5.5.20}$$

where

$$c = 1/(K.x)$$

$$D(t) = -\frac{1}{K.x}.Q - \frac{K(1-x)}{K.x}.Q'(t)$$

Equation (5.5.20) is very similar to the example discussed above, i.e.: $y' = x - y$. Hence, the way to solve eq.(5.5.20) is the same as that in the example. The difference is that the value of D(t) should be determined by using the spline function. The value Q(t) is obtained using eq.(5.5.9) while the value of Q'(t) is obtained using eq.(5.5.10). It should be emphasized that since the spline function results in a different polynomial equation for every subinterval, it is very essential to check thoroughly whether or not the appropriate polynomial equation is used according to the corresponding time interval.

Numerical experiments were done using the downstream hydrograph taken from ARR87 Table 7.1 page 134 with K = 66 hours, Δt = 24 hours and various parameter x values (0, 0.1, ..., 0.5). Computation shows that only x = 0.0 gives adequately satisfactory result, the rest of parameter x values result in divergence, even though the very small h value (h = 0.1 hours) and quite large h value (h = Δt = 24 hours) are used (in this context, h is the subinterval of time step Δt, see section 5.5.2).

If parameter x = 0.0, the Runge-Kutta method is not applied. This is because eq.(5.5.3) does not have a term dI/dt. The spline function is still applied to obtain the value of dQ/dt (and Q) at any time required in the computation. Figure 5.5.1 shows the result for parameter x = 0.0.

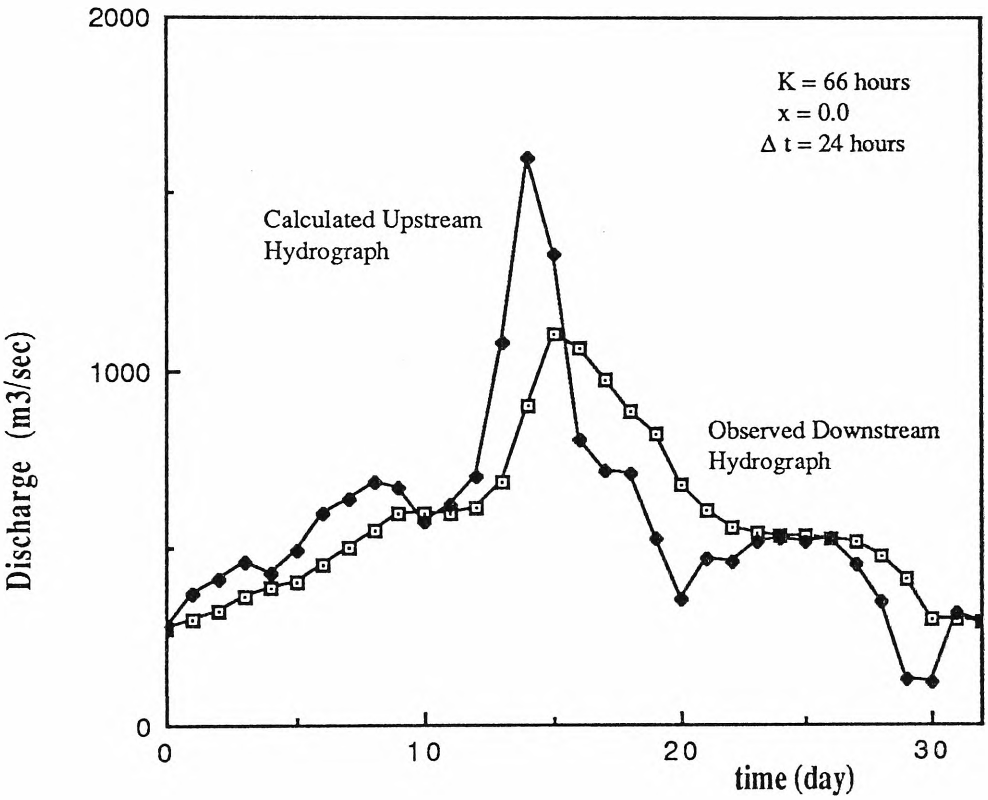


Figure 5.5.1 Upstream Routing with Spline Function

Based on experiments, it can be concluded that the Runge-Kutta combined with cubic spline fitting methods does not yield satisfactory results, the computation diverges rapidly. The only x which makes the computation converge is x = 0.0, since the computation does not need the Runge-Kutta method. However, the result is not satisfactory, oscillations will most probably occur, as can be seen in Fig. 5.5.1.

5.6 SUMMARY

It has been shown that the reverse application of the conventional Muskingum routing procedure to obtain an upstream hydrograph yields unsatisfactory results (chapter 4). Very rapid divergence occurs since the computation is numerically unstable. However, re-arrangement of the formulation to use an iterative solution combined with a smoothing algorithm and a weighting factor α can replace that method. Very good estimates of upstream hydrograph I are obtained if the correct choice of time step Δt is applied (section 5.1).

The problem encountered by the iterative method is how to determine the derivative storage S at the end of hydrograph (at time i = N). In conjunction with that, several approximations which have been investigated indicate that the most accurate estimates of I are obtained by assuming the derivative S at time i = N+1 to be equal to the one at time i = N, so that a central finite difference can be used to calculate the derivative S at time i = N. First order backward difference also gave satisfactory results, but second order backward difference and Newton backward formula did not.

The use of a smoothing algorithm in the iteration process is for removing oscillations which are likely to occur in the computation, while the use of a weighting factor α is for accelerating the iteration process so that the required number of iterations decreases greatly.

Re-arrangement of the usual finite difference form of the Muskingum equation to solve for I_i given I_{i+1} (i.e.: upstream routing moving backward in time) ensures that the solution converges and very accurate estimates of the upstream hydrograph are obtained (section 5.4).

The cubic spline combined with the Runge-Kutta method does not yield satisfactory results. According to the numerical experiments, the computation diverges for any time step Δt, except for parameter $x = 0$, and even this has oscillations. It should be noted that if parameter $x = 0$, only the cubic spline (without Runge-Kutta) is applied in the computation since the term dI/dt does not appear in the equation.

Chapter Six

Downstream Routing Using Iterative Method

6.0 INTRODUCTION

As discussed in chapter 5, it is clear that the iterative method can overcome the inability of upstream routing derived from the standard Muskingum equation to give satisfactory results.

This chapter is intended to describe briefly how the iterative method can be applied not only for upstream routing but also for downstream routing. Samples of computations using both the iterative method and conventional downstream routing are compared. The results of the iterative method cannot agree exactly with those of conventional downstream routing, since the approaches used are different. Nevertheless, the results of both methods have been shown to agree reasonably well with the observed downstream hydrograph.

As has been described in chapter 5, a problem with the iterative method is how to determine the derivative of storage S at the tail of the hydrograph (at time i = N) since the value of S_{N+1} is not known. Some approaches were investigated in conjunction with that problem. Computations showed that the best results were obtained by assuming S_{N+1} equal to S_N. Therefore, in this chapter this assumption is adopted. The other approaches, i.e.: backward differences, Newton backward formula and Runge-Kutta and cubic spline are no longer discussed.

6.1 COMPUTATION PROCEDURE

The computation procedure of the iterative method for downstream routing is the same as that for upstream routing. The procedure is discussed briefly.

The equation of conservation of mass [eq.(2.1.1)] is re-arranged into

$$Q_i = I_i - \frac{dS}{dt}\Big|_i \qquad (6.1.1)$$

where the subscript i refers to the time i. The derivative of S is expressed in central finite differences using the simplest two point scheme

$$\frac{dS}{dt}\Big|_i = (S_{i+1} - S_{i-1})/(2.\Delta t) \qquad (6.1.2)$$

while the storage S at any specified discharge is expressed by

$$S = K.[x.I + (1-x).Q] \qquad (6.1.3)$$

Equation (6.1.1) is not used to calculate the value of Q_0 at time i = 0. The value of Q_0 must be given an initial value, since eq.(6.1.1) is a differential equation. The assumption usually made is

$$Q_0 = I_0 \qquad (6.1.4)$$

but any value of Q_0 can be used.

As has been discussed in chapter 5, the storage S at time i = N+1 is assumed to be equal to that at time i = N ($S_{N+1} = S_N$), therefore eq.(6.1.2) for calculating $dS/dt|_N$ becomes

$$\frac{dS}{dt}\Big|_N = (S_N - S_{N-1})/(2.\Delta t) \qquad (6.1.5)$$

Combining eqs.(6.1.1), (6.1.2) and (6.1.3) yields an implicit equation, since the storage S is expressed in terms of the downstream discharge Q, the value of which itself is being sought.

The implicit equation is solved using an iterative process with instantaneous discharges. The method of solution used is to

- adopt the upstream hydrograph ordinates I as the first estimate of the downstream hydrograph ordinates Q, give an initial value at time i = 0 to Q_0 which remains unchanged throughout the iterative process,
- use eq.(6.1.3) to calculate the values of storage S,
- use eqs.(6.1.2) and (6.1.5) to determine the derivative dS/dt
- then use eq.(6.1.1) to make an improved estimate of Q.

These steps are repeated until successive calculated downstream hydrographs converge.

For clarity, it is more convenient to describe the steps in the computation procedure with help of a flow chart (see Fig. 6.1.1). They are explained below.

<u>Step 1</u>

Initialize iteration k = 1.

<u>Step 2</u>

Give an initial value at time i = 0 to Q (Q_0) [eq.(6.1.4)] which remains unchanged throughout the required number of iterations to converge and adopt the upstream

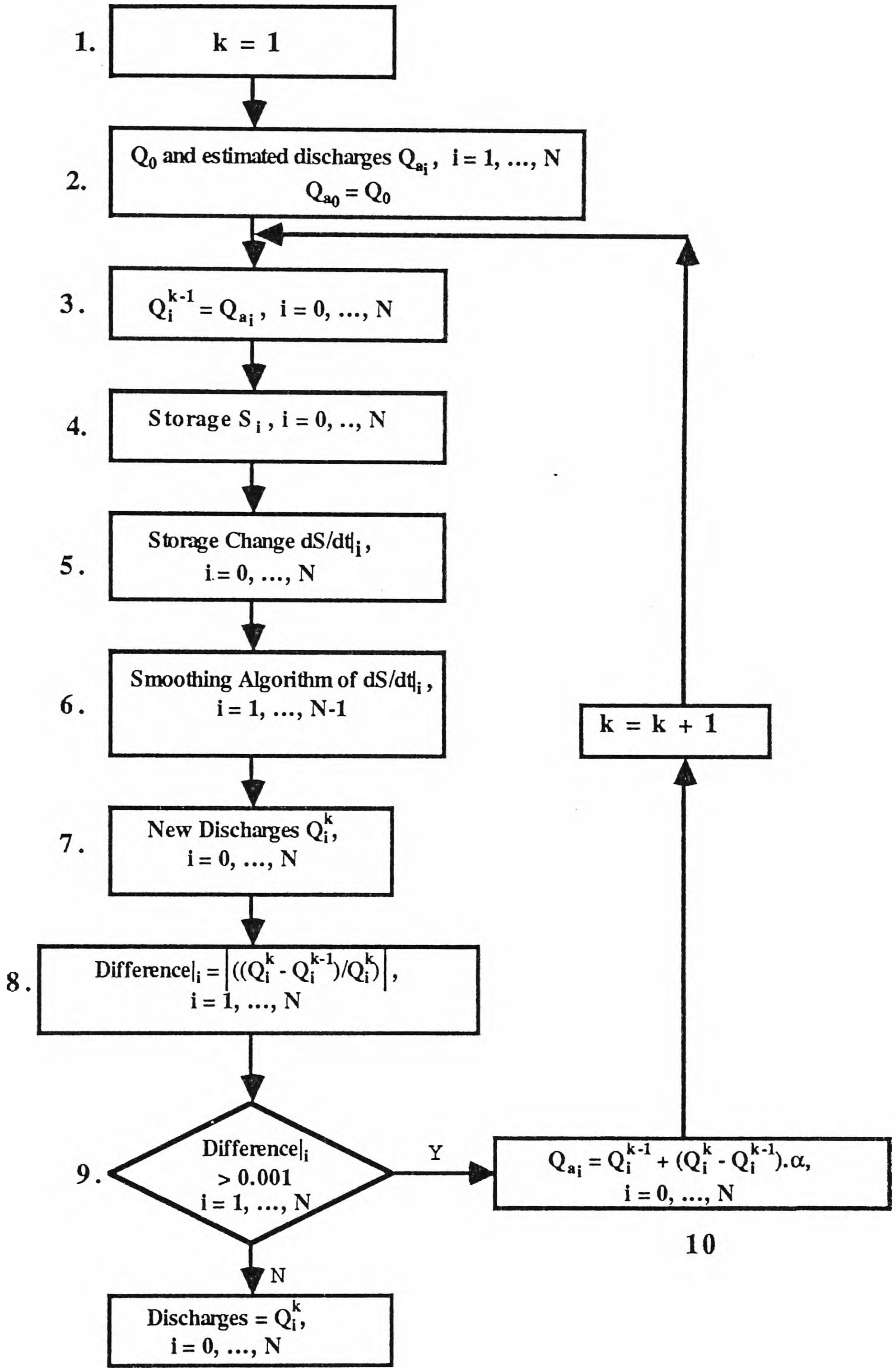


Figure 6.1.1 Flow Chart of the Computation

hydrograph ordinates as the first estimate of the downstream hydrograph ordinates (Q_a).

Step 3

Equate Q^{k-1} (downstream hydrograph ordinates at iteration k-1) with Q_a.

Step 4

Calculate storage S for all ordinates throughout the flood according to the given data I and the values of Q obtained in step 3 using eq.(6.1.3).

Step 5

Calculate storage change dS/dt using eq.(6.1.2) for all ordinates, except for the first and the last ordinates. Since the value of Q at time i = 0 (Q_0) is assumed, $dS/dt|_0$ is calculated using

$$\frac{dS}{dt}\Big|_0 = I_0 - Q_0 \tag{6.1.6}$$

The value of dS/dt at the last ordinate is calculated using eq.(6.1.5). It should be noted that the value of $dS/dt|_0$ is used in the smoothing algorithm (step 6) when time i = 1.

Step 6

Because oscillations are likely to occur in the estimated downstream hydrograph (as discussed in chapter 5), the smoothing algorithm

$$\frac{dS}{dt}\Big|_i^* = \left(\frac{dS}{dt}\Big|_{i-1}^* + 2.\frac{dS}{dt}\Big|_i + \frac{dS}{dt}\Big|_{i+1}\right)/4 \tag{6.1.7}$$

is applied to eradicate them. The subscript * refers to the value which has been or is being smoothed. The smoothing algorithm is carried out from the derivative at time i = 1 up to time i = N-1.

Step 7

Calculate new downstream hydrograph ordinates Q^k using eq.(6.1.1).

Step 8

Since the upstream hydrograph ordinates I are adopted as the first estimate of the downstream hydrograph ordinates to be used to estimate the ordinates Q at the next iteration, and the iteration is repeated until successive calculated downstream hydrographs converge, it is necessary to adopt a convergence criterion for terminating the iteration. If the relative change in each value of Q from one iteration to the next is expressed as

$$d_i = \left| \frac{Q_i^k - Q_i^{k-1}}{Q_i^k} \right| \qquad i = 1, 2, \ldots, N \qquad (6.1.8)$$

where superscript k refers to the value of Q at iteration k and superscript k-1 refers to the value of Q at iteration k-1, convergence can be said to have been reached when each d_i is equal to or less than some specified small quantity. In this project, as used in chapter 5, the convergence criterion is taken as

$$d_i \leq 0.001 \qquad (6.1.9)$$

Step 9

Check the values of d_i obtained from eq.(6.1.8) in step 8 using condition (6.1.9). If they are all equal to or less than 0.001, the downstream hydrograph ordinates are set equal to the Q^k ordinates and the process is finished. If not, continue to step 10.

Step 10

In order to improve the results dramatically, with fewer iterations required, the downstream hydrograph ordinates at iteration k are combined with those at iteration k-1 as a weighted average to make a new estimate of downstream hydrograph ordinates (Q_a). This condition is expressed as

$$Q_{a_i} = Q_i^{k-1} + (Q_i^k - Q_i^{k-1}).\alpha \qquad (6.1.10)$$

where i = 0,1, 2, ..., N and $0 < \alpha < 1$. It was found by numerical experiments that the effective α lies between 0.1 and 0.7. After calculating a new estimate of downstream hydrograph ordinates Q using eq.(6.1.10) return to step 3 to get into the next iteration (k+1).

Further explanation about the weighting factor α is discussed in section 6.3 of this chapter.

6.2 CONDITION TO CONVERGE

Equations (6.1.1), (6.1.2) and (6.1.3) can be combined to yield

$$Q_i = I_i - \frac{K}{2.\Delta t}\left[x.I_{i+1} + (1-x).Q^*_{i+1} - x.I_{i-1} - (1-x).Q^*_{i-1}\right] \qquad (6.2.1)$$

Superscript * refers to the values which are assumed for the trial or obtained from the previous iteration.

Convergence can be reached as long as the absolute value of the multiplying factor related to the unknown variable Q is less than 1.0 (as discussed similarly for upstream routing). This condition is expressed from eq. (6.2.1) as

$$K.(1-x)/(2.\Delta t) < 1, \text{ or}$$

$$\Delta t > K.(1-x)/2 \qquad (6.2.2)$$

The larger the time step Δt used in the computation, the fewer the number of iterations required. However, if the time step Δt is too large, not all points on the hydrograph are considered and the peak may be missed.

In practice, the limiting time step Δt required to converge is somewhat larger than that given by condition (6.2.2). This can be noticed more clearly from Figure 6.2.1.

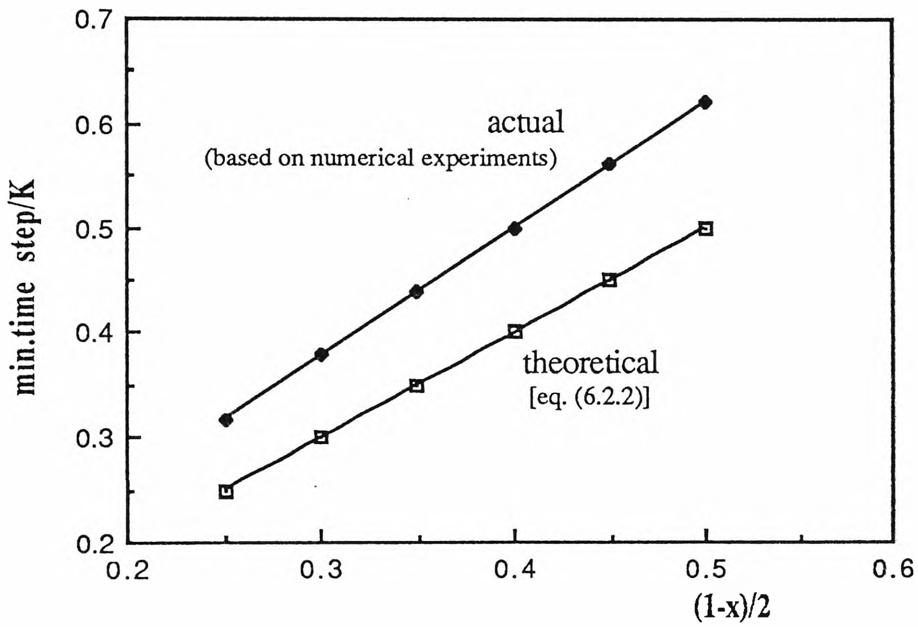


Figure 6.2.1 Graphic (1-x)/2 vs. Min.Time Step/K

The values in the actual line (Fig.6.2.1) were obtained by trial and error computations using the data taken from ARR87 Table 7.1 page 134. These values can be reduced down to those in the theoretical line, if a weighting factor is applied, as discussed in the next section.

6.3 WEIGHTING FACTOR (α)

As mentioned in section 6.1 step 10, applying a weighting factor α [eq.(6.1.10)] can improve the results, with fewer iterations required. The other advantage of applying a weighting factor α (as also mentioned in chapter 5) in the iterations is that the actual limiting time steps Δt can be reduced down to those in the theoretical line given by condition (6.2.2) or even to certain values of Δt which are less than those in the theoretical line if the appropriate weighting factor α is used. The particular values of Δt that can be reached should be determined by numerical experiments. For example if parameter x = 0 and K = 66 hours, then using condition (6.2.2), Δt > 33 hours. In practice, the minimum Δt which still can make the process converge without weighting factor (i.e. α = 1) is 41 hours. If a weighting factor α = 0.4 is applied, the time step Δt can be reduced down to 23 hours which is less than that given by condition (6.2.2).

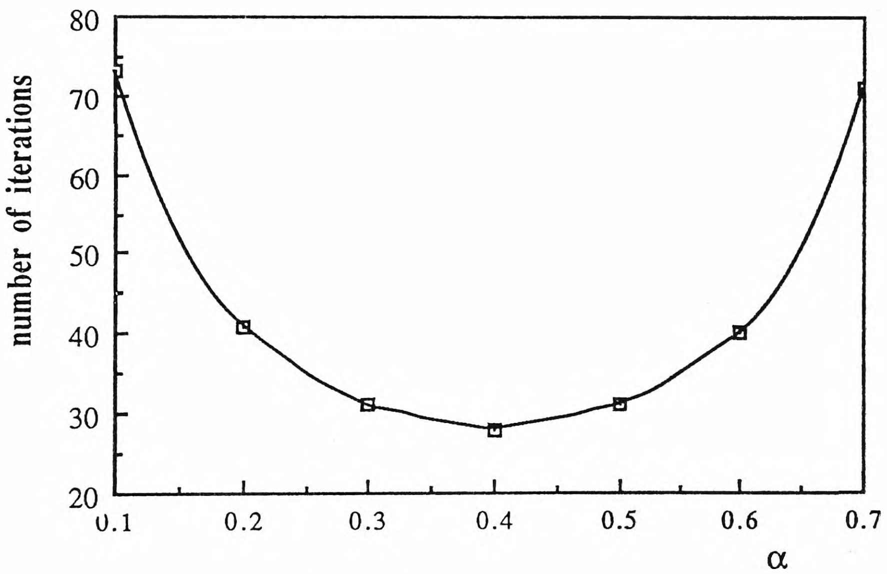


Figure 6.3.1 Graphic α Vs. Number of Iterations for Parameter x = 0.3

Based on numerical experiments using various values of parameter x, with K = 66 hours and $\Delta t > K.(1-x)/2$ and condition (6.1.9) for terminating the iteration, the optimum α which gives the fewest number of iterations is $\alpha = 0.4$ or a value which is close to 0.4 (see Fig. 6.3.1). Other values of parameter x result in similar graphics to that in Fig. 6.3.1 which is given by x = 0.3.

6.4 TESTS OF COMPUTATIONS

Tests of computations have been carried out in chapter 5 using the observed upstream hydrograph taken from ARR87 Table 7.1 page 134, see Tables V.1.1 column 3 and also Figs. 5.1.5 in this thesis.

In order to see the order of accuracy of the iterative method for downstream routing compared with that of the standard Muskingum method, results using both methods with the observed upstream hydrograph taken from ARR87 Table 7.1 page 134, parameter x = 0.45, K = 66 hours and Δt = 24 hours are presented in Fig. 6.4.1.

It can be noticed from Fig. 6.4.1 that the decreasing discharge which occurs on day 12 in the hydrograph obtained from the standard Muskingum method, occurs in the hydrograph obtained from the iterative method on day 11 and is 'deeper' than that on day 12. On the other hand, the peak of the hydrograph (the magnitude and the time at which it occurs) obtained from the iterative method is much closer to the recorded one compared with that obtained from the standard Muskingum method. It can be concluded that notwithstanding the decreasing discharge on day 11, the result obtained from the iterative method for downstream routing agrees as well with the recorded one as does the standard Muskingum method.

It should be noted that the decreasing discharge at the start of rise of the major peak in the downstream hydrograph occurs for all values of parameter x in the range $0 \le x \le 0.5$. The decreasing discharge becomes 'deeper' as the parameter x value increases.

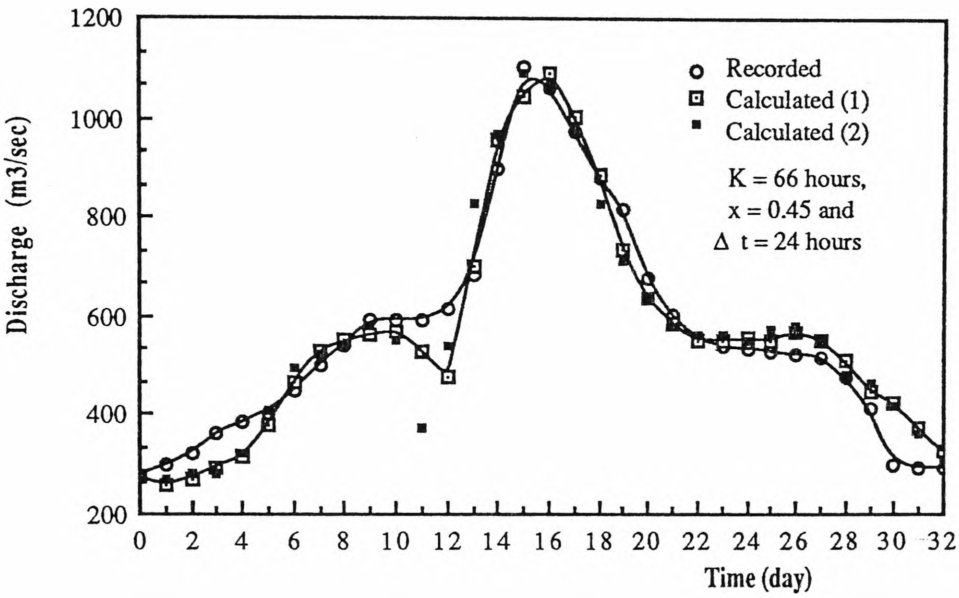


Figure 6.4.1 Downstream Routing to Obtain Downstream Hydrograph Using the Standard Muskingum and Iterative Methods

Note : Calculated (1) : calculated using the standard Muskingum method
Calculated (2) : calculated using the iterative method

6.5 SUMMARY

It has been shown that the iterative method developed in this study for the case of upstream routing can also be applied to downstream routing. It can be seen, by comparing the results obtained from the standard Muskingum and iterative methods with the observed downstream hydrograph, that the results obtained from the iterative method agree reasonably well with the observed hydrograph and also with the standard Muskingum method.

In order to converge, the iterative method has a condition for choosing the time step Δt [eq.(6.2.2)]. The number of iterations are reduced if a time step larger than this value of Δt is used. However, if the time step Δt chosen is too large, the shape of the calculated hydrograph is not adequately defined, and the peak may be missed.

If a weighting factor $\alpha = 0.4$ is used, the iteration converges much faster, and a smaller time step can be used.

Chapter Seven

Conclusions

Conventional application of the Muskingum method, to calculate the flood hydrograph at a downstream station on a river from a known hydrograph at an upstream station, has been shown to work satisfactorily. However, when the method is applied in reverse, to calculate the upstream hydrograph from a known hydrograph at a downstream station, the process has been found to be computationally unstable and the calculation diverges from the true solution. This has been investigated and found to be due to the values of the coefficients appearing in the equation.

The computational instability can be overcome by adopting an alternative finite difference approximation to the differential equation of conservation of mass, and solving the problem iteratively, in which the required values for each trial are set equal to the calculated values from the previous trial. The method has been found to converge to the correct solution, depending on the time Δt used, and this value of Δt depends on the values of the model parameters K and x. More rapid convergence occurs if a smoothing algorithm is applied to the derivative of storage S and a weighting factor (α) is applied to combine the calculated values from the last two trials as a weighted average to yield a new estimate for the subsequent trial.

The advantage of applying the weighting factor (α) in the calculations is in not only reducing the total number of iterations required but also reducing the limiting time step Δt which still can make the process converge.

Several variations of the iteration method have been investigated, including the use of backward differences and the Newton backward formula for estimating values at the end of the hydrograph. However best results were obtained when a simple two point central difference scheme, plus smoothing and weighting was used.

The computational instability of the Muskingum method to calculate the upstream hydrograph from a known hydrograph at a downstream station can also be overcome if the Muskingum equation is re-arranged to solve for I_i given I_{i+1} (i.e.: moving backward in time). The solution has been found to converge and yield very accurate estimates of the upstream hydrograph.

The application of cubic spline combined with the Runge-Kutta method to calculate the upstream hydrograph from a known downstream hydrograph does not yield satisfactory results. The computations have been found to diverge rapidly for any time step Δt, except for parameter $x = 0$ when only the cubic spline (without Runge-Kutta) is applied in the computation.

Computer programs have been developed which allow normal downstream routing calculation, upstream and downstream routing using the iterative method, and upstream routing moving backward in time. These computer programs contain graphical output. Examples of running the programs are given in Appendix A.

References

Bellman, R.E. and Roth, R.S. (1986). "Methods in Approximation". D. Reidel Publishing Co., Holland.

Borland International (1985). "Turbo Graphix Toolbox". Version 1.0, First Edition. U.S.A.

Boyd, M.J., Pilgrim, D.H., Knee, R.M. and Budiawan, D. (1989). "Reverse Routing to Obtain Upstream Hydrographs". Inst. Engineers Australia, Hydrology and Water Resources Symposium, National Conference Publ. 89/19, pp. 372-376.

Butler, S.S. (1957). "Engineering Hydrology". Prentice-Hall, U.S.A.

Cunge, J.A. (1969). "On the Subject of a Flood Propagation Method". Jour. of Hydraulic Research, Intl Assoc. of Hydraulic Research. Vol. 7, pp. 205-230.

De Vahl Davis, G. (1986). "Numerical methods in Engineering and Science". Allen and Unwin Ltd., U.K.

Diskin, M.H. (1967). "On the Solution of the Muskingum Method of Flood Routing Equation". Jour. of Hydrology, Vol. 5, pp. 286-289.

Dooge, J.C.I. (1973). "Linear Theory of Hydrologic Systems". U.s. Dep. Agric. Res. Serv. Washington., D.C., Tech. Bull. 1468.

Gill, M.A. (1978). "Flood Routing by the Muskingum Method". Jour. of Hydrology, Vol. 36, pp. 353-363.

Gill, M.A. (1979a). "Translatory Characteristics of the Muskingum Method of Flood Routing". Jour. of Hydrology, Vol. 40, pp. 17-29.

Gill, M.A. (1979b). "Flood routing by the Muskingum Method - Reply". Jour. of Hydrology, Vol. 41, pp. 169-170.

Grove, W.E. (1966). "Brief Numerical Methods", Prentice-Hall, N.J.

Institution of Engineers, Australia (1987). "Australian Rainfall and Runoff. A Guide to Flood Estimation". Vol. 1.

Jones, S.B. (1981). "Choice of Space and Time Steps in the Muskingum-Cunge Flood Routing Method". Proc. ICE, Part 2, Vol. 71, pp. 759-772.

Kulandaiswamy, V.C. (1966). "A Note on Muskingum Method of Flood Routing". Jour. of Hydrology, Vol. 4. pp. 273-276.

Linsley, R.K., Kohler, M.A. and Paulhus, J.L.H. (1982). "Hydrology for Engineers". 3rdEd. McGraw-Hill, New york.

Nash, J.E. (1959). "A Note on the Muskingum Flood Routing Method". Jour. of Geophysical Research, Vol. 64, pp. 1053-1056.

Pilgrim, D.H. and Watson, K.K (1967). " A Comparative Study of Response Correction Methods for Linear Ratemeters". Nuclear Instruments and Methods, Vol. 46, pp. 77-85.

Ponce, V.M., Yevjevich, V. and Simons, D.B. (1978). "The Numerical Dispersion pf the Muskingum Method". Proc. 26 th Hydraulic Div. Specialty Conf. ASCE, College Park Md.

Price, R.K. (1973a). "Flood Routing Methods for British Rivers". Proc. ICE, Part 2, Vol. 55, pp. 913-930.

Price, R.K. (1973b). "Variable Parameter Diffusion Method for Flood Routing". Hydr. Res. Station, Wallingflord, U.K. Rep. INT 115.

Raudkivi, A.J. (1979). "Hydrology - An Advanced Introduction to Hydrological Processes and Modelling". William Clowes (Beccles) Limited, Beccles and London.

Salvadori, M.G. and Baron, M.L (1964). "Numerical Methods in Engineering". Prentice Hall, N.J.

Savitch, W.J. (1986). "An Introduction to the Art and Science of Programming: Turbo Pascal Edition". The Benjamin/Cummings Publishing Co., Inc.

Scheid, F. (1968). "Theory and Problems of Numerical Analysis". Schaum's Outline Series. McGraw-Hill, U.S.A.

Schneider, G.M., Weingart, S.W., Perlman, D.M. (1982). "An Introduction to Programming and Problem Solving with Pascal". John Wiley and Sons.

Sharp, J.J. and Sawden, P. (1984). "Basic Hydrology". Butterworth and Co. Ltd., U.K.

Singh, V.P. and McCann, R.C. (1979). "Quick Estimation of Parameters of Muskingum Method of Flood Routing". Proc. 14 th Annu. Mississippi Water Resour. Conf., Jackson, Miss., Sept. 24-25, 1979, pp.65-70.

Singh, V.P. and McCann, R.C. (1980). "Some Notes on Muskingum Method of Flood Routing". Jour. of Hydrology, Vol. 48, pp. 343-361.

Stephenson, D. (1979). "Direct Optimization of Muskingum Routing Coefficients". Jour. of Hydrology, Vol. 41, pp. 161-165.

Steve W. (1986). "Using Turbo Pascal". Osborne McGraw-Hill.

Strupczewski, M. and Kundzewicz, Z. (1980a). "Muskingum Method Revisited". Jour. of Hydrology, Vol. 48, pp. 327-342.

Strupczewski, M. and Kundzewicz, Z. (1980b). "Translatory Characterictics of the Muskingum Method of Flood Routing - A Comment". Jour. of Hydrology, Vol. 48, pp. 363-368.

Venetis, C. (1969). "The IUH of the Muskingum Channel Reach". Jour. of Hydrology, Vol. 7, pp. 444-447.

Young, D.M. (1972). "A Survey of Numerical Mathematics". Volume 1. Addison-Wesley, U.S.A.

Appendix A

Examples of Running Programs

A.0 INTRODUCTION

The aim of these examples is to provide a set of easy to follow instructions on how to use the programs, on the function of each program and the data and information that each program requires.

The programs are written in **Turbo Pascal** language and allow for the input of upstream and downstream hydrograph data to be used for downstream or upstream routing. To describe results of computations more clearly, graphic programs are also provided. These programs comprise several sub-programs taken from 'Turbo Graphix Toolbox' by Borland International (1985). Some modifications to those sub-programs were made to suit the need of the numerical analysis. These graphic programs have some limitations, namely:

- Spline function which is used for fitting polynomials through the observed or calculated data points cannot be expected to work satisfactorily when the number of data points is very few (e.g. 3 data points). This is because the spline function is not fitted to endpoints. To overcome this problem, linear curves are used to replace spline function at end intervals.

- The maximum number of data points which can be plotted is approximately 60. This limitations is due to the size of matrix used in the spline computation in 'Turbo Graphix Toolbox'.

All of the programs in the diskette enclosed with this thesis are in machine code language. This means that to run the program, it is not necessary to first load the Turbo Pascal language into the memory of the computer. The programs can be run directly from DOS. The advantage of this is that the program will run much faster, because the computer does not have to change the Turbo Pascal commands into executable machine code commands while it is running.

A.1 HOW TO RUN THE PROGRAM

First of all, if the computer is off, place the diskette in disk drive A and switch on the computer. If everything is working correctly, there will simply be a prompt like 'A>'. If the computer is already on, place the diskette in disk drive A and depress <ctrl><Alt><Del> simultaneously. Now type 'INITIAL' and press <RETURN>. This will load the menu program, so on running the program the opening screen will look like this:

Flood Routing Program
M U S K I N G U M

by :
DEDI BUDIAWAN
Student id. no. 8900066

Department of Civil & Mining Engineering
The University of Wollongong
N.S.W. - A U S T R A L I A

Figure A.1.1

And then after few seconds,

MUSKINGUM
FLOOD ROUTING PROGRAM

MAIN MENU

[a] Downstream Routing
[b] Upstream Routing
[Q] Exit to System

Press the appropriate character and <RETURN> !

Your choice ? _

Figure A.1.2

This is the main menu of the programs. Once the user has decided which program to use, simply enter the character of the program followed by pressing <RETURN>.

A.2 WORKED EXAMPLES

The data used in these examples are the same as those used in the main chapters, i.e. the data taken from ARR87 Table 7.1 page 134 (Pilgrim, I.E.,Australia, 1987). See chapter 1 in this thesis.

Note: the character(s) written in italics is(are) entered by the user.

A.2.1 DOWNSTREAM ROUTING

If option [a] in Main Menu (Fig. A.1.2) is chosen, downstream routing calculation will be carried out. The following menu will come up:

```
PROGRAM OF DOWNSTREAM ROUTING

[a] Conventional Muskingum Method
[b] Iterative Method
[R] Return to Main Menu
[Q] Exit to System

Press the appropriate character and <RETURN> !

your choice ? _
```

Figure A.2.1

There are two methods of downstream routing, namely:

a. Conventional Muskingum method

b. Iterative method.

A.2.1.1 Downstream Routing Using Conventional Muskingum Method

If option [a] in the menu above (Fig. A.2.1) is chosen, the following menu will come up:

```
Downstream Routing Using Conventional Muskingum Method

              M E N U

              [a] Store data into a file
              [b] Change data
              [c] Run program
              [d] Graphic
              [e] Erase data file
              [R] Return to Main Menu
              [Q] Exit to System

    Press the appropriate character and <RETURN> !

              Your choice ? _
```

Figure A.2.2

Below is the description of main options of the menu in Fig. A.2.2.

[a] Store data into a file

After choosing [a] in the menu (Fig. A.2.2), the opening screen will be:

Name of data file to store observed upstream hydrograph data : *DATA1* <RETURN>

The next screen is:

The unit of inflow discharge has to be in m^3/sec.
Information :
The things which should be noticed are that:
1. Number of data is unlimited
2. As the last datum, simply write "30303"
3. The value of alfa (0<alfa<1) plays no part in
   Conventional Muskingum downstream routing

Routing Period T [hours] = *24*
Average Travel Time K [hours] = *66*
Parameter x = *0.00*
alfa = *0.40*

Inflow [0] = *274*
Inflow [1] = *314*
Inflow [2] = *355*
Inflow [3] = *404*
:
.
Inflow [30] = *301*

Inflow [31] = *274*
Inflow [32] = *271*
Inflow [33] = *30303*

Note: Inflow [0] = 274 denotes the value of inflow at time i = 0

Inflow [32] = 271 denotes the value of inflow at time i = 32

Inflow [33] = 30303 does not denote the value of inflow at time i = 33. This is used for terminating data input. The data in this example are taken from chapter 1 Table I.2.1 column (2) of this thesis.

After Inflow [33] = 30303, the screen will return to the menu in Fig. A.2.2.

<u>[b] Change data</u>

After choosing [b] in the menu (Fig. A.2.2), the opening screen will be:

Name of file of which data will be changed : *DATA1*

The next screen is:

INFORMATION
Name of file of which data will be changed : DATA1

PD	DATA	PD	DATA
0	24.000	1	66.000
2	0.000	3	0.400
:			
.			
etc.			

(Press <RETURN>)

PD is datum position number on which the change of datum is based.

After pressing <RETURN>, the next screen will be:

INFORMATION :

You will change the data in a file named : DATA1
if there is no datum changed, give 30303 to PD !

Position number of datum (PD) which is changed : *2*
Old datum : 0.00
New datum : *0.45*

Do you want to change more data ? [Y/N] *N*

After pressing 'N' and <RETURN>, the program will return to the menu in Fig. A.2.2.

[c] Run Program

After choosing [c] in the menu (Fig. A.2.2), the opening screen will be :

INFORMATION :
You must have the inflow hydrograph data
stored in a file
if not, choose [a] in M E N U

Do you have an inflow hydrograph data file ? [Y/N] *Y*

The next line is :

Name of inflow hydrograph data file : *DATA1*

The next screen will be:

The current values of T, K, x and alfa are:
Routing Period T = 24.00 hours
Average Travel Time K = 66.00 hours
Parameter x = 0.00
alfa = 0.40
Note: The value of alfa (0<alfa<1) plays no part in this method

Do you want to make any changes to K and x values ? [Y/N] *Y*

The next lines will be:

New Average Travel Time K [hours] = *66*
New Parameter x = *0.45*

In this case, only the parameter x value is changed.

The next line is:

Do you want to change T value ? [Y/N] *N*

If the answer is 'Y' then the program will ask for the new routing period. Based on this new routing period, the data are interpolated. Then, the program will ask for a data filename to put these interpolated data. They are needed in the graphic program.

The next screen will be:

Is starting outflow value the same as starting inflow value ? [Y/N] *Y*

The program will then display the result of the computation:

Q outflow [0] = 274.000
Q outflow [1] = 259.342
Q outflow [2] = 271.476
Q outflow [3] = 295.022
:
.
etc.

Press any key to continue !

After displaying the result of the computation, the next screen will be:

Total volume of inflow hydrograph = 1583064000.00 m^3
Total volume of outflow hydrograph = 1576724760.40 m^3
Relative difference between these total volumes = 0.400 %

Name of data file to store result matrix : *BBB*

Note: Relative difference between the total volumes above is obtained using:

$$D_R = \frac{|V_{in} - V_{out}|}{V_{in}} \cdot 100\%$$

where: V_{in} : total volume of inflow hydrograph

V_{out} : total volume of outflow hydrograph, and

D_R : relative difference between these total volumes

The next screen is:

Input & output will be printed ? [Y/N] *N*

If the answer is 'Y', make sure that the printer is already on. The results of the computations including the data used are printed in a tabular form as presented in the former chapters.

The last screen of this program is:

Return to MENU ? [Y/N] *Y*

If the answer is 'N' then it will exit to DOS. Since the answer is 'Y', the program will return to menu in Fig.A.2.2, and the next option is ready to be chosen.

[d] Graphic

This graphic program can be run provided that program [c] has been run to obtain the result of computation stored in a file.

Below are the instructions after choosing [d] in the menu (Fig. A.2.2).

Do you have RESULT FILE, obtained by
running program [c] in MENU ? [Y/N] *Y*

If the answer is 'N', the program will ask the user to return to MENU.

The next screen is:

Was your result file obtained by iterative method ? [Y/N] *N*

This question determines whether or not the value of alfa is put in the graphic. In this method the value of alfa plays no part so that the answer is 'N'.

The next screen will be:

Name of the calculated outflow data file : *BBB*

and then

Name of the inflow hydrograph data file used
for calculation : *DATA1*

It should be noted that if the data have been interpolated according to the new value of T before being processed, the name of the inflow hydrograph data file must be the name of the file in which the interpolated data are stored (see part '[c] Run Program')

The next screen is :

Graphic will be printed ? [Y/N] *N*

If the answer is 'Y', make sure that the printer is already on.

After a few moments, the screen will display the graphic. To return to text mode, simply press any key.

The last screen of this program is:

Return to MENU ? [Y/N] *Y*

The program will return to menu in Fig. A.2.1. If the answer is 'N', it will exit to DOS.

A.2.1.2 Downstream Routing Using Iterative Method

If option [b] in the menu (Fig. A.2.1) is chosen, the following menu will come up:

```
Downstream Routing Using Iterative Method

  M E N U

  [a] Store data into a file
  [b] Change data
  [c] Run program
  [d] Graphic
  [e] Erase data file
  [R] Return to Main Menu
  [Q] Exit to System

Press the appropriate character and <RETURN> !

  Your choice ? _
```

Figure A.2.3

Except option '[c] Run Program', the other options in Fig. A.2.3 will not be discussed any longer since they are similar to those in section A.2.1.1.

<u>[c] Run Program</u>

After choosing [c] in the menu (Fig. A.2.3), the opening screen will be:

```
INFORMATION :
You must have the inflow hydrograph data
stored in a file
if not, choose [a] in M E N U

Do you have an inflow hydrograph data file ? [Y/N] Y
```

The next line is :

Name of inflow hydrograph data file : *DATA1*

The next screen will be:

The current values of T, K, x and alfa are:
Routing Period T = 24.00 hours
Average Travel Time K = 66.00 hours
Parameter x = 0.00
alfa = 0.40
Do you want to make any changes to K, x and alfa values ? [Y/N] *Y*

The next lines will be:

New Average Travel Time K [hours] = *66*
New Parameter x = *0.45*
New alfa = *0.40*

Only the parameter x value is changed in this case.

The next lines are :

Based on K and x values in order to converge,
Routing Period T should be > 18.15 hours

Do you want to change T value ? [Y/N] *N*

If the answer is 'Y' then the program will ask for the new routing period. If the new routing period is still ≤ 18.15 hours, the program will warn that it may lead to divergence and it will ask the user whether or not to correct the routing period T again. Based on the new routing period, the data are interpolated. Then, the program will ask for a data filename to put these interpolated data. They are needed in the graphic program.

The next screen will be:

Is starting outflow value the same as starting inflow value ? [Y/N] *Y*

The program will then display:

Iteration 21

Process has been finished !
Press any key to continue !

The next screen will be the result of the computation:

Q outflow [0] = 274.000
Q outflow [1] = 269.450
Q outflow [2] = 283.490
Q outflow [3] = 280.261
Q outflow [4] = 319.690

:
.
etc.

Press any key to continue !

After displaying the result of the computation, the next screen will be:

Total volume of inflow hydrograph = 1583064000.00 m^3
Total volume of outflow hydrograph = 1578855653.00 m^3
Relative difference between these total volumes = 0.266 %

Name of data file to store result matrix : *BBB*

Note: The relative difference between the total volumes above is obtained using the same formula given in section A.2.1.1 part '[c] Run Program'.

The next screen is:

Input & output will be printed ? [Y/N] *N*

If the answer is 'Y', make sure that the printer is already on. The results of the computations including the data used are printed in a tabular form as presented in the former chapters.

The last screen of this program is:

Return to MENU ? [Y/N] *Y*

If the answer is 'N' then it will exit to DOS. Since the answer is 'Y', the program will return to menu in Fig.A.2.3.

A.2.2 UPSTREAM ROUTING

If option [b] in Main Menu (Fig. A.1.2) is chosen, upstream routing calculation is ready to be carried out. The following menu will come up:

```
PROGRAM OF UPSTREAM ROUTING

[a] Moving Backward in Time
[b] Iterative Method
[R] Return to Main Menu
[Q] Exit to System

Press the appropriate character and <RETURN> !

your choice ? _
```

Figure A.2.4

There are two methods of upstream routing, namely:

a. moving backward in time

b. iterative method

The worked examples for storing data into a file, changing data and graphic will not be described any longer in this section, since they are similar to those in section A.2.1.1. The only option which will be described is '[c] Run Program'. The data used in these examples are taken from chapter 1 Table I.2.1 column (3). These data were stored using option [a] in the menu presented later below with file name: DATA2.

A.2.2.1 Upstream Routing Moving Backward in Time

If option [a] in the menu (Fig. A.2.4) is chosen, the following menu will come up:

Upstream Routing Moving Backward in Time

M E N U

[a] Store data into a file
[b] Change data
[c] Run program
[d] Graphic
[e] Erase data file
[R] Return to Main Menu
[Q] Exit to System

Press the appropriate character and <RETURN> !

Your choice ? _

Figure A.2.5

Below is the description of option [c] of the menu in Fig. A.2.5.

[c] Run Program

After choosing [c] in the menu (Fig. A.2.5), the opening screen will be:

INFORMATION :
You must have the outflow hydrograph data
stored in a file
if not, choose [a] in M E N U

Do you have an outflow hydrograph data file ? [Y/N] *Y*

The next line is :

Name of outflow hydrograph data file : *DATA2*

The next screen will be:

The current values of T, K, x and alfa are:
Routing Period T = 24.00 hours
Average Travel Time K = 66.00 hours
Parameter x = 0.00
alfa = 0.40
Note: The value of alfa (0<alfa<1) plays no part in this method

Do you want to make any changes to K and x values ? [Y/N] *Y*

The next lines will be:

New Average Travel Time K [hours] = *66*
New Parameter x = *0.45*

In this case, only the parameter x value is changed.

The next line is:

Do you want to change T value ? [Y/N] *N*

As has been mentioned in the previous section, if the answer is 'Y' then the program will ask for the new routing period. Based on this new routing period, the data are interpolated. Then, the program will ask for a data filename to put these interpolated data. They are needed in the graphic program.

The next screen will be:

Is starting inflow value the same as starting outflow value ? [Y/N] *Y*

If the answer is 'N', the program will ask for the new starting inflow value. It should be carefully noted that since this method is moving backward in time, the starting inflow is the value at the end of hydrograph (at time i = N).

The program will then display the result of the computation:

Q inflow [0] = 343.920
Q inflow [1] = 373.235
Q inflow [2] = 415.214
Q inflow [3] = 432.436
Q inflow [4] = 469.265

:
.
etc.

Press any key to continue !

After displaying the result of the computation, the next screen will be:

Total volume of inflow hydrograph = 1579346579.70 m^3
Total volume of outflow hydrograph = 1583020800.00 m^3
Relative difference between these total volumes = 0.232 %

Name of data file to store result matrix : *CCC*

Note: Relative difference between the total volumes above is obtained using:

$$D_R = \frac{|V_{in} - V_{out}|}{V_{out}} \cdot 100\%$$

where: V_{in} : total volume of inflow hydrograph

V_{out} : total volume of outflow hydrograph, and

D_R : relative difference between these total volumes

The next screen is:

Input & output will be printed ? [Y/N] *N*

If the answer is 'Y', make sure that the printer is already on. The results of the computations including the data used are printed in a tabular form as presented in the former chapters.

The last screen of this program is:

Return to MENU ? [Y/N] *Y*

If the answer is 'N' then it will exit to DOS. Since the answer is 'Y', the program will return to menu in Fig.A.2.5.

A.2.2.2 Upstream Routing Using Iterative Method

If option [b] in the menu (Fig. A.2.4) is chosen, the following menu will come up:

```
Upstream Routing Using Iterative Method

  M E N U

  [a] Store data into a file
  [b] Change data
  [c] Run program
  [d] Graphic
  [e] Erase data file
  [R] Return to Main Menu
  [Q] Exit to System

Press the appropriate character and <RETURN> !

  Your choice ? _
```

Figure A.2.6

Below is the description of option [c] of the menu in Fig. A.2.6.

[c] Run Program

After choosing [c] in the menu (Fig. A.2.6), the opening screen will be:

INFORMATION :
You must have the outflow hydrograph data
stored in a file
if not, choose [a] in M E N U

Do you have an outflow hydrograph data file ? [Y/N] *Y*

The next line is :

Name of outflow hydrograph data file : *DATA2*

The next screen will be:

The current values of T, K, x and alfa are:
Routing Period T = 24.00 hours
Average Travel Time K = 66.00 hours
Parameter x = 0.00
alfa = 0.40
Do you want to make any changes to K, x and alfa values ? [Y/N] *Y*

The next lines will be:

New Average Travel Time K [hours] = *66*
New Parameter x = *0.45*
New alfa = *0.40*

Only the parameter x value is changed in this case.

The next lines are :

Based on K and x values in order to converge,
Routing Period T should be > 14.85 hours

Do you want to change T value ? [Y/N] *N*

As has been similarly mentioned in section A.2.1.2, if the answer is 'Y', the program will ask for the new routing period. If the new routing period is still $\leq$ 14.85 hours, the program will warn that it may lead to divergence and it will ask the user whether or not to correct the routing period T again. Based on the new routing period, the data are interpolated. Then, the program will ask for a data filename to put these interpolated data. They are needed in the graphic program.

The next screen will be:

Is starting inflow value the same as starting outflow value ? [Y/N] *Y*

The program will then display:

Iteration 16

Process has been finished !
Press any key to continue !

The next screen will be the result of the computation:

Q inflow [0] = 274.000
Q inflow [1] = 378.502
Q inflow [2] = 406.226
Q inflow [3] = 445.065
Q inflow [4] = 469.599

:
.
etc.

Press any key to continue !

After displaying the result of the computation, the next screen will be:

Total volume of inflow hydrograph = 1575918712.50 m^3
Total volume of outflow hydrograph = 1583020800.00 m^3
Relative difference between these total volumes = 0.449 %

Name of data file to store result matrix : *CCC*

Note: The relative difference between the total volumes above is obtained using the same formula given in section A.2.2.1.

The next screen is:

Input & output will be printed ? [Y/N] *N*

If the answer is 'Y', make sure that the printer is already on. The results of the computations including the data used are printed in a tabular form as presented in the former chapters.

The last screen of this program is:

Return to MENU ? [Y/N] *Y*

If the answer is 'N' then it will exit to DOS. Since the answer is 'Y', the program will return to menu in Fig.A.2.6.

wabash DATATECH®