

2002

## Sensitivity analysis of satellite trajectory under gravitational and aerodynamic perturbations

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Ribicic, Mirela M., Sensitivity analysis of satellite trajectory under gravitational and aerodynamic perturbations, Master of Engineering (Hons.) thesis, Department of Mechanical Engineering, University of Wollongong, 2002. <https://ro.uow.edu.au/theses/2526>

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# **SENSITIVITY ANALYSIS OF SATELLITE TRAJECTORY UNDER GRAVITATIONAL AND AERODYNAMIC PERTURBATIONS**

by

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(BE)**

Thesis submitted in fulfilment of the requirements for the award of the  
degree

**Master of Engineering  
(HONS)**

**from: University of Wollongong  
Supervisor: Prof. M. P. West  
Department of Mechanical Engineering**

## **ACKNOWLEDGMENTS**

First of all I would like to express gratitude to my supervisor Professor M.P. West. Without his guidance, help, enthusiasm, patience and knowledge this work would never be completed.

My gratitude is extended to The University of Wollongong for all support given. I am very thankful to my friends, especially David, for their great interest in this project.

Finally I would like to thank to my mother and brother for their continuous encouragement and support, for all creative ideas, for understanding and sharing with me every bad and good moment during the past two years.

## ABSTRACT

The trajectory analysis of a celestial object orbiting another body is the basis for all other, more complex theories of spacecraft motion. It is also highly important for the design and construction of spacecraft.

New computer software SATELIGHT is developed for computing the satellite coordinates with respect to the Earth centre or to the point on its surface, in orbit plane or in right ascension system.

A dynamic model is presented by second order differential equations solved numerically. The numerical method used is Runge Kutta IV order. Initial conditions are based on the orbit characteristics – shape and orientation, and are result of mission objectives and constraints analysis.

The model is developed gradually. The starting stage is solving the so-called Kepler's two body problem which includes only gravitational force without any perturbing forces. This model is further modified for anomalies of the Earth gravitational field, atmospheric drag and three body problem – influence of Moon on the trajectory of the Earth satellite. The model for the Three-Body perturbation gives solution for any situation in the space and computes change in the orbit inclination angle.

The coordinates are obtained in numerical form with adjustable precision, depending on the computer capability. Results could be transferred to the Excel and by using a particular program could be imported into ACAD and plotted as a drawing file. This gives great visual presentation in two dimensions, with opportunity to effectively compare, measure and further manipulate imported data.

This work is primarily concerned with unmanned Earth orbiting spacecraft but the basic principles are sufficiently broad to be applicable to any situation. The advantage of this software is its flexibility to be modified for any specific situation required by initial or environmental conditions.

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## NOMENCLATURE

$a$  – semimajor axis of the ellipse

$a$  – acceleration

$a_x$  - the acceleration component in  $x$  direction

$a_y$  - the acceleration component in  $y$  direction

$a_z$  - the acceleration component in  $z$  direction

$b$  – semiminor axis of the ellipse

$c$  – velocity of light in vacuum,  $c = 2.99 \times 10^8 \frac{m}{s}$

$C_D$  - Air Drag coefficient

$C_{Dn}$  - Air Drag coefficient for a compressible flow

$C_m$  - Centre of mass

$d$  – distance between masses

$D$  – drag component of the Aero-dynamic force

$e$  – eccentricity of the orbit

$E$  – eccentric anomaly

$E$  – internal energy

$F$  – focus

$F$  – hyperbolic anomaly

$F_s$  - total external force applied to the system

$F_g$  - gravitational attraction force between bodies

$F_{g_x}$  - the component of gravitational attraction force in  $x$  direction

$F_{g_y}$  - the component of gravitational attraction force in  $y$  direction

$F_{g_z}$  - the component of gravitational attraction force in  $z$  direction

$g$  – Earth gravitational field strength

$g_0$  - acceleration due to Earth gravitational field measured at the equator

$G$  - Universal gravitation constant,  $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

$h$  - altitude

$h$  - time step for the numerical method

$i$  - inclination

$i$  - unit vector in  $x$  direction

$I$  - inertia tensor

$j$  - unit vector in  $y$  direction

$J$  - coefficient of zonal harmonics, tesseral harmonics and sectorial harmonics

$k$  - unit vector in  $z$  direction

$m_1, m_2 \dots m_n$  - point masses

$M_E$  - mass of the Earth,  $M_E = 5.976 \times 10^{24}$

$M$  - mean anomaly

$N$  - normal force

$n$  - unit normal

$O$  - origin of the reference frame

$p$  - pressure

$Q$  - heat

$\bar{r}$  - radial component or displacement

$\dot{\bar{r}}$  - radial component of velocity,  $\dot{\bar{r}} = \frac{d\bar{r}}{dt}$

$\ddot{\bar{r}}$  - radial component of acceleration,  $\ddot{\bar{r}} = \frac{d^2\bar{r}}{dt^2}$

$r_a$  - radius of apogee

$r_p$  - radius of perigee

$\vec{R}$  - position vector

$R$  - perturbing or disturbing potential

$R$  - Sun radius  $R=93$  million miles

$R_e$  - Reynold's number

$R_E$  - mean equatorial Earth radius

$R_0$  - mean Earth radius,  $R_0 = 6378$  km

$R_p$  – Earth polar radius

$S$  – inertial reference frame

$S$  – reference area

$t$  – time

$t_f$  - time of flight

$T$  - period of orbit

$T$  – temperature

$U$  – potential function; potential energy per unit mass

$V_x$  - velocity component in  $x$  direction

$V_y$  - velocity component in  $y$  direction

$V_z$  - velocity component in  $z$  direction

$V_c$  - local velocity in a circular trajectory

$V_e$  - local velocity in a elliptical trajectory

$V_p$  - local velocity in a parabolic trajectory

$V_h$  - local velocity in a hyperbolic trajectory

$V_{esc}$  - escape velocity

$x$  – the displacement along  $x$  axis

$x$  –  $x$  axis of a reference frame

$\dot{x}$  - velocity component in  $x$  direction

$\ddot{x}$  - acceleration component in  $x$  direction

$y$  – the displacement along  $y$  axis

$y$  –  $y$  axis of a reference frame

$\dot{y}$  - velocity component in  $y$  direction

$\ddot{y}$  - acceleration component in  $y$  direction

$z$  - the displacement along  $z$  axis

$z$  –  $z$  axis of a reference frame

$\dot{z}$  - velocity component in  $z$  direction

$\ddot{z}$  - acceleration component in  $z$  direction

$\alpha$  - right ascension

$\beta$  - latitude angle

$\gamma$  - Vernal Equinox direction

$\delta$  - declination angle

$\theta$  - Euler angle

$\lambda$  - longitude angle

$\mu$  - attraction parameter,  $\mu = GM_E$

$\rho$  - atmospheric density

$\varpi$  - angular velocity

$\Omega$  - longitude of ascending node

# **CHAPTER 1**

## **Introduction**

## Chapter 1: Introduction

The most important element of any project in modern aerospace vehicle development is the engineering simulation model of the flight dynamics of the vehicle.

To describe and predict the resulting motion of the satellite, it is necessary to understand the physical processes by which forces cause objects to move. The analysis of the forces acting on the satellite starts with the most basic type of motion in the space – *the two body problem*. This analysis, of the laws that govern motion in a central force field, is presented in Chapter 2 of the thesis. The establishment of a mathematical model that describes the observed motion concludes in the final stage of Chapter 2. The second order ordinary differential equation, projected onto two directions of a right-angled rectangular Cartesian coordinate system, is further solved by a numerical method – Runge – Kutta IVth order. To enable the application of the numerical method to this problem, a software system named SATELIGHT is developed, in FORTRAN. The derivation and discussion of the numerical method, including error analysis and improvement of the results, is presented in Chapter 3.

After obtaining the method for computing with the two-body orbit model, the analysis is continued in two, more specific, directions:

1. The first one is to incorporate the effect of Aerodynamic Forces into the two-body model. The satellites observed here are Earth satellites, and Aerodynamic Forces are due to the Earth atmosphere. The intensity of these forces varies directly with respect to the Atmosphere characteristics and to geometrical features of the satellite.



2. The second objective is to include gravitational perturbations that have significant effects on the satellite trajectory. These derive from the Earth Gravitational Field Anomalies and the Moon's impact on the observed orbit.

The results of this theory are analysed in Chapter 4.

A particular problem that is solved by the software developed is the effect of the Gravitational perturbations on geosynchronous orbits. A satellite that is placed in this orbit remains above the same point at the Earth's surface during its rotation. In reality, satellites in geosynchronous orbit achieve a mean motion from 0.9 to 1.1 revolutions per day. Depending on the satellite purpose, a particular tolerance for the satellite's orbital deviation is specified, which, when exceeded, is corrected by the utilization of firing devices – thrusters. The effects of the gravitational perturbations included in the model on the geosynchronous orbit are analysed and discussed in Chapter 5.

The theory of the Atmospheric Drag effects is applied to the International Space Station orbital motion, including an analysis of the Air Drag Coefficient. This coefficient depends on the geometrical characteristics of the station and is based on the station photographs obtained from NASA. The determination of the Aerodynamic Forces depends on the evaluation of the air density at a particular altitude. The model of the Earth's atmosphere is also produced as a part of this research and is based on real measurements collected by a satellite that already had been in the orbit [7]. A predicted degradation of the Orbital Altitude of the International Space Station is also calculated and presented in Chapter 5.

The next introductory section is based on the Space Mission Design methods applied at NASA [8]. It explains the importance and the purpose of the Spacecraft Dynamics Analysis.

## 1.1. Introduction to the Space Flight Dynamics

One of the most systematic ways to perform an analysis of the proposed space flight and to determine the trajectory characteristics is to divide the analysis into four discrete, but integrated areas:

1. Mission Design and Analysis
2. Orbit Engineering
3. Attitude Engineering
4. Tracking Network Support

Each of the noted areas is more or less directly related to the space trajectory analysis. These relationships are discussed in more details in further text. Trajectory analysis is one of the most important aspects of any Space Flight and affects all other Space Flight Mission elements.

To make a short overview of the importance of the trajectory construction and later satellite trajectory monitoring more general approach is used.

For comparison, another approach is to set the phases as:

1. Launch – The phase that starts by lift-off and completes by the end of powered flight in a preliminary Earth orbit
2. Acquisition – While in preliminary orbit, the number of corrective manoeuvres is performed with the aim to achieve a desired orbit shape and altitude. This is also the stage in which all hardware is tested.
3. Mission Operations – The phase in which vehicle actually fulfils the concrete mission for which the flight is intended.

In this case, the 2<sup>nd</sup> and 3<sup>rd</sup> phases are based on the trajectory analysis. Many of the principles applied in the last two phases are also applicable for the Launch phase – example air drag.

### 1.1.1. Mission Design and Analysis

This could be observed as the initial stage of a flight ‘construction’ procedure, where trajectory design, orbit analysis and evolution and finally, an appropriate analysis tool are defined.

All of these elements are in some degree directly related to the actual aim of the mission.

These activities are actually contained in the pre-launch preparation phase, and later followed by in-flight operational support and ‘proof of concept’ studies for future missions.

Throughout the complete mission life cycle, project managers and principal investigators provide practical value and expertise. It is very important to maintain close cooperation with stakeholders, from conceptualisation, through to the post-mission analysis stage, to ensure cost-effective outcomes.

One of the first actions in planning a mission is to determine the type of the orbit to be achieved. There are a variety of missions that are designed for a particular application.

Some of them are represented below:

- Low Earth
- Geosynchronous
- Highly eccentric
- Libration point
- Lunar
- Interplanetary trajectories

Each of these missions requires a special orbit type. These orbits have to be constructed, initial conditions selected and the applicable software used for final computation. SATELIGHT is developed on the sense to provide a solution for each of the noted orbits. Selection of the initial conditions is given in Chapter 4, the software practical interpretation results in Chapter 5 and the software development and characteristics in Chapters 2 and 3. It could be seen from the above text that all other mission elements depend heavily on the orbit type that has to be achieved.

Starting by the goal definition, the whole process of developing the mission strategy could be subdivided into the following segments:

A. Establishing initial-viable trajectory composed of:

1. Defining scientist's mission goals
2. Producing a viable trajectory
3. Providing input to subsystem designers (power, thermal, propulsion, communications, attitude control)

Some of the special types of trajectory design are:

- Formation Flying – a new concept created with the aim to fly several satellites in formation form to observe object in space with much larger sensors than could be flown on a single satellite. This is one of the areas that is developing very fast, with the number of new exciting ideas but also with realistic projects already at an advanced stage.
- Constellation Design – the concept that is based on the idea of making a grid over the part of Earth or very often the entire Earth. In other words each point on the Earth's surface should be able to contact one satellite at each point of time. Constellation design differs from single orbit construction, but is heavily based on it. Constellation design incorporates basic shape of the orbit (circular, elliptic, synchronous, etc) with for example dynamic effects of Earth's gravitational anomalies. Usually there is no single answer on observed problem, but an array of constellations with specific properties to support various mission constraints.
- Aerobraking – using a planetary atmosphere to change the orbit of a spacecraft. Some of the satellite orbital energy is changed into thermal energy due to the interaction with the atmospheric particles. This concept is particularly effective way to mitigate the total mass problem, ie. if aerobraking is used the cchemical propulsion system on the spacecraft can be much smaller.

A number of characteristics of a given trajectory, segment by segment, affect further development. Some of the items provided to scientists and spacecraft designers are:

- Propellant loading studies – affected by trajectory correction method
- Lifetime analysis – depends on aero drag analysis
- Trajectory error analysis – depends on the numerical method applied
- Orbital event analysis – determined by the aim of the mission
- Eclipse profiles – affected by the orbital elements

- Station contact data – also affected by the orbital elements
- Launch vehicle dispersion analysis – affected by parking orbit
- Reentry and disposal orbit planning – determined by Aerodynamic perturbations/ effects and orbit elements characteristics

**B.** In this stage the trajectory is refined, necessary changes are introduced to the initial model and more detailed analysis is applied. The final results are utilised further in the appropriate sections. This process is divided into:

1. Design of subsystems/elements
2. Construction
3. Testing

**C.** The mission trajectory is subject to changes, both planned and contingency. Various strategies are applied to cope with the effects of these changes on mission goals. Placing the vehicle in a desired orbit completes the final stage and could be divided into:

1. Ascent from the injection orbit to the mission orbit
2. Maintaining the mission orbit
3. End-of-life disposal, which has to meet particular requirements.

This outline of the flight design process indicates how important it is to determine precise models of the motion in space. Accurate prediction of the drift from an ideal trajectory due to particular perturbing force will largely affect the design stage of the vehicle, but also flight control methods.

### **1.1.2. Mission Analysis Tools**

The Mission Analysis Tools define the software systems implied for trajectory design and orbit mission analysis.

There are a number of different software systems available commercially, each of them developed for particular conditions, and providing a particular accuracy level. In the process of selection of the appropriate software, particular emphasis is placed on the accuracy of the system, as the trajectory design depends heavily on the developed model. Methods based on Numerical Integration give one of the most accurate solutions to the problem of motion in the space. One of these, a step-by-step method, is applied in

this project, and used as a base for further software development. Error analysis of the method is presented in the Chapter 3.

It is very important to understand and to select the most appropriate software, or a combination of the software tools, to solve a given problem for a particular Mission in Space.

### **1.1.3. Orbit Engineering**

Orbit Engineering consists of orbit determination prior to launch of the spacecraft, and maintaining and controlling the orbit after the launch.

During the first phase, science mission objectives and constraints are analysed, and as a result, orbit shape and orientation are determined.

After launch, the satellite transmits data back to the control centre, known as The Space Network or The Ground Network. This data is used to compute the distance to the spacecraft (range), the rate at which this distance is changing (range rate) and the direction in which the tracking station antenna is pointed while it communicates with the satellite. The orbit generated from the acquired data from a satellite is compared to the theoretical orbit, and based on these results the predicted orbit is further computed.

Besides the position, that is a subject of calculation, the associated velocity at any time is also determined.

The predicted orbit is used further for producing so-called scheduling aids. Scheduling aids indicate spacecraft environmental conditions (for example sun or shadow, interference regions and altitude) as well as all potential station-to-spacecraft contact times (view periods). Based on the determined environmental conditions, plans for

scientific data collection can be established. From the estimated station view periods, Control Centre or Project Operations Control Centres can select the times needed to meet mission communications requirements.

Most of the satellites will experience some orbit transformation during their lifetime. These transformations can occur with the aim to correct disturbances due to perturbing forces, or simply to achieve desirable orbit shape, size or inclination – these orbit elements are defined in Chapter 2.

Thrusters are the orbit – adjust engines used to achieve the required orbit transfer. Elements that have to be determined for successful orbit change are direction, duration and particular times at which thruster firing will occur. Computed thruster firing elements are converted into spacecraft-recognizable commands, and transmitted from the Project Operations Control Centres to the Tracking Station, and then to the satellite for execution.

For the satellites (elliptical or circular orbit) that do not have installed thrusters, or for those which have run out of fuel, the orbit cannot be corrected, and the satellite will lose its altitude gradually because of the drag of the Earth's atmosphere (orbit decay). Details about methods by which a perturbed orbit could be altered are presented in Chapter 5.

Eventually, the satellite will re-enter the Earth's atmosphere, and one of the most important applications of the predicted orbits is to determine the time and the place of the re-entry.

#### **1.1.4. Attitude Engineering**

Attitude of the satellite is the direction of the orientation with respect to the particular object whose position is already defined. To determine the attitude means compute the set of parameters that describe satellite orientation with respect to the chosen origin.

The first set of signals sent from the satellite is converted into data and processed further. The result of this analysis is the actual position of the satellite at a particular time. Attitude control and adjustment of the existing attitude is performed, for example, if the latest computed attitude indicates that the satellite has drifted too far to observe its current science target, or the satellite must be re-oriented for orbit manoeuvres, or if a new target is to be observed.

The desired attitude is transformed into required attitude control commands and transmitted to the satellite.

According to NASA [24] one of the most effective ways to approach the Attitude Analysis is to subdivide it into the sections concerned with an Attitude Determination Error Analysis, Attitude Dynamics Studies, Advanced Attitude Determination Techniques, Attitude Sensor Studies and SKYMAP Star Catalog.

### **1.1.5. Tracking Network Support**

Tracking Data is intensively analysed with the aim of assisting in identifying any defective tracking equipment. Analysis also includes the data obtained from the predicted orbit defined previously. This analysis results in acquisition data that precisely indicates where and when tracking stations must point antennas to track a particular satellite. Acquisition data is distributed from the Flight Dynamics Facility to all tracking stations. Predicted orbit data is sent to stations before a satellite launch. Flight Dynamics Facility acquisition support extends throughout a mission lifetime.

For launch vehicles, tracking data and onboard directional and acceleration (inertial guidance) data is processed to monitor the powered flight (thrusting) of the rocket. Real data, transmitted from the satellite, is continuously compared to the predicted data. The



results are used to further update the acquisition data, based on actual vehicle performance.

To provide tracking support for a tracking data and relay satellite, a mini – control centre is utilized. This centre is named the Bilateral Ranging Transponder system, and consists of several unmanned ground based transponders at various locations around the world. Satellite orbits are accurately computed as the transponder positions are precisely known.

## **1.2. Orbit Properties and Terminology**

There are different classifications of the orbits that a heavenly body can move in, under the action of the gravitational central force. Which orbit will be obtained depends on the initial conditions and the mission requirements.

For example, the classification based on altitude:

- High Altitude Orbits, with the altitude above 800km.
- Low Altitude Orbits, with the altitude from 300 to 800km.

Another classification is based on the orbit shape:

- Elliptical
- Circular (as the special case of the elliptical orbit)
- Parabolic
- Hyperbolic

All of the above curves can be constructed by a cone intersected by a plane. The properties of the particular orbit type are discussed in Chapter 2, as a part of the section on celestial mechanics.

Some special purpose orbits are:

- Polar orbit – An orbit of a satellite which passes above the Earth's poles during one rotation,
- Sun synchronous orbit - A satellite orbit which always contains the Earth – Sun line (special case of the polar orbit). Its orientation is nearly fixed relative to the Sun as the Earth moves in its orbit.
- Geostationary orbit – An orbit in which the satellite remains above the same point at the Earth's surface.

A particular emphasis is placed on the geostationary orbit, which is used as a subject of further analysis for determining the sensitivity of a satellite orbit under the atmospheric drag and gravitational perturbations.

### **1.2.1. Orbit Elements**

The orbit is completely defined by seven numbers, which are, for the idealized Kepler's model, constant with respect to time. These numbers are known as orbit elements or Keplerian elements.

These elements define the shape of the orbit, its position and orientation in space, and also, the position of a satellite in that orbit at particular time.

In reality, the shape of the orbit and its orientation change under the action of different perturbation forces.

The definition of these seven elements, which are used as the basis for development of a general orbit model, is presented below.

## 1. Epoch

Epoch is the instant at which the set of other parameters was taken. It is simply the number that specifies the time.

## 2. Orbital Inclination

Every orbit is contained by a flat plane in a space. For an Earth satellite, the orbital plane contains the centre of the Earth. Another plane that also contains Earth's centre, but whose position in space is already defined, is the *equatorial plane*. The smallest angle measured between the positive direction of the normal vector of the equatorial plane and the satellite's orbital angular momentum vector is named the *inclination angle*.

Please refer to the Fig. 1.2.1.1. presented below:

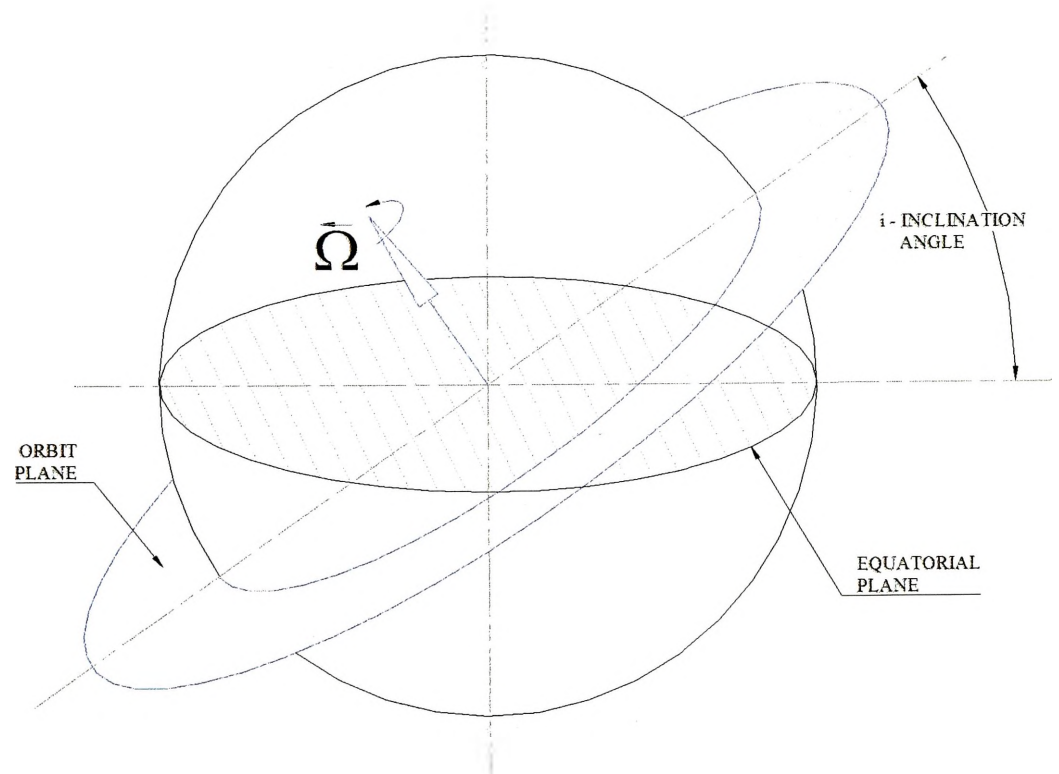


Fig. 1.2.1.1. Inclination angle of the orbit

Orbits with  $0^\circ$  inclination angle are named equatorial orbits – the orbit plane is almost coincident with the equatorial plane. If the inclination angle approaches  $90^\circ$ , the orbit is named polar – the satellite passes over (or nearly over) the North and South poles.

### 3. Right Ascension of Ascending Node

It was already mentioned that an orbit plane passes through the Earth's centre and intersects the equatorial plane. The intersection of these two planes results in a line, in this case named the *line of nodes*. Refer to Fig. 1.2.1.2.

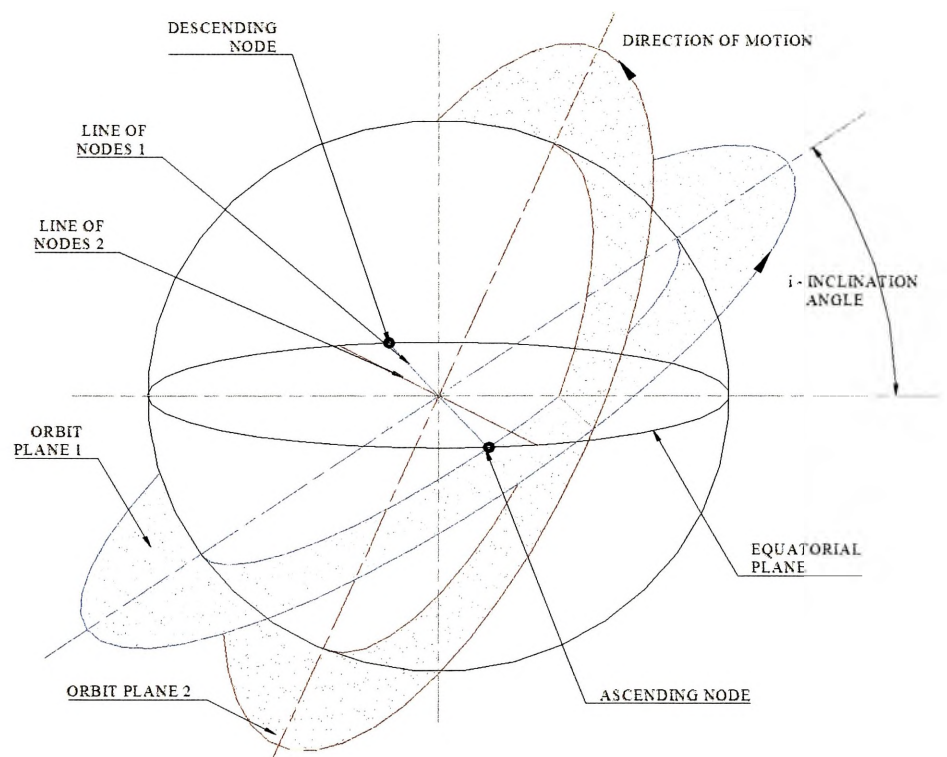


Fig. 1.2.1.2. The Ascending and Descending nodes

It is seen from the above figure that the orbit plane can intersect the equatorial plane in an infinite number of lines and still satisfy the requirement determined by the

inclination angle. The real orbit plane of a particular satellite can have only one of these possibilities, therefore the element that will specify the exact position of the line of nodes has to be defined.

As both the orbital plane and the equatorial plane pass through the Earth's centre, their intersecting line also passes through the Earth's centre.

The conclusion is, at this stage, only one point, different than Earth centre, is enough to define the line of nodes.

The *ascending node* is the node where satellite crosses the equator while 'climbing' from south to north.

Another point, at the opposite end of the line of nodes, named *descending node*, is the intersection of the satellite path from north to south, and the equatorial plane.

By convention, the location of the ascending node is used for the orbit definition. One of the methods to specify the ascending node is to use the reference system that does not rotate with the Earth. This system is widely used by astronomers and is known as the right ascension/declination coordination system.

Right ascension of the ascending node is the angle that lies in the equatorial plane, measured from the point defined as the Vernal Equinox, to the ascending node, with the apex at the centre of the Earth.

The Vernal Equinox is actually the ascending node of the Sun rising from the south to the north in its orbit about Earth. By convention, right ascension of the ascending node can take any value between  $0^\circ$  and  $360^\circ$ .

To summarize the first three elements, a short review will be made. At a particular instant, there are specific values of an inclination angle and right ascension of ascending node, constant in time, which are enough to orient the orbit in space. Refer to figure 1.2.1.3. below:

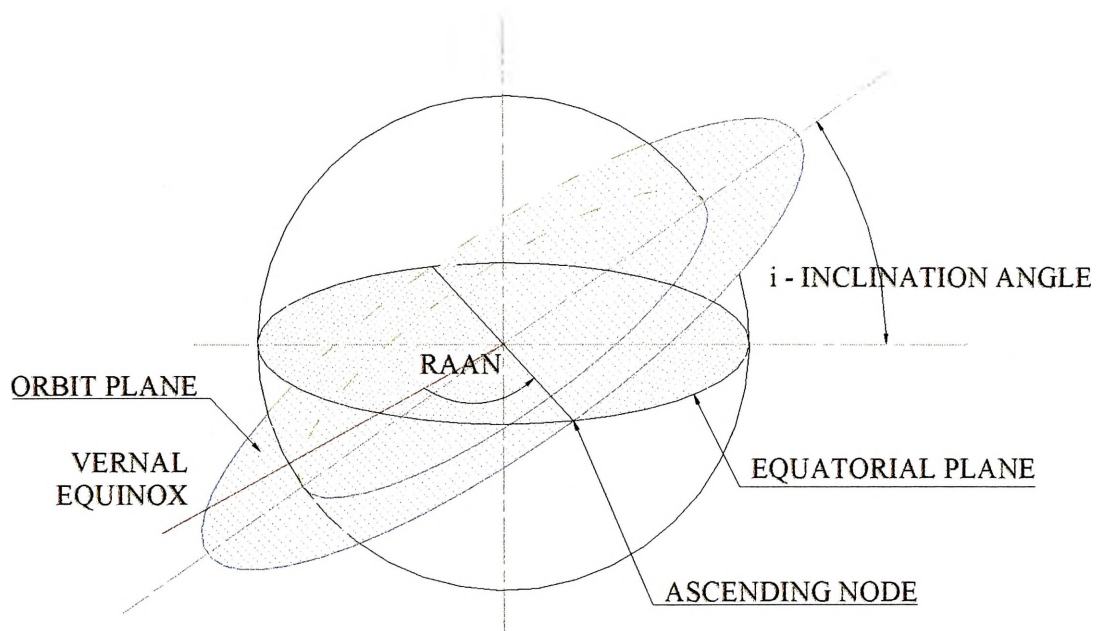


Fig. 1.2.1.3. Orbit defined with the first three elements

The next step is to define the orbit orientation in its plane. The elliptic orbit will be taken as the object of interest as the most common case for earth satellite orbits (a circular orbit is a special case of the elliptic orbit). The following elements are used for definition of orbit shape and orientation in its plane.

#### 4. Argument of Perigee

The point at which the satellite is the closest to the Earth is called its perigee. The distance from the centre of the Earth to this point is used as one of the initial conditions for software developed in this study. Another characteristic used also as the initial condition for trajectory generation is the velocity of the satellite at this point.

For an elliptic orbit, the centre of attraction is placed at one of its foci, Figure 1.2.1.4:

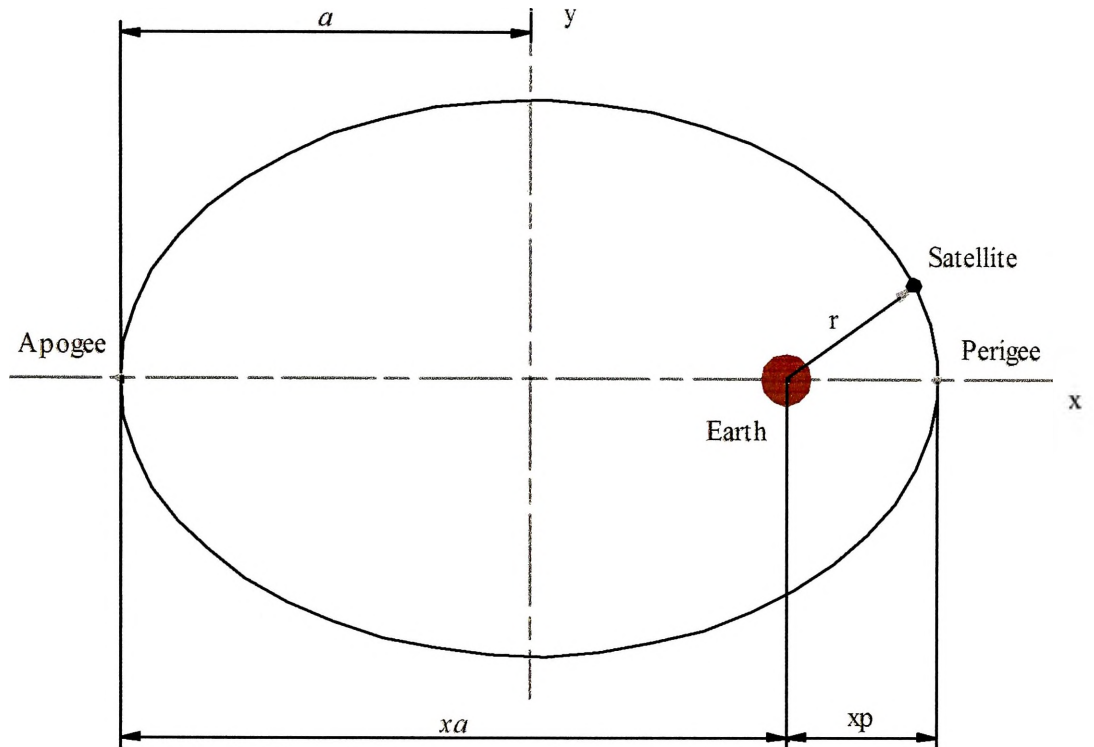


Fig. 1.2.1.4. Elliptic Orbit Geometrical Properties

There are two extreme positions of the orbiting satellite, the one closest to the attracting body – perigee, and the other one that is furthest away – apogee.

Kepler's second law states that 'The radius vector of the orbiting planet with the Sun at the origin sweeps out equal areas in equal times' [20]. This statement is illustrated in the Fig.1.2.1.5.

Kepler's second law is actually based on the property of the conic section, and is a consequence of Kepler's first law. These laws, that are results of an abundance of observations and measurements, are explained and analysed in Chapter 2, as the introductory section of Celestial Mechanics.

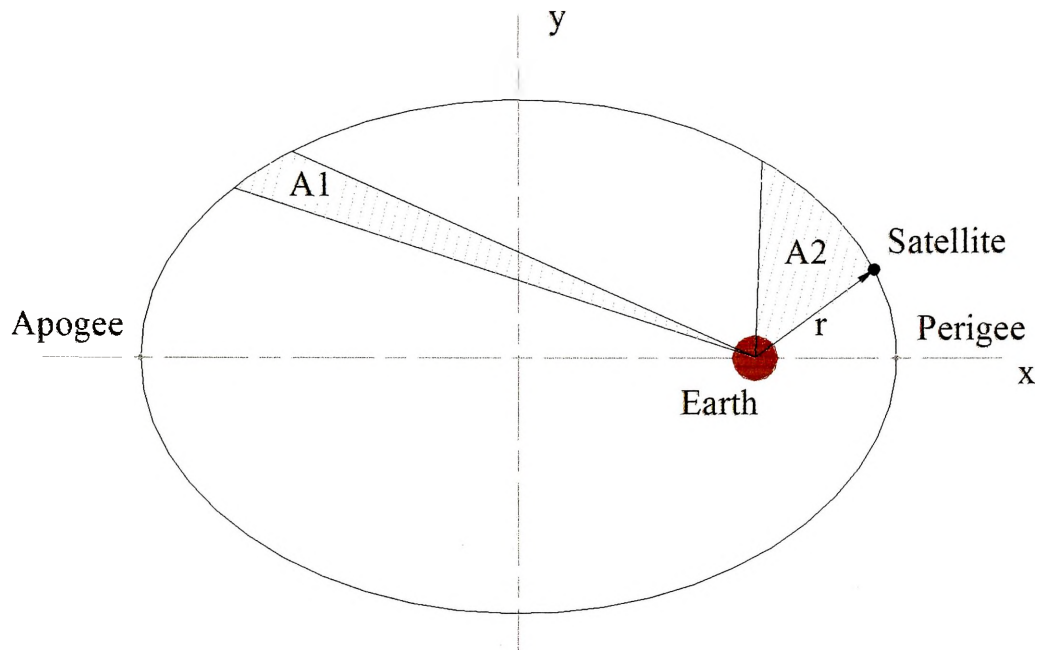


Fig. 1.2.1.5. The Law of areas

Celestial Mechanics has another approach to the problem of planetary motion, but its results prove Kepler's laws to be true.

The dynamic analysis of an orbiting body is also presented in Chapter 2.

For now, it is important to note that the closest point to the Earth has the greatest velocity and vice-versa, the farthest point from the Earth's centre has the lowest velocity.

The position of the perigee in the orbit plane is defined by the angle measured from the line of nodes (contained by the Equator plane, passing through the Earth's centre) to the *line of apsides*.

The line of apsides is on the major axis of the elliptic orbit, containing the perigee and apogee, and passing through the Earth centre, obviously lying in the orbital plane.

By convention the argument of perigee is an angle between  $0^\circ$  and  $360^\circ$ .



## 5. Eccentricity

The most fundamental classification of conic sections is based on their eccentricity.

$$\left. \begin{array}{l} \text{Ellipse } 0 < e < 1 \\ \text{Circle } e = 0 \\ \text{Hyperbola } e > 1 \\ \text{Parabola } e = 1 \end{array} \right\} \quad (1.2.1.1.)$$

All of the noted conic sections have one common characteristic: The ratio of radius, measured from the focus of the section to the point of the locus, and the respective shortest distance to the directrix is known as the eccentricity and is always constant for a particular shape of the section. The directrix is a line normal to the axis of symmetry of the section.

Graphically presented:

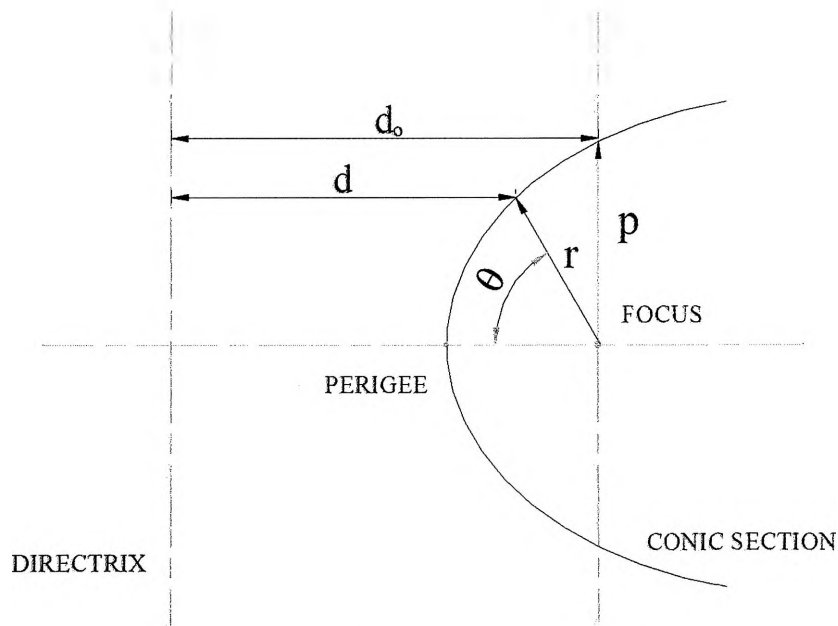


Fig. 1.2.1.6. General Conic Section

Eccentricity of a conic section is expressed mathematically as:

$$e = r / d = \text{const} \quad (1.2.1.2)$$

From the above figure

$$d_0 = d + r \cos \theta \quad (1.2.1.3)$$

and

$$\frac{p}{d_0} = \frac{r}{d} = e \quad (1.2.1.4)$$

Therefore,

$$\frac{p}{e} = d_0 = d + r \cos \theta = \frac{r}{e} + r \cos \theta = \frac{r + re \cos \theta}{e} \quad (1.2.1.5)$$

From the last expression, the general equation for a conic section radius-vector is:

$$r = \frac{p}{1 + e \cos \theta} \quad (1.2.1.6)$$

As it can be seen from Fig. 1.2.1.6, for  $\theta = 90^\circ$ , the radius is denoted by  $p$  and is called the semi-latus rectum.

The first stated characteristic of conic sections holds here as well, so equation (1.2.1.2) is applicable.

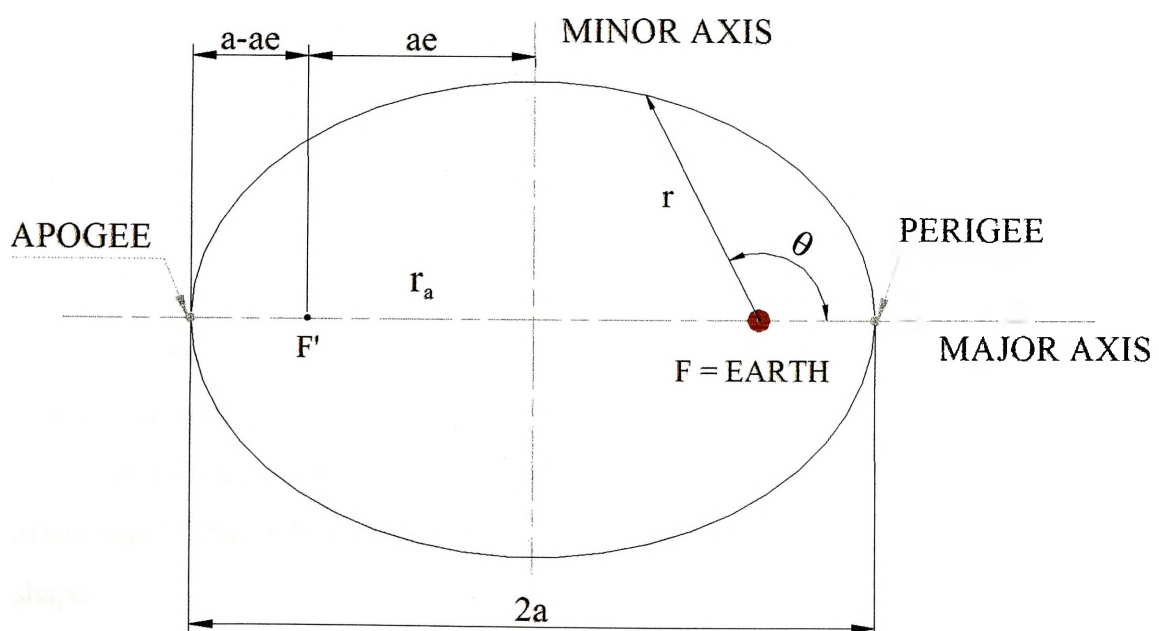


Fig. 1.2.1.7. Ellipse

Another feature of the ellipse is that, the sum of the distances of any point on its locus, from two foci is a constant and equal to  $2a$  – the major axis of the ellipse.

From (1.2.1.6) it can be concluded that the minimum radius is obtained for  $\theta = 0^\circ$ :

$$r = r_{\text{perigee}} = \frac{p}{1+e} \quad (1.2.1.8)$$

The maximum value of the radius is:

$$r = r_{\text{apogee}} = \frac{p}{1-e} \quad \text{for } \theta = 180^\circ \quad (1.2.1.8)$$

From Fig. 1.2.1.7.

$$a = \frac{1}{2}(r_p + r_a) = \frac{1}{2} \left( \frac{p}{1+e} + \frac{p}{1-e} \right) = \frac{p}{1-e^2} \quad (1.2.1.9)$$

$$e = \frac{r_a - r_p}{r_a + r_p} \quad (1.2.1.10)$$

$$\overline{F^1 F} = 2f = 2a - 2rp = \frac{2pe}{1-e^2} = 2ae \quad (1.2.1.11)$$

$$b = \sqrt{a^2 - f^2} = a\sqrt{1-e^2} \quad (1.2.1.12)$$

The observation for a parabola is very similar to the previous one.

From the theory presented, it is clear that eccentricity determines the shape of the orbit. Typically, tracking programs are not programmed to compute orbits different than an ellipse or a circle. The software developed here can determine properties of the orbit in a two-body problem with any eccentricity. The rest of the programs are used for elliptical orbits mainly, but only a small modification is required to adapt them for any other orbit shape.

The orbit model, defined with the five elements already described is:

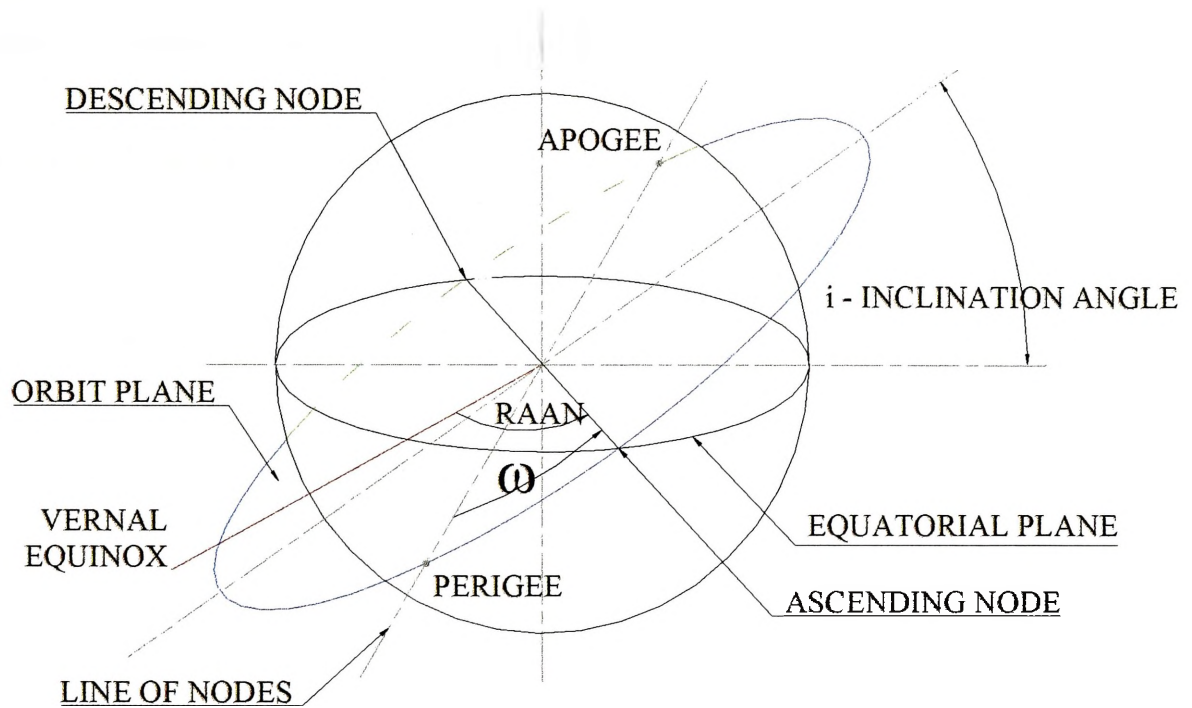


Fig. 1.2.1.8. Orbit defined with: inclination angle  $i$ , right ascension of ascending node RAAN, argument of perigee  $\omega$  and perigee/apogee values (eccentricity) at a particular instant – epoch.

These elements could be classified further as:

1. Elements that that define the orbit's orientation in space:

- inclination angle
- right ascending node

2. Elements that define orientation in orbit plane

- argument of perigee

3. Elements for orbit shape definition:

- eccentricity

The last classification is emphasised because of its importance for the system transformations that will be performed in the one of the following steps.

The next element will determine the size of the orbit.

## 6. Mean motion

Based on the Kepler's third law, the velocity and radius vector of the orbiting body – satellite are directly related.

In other words, when one of these values is known the other one could be determined. For a circular orbit the radius is constant, therefore the orbiting speed is also constant. For orbits with  $e > 0$  smaller radius means greater speed and vice-versa.

*Mean motion* is the average speed determined for a particular orbit. For example, satellites typically have mean motions in the range of 1 rev/day to 16 rev/day.

## 7. Mean anomaly

The last element, named *mean anomaly*, specifies the position of the satellite in the orbit.

The first element described here, *epoch*, is the particular time that gives a particular set of values including the position of the satellite in the orbit. A concept similar to the polar angular coordinate used for description of the object position on its trajectory is used also for the mean motion in a circular orbit. The mean motion is the angle whose vertex is placed at the centre of the Earth – one of the foci of the orbital plane. It is  $0^\circ$  at perigee and  $180^\circ$  at apogee. Its rate of change is constant during the orbiting, as it is based on the mean velocity of the satellite. The value of mean motion, by the convention, varies from  $0^\circ$  to  $360^\circ$ .

Some authors include air drag as an eighth element that describes the orbit. In this project air drag is not treated as an orbit element, because of two reasons. The first one is, the air drag effect on the orbit is studied in greater depth through the following chapters of this work. The second reason is, the orbit elements are also known as Kepler's elements, which are automatically associated to the values constant in time. The air drag varies during the time and alters other elements to change, therefore does not contain the basic definition of these elements.

The alteration of orbit elements by the air drag effects is the main subject of Chapters 3 and 5.

### **1.3. Obtaining the Trajectory**

The trajectory of the satellite or a spacecraft will mainly depend on the mission objective, but there are other factors that, in different ways, have an effect on the final trajectory, or have to be chosen in such way as to achieve set goals.

In general, as it was mentioned in first section of this chapter, most missions have in common the following three phases:

1. Launch
2. Acquisition
3. Mission Operations

The first phase starts with the appropriate selection of the launching site. Because of the Earth's rotation, the final orbit of the satellite will depend on the latitude and longitude of the launching site. For example, in the United States in the past, most launches for equatorial orbits occurred from the Eastern Test Range at Cape Canaveral, Florida. For polar orbits, rocket launching was performed from the Western Test Range at Vandenberg Air Force Base, California.

This project does not observe the actual dynamics of the flight during the launch phase. The acquisition phase starts by separation of the satellite from most of the launch vehicle. This phase includes a number of tests, and can last from a few minutes to several months. The definition of this phase depends on details of the mission. For example, the time and the process for testing and manoeuvring of the satellite for communications purposes is different from the satellite involved in collection of scientific data.

By completion of all required checks and tests, after a particular time, the proper orbit is achieved and normal function of the satellite begins.

To illustrate the whole process of placing a satellite into the orbit, a particular mission will be used as an example.

**Communication Technology Satellite** CTS, was launched from Eastern Test Range at 23:28 Universal Time (UT) (18:28EST) on Jan 17, 1976. The launch vehicle used was Delta 2914.

This project was performed in cooperation with the Canadian Department of Communications and the United States National Aeronautics and Space Administration. NASA provided the launch vehicle, launch facilities and operational support through the acquisition phase of the mission. The satellite itself was built and operated by the Canadian Department of Communications. The purpose of the mission was to improve communications by tests performed in a high power television relay from portable transmitters operating at a frequency of 14 GHz to low – cost 12 GHz receivers.

The orbit required for achieving this task was geo-synchronous (this type of the orbit is discussed later) and the satellite was supposed to remain stationary above a point at the equator and at 114° West Longitude.

This location was chosen as it permitted television transmission to remote regions of both Canada and Alaska.

The total mass of the CTS spacecraft was 676kg at lift off. About 340kg of this value was in the weight of a rocket motor, known as the apogee – boost motor, required for achieving proper orbit.

### 1.3.1. The Geometrical characteristics of the satellite

The satellite's body was approximately cylindrical, 1.88m high and 1.83m in diameter. The main source of the power necessary for operating the satellite is solar energy converted by solar cells to electric power. Two extendable solar arrays were each 6.20m long and 1.30m wide with mass 15kg and power output of 600watts per array. The attitude control was performed by utilization of 11 sensors and 3 gyroscopes.

Orbit control equipment included one big apogee boost motor used in the acquisition phase, and 18 smaller rocket motors - 2 high thrust and 16 low thrust for both orbit and attitude manoeuvres. The trajectory development is presented in the figure below:

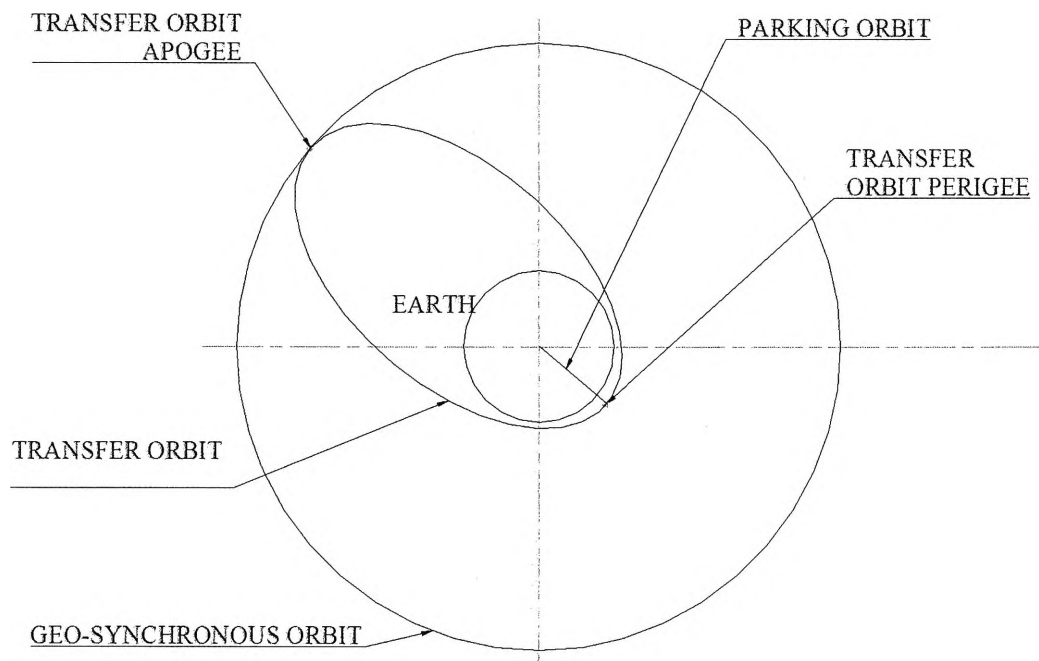


Fig. 1.3.1. Achieving the CTS orbit



### 1.3.2. Obtaining the Orbit

CTS was launched into an initial 185km parking orbit. This orbit was maintained for about 15 min until the satellite crossed the Equator.

The second manoeuvre was to inject the spacecraft into 'transfer-orbit', in this case elliptical orbit with its perigee at parking orbit altitude, and apogee at the geo-synchronous orbit altitude (which is almost constant as the orbit is of very small eccentricity).

This example clearly indicates the boundaries between the three, earlier defined referred, phases of the mission.

The launching phase was completed by the injection of the satellite into the transfer orbit.

The injection of the satellite occurred in such way to provide an appropriate velocity direction and intensity for achieving elliptical orbit with apogee altitude equal to the altitude of the geo-synchronous orbit.

Further transfer from elliptical to geo-synchronous orbit was achieved by firing the apogee boost motor at an appropriate time and in a precisely determined direction and intensity.

To achieve the required precision for orienting the boost motor, therefore orienting the spacecraft, all attitude control sensors had to be tested and calibrated.

This stage required about 6.5 transfer orbits, or about 3 days. The precision of positioning apogee boost motor was  $\pm 1^\circ$ . The total angle of rotation from initial orientation to the apogee motor firing attitude was  $225^\circ$ . This rotation was performed by firing two high thrust rocket motors.

After achieving the desired position of the satellite in the transfer orbit and firing the boost motor, the satellite was placed into an orbit of about 23 hours and 15 minutes, so

the spacecraft drifted slowly westward relative to the Earth's surface. When the satellite was at a desired longitude, characteristics of motion were altered further by firing thrusters, until its period was nearly identical to the Earth's rotation period.

This orbit refinement was carried out for period of about 9 days and consisted of five progressively smaller orbit manoeuvres. The lifetime of the satellite was about 2 years.

### **1.3.3. Geosynchronous Orbit**

Most satellites are placed into geo-synchronous orbits using so called geostationary transfer orbit.

Geostationary transfer orbit is an initially elliptic orbit of about 300 by 36000km, perigee to apogee size. After achieving this orbit the next transformation leads to new orbit whose perigee is placed at 36000km.

In fact, the initial orbit could be defined as the orbit of 300km to 82000km. A series of burns are applied further to lower the apogee and raise the perigee, until the final orbit (with close to zero eccentricity) of 36000km altitude is attained.

Another possible approach is to use supersynchronous insertion method - the transfer orbit scenario that optimizes the combined propulsion capabilities of the launch vehicle and the spacecraft.

## **CHAPTER 2**

# **Model of the Motion in the Space Environment**

## Chapter 2: Model of the Motion in the Space Environment

This chapter contains the main principles and laws that have to be followed during the construction of the mathematical model for orbital motion in space. It provides the necessary theoretical basis, with some analytical solutions, to the dynamic analysis that starts with purely physical laws, now observed from a slightly different point of view. This analysis provides the final result in the form of a system of 2<sup>nd</sup> Order DE's that describes the observed motion.

### 2.1. Celestial Mechanics

Celestial mechanics would be the foundation stone for any space motion-related analysis. The simplest and most general approach to define the term *celestial mechanics* would be:

*The study of the movement of celestial bodies, including the objects launched by man, observing the various forces influencing its movement.* [14]

The force that has the biggest influence on the motion of the Earth satellite is the Earth's gravitational force. Other gravitational forces present because of different celestial bodies, and other non-gravitational forces, are treated as perturbing forces.

To predict motion in space, basic physical laws will be introduced.

### 2.1.1. Kepler's laws

The laws were established in about the 16<sup>th</sup> century [6], and are based on the accurate records that have been collected by Kepler's teacher, Danish astronomer Tycho Brahe. The object of the observation was the planet Mars.

The laws that resulted are the three famous laws [15], outlined below:

1. The orbit of each planet lies in a fixed plane, and is of an elliptical shape, with the Sun in one of its foci.

In general, the orbit of the body that is subjected to the central force field is a conic section with a focus at the centre of attraction. In reality, due to different perturbing forces the position of the orbit plane is not fixed and this will be a subject of further analysis.

2. The radius vector of the orbiting planet, with the Sun at the origin, sweeps out equal areas in equal times. This law is actually a consequence of the first law, as it states the property of the conic sections.

3. The square of the orbiting period of the planet is proportional to the cube of the semi-major axis of that orbit. For two orbiting planets, the square of the ratio of their periods is equal to the cube of their major axes ratio.

### 2.1.2. Newton's Laws of Mechanics

1. The first law of mechanics states that a body free of any external force continues in its state of uniform motion in a straight line (or rest).

2. The second law is mathematically expressed as:

$$\vec{F} = \frac{d}{dt}(mv) \quad (2.1.2.1.)$$

where  $mv$  is the linear momentum of the observed particle.

This mathematical expression, interpreted in words, would be that the rate of change of linear momentum is proportional to the applied force, and takes place in the direction in which the force acts.

3. The third law of mechanics states that, for every action there is an equal and opposite reaction. It is clear the action and reaction forces are collinear, because only in that case mechanical angular momentum of an isolated system could be conserved.

### 2.1.3. Newton's Law of Gravitation

The law of gravitation, developed by Sir Isaac Newton, states that any two particles in space attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line which joins the particles, Fig 2.1.3.1.

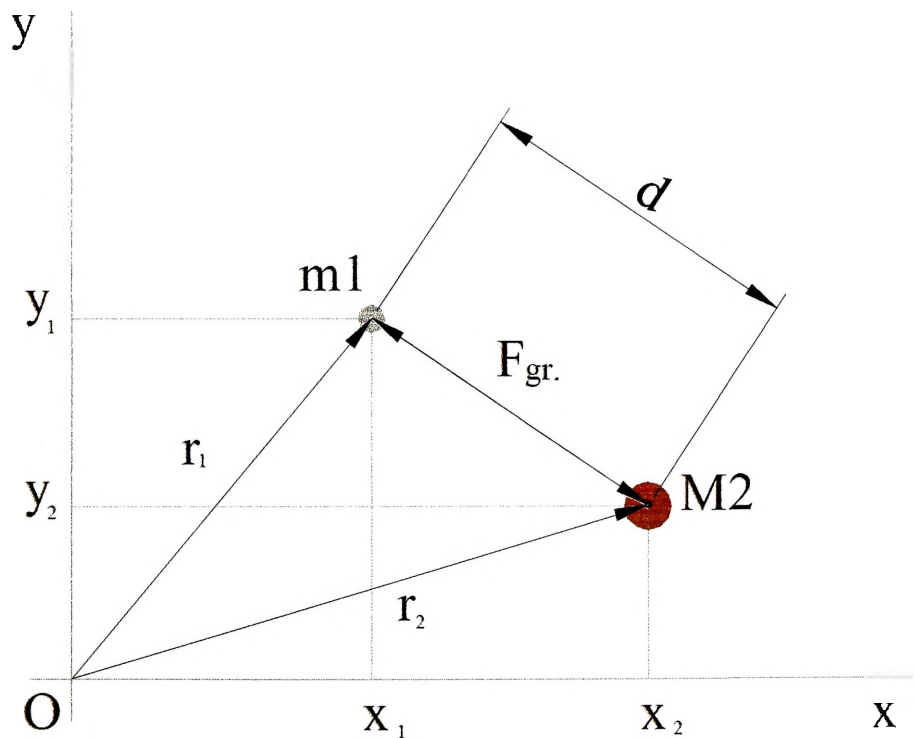


Fig. 2.1.3.1. Gravitational force acting on two particles with masses  $m_1$  and  $M_2$ , at the distance  $d$

Mathematically expressed:

$$\vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^3} \vec{r}_{12} \quad (2.1.3.1)$$

where:

$\vec{F}_{12}$  – is a force that particle 2 exerts on the particle 1

$\vec{r}_{12}$  – is the vector of magnitude  $|\vec{r}_{12}|$ , which defines the position of particle 2 with respect to particle 1.

G – is a constant, named Universal Gravitational constant, equal to  $6.6732 \times 10^{-11} \text{ m}^3/\text{kgs}^2$

## 2.2. Discussion and Definition

In order to apply the above laws, and to make the introduction into deeper analysis of a satellite motion, it is necessary to define and clarify some terms.

The speeds observed in this case are much smaller than the speed of light, therefore Newton Laws of Mechanics could be applied in further analysis. All forces observed are behaving as vectors and will be represented according to this statement.

To analyse the motion of the satellite, the first step is to select the appropriate reference point, which becomes the origin of the coordinate system used in further analysis. The established reference system has to obey the laws previously defined.

The first law of mechanics is obeyed only in *Inertial Systems*. Such a system moves with a constant velocity, or is stationary. If it is noted that it is hard (impossible) to specify the body or point in the Universe that is absolutely static, this problem does not look trivial.

The inertial system will obey all of Newton's Laws of Mechanics, therefore all observed forces acting on the body would be balanced. Newton's second law involves only the acceleration of the observed body, but not its velocity. This indicates that the absolute velocity of the body can not be determined since there is no way to determine which system is in rest.

If one inertial system is defined, all other systems that are stationary or move in a straight line with constant velocity with respect to the defined inertial system are also inertial systems. The proof for this statement is based on Galileo's principle of relativity and is presented in the Appendix 2.1.

The system that undergoes acceleration is not inertial. An example is a rotating system. To apply Newton's laws in accelerated systems of reference, appropriate modifications have to be performed. It was mentioned, in the above text, that in inertial systems all forces are balanced. In the non-inertial systems there exists an unbalanced force, or acceleration with no apparent force responsible for it. To achieve the balance in such systems so called *pseudo – forces* are introduced.

In a rotating system one pseudo-force is centrifugal force, which has no physical origin, but is a consequence of applying Newton's laws in a non-inertial system.

To continue further analysis of orbital motion, with Newtonian mechanics theory as a base, the systems '0' and '1' are introduced. Both systems are rectangular Cartesian systems with positive orientations. The origins of both systems coincide with the centre of the Earth. The plane  $x_0y_0$  of '0' system lies in the equatorial plane, with  $x_0$  axis passing through the point defined as *vernal equinox* - defined in Chapter 1. It is usual practice in astronomy to assume this point as static in space and in time. The second axis  $y_0$ , contained by  $x_0y_0$  plane is perpendicular to the axis  $x_0$ . Third axis  $z_0$  is selected in such way to form right hand orientated rectangular Cartesian system, referring to Figure 2.2.1.  $\vec{i}_0, \vec{j}_0, \vec{k}_0$  are the unit vectors attached to the axes of the inertial system  $\vec{x}_0, \vec{y}_0, \vec{z}_0$  respectively.



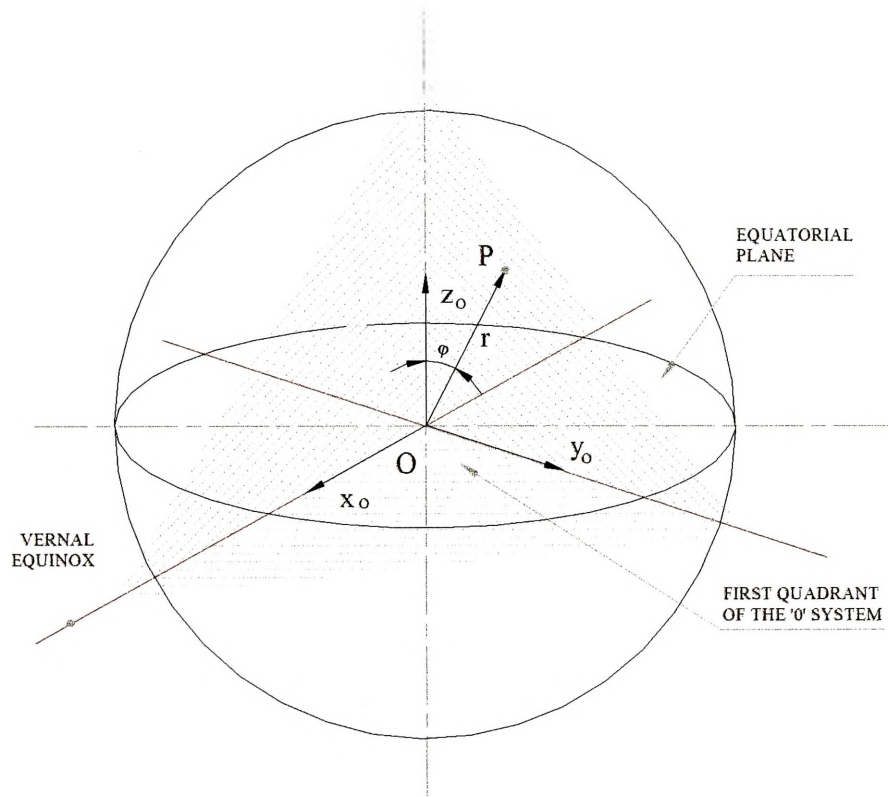


Fig. 2.2.1. Inertial System 'O' with major plane containing the Equatorial plane

Point P represents a point on the Earth's surface, and it is assumed for the time  $t = t_0 = 0$ , P is stationary with respect to O.

Another system 'I' for the initial time  $t = t_0 = 0$  coincides with the system 'O'. For time  $t = t_1$  system 'I' rotates with respect to system 'O' about axis  $z_1 \equiv z_0$  by the

angular velocity  $\omega = \frac{1 \text{ rev} \times 2\pi}{24 \times 60 \times 60}$  where  $\omega = \|\vec{\Omega}_E\|$  and  $\vec{\Omega} = \omega \vec{k}_0$  figure 2.2.2. The

point P rotates together with the coordinate system 'I'. Unit vectors  $\vec{i}_1, \vec{j}_1, \vec{k}_1$  are attached to the axes of the rotating system  $\vec{x}_1, \vec{y}_1, \vec{z}_1$  respectively.

After time  $\Delta t$ , the position of the systems could be illustrated as:

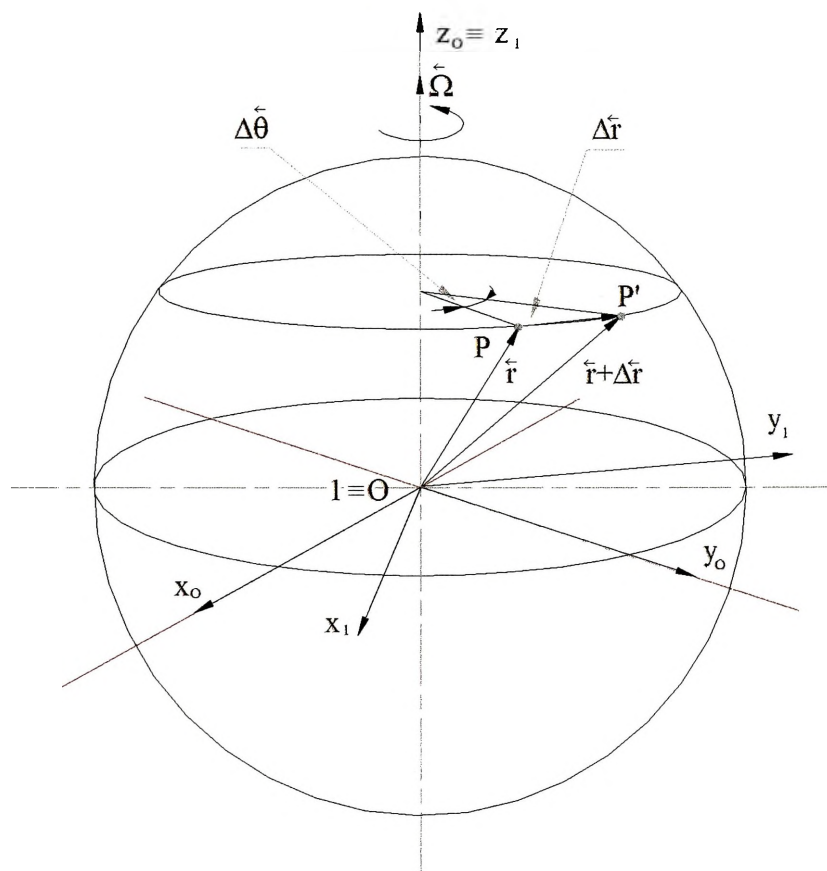


Fig. 2.2.2. The position of the rotating system '1' with respect to the system '0' after a time interval  $\Delta t$

The point P is defined as the point fixed to the system '1', therefore rotating with the same angular velocity  $\omega$  with respect to the system '0'.

After time  $\Delta t$  point P will have a trajectory – part of a circle with radius  $r \sin \phi$  lying in the plane perpendicular to the axis of rotation, Fig. 2.2.2.

Position vector of P with respect to  $x_1, y_1$  and  $z_1$  is defined as  $\vec{r}_1$  and has a constant value with respect to the time.

It could be stated:

$$\begin{aligned}\vec{i}_1 &= \vec{i}_0 \cos \omega t + \vec{j}_0 \sin \omega t \\ \vec{j}_1 &= -\vec{i}_0 \sin \omega t + \vec{j}_0 \cos \omega t \\ \vec{k}_1 &= \vec{k}_0\end{aligned}\tag{2.2.1}$$

The position vector of the point P is defined in rotating system '1' as:

$$\vec{r}_1 = x_1 \vec{i} + y_1 \vec{j} + z_1 \vec{k}\tag{2.2.2}$$

where  $x_1, y_1, z_1$  are the constants. This expression is transformed into:

$$\vec{r}_1 = x_1(\cos \omega t \vec{i}_0 + \sin \omega t \vec{j}_0) + y_1(-\sin \omega t \vec{i}_0 + \cos \omega t \vec{j}_0) + z_1 \vec{k}_0 \quad (2.2.3)$$

therefore

$$\vec{r} = (x_1 \cos \omega t - y_1 \sin \omega t) \vec{i}_0 + (x_1 \sin \omega t + y_1 \cos \omega t) \vec{j}_0 + z_1 \vec{k}_0 \quad (2.2.4)$$

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \omega(-x_1 \sin \omega t - y_1 \cos \omega t) \vec{i}_0 + \omega(x_1 \cos \omega t + y_1 \sin \omega t) \vec{j}_0 + z_1 \vec{k}_0 \\ &= \omega x_1(-\sin \omega t \vec{i}_0 + \cos \omega t \vec{j}_0) + \omega y_1(-\cos \omega t \vec{i}_0 - \sin \omega t \vec{j}_0) \\ &= \omega x_1 \vec{j}_1 - \omega y_1 \vec{i}_1 \end{aligned} \quad (2.2.5)$$

$$\text{Let } \vec{\omega} = \omega \vec{k}_1 \quad (2.2.6)$$

$$\vec{r}_1 = x_1 \vec{i}_1 + y_1 \vec{j}_1 \quad (2.2.7)$$

$$\vec{\omega} \times \vec{r}_1 = \begin{vmatrix} \vec{i}_1 & \vec{j}_1 & \vec{k}_1 \\ 0 & 0 & \omega \\ x_1 & y_1 & 0 \end{vmatrix} = -\omega y_1 \vec{i}_1 + \omega x_1 \vec{j}_1 \quad (2.2.8)$$

$$\vec{V}_p = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}_1 \quad (2.2.9)$$

is the velocity of the point P measured in the inertial system.

From the vectorial product above, it is seen the direction of  $\vec{V}_p$  is along the vector perpendicular to the plane defined by the vectors  $\vec{\omega}$  and  $\vec{r}_1$ . This vector is represented by  $\Delta r$  in the Fig. 2.2.2.

Expression 2.2.9 is particularly important because it is used for obtaining the formula, which relates the time derivative of the same vector in a system '0'.

### 2.3. Theory Applied to the Orbital Motion

Assume, the origin of the system 'I' is placed at the Earth's centre, with  $x_I y_I$  plane lying in the Equatorial plane with  $x_I$  axis pointed through the Greenwich. This system

rotates together with the Earth with the angular velocity  $\omega_E$  as defined in the previous section.

The origin of the system '2' is also placed at the Earth centre (Kepler's first law), with  $x_2y_2$  plane coinciding with the satellite's orbital plane (also Kepler's first law which states that the orbital motion occurs in the 'fixed' plane). Axis  $x_2$  passes through the perigee of the elliptical orbit which is treated as the fixed point in the space (perturbations are introduced later). For the time  $t=0$  satellite is placed at the orbit perigee, therefore its total velocity is projected along system's second axis  $y_2$  contained by the orbit plane and perpendicular to the axis  $x_2$ . Third axis of this inertial system  $z_2$  is perpendicular to the orbit plane. This system is an inertial system.

Based on (2.2.5) for the satellite moving in the orbital plane about  $z_2$  axis with the angular velocity  $\vec{\omega}_s$  it is stated:

$$\vec{\omega}_s = \omega_s \vec{k}_1 \quad (2.3.1)$$

$$\vec{r}_1 = x_1 \vec{i}_1 + y_1 \vec{j}_1 \quad (2.3.2)$$

$$\vec{\omega} \times \vec{r}_1 = \begin{vmatrix} \vec{i}_1 & \vec{j}_1 & \vec{k}_1 \\ 0 & 0 & \omega \\ x_1 & y_1 & 0 \end{vmatrix} = -\omega y_1 \vec{i}_1 + \omega x_1 \vec{j}_1 \quad (2.3.3)$$

so

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}_1 \quad (2.3.4)$$

Last expression is a velocity measured in the inertial system.

To develop an expression for an acceleration of the point P seen by the observer attached to the inertial frame, more general case will be introduced.

Two systems are defined,  $xyz$  and  $XYZ$ . Both of these systems are rectangular De Cartes coordinate systems. System  $XYZ$  is a stationary system, while  $xyz$  moves with respect to  $XYZ$  system by the motion that involves both, translation and rotation.

State that the motion of P is known with respect to the non-inertial system  $xyz$ :

$$\vec{R}_p = \vec{R}_o + \vec{R} \quad (2.3.5)$$

where  $\vec{R}_p$  represents the radius vector of the point P as seen by the observer placed at the origin of the inertial system XYZ.  $\vec{R}_o$  is the vector that describes the position of the origin of the moving system xyz with respect to the origin of the fixed system XYZ.  $\vec{R}$  is the radius vector of the point P seen from the moving system xyz.

Using the same approach to the one that was defined in the first part of this section, unit vectors  $\vec{i}, \vec{j}$  and  $\vec{k}$  are attached to the moving system axes  $x, y$  and  $z$  respectively.

$$\vec{R} = x\vec{i} + y\vec{j} + z\vec{k} \quad (2.3.6)$$

The absolute velocity of the point P is defined with respect to the stationary system and is obtained by differentiating vector  $\vec{R}_p$  with respect to the time. From (2.3.6)

$$\vec{V}_p = \dot{\vec{R}}_p = \dot{\vec{R}}_o + \dot{\vec{R}} \quad (2.3.7)$$

From (2.3.6)

$$\dot{\vec{R}} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} + x\frac{d\vec{i}}{dt} + y\frac{d\vec{j}}{dt} + z\frac{d\vec{k}}{dt} \quad (2.3.8)$$

First term of the expression (2.3.8) describes the change of the radius vector defined in the moving system. The second term contains first time derivative of the unit vectors  $\vec{i}, \vec{j}$  and  $\vec{k}$ .

Based on the equation (2.3.4)

$$\begin{aligned} \dot{\vec{i}} &= \vec{\omega} \times \vec{i} \\ \dot{\vec{j}} &= \vec{\omega} \times \vec{j} \\ \dot{\vec{k}} &= \vec{\omega} \times \vec{k} \end{aligned} \quad (2.3.9)$$

where  $\vec{\omega}$  represents angular velocity of the moving coordinate system  $xyz$  relative to the fixed system  $XYZ$ .

When substituted:

$$\begin{aligned} x\dot{\vec{i}} + y\dot{\vec{j}} + z\dot{\vec{k}} &= x(\vec{\omega} \times \vec{i}) + y(\vec{\omega} \times \vec{j}) + z(\vec{\omega} \times \vec{k}) \\ &= \vec{\omega} \times (x\vec{i} + y\vec{j} + z\vec{k}) \end{aligned} \quad (2.3.10)$$

$$x\dot{\vec{i}} + y\dot{\vec{j}} + z\dot{\vec{k}} = \vec{\omega} \times \vec{R}$$

Finally, when the last equation is substituted in the expression for the absolute velocity:

$$\dot{\vec{R}} = \vec{V} + \vec{\omega} \times \vec{R} \quad (2.3.11)$$

If  $\vec{V}_O = \dot{\vec{R}}_O$ , then:

$$\vec{V}_P = \vec{V}_O + \vec{V} + \vec{\omega} \times \vec{R} \quad (2.3.12)$$

where

$\vec{V}_P$  - the velocity of the point P in the  $XYZ$  system,

$\vec{V}_O$  - the velocity of the origin of the  $xyz$  system with respect to the system  $XYZ$ .

$\vec{V}$  - the velocity of P relative to  $xyz$  system

$\vec{\omega}$  - angular velocity of the system  $xyz$  relative to the  $XYZ$  system

$\vec{R}$  - distance from origin of  $xyz$  system to P

If the last expression is differentiated again with respect to the time:

$$\vec{A}_P = \dot{\vec{V}}_P = \dot{\vec{V}}_O + \dot{\vec{V}} + \dot{\vec{\omega}} \times \vec{R} + \vec{\omega} \times \dot{\vec{R}} \quad (2.3.13)$$

From the expression for the velocity, follows:

$$\dot{\vec{V}} = ((\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}) + (\dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}} + \dot{z}\dot{\vec{k}})) \quad (2.3.14)$$

The acceleration of P with respect to xyz system is defined by the component:

$$\vec{A} = ((\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k})) \quad (2.3.15)$$

Second term in the equation (2.3.14) could be also defined as:

$$\begin{aligned} \dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}} + \dot{z}\dot{\vec{k}} &= x(\dot{\vec{\omega}} \times \vec{i}) + y(\dot{\vec{\omega}} \times \vec{j}) + z(\dot{\vec{\omega}} \times \vec{k}) \\ &= \dot{\vec{\omega}} \times (x\vec{i} + y\vec{j} + z\vec{k}) \end{aligned} \quad (2.3.16)$$

Also, from:

$$\vec{V} = (x\dot{\vec{i}} + y\dot{\vec{j}} + z\dot{\vec{k}})$$

is derived:

$$\dot{\vec{\omega}} \times \vec{V} = (\dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}} + \dot{z}\dot{\vec{k}}) \quad (2.3.17)$$

and:

$$\dot{\vec{V}} = \vec{A} + \dot{\vec{\omega}} \times \vec{V} \quad (2.3.18)$$

From equation (2.3.11)

$$\dot{\vec{\omega}} \times \vec{R} = \dot{\vec{\omega}} \times \vec{V} + \dot{\vec{\omega}}(\dot{\vec{\omega}} \times \vec{R}) \quad (2.3.19)$$

The final expression for the acceleration of the point P with respect to the XYZ system is:

$$\vec{A}_P = \vec{A}_O + \vec{A} + 2\dot{\vec{\omega}} \times \vec{V} + \dot{\vec{\omega}} \times \vec{R} + \dot{\vec{\omega}} \times (\dot{\vec{\omega}} \times \vec{R}) \quad (2.3.20)$$

where:

$2\vec{\omega} \times \vec{V}$  - the Coriolis component of the acceleration

$\vec{A}_p$  - acceleration of P with respect to the system  $xyz$

$\vec{A}_O$  - acceleration of  $xyz$  with respect to the system  $XYZ$  (to specify the normal and tangential components of  $\vec{A}$  the path of P relative to the  $xyz$  system must be known).

$\vec{\omega}$  - angular velocity of  $xyz$  system relative to  $XYZ$  system

$\vec{V}$  - the velocity of P relative to  $xyz$  system

$\vec{R}$  - distance from origin of  $xyz$  system to the point P

To define Newton's laws of mechanics in non-inertial frame start from the second law:

$$\vec{F} = m\vec{a}_{abs} \quad (2.3.21.)$$

where

$\vec{a}_{abs}$  is the acceleration in an inertial frame

$m$  is the mass, defined as an inertial mass which, based on the experimental results by Roll and latter by Braginsky and Panov [11], differs from gravitational mass by a coefficient of the order of  $10^{-11}$ . Hence, in further analysis both masses will be treated as equal.

The same problem is defined and solved by the next polar coordinates theory:

Observe the particle S that moves with translational and rotational motion with respect to the stationary system  $O$ . There is another system  $O'$  that is moving on the same trajectory as point S, so S remains static with respect to  $O'$ .



The position of S is defined by  $\vec{r}$  and  $\vec{\rho}$  in  $O$  and  $O'$  respectively, Fig. 2.2.3:

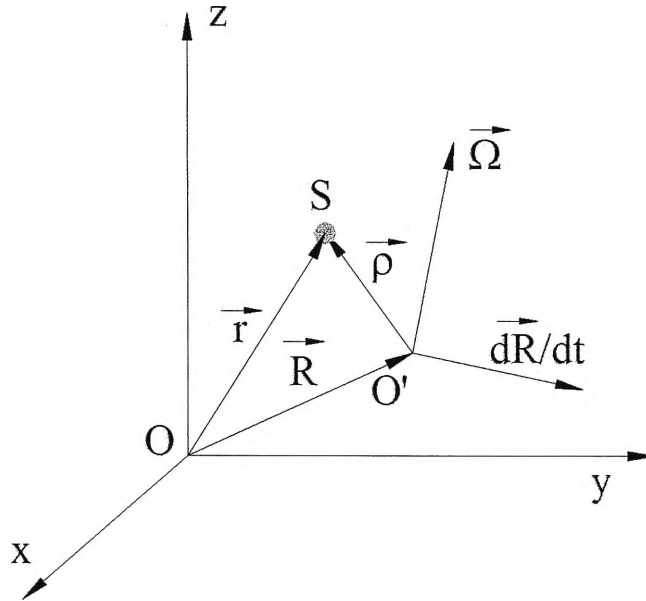


Fig. 2.2.3. Particle S is stationary with respect to system  $O'$  and moves with respect to the inertial system  $O$

The acceleration of the point S, in inertial system, is expressed as:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{l}}{dt^2} \quad (2.3.22.)$$

From the Fig. 2.2.3, absolute position vector of S is:

$$\vec{r} = \vec{R} + \vec{\rho} \quad (2.3.23.)$$

Absolute velocity of S is:

$$\vec{V}_{abs} = \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} + \frac{d\vec{\rho}}{dt} \quad (2.3.24.)$$

$\frac{d\vec{\rho}}{dt}$  is composed of the rate of change of its magnitude defined by  $\frac{d\rho}{dt}$  and direction

defined by  $\vec{\Omega} \times \vec{\rho}$ .

Therefore:

$$\vec{V}_{abs} = \frac{d\vec{R}}{dt} + \vec{\Omega} \times \vec{\rho} + \frac{\partial \vec{\rho}}{\partial t} \quad (2.3.25.)$$

The component  $\frac{d\vec{R}}{dt} + \vec{\Omega} \times \vec{\rho}$  describes the velocity of the particle with respect to  $O$  due to the motion of system  $O'$ .

If the observed particle is a satellite, and the systems  $O$  and  $O'$  have the same properties as previously defined systems '1' and '2' respectively, then  $R=0$  and equation 2.2.10. has the form:

$$\vec{V}_{abs} = \vec{\Omega} \times \vec{\rho} + \frac{\partial \vec{\rho}}{\partial t} \quad (2.3.26.)$$

Assume, at  $t=0$  point  $S$  coincides with the point  $S_p$  – orbit perigee. During the time interval  $\Delta t$  point  $S$  will move on the fragment of elliptical trajectory with respect to the point  $S_p$ , Fig. 2.2.4:

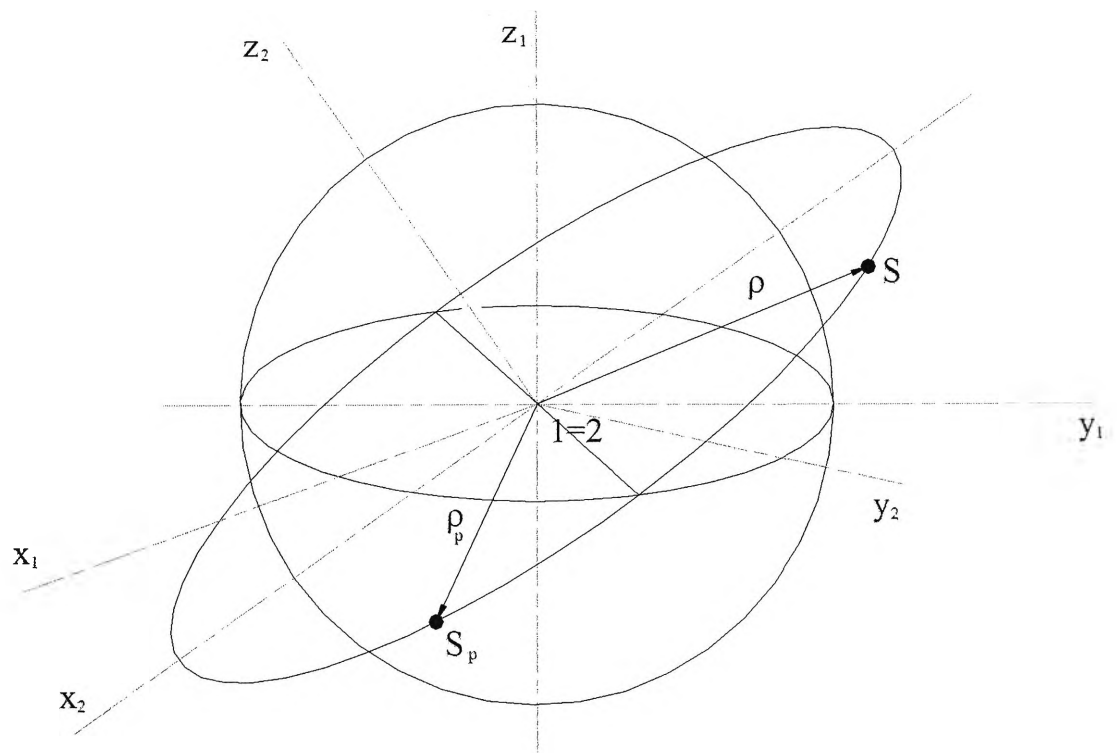


Fig. 2.2.4. The representation of orbit and Earth attached systems

The velocity of the point  $S_p$  relative to '1' (fixed to Earth, rotating with it) is given by the component  $\vec{\Omega}_E \times \vec{\rho}$ . The velocity of the point S with respect to the point  $S_p$  is

given by  $\left(\frac{\partial \vec{\rho}}{\partial t}\right)_2$ . The expression for the absolute velocity, based on (2.3.26.) is given by:

$$\vec{V}_{abs} = \vec{\Omega}_E \times \vec{\rho} + \left(\frac{\partial \vec{\rho}}{\partial t}\right)_2 = \vec{\Omega}_E \times \vec{\rho} + \vec{V}_{rel} \quad (2.3.27.)$$

Therefore:

$$\vec{V}_{rel} = \vec{V}_{abs} - \vec{\Omega}_E \times \vec{\rho} \quad (2.3.28.)$$

The last expression defines the velocity of the point S (satellite) seen by an observer in '2'. If the expression (2.3.28.) is again differentiated with respect to time, next expression is obtained:

$$\vec{a}_{abs} = \frac{d\vec{\Omega}_E}{dt} \times \vec{\rho} + \vec{\Omega}_E \times \frac{d\vec{\rho}}{dt} + \frac{\partial^2 \vec{\rho}}{\partial t^2} \quad (2.3.29.)$$

By the same analogy leading to (2.3.11.) and (2.3.29.) follows:

$$\vec{a}_{abs} = \frac{d\vec{\Omega}_E}{dt} \times \vec{\rho} + \vec{\Omega}_E \times (\vec{\Omega}_E \times \vec{\rho}) + 2\vec{\Omega}_E \times \frac{\partial \vec{\rho}}{\partial t} + \frac{\partial^2 \vec{\rho}}{\partial t^2} \quad (2.3.30.)$$

All elements in the last expression that are based on the vector product of the vector  $\rho$  describe the acceleration component of the point S with respect to the point  $S_p$ . These two components (in general case for  $R \neq 0$  there are three components), are named *dragging acceleration*.

The acceleration in the last expression is further classified as:

- Centripetal acceleration which results from the rotation of the system '2' and is equal to  $\Omega \times (\Omega \times \rho)$

**Example 2.3.1:**

Earth orbiting the Sun by the angular velocity defined as:

$$\varpi = \frac{1rev \times 2\pi}{365 \times 24 \times 60 \times 60sec} \quad (2.3.1.1)$$

Earth's orbiting radius is:

$$R = 93million \text{ miles} = 1.498 \times 10^{11} m \quad (2.3.1.2)$$

Therefore, centripetal acceleration that results from the Earth's rotation is equal to:

$$\varpi^2 R = 0.006 \frac{m}{sec^2} \quad (2.3.1.3)$$

Result in (2.3.1.3) is more often defined in units known as *milig*, therefore

$$\varpi^2 R = 0.6milig \quad (2.3.1.4)$$

- Tangential acceleration results from the change in rotational velocity with respect to the time, and is equal to  $2\Omega \times \partial\rho/\partial t$ .

The acceleration of the point S with respect to the point  $S_p$  is given by the last two elements in the expression (2.3.30.). The first component is the Coriolis acceleration and the second component is therefore the relative acceleration of S with respect to '2'. From Newton's second law follows:

$$\vec{F} = m\vec{a}_{abs} = m\vec{a}_{dr} + m\vec{a}_c + m\vec{a}_{rel} = -\vec{F}_{dr} - \vec{F}_c + m\vec{a}_{rel} \quad (2.3.31.)$$

$$\therefore m\vec{a}_{rel} = \vec{F} + \vec{F}_{dr} + \vec{F}_c = \vec{F}_{tot} \quad (2.3.32.)$$

The expression (2.3.32.) states the condition that has to be applied when Newton's law of motion is used in non inertial frame. The total force expressed in (2.3.32.) is the sum of an external force and two apparent forces.

**2.4. Discussion of Newton's Gravitational Law**

There is the number of assumption, related to the gravitational law, which alter the obtained motion model. Some of them are discussed in more details and the appropriate corrections are included in the final model. The assumption for equality of

the gravitational and the inertial masses of the observed bodies is the example of the assumption that will not be treated further, as the error produced by this action has a very low value.

The mass of the Earth is treated as the particle mass, in another words, the total mass of the Earth is concentrated at one point – the centre of the Earth. This assumption is further approximated by more real model – the mass is symmetrically distributed in the sphere. There is a mathematical proof that states, no error is produced if the symmetrical sphere is observed as a particle. The next step in the improvement of the obtained model is to compare real Earth's mass distribution to the assumed symmetrical distribution. This problem is a subject of detailed analysis and is incorporated as one of the perturbations into idealised Kepler's model.

If all existing perturbations are ignored, including perturbations due to gravitational forces, the trajectory of the orbiting satellite is ellipse with the Earth in one of its foci (Kepler's first law).

To apply Newton's law of gravitation, for the start, assume that a satellite and the Earth are presented by two particles, on the distance  $r$ .

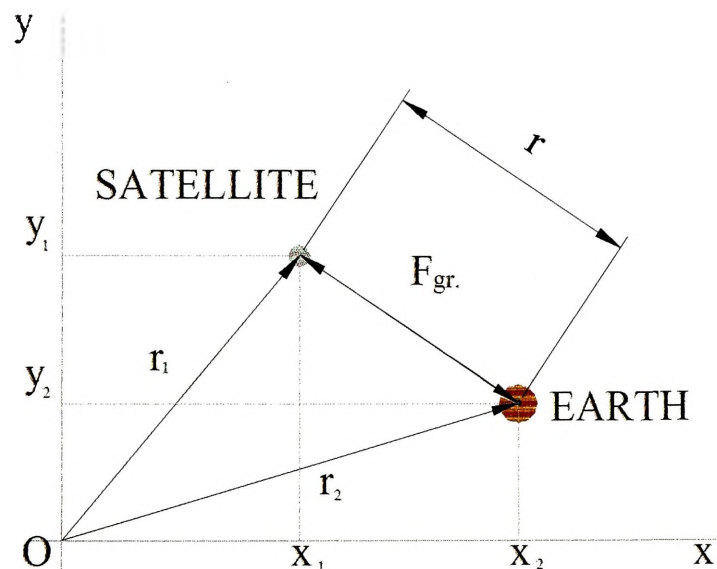


Fig. 2.4.1. Newton's law of gravitation applied to the Earth-satellite system

After observing possible values of the orbiting radii  $r$ , Earth is treated as a sphere and a satellite as a particle. If the satellite's dimensions are compared to the other dimensions, masses and distances in the system, following conclusion is, its mass could be treated as the particle mass with no further discussion.

The proof for observing Earth as a particle instead of a sphere is based on the method by which the observed sphere is divided into an infinite number of infinitesimal particles. The force implied by such infinitesimal particle on another observed particle (satellite) is:

$$d\vec{F} = G \frac{m_s dm}{r^3} \vec{r} \quad (2.4.1.)$$

Total force exerted on a satellite is defined by an integral of the equation (2.4.1.). Further mathematical procedure is presented in the Appendix 2.4.1.

The conclusion after mathematical analysis is the gravitational field of the sphere with a radius  $r_E$  is the same if the whole mass is concentrated at the centre of the sphere or if it is symmetrically distributed in its volume.

Based on the last conclusion, Newton's law of gravitation is applied on the system Earth – satellite, with the aim to establish the equations of satellite motion.

Another assumption applied to this theory is regarding the invariance of Newton's second law, which holds only if the observed mass in motion is constant. Mathematical analysis of this problem is presented in the Appendix 2.4.2.

Since the main terms that will be incorporated into further analysis have been defined, and physical laws that will be applied have been discussed and assumptions are stated, the next step is to structure all of these into the first and simplest orbital motion model – *two body problem*.

## 2.5. Two body problem

Kepler's laws are actually defined on the two-body problem. From one aspect, this is simplified orbital motion analysis that ignores all perturbing effects. From another

aspect, this motion is just the special case of the motion of  $n$  bodies under their mutual gravitational attractions.

The analytical solution for the general  $n$ -body problem is not defined yet; the two-body problem is the only one that has a closed form analytical solution. The bodies observed are supposed to possess a spherical symmetry, so they could be treated as point masses.

Based on Kepler's three laws of planetary motion, Newton's law of Universal gravitation and *Vis-Viva* equation, the basic orbital elements could be determined.

*Vis-Viva* (living force) equation is represented as:

$$V^2 = G(M_2 + m_1) \left( \frac{2}{r} - \frac{1}{a} \right) \quad (2.5.1.)$$

In 1673 Christian Huygens introduced the quantity  $1/2m_1V^2$ , and gave to it the name *Vis-Viva* or Living Force [11]. The expression was introduced to explain the motion of the compound pendulum. This concept was developed further by Leibnitz, and the conclusion was that, the measure of the effect of the force  $F$  is given by:

$$F\Delta x = \Delta(1/2m_1V^2) \quad (2.5.2.)'$$

On the other side, analysis based on Galileo-Newton observations led to the expression  $F\Delta t = \Delta(m_1V)$  given as the measure for the same variable.

The controversy was resolved in 1743 by Jean D'Alembert, which proved that both measures were correct and not equivalent. This problem is observed in [7] Girvin, H. The application of this kinetic energy theory to celestial mechanics leads to the one of fundamental relationships of two-body problem (an elliptical orbit):

$$V^2 = G(M_2 + m_1) \left( \frac{2}{r} - \frac{1}{a} \right) \quad (2.5.1.)$$

where:

$m$ -mass of the satellite

$M_2$ -mass of the Earth

$G$ -Universal constant

$r$ -the distance at a particular time – orbit radius

$a$ -semi major axis of the elliptical orbit.

The value of  $ml$  is extremely small compared to  $M2$  and it will be ignored in further calculations for the sake of clarity. The equation is re-written as:

$$V^2 = GM2 \left( \frac{2}{r} - \frac{1}{a} \right) \quad (2.5.2.)$$

$$\frac{1}{2} V^2 = \frac{GM2}{r} - \frac{GM2}{2a} \quad (2.5.3.)$$

$$\frac{1}{2} V^2 - \frac{GM2}{r} = -\frac{GM2}{2a} \equiv E \quad (2.5.4.)$$

where  $E$  is the total energy per unit mass, as the left hand side of the equation contains kinetic and potential energy terms.

An important conclusion from the last equation is that, the semimajor axis of the orbit is a function of the total energy only, which has great importance for the observation of air-drag effect and orbit transfer/correction manoeuvres.

The semimajor axis depends on the speed at which the satellite is injected into the orbit, and therefore is in relation to the eccentricity of the orbit.

If the total energy of the launched satellite  $E$ , expressed by (2.5.4.), exceeds zero the achieved orbit is hyperbolic. If  $E < 0$  the trajectory of the satellite is elliptical (or circular as a special case of the ellipse). For  $E = 0$  the result is a parabolic trajectory – the orbit with infinite semimajor axis. The velocity required for the parabolic orbit is also known as the escape or parabolic velocity  $V_e$  - refer to 2.5.4. Mathematically expressed:

$$V_e = (2GM2/R)^{1/2} \quad (2.5.5.)$$

where  $R$  is a new notation for the orbit radius that describes any distance from the centre of the spherically symmetric object. A satellite launched with this velocity in any direction will escape the gravitational field of the observed attracting body, assuming there is no presence of the other forces.

If  $R = a$ :



$$V_c = (GM_2/R)^{1/2} \quad (2.5.6.)$$

is the velocity of a circular orbit of a radius  $R$ .

The velocity of an orbiting object infinitely far away from the attracting body is

$$V_h = (2E)^{1/2} = (V^2 - 2GM_2/R)^{1/2} \quad (2.5.7.)$$

where  $V_h$  is the instantaneous velocity of an object in the hyperbolic orbit at arbitrary distance  $R$  from the centre of the attraction.

Previous analysis proves that the magnitude of the velocity  $V$  at some instant, for a distance  $R$  determines the shape of the orbit.

Orbit properties could be also analysed based on their geometrical characteristics. It was mentioned the orbit of central field motion would always be some conical section, therefore an ellipse (circle), parabola, hyperbola or even a straight line as a special case.

The properties of the conic sections are discussed in Chapter 1.

Combining geometrical analysis and Vis-Viva integral [9], leads to the next expressions:

$$V = (GM_2/R)^{1/2} \quad \text{The injection velocity required for a circular orbit}$$

$$V = (GM_2(2/R - 1/a))^{1/2} \quad \text{The injection velocity required for an elliptical orbit}$$

$$V = (2GM_2/R)^{1/2} \quad \text{The injection velocity required for a parabolic orbit}$$

$$V = (GM_2(2/R + 1/a))^{1/2} \quad \text{The injection velocity required for a hyperbolic orbit}$$

The above equations are designated as (2.5.8.)

The period of an orbit is one of Kepler's elements, already explained in Chapter 1. To derive the expression that mathematically describes this element, the elliptical orbit will be considered.

The area of an ellipse is given by:

$$A = ab\pi = a^2\pi\sqrt{1-e^2} \quad (2.5.9.)$$

Areal velocity  $dA/dt$  is:

$$\frac{dA}{dt} = \frac{1}{2}\sqrt{GM2p} = \frac{1}{2}\sqrt{GM2a(1-e^2)} \quad (2.5.10.)$$

Period  $T$  is equal to:

$$T = \frac{A}{\frac{dA}{dt}} = \frac{2a^2\pi}{\sqrt{GM2a}} = 2\pi\sqrt{\frac{a^3}{GM2}} \quad (2.5.11.)$$

This formula will be used for the analytical determination of the periods of different orbits and then compared to the values computed by the SATELIGHT – program developed here.

## 2.6. Two Body Problem Equations of Motion Referred to an Inertial Coordinate System

Newton's gravitational law, previously defined, states:

$$\vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^3} \vec{r}_{12} \quad (2.6.1.)$$

The particle near the Earth's surface experiences an accelerating force due to the attraction of the Earth's gravitational field. The acceleration is given by:

$$g = \frac{GM_2}{R_E^2} \quad (2.6.2.)$$

The value of this acceleration is accurately measured, and is used for the determination of other constants:

$$R_E^2 g = GM_2 = k \quad (2.6.3.)$$

where:

$R_E=6366.2\text{km}$  is the adopted value of the Earth's radius

$g=9.81\text{km/s}^2$  is the mean value of the acceleration due to the Earth's gravity force when substituted:

$$\begin{aligned} k=GM_2 &= (6366.2)^2 \text{km}^2 \times 9.81 \times 10^{-3} \text{km/s}^2 \\ &= 397584.6089 \text{km}^3/\text{s}^2 \end{aligned} \quad (2.6.4.)$$

A system used to define the position of the observed bodies is attached to the centre of the Earth. Its principal plane of reference is the  $xy$  plane, which coincides with the plane of the satellite's orbit. This plane is inclined for an angle 'i' – inclination angle with respect to the Equatorial plane.  $x$  axis is oriented along the line that passes through the Earth's centre and the orbit perigee, therefore it also contains the apogee of the orbit. The position of this line is considered to be stationary in the space.  $y$  axis is contained by the same orbit plane, as it was defined previously, and is perpendicular to the  $x$  axis.  $z$  axis forms the right-angled rectangular Cartesian system with the other two axes and is directed along the vector of angular momentum of the satellite's motion.

The methods for transformation of the results obtained in the orbit plane to the system placed at the same origin, but with the  $xy$  plane coinciding with the infinitely extended Equatorial plane are discussed in Chapter 4.

If Earth is observed as a point mass, (it was proven this could be done with the introduction of few assumptions), Newton's law of gravitation could be applied. The

values that are used in this law are vectors, therefore they could be projected onto the three axes of the defined coordinate system.

The Newton laws, applied to the motion of the satellite in the Earth's gravitational field, referred to the inertial coordinate system, are expressed as:

$$\vec{F}_{12} = G \frac{m_1 M_2}{r_{12}^3} \vec{r}_{12} \quad (2.6.5.)$$

and

$$\vec{F} = m_1 \vec{a} \quad (2.6.6.)$$

The force that acts on the satellite is equalised, therefore:

$$\vec{a} = GM_2 \frac{\vec{r}_{12}}{r_{12}^3} \quad (2.6.7.)$$

The acceleration of the body is equal to the second derivative of its position vector, therefore:

$$\frac{d^2 \vec{r}}{dt^2} = GM_2 \frac{\vec{r}_{12}}{r_{12}^3} \quad (2.6.8.)$$

When projected on the three axes of the selected reference system, the equations are:

$$\frac{d^2 x}{dt^2} = GM_2 \frac{x}{r_{12}^3} \quad (2.6.9.)$$

$$\frac{d^2 y}{dt^2} = GM_2 \frac{y}{r_{12}^3} \quad (2.6.10.)$$

$$\frac{d^2 z}{dt^2} = GM_2 \frac{z}{r_{12}^3} \quad (2.6.11.)$$

where

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (2.6.12.)$$

Based on Kepler's first law, orbital motion occurs in a plane, therefore the third dimension –  $z$  could be completely ignored at this stage.

It is seen that the motion of the two-body problem is represented by the set of the three (two) second-order differential equations. The method of solving these equations is discussed in the next chapter.

### 2.7. Equations of motion for $n$ -body problem

The system observed consists of  $n$  bodies, according to the Kepler's first law, all contained by the plane  $xy$  of previously defined  $xyz$  inertial system. This is actually a special case of the more general problem. The system is presented in the figure 2.7.1:

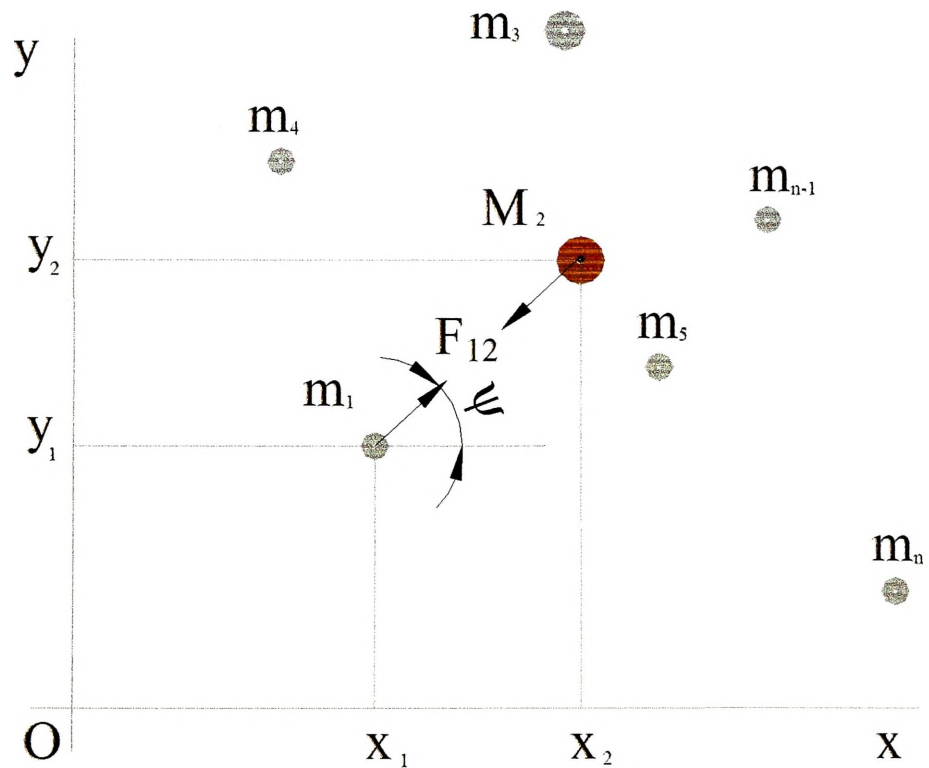


Fig. 2.7.1.  $n$ -body system in the  $xy$  plane of the  $xyz$  system

Force  $F_{12}$  could be projected on the axes of the system, producing two components:

$$F_{12_x} = F_{12} \cos \psi = F_{12} \frac{x_2 - x_1}{r_{12}} \quad (2.7.1.)$$

$$F_{12_y} = F_{12} \sin \psi = F_{12} \frac{y_2 - y_1}{r_{12}} \quad (2.7.2.)$$

where

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (2.7.3.)$$

In the further analysis only the  $x$  component will be observed, with the note that all developed theory in the same way applies to the component projected to the  $y$  axis.

Further:

$$F_{12_x} = \frac{Gm_1M_2}{r_{12}^2} \frac{x_2 - x_1}{r_{12}} \quad (2.7.3.)$$

$$\therefore F_{12_x} = \frac{Gm_1M_2}{r_{12}^3} (x_2 - x_1) \quad (2.7.4.)$$

By the same analogy, the  $x$  component of the force acting on  $m_3$  due to the gravitational force of the body  $m_1$  is expressed as:

$$F_{13_x} = \frac{Gm_3m_1}{r_{13}^3} (x_3 - x_1) \quad (2.7.5.)$$

For the body  $n$  could be stated under the action of the force inserted by the body  $m_1$  follows:

$$F_{1n_x} = \frac{Gm_n m_1}{r_{1n}^3} (x_n - x_1) \quad (2.7.6.)$$

The total force on body  $m_1$  in the  $x$  direction due to  $n$  bodies is:

$$F_{1_x} = F_{12_x} + F_{13_x} + \dots + F_{1n_x} \quad (2.7.7.)$$

which could be arranged as:

$$F_{1x} = G \sum_{j=2}^n m_1 m_j \frac{(x_j - x_1)}{r_{1j}^3} \quad (2.7.8.)$$

The force inserted on some arbitrary body  $m_i$  is:

$$F_{ix} = G \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^n m_i m_j \frac{(x_j - x_i)}{r_{ij}^3} \quad (2.7.9.)$$

Newton's second law states that the unbalanced force on a body in the  $x$  direction is given by:

$$F_{ix} = m_i \frac{d^2 x_i}{dt^2} \quad (2.7.10.)$$

substituting in (2.7.9.)

$$m_i \frac{d^2 x_i}{dt^2} = G m_i \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^n m_j \frac{(x_j - x_i)}{r_{ij}^3} \quad (2.7.11.)$$

By repeating the same analogy for  $y$  and  $z$  components (the  $z$  component in this special case is again equal to zero, as all observed bodies lie in the plane) the general equation is obtained:

$$m_i \frac{d^2 r_i}{dt^2} = G m_i \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^n m_j \frac{(r_j - r_i)}{r_{ij}^3} \quad (2.7.12.)$$

It is obvious that the term  $m_i$  could be cancelled from the both sides of the last equation therefore the mass of the observed body does not have any effect on the resulting force.

The observed problem is just the special case of the  $n$ -body problem as all observed bodies are contained by the same plane. This problem is modified further in this project with the aim to define the lunar effect on the Earth's satellite orbit. The number of bodies in the system is restricted to three – Earth, satellite and Moon. The problem is solved numerically, therefore there is no need to search for the special case

that will give closed solution to the analytically formulated problem. The bodies do not lay in the same plane, and the resulting force has three components. This problem is explained in more details further.

By this section the analysis of the unperturbed orbit and the introduction to the three-body problem is completed. The next stage is to incorporate the perturbing effects into already established equations of motion.

## 2.8. Perturbation theory

The motion in the central force field has constant elements that are the set of seven defined orbital elements, which do not depend on time.

This model, defined as the two-body motion model, is sufficient for some applications, especially if two very close points on the trajectory are observed. There are other applications that require a more accurate model, especially for the long time periods. The major difference between two-body model and real motion are so called perturbative elements that deviate the orbit from its theoretical model.

### 2.8.1. Introduction

The examples of the causes for the perturbing effects are

- (1) the force field of the primary body, which is the centre of the attraction, is not truly of an inverse square function;
- (2) the aerodynamic forces due to the Earth's atmosphere interaction;
- (3) the closeness of a neighbouring celestial body, etc. The dependency of an observed motion  $q$ , upon perturbing forces could be represented as:

$$q = q(F_1, F_2, F_3, \dots, F_n) \quad (2.8.1.)$$



where  $F_i$  are the different perturbing forces. These forces are always functions of the satellite position and velocity vectors. In turn, it is stated that, the position and velocity vectors of the satellite are actually functionally dependent on the perturbing forces. The last statement is actually the basis for the theory developed here to incorporate perturbing effects into the model of motion.

One of the methods to express the perturbations of the orbit elements is by utilisation of Taylor's series:

$$a = a_o + \frac{da_o}{dt}(t-t_o) + \frac{1}{2!} \frac{d^2a_o}{dt^2}(t-t_o)^2 + \dots \quad (2.8.2.)$$

Taylor's series is expanded about some value  $a_o$ . The same approach is used for all other elements, where the time derivatives of these elements depend on the perturbing force  $F_i$ . The development of infinite series, as the example above illustrates, is known as a general perturbation method [15]. Another method, applied here, is known as special perturbation method and is based on a numerical approach.

The first method gives good results only for a short period of time, if the appropriate function is chosen, for example trigonometric terms in a Fourier expansion. This method does not produce real fluctuation of orbit elements.

The second method gives more accurate results over longer periods of time and could be developed further, to describe the specific kind of motion for specific initial conditions. This method is very effective for orbit determination of lunar and interplanetary flights, but also for comets and planets.

The process of determining the initial conditions is discussed in Chapter 4, and analysis of the numerical method is presented in Chapter 3.

The method used here to treat special perturbations is known as Cowell's method and is based on step-by-step integration of the total acceleration, central forces as well perturbations. .

The applied procedure is simple and very straight-forward. In the three-body problem this method is implemented and used for observation of the orbit's inclination angle.

The mathematical expressions, based on SATELIGHT results, for all other orbit elements are defined as well.

The equations of motion, that contain perturbation elements, have to be integrated twice to give the position of the observed satellite.

This method is mathematically described as:

$$\frac{d^2\vec{r}}{dt^2} + \frac{GM_2}{r^3}\vec{r} = -\nabla\Phi + a \quad (2.8.3.)$$

where

$\Phi$ -the perturbing potential that includes all perturbing forces which could be defined by a potential function

$a$  - includes all perturbing forces which can not be written as the gradient of a scalar function.

Prior to actual analysis of a method of solving the final equations of motion, perturbing forces are discussed separately.

### **2.8.2. Short overview of the analytical approach to the perturbation problem and discussion on the selected numerical method**

One of the best known methods for determining the effect of the perturbations on the orbit is presented by Lagrange's planetary equations [20]. These are six simultaneous first-order differential equations, expressing the effects of a perturbing potential on the orbital elements. The disadvantages of this method are:

1. They could be applied only on the forces that are derived from a potential function, therefore the air drag (and rocket thrust) could not be included.
2. In general, no exact analytical solution of these equations can be obtained

If the Lagrange's method is further modified to be solved by a numerical method, and compared with the equations solved by Cowell's method, the only difference was that Lagrange's equations allow a larger integration step. Still, the integration step has to be kept reasonably small because of the other factors that do not figure in the Lagrange's equations; therefore it is decided to use Cowell's method instead.

Another advantage of the selected method is its simplicity.

### 2.8.3. Anomalies of the Earth's gravitational field

The assumptions introduced to allow the application of Newton's law of gravitation were already analysed in a previous section of this chapter. It was also shown that this law could be applied on the symmetrically mass-distributed sphere. The real shape of the Earth does not completely match this description, and therefore some results achieved deviate from its true value.

To observe the more accurate variation of  $g$ , a short introduction is presented below. Some of the equations are repeated, for the sake of clarity and compactness.

#### 2.8.3.1. Introduction

The gravitational force at the Earth's surface is given by:

$$F=GM2m1/R^2 \quad (2.8.3.1.)$$

Where

$M2$  – is the mass of the Earth, currently estimated as  $5.97 \times 10^{24}$  kg.

$R$ - is the radius of the Earth

$G$  – the Universal Gravitational Constant, first measured by Cavendish in 1798. Its accepted value today is  $G=6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$  in SI units.

The gravitational acceleration experienced by the body is

$$g=F/m1=GM2/R^2 \quad (2.8.3.2.)$$

The average density of the Earth, calculated by dividing the mass of the Earth (obtained from 2.8.3.2.) by the volume of the Earth, is represented as:

$$\rho_e = 3g/4\pi RG = 5.52 \text{g/cm}^3 \quad (2.8.3.3.)$$

The first value of the density, determined by Cavendish, indicated that the interior of the Earth must be of much higher density than the surface rocks.

The moment of Inertia,  $I$ , of the Earth is actually the measure of its mass distribution, which affects, in high degree, the rotational behaviour of the Earth and its gravitational attraction. The value of  $I$  could be determined from the combination of measurements including the observation of the perturbation of satellite orbit and precession of the Earth's rotation axis.

$I$  – the moment of inertia of a uniform sphere is  $0.4MR^2$

$I_e$  – the moment of inertia estimated for the Earth is  $0.331 MR^2$  ... [59]

The study of seismology provides the best estimates of the density in the interior of the Earth [54].

### 2.8.3.2. Variation in $g$ over the Earth's surface

The value of  $g$  measured at the poles is about  $9.83 \text{m/s}^2$  and at the Equator  $9.78 \text{m/s}^2$ . Total variation is about 0.5%. The main causes of this variation are the Earth's rotation and shape. With the aim to define the relationship that describes the value of  $g$  as a function of the satellite's position, the definition of major contributors to this variation is presented below:

1. Rotation of the Earth. The effect of the Earth's rotation at its poles is almost zero, while at the Equator it has a significant value. There must exist the force at the Equator, which keeps a body on the Earth's surface to rotate with it. This force is actually a part of the gravitational acceleration, which could be alternatively observed as the apparent 'centrifugal' force that reduces gravity.

The fact of interest here is that, the reduction of gravity depends on latitude. The contribution of rotation is to reduce  $g$  by  $0.0339\text{m/s}^2$  at the Equator.

2. Ellipsoidal shape of the Earth. The Equatorial radius of the Earth is 6378.14km and the polar radius is 6356.75km [54]. The difference is about 21km. The pole is closer to the centre of the Earth; therefore the value of the gravitational acceleration  $g$  measured is greater than the value measured at the Equator by about  $0.0663\text{ m/s}^2$ . The mass shape factor, due to extra mass in the equatorial bulge, reduces the last value by  $0.0485\text{ m/s}^2$ . The overall effect of the ellipsoidal shape of the Earth is to reduce  $g$  at the Equator by  $0.0178\text{ m/s}^2$ .
3. Lateral density variation. This effect is the smallest effect of these three and represents the deviation in  $g$  at regions of excess or deficit mass, caused by contrasts in density. It is a practice to compare the expected and measured value of  $g$ , take into account the effects of other two elements, and define the remaining difference as *gravity anomaly* [54].

### 2.8.3.3. Spheroid and Geoid

Gravity studies use the sea level as the reference surface, because it is an equipotential surface when undisturbed by winds and tides [55].

To represent and observe any field function, including gravitational, the concept of potential is introduced. This approach is much more effective than to explicitly use the forces involved. A mass, placed in the gravitational field of an attracting body, gains potential energy by virtue of that attraction. The definition of gravitational potential  $U$  is: 'The work done by gravity on a body of unit mass in bringing it from infinity to its present position, a distance  $r$  from the centre of the field.' [17]

$$U=GM^2/r \quad (2.8.3.4.)$$

The potential is a scalar quantity, whose concept is applied to the gravitational field. Earth's gravitational field is represented by surfaces over which the potential is constant, known as equipotential surfaces. The forces that act in these surfaces are always perpendicular to them; therefore there is no force component that lies in the

surface plane. The intensity of the force is determined by the space between the equipotential surfaces, which is known as the gradient of the potential. The example is, the surface of a liquid. It has to be an equipotential surface; otherwise it would flow from one space to another. If the Earth were a uniform, non rotating sphere, the gravitational equipotential surface would be a perfect sphere. But Earth is rotating, it is not of spherical shape, and its mass is not uniformly distributed, so the real equipotential surface is defined as *geoid*. Its main characteristics are:

- Geoid is defined as sea-level at oceanic regions.
- The value of  $g$  varies over the geoid from 9.78 to 9.83m/s<sup>2</sup>.

The geoid is warped upward under continental masses due to the attracting material above, and is warped downwards over ocean basins. The lowest point of the geoid is in the Indian Ocean – Srilanka (93m below the reference spheroid) and the highest point is at Papua New Guinea (76m above the reference ellipsoid). The reference spheroid is a mathematical approximation to the shape of the geoid with all the highs and lows removed.

#### 2.8.3.4. International Gravity Formula [55]

The *international gravity formula* provides the value of  $g$  as a function of geographical latitude on the reference spheroid; in another words, at sea level. Its value was changed between 1967 and 1971, and the current expression used for the international gravity formula is:

$$g = g_0(1 + \beta_1 \sin^2 \phi + \beta_2 \sin^2 2\phi) \quad (2.8.3.5.)$$

Where

$$\beta_1 = 0.0053024$$

$$\beta_2 = -0.0000059$$

$g_0 = 9.780318\text{m/s}^2$  is the equatorial gravity

The error produced by using the spheroid as a reference, instead of geoid, is small enough that there is no need to introduce any corrective methods. For illustration, the maximum difference between spheroid and geoid is not greater than 93m. The rate of change of difference between the two is on the scale of the exploration surveys.

The method of incorporating this formula in the equations of motion is discussed in later text.

#### **2.8.4. Atmospheric drag**

To make any theoretical approach to analytical determination of atmospheric drag , some model of the upper Earth's atmosphere must be assumed.

The analysis of the effects of the atmospheric drag is performed through the next stages:

1. Evaluate the effects during a single revolution
2. Determine long term changes on the orbit properties

##### **2.8.4.1. Aerodynamic forces acting on a satellite**

Observe an object that moves with the velocity  $V$  relative to the ambient air. This motion results in aerodynamic forces, which could be represented as the vectors and resolved in the two components. The first component is of the same direction as body's velocity vector, but of the opposite orientation, and is known as *air drag*.

The second one is perpendicular to the first component and is known as *air lift*.

Aerodynamic drag could be represented mathematically as:

$$D = \frac{1}{2} \rho V^2 S C_D \quad (2.8.4.1.)$$

where:

$\rho$  - the density of the ambient air

$V$  - the relative velocity of the body to the ambient air

$S$  - a reference area, which is actually the cross-sectional area of the object perpendicular to the direction of motion

$C_D$  – the non-dimensional *drag coefficient*

The term  $1/2\rho V^2$  indicates the increase in pressure when low-speed air is brought to rest. This expression could be applied to the very different conditions of satellite motion. The second component of the aerodynamic force does not pass through the centre of the mass of the observed body, therefore could be divided into a force passing through the centre of the mass and the turning moment  $M$  about the centre of the mass, is illustrated on Fig. 2.8.4.1.

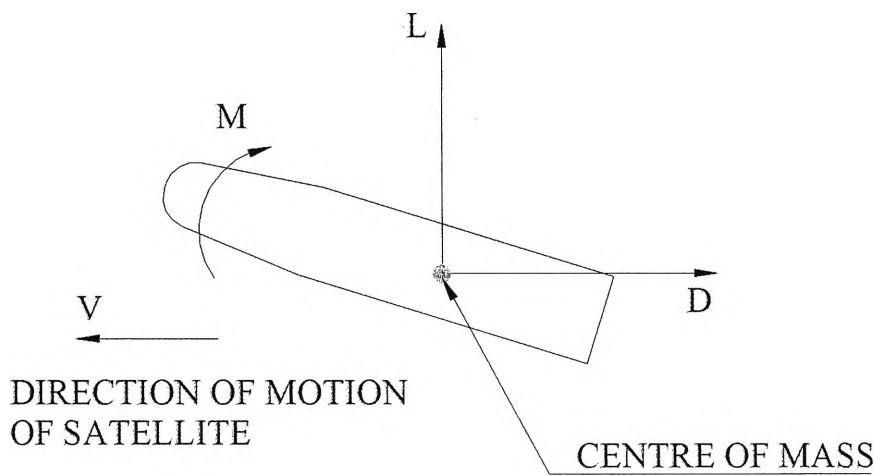


Fig. 2.8.4.1. The forces acting on the satellite

The total turning moment  $M$ , it may be assumed, includes torques due to the gravity gradient, the Earth's magnetic field, as well as aerodynamic torques. In practice, the turning moment on a satellite could be balanced if a particular application requires that, by so named control-jets. The equilibrium position is as the one shown in the Fig. 2.8.4.1. with an established steady lift  $L$ .

In this observation it is assumed no such control is applied on the spacecraft, therefore moment  $M$  will tend to tumble the satellite end-over-end. Experiments have proved that, in such a case, the satellite reverses itself once every 5 seconds, thus its value is taken to be zero [10]. With regard to the geometry of the body, the assumption of the lift force equal to zero is fully justified for the bodies of the near – spherical shape and for cylinder with a length/diameter ratio approaching 1. Even for the ratio of  $l/d$  much less than 1, spin about its axis in a fixed direction, and even for the disc-like satellites the effect of lift is usually negligible. It is important to emphasise that, this assumption does not apply on all satellites, even if they do satisfy geometrical requirements. This will depend primarily on the satellite major purpose with a particularly critical effect at satellite re-entry, known as a 'skip' caused by the air lift. The equation for air drag



indicates that the drag is directly proportional to the air density, which in turn decreases exponentially with the altitude.

The effect of the air drag in the elliptical orbit will be the biggest at its perigee – the point closest to the Earth. The drag is the force opposing the velocity of the spacecraft, applied near the perigee, therefore having a similar effect to an impulsive in-plane transfer manoeuvre, already explained in the introductory section. The result of this effect on the orbit is analysed in Chapter 5.

#### 2.8.4.2.1. Aerodynamic Forces Acting on a Satellite defined with respect to the Compressible Flow

The flows that experience high change in the density are named *compressible*. Usually, gases flows would satisfy this description while liquids are treated as incompressible. Gas flows at speeds low compared to the speed of sound with negligible heat transfer may be considered as incompressible.

Flow characteristic known as Mach number defines the ratio of the flow speed  $V$  to the local speed of sound  $c$ :

$$M = \frac{V}{c} \quad (2.8.4.2.1.)$$

For  $M < 0.3$  the max density variation is less than 5% [61] – therefore the flow is treated as incompressible. For  $M = 0.3$  in air, at standard conditions, corresponds to a speed of about  $100\text{m/s}$ .

To determine what effect gas compressibility has on the satellite motion, two main factors are observed:

1. The velocity of the satellite at perigee for up to 2000km altitude
2. The properties of the atmosphere at the particular altitude that will affect the speed of the sound.

To proceed with the application of the Mach number to the satellite motion next definition is introduced:

*Mach number is a key parameter that characterizes compressibility effect in a flow and mathematically is described as:*

$$M = \frac{V}{c} = \frac{V}{\sqrt{\frac{dp}{d\rho}}} \quad (2.8.4.2.2.)$$

and is interpreted as a ratio of inertia forces to forces due to compressibility. Flows for which  $M < 1$  are subsonic and those with  $M > 1$  are supersonic. The most important flow for this analysis is hypersonic flow which starts at  $M \sim 5$ .

For example the Mach number for the satellite in the orbit with perigee height of 1000km and velocity of 7.5km/s:

$c = \sqrt{kRT}$  - Theoretical prediction of the speed of sound as the function of temperature, confirmed by the experiments to produce an error of less than 5% [35].

$$k = \frac{c_p}{c_v} = 1.4 \text{ for air}$$

$$R = 287 \text{ Nm/kgK}$$

$T = 1000\text{K}$  at altitude of 1000km, according to the data given in table L-6, p820 [29]

$$c = (1.4 \times 287 \frac{\text{Nm}}{\text{kgK}} \times 10^3 \text{ K})^{1/2}$$

$$c = 633.877 \text{ m/s}$$

$$M = \frac{V}{c} = 11.83 \text{ - Hypersonic flow}$$

The National Aerospace Plane [35] would cruise at Mach numbers approaching 20.

If a point sound source, that emits instantaneous infinitesimal disturbances which propagate in all directions with speed  $c$ , is observed at any time  $t$  the location of the wave front from the disturbance emitted at time  $t_0$  is represented by a sphere, with radius  $c(t-t_0)$ , whose centre coincides with the location of the disturbance at time  $t_0$ .

The locus of leading surfaces of the sound waves will be a cone. No sound will be heard in front of the cone.

#### 2.8.4.2.2. Aerodynamic drag coefficient dependency with respect to the Mach number

To determine the forces acting on a satellite in a supersonic flow the first step is to determine the nature of the disturbance propagation. Here,  $v > c$  so the locus of leading surfaces of the sound waves will be a cone, as concluded in the previous heading. Represented graphically:

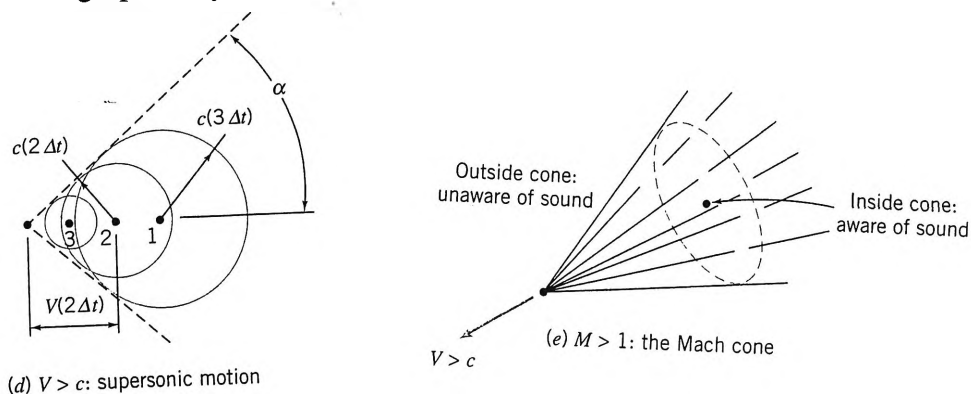


Figure 2.8.4.2.2.1: Propagation of sound waves from a moving source: The Mach cone

Next step is to observe a hypersonic boundary layer formed at the surface of a moving body. This analysis is much simplified by the assumption that the satellite is a 2-dimensional body, whose 3<sup>rd</sup> dimension is much smaller. For both observed shapes: plate and cylinder this assumption does not deviate much from the reality. The concept of the thin boundary layer, within which the effects of viscosity are largely confined in the flow of a gas over a body, established by Prandtl [66], states that the boundary layer has a displacement that changes the effective shape of the body. This change is seen as an increase of the body dimensions and is a result of reduced mass flux within the boundary layer. Mathematically described:

$$\rho_e u_e (\delta - \delta^*) = \int_0^{\delta} \rho u dy, \quad \delta \rightarrow \infty \quad (2.8.4.2.3)$$

$$\delta^* = \int_0^{\infty} \left( 1 - \frac{\rho u}{\rho_e u_e} \right) dy$$

Last expression actually states that the flow external to the boundary layer is displaced for the distance  $\delta^*$  due to the diminution of the mass flux within the boundary layer, where  $\rho$  represents gas density,  $u$  relative velocity and  $e$  is the subscript for local values at the edge of the boundary layer.

One of the characteristics of the hypersonic flow is a high temperature of the walls of the observed moving body, which affects the density of surrounding gas and therefore the displacement thickness. Still, this effect does not have particular importance except at the object's leading edge, where the transition of the flow external to the boundary layer causes an initial compression accompanied by a shock wave which becomes very severe at high Mach numbers. The compression is followed by the flow expansion which affects the build-up of the boundary layer, so now the interest is placed onto the interaction between the external stream and the boundary layer. Presented graphically:

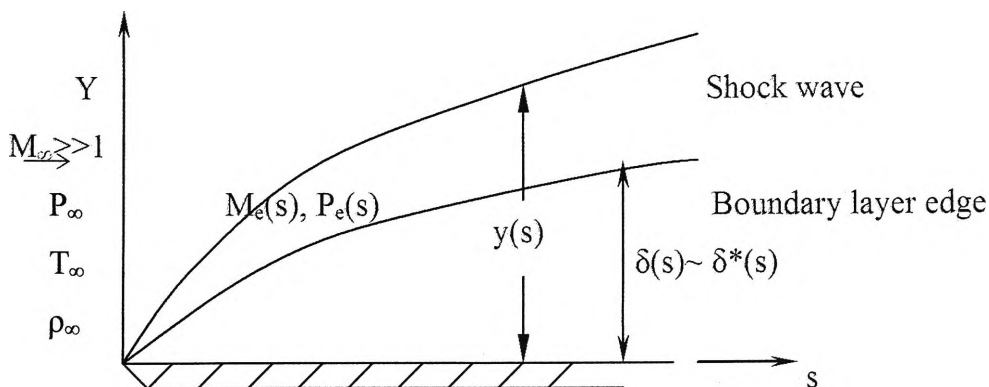


Figure 2.8.4.2.2.2: Shock wave boundary layer at Hypersonic flow

The region between the edge of the boundary layer and the shock wave has the Mach no. and the pressure that change with  $s$  in the same sense as if the flat plate is replaced by the shape defined by the boundary layer edge (actually the locus of the displacement thickness) in an inviscid hypersonic gas stream. Based on this discussion it is assumed for the vehicle half cone to be  $\theta=60^\circ$ , the angle used in further analysis. Finally, the drag of the spacecraft could be divided into the following components:

- *Wave drag* – due to the presence of shock waves; dependent on the Mach number.
- *Viscous drag* – due to friction – Section 2.8.4.4.
- *Induced drag* – due to the generation of the lift; here ignored, should be observed for the re-entry,
- *Base drag* – due to the wake behind the vehicle – ignored for the case of the uncontrolled satellite
- *Interference drag* – due to the interaction of various flow fields, here neglected, should be developed further, especially because of the new discoveries in this field, and new data collected about the conditions in the space
- *Roughness drag* – due to the surface roughness such as rivets and welds [60].

The component wave drag is described based on the theory of a conical body as [61] and [66]:

$$C_{Dn} = \left(0.083 + \frac{0.096}{M^2}\right)(5.73\theta)^{1.69}$$

where  $\theta$  is the vehicle half cone angle in radians, here assumed to be  $60^\circ$ .

Therefore:

$$C_{Dn} = 1.73$$

### 2.8.4.3. The properties of upper atmosphere – evaluation of $\rho$ [36]

The properties of the Earth atmosphere vary exponentially with the altitude. This fact would be the starting point for atmosphere properties analysis. There is a direct relation between the temperature and the density of the atmosphere. Particular regions are introduced and named according to the temperature profile. A mathematical expression (the simplest one) that describes the change of the air density as a function of altitude is given as:

$$\rho = \exp(-mgz/KT) \quad (2.8.4.2.)$$

where:

$z$ -the altitude in km

Quantity  $KT/mg$  is known as the scale height where:

$m$ - molecular weight

$g$ - acceleration due to gravity

$T$ -temperature

$K$ -Boltzmann's constant

This equation could be applied for the heights between 100 and 1000km, with large deviations from the real values due to the inaccuracy in the model. The major atmospheric constituents below  $10^3$ km are  $O_2$ ,  $N_2$ ,  $O$  and  $He$ . Minor constituents are  $O_3$ ,  $CO_2$ ,  $H_2O$ ,  $NO$ , electrons and positive and negative ions. Another kind of variations in the density is divided into six types [36]:

1. diurnal
2. 27 day period variation
3. seasonal-latitudinal
4. semi-annual
5. 11-year period variation
6. geomagnetic

This short introduction is a very brief outline of properties of the atmosphere. The concrete problem here is to define the method for obtaining the model that will describe the atmosphere density, necessary to determine the air-drag effects. A number of analytical methods exist at a moment, but even with all complexity, these are still based on a number of assumptions. The results obtained deviate from the real values to a large degree. This is a reason for selecting an approach based on real values measured by the satellite already in the orbit. This method is also very easy to improve further, and what is more important, it is based on true values.

Data collected in space are transferred to the Earth, and treated by statistical methods. There is an appropriate set of values for every particular condition, referring to the variations mentioned above. Values selected in this work are the average values of a medium density atmosphere as the most general case. Results of similar measurements are collected by Mr. Francis S. Johnson and published in the book, ref [36].

The table 2.8.4.3. gives the density as the function of the altitude, which ranges from 105km to 2500km:

No.	density (kg/m <sup>3</sup> )	h (km)
1	2.14E-07	105
2	9.80E-08	110
3	2.45E-08	120
4	6.58E-09	130
5	2.60E-09	140
6	1.40E-09	150
7	9.00E-10	160
8	4.58E-10	180
9	2.67E-10	200
10	1.66E-10	220
11	1.07E-10	240
12	7.10E-11	260
13	4.80E-11	280
14	3.30E-11	300
15	1.38E-11	350
16	6.23E-12	400
17	2.97E-12	450
18	1.48E-12	500
19	4.05E-13	600
20	1.21E-13	700
21	3.85E-14	800
22	1.32E-14	900
23	5.05E-15	1000
24	3.57E-16	1500
25	1.17E-16	2000
26	4.91E-17	2500

Table 2.8.4.3. Atmosphere density values at a given altitudes [36]

#### 2.8.4.4. Evaluation of cross-sectional area

The cross-sectional area of interest is one that is perpendicular to the direction of motion. For spherical spacecraft it is easy to determine this area, but there are other shapes, mainly cylindrical, that are used for the spaceflights. It was already mentioned, if the motion of the spacecraft is uncontrolled, there exists an uncontrolled rotation about an axis of maximum moment of inertia, as this is the mode of motion in which the rotational energy is a minimum for a given angular momentum.

Precession of the axis of spin occurs slowly under the influence of the small torques. The rate of spin of the satellite decreases with time under the action of magnetic and aerodynamic damping. Most of the satellites are cylindrical in shape with ratio length/diameter of about 2. It is most likely the axis of maximum moment of inertia to be transverse axis. The two possible motions of the satellite, in accordance to the previous theory, are:

1. rotating in the same manner as the airplane propeller
2. tumbling end-over-end

The first motion has an axis of spin aligned with the direction of motion, and the second one moves in a direction that is perpendicular to the axis of spin. In reality motion can occur in any angle in between these two extreme modes of motion. If the length of a cylinder is designated with  $l$  and its diameter by  $d$  the cross-sectional area in the first case is  $lxd$  and in the second one is  $2/\pi(ld+\pi d^2/4)$ . The mean value of the above expressions gives:

$$S = ld \left( 0.818 + 0.25 \frac{d}{l} \right) \quad (2.8.4.2.)$$

Particularly interesting is a value of  $S$  during perigee pass. In this situation the direction of motion of the spacecraft changes while its spin axis stays fixed in direction, in space. The change in  $S$  would be very small, and  $S$  will never reach any of the two extreme values. The expression for  $S$  will not produce error greater than 5%. For the satellites  $l$  to  $d$  ratio between 2 and 8,  $S$  remains almost constant. As this ratio decreases, expression for  $S$  is more true, and for sphere is precisely true.

#### 2.8.4.5. Evaluation of Drag Coefficient $C_D$

The drag coefficient of the bodies of various shapes at varying angles to the air flow, were determined by Cook (1960) [11].

To determine value of  $C_D$  the number of assumptions were made. The model was based on:

1. Satellites with perigee heights between 180 and 500 km
2. orbital eccentricities between 0 and 0.2

It was clear that, for an altitude from 150 – 200km the ordinary continuum flow theory of a conventional aerodynamics produces large error. The approach that gives better results is known as free-molecular flow, which is applied when the mean free path of the molecules greatly exceeds a linear dimension of the satellite.

An important fact is that, the fully developed free molecular flow does not exist when the mean free path of the molecules is less than twice the maximum linear dimension of satellite. For an example, the mean free path increases from 2 metres at 120km altitude to the 50m at about 160km altitude. The length of the satellites launched vary from 1 to 25m, which implies that, the small satellites can experience free-molecular flow until almost their last revolution, while large satellites can be in the transition region from free – molecular to an intermediate flow regime for the last day or two of their life time. Free molecular flow analysis is based on the next assumptions:

1. The satellite is assumed to be static and the air molecules are flowing past
2. Flowing molecules have a Maxwellian distribution of thermal velocity superposed on their uniform velocity  $v$ .
3. The molecules hitting the surface, are temporarily captured and then re-emitted
4. Possible collisions between bombarding and re-emitting molecules are ignored. The mode of re-emission is of high importance for this problem. It is certain that the molecules are not simply reflected from the surface as from a mirror, but they behave in accordance to the so-called Knudsen cosine law – the number of molecules emitted in directions making angles between  $\theta$  and  $\theta + \delta\theta$  with the normal to the surface being proportional to  $\cos\delta\theta$ .



- The most uncertain quantity is the temperature of the emission, but it is widely adopted that, re-emitted molecule has the same temperature as the surface from which is it being emitted.

The values of  $C_D$  for a particular shapes are represented in the Table below [32]:

Shape	$Cd$
Sphere	2.10-2.20
Cylinder-Inclined to the air - flow	2.10-2.20
Cylinder-Tumbling end over end	2.15
Flat Plate-Perpendicular to the air-flow	2.20
Cones-With semi-apex angle 15-20deg	2.10

Final selection for  $C_D$  is 2.2 for the mean area perpendicular to the direction of motion, with an error (standard deviation), which will not exceed 5%.

Remembering that the first component of the air drag coefficient was determined in section 2.8.4.2.2 by including the geometrical air drag coefficient, total air drag coefficient was determined. Based on the analysis not applied at the steady state flight of an aerodynamically stable vehicle [62] total coefficient is equal to:

$$C = C_{Dn} + C_D = 3.73$$

#### 2.8.4.6. Aerodynamic Drag in terms of velocity – Conclusion

In the equation for Drag:

$$D = \frac{1}{2} \rho V^2 S C_D \quad (2.8.4.1.)$$

term  $V$  represents the velocity of the satellite relative to the ambient air. This velocity could be further expressed in terms of  $v$  – velocity of the satellite relative to the Earth’s centre. This velocity is computed by SATELIGHT and is determined in both, orbital plane and the system that contains equatorial plane as  $xy$  plane (global system). The velocity of the air, relative to the Earth’s centre is assumed to be west to east and is designated by  $V_A$ . Therefore:

$$V = v - V_A \quad (2.8.4.3.)$$

Or, in scalar expression

$$V^2 = v^2 + V_A^2 - 2vV_A \cos \gamma \quad (2.8.4.4.)$$

where  $\gamma$  is the angle between  $V_A$  and  $v$ . It is assumed that, the atmosphere rotates with angular velocity  $\omega$  about the Earth's axis, therefore:

$$V_A = r\omega \cos \phi \quad (2.8.4.5.)$$

where

$r$ -the distance from the Earth's centre

$$r = \sqrt{x^2 + y^2} \quad (2.8.4.6.)'$$

$\phi$ - The geocentric latitude

$\omega$ - Earth's rotational speed 1/sec

Because of the properties of the atmosphere, the effect of the air drag is the most significant at the heights near the perigee. At this region, the satellite is travelling almost horizontally – its climb angle never exceeds  $10^\circ$  at heights up to  $2x0.01r_p$  above perigee, where  $0.01=H/r_p$  the angle difference between actual angle  $\gamma$  between  $V_A$  and  $v$  and assumed angle  $\gamma'$  between  $V_A$  and the horizontal component  $v_h$  of  $v$  with an error in  $\cos \gamma$  of less than 1% [11].

where

$r_p$  - the altitude at the perigee

$H$  – a constant whose value varies between 30-50 km for the satellites with perigee altitudes in the region of 200 – 300km [32].

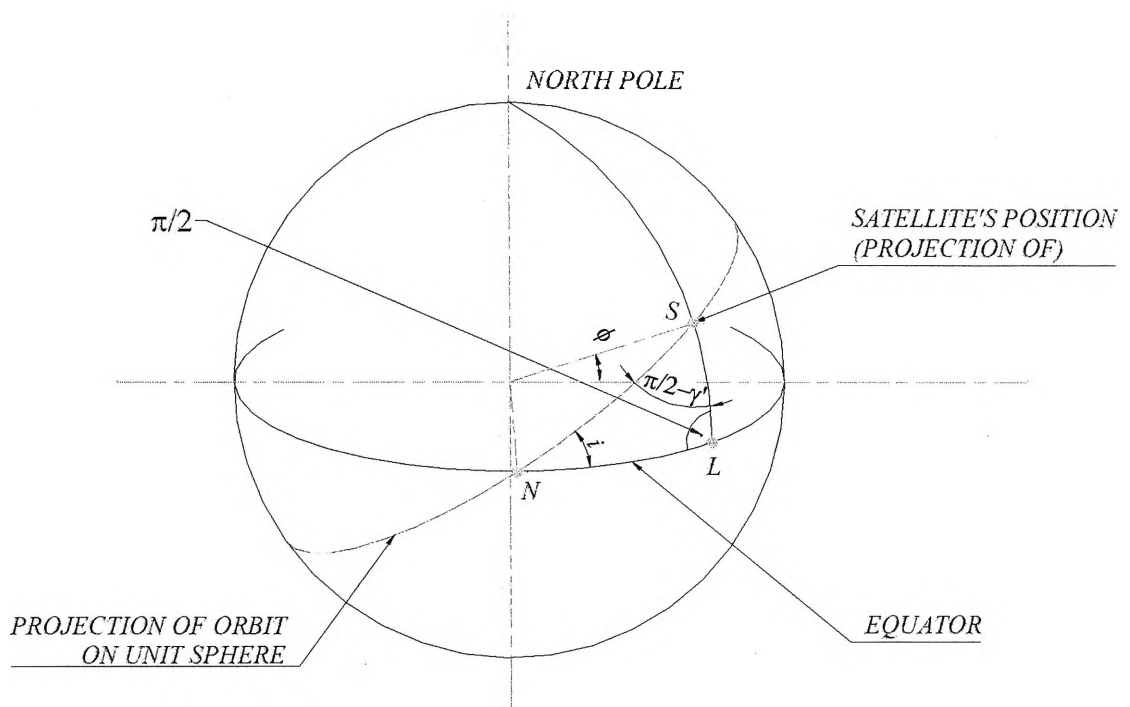


Fig. 2.8.4.2. The illustration of angles  $\gamma'$  and  $\phi$

From figure 2.8.4.2. it is concluded: for the triangle  $SLN$  and based on spherical trigonometry:

$$\cos \gamma' \cos \phi = \cos i \quad (2.8.4.6.)$$

and if the error of taking  $\cos \gamma$  equal to  $\cos \gamma'$  is less than 1%,

$$V_A = r \omega \cos i / \cos \gamma' \quad (2.8.4.7.)$$

$$V_A \cos \gamma' = r \omega \cos i \quad (2.8.4.8.)$$

$$V_A \cos \gamma = r \omega \cos i (1 + 0(0.01)) \quad (2.8.4.9.)$$

When substituted further:

$$V^2 = v^2 \left[ 1 - \frac{r \omega}{v} \cos i (1 + 0.0(0.01)) \right]^2 + r^2 \omega^2 (\cos^2 \phi - \cos^2 i) \quad (2.8.4.10.)$$

The effect of atmospheric rotation on the drag is small and  $r^2 \omega^2 < 0.005 V^2$  if  $\omega$  is of the same order as the Earth's angular velocity, therefore the term  $r^2 \omega^2$  is neglected.

Another approximation is to ignore the variance of the inclination angle  $i$ , which according to [12] varies by less than  $0.3^\circ$  during satellite's life.

Finally:

$$V \approx v \left( 1 - \frac{r\omega}{v} \cos i_o \right) \quad (2.8.4.11.)$$

Therefore, a resultant drag force is acting parallel to  $v$  and is equal to:

$$D = \frac{1}{2} \rho v^2 \left( 1 - \frac{r\omega}{v} \cos i_o \right)^2 l d \left( 0.818 + 0.25 \frac{d}{l} \right) 2.2 \quad (2.8.4.12.)$$

The element that contains biggest error in the equation is  $\omega$ , as the exact speed of the rotation of the atmosphere is both variable and unknown and the accuracy of this calculation could be increased if this element is determined by better approximated value [32].

### 2.8.5. Three body problem [32]

This problem is still the most challenging problem in analytic celestial mechanics theory. The general solution to this problem, which requires twelve arbitrary constants, is not possible. There are special cases that could be observed and solved analytically. These cases are:

1. Three bodies placed in a straight line
2. Three bodies at the vertices of an equilateral triangle
3. The trivial case of the three bodies all at one point

There also exists a solution involving a physically meaningless system of fixed masses, set by Euler.

1. The problem of three bodies in a straight line could be solved analytically if the next assumptions are adopted:

- a) zero mass of the satellite
- b) all other perturbations are neglected except those due to the Moon
- c) the orbit of Moon is taken to be circular

This problem can have five different possible cases, presented in the figure 2.8.5.1:

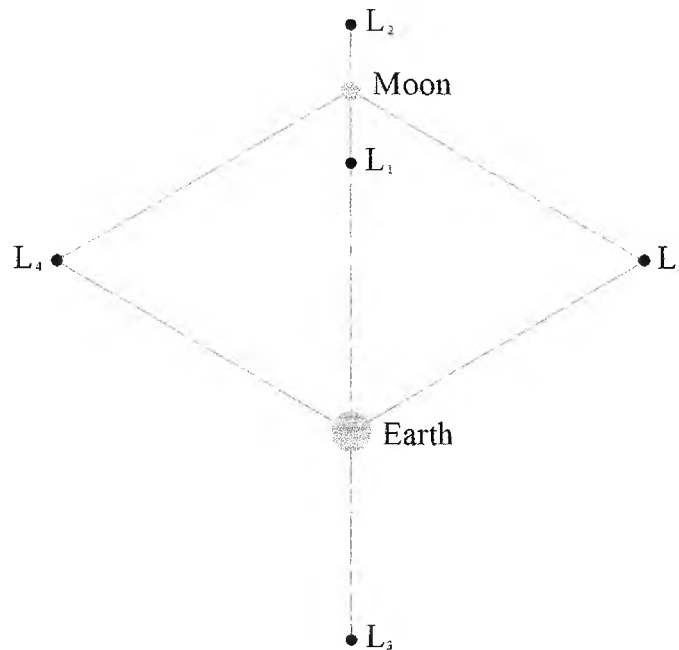


Fig. 2.8.5.1. Lagrangian points (not to scale)

The case in which the satellite is at inferior conjunction, ie, between the Earth and Moon on their line of centres, is used as an example. Note that this is not a stable point.

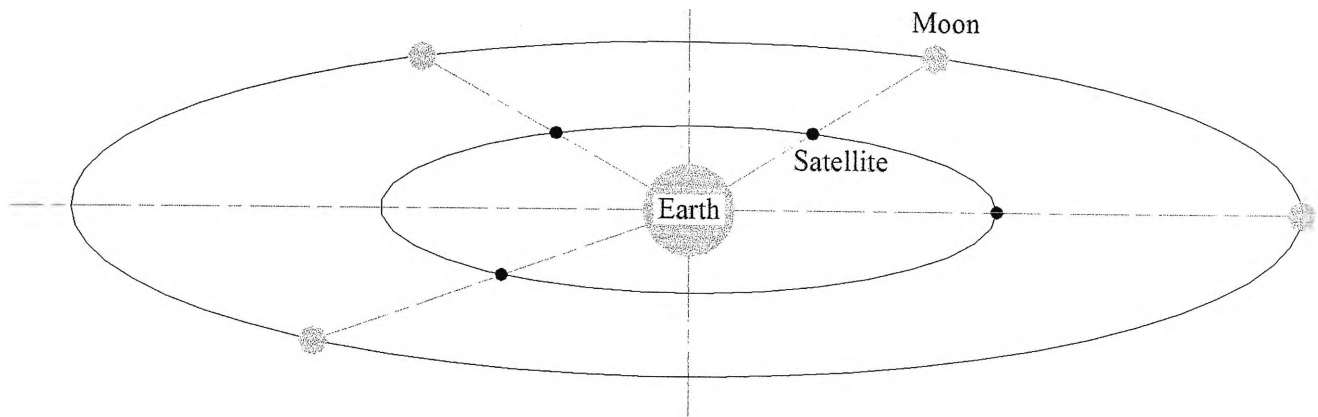


Fig. 2.8.5.2. Synodic satellite – Satellite that always lies on the line which passes through the Earth and the Moon

The appropriate distances, as well as the relative position of the satellite with respect to the Earth-Moon system are illustrated below:

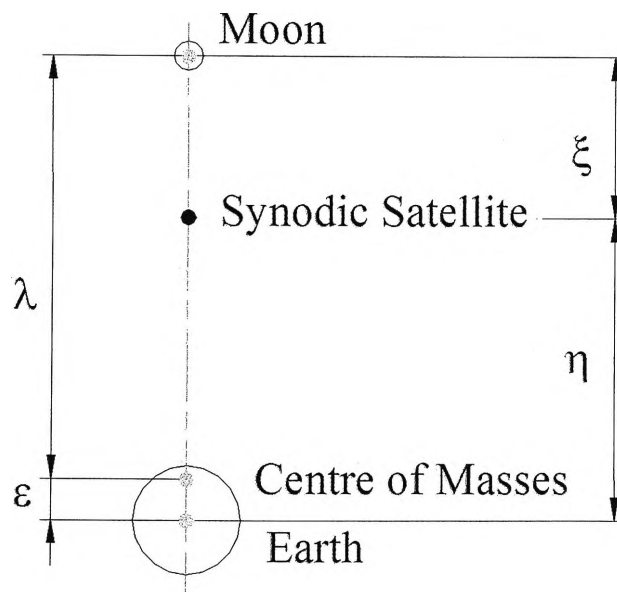


Fig. 2.8.5.3. Synodic satellite relative distances

From the above figure and the definition for the centre of masses:

$$\varepsilon/\lambda = \text{Moon mass}/\text{Earth mass} = 0.012\ 288\ 800 \quad (2.8.5.1)$$

The system selected is a rotating system with angular velocity  $\omega_M$  in rad/min placed at the centre of the masses. The forces acting on the satellite are:

$$\frac{k_e^2 m_s m_E}{\eta^2} = \frac{k_e^2 m_s m_m}{\xi^2} + \omega_M^2 m_s (\eta - \varepsilon) \quad (2.8.5.2.)$$

From the vis-viva equation for a circular orbit with  $r=a$ :

$$\left(\frac{ds}{dt}\right)_M^2 = \frac{k_e^2 (m_E + m_m)}{\lambda + \varepsilon} \quad (2.8.5.3.)$$

also:

$$\omega_M^2 = \frac{\left(\frac{ds}{dt}\right)^2}{(\lambda + \varepsilon)^2} = \frac{k_E^2 m_E \left(1 + \frac{\varepsilon}{\lambda}\right)}{(\lambda + \varepsilon)^3} = \frac{k_E^2 m_E}{\lambda} \quad (2.8.5.4.)$$

From (2.8.5.1.) and (2.8.5.4.)

$$\frac{k_E^2 m_E}{\eta^2} = \frac{k_E^2 m_M}{(1-\eta)^2} + \frac{k_E^2 m_E}{\lambda} (\eta - \varepsilon) \quad (2.8.5.5.)$$

$$\frac{1}{\eta^2} = \frac{\varepsilon}{\eta} \frac{1}{(1-\eta)^2} + \frac{(\eta - \varepsilon)}{\lambda} \quad (2.8.5.6.)$$

Last expression is of 5<sup>th</sup> order, giving one real solution for  $\eta$ . Calculated value of  $\eta$  is 0.84910870 (bearing on mind that total distance from Earth to Moon centre was assumed to be unity).

The period of satellite is determined from

$$T = \frac{2\pi a^{3/2}}{k_E \sqrt{\mu}} \quad (2.8.5.7.)$$

where:

$$a = 60.26818 \text{ Earth radii}$$

$$k_E = 0.07436574$$

$$\mu = m_E + m_M = 1 + 0.0122888$$

2. The example for the triangle solution is the Trojan group of minor planets. This group of natural satellites belong to the Jupiter – Sun system. There are two characteristic points in the Jupiter's orbit about Sun. According to Lagrange, an infinitesimal body placed at either of these points will form a stable orbit around the Sun in the same period as Jupiter. In 1906 was discovered small planet situated near the equilateral triangle point preceding Jupiter by  $60^\circ$  [13]. A small torque applied on the planet or spacecraft placed in one of the equilateral triangle points (points  $L_4$  and  $L_5$  on Fig. 2.8.5.1.) will cause observed body to oscillate about the equilibrium point, also known as the libration point. This problem of oscillating bodies near libration points could be analysed by the standard methods for the treatment of small oscillations in mechanical problems.

Analysis has shown [14] that for a Trojan minor planet, the motion about the equilibrium point consists of a vibration with a period of about 147 years and a more rapid superimposed oscillation with a period slightly in excess of the period of Jupiter (about 11 years).

## 2.9. Conclusion

The equations of motion for the system of the three bodies including external perturbing forces, presented in its most general form and based on the above theory and arranged for numerical computation are.



$$m_i \frac{d^2 \vec{r}_i}{dt^2} = G \sum_{j=1, j \neq i}^3 m_i m_j \frac{\vec{r}_{ij}}{r_{ij}^3} + P_i \quad (2.9.1.)$$

This general equation is further projected on  $x$  and  $y$  axes of the rectangular De Cartes coordinate system.

Particular perturbing effects are observed for particular altitude values, that is, not all three major perturbing forces figure in the same model. There are two different cases observed and modelled:

1. Satellites' orbits with initial perigee value above 2500km. For this type of orbit, gravitational perturbations due to the Moon influence on the Earth's satellite trajectory is one of the elements included in the model. The second one is the effect on trajectory due to the anomalies of the Earth's gravitational field.
2. For satellites launched into the orbits with the perigee less than 2500km, or for the spacecrafts that enter this region after a particular time in space, the major perturbing torques are: *The anomalies of Earths gravitational field and the air drag.*

According to this selection, appropriate equations with discussions are presented in the following text.

### 2.9.1. High altitude satellite motion Model Equations

- a. Three-body problem only, excluding gravitational perturbations, observing satellite placed in between Moon and the Earth. The shape and the altitude of the orbit can vary, according to the mission requirements. The problem is solved by program TBP.FOR. More details about the initial conditions, method of integration and other

information regarding the program are provided in Chapter 4. At this stage, applied equations of motion are presented as:

$$\left. \begin{aligned} \frac{d^2 x_s}{dt^2} &= \frac{Gm_E}{r_{SE}^3} (x_E - x_s) + \frac{Gm_M}{r_{SM}^3} (x_M - x_s) \\ \frac{d^2 y_s}{dt^2} &= \frac{Gm_E}{r_{SE}^3} (y_E - y_s) + \frac{Gm_M}{r_{SM}^3} (y_M - y_s) \end{aligned} \right\}$$

is the first set of equations for satellite motion.

$$\left. \begin{aligned} \frac{d^2 x_E}{dt^2} &= \frac{Gm_M}{r_{EM}^3} (x_M - x_E) + \frac{Gm_S}{r_{ES}^3} (x_S - x_E) \\ \frac{d^2 y_E}{dt^2} &= \frac{Gm_M}{r_{EM}^3} (y_M - y_E) + \frac{Gm_S}{r_{ES}^3} (y_S - y_E) \end{aligned} \right\}$$

are the second two equations that describe the motion of the Earth about the centre of masses.

$$\left. \begin{aligned} \frac{d^2 x_M}{dt^2} &= \frac{Gm_S}{r_{MS}^3} (x_S - x_M) + \frac{Gm_E}{r_{ME}^3} (x_E - x_M) \\ \frac{d^2 y_M}{dt^2} &= \frac{Gm_S}{r_{MS}^3} (y_S - y_M) + \frac{Gm_E}{r_{ME}^3} (y_E - y_M) \end{aligned} \right\}$$

are the last two equations that describe the motion of the Moon in the system of three bodies: Earth, Moon and a satellite.

The values  $r_{ij}$  are represented as:

$$\begin{aligned} r_{12} &= \left( (x_2 - x_1)^2 + (y_2 - y_1)^2 \right)^{1/2} \\ r_{23} &= \left( (x_3 - x_2)^2 + (y_3 - y_2)^2 \right)^{1/2} \\ r_{13} &= \left( (x_3 - x_1)^2 + (y_3 - y_1)^2 \right)^{1/2} \end{aligned}$$

Additional observation of the motion of the bodies that move in the space (not in a plane, therefore satellite orbit could be of any inclination angle, while the inclination angle of the Moon orbit is taken to be about  $6^\circ$ ) is presented in Chapter 5.

Only for the comparison to the analytical method, this method will provide a solution for any case (not only for the special cases), includes the satellite mass in the equations and could be combined with other gravitational and non-gravitational perturbing effects. The exact values for particular constants and the values of required initial conditions are discussed in Chapter 4.

b. The *effects of the three-body problem combined with the effects of the perturbing forces due to anomaly of Earth's gravitational field* are incorporated in the equations of motion as:

$$\left. \begin{aligned} \frac{d^2 x_s}{dt^2} &= g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr_{SE}^3} (x_E - x_S) + \frac{Gm_M}{r_{SM}^3} (x_M - x_S) \\ \frac{d^2 y_s}{dt^2} &= g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr_{SE}^3} (y_E - y_S) + \frac{Gm_M}{r_{SM}^3} (y_M - y_S) \end{aligned} \right\}$$

These are the equations that describe the motion of the satellite.

$$\left. \begin{aligned} \frac{d^2 x_E}{dt^2} &= \frac{Gm_M}{r_{EM}^3} (x_M - x_E) + \frac{Gm_S}{r_{ES}^3} (x_S - x_E) \\ \frac{d^2 y_E}{dt^2} &= \frac{Gm_M}{r_{EM}^3} (y_M - y_E) + \frac{Gm_S}{r_{ES}^3} (y_S - y_E) \end{aligned} \right\}$$

are the equations for the Earth's motion, which are not changed compared to the previous set of the equations.

$$\left. \begin{aligned} \frac{d^2 x_M}{dt^2} &= \frac{Gm_S}{r_{MS}^3} (x_S - x_M) + g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr_{ME}^3} (x_E - x_M) \\ \frac{d^2 y_M}{dt^2} &= \frac{Gm_S}{r_{MS}^3} (y_S - y_M) + g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr_{ME}^3} (y_E - y_M) \end{aligned} \right\}$$

are the equations of the Moon's motion in three-body system, including anomalies of Earth's gravitational field.

### 2.9.2. Satellite orbits with the altitudes less than 2500km

The effects that alter the motion of the satellites in these orbits are the air drag perturbing effects and perturbations due to the anomalies of the Earth's gravitational field.

Air drag is incorporated in the final equations as:

$$\left. \begin{aligned} \frac{d^2 x}{dt^2} &= \frac{Gm_E}{r^3} x + \frac{1}{2} \rho v_x^2 \left( 1 - \frac{r\omega}{v_x} \cos i_o \right)^2 l d \left( 0.818 + 0.25 \frac{d}{l} \right) 2.2 \\ \frac{d^2 y}{dt^2} &= \frac{Gm_E}{r^3} y + \frac{1}{2} \rho v_y^2 \left( 1 - \frac{r\omega}{v_y} \cos i_o \right)^2 l d \left( 0.818 + 0.25 \frac{d}{l} \right) 2.2 \end{aligned} \right\}$$

It is seen from the above equations, the air drag force components are based on the satellite velocity components at a particular position, determined in the program. All values that depend on the actual latitude of the satellite are also based on the values computed in the program, involving a method for space systems transformations analysed in the Chapter 4.

To include Earth's gravitational field anomalies, these equations are modified further as:

$$\left. \begin{aligned}
 \frac{d^2x}{dt^2} &= g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr^3} x \\
 &\quad + \frac{1}{2} \rho v_x^2 \left( 1 - \frac{r\omega}{v_x} \cos i_o \right)^2 ld \left( 0.818 + 0.25 \frac{d}{l} \right) 2.2 \\
 \\
 \frac{d^2y}{dt^2} &= g_0 (1 + 0.0053024 \sin^2 \phi - (5.9E - 6 \sin^2(2\phi))) \frac{Gm_E}{gr^3} y \\
 &\quad + \frac{1}{2} \rho v_y^2 \left( 1 - \frac{r\omega}{v_y} \cos i_o \right)^2 ld \left( 0.818 + 0.25 \frac{d}{l} \right) 2.2
 \end{aligned} \right\}$$

where:

$g_0$ - equatorial gravity

$\phi$  - latitude

$m_E$  – mass of the Earth

$m_M$  – mass of the Moon

For the definition of other values refer to section 2.8.

The next stage is to choose a numerical method to solve these equations, introduce the model for air density, observe the error due to numerical method, and introduce two case studies that will be used for the results comparison – Chapter 3.

# **CHAPTER 3**

## **Numerical Method Analysis**

## Chapter 3: Numerical Method Analysis

The main numerical method applied for solving the second order differential equations of satellite motion is *Runge Kutta* fourth order step-by-step method. To model atmosphere density variation, with respect to the altitude, *linear interpolation* method is used. The error analysis is also included in this section.

### 3.1. Runge – Kutta IVth Order numerical method

This method is based on a Taylor series approach, and is applied without requirement for the calculation of higher derivatives. A differential equation to be solved is given by

$\frac{dy}{dx} = f(x, y)$ . The general form of the Runge – Kutta method, referred to the given

differential equation, could be represented as:

$$y_{i+1} = y_i + \phi(x_i, y_i, h)h \quad (3.1.1.)$$

where

$\phi(x_i, y_i, h)$  is actually an incremental function which could be defined as a representative slope over observed interval. This function is mathematically defined as:

$$\phi = a_1k_1 + a_2k_2 + \dots + a_nk_n \quad (3.1.2.)$$

where

$a_i$  – are the constants

$k_i$  – Runge Kutta coefficients given as:

$$k_1 = f(x, y)$$

$$k_2 = f(x + b_2h, y + C_2k_1)$$

.

.

.

$$k_n = f(x_i + b_nh, y_i + C_nk_{n-1}) \quad (3.1.3.)$$

In words, the above equation, could be described as:

New function value is equal to an old value plus the slope multiplied by a step size.

Graphically:

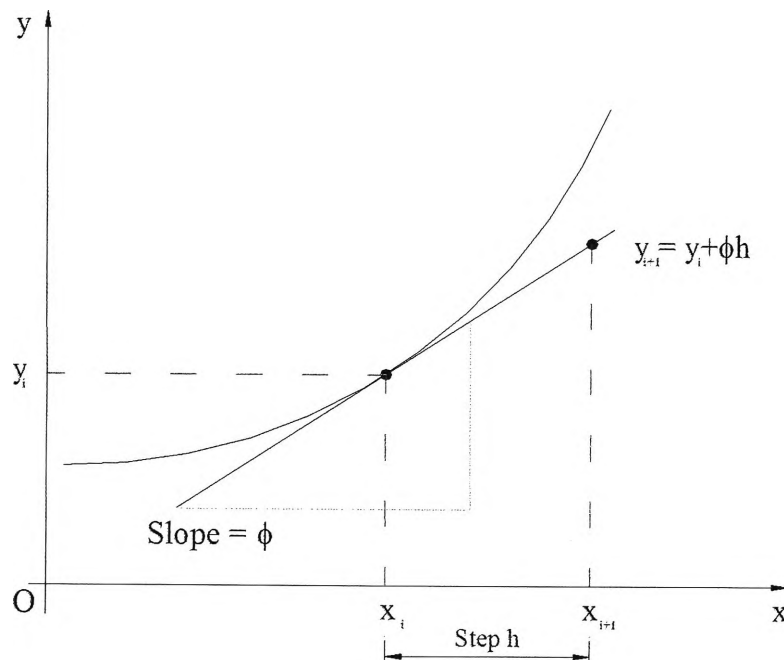


Fig. 3.1. The principle of Runge-Kutta method

This step by step same formula is used to compute future values of the function, based on its previous values.

This method is very efficient for computer calculations, because of the recursive relationship of  $k_i$  coefficients. For example,  $k_1$  appears in an equation for  $k_2$ , which appears in the expression for  $k_3$  and so on. Different numbers of  $k$  factors that are applied in the increment function actually determines the particular type of the method.



For  $n=1$  the first order method is derived, which is also known as Euler's method. For the differential equation of the general form

$$y' = f(x, y) \quad (3.1.4.)$$

Taylor's series is expanded about a starting value  $(x_i, y_i)$  as:

$$y_{i+1} = y_i + y'_i h + \frac{y''_i}{2} h^2 + \dots + \frac{y_i^{(n)}}{n!} h^n + R_n \quad (3.1.5.)$$

where

$$h = x_{i+1} - x_i$$

$$R_n = \frac{y^{(n+1)}(\xi)}{(n+1)!} h^{n+1} \quad (3.1.5.)$$

where  $\xi$  lies somewhere in the interval from  $x_i$  to  $x_{i+1}$ .

Expressed in the accordance to the equation (3.1.4.)

$$y_{i+1} = y_i + f(x_i, y_i)h + \frac{f'(x_i, y_i)}{2} h^2 + \dots + \frac{f^{(n-1)}(x_i, y_i)}{n!} h^n + O(h^{n+1}) \quad (3.1.6.)$$

where  $O(h^{n+1})$  specifies that the local truncation error is proportional to the step size raised to the  $(n+1)$ th power. For example, for the second order method, the local truncation error is  $O(h^2)$ . Further error analysis, including explanation of the terms mentioned above is presented at the end of this chapter.

From the equation (3.1.6.) it is further derived:

i) first order method:

$$y_{i+1} = y_i + f(x_i, y_i)h \quad (3.1.6.)'$$

where the first derivative  $\phi = f(x_i, y_i)$  provides a direct estimate of the slope at  $x_i$ .

Graphically:

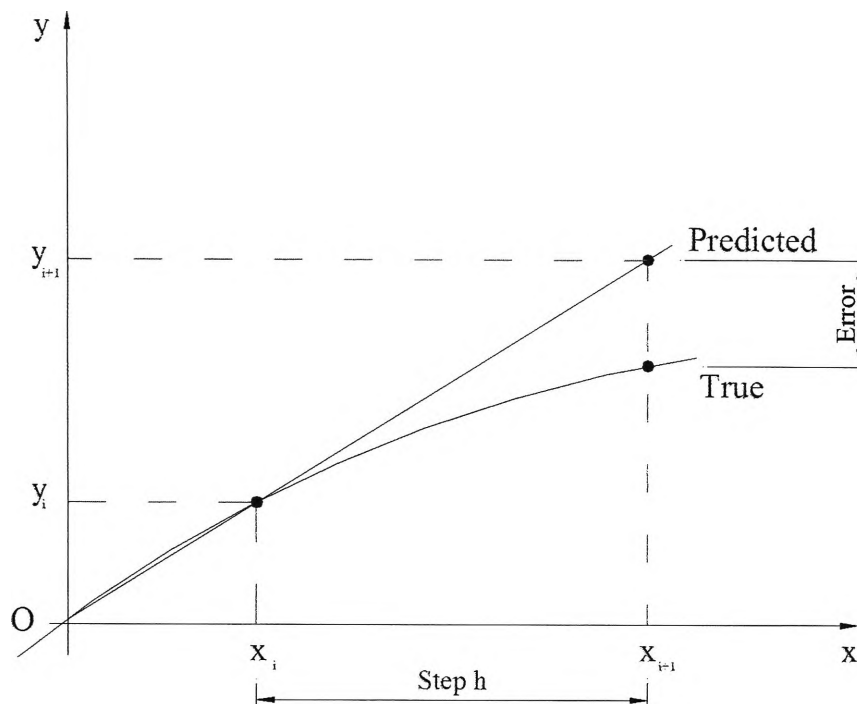


Fig. 3.2. Uncorrected prediction for the function value

From the above equation and the graph it is seen that, new value of  $y$  is predicted using the slope  $\phi$  to extrapolate linearly over the step size  $h$ . It is obvious, this approach will produce a relatively large error.

Runge Kutta methods are based on the assumptions that there are existing values of  $a_i$  and  $k_i$ , already defined, which could improve the accuracy of the solution. It is required to determine constants  $a_i$  and  $k_i$  as the functions of  $b_i$  and  $C_i$  to get as close as possible to the Taylor's series, up to some specified number of terms and without further differentiation.

To achieve this, the biggest difficulties are due to excessive algebra and there are too many possible solutions. According to this, the solution that will lead to the easier further calculations are selected.

This procedure starts with the Taylor's Theorem For Many Variables [31].

If  $f=f(x,y,z)$  and all its partial derivatives are continuous through order  $n$  in neighbourhood  $N$ , then:

$$f(x+H) = f(x) + (H \cdot \nabla)f(x) + \frac{1}{2!}(H \cdot \nabla)^2 f(x) + \frac{1}{3!}(H \cdot \nabla)^3 f(x) + \dots + E_n \quad (3.1.7.)$$

where

$$E_n = \frac{1}{n!}(H \cdot \nabla)^n f(x) \quad (3.1.8.)$$

$$x=(x,y,z, \dots)$$

$$H=(\Delta x, \Delta y, \Delta z, \dots)=(h,k,l, \dots)$$

$$\nabla=(\partial/\partial x, \partial/\partial y, \partial/\partial z, \dots)$$

Proof of this theorem is based on Taylor's theorem for one variable.

### The P Theorem

One-variable Taylor's series, presented below, need to be compared to the terms of partial derivatives in Runge Kutta formulas. If the assumption is adopted that the partial derivatives exist and are continuous, it follows:

$$y' = \frac{dy}{dx} = f(x, y) \quad (3.1.9.)$$

$$dy' = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \quad (3.1.10.)$$

$$\therefore y'' = \frac{dy'}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx} \quad (3.1.11.)$$

$$y'' = \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) f = Pf \quad (3.1.12.)$$

$$\frac{dy''}{dx} = \frac{\partial y''}{\partial x} + \frac{\partial y''}{\partial y} \frac{dy}{dx} \quad (3.1.13.)$$

$$y''' = \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) y'' \quad (3.1.14.)$$

By substituting (3.1.12.) into (3.1.14.)

$$y''' = \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) f \quad (3.1.15.)$$

which could be represented as

$$y''' = P^2 f \quad (3.1.15.)$$

The final expression is easily generated by mathematical induction, therefore

$$y^{(n)} = P^{n-1} f \quad (3.1.16.)$$

where

$$P = \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) \quad (3.1.17.)$$

According to the previous theorem

$$y_{i+1} = y_i + hy'_i = y_i + k_1 \quad (3.1.18.)$$

$$\begin{aligned} y_{i+1} = y_i + hf + \frac{1}{2!} h^2 Q + \frac{1}{3!} h^3 (Q_2 + f_y Q) \\ + \frac{1}{4!} [Q_3 + f_y Q_2 + f_y^2 Q + 3(f_{xy} + ff_{yy}) Q] + O(h^5) \end{aligned} \quad (3.1.19.)$$

where:

$$Q = f_x + ff_y \quad (3.1.20.)$$

$$Q_2 = f_{xx} + 2ff_{xy} + f^2 f_{yy} \quad (3.1.21.)$$

$$Q_3 = f_{xxx} + 3ff_{xxy} + 3f^2 f_{xyy} + f^3 f_{yyy} \quad (3.1.22.)$$

$$Q_4 = f_{xxxx} + 4ff_{xxx} + 6f^2 f_{xxy} + 4f^3 f_{xyy} + f^4 f_{yyy} \quad (3.1.23.)$$

and from binomial expansion:

$$y' = f \quad (3.1.24.)$$

$$y'' = Q \quad (3.1.25.)$$

$$y''' = Q_2 + f_y Q \quad (3.1.26.)$$

$$y^{IV} = Q_3 + f_y Q_2 + f_y^2 Q + 3(f_{xy} + ff_{yy}) Q \quad (3.1.27.)$$

From

$$y_{n+1} = y_n + hy'_n + \frac{h^2}{2!} y''_n + \dots + \frac{h^m}{m!} y_n^{(m)} + R \quad (3.1.28.)$$

$$y_{n+1} = y_n + a_1 k_1 + a_2 k_2 + a_3 k_3 + a_4 k_4 \dots \quad (3.1.29.)$$

$$k_1 = hf_n \quad (3.1.30.)$$

$$k_2 = hf(x_n + b_2 h, y_n + b_2 k_1) \quad (3.1.31.)$$

$$k_3 = hf(x_n + b_3 h, y_n + b_3 k_2) \quad (3.1.32.)$$

$$k_4 = hf(x_n + b_4 h, y_n + b_4 k_3) \quad (3.1.33.)$$

This has to agree with Taylor series up to  $h^4$  terms. Based on Taylor's theorem these terms are expanded further as:

$$k_2 = h \left[ f_n + \left( b_2 h \frac{\partial}{\partial x} + b_2 k_1 \frac{\partial}{\partial y} \right) f_n + \frac{1}{2!} \left( b_2 h \frac{\partial}{\partial x} + b_2 k_1 \frac{\partial}{\partial y} \right)^2 f_n + \frac{1}{3!} \left( b_2 h \frac{\partial}{\partial x} + b_2 k_1 \frac{\partial}{\partial y} \right)^3 f_n + \dots \right] \quad (3.1.34.)$$

$$k_3 = h \left[ f_n + \left( b_3 h \frac{\partial}{\partial x} + b_3 k_2 \frac{\partial}{\partial y} \right) f_n + \frac{1}{2!} \left( b_3 h \frac{\partial}{\partial x} + b_3 k_2 \frac{\partial}{\partial y} \right)^2 f_n + \frac{1}{3!} \left( b_3 h \frac{\partial}{\partial x} + b_3 k_2 \frac{\partial}{\partial y} \right)^3 f_n + \dots \right] \quad (3.1.35.)$$

$$k_4 = h \left[ f_n + \left( b_4 h \frac{\partial}{\partial x} + b_4 k_3 \frac{\partial}{\partial y} \right) f_n + \frac{1}{2!} \left( b_4 h \frac{\partial}{\partial x} + b_4 k_3 \frac{\partial}{\partial y} \right)^2 f_n + \frac{1}{3!} \left( b_4 h \frac{\partial}{\partial x} + b_4 k_3 \frac{\partial}{\partial y} \right)^3 f_n + \dots \right] \quad (3.1.36.)$$

The last expressions are further transformed, based on the P theorem, into:

$$k_1 = h f_n \quad (3.1.37.)$$

$$\begin{aligned} k_2 &= h f_n + b_2 h^2 \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right) f_n + \frac{1}{2!} b_2^2 h^3 \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right)^2 f_n + \frac{1}{3!} b_2^3 h^4 \left( \frac{\partial}{\partial x} + f \frac{\partial}{\partial y} \right)^3 f_n \\ &= h f_n + b_2 h^2 P f_n + \frac{1}{2!} b_2^2 h^3 P^2 f_n + \frac{1}{3!} b_2^3 h^4 P^3 f_n \\ &= h f_n + b_2 h^2 y'' + \frac{1}{2!} b_2^2 h^3 y''' + \frac{1}{3!} b_2^3 h^4 y^{IV} \\ &= h f_n + b_2 h^2 Q + \frac{1}{2!} b_2^2 h^3 (Q_2 + f_y Q) \\ &\quad + \frac{1}{3!} b_2^3 h^4 (Q_3 + f_y Q_2 + f_y^2 Q + 3(f_{xy} + f f_{yy}) Q) \quad (3.1.38.) \end{aligned}$$

In a similar manner:

$$\begin{aligned} k_3 &= h f_n + b_3 h^2 Q + \frac{1}{2!} h^3 (b_3^2 Q_2 + 2b_2 b_3 f_y Q) \\ &\quad + \frac{1}{3!} h^4 (b_3^3 Q_3 + 3b_3^2 b_2 f_y Q_2 + 6b_3 b_2^2 [f_{xy} + f f_{yy}] Q) + \dots \quad (3.1.39.) \end{aligned}$$

and:

$$\begin{aligned} k_4 &= h f_n + b_4 h^2 Q + \frac{1}{2!} (b_4 Q_2 + 2b_3 b_4 f_y Q) \\ &\quad + \frac{1}{3!} (b_4 Q_3 + \dots) + \dots \quad (3.1.40.) \end{aligned}$$

Last expressions are compared with the equation (3.1.19.):

$$\begin{aligned}
 a_1 + a_2 + a_3 + a_4 &= 1 \\
 a_2 b_2 + a_3 b_3 + a_4 b_4 &= \frac{1}{2} \\
 a_2 b_2^2 + a_3 b_3^2 + a_4 b_4^2 &= \frac{1}{3} \\
 a_2 b_2^3 + a_3 b_3^3 + a_4 b_4^3 &= \frac{1}{4} \\
 a_3 b_2 b_3 + a_4 b_3 b_4 &= \frac{1}{6} \\
 a_3 b_2 (b_3)^2 + a_4 b_3 (b_4)^2 &= \frac{1}{8} \\
 a_3 (b_2)^2 b_3 + a_4 (b_3)^2 b_4 &= \frac{1}{12} \\
 a_4 b_2 b_3 b_4 &= \frac{1}{24}
 \end{aligned} \tag{3.1.37.}$$

The system has one degree of freedom, therefore it will be chosen  $a_1=1$

Final result is:

$$\tilde{y}(x_n + h) = y(x_n) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3.1.38.}$$

where:

$$k_1 = hf(x_n, y_n) \tag{3.1.39.}$$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right) \tag{3.1.40.}$$

$$k_3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right) \tag{3.1.41.}$$

$$k_4 = hf(x_n + h, y_n + k_3) \tag{3.1.42.}$$

To apply this method on the equations of a satellite motion, it is further modified for solving the system of two differential equations in  $x$  and  $y$  direction.

Original equations to be solved are again noted here:

$$\frac{d^2 x}{dt^2} = -GM2 \frac{x}{(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2})^3}$$

$$\frac{d^2 y}{dt^2} = -GM2 \frac{y}{(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2})^3}$$

The final set of equations, prepared for computer application is:

$$x = x + h \times V_x$$

$$y = y + h \times V_y$$

$$k_1 = hf(t, x, y)$$

$$l_1 = hg(t, x, y)$$

$$k_2 = hf(t + 0.5h, x + 0.5k_1, y + 0.5l_1)$$

$$l_2 = hg(t + 0.5h, x + 0.5k_1, y + 0.5l_1)$$

$$k_3 = hf(t + 0.5h, x + 0.5k_2, y + 0.5l_2)$$

$$l_3 = hg(t + 0.5h, x + 0.5k_2, y + 0.5l_2)$$

$$k_4 = hf(t + 0.5h, x + 0.5k_3, y + 0.5l_3)$$

$$l_4 = hg(t + 0.5h, x + 0.5k_3, y + 0.5l_3)$$

$$V_x = V_x + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$V_y = V_y + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$t = t + h$$

$$f = -GM \frac{x}{(\sqrt{x^2 + y^2})^3}$$

$$g = -GM \frac{y}{(\sqrt{x^2 + y^2})^3}$$

To apply presented equations next initial values are required:  $x_o, y_o, h, V_{x_o}, V_{y_o}$  and  $N$ .

### 3.2. Linear Interpolation Method

To apply the theory derived for the air drag effects it is necessary to produce some kind of a model for the upper Earth's atmosphere. It had been already discussed why numerical approach based on the true data was selected as the method for solving this problem.

The next problem is to select the method that will describe observed model in the most appropriate way. Possible methods will approach the same problem from a different aspect, and will produce a different kind of error. This statement is illustrated in the figure below:

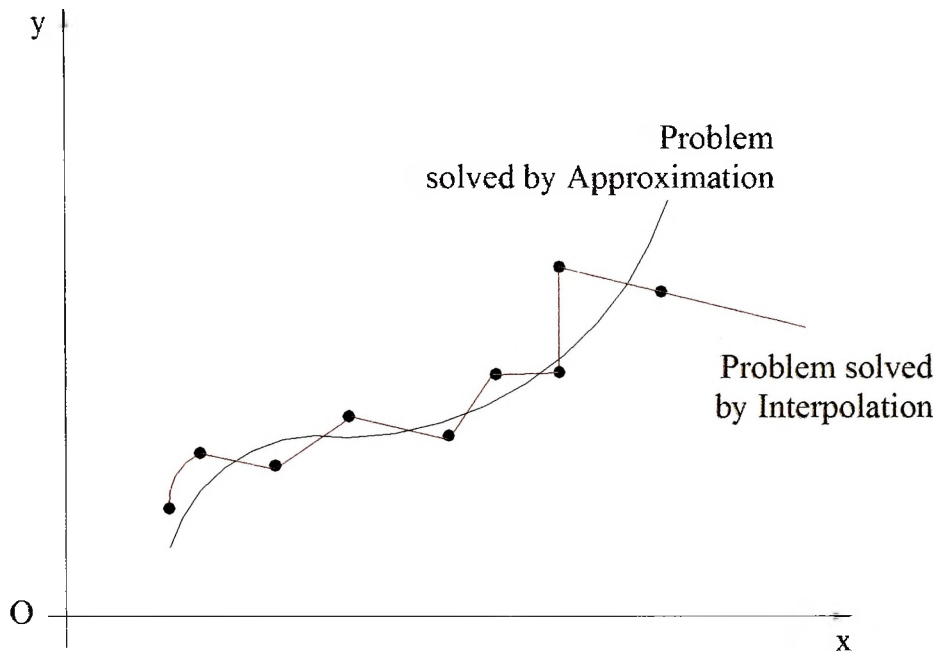


Fig. 3.3. The results of the two different methods applied to a same problem: to describe the functional dependency of  $x$  to  $y$ , based on the table values for a points given

The method selected in this case is the Interpolation, because of the next properties, that are considered as the advantages for this application:

- The variation of the data values in one direction (density) oscillates by very small amount for the first eleven points. At the same time for these points another dimension (altitude) also oscillates for a relatively small amount. The error produced by connecting actual points will not be large, and would be of about the same order as if the method of approximation is applied.
- For the points from 11 to 26, the variation in altitude is that large, even the approximation would give almost linear dependency.
- The properties of the upper atmosphere are not clearly defined yet theoretically, therefore there is no point in deviating from the measured value with the aim to approximate the values in between.



- Linear interpolation is far simpler than approximation method, therefore is much faster for computer application
- If more accurate measurements are applied, it is very easy to improve this method by Lagrange polynomial, also explained in this section.

Approximation is the numerical method that provides functional dependency of the variables given in the table form. As it is seen from Fig. 3.3. this method approximates values that are given by the table – passes between the values finding the best ‘fit’ line. There are given table values for  $o$  and  $p$ , as:

$O$	$O_0$	$O_1$	$O_2$	$O_3$	...	$O_n$
$P$	$P_0$	$P_1$	$P_2$	$P_3$	...	$P_n$

Where  $p=f(o)$ .

If the problem is based on the function value of the argument  $O$ , for  $O$  between  $O_1$  and  $O_2$ , the simplest way to determine required function value is to perform so-known linear interpolation. By this method, function  $f(o)$  on the region  $O_1$  to  $O_2$  is represented by the first order polynomial:

$$f(o) \approx P_1(o) = a_0 + a_1 o \quad (x_1 \leq x \leq x_2) \quad (3.2.1.)$$

The coefficients  $a_0$  and  $a_1$  are determined from the requirement that the polynomial has to pass through the points  $M_1(o_1, p_1)$  and  $M_2(o_2, p_2)$ :

$$\begin{aligned} p_1 &= a_0 + a_1 o_1, \\ p_2 &= a_0 + a_1 o_2 \end{aligned} \quad (3.2.2.)$$

The above equations are solved for  $a_0$  and  $a_1$ :

$$a_0 = p_1 - o_1(p_2 - p_1)/(o_2 - o_1) \quad \text{and} \quad a_1 = (p_2 - p_1)/(o_2 - o_1) \quad (3.2.3.)$$

Therefore:

$$p = P_1(x) = p_1 + (p_2 - p_1)(o - o_1)/(o_2 - o_1) \quad (3.2.4.)$$

The expression 3.2.4. is actually an equation of the straight line that passes through the points  $M_1$  and  $M_2$ .

If the solution is applied to the problem altitude – density, where  $p=\rho$  (density) and  $o=h$  (altitude), then:

$$\rho=P_1(h)=\rho_1+(\rho_2-\rho_1)(h-h_1)/(h_2-h_1) \quad (3.2.4.)$$

The model could be improved if instead of linear, higher order interpolation is applied. It is possible to describe the whole table of given values by only one polynomial. High order polynomial interpolation gives very large oscillations between data points, so they are dangerous to use. This implies that, for  $n$  table values, the defined polynomial would be of  $n$ th order, with  $n+1$  linear equations needed to solve for determining coefficients  $a_0$  to  $a_1$ . Solution for this system is possible, derived by one of numerical methods, but the matrix used in the calculation would be constantly crowded and will significantly reduce the available memory of the computer. The solution to this problem is to use Lagrange polynomial instead of the ordinary one. Lagrange interpolating polynomial, applied on the values given in table 2.8.4.2. has a form:

$$P_n(h)=L_0(h)\rho_0+L_1(h)\rho_1+\dots+L_{26}(h)\rho_{26} \quad (3.2.5.)$$

where

$$L_i(h)=\frac{(h-h_0)(h-h_1)\dots(h-h_{i-1})(h-h_{i+1})\dots(h-h_{26})}{(h_i-h_0)(h_i-h_1)\dots(h_i-h_{i-1})(h_i-h_{i+1})\dots(h_i-h_{26})} \quad (3.2.6.)$$

The most important characteristic of this polynomial is that, it is passing through all known points.

At this stage of model development, it is more important to analyse the error due to method applied for solving differential equations of motion. Air density model could be improved when more details about upper atmosphere properties are revealed.

### 3.3. Error Analysis

The numerical method applied for solving ordinary second order differential equations produces two types of error:

1. Truncation or discretization errors caused by the characteristics of the method employed to approximate required functions.
2. Round-off errors, caused by the limited numbers of significant digits that are retained by a computer.

First type of error is composed of two parts.

- i) local truncation error results from method applied over a single step.
- ii) Propagated truncation error results from the approximations produced during the previous step.

Global truncation error is actually the sum of the two.

Local error is directly related to the truncation of the Taylor's series, which is discussed earlier. The conclusion regarding error analysis in this case is:

1. The Taylor series only provide an estimate of the local truncation error. Propagated and hence global truncation errors are not related to this approach. In the most cases, local error does not exceed 33% of total global error. To determine the exact global error, real solution has to be known [50].
2. Approach used here is to analytically determine period of a single orbit and compare it to the result obtained numerically.

As an example Kepler's model orbit is used (because it is possible to analytically determine the period of such an orbit, and because at this moment, the only subject of

concern is the accuracy of the numerical method which is in the first plane if perturbations are ignored). Orbit characteristics are:

Perigee 4000km

Apogee 8000km

From Kepler's third law:

$$T^2 = \frac{4\pi^2}{GM_E} a^3 \quad (3.3.1.)$$

where  $a$  – semimajor axis:

$$a = \frac{r_{\max} + r_{\min}}{2} \quad (3.3.2.)$$

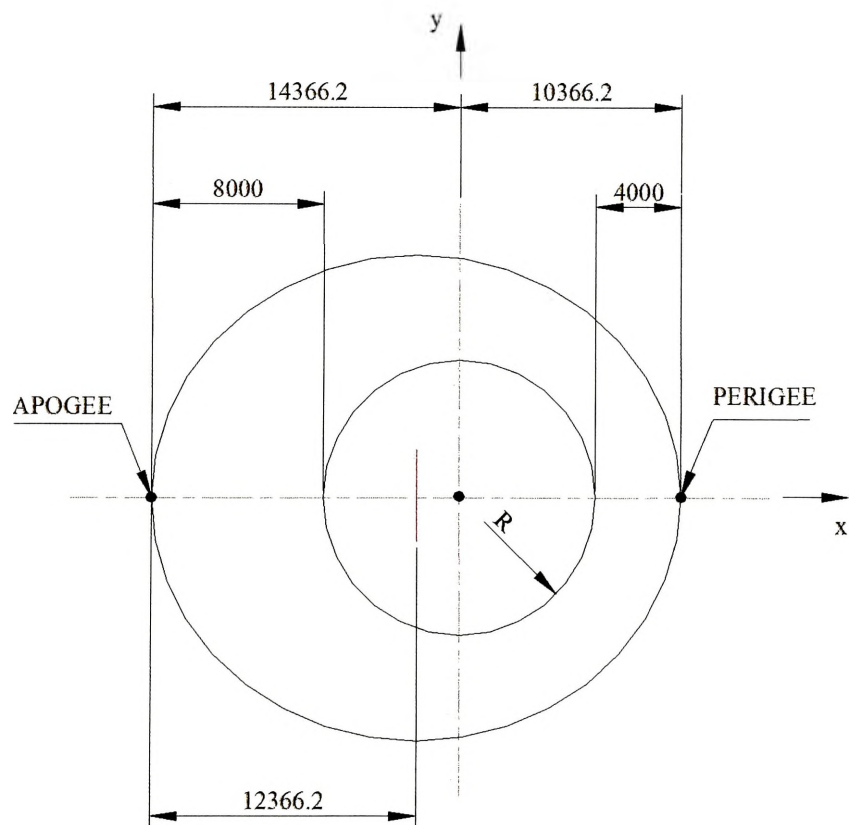


Fig. 3.3.1. Elliptical Orbit 4000/8000km, all dimensions in km

From the Fig. 3.3.1.

$$a = 12\,366.2 \text{ km}$$

$$T = \sqrt{\frac{4\pi^2}{399059.852} 12366.2^3} = 13677.7685 \text{ sec} \approx 3.8 \text{ hours} \quad (3.3.3.)$$

For an analytical solution, after one orbit, the value of  $x$  is equal to the initial value at perigee:  $x_p = 10366.2 \text{ km}$ . The period of the orbit  $T$  at the value closest to the  $x_p$  will be determined, and the relative error calculated according to:

$$\text{relative Error}(\%) = 100 \frac{|nT_{\text{analyt.}} - T_{\text{compt. } n}|}{nT_{\text{analyt.}}} \quad (3.3.4.)$$

The computed value of  $T$ , for the step of integration of  $5 \text{ sec}$ , single precision was:

Orbit 1:

$$x = 10366.2 \text{ km}, y = 43.71 \text{ km}$$

$$T = 2716 * 5 = 13580 \text{ s}, \text{ Error} = 0.716\%$$

Orbit 2:

$$x = 10366.17 \text{ km}, y = 54.10 \text{ km}$$

$$T = 5431 * 5 = 27155 \text{ s}, \text{ Error} = 0.735\%$$

Orbit 3:

$$x = 10366.22 \text{ km}, y = 31.04 \text{ km}$$

$$T = 8144 * 5 = 40720 \text{ s}, \text{ Error} = 0.765\%$$

Orbit 4:

$$x=10366.20km, y=7.78 km$$

$$T=10859*5=54295s, Error=0.762\%$$

Orbit 5:

$$x=10366.10km, y=18.64 km$$

$$T=13574*5=67870s, Error=0.760\%$$

Orbit 6:

$$x=10366.10km, y=29.30 km$$

$$T=16289*5=81445s, Error=0.759\%$$

Orbit 7:

$$x=10366.10km, y=6.56km,$$

$$T=19003*5=95015s, Error=0.763\%$$

Orbit 8:

$$x=10366.10km, y=16.97 km$$

$$T=21718*5=108590s, Error=0.762\%$$

Orbit 9:

$$x=10366.10km, y=27.70 km$$

$$T=24433*5=122165s, Error=0.761\%$$

Orbit 10:

$$x=10366.10km, y=4.87 km,$$

$$T=27147*5=135735s, Error=0.764\%$$

Errors are presented graphically in the Fig. 3.3.2.

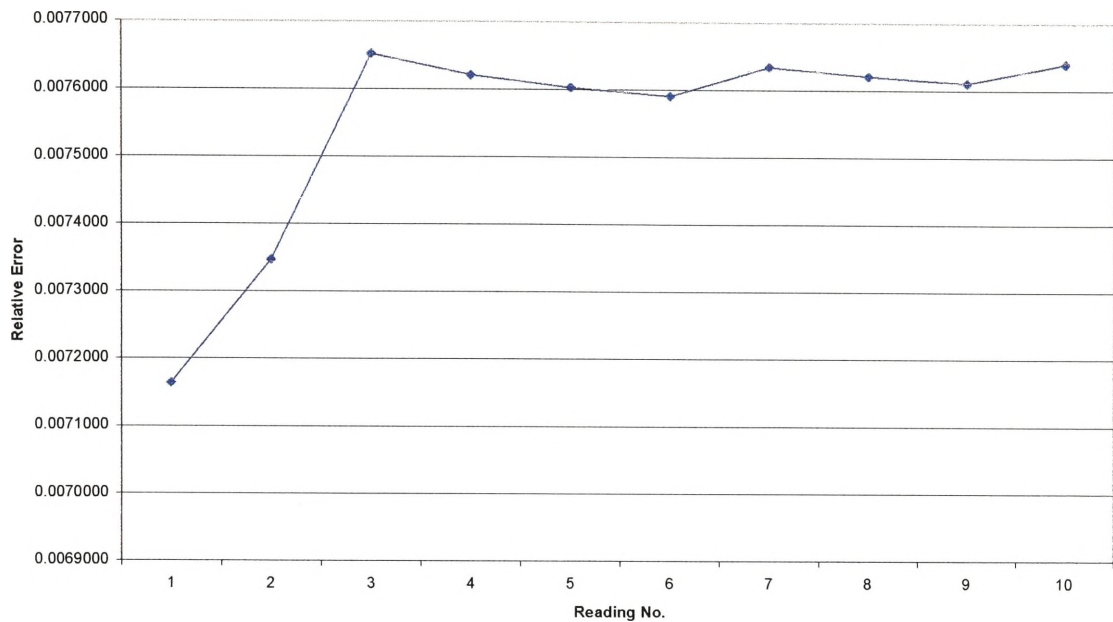


Fig. 3.3.2. The error in 4000/8000km Orbit, 5sec step, due to the numerical method applied for solving differential equations of motion

Simple computer application in Excel, named NumError is produced to calculate the relative error observed above, requiring the value of  $T$  calculated for a particular Orbit No.

From the analysis of the local truncation error it is seen that it is proportional to the fourth order of the integration step size. This suggests that, for higher order Runge-Kutta methods local error is decreased by the decrease in the step size.

This fact is applied on the problem of satellite motion in the orbit already observed for 5sec step with apogee to perigee ratio 8000/4000. Step sizes that will be observed are 2sec, 3sec and 8sec. Relative error is calculated for all cases and compared.

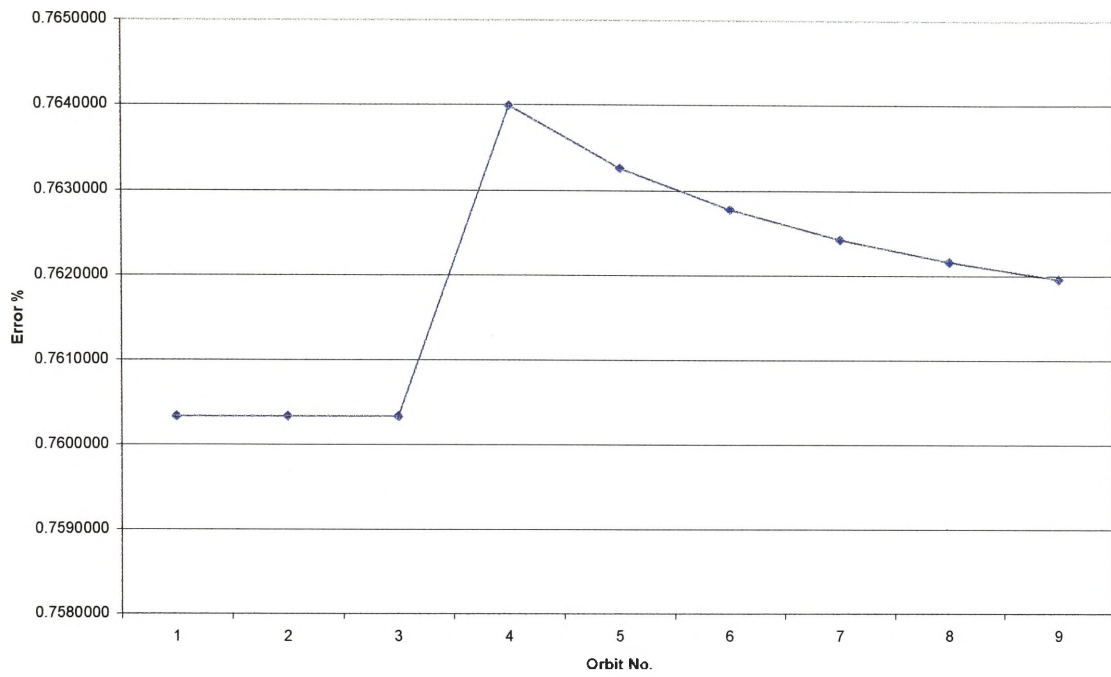


Fig. 3.3.3. The error(%) in 4000/8000km Orbit, 2sec step, due to the numerical method applied for solving differential equations of motion

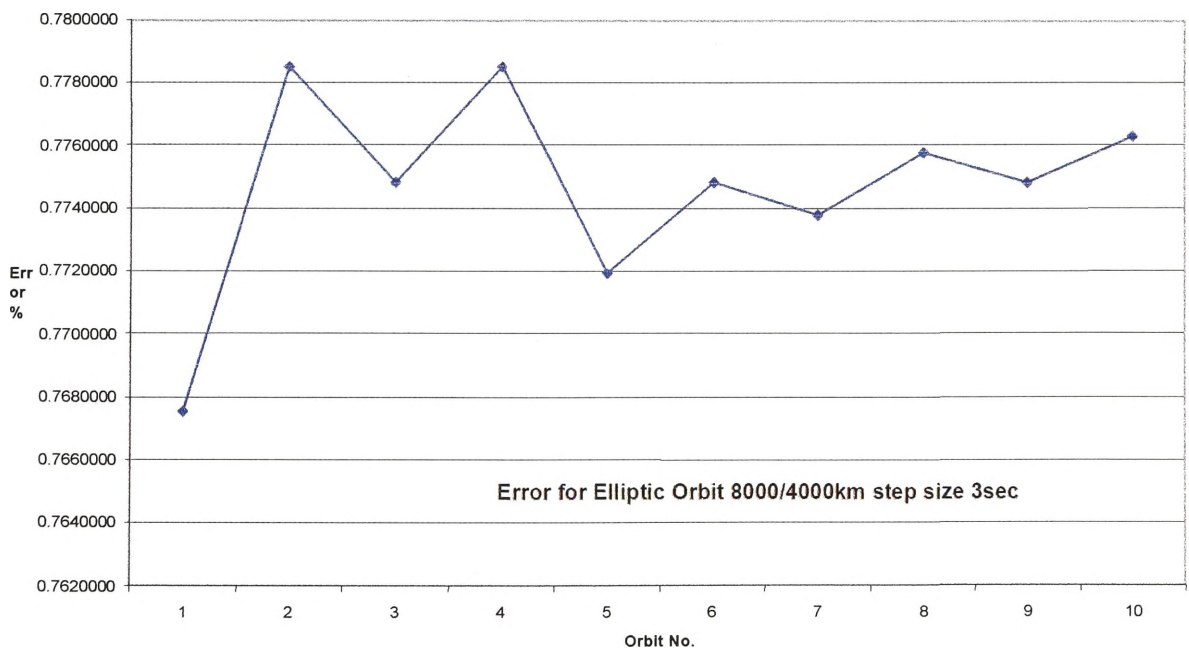


Fig. 3.3.4. The error(%) in 4000/8000km Orbit, 3sec step, due to the numerical method applied for solving differential equations of motion



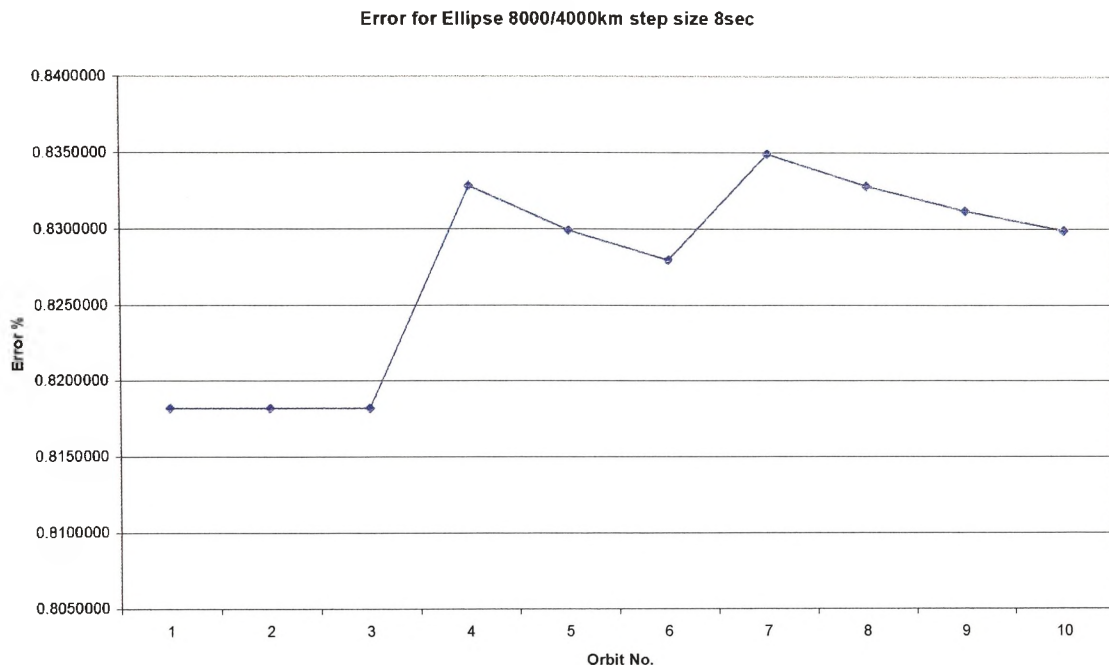


Fig. 3.3.5. . The error(%) in 4000/8000km Orbit, 8sec step, due to the numerical method applied for solving differential equations of motion

Program *Kepler2precfinalGrTr.f90* was modified in the sense to compute results with the double precision. The error determined for the first 10 orbits with the same orbital elements as in the previous case (Perigee altitude=4000km, apogee altitude=8000km,  $H = 5\text{sec}$ ,  $RAAN = 120^\circ$ ,  $i = 60^\circ$ ,  $AOP = 90^\circ$ ) was:

Orbit 1:

$$x=10366.20\text{km}, y=43.58\text{km}$$

$$T=2716*5=13580\text{s}, \text{Error}=0.716\%$$

Orbit 2:

$$x=10366.20\text{km}, y=54.92\text{km}$$

$$T=5431*5=27155\text{s}, \text{Error}=0.735\%$$

Orbit 3:

$$x=10366.20\text{km}, y=32.32\text{km}$$

$$T=8145*5=40725\text{s}, \text{Error}=0.757\%$$

Orbit 4:

$$x=10366.20\text{km}, y=43.09 \text{ km}$$

$$T=10860*5=54300\text{s}, \text{Error}=0.751\%$$

Orbit 5:

$$x=10366.20\text{km}, y=53.87 \text{ km}$$

$$T=13575*5=67875\text{s}, \text{Error}=0.751\%$$

Orbit 6:

$$x=10366.20\text{km}, y=31.26 \text{ km}$$

$$T=16289*5=81445\text{s}, \text{Error}=0.758\%$$

Orbit 7:

$$x=10366.20\text{km}, y=42.04\text{km},$$

$$T=19004*5=95020\text{s}, \text{Error}=0.757\%$$

Orbit 8:

$$x=10366.20\text{km}, y=52.81 \text{ km}$$

$$T=21719*5=108595\text{s}, \text{Error}=0.756\%$$

Orbit 9:

$$x=10366.20\text{km}, y=30.21 \text{ km}$$

$$T=24433*5=122165\text{s}, \text{Error}=0.760\%$$

Orbit 10:

$$x=10366.20\text{km}, y=40.98 \text{ km},$$

$$T=27148*5=135740\text{s}, \text{Error}=0.759\%$$

Conclusion:

The decrease in the step size improves accuracy of the method. During a different operation performed on the results, it was noted that important element is round-off error. For the smaller step sizes, due to the extremely fine scale of next function value generation, even small round-off error will cause large deviations from the true value.

Another problem that could arise is due to large difference in values of computed variables. If one of the values becomes extremely small compared to the other configuring values, its accuracy will be significantly reduced. This is particularly a case with the Three Body Problem. The solution to this problem is known as a step size control. The biggest purpose of estimating the error is actually establishing the limits when to adjust the step size.

According to the above figures, the step of integration will be increased if the error is too small, and decreased if the error is too large.

For example, in the case of Three Body Problem, it is enough to observe only radius vectors that describe the positions of the bodies in the system. Adjustable step size is defined as [21]:

$$h = \frac{h_{scale}}{r_{12}^{-2} + r_{23}^{-2} + r_{13}^{-2}} \quad (3.3.5.)$$

where  $h_{scale}$  is the fixed step value selected at the beginning of the program, and

$$\begin{aligned} r_{12} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ r_{23} &= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \\ r_{13} &= \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \end{aligned} \quad (3.3.6)$$

The accuracy of the computed results was also increased by increasing the single precision to double precision for the program calculations. The average improvement was for about 0.03%, for first ten orbits, step 5 sec. For integration steps smaller than 5 sec the improvement in accuracy would increase further.

It was particularly interesting to make another approach to results testing, this time performed with respect to another software program. The program named WINORBIT version 3.6, Satellite Orbital Prediction and Display, author C.D. Gregory was used as a reference to which SATELIGHT was compared [63].

It was assumed the analytical solution is at 0 at the error scale. WINORBIT program gives solution for the particular satellites, therefore one of them will be chosen, and its orbit elements will be used for the orbit generation by SATELIGHT.

Selected satellite is **METEOR 3-5**.

**About Satellite:** Weather Satellites Meteor series are LEO, polar-orbiting, satellites with cameras for cloud pictures in the visible (VIS) or infrared (IR) bands. Some of the satellites from this group are APT (automatic picture transmission) birds, and can be received with inexpensive home equipment. Not all satellites are operational at the same time.

Meteor 3/5 - APT on 137.85 MHz FM. [25].

Kepler's Elements: Epoch = 6/01/2000 05:13:15

Mean Anomaly = 197.435

Mean Motion = 13.16889074

SemiMajor Axis = 7574.72 km

Alt. Perigee = 1197.55 km

Alt. Apogee = 1217.88 km

Inclination = 82.5541

Eccentricity = 0.00134

Argument of Perigee = 162.723

R.A. of Node = 123.6893

Based on the above values the analytical orbit period would be:

$$T = \sqrt{\frac{4\pi^2}{399059.85} \left( \frac{7563.75 + 7584.08}{2} \right)^3}$$

$$= 6556.04 \text{ sec}$$

Period determined by WINORBIT, measured at the middles of the intervals during perigee pass (as the altitude is determined as the whole number) is

$T=6280\text{sec}$ , Relative Error= 4.21%

Single orbit period determined by SATELIGHT for 5sec step:

$T=6540\text{sec}$ , Relative Error=0.24%

The number of steps for which single orbit is completed, for a particular step size is:

Step Size 1sec:  $N=6556$ , Relative Error=0.381%

Step Size 2sec:  $N=3278$ , Relative Error=0.274%

Step Size 5sec:  $N=1331$ , Relative Error=0.397%

Step Size 10sec:  $N=656$ , Relative Error=0.609%

Step Size 15sec:  $N=437$ , Relative Error=1.144%

The above results prove the previously derived conclusion as valid. Final statements that could be still derived from the above observations are:

1. SATELIGHT's accuracy increases for the step sizes 2 to 5 seconds, compared to the analytical solution
2. Lower eccentricity of the orbit, higher accuracy of the results, directly related to the discussion of the step size control.

Discussion regarding input conditions and software description is presented in the next Chapter.

## **CHAPTER 4**

### **Initial Conditions Definition**

## Chapter 4: Initial Conditions Definition and SATELIGHT Software Analysis

### 4.1. Initial conditions definition

The most general case of motion, according to the Chapter 2, is presented by the system of the three, second order, differential equations.

First integration of this coupled system results in solution that contains three constants. Second integration would produce next three constants, therefore there are six constants in total.

These constants are, for a particular time – epoch time  $t_o$ :

$X_o, Y_o, Z_o, dx/dt, dy/dt, dz/dt$ . Described by the words these are components of the position and velocity vectors evaluated at  $t_o$ . These elements are not directly observable and are the subject of preliminary orbit determination scheme, presented later.

The inertial system, placed at the centre of the Earth, containing equatorial plane and already defined at Chapter 2, is used at the final stage of satellite position determination. The initial system applied is placed in Earth's centre with its  $xy$  plane containing orbit plane, therefore inclined to the equatorial plane for the angle  $i$ .

The unperturbed orbit of the satellite lies in the plane, which also contains  $xy$  plane of the inertial system. For elliptical orbit  $x$  axis passes through the orbit perigee. At this

point, satellite has  $x$  coordinate equal to the perigee altitude determined from the mission requirement, and  $y$  coordinate equal to zero. Velocity at any point is a tangent on the trajectory at that point, in this case, because  $y$  position coordinate is equal to zero, total velocity will be projected in only one direction. The velocity at orbit perigee is normal to the  $x$  axis (more precisely to the radius vector of the perigee) and therefore its total intensity is projected to the  $y$  axis. Refer to the Fig. 4.1.1.

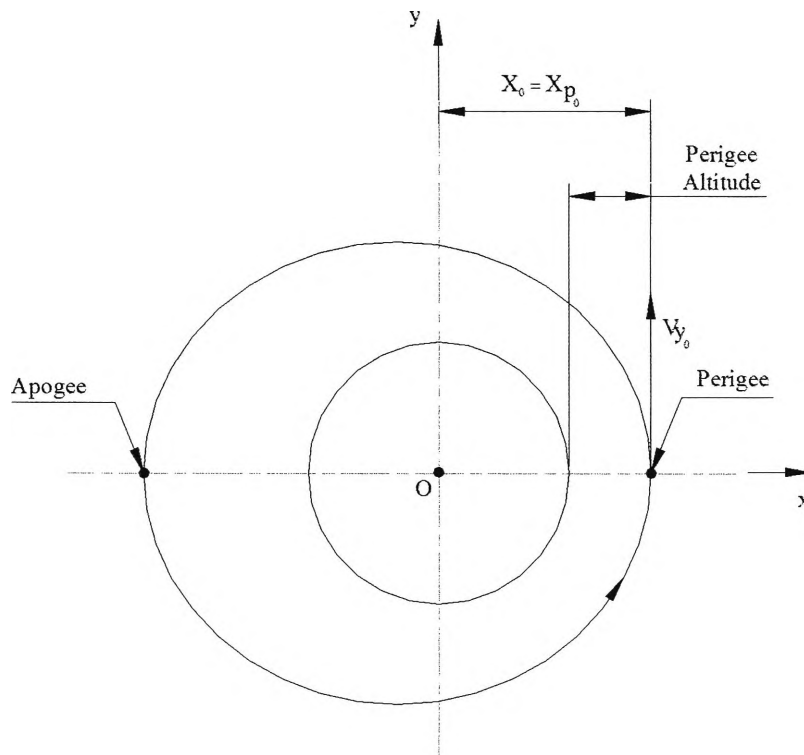


Fig. 4.1. Initial Conditions for elliptical Ellipse

Program SATELIGHT Orbit requests next elements as the input:

$T, X_o, Y_o, H, N, VX_o, VY_o.$

$T$  is the epoch time at which orbit starts to generate. This element is chosen according to the user requirements. There is no restraint that determines the value of  $T$ , apart from that other elements have to have its value determined properly. For the first orbit, if the calculation starts from the perigee (it is the initial point at which mission operations start, for example)  $T$  is equal to zero, and by the completion of the first orbit it is equal to the period of orbit. It is calculated for each step of integration in the increments equal to the time step  $H$ . The program will generate orbit from any point on the trajectory, as long as all requested initial elements are provided. This feature is very useful for constructing inter-planetary trajectories, as they are composed of different orbit types.  $T$



should be given in seconds and its final result represents the amount of time that satellite spent in the orbit.

$X_o$  is the value equal to Earth radius subtracted to the perigee altitude. This value should be provided in km.

$Y_o$  is the value of satellite  $y$  coordinate, and in this case is equal to 0.

$H$  is the size of the integration step in seconds and could be chosen arbitrarily (it was shown the most accurate results are achieved for steps 2-5sec; bigger step reduces accuracy but requires less calculation time and less memory).

$N$  is the number of steps for which an orbit is going to be generated. It is based on user's requirement and the capabilities of the applied computer. If single orbit is required, the number of steps is equal to the period divided by a step size:

$$N = T_p / H \quad (4.1.)$$

For circular orbit:

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_E}} \quad (4.2.)$$

where  $R$  is the radius of circular orbit.

For elliptical orbit:

$$T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} \quad (4.3.)$$

$$a = \frac{r_{\min} + r_{\max}}{2} \quad (4.4.)$$

$r_{\min}$  = radius vector at perigee

$r_{\max}$  = radius at apogee

For parabolic and hyperbolic orbits there is no same concept of a period, as these are not closed orbits.

$VX_0$  for this point (perigee, with  $x$  axis passing through it) is equal to zero, as the total velocity, perpendicular to the radius vector, is projected to  $y$  axis.

$VY_0$  is different for different orbit types, and this is actually the value that will determine the shape of the orbit. It is determined from the equations derived at Chapter 2:

Circle:

$$v = \sqrt{\frac{GM_E}{R}} \quad (4.5.)$$

Ellipse:

$$v_{perigee} = \sqrt{\frac{GM_E}{a} \left( \frac{r_{max}}{r_{min}} \right)} \quad (4.6.)$$

$$a = \frac{r_{min} + r_{max}}{2}$$

Parabola:

$$v_{perigee} = \sqrt{\frac{2GM_E}{r_{perigee}}} \quad (4.7.)$$

Hyperbola:

$$v_{perigee} > \sqrt{\frac{2GM_E}{r_{perigee}}} \quad (4.8.)$$

The velocity is in km/s, input and output.

Results could be transferred to Excel – the program requires the name of the file before than major loop is performed. Name of the file has to satisfy basic FORTRAN requirements and could contain any extension including .xls. Excel provides facility for

graph generation, based on the stored results. The output of the program are values:  $T[s]$ ,  $x[km]$ ,  $y[km]$ ,  $V_x[km/s]$ ,  $V_y[km/s]$ . Program Orbit is presented in Appendix 4.1.

Another possibility is to apply modified program version, which arranges satellite coordinates in LISP accepted format (presented here), so the generated orbit could be inserted and viewed in ACAD. This is particularly useful for comparing perturbed orbits, as particular orbit sections could be zoomed and directly measured. The disadvantage is, the huge amount of transferred data significantly slows a computer.

There is also a number of additional Excel based programs, developed for calculating initial conditions, error analysis, etc.

Concrete problem, solved by the above method is:

Perigee altitude: 850km.

For elliptic orbit, apogee altitude: 1500km. Fig. 4.2. is a result of a program Orbit LISP as a part of SATELIGHT software, presented in the Appendix 4.2.

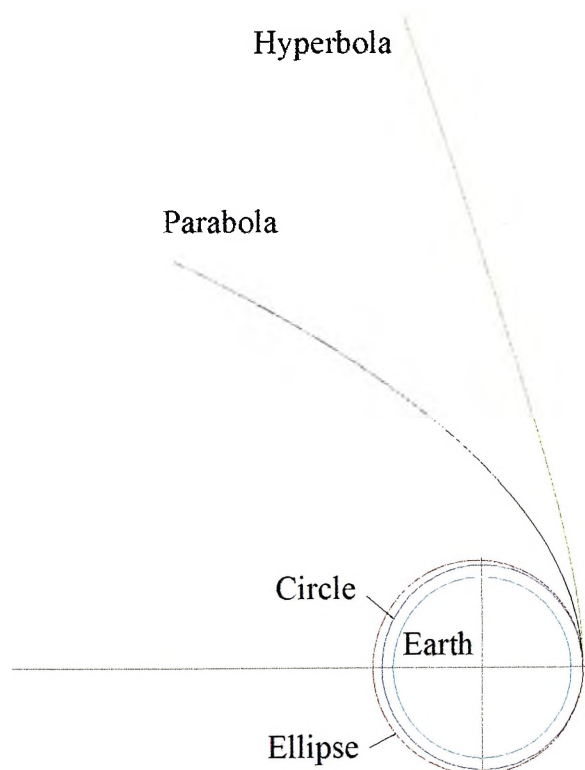


Fig. 4.2. Orbits, according to the initial conditions above, transferred to ACAD, calculated by SATELIGHT Software developed here

**Ground Track:**

The orbit starts to be generated from the point that is just vertically above the launching point. The dynamics of the launching phase was not subject of this program.

Ground track of the satellite trajectory depends on the orbit elements/ initial conditions, which are actually determined from the task of the mission.

Next elements are defined prior to ground track generation:

1. The world map used to refer the satellite trajectory to is the Mercator projection
2. The relative position in between non-rotating frame, whose  $x$  axis passes through the vernal equinox (chapter 2), and rotating system attached to the rotating Earth, whose  $x$  axis passes through the Greenwich, is defined as *Mean sidereal (equinoctial) time*.

Referring to the figure 2.2.2 there exists a unique angle between the axis passing through Greenwich prime meridian, and an inertial  $x$  axis passing through the vernal equinox. This angle is here denoted by  $\theta_g$  and is defined as the sidereal time of the Greenwich prime meridian.

Practical calculation of Greenwich sidereal time at 12 midnight, or 0 hours Universal Time, is given by formula:

$$\theta_{g0} = 99.6909833^\circ + 36000.7689^\circ T_u + 0.00038708^\circ T_u^{**2}$$

where the time is measured in centuries as:

$$Tu = (\text{Julian Date} - 2415020.0)/36525$$

Based on The American Ephemeris and Nautical Almanac, U.S. Government Printing Office, Washington D.C., Julian day number for years 2000 and 2001 is given as:

	Year 2000	Year 2001
January	2541544	2541910
February	2541575	2541938
March	2541604	2541969
April	2541635	2541999
May	2541665	2542030
June	2541696	2542060
July	2541726	2542091
August	2541757	2542122
September	2541788	2542152
October	2541818	2542183
November	2541849	2542213
December	2541879	2542244

Therefore, Greenwich sidereal time is calculated from:

$$\theta_g = \theta_{g0} + (t - t_0)d\theta/dt$$

For any other point on the Earth surface, including launching point, local sidereal time is given by:

$$\theta_g = \theta_{g0} + \lambda E$$

where  $\lambda E$  is the east longitude, the angle measured eastward in the equatorial plane from Greenwich to the observed meridian.

By this way the position of the launching point of the satellite is defined. For simplicity, all calculations in SATELIGHT start from this point. Time of the launch is transformed to the Universal Time by application of the above theory.

Let summarise the process of the satellite coordinates generation:

The first satellite coordinates are determined with respect to the inertial orbit plane, then by application of space transformations are brought to the rectangular DeCarte's system, with origin at the Earth centre and  $xy$  plane contained by equatorial plane, with  $x$  axis passing through the Greenwich meridian, section 4.2.

Utilising spherical coordinates, satellite position is determined by two angles and radius. More precisely, radius of the satellite position is the vector from the Earth's centre to the satellite, and two angles are longitude and latitude of the satellite's current position.

To start to generate ground track of the desired satellite, initial conditions have to be supplied to the program. These initial conditions will depend on the mission requirements and are calculated for each satellite.

Illustrative Example:

Determine the orbital elements of a satellite that will pass directly over Sydney at an altitude 1000km exactly three days after injection.

Injection occurred from Woomera at midnight ( 0 hour) on November 26, 2001.

The coordinates of Sydney are:

$\phi=34^\circ$  - Latitude

$\lambda E = 151^\circ 14'$  - Eastern longitude

$H = 0$  - elevation

Values adopted [20]:

$f= 1/298.3$  – flattening of the Earth ellipsoid

$Re = 6378.15$  km – Equatorial radius of Earth

$ke = 0.07436574 * (Re)**3/4$

$d\theta/dt = 4.375269 \cdot 10^{-3}$  rad/min – sidereal rate of change [20]

$t-t_0=4320$  min

By applying above equations, next values are determined:

Orbit inclination angle,  $i= 33.822^\circ$

and  $RAAN= 55.999^\circ$

These two elements are enough to determine the orientation of the orbit, therefore third element argument of perigee will be determined arbitrarily as  $AOP=61^\circ$ .

To satisfy the altitude requirement of the fly-over problem, semimajor axis of the orbit needs to be determined.

Expression used:

$$a=r/(1-e*\cos E)$$

where:

$$r=\sqrt{rc^{**2}+Hs^{**2}+2*rc*Hs*\cos(\phi s-\phi s')}$$

$$-\phi s'=inv(\tan(1-f)**2\tan\phi s))$$

$$a=9192.992\text{km}$$

Ground track for these initial conditions is represented in the figure 4.1.1.

Program AIRDRAG includes perturbations due to the air drag, explained in Chapter 2.

Program is composed of:

Main program: ADMAINI.for

    Subroutine: AS3I.for

        Subroutine: I2I1.for

        Linked to DATA.dat

This software portion contains a model of air density and includes air perturbations only for altitudes 100km to 2500km.

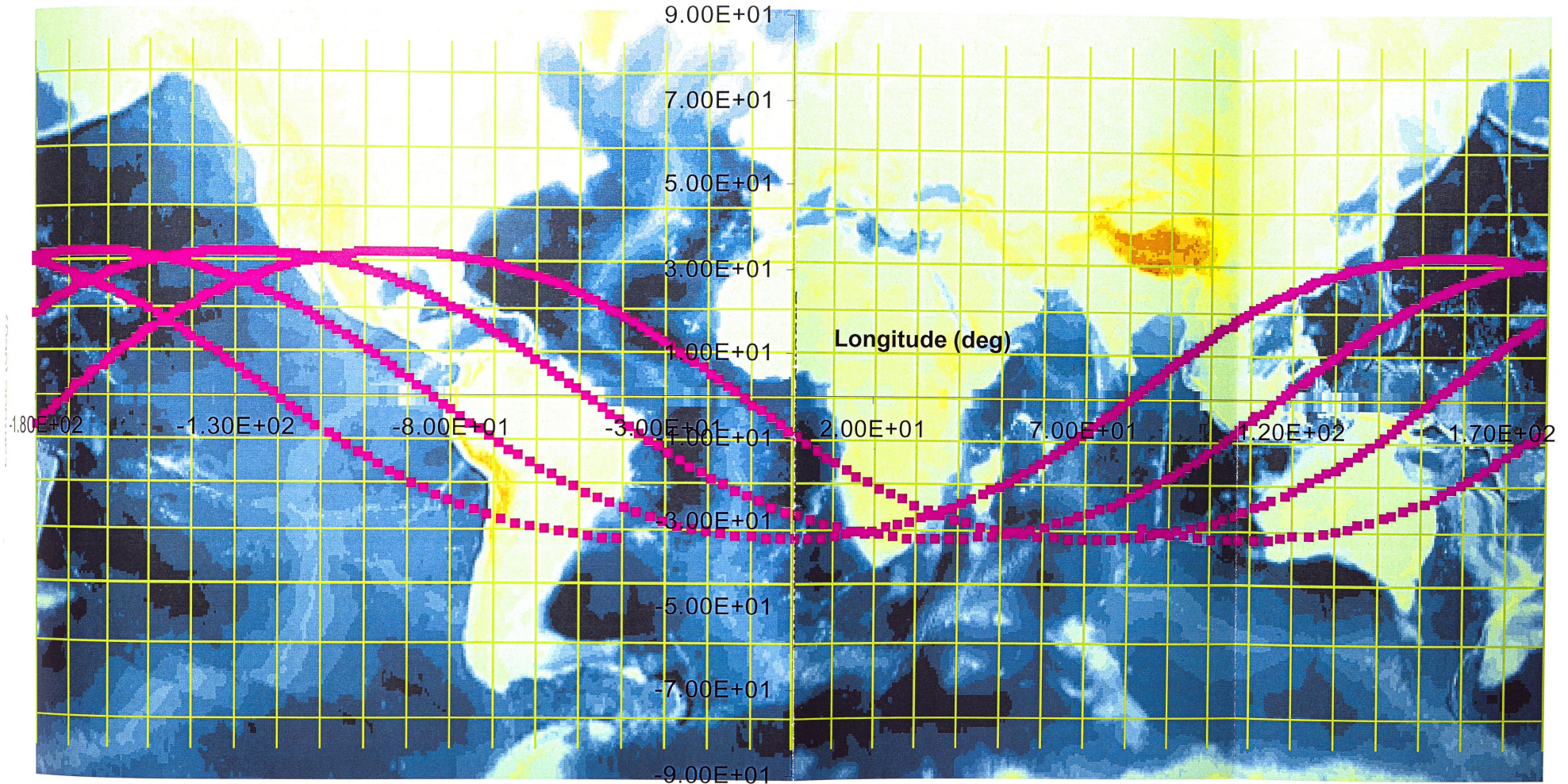
## **Figure 4.1.1: Orbit elements derivation**



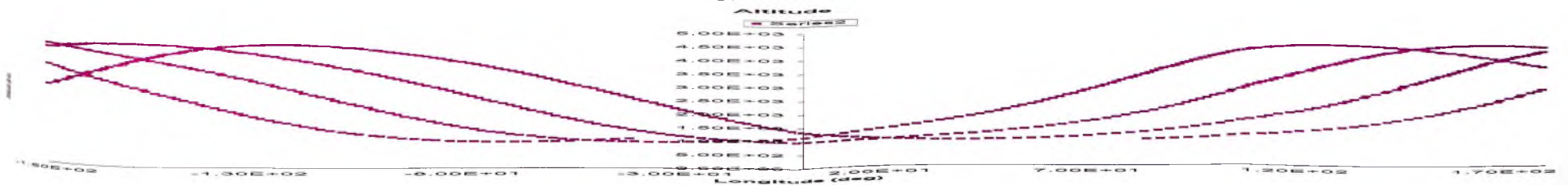
# SENSITIVITY ANALYSIS OF SATELLITE TRAJECTORY UNDER GRAVITATIONAL AND AERODYNAMIC PERTURBATIONS

STUDENT: M. M. RIBICIC • SUPERVISOR: PROF. M.P. WEST • UNIVERSITY OF WOLLONGONG, DECEMBER 2001

■ Longitude vs Latitude



**Ground Track of the orbit with the initial conditions: AP = 976.244km, AA = 4653.44km, RAAN = 55.99deg, i=33.82deg, AOP = 61deg**



Input required by this application is:

$T, X_o, Y_o, H, N, VX_o, VY_o$  already explained, and Latitude in [deg] and Inclination [deg]. Last two values are related to the system transformations and will be explained at the end of this chapter.

All programs coupled in *Air Drag* program set are represented in Appendix 4.3.

Gravitational perturbations due to the anomalies of the Earth gravitational field are included in the program ORBITANOM.for, represented in the Appendix 4.4.

Input values are  $T, X_o, Y_o, H, N, VX_o, VY_o$  and  $ZBI$  – initial latitude at which satellite starts orbiting motion (perigee in most cases).

Three Body Problem is presented in Appendix 4.5. in program named Three Body Problem with input values:

$T$ , perigee altitude, apogee altitude – defined previously

$XE, YE$  – coordinates of the Earth centre, calculated from the law of centre of masses, taken at any moment, according to the application. Program will work if these values are taken as zeros.

$XM0$  –  $x$  coordinate of the Moon's centre, usually taken as 384749.9km

$YM0$  –  $y$  coordinate of the Moon's centre. For the above value of  $XM0$ ,  $YM0$  is 0.

$H$  – step of the integration

$VxE, VyE$  – the speed of the Earth, for now taken as zeroes.

$VxM0$  –  $x$  component of the speed, according to the position coordinates equal to zero.

$VyM0$  –  $y$  and also total orbiting speed of the Moon, equal to 1.024km/s

$SMS$  – mass of the satellite, could be chosen as 0

$RAAN$ , Inclination and Argument of perigee – values related to the system transformations; they describe initial position of the satellite, as many physical properties depend on the satellite position in the space.

$N$  – in this case is the number of orbits required, could be chosen arbitrarily. For LISP application every new orbit is generated in different colour.

It could be noted some of the standard input values are not requested in this case. The reason is, this time they are calculated as the part of the program. Another advantage of this program is, it calculates deviation in an inclination angle after each orbit. This opens a possibility for completely new aspect of SATELIGHT utilisation, explained in the last section of this chapter.

Program that calculates satellite position referred to the rotating point at the Earth's surface is named GTRTGPH and is represented in Appendix 4.6. Its initial conditions do not have new elements, compared to the previous programs.

Last step in software development is final combination of a particular perturbing elements.

Combination of Air Drag and anomalies of the Earth's gravitational field is represented in Appendix 4.7.

Combined Three Body Problem with the effects due to the anomalies of the Earth's gravitational field is represented in Appendix 4.8.

## **4.2. System Transformations**

To transform coordinates from the orbit plane to the other system of interest, so named Space Transformations are applied. Another transformation is of the Moon orbit plane to the satellite orbit plane otherwise all three bodies would be placed in the same plane, which is just a special case. This approach gives the opportunity to generalize Three Body Problem one step further – section 4.2.2.

### **4.2.1. System Transformation from Orbit to Right Ascension System**

Transformation from the system coinciding with the orbit plane, in which all calculations were referred to, into the system that contains Equatorial plane is performed by three rotations:

1. First rotation is about z axis of the initial system, for the angle  $\alpha$  equal to the longitude of the orbit element defined as right ascension of Ascending Node.
2. Second rotation is performed about  $x'$  axis (therefore about perigee-apogee line) for the angle of inclination  $\beta$ .
3. Third rotation is performed about  $z''$  axis for an angle  $\gamma$  equal to the Argument of Perigee, with the aim to bring the perigee to its final position.

Transformations from Right Ascension system to the orbit plane system are performed by utilisation of matrices:

1.

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (4.2.1.)$$

2.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad (4.2.2.)$$

3.

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \quad (4.2.3.)$$

To get reversed system transformations matrices are transposed, by taking care of order of multiplication. The final system is:

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix}^T \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (4.2.4.)$$

After developing the above system:

$$\begin{aligned}
 x_3 &= (\cos\alpha \cos\gamma - \sin\alpha \cos\beta \sin\gamma)x + (-\cos\alpha \sin\gamma - \sin\alpha \cos\beta \cos\gamma)y + \\
 &\quad (\sin\alpha \sin\beta)z \\
 y_3 &= (\sin\alpha \cos\gamma + \cos\alpha \cos\beta \sin\gamma)x + (-\sin\alpha \sin\gamma + \cos\alpha \cos\beta \cos\gamma)y + \\
 &\quad (\sin\beta \cos\gamma)z \\
 z_3 &= (\sin\beta \sin\gamma)x + (\sin\beta \cos\gamma)y + (\cos\beta)z \quad (4.2.5.)
 \end{aligned}$$

Final equations (4.2.5.) are applied in all programs to refer satellite position and velocity to the new system. Values requested by the input, RAAN, inclination angle and argument of perigee, are therefore the orbit elements specified by user.

#### 4.2.2. System Transformation from Moon Orbit to the Orbit Plane System

The coordinates of the satellite are calculated in the orbit plane. Moon's orbit plane does not coincide with the satellite orbit plane, but the forces of Moon's gravitational field do have effect to the satellite motion. If all three bodies are contained by the same plane, satellite would drift from its orbit, for a particular value in the plane, toward Moon, and then return to its unperturbed trajectory when the Moon is far enough. If the inclination angle of the satellite orbit is for example less than the inclination angle of the Moon, satellite orbit will also experience change in the inclination angle, which is not in the plane.

It is clear, for a definition of a plane, three points, which could be centres of the observed bodies, are enough. It would be, theoretically, necessary to define the system that moves in such manner, to contain these points all the time.

In that case, special case of three bodies would be solved in the more general terms. Besides the fact it is very hard to define such system, and observe all laws validity in it, final results would be again transformed to some more appropriate system of reference.

To avoid complexity, it is much easier if the coordinates of the Moon are at the start transferred to the Right Ascension System, and then to the Orbit Plane system. Final

transformation gives three components, two of them affect the motion. Third one is normal to the orbit plane, and it is assumed, it does not have any effect on the orbit plane. Same transformations are applied to the velocities calculated in the Moon orbit. The element that was observed further is inclination angle, recalculated after each orbit as:

$$i = \arccos\left(\frac{xV_y - yV_x}{H}\right), \quad 0 \leq i \leq 180^\circ \quad (4.2.6.)$$

It was shown, the inclination angle increased slowly toward Moon. Other orbit elements could be calculated in a similar way, from the numerically determined values that include different perturbing elements.

This is an excellent tool for observing different effects on different shaped and oriented orbits including different spacecraft geometries, masses, etc.

Moon transformations are performed as:

1. Transformation from Moon orbit to Right Ascension system, according to the equations:

$$\begin{aligned} XM00 &= (\cos(AM) \cdot \cos(GRM) - \sin(AM) \cdot \cos(BM) \cdot \sin(GRM)) \cdot XM0 + \\ &\quad (-\cos(AM) \cdot \sin(GRM) - \sin(AM) \cdot \cos(BM) \cdot \cos(GRM)) \cdot YM0 \\ YM00 &= (\sin(AM) \cdot \cos(GRM) + \cos(AM) \cdot \cos(BM) \cdot \sin(GRM)) \cdot XM0 + (-\sin(AM) \cdot \\ &\quad \sin(GRM) + \cos(AM) \cdot \cos(BM) \cdot \cos(GRM)) \cdot YM0 \\ ZM00 &= (\sin(BM) \cdot \sin(GRM)) \cdot XM0 + (\sin(BM) \cdot \cos(GRM)) \cdot YM0 \end{aligned}$$

$$\begin{aligned} VXM00 &= (\cos(AM) \cdot \cos(GRM) - \sin(AM) \cdot \cos(BM) \cdot \sin(GRM)) \cdot VXM0 + \\ &\quad (-\cos(AM) \cdot \sin(GRM) - \sin(AM) \cdot \cos(BM) \cdot \cos(GRM)) \cdot VYM0 \\ VYM00 &= (\sin(AM) \cdot \cos(GRM) + \cos(AM) \cdot \cos(BM) \cdot \sin(GRM)) \cdot VXM0 \\ &\quad (-\sin(AM) \cdot \sin(GRM) + \cos(AM) \cdot \cos(BM) \cdot \cos(GRM)) \cdot VYM0 \\ VZM00 &= (\sin(BM) \cdot \sin(GRM)) \cdot VXM0 + (\sin(BM) \cdot \cos(GRM)) \cdot VYM0 \end{aligned}$$

2. Second set of rotations transforms system from Right Ascending to the satellite orbit plane system, by the set of next equations:

$$\begin{aligned} XM &= (\cos(GR) \cdot \cos(A) - \sin(A) \cdot \sin(GR) \cdot \cos(B)) \cdot XM00 + (\sin(GR) \cdot \cos(B) \cdot \\ &\quad \cos(A) + \cos(GR) \cdot \sin(A)) \cdot YM00 + (\sin(GR) \cdot \sin(B)) \cdot ZM00 \end{aligned}$$

$$\begin{aligned}
 YM &= (-\sin(GR) \cdot \cos(A) - \sin(A) \cdot \cos(B) \cdot \cos(GR)) \cdot XM00 + (-\sin(GR) \cdot \sin(A) + \\
 &\quad \cos(B) \cdot \cos(GR) \cdot \cos(A)) \cdot YM00 + (\cos(GR) \cdot \sin(B)) \cdot ZM00 \\
 ZM &= (\sin(A) \cdot \sin(B)) \cdot XM00 + (-\sin(B) \cdot \cos(A)) \cdot YM00 + \cos(B) \cdot ZM00
 \end{aligned}$$

$$\begin{aligned}
 VXM &= (\cos(GR) \cdot \cos(A) - \sin(A) \cdot \sin(GR) \cdot \cos(B)) \cdot VXM00 + (\sin(GR) \cdot \\
 &\quad \cos(B) \cdot \cos(A) + \cos(GR) \cdot \sin(A)) \cdot VYM00 + (\sin(GR) \cdot \sin(B)) \cdot VZM00 \\
 VYM &= (-\sin(GR) \cdot \cos(A) - \sin(A) \cdot \cos(B) \cdot \cos(GR)) \cdot VXM00 + (-\sin(GR) \cdot \\
 &\quad \sin(A) + \cos(B) \cdot \cos(GR) \cdot \cos(A)) \cdot VYM00 + (\cos(GR) \cdot \sin(B)) \cdot VZM00 \\
 VZM &= (\sin(A) \cdot \sin(B)) \cdot VXM00 + (-\sin(B) \cdot \cos(A)) \cdot VYM00 + \cos(B) \cdot VZM00
 \end{aligned}$$

By these equations, Moon orbit plane transformations are completed. Next section represents formulas that could be used for orbit elements' change observation.

#### 4.2.3. Orbit Elements derived from numerical results

Radius vector:

$$r = \sqrt{x^2 + y^2 + z^2} \quad (4.2.7.)$$

Total velocity:

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2} \quad (4.2.8.)$$

Eccentricity is determined from the coupled equations:

$$e \sin E = \frac{1}{\sqrt{GM_E a}} (xV_x + yV_y + zV_z) \quad (4.2.9.)$$

$$e \cos E = 1 - \frac{r}{a} \quad (4.2.10.)$$

Argument of Perigee:

$$\sin(\omega + \theta) = \frac{z}{r \sin i} \quad (4.2.11.)$$

$$\cos(\omega + \theta) = \frac{y}{r} \sin \Omega + \frac{x}{r} \cos \Omega \quad (4.2.12.)$$

$$\cos \Omega = \frac{xV_z - zV_x}{H \sin i} \quad (4.2.13.)$$

## **CHAPTER 5**

### **Practical Interpretation Results**



## Chapter 5: Practical Interpretation of the Results

### 5.1. Introduction

Different perturbations have the effect on the different orbit elements. With the aim to analyse these changes more effectively short reflection to the analytical method is also introduced.

The 'general perturbations' approach is based on the analytical method, developed from the theory of infinite series. For some special cases it is possible to obtain a closed solution.

Special Perturbations are derived from the numerical approach, and are used in this project for determining deviations from the ideal satellite trajectory.

In both cases, changes that can occur to a particular orbit element could constantly to increase or decrease, or periodically vary (fluctuate) with respect to a chosen reference value. Some elements possess only periodic variations, which occur about their mean value.

There are also short period variations and long period variations. For example, due to the effect of the Earth's oblateness next variations can occur:

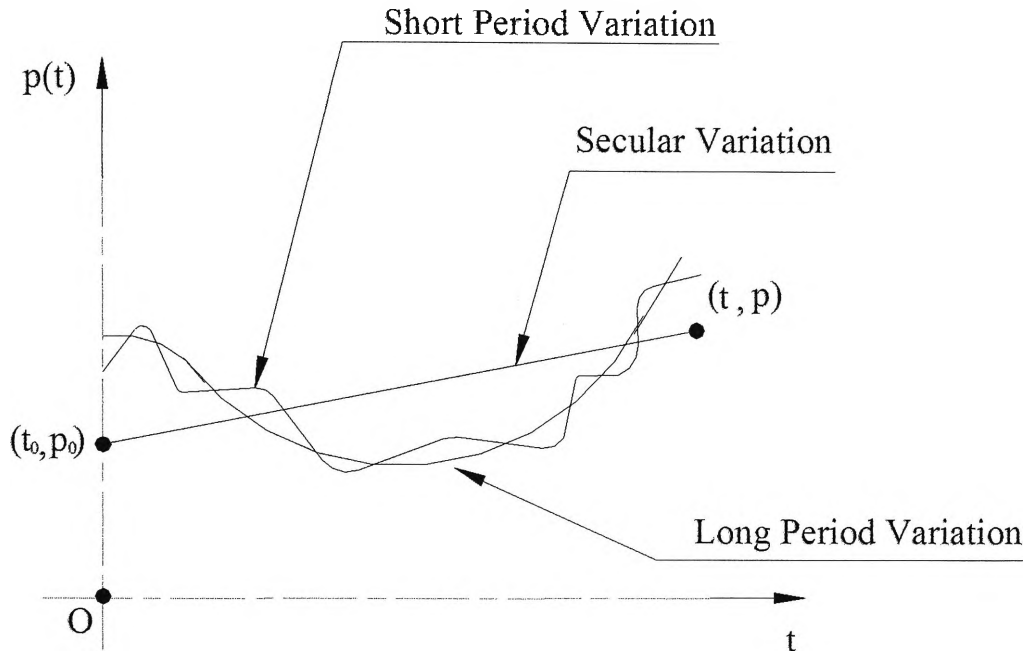


Fig. 5.1. Character of Perturbing elements

Secular variations are associated with a steady, non-oscillatory continuous drift of an element from the adopted epoch value. Short period variations are produced by trigonometric functions of linear combinations of  $v$  and  $\omega$ .

Long period variations are associated with trigonometric functions of  $\omega$  and multiples of  $\omega$ .

According to the above definition, the total variance of an element  $p$  represented by the hypothetical relation is:

$$p = p_0 + p'_0(t - t_0) + k_1 \cos(2\omega) + k_2 \sin(2v + 2\omega) \quad (5.1.)$$

where the first element is the adopted epoch mean element, second term is the secular variation, the third term – long period variation and the last term – the short period variation.

The atmospheric drag effect on the orbits manifests itself as secular variation [28]. The change occurs in eccentricity, semimajor axis and inclination.

## 5.2. The Effect of the Air Drag on the satellite Orbit

To evaluate the effect of the atmospheric drag on a satellite orbit next approach is used: The number of single-orbit observations is performed, and as a result a general solution for the variation of perigee distance with eccentricity is provided. These two elements, and orbit period, are the only subjects of the analysis, as other elements do not vary significantly.

This procedure is hard to obtain but gives very accurate results.

For an elliptical orbit, of a high eccentricity, drag effect is the largest on the small orbit portion near the perigee. This is due to, as it was already explained, exponential change of the air density with respect to the altitude.

The result of this effect is retardation of a satellite speed at perigee, similar to the jet firing for manoeuvring purpose, affects apogee height, while perigee height changes much slower. This means that, the orbit contracts, therefore becomes more nearly circular, with both  $e$  and  $a$  elements decreasing steadily.

The argument of perigee and inclination stay constant, because the force of the air drag lies within the orbit plane.

Lowering of the apogee height actually indicates the process of the reduction in the total energy of the spacecraft and is named orbital decay.

### 5.2.1. Case Study The Effects of the Air Drag to the International Space Station

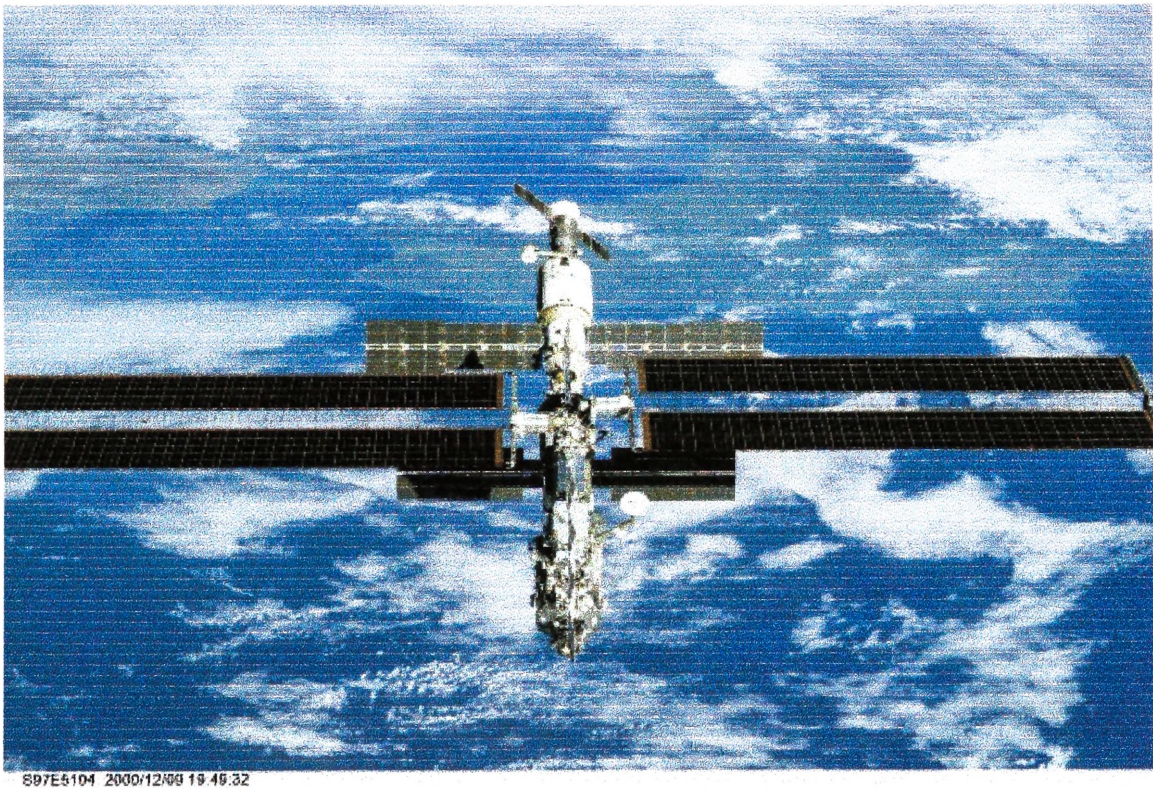


Fig. 5.2.1. International Space Station Photo 1

The International Space Station, refer to the figures 5.2.1. – 5.2.5., is placed in near circular orbit of altitude 402km.

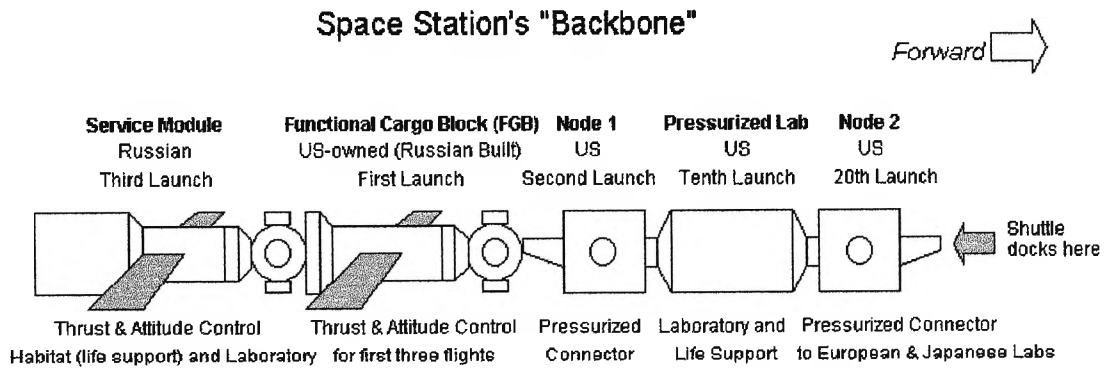


Fig.5.2.2.

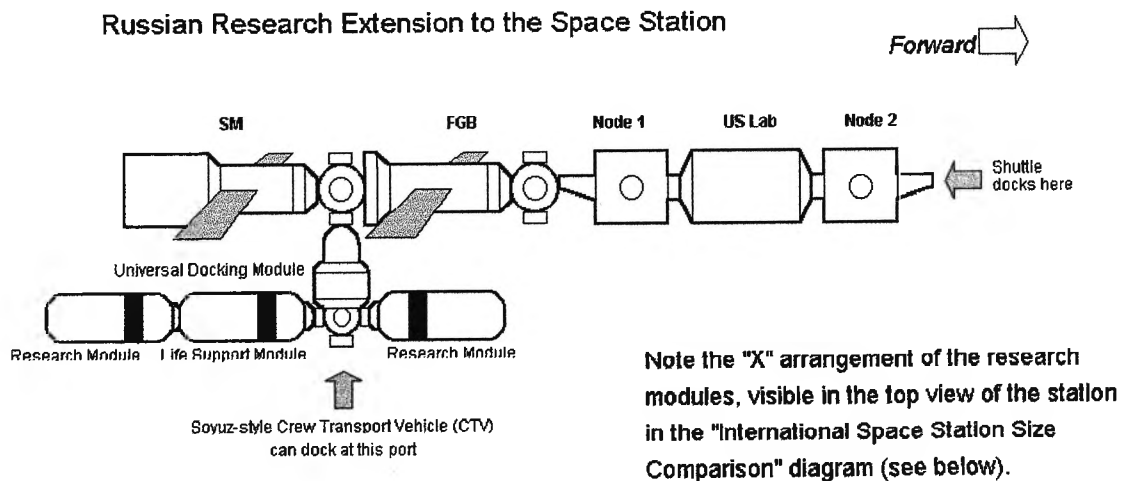


Fig. 5.2.3.

## International Space Station Size Comparison

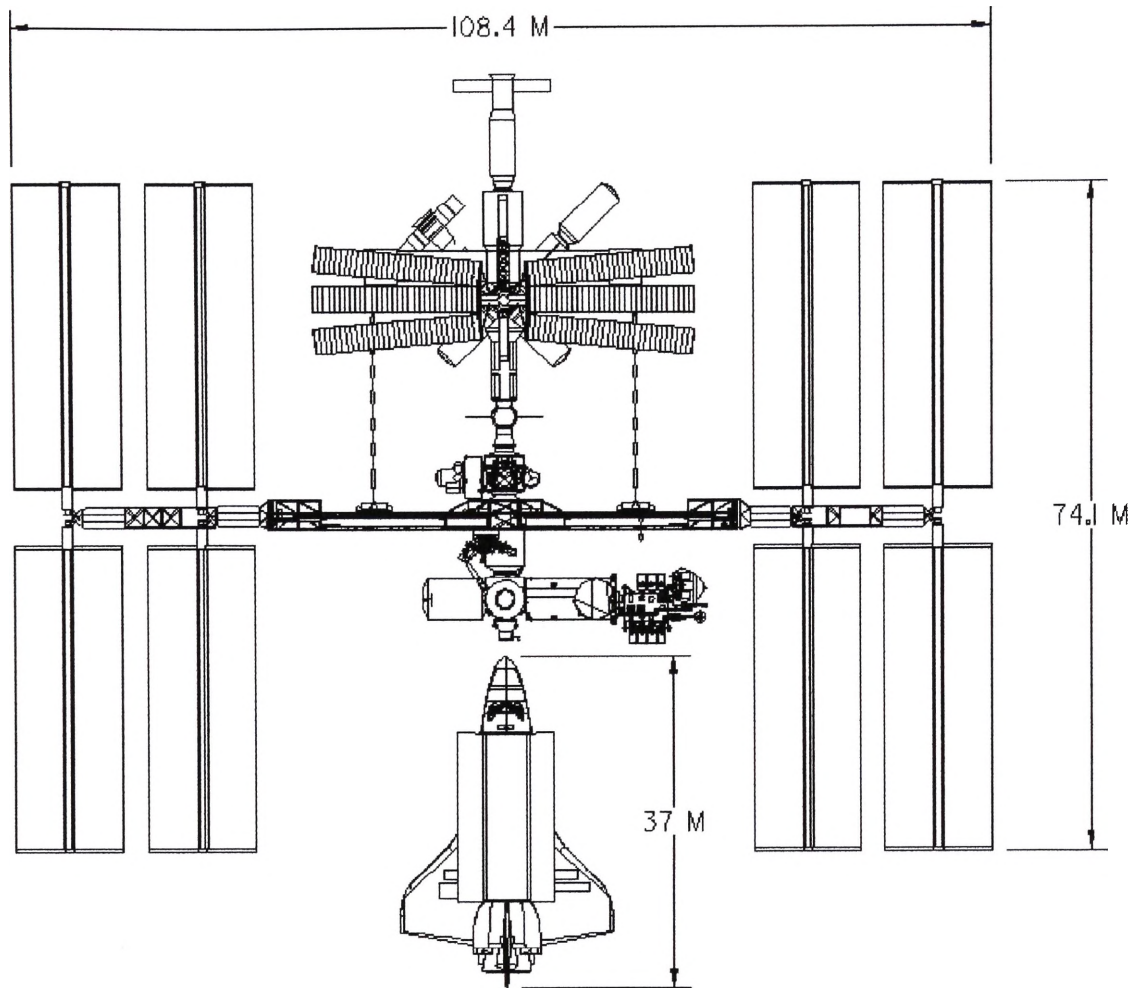


Fig. 5.2.4. This representation was used as the guide for the shape approximation in air-drag calculations

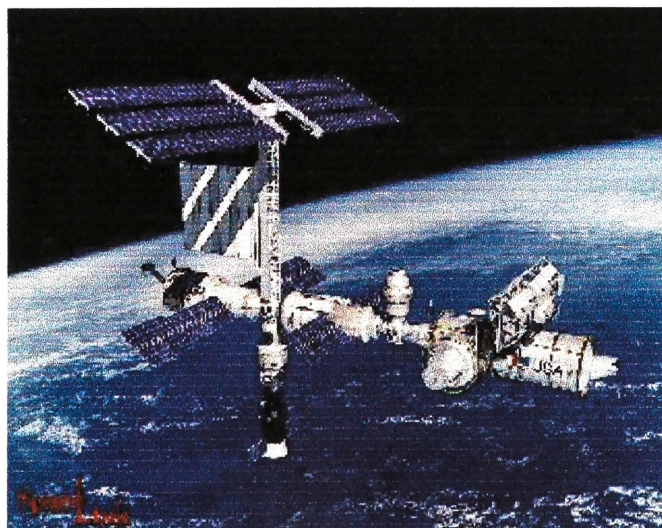


Fig. 5.2.5. International Space Station Photo 2

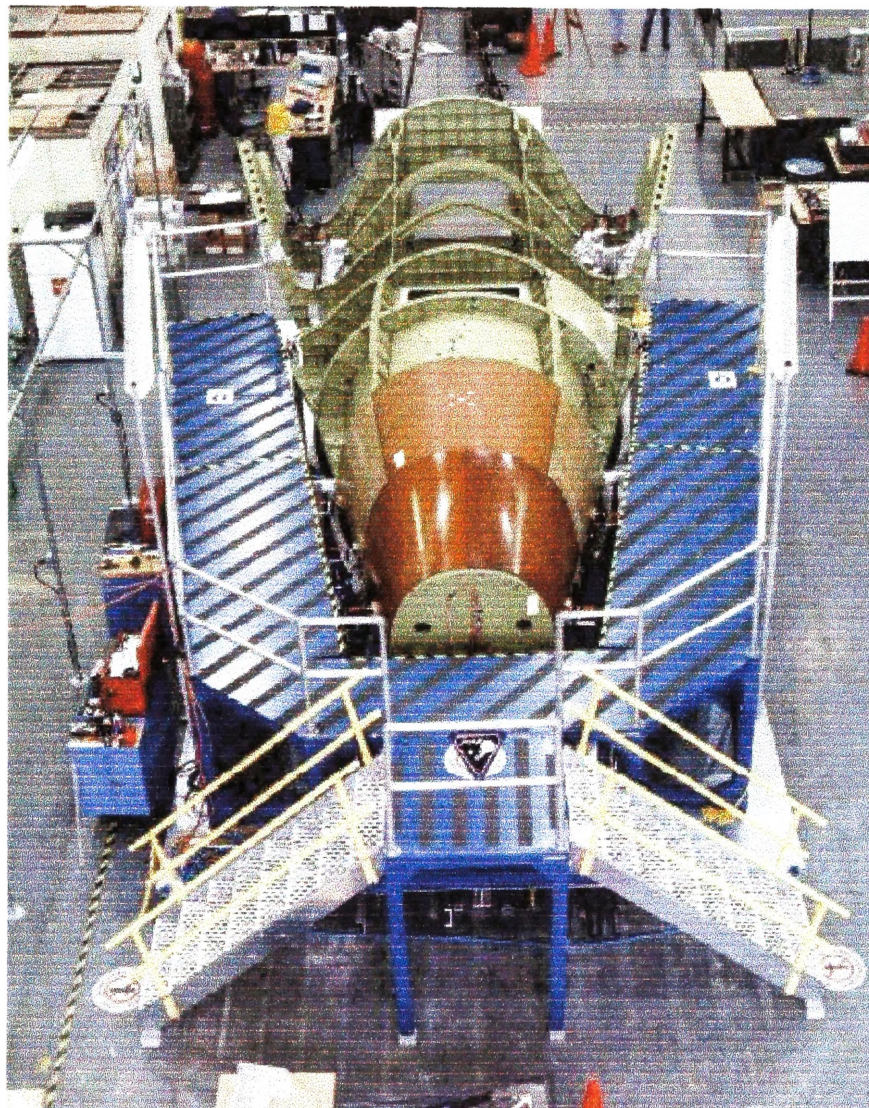


Fig. 5.2.6. NASA's workshop

For the circular orbit, of the altitude 402km and for the program ADMAINI initial conditions were selected to be:

Orbit Radius = 402km

H=5sec

RAAN = 30deg

Inclination = -51.6deg, real inclination angle

AOP = 30deg

Results represented in the tables 5.2.0.1 and 5.2.0.2 show the variation of the perigee/apogee values for 60 successive orbits, for the two basic cases, with respect to the shape of the station:

1. Cylinder of the length 89m and diameter 18m. These values are substituted in the expression for determination of the cross section (2.8.4.2.). Air drag coefficient for cylinder does not change and is equal to 2.2 [32].
2. Flat plate, that actually represents the predominant shape of the solar panels, with dimensions 74.1m width, 108.4m length and 0.3m thickness. Air drag coefficient for a flat plate is 2.2 [32].

TIME= 0.0  
 AltPerigee= 402.0km  
 AltApogee= 402.0km  
 H= 5.00  
 RAAN= 30.00  
 INCLINATION=-52.51  
 AOP=30.00  
 1stOrbitNo 1  
 2ndOrbitNo 30  
 3rdOrbitNo 60

ALT PERIGEE 394.83243	ALT APOGEE 413.87586
ALT PERIGEE 394.83247	ALT APOGEE 413.81177
ALT PERIGEE 394.83248	ALT APOGEE 413.74769
ALT PERIGEE 394.83247	ALT APOGEE 413.68361
ALT PERIGEE 394.83254	ALT APOGEE 413.61953
ALT PERIGEE 394.83249	ALT APOGEE 413.55545
ALT PERIGEE 394.83254	ALT APOGEE 413.49138
ALT PERIGEE 394.83253	ALT APOGEE 413.42732
ALT PERIGEE 394.83254	ALT APOGEE 413.36326
ALT PERIGEE 394.83259	ALT APOGEE 413.29920
ALT PERIGEE 394.83255	ALT APOGEE 413.23516
ALT PERIGEE 394.83261	ALT APOGEE 413.17112
ALT PERIGEE 394.83259	ALT APOGEE 413.10708
ALT PERIGEE 394.83260	ALT APOGEE 413.04306
ALT PERIGEE 394.83265	ALT APOGEE 412.97905
ALT PERIGEE 394.83262	ALT APOGEE 412.91505
ALT PERIGEE 394.83269	ALT APOGEE 412.85106
ALT PERIGEE 394.83265	ALT APOGEE 412.78708



ALT PERIGEE 394.83267	ALT APOGEE 412.72311
ALT PERIGEE 394.83270	ALT APOGEE 412.65916
ALT PERIGEE 394.83268	ALT APOGEE 412.59522
ALT PERIGEE 394.83276	ALT APOGEE 412.53130
ALT PERIGEE 394.83271	ALT APOGEE 412.46739
ALT PERIGEE 394.83274	ALT APOGEE 412.40350
ALT PERIGEE 394.83275	ALT APOGEE 412.33962
ALT PERIGEE 394.83275	ALT APOGEE 412.27577
ALT PERIGEE 394.83282	ALT APOGEE 412.21193
ALT PERIGEE 394.83277	ALT APOGEE 412.14811
ALT PERIGEE 394.83282	ALT APOGEE 412.08431
ALT PERIGEE 394.83281	ALT APOGEE 412.02053
ALT PERIGEE 394.83281	ALT APOGEE 411.95677
ALT PERIGEE 394.83287	ALT APOGEE 411.89303
ALT PERIGEE 394.83283	ALT APOGEE 411.82932
ALT PERIGEE 394.83289	ALT APOGEE 411.76563
ALT PERIGEE 394.83287	ALT APOGEE 411.70197
ALT PERIGEE 394.83288	ALT APOGEE 411.63833
ALT PERIGEE 394.83292	ALT APOGEE 411.57471
ALT PERIGEE 394.83289	ALT APOGEE 411.51112
ALT PERIGEE 394.83296	ALT APOGEE 411.44756
ALT PERIGEE 394.83292	ALT APOGEE 411.38402
ALT PERIGEE 394.83295	ALT APOGEE 411.32052
ALT PERIGEE 394.83298	ALT APOGEE 411.25704
ALT PERIGEE 394.83296	ALT APOGEE 411.19359
ALT PERIGEE 394.83304	ALT APOGEE 411.13018
ALT PERIGEE 394.83298	ALT APOGEE 411.06679
ALT PERIGEE 394.83302	ALT APOGEE 411.00344
ALT PERIGEE 394.83303	ALT APOGEE 410.94012
ALT PERIGEE 394.83302	ALT APOGEE 410.87683
ALT PERIGEE 394.83310	ALT APOGEE 410.81358
ALT PERIGEE 394.83304	ALT APOGEE 410.75036
ALT PERIGEE 394.83309	ALT APOGEE 410.68718
ALT PERIGEE 394.83308	ALT APOGEE 410.62403
ALT PERIGEE 394.83309	ALT APOGEE 410.56092
ALT PERIGEE 394.83315	ALT APOGEE 410.49785
ALT PERIGEE 394.83310	ALT APOGEE 410.43482
ALT PERIGEE 394.83317	ALT APOGEE 410.37182
ALT PERIGEE 394.83314	ALT APOGEE 410.30887
ALT PERIGEE 394.83316	ALT APOGEE 410.24595
ALT PERIGEE 394.83320	ALT APOGEE 410.18308

Table 5.2.0.1. Altitude in (km) at perigee/apogee of the ISS orbit perturbed by the aerodynamic drag, for **cylinder**

T = 0.0

AltPer= 402.0km

AltAp= 402.0km

H= 5.00

RAAN= 30.00

INCLINATION=-51.6

AOP=30.00

1stOrbitNo 1

2ndOrbitNo 30

3rdOrbitNo 60

ALT PERIGEE 394.83248	ALT APOGEE 413.87592
ALT PERIGEE 394.83258	ALT APOGEE 413.81191
ALT PERIGEE 394.83265	ALT APOGEE 413.74789
ALT PERIGEE 394.83270	ALT APOGEE 413.68387
ALT PERIGEE 394.83283	ALT APOGEE 413.61985
ALT PERIGEE 394.83284	ALT APOGEE 413.55583
ALT PERIGEE 394.83295	ALT APOGEE 413.49182
ALT PERIGEE 394.83300	ALT APOGEE 413.42781
ALT PERIGEE 394.83306	ALT APOGEE 413.36380
ALT PERIGEE 394.83318	ALT APOGEE 413.29980
ALT PERIGEE 394.83320	ALT APOGEE 413.23580
ALT PERIGEE 394.83332	ALT APOGEE 413.17181
ALT PERIGEE 394.83335	ALT APOGEE 413.10782
ALT PERIGEE 394.83342	ALT APOGEE 413.04384
ALT PERIGEE 394.83353	ALT APOGEE 412.97987
ALT PERIGEE 394.83355	ALT APOGEE 412.91591
ALT PERIGEE 394.83369	ALT APOGEE 412.85195
ALT PERIGEE 394.83371	ALT APOGEE 412.78801
ALT PERIGEE 394.83379	ALT APOGEE 412.72408
ALT PERIGEE 394.83388	ALT APOGEE 412.66016
ALT PERIGEE 394.83392	ALT APOGEE 412.59625
ALT PERIGEE 394.83406	ALT APOGEE 412.53235
ALT PERIGEE 394.83406	ALT APOGEE 412.46847
ALT PERIGEE 394.83416	ALT APOGEE 412.40461
ALT PERIGEE 394.83423	ALT APOGEE 412.34075
ALT PERIGEE 394.83428	ALT APOGEE 412.27692
ALT PERIGEE 394.83441	ALT APOGEE 412.21310
ALT PERIGEE 394.83442	ALT APOGEE 412.14930
ALT PERIGEE 394.83452	ALT APOGEE 412.08551
ALT PERIGEE 394.83458	ALT APOGEE 412.02175
ALT PERIGEE 394.83464	ALT APOGEE 411.95800
ALT PERIGEE 394.83476	ALT APOGEE 411.89427
ALT PERIGEE 394.83477	ALT APOGEE 411.83057
ALT PERIGEE 394.83489	ALT APOGEE 411.76688
ALT PERIGEE 394.83493	ALT APOGEE 411.70322
ALT PERIGEE 394.83500	ALT APOGEE 411.63958
ALT PERIGEE 394.83511	ALT APOGEE 411.57597
ALT PERIGEE 394.83513	ALT APOGEE 411.51238
ALT PERIGEE 394.83526	ALT APOGEE 411.44881
ALT PERIGEE 394.83528	ALT APOGEE 411.38527
ALT PERIGEE 394.83537	ALT APOGEE 411.32176

Table 5.2.0.2. Altitude in (km) at perigee/apogee of the ISS orbit perturbed by the aerodynamic drag, for **flat plate**

Results indicate that the altitude at perigee stays almost unchanged for the different orbits, while the altitude at apogee decreases. These results are in accordance to the theory analysed in Chapter 2.

Ground track and altitude are represented graphically on the Figure 5.2.7.

### 5.3. The Effect of the Earth's Gravitational Field Anomalies

This effect is very significant for the altitudes 500-600km. The elements  $a, e$  and  $i$  experience short period variation, that averages to zero over one complete orbit. Other three elements undergo cumulative secular variations in which the average value of the parameters changes monotonically. Total energy, the mean values of the semimajor axis, the apogee and perigee heights and the eccentricity do not change. To understand the causes of secular variations it is convenient to imagine Earth as a point mass and a ring of uniform density in the Equatorial plane, representing the Earth's Equatorial bulge. The example of the rotation of the orbit plane is represented as:

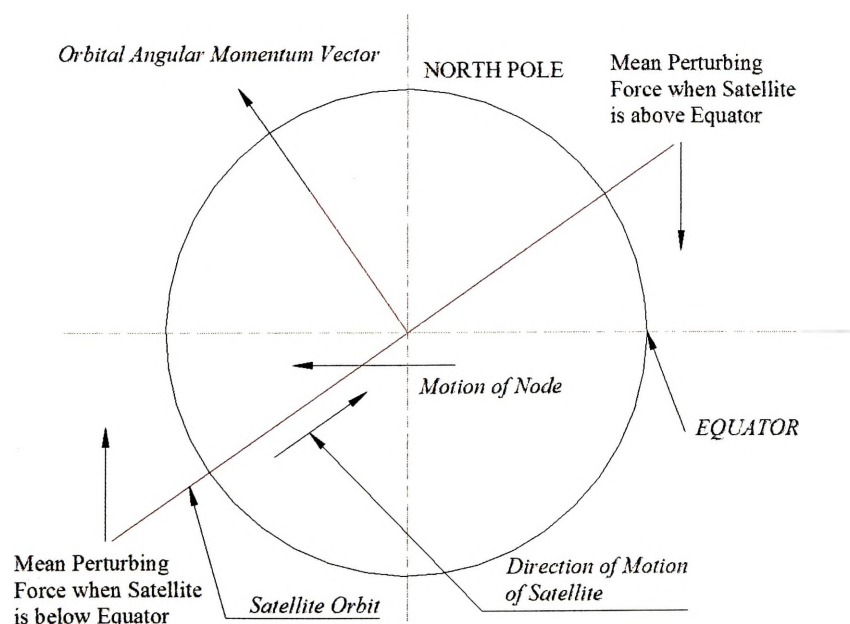


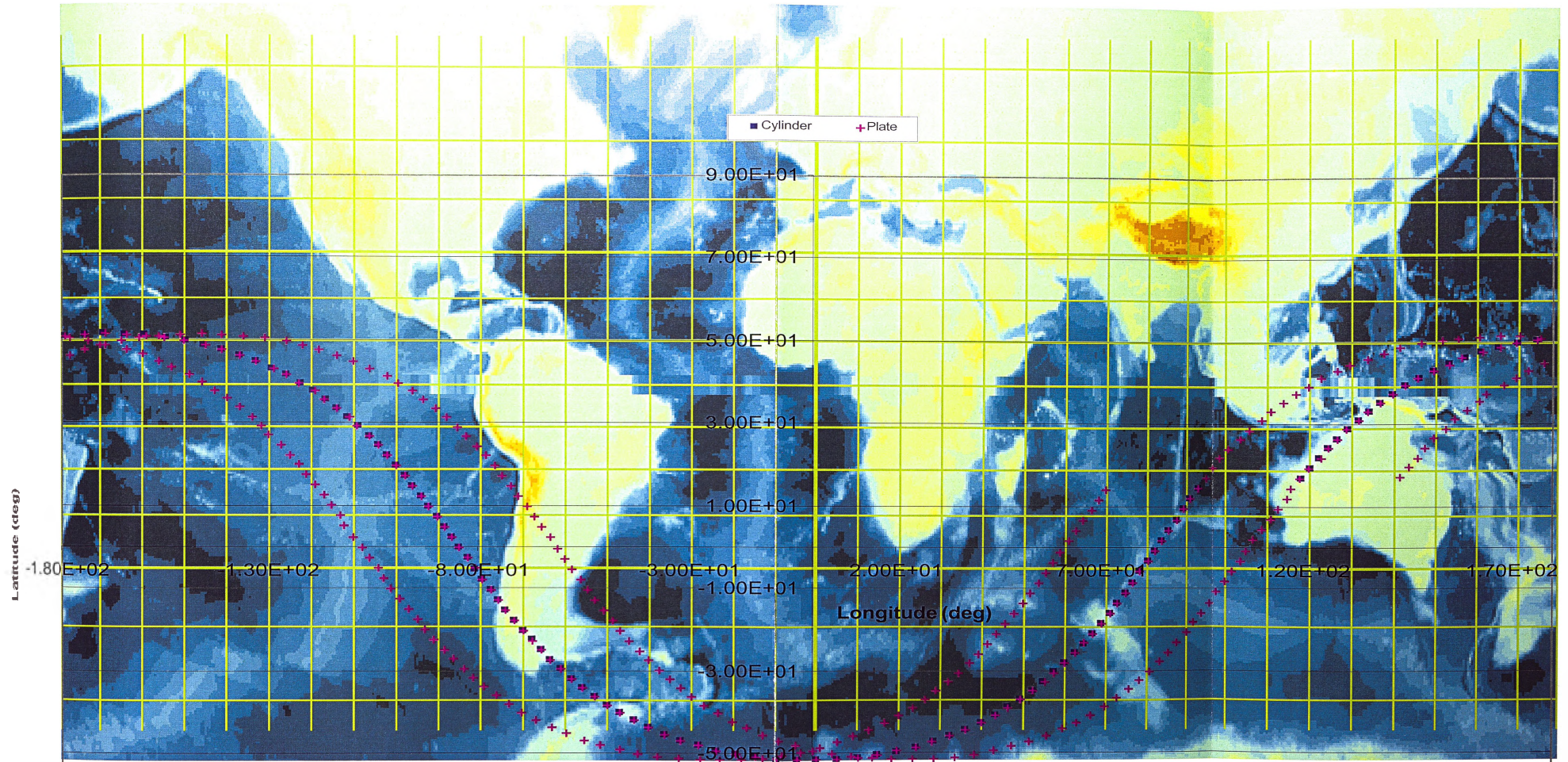
Fig. 5.3.1. Earth's Oblateness effect on the Orbit

**Figure 5.2.1.a:**

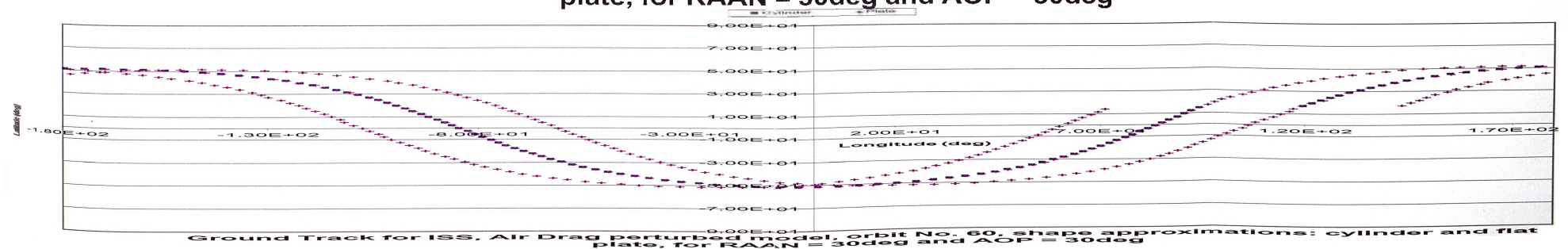
**ISS Orbit – Air drag perturbation included**

# SENSITIVITY ANALYSIS OF SATELLITE TRAJECTORY UNDER GRAVITATIONAL AND AERODYNAMIC PERTURBATIONS

STUDENT: M. M. RIBICIC • SUPERVISOR: PROF. M.P. WEST • UNIVERSITY OF WOLLONGONG, DECEMBER 2001



Ground Track for ISS, Air Drag perturbed model, orbit No. 60, shape approximations: cylinder and flat plate, for RAAN = 30deg and AOP = 30deg



Ground Track for ISS, Air Drag perturbed model, orbit No. 60, shape approximations: cylinder and flat plate, for RAAN = 30deg and AOP = 30deg

The line of node is also changed, for example, for  $I=0$  gravitational force is stronger than in Kepler's model, therefore the orbit will 'curve' more strongly and angular velocity of the satellite will increase.

The effect is, each successive perigee and apogee will occur further around than previously and the line of nodes rotates in the direction of the satellite's motion.

For satellites in polar orbits, the gravitational force reduces while the satellite is above the pole, therefore the orbit curve less and line of nodes rotate opposite the direction of satellite motion.

In a polar orbit, the net effect is rotation of the line of nodes opposite the direction of motion, although the rate of rotation is less than for equatorial orbits [29].

#### **5.4. Third Body Interaction**

The relative importance of the disturbing force increases with the third power of satellite distance [31].

Third body, in this case Moon, pulls satellite orbit toward its centre, therefore the inclination angle changes. Other elements that experience small changes are  $a, e, \omega$  and  $\Omega$ .

This effect is particularly interesting to observe when applied on geosynchronous orbit.

Geosynchronous Orbit is very sensitive on the so-named tesseral and sectorial harmonics, or in other words deviations from Kepler's orbit due to the ellipticity of

the Earth's equator and the pear – shaped Earth's appearance, respectively, analysed in Chapter 2.

Program TBPXYZLS solves this problem, including the observation of an inclination angle change.

The illustrative example used here is the Geosynchronous orbit with next characteristics:

Altitude Perigee= 35796.86km

Altitude Apogee= 35796.86km

H=5sec

RAAN=322.923deg

Inclination Angle=30deg

AOP=96deg

No of Orbits=9

Representations of the Ground Track and Altitude with respect to the time and longitude are given in Figures 5.4.1 and 5.4.2.

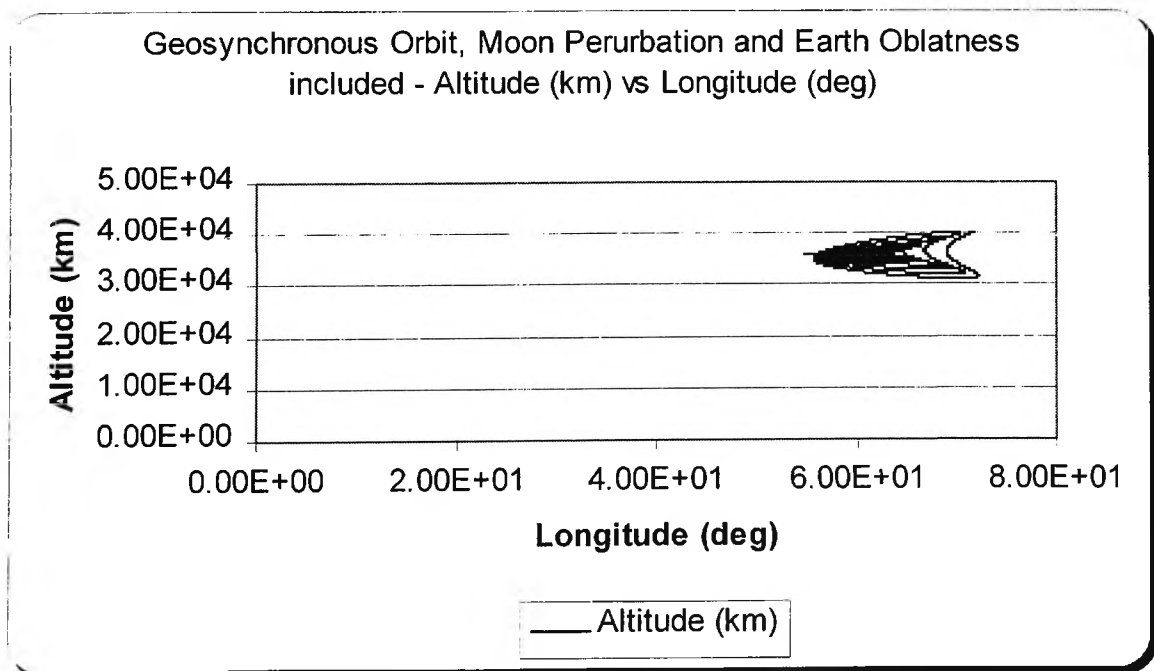
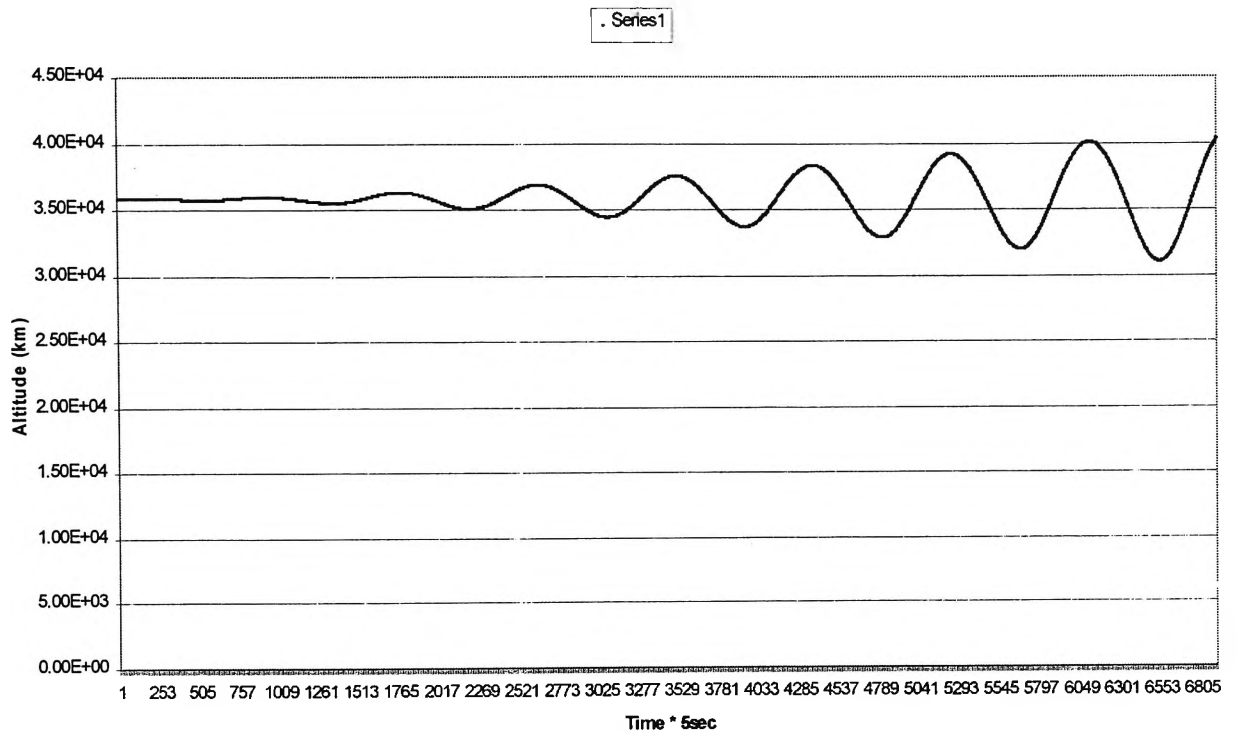


Figure 5.4.1, Third Body (Moon) effect on the Earth's satellite in Geosynchronous orbit with RAAN=322.9234deg, AOP=96deg and  $i=30$ deg: Altitude (km) vs Longitude (deg)



Geosynchronous Orbit: Moon Perurbation and Earth Oblatness included - Altitude (km) vs Time (sec\*5)

Figure 5.4.2, Third Body (Moon) effect on the Earth’s satellite in Geosynchronous orbit with RAAN=322.9234deg, AOP=96deg and i=30deg:  
Altitude (km) vs Time (sec\*5)

### 5.5. Ground Track

The coordinates of the Earth satellite were calculated with respect to the inertial system, whose *xy* plane was coincident with the orbit plane, and whose origin was placed at the Earth centre.

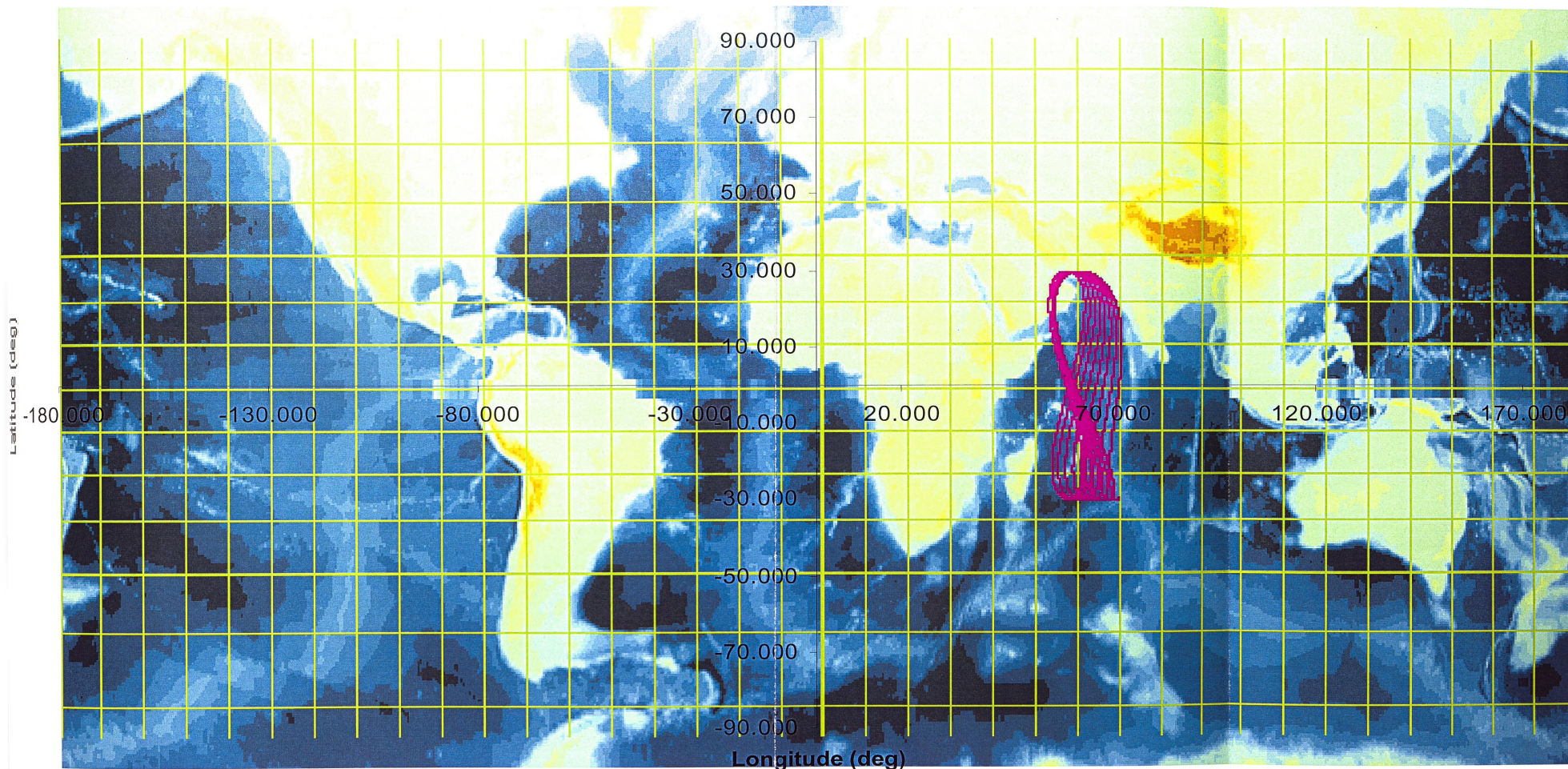
These coordinates were transformed to the values referred to the rotating rectangular coordinate system, attached to the Earth and rotating with it at the same angular speed – Chapter 2.



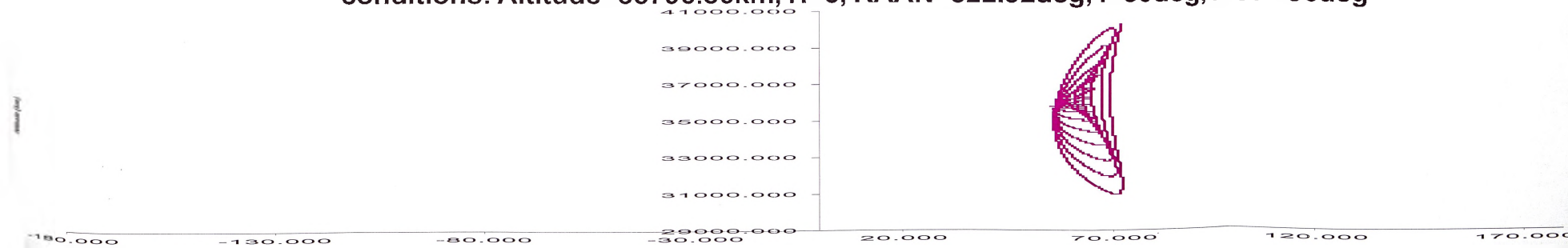
## **Figure 5.4.2.1: Moon Perturbation**

# SENSITIVITY ANALYSIS OF SATELLITE TRAJECTORY UNDER GRAVITATIONAL AND AERODYNAMIC PERTURBATIONS

STUDENT: M. M. RIBICIC • SUPERVISOR: PROF. M.P. WEST • UNIVERSITY OF WOLLONGONG, DECEMBER 2001



**Three Body Problem (Earth-Moon-satellite), double precision, Geosynchronous orbit with initial conditions: Altitude=35796.86km, H=5, RAAN=322.92deg, i=30deg, AOP=96deg**



These values were further transformed into spherical coordinates – Chapter 4, so the values for satellite longitude and latitude were calculated, with respect to the Earth's centre. The third dimension of this system represents the altitude.

Longitude vs Latitude values are plotted above world's map, giving the clear information for the satellite position at any point of time. The ground tracks were represented as the part of result analysis for three particular cases:

1. Air drag perturbed orbit of ISS
2. As a part of the initial conditions representation for orbit in section 4
3. Geosynchronous orbit.

## **CHAPTER 6**

### **Conclusion and Recommendation**

## **Chapter 6: Conclusion and Further Work Recommendations**

### **6.1. Conclusion**

The main objective of this work is to analyse the dynamics of motion in space, and to develop a method to define the position of a satellite at any time. Some of the physical laws applied in this model were discovered centuries ago; the rest of the laws are now well understood, and have been applied in many scientific models of spacecraft motion. However, the emphasis of this work is to ‘combine’ the existing models and the theories in a way that will lead to the most accurate results. A particularly interesting part of this work is the way that the three body problem was approached. One of the segments of SATELIGHT software developed in this thesis opens a new level for calculation of the change in orbital elements due to the perturbation forces.

Improvements to this model could be easily performed each time a new discovery regarding perturbation forces is made, or when different conditions are to be specified. Solutions can then be obtained for the most general problems of spacecraft flight that could occur in reality. The advantage of the method, with regard to the user, is that, it could be simply, by selecting appropriate initial conditions, adopted for any special purpose.

The introduction of gravitational and non-gravitational perturbations opens a number of possible methods for application of numeric mathematics that analytically could not be solved.

This work synthesises few very different fields that after performed analysis lead to the software SATELIGHT.

Its results are presented in the different formats, and could be manipulated further very effectively.

## **6.2. Further Work Recommendations**

There is the number of possible improvements and directions of development to the model and to software, for example:

1. Air Density model improved by the application of Lagrange polynomials and more sophisticated initial data
2. Orbital elements could be calculated from the new determined numerical values which contain modifications due to the perturbations effects – similar to the method applied to an inclination angle
3. Accuracy of the Runge – Kutta method could be approved further by application of different polynomials (increasing the order) and further error analysis
4. Find the solution to reduce memory required for data transfer from FORTRAN to ACAD.
5. Include next set of perturbative elements: Solar radiation Pressure, Magnetic Disturbances and Micro-meteorites.

6. Relate the deviations from theoretical trajectory, due to a particular perturbation, to the fuel required for orbit correction.
7. Three-dimensional models could be developed further.
8. Observe generation of the Air Drag Coefficient  $C_D$  for the different geometrical shapes.
9. Develop SATELIGHT for orbit transfer and correction computations (related to fuel consumption).

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# **APPENDICES**

**Inertial system defined with respect to another inertial system, if the first one is moving with constant velocity**

Let observe inertial system  $S$  and another system  $S1$  which translates uniformly with velocity  $V=C$ , with respect to the system  $S$ .

It has to be proven here, the system  $S1$  is also inertial system.

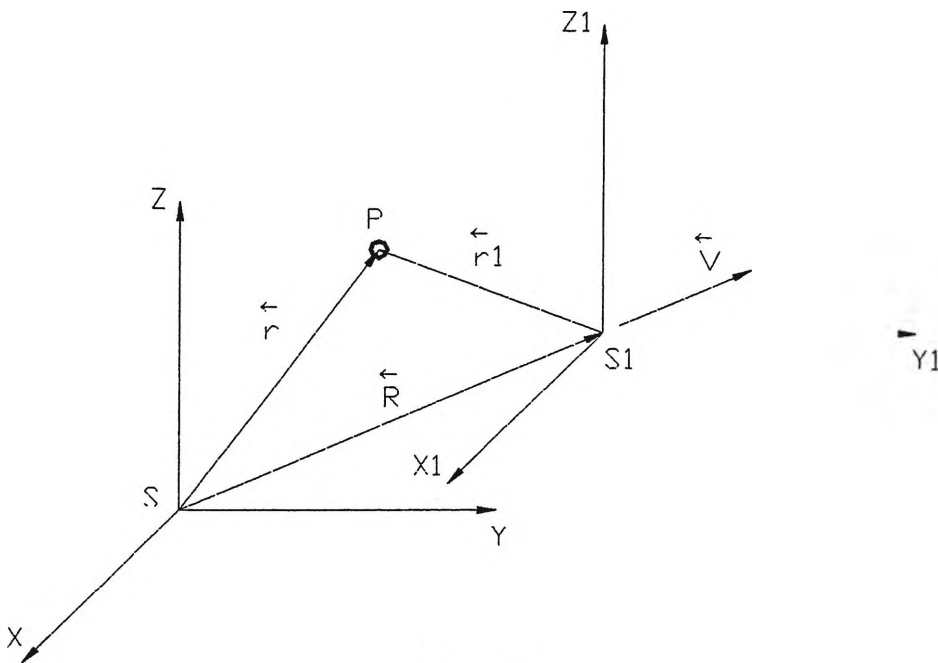


Fig. A2.1

Note that the time in  $S1$  differs only by constant from the time in  $S$ .

The transformation of space from system  $S$  to  $S1$  can be described mathematically as:

$$\vec{r}_1 = \vec{r} - \vec{V}t - \vec{R} \quad (\text{A1.1})$$

which, after derivation gives:

$$\frac{d\vec{r}_1}{dt} = \frac{d\vec{r}}{dt} - \vec{V} - \frac{d\vec{V}}{dt}t - \frac{d\vec{R}}{dt} \quad (\text{A1.2})$$

$\vec{r}_1$  and  $\vec{r}$  are the position vectors of a particle P, with respect to frames origins  $S1$  and  $S$  respectively.

$\vec{R}$  is the constant vector that defines the position of system  $S1$  with respect to system  $S$  at time  $t = 0$ . The transformation of time from  $S$  to  $S1$  is given by:

$$t_1 = t + T \quad (\text{A1.3})$$

where  $T$  is constant time difference between  $S$  and  $S1$ .

If for particle  $P$  is assumed that there is no external force acting on it, than it could be stated:

$$\vec{V}_P^S = \frac{d\vec{r}}{dt} \quad (\text{A1.4})$$

and

$$\vec{V}_P^{S1} = \frac{d\vec{r}1}{dt1} \quad (\text{A1.5})$$

Therefore:

$$\vec{V}_P^{S1} = \frac{d\vec{r}1}{dt} \frac{dt}{dt1} = \frac{d\vec{r}1}{dt} = \frac{d\vec{r}}{dt} - \vec{V} = \vec{V}_P^S - \vec{V} \quad (\text{A1.6})$$

As both of these velocities are constants, then velocity of the particle  $P$  with respect to the system  $S1$  is constant as well.

### Proof of the invariance of Newton's second law

Consider systems S and S1 where it is supposed that S is inertial system, and S1 translates uniformly with velocity V with respect to S. The time in S1 differs only for a constant from time in S. It has to be proved, the second law of mechanics is obeyed in system S1. In previous attachment it was proved, the system which moves with constant velocity with respect to inertial system is also inertial system, therefore first law is already obeyed. Let define the transformations of space and time from S to S1:

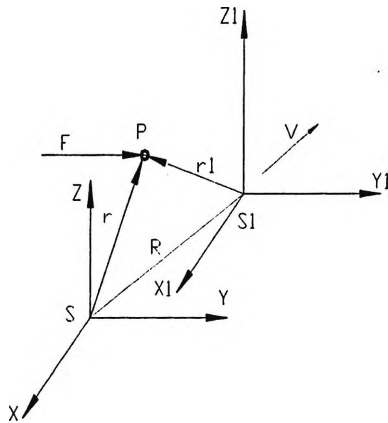


Fig. D2

$$r_1 = r - Vt - R$$

$$t_1 = t + T$$

$$V_r = \frac{dr}{dt} \quad (D.2.1)$$

$$V'_r = \frac{dr_1}{dt_1}$$

If F is force exerted on the particle P in S, and F1 is the force exerted on the same particle, but in the system S1, it will be stated the force is invariant, no matter in which system is it observed.



Therefore

$$F=F1 \quad (D.2.2)$$

iv)

From (D.2.1),

$$r=r1+Vt+R$$

after differentiating and multiplying by m-mass of the particle,

$$m \frac{dr}{dt} = m \frac{dr1}{dt} + mV \quad (D.2.3)$$

Further, change of the linear momentum with respect to the time, from (D.2.3), is

$$\frac{d}{dt} \left( m \frac{dr}{dt} \right) = \frac{d}{dt1} \left( m1 \frac{dr1}{dt1} \right) + V \frac{dm1}{dt1} \quad (D.2.4)$$

By comparing mathematical expression (D.2.4) with next relations:

$$\text{For system S} \quad F = \frac{d}{dt} \left( m \frac{dr}{dt} \right)$$

$$\text{For system S1} \quad F1 = \frac{d}{dt} \left( m1 \frac{dr1}{dt1} \right) + V \frac{dm1}{dt1}$$

it can be concluded the second law is invariant only if the mass m is constant.

### The proof for *Particle Mass* assumption

Observe the system composed of only two particles. The first particle P has a mass  $m_1$  and second particle P1 has a unit mass. The distance between them is defined by vector  $\vec{r}$ , directed from P to P1. Then the force exerted by P on P1 is

$g = -\frac{m}{r^3} \vec{r}$ , which is recognised as the force due to the gravitation. Now, let isolate the particle with the unit mass, P1 and place it into three-dimensional space. At that initial moment, there is no external force acting toward this particle.

Now bring another particle P, with the mass  $m$  into the space where P1 is already placed. At this moment there exists definite force acting from P toward P1. The conclusion is, the space around P has been changed and the change is manifested by the so named, gravitational field.

The strength of the field is characterised by the force exerted on the unit mass. From this fact it also can be concluded, the gravitational field is a vector field, or more precisely force field. To bring this idea closer, the concrete example is introduced.

Suppose that a mass P is fixed at the origin of the three dimensional DeCartesian inertial frame. When P1 was brought at the point  $(x,y,z)$  of the system, where  $x,y,z \neq 0$ , the force F was measured. This force is, as it was mentioned before, the force of gravitational attraction directed toward the particle P, with mass M, placed at the origin. Its magnitude is

$$F(x, y, z) = -\frac{k\vec{r}}{r^3}$$

where

$k=GM$  and  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  therefore  $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ . It has to be noted that,

the force is not defined at  $\bar{r} = 0$  and if  $\bar{r} \rightarrow 0$  then  $\bar{F} \rightarrow \infty$ . This force field is known as an inverse - square force field, and it is illustrated at the Fig. 4.4.2. below:

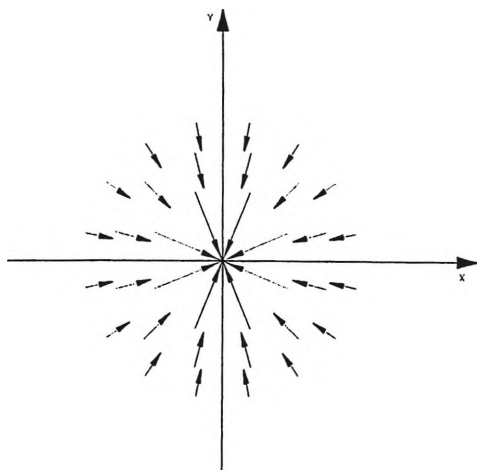


Fig. 2.4.2 Gravitational Force-Field or Vector Field

Let introduce the new terms *Conservative Fields and Potential Functions*. By definition the vector field  $F$  defined on a region  $D$  is called conservative provided that there exists a scalar function  $f$  defined on  $D$  such that

$$\bar{F} = \nabla f \quad (2.4.3)$$

at each point of  $D$ . In this case  $f$  is called a potential function for the vector field  $F$ , where,

$$\nabla = \bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z} \quad (2.4.4)$$

In other words, a field is conservative if the line integral  $\int_A^B \bar{g} d\bar{r}$  is a function of the end points only, which implies that  $\oint \bar{g} d\bar{r} = 0$ . The line integral represents the work done by the force field, as the particle of unit mass moves from  $A$  to  $B$ . As the integrand of this integral is an exact differential it can be stated:

$$\bar{g} d\bar{r} = -dU \quad (2.4.5)$$

$U$  is a scalar quantity called the Potential energy per unit mass, and can be expressed in Cartesian coordinates, as it is a function of position only:

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz \quad (2.4.6)$$

Also, it could be stated

$$\begin{aligned} \vec{r} &= x\vec{i} + y\vec{j} + z\vec{k} \\ \vec{g} &= g_x\vec{i} + g_y\vec{j} + g_z\vec{k} \end{aligned}$$

where vectors  $r$  and  $g$  are projected onto three direction of rectangular system, then

$\vec{g}d\vec{r} = g_x dx + g_y dy + g_z dz$  and combining further

$$g_x = -\frac{\partial U}{\partial x}, g_y = -\frac{\partial U}{\partial y}, g_z = -\frac{\partial U}{\partial z} \quad \text{or } \vec{g} = -\nabla U \quad (2.4.7)$$

which is the same form of the expression as  $\vec{F} = \nabla f$  which defines conservative field.

For gravitational field

$$\int_A^B \vec{g}d\vec{r} = -\int_A^B \frac{Gm}{r^3} \vec{r}d\vec{r} = -\int_A^B \frac{Gm}{r^2} d\vec{r} = Gm \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \quad (2.4.8)$$

Further,

$$U = -\frac{Gm}{r} + C \quad (2.4.9)$$

where  $C$  is chosen to be 0. From the expression (2.4.9) it could be seen the potential energy approaches zero as  $\vec{r} \rightarrow \infty$ .

To find the gravitational force between a sphere and a particle it will be assumed the surface of the sphere  $S$  encloses its volume  $V$  and contains a particle of mass  $m$ . By Gauss Theorem,

$$\int_V \nabla \vec{g} dV = \int_S \vec{g} \cdot \vec{n} ds \quad (2.4.10)$$

Introduce spherical coordinates

$$\begin{aligned} x &= \rho \sin \phi \cos \theta \\ y &= \rho \sin \phi \sin \theta \\ z &= \rho \cos \phi \end{aligned} \quad (2.4.11)$$

the divergence of the vector is given by:

$$\nabla \vec{L} = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} (\rho^2 L_\rho) + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \theta} (L_\theta \sin \theta) + \frac{1}{\rho \sin \theta} \frac{\partial L_\phi}{\partial \phi} \quad (2.4.12)$$

If now,  $\vec{L}$  is compared to  $\vec{g}$  from the expression  $\vec{g}_r = -\frac{Gm}{r^3}\vec{r}$  it follows  $g_\theta = g_\varphi = 0$ , and

$$\begin{aligned}\nabla\vec{g} &= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \left( -\frac{Gm}{r^2} \right) \right) \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} (-Gm) = -\frac{G}{r^2} \frac{\partial m}{\partial r}\end{aligned}\tag{2.4.13}$$

In equation (2.4.10) the volume element can be represented by  $dV = r^2 dr d\Omega$ , where  $d\Omega$  is so-called solid angle, and then

$$\int_S \vec{g} \cdot \vec{n} dS = -G \int_V \frac{1}{r^2} \frac{\partial m}{\partial r} r^2 dr d\Omega = -4\pi Gm\tag{2.4.14}$$

The integral for a number of the particles within S with total mass M is

$$\int_S \vec{g} \cdot \vec{n} dS = -4\pi Gm\tag{2.4.15}$$

It can be seen from this expression,  $\vec{g}$  is the field strength due to all particles contained in S. Assume now that there is another sphere S1 with mass M, concentric with S and with radius  $\bar{r}_1 < \bar{r}$ . If the density of S1 is a function of the distance from the sphere center and is symmetric (what can be adopted for the Earth's sphere, for the first approximation), then  $\vec{g}$  is directed normal to S, has the same magnitude everywhere on S and is directed toward sphere centre. The surface integral  $\iint_S f(x, y, z) dS$  for  $\int_S \vec{g} \cdot \vec{n} dS$  is calculated to be  $-4\pi r^2 g$ . If  $g_r = g\bar{r}$  where  $\bar{r}$  is a radius of sphere S, then

$$\vec{n} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2 + y^2 + z^2}} = \frac{1}{r} (x\vec{i} + y\vec{j} + z\vec{k})\tag{2.4.16}$$

By comparing

$$-4\pi r^2 g = -4\pi GM \Rightarrow g = \frac{GM}{r^2};$$

and finally

$$\vec{g} = \frac{GM}{r^3} \vec{r}\tag{2.4.17}$$

Therefore, the conclusion is, the gravitational field is the same for a particular radius greater than  $r$  in both cases, when the whole mass is concentrated at the centre of the sphere, or if it is symmetrically distributed in its volume. This implies, Newton's law of Gravitation can be applied for Earth's satellites.

## Determination of First and Second Constant of Motion

Let multiply equation (2.5.2) vectorially by  $\vec{r}$

$$\vec{r} \times \frac{d^2 \vec{r}}{dt^2} = \vec{r} \times \left( \frac{-GM}{r^3} \vec{r} \right) \quad (2.5.3)$$

From the analytical geometry it is known, the vector product of two collinear vectors is zero, therefore

$$\vec{r} \times \frac{d^2 \vec{r}}{dt^2} = 0 \quad (2.5.4)$$

The expression (2.5.4) could be expressed in a different way:

$$\frac{d}{dt} \left( \vec{r} \times \frac{d\vec{r}}{dt} \right) = 0$$

so,

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{r} \times \vec{V} = \vec{L} = \text{const} \quad (2.5.5)$$

$\vec{L}$  is the angular momentum per unit mass of the particle moving due to exerted gravitational field. From the characteristics of the vectorial product it is known, the two vectors form the plane and their vectorial product is perpendicular to that plane. From last expression it is obvious, the motion occurs in the plane determined by the two vectors,  $\vec{r}$  and  $\vec{V}$ .

To observe the angular momentum of the particle, it is very suitable to introduce the polar coordinates in the plane of motion. The polar coordinates are defined as:

$\vec{r}$  - radius vector of the particle trajectory in the plane of motion

$\varphi$  - the angle that defines the position of the radius vector in the plane.

The tangential velocity in polar coordinates is represented by

$$V_{\varphi} = r \frac{d\varphi}{dt} \quad (2.5.6)$$

Let introduce new term, flight path angle  $\gamma$ , Fig. 2.5.1. This is the angle between the velocity vector  $V$  of the satellite and the plane whose normal is collinear with radius vector of the satellite. With so defined flight path angle and the normal plane, the new expression for the tangential velocity is

$$V_{\phi} = V \cos \gamma \tag{2.5.7}$$

$V_{\phi}$  lies in the plane which is normal on the radius vector, and is also a tangent on the trajectory at that point. Based on (2.5.6) new expression for the magnitude of the angular momentum per unit mass of the particle is:

$$|\vec{L}| = |\vec{r} \times \vec{V}| = rV \cos \gamma = rV_{\phi} = r^2 \frac{d\phi}{dt} \tag{2.5.8}$$

In the case of the orbiting satellite, velocity can be described by magnitude of the velocity vector and two angles which determine the direction of that vector. Set first inertial coordinate system  $X,Y,Z$  in the centre of the Earth, Fig.2.5.1, represented below,

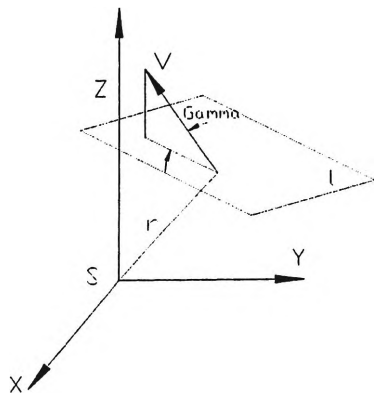


Fig. 2.5.1. Flight angle representation

The position of the satellite is defined by the vector  $\vec{r}$ . From another aspect, the area swept out by the radius vector  $\vec{r}$ , during the time interval  $\Delta t$  is given by:

Fig.2.5.2



$$\Delta A = \frac{1}{2} r^2 \Delta \phi + 0(r \Delta \phi \Delta r) \quad (2.5.9)$$

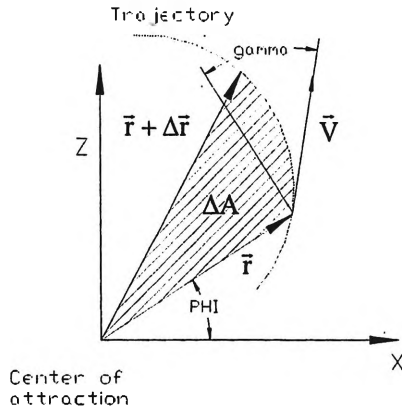


Fig. 2.5.2 Velocity vector of the satellite

If equation 2.5.9. is divided by  $\Delta t \rightarrow 0$ , then

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} \text{ and when compared to (2.5.8) then}$$

$$\frac{dA}{dt} = \frac{1}{2} |\vec{L}| \quad (2.5.10)$$

In words, the change of the area with respect to the time is proportional to the angular velocity which is constant. This statement is in accordance to the second Kepler's law and represents first constant of motion.

Second constant of motion is derived from equation (2.5.2) by scalar multiplication

with  $\frac{d\vec{r}}{dt}$ :

$$\frac{d\vec{r}}{dt} \cdot \frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^3} \vec{r} \cdot \frac{d\vec{r}}{dt} \quad (2.5.11)$$

or

$$\frac{d\vec{r}}{dt} \cdot \frac{d}{dt} \left( \frac{d\vec{r}}{dt} \right) = -\frac{GM}{r^3} \vec{r} \cdot \frac{d\vec{r}}{dt} \quad (2.5.12)$$

$$\frac{1}{2} \frac{d}{dt} \left( \frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt} \right) = -\frac{GM}{r^3} \frac{1}{2} \frac{d}{dt} (\vec{r} \cdot \vec{r}) \quad (2.5.13)$$

From the right side of the last equation, follows:

$$RH = \frac{d}{dt} (GM / r) \quad (2.5.14)$$

Therefore

$$\frac{1}{2} \frac{d}{dt} \left( \frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt} \right) = \frac{d}{dt} \left( \frac{GM}{r} \right)$$

When the last expression is integrated

$$\frac{1}{2} V^2 - \frac{GM}{r} = K = \text{const}$$

The last expression states that the particle possesses a constant difference in energy, between kinetic energy and potential energy per unit mass, therefore this is the law of conservation of energy.

# **APPENDIX 4.1.**

## **Program Orbit**

```
PROGRAM orbit
```

```
! Runge Kutta method IV order used for orbit calculation
```

```
REAL K1,K2,K3,K4,L1,L2,L3,L4
```

```
C READ (5,*) T,X,Y,H,N,VX,VY
```

```
10 FORMAT (I5,2F10.3,2I5,2F10.3)
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE T '
```

```
READ(*,*) T
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE X '
```

```
READ(*,*) X
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE Y '
```

```
READ(*,*) Y
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE H '
```

```
READ(*,*) H
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE N '
```

```
READ(*,*) N
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE VX '
```

```
READ(*,*) VX
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE VY '
```

```
READ(*,*) VY
```

```
WRITE (6,21)
```

```
21 FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
```

```
WRITE (6,22)
```

```
22 FORMAT(T2,'ORBIT')
```

```
DO 100 I=1,N
```

```
X=X+H*VX
```

```
Y=Y+H*VY
```

```
K1=H*F(T,X,Y)
```

```
L1=H*G(T,X,Y)
```

```
K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1)
```

```
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1)
```

```
K3=H*F(T+0.5*H,X+0.5*K2,Y+0.5*L2)
```

```
L3=H*G(T+0.5*H,X+0.5*K2,Y+0.5*L2)
```

```
K4=H*F(T+0.5*H,X+0.5*K3,Y+0.5*L3)
```

```
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3)
```

```
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
```

```
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
```

```
T=T+H
```

```
WRITE (9,23) I,T,X,Y,VX,VY
```

```
23 FORMAT (5X,I5,5E14.6)
```

```
100 CONTINUE
```

```
STOP
```

```
END
```

```
FUNCTION F(T,X,Y)
```

```
F=(-X)*399059.852/(SQRT(X*X+Y*Y))**3
```

```
RETURN
```

```
END
```

```
FUNCTION G(T,X,Y)
```

```
G=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3
```

```
RETURN
```

```
END
```

## **APPENDIX 4.2.**

**Program Orbit LISP**

```
PROGRAM orbitlsp
! Runge Kutta method IV order used for orbit calculation
REAL K1,K2,K3,K4,L1,L2,L3,L4
READ (5,*) T,X,Y,H,N,VX,VY
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE T '
READ(*,*) T
WRITE(*,'(A\)' ) ' ENTER VALUE X '
READ(*,*) X
WRITE(*,'(A\)' ) ' ENTER VALUE Y '
READ(*,*) Y
WRITE(*,'(A\)' ) ' ENTER VALUE H '
READ(*,*) H
WRITE(*,'(A\)' ) ' ENTER VALUE N '
READ(*,*) N
WRITE(*,'(A\)' ) ' ENTER VALUE VX '
READ(*,*) VX
WRITE(*,'(A\)' ) ' ENTER VALUE VY '
READ(*,*) VY
```

```
10 FORMAT (I5,2F10.3,2I5,2F10.3)
WRITE (6,21)
21 FORMAT (/ ,5X,'I' ,5X,'T' ,5X,'X' ,5X,'Y' ,5X,'VX' ,5X,'VY' ,/)
WRITE (6,22)
22 FORMAT(T2,'ORBIT')
```

```
WRITE (9,33)
33 FORMAT('(' 'Setq' )
```

```
DO 100 I=1,N
X=X+H*VX
Y=Y+H*VY
K1=H*F(T,X,Y)
L1=H*G(T,X,Y)
K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1)
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1)
K3=H*F(T+0.5*H,X+0.5*K2,Y+0.5*L2)
L3=H*G(T+0.5*H,X+0.5*K2,Y+0.5*L2)
K4=H*F(T+0.5*H,X+0.5*K3,Y+0.5*L3)
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3)
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
T=T+H
```

```
IF (I.LE.9) WRITE (9,23) I,X,Y
```

```
23 FORMAT ('pt'I1 ' ' ('(F14.3) (F14.3))' )')
```

```
IF (I.LE.99.AND.I.GT.9) WRITE (9,27)I,X,Y
```

```
27 FORMAT ('pt'I2 ' ' ('(F14.3) (F14.3))' )')
```

```
IF (I.LE.999.AND.I.GT.99) WRITE (9,30)I,X,Y
```

```
30 FORMAT ('pt'I3 ' ' ('(F14.3) (F14.3))' )')
```

```
IF (I.GT.999) WRITE (9,32)I,X,Y
32  FORMAT ('pt'I4 ' '(F14.3) (F14.3))')
```

```
100 CONTINUE
```

```
WRITE (9,34)
34  FORMAT(' ',/, ' (' 'command "line" ')
```

```
DO 200 J=1,N
```

```
IF (J.LE.9) WRITE (9,35)J
```

```
35  FORMAT ('pt'I1)
```

```
IF (J.LE.99.AND.J.GT.9) WRITE (9,36)J
```

```
36  FORMAT ('pt'I2)
```

```
IF (J.LE.999.AND.J.GT.99) WRITE (9,37)J
```

```
37  FORMAT ('pt'I3)
```

```
IF (J.GT.999) WRITE (9,38)J
```

```
38  FORMAT ('pt'I4)
```

```
200 CONTINUE
```

```
WRITE (9,39)
```

```
39  FORMAT(' ')
```

```
STOP
```

```
END
```

```
FUNCTION F(T,X,Y)
```

```
F=(-X)*399059.852/(SQRT(X*X+Y*Y))**3
```

```
RETURN
```

```
END
```

```
FUNCTION G(T,X,Y)
```

```
G=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3
```

```
RETURN
```

```
END
```

## **APPENDIX 4.3.**

### **Programs**

- **ADMAINI**
- **AS3I**
- **I2I1**



PROGRAM admaini  
! Runge Kutta method IV order used for orbit calculation

IMPLICIT REAL\*8 (A-H,K,L,O-Z)

REAL K1,K2,K3,K4,L1,L2,L3,L4

C READ (5,\*) T,X,Y,H,N,VX,VY,ZB1,ZINCL  
WRITE(\*,'(A\)' ) ' ENTER VALUE T ' '  
READ(\*,\*) T  
WRITE(\*,'(A\)' ) ' ENTER VALUE X ' '  
READ(\*,\*) X  
WRITE(\*,'(A\)' ) ' ENTER VALUE Y ' '  
READ(\*,\*) Y  
WRITE(\*,'(A\)' ) ' ENTER VALUE H ' '  
READ(\*,\*) H  
WRITE(\*,'(A\)' ) ' ENTER VALUE N ' '  
READ(\*,\*) N  
WRITE(\*,'(A\)' ) ' ENTER VALUE VX ' '  
READ(\*,\*) VX  
WRITE(\*,'(A\)' ) ' ENTER VALUE VY ' '  
READ(\*,\*) VY  
WRITE(\*,'(A\)' ) ' ENTER VALUE LATITUDE ' '  
READ(\*,\*) ZB1  
WRITE(\*,'(A\)' ) ' ENTER VALUE INCLINATION ' '  
READ(\*,\*) ZINCL

10 FORMAT (I5,2F10.3,2I5,3F10.3)

WRITE (6,21)

21 FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)

WRITE (6,22)

22 FORMAT(T2,'ORBIT')

L=1

DO 100 I=1,N

R=(SQRT(X\*X+Y\*Y))-6366.2

IF(R.GE.6471.AND.R.LE.8866)GO TO 9

X=X+H\*VX

Y=Y+H\*VY

K1=H\*F(T,X,Y)

L1=H\*G(T,X,Y)

K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)

L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)

K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)

L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)

K4=H\*F(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3)

L4=H\*G(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3)

VX=VX+1./6.\*(K1+2.\*K2+2.\*K3+K4)

VY=VY+1./6.\*(L1+2.\*L2+2.\*L3+L4)

T=T+H

IF (R.LT.6471.AND.R.GT.8866) WRITE (10,23)T,X,Y,R

REWIND 9

23 FORMAT (5X,4E14.6)

```
IF(R.LT.6471.AND.R.GT.8866)GO TO 99
```

```
9  CALL AS3I(X,Y,VX,VY,H,ZINCL)
   WRITE(*,*) ' IT IS INSIDE THE 99 LOOP '
99  L=L+1

100 CONTINUE
    STOP
    END
```

```
FUNCTION F(T,X,Y)
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
F=(-X)*399059.852/(SQRT(X*X+Y*Y))**3
RETURN
END
```

```
FUNCTION G(T,X,Y)
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
G=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3
RETURN
END
```

□

```
SUBROUTINE AS3I (X, Y, VX, VY, H, ZINCL)
```

```
! Runge Kutta method IV order used for orbit calculation
! INCLUDING AIRDRAG PERTURBATION FOR ALTITUDES 105-2500km
IMPLICIT REAL*8 (A-H, O-Z)
```

```
X=X+H*VX
Y=Y+H*VY
RK1=H*F1 (T, X, Y, DRAGX)
RL1=H*G1 (T, X, Y, DRAGY)
RK2=H*F1 (T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGX)
RL2=H*G1 (T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGY)
RK3=H*F1 (T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGX)
RL3=H*G1 (T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGY)
RK4=H*F1 (T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGX)
RL4=H*G1 (T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGY)
VX=VX+1./6.*(RK1+2.*RK2+2.*RK3+RK4)
VY=VY+1./6.*(RL1+2.*RL2+2.*RL3+RL4)
T=T+H
```

```
R=SQRT (X*X+Y*Y) -6366.2
R1=SQRT (X*X+Y*Y)
VTOT=SQRT (VX*VX+VY*VY)
```

```
VXANG=VX/VTOT
VYANG=VY/VTOT
```

```
FI=1-R1*6.63146E-04/VTOT*COS (ZINCL)
F4=FI*FI
```

```
SD=(0.818+0.25/8)/8*10E-6
```

```
! SD IS GEOMETRICAL CHARACTERISTIC OF THE SPACECRAFT
! IN THIS CASE TAKEN AS CYLINDAR, 1m LONG 1/8m DIA
! CALCULATED AS THE PROJECTION OF THE MEAN AREA,
! AS THE SPACECRAFT
! IS ROTATING IN SPACE
```

```
! ANOTHER CHARACTERISTIC THAT DEPENDS ON THE GEOMETRY IS
! CD - AIRDRAG COEFFICIENT, HERE 2.2 BASED ON EXPERIMENTS
```

```
CALL I2I1 (R, DENSY)
DENSY=2.14E-7
```

```
DRAG=0.5*DENSY*VTOT*VTOT*F4*SD*2.2
DRAG=0.5*1E-9*VTOT*VTOT*F4*SD*2.2
DRAGX=DRAG*VXANG
DRAGY=DRAG*VYANG
```

```
WRITE (10, 23) T, X, Y, R
```

```
FORMAT (5X, 4E14.6)
REWIND 9
```

```
RETURN
```

END

```
FUNCTION F1 (T, X, Y, DRAGX)
  IMPLICIT REAL*8 (A-H, O-Z)
  F1 = (-X) * 399059.852 / (SQRT (X*X+Y*Y) ) **3 + DRAGX
  RETURN
END
```

```
FUNCTION G1 (T, X, Y, DRAGY)
  IMPLICIT REAL*8 (A-H, O-Z)
  G1 = (-Y) * 399059.852 / (SQRT (X*X+Y*Y) ) **3 + DRAGY
  RETURN
END
```

□

```
SUBROUTINE I2I1(Z,DENSY)
```

```
! LINEAR INTERPOLATION USED FOR DETERMINING
! CHARACTERISTICS OF THE ATMOSPHERE
```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
DIMENSION DENS(26,2)
```

```
M1=0
```

```
OPEN(UNIT=9, FILE='DATA.DAT')
```

```
DO 304 J=1,2
```

```
DO 303 I=1,26
```

```
READ(9,10) DENS(I,J)
```

```
10  FORMAT(E16.8)
```

```
M=M+1
```

```
IF(Z.GE.DENS(I-1,2).AND.Z.LE.DENS(I,2))M=M1
```

```
303 CONTINUE
```

```
304 CONTINUE
```

```
C DO 300 I=1,26
```

```
C M11=I
```

```
C IF(Z.EQ.DENS(I,2))GO TO 91
```

```
C 300 CONTINUE
```

```
C M=0
```

```
C M1=0
```

```
C DO 100 I=1,26
```

```
C M=M+1
```

```
C IF(Z.GE.DENS(I,2).AND.Z.LE.DENS(I+1,2))M=M1
```

```
C 100 CONTINUE
```

```
C M=M1
```

```
ALTN=DENS(M1+2,2)-DENS(M1+1,2)
```

```
ALTB=Z-DENS(M1+2,2)
```

```
DENSF=DENS(M1+2,1)-DENS(M1+1,1)
```

```
C DENSY=ALTB*DENSF*10E6
```

```
DENSY=DENS(M1+1,1)+ALTB*DENSF/ALTN
```

```
C WRITE(10,231) DENSY, DENS(M1+2,1), DENS(M1+1,2), ALTB, DENSF, ALTN, Z, M1
```

```
C 231 FORMAT(7E14.6,I5)
```

```
C 91 IF(Z.EQ.DENS(M11,2))DENSY=DENS(M11,1)
```

```
RETURN
```

```
END
```

0.000000214  
0.000000098  
2.45E-08,  
6.58E-09,  
2.60E-09,  
1.40E-09,  
9.00E-10,  
4.58E-10,  
2.67E-10,  
1.66E-10,  
1.07E-10,  
7.10E-11,  
4.80E-11,  
3.30E-11,  
1.38E-11,  
6.23E-12,  
2.97E-12,  
1.48E-12,  
4.05E-13,  
1.21E-13,  
3.85E-14,  
1.32E-14,  
5.05E-15,  
3.57E-16,  
1.17E-16,  
4.91E-17,  
1.05E+02,  
1.10E+02,  
1.20E+02,  
1.30E+02,  
1.40E+02,  
1.50E+02,  
1.60E+02,  
1.80E+02,  
2.00E+02,  
2.20E+02,  
2.40E+02,  
2.60E+02,  
2.80E+02,  
3.00E+02,  
3.50E+02,  
4.00E+02,  
4.50E+02,  
5.00E+02,  
6.00E+02,  
7.00E+02,  
8.00E+02,  
9.00E+02,  
1.00E+03,  
1.50E+03,  
2.00E+03,  
2.50E+03

□

## **APPENDIX 4.4.**

### **Program Orbitanom**

PROGRAM orbitanom

! Runge Kutta method IV order used for orbit calculation  
! including anomalies of Earth gravitational field

REAL K1,K2,K3,K4,L1,L2,L3,L4,LZB2,LZB21,LZB3,LZB31,LZB4,LZB5  
READ (5,\*) T,X,Y,II,N,VX,VY,ZB1

10 FORMAT (I5,2F10.3,2I5,3F10.3)  
WRITE (6,21)  
21 FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',5X,'LAT',/)  
WRITE (6,22)  
22 FORMAT(T2,'ORBITANOM')

DO 100 I=1,N  
X=X+H\*VX  
Y=Y+H\*VY  
K1=H\*F(X,Y,LZB5)  
L1=H\*G(X,Y,LZB5)  
K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)  
L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)  
K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)  
L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)  
K4=II\*F(T+0.5\*II,X+0.5\*K3,Y+0.5\*L3)  
L4=H\*G(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3)  
VX=VX+1./6.\*(K1+2.\*K2+2.\*K3+K4)  
VY=VY+1./6.\*(L1+2.\*L2+2.\*L3+L4)  
T=T+H

ZB1=(ZB1+360./N)\*3.14/180  
LZB2=DSIN(ZB1)  
LZB21=DSIN(2\*ZB1)  
LZB3=LZB2\*LZB2  
LZB31=LZB21\*LZB21  
LZB4=1+0.0053024\*LZB3-0.0000059\*LZB31  
LZB5=LZB4\*397852.4214

WRITE (9,23) I,T,X,Y,VX,VY,LZB4,LZB5,LZB2,LZB21,LZB3,LZB31  
23 FORMAT (5X,I5,11E14.6)

100 CONTINUE  
STOP  
END

! 'GRAVITY' is CHANGED ACCORDING TO  
! THE INTERNATIONAL FORMULA

FUNCTION F(X,Y,LZB5)  
REAL LZB5  
F=(-X)\*LZB5/(SQRT(X\*X+Y\*Y))\*\*3  
RETURN  
END

FUNCTION G(X,Y,LZB5)  
REAL LZB5  
G=(-Y)\*LZB5/(SQRT(X\*X+Y\*Y))\*\*3  
RETURN  
END



## **APPENDIX 4.5.**

**Program Three Body Problem**

PROGRAM THREE BODY PROBLEM

! Runge Kutta method IV order used for orbit calculation

DIMENSION RLONGS(30000), NUM(30000), SFSLGRT(30000)

REAL K1, K2, K3, K4, L1, L2, L3, L4, K1E, K2E, K3E, K4E, L1E, L2E, L3E,  
+L4E, K1M, K2M, K3M, K4M, L1M, L2M, L3M, L4M, LATS, LONGREL, LONGGR

C READ (5, \*) T, X, Y, XE, YE, XM, YM, H, N, VX, VY, VXE, VYE, VXM,  
C +VYM, SMS

WRITE(\*, '(A\)' ) ' ENTER VALUE T '  
READ(\*, \*) T

WRITE(\*, '(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE '  
READ(\*, \*) AP

WRITE(\*, '(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '  
READ(\*, \*) AA

WRITE(\*, '(A\)' ) ' ENTER VALUE XE '  
READ(\*, \*) XE  
WRITE(\*, '(A\)' ) ' ENTER VALUE YE '  
READ(\*, \*) YE

WRITE(\*, '(A\)' ) ' ENTER VALUE XMO '  
READ(\*, \*) XMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE YMO '  
READ(\*, \*) YMO

WRITE(\*, '(A\)' ) ' ENTER VALUE H '  
READ(\*, \*) H

WRITE(\*, '(A\)' ) ' ENTER VALUE VXE '  
READ(\*, \*) VXE  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYE '  
READ(\*, \*) VYE

WRITE(\*, '(A\)' ) ' ENTER VALUE VXMO '  
READ(\*, \*) VXMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYMO '  
READ(\*, \*) VYMO

WRITE(\*, '(A\)' ) ' ENTER VALUE SMS '  
READ(\*, \*) SMS

WRITE(\*, '(A\)' ) ' ENTER VALUE RAAN '  
READ(\*, \*) ALFA

WRITE(\*, '(A\)' ) ' ENTER VALUE INCLINATION '  
READ(\*, \*) BETA

WRITE(\*, '(A\)' ) ' ENTER VALUE ARG OF PER '  
READ(\*, \*) GAMA

## Program Three Body Problem

```
WRITE(*, '(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*, *) N1
```

```
C OPEN (M1)
```

```
! CALCULATE THE POSITION OF THE CENTRE OF THE MASS, COMPARE IT WITH
ASSUMED
! CENTRE OF THE SATELLITE ORBIT- CN'N: WHAT ABOUT AXES MODIFICATION?
```

```
EMS=5.9761E24
RMMS=7.3534E22
```

```
! THE PART OF THE PROGRAM THAT WILL RECALCULATE VALUES OF THE MOON
POSITION
! W.R.T. THE SYSTEM IN SATELLITE ORBIT PLANE CONSISTS OF:
! 1. DEFINE POSITION OF THE MOON IN ITS ORBIT PLANE, IE: X=384749.9km,
Y-0, Z-0
! THERE IS ALSO A PARTICULAR VELOCITY ASSOCIATED TO THIS POSITION WHICH
WILL BE
! TRANSFORMED BY THE SAME EQUATIONS.
! 2. LET IGNORE THE OTHER TWO ROTATIONS, AND PERFORM ONLY THAT ONE FOR THE
! INCLINATION ANGLE (6.65DEG RELATIVE TO THE EQUATOR, ACCORDING TO THE
HANDBOOK)
! 3. NEXT ROTATION WOULD BRING THE MOON PLANE COORDINATES TO THE SATELLITE
SYSTEM
! PERFORMED BY THE SAME SPACE TRANSFORMATIONS, BUT WITH TRANSPOSED
EQUATIONS
! THEORETICAL APPROACH IS IN THE THESIS 'THREE BODY PROBLEM' SECTION
! ONCE THE INITIAL VALUE IS GIVEN, ONE COMPLETE SATELLITE ORBIT WILL BE
COMPUTED, AND
! ON THE BEGINNING OF THE NEXT ONE NEW INCLINATION ANGLE WILL BE
DETERMINED
```

```
ALFA1=0
BETA1=6.65
GAMA1=0
! MOON ORBIT ELEMENTS
```

```
AM=ALFA1*0.01744444
BM=BETA1*0.01744444
GRM=GAMA1*0.01744444
```

```
XM00=(COS(AM)*COS(GRM)-SIN(AM)*COS(BM)*SIN(GRM))*XM0+
+(-COS(AM)*SIN(GRM)-SIN(AM)*COS(BM)*COS(GRM))*YM0
YM00=(SIN(AM)*COS(GRM)+COS(AM)*COS(BM)*SIN(GRM))*XM0+(-SIN(AM)*
+SIN(GRM)+COS(AM)*COS(BM)*COS(GRM))*YM0
ZM00=(SIN(BM)*SIN(GRM))*XM0+(SIN(BM)*COS(GRM))*YM0
```

```
VXM00=(COS(AM)*COS(GRM)-SIN(AM)*COS(BM)*SIN(GRM))*VXM0+
+(-COS(AM)*SIN(GRM)-SIN(AM)*COS(BM)*COS(GRM))*VYM0
VYM00=(SIN(AM)*COS(GRM)+COS(AM)*COS(BM)*SIN(GRM))*VXM0+
+(-SIN(AM)*SIN(GRM)+COS(AM)*COS(BM)*COS(GRM))*VYM0
```

VZM00=(SIN(BM)\*SIN(GRM))\*VXM0+(SIN(BM)\*COS(GRM))\*VYM0

XM=(COS(GR)\*COS(A)-SIN(A)\*SIN(GR)\*COS(B))\*XM00+(SIN(GR)\*COS(B)\*  
 +COS(A)+COS(GR)\*SIN(A))\*YM00+(SIN(GR)\*SIN(B))\*ZM00  
 YM=(-SIN(GR)\*COS(A)-SIN(A)\*COS(B)\*COS(GR))\*XM00+(-SIN(GR)\*SIN(A)  
 ++COS(B)\*COS(GR)\*COS(A))\*YM00+(COS(GR)\*SIN(B))\*ZM00  
 ZM=(SIN(A)\*SIN(B))\*XM00+(-SIN(B)\*COS(A))\*YM00+COS(B)\*ZM00

VXM=(COS(GR)\*COS(A)-SIN(A)\*SIN(GR)\*COS(B))\*VXM00+(SIN(GR)\*  
 +COS(B)\*COS(A)+COS(GR)\*SIN(A))\*VYM00+(SIN(GR)\*SIN(B))\*VZM00  
 VYM=(-SIN(GR)\*COS(A)-SIN(A)\*COS(B)\*COS(GR))\*VXM00+(-SIN(GR)\*  
 +SIN(A)+COS(B)\*COS(GR)\*COS(A))\*VYM00+(COS(GR)\*SIN(B))\*VZM00  
 VZM-(SIN(A)\*SIN(B))\*VXM00+(-SIN(B)\*COS(A))\*VYM00+COS(B)\*VZM00

Y=0

X=AP+6366.2  
 XM1=AA+6366.2

RMIN=X  
 RMAX=XM1

N=SQRT(((X+XM1)\*\*3)\*1.2411954E-5)/H

VX=0

VY=SQRT((0.00981\*(6366.2\*\*2)\*XM1)/((X+XM1)/2\*X))  
 VYM=SQRT((198792.3045\*X)/((X+XM1)\*XM1))

! try to determine the satellite position for the geosynchronised orbit  
 ! the initial condiditons are: T=0, (X=42205.1713km, Y=0), XE=0, YE=0,

H=5,

! N=17280, VX=0, VY=3.06925, VXE=0, VYE=0, VXM=0, VYM=1.024, SMS=0

! the observed orbit is of very high altitude, therefore the effect due to  
 ! atmospheric drag and gravitational anomalies will be ignored and only

Lunar

! impact will be observed

10 FORMAT (I5,6F10.3,2I5,7F10.3)  
 WRITE (6,21)  
 21 FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)  
 WRITE (6,22)  
 22 FORMAT (T2,'ORBIT')

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE  
 !TRACKS COULD BE PLOTTED

RPER=SQRT(9.87\*(RMIN+RMAX)\*\*3/397584.3246)

RNUM=RPER/H

M=0

M12=0

NUM=N/RNUM

WRITE (9,33)

FORMAT('(' 'Setq' )

DO 200 I1=1,N1

M12=M12+1

DO 100 I=1,N

BR=RNUM/4

M=M+1

X=X+H\*VX

Y=Y+H\*VY

XE=XE+H\*VXE

YE=YE+H\*VYE

XM=XM+H\*VXM

YM=YM+H\*VYM

K1=H\*F(T,X,Y,XE,YE,XM,YM)

L1=H\*G(T,X,Y,XE,YE,XM,YM)

K1E=H\*FE(T,X,Y,XE,YE,XM,YM,SMS)

L1E=H\*GE(T,X,Y,XE,YE,XM,YM,SMS)

K1M=H\*FM(T,X,Y,XE,YE,XM,YM,SMS)

L1M=H\*GM(T,X,Y,XE,YE,XM,YM,SMS)

K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M)

L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M)

K2E=H\*FE(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

L2E=H\*GE(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K2M=H\*FM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

L2M=H\*GM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E  
+,XM+0.5\*K2M,YM+0.5\*L2M)

L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E  
+,XM+0.5\*K2M,YM+0.5\*L2M)

## Program Three Body Problem

```
K3E=H*FE (T+0.5*H,X+0.5*K2,Y+0.5*L2,XE+0.5*K2E,YE+0.5*L2E
+,XM+0.5*K2M,YM+0.5*L2M,SMS)
L3E=H*GE (T+0.5*H,X+0.5*K2,Y+0.5*L2,XE+0.5*K2E,YE+0.5*L2E
+,XM+0.5*K2M,YM+0.5*L2M,SMS)
```

```
K3M=H*FM (T+0.5*H,X+0.5*K2,Y+0.5*L2,XE+0.5*K2E,YE+0.5*L2E
+,XM+0.5*K2M,YM+0.5*L2M,SMS)
L3M=H*GM (T+0.5*H,X+0.5*K2,Y+0.5*L2,XE+0.5*K2E,YE+0.5*L2E
+,XM+0.5*K2M,YM+0.5*L2M,SMS)
```

```
K4=H*F (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3M,YM+0.5*L3M)
L4=H*G (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3M,YM+0.5*L3M)
```

```
K4E=H*FE (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3E,YM+0.5*L3M,SMS)
L4E=H*GE (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3M,YM+0.5*L3M,SMS)
```

```
K4M=H*FM (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3M,YM+0.5*L3M,SMS)
L4M=H*GM (T+0.5*H,X+0.5*K3,Y+0.5*L3,XE+0.5*K3E,YE+0.5*L3E
+,XM+0.5*K3M,YM+0.5*L3M,SMS)
```

```
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
```

```
VXE=VXE+1./6.*(K1E+2.*K2E+2.*K3E+K4E)
VYE=VYE+1./6.*(L1E+2.*L2E+2.*L3E+L4E)
```

```
VXM=VXM+1./6.*(K1M+2.*K2M+2.*K3M+K4M)
VYM=VYM+1./6.*(L1M+2.*L2M+2.*L3M+L4M)
```

```
XC= ((SMS*X+EMS*XE+RMMS*XM) / (SMS+EMS+RMMS))
YC= ((SMS*Y+EMS*YE+RMMS*YM) / (SMS+EMS+RMMS))
```

```
A=ALFA*0.01744444
B=BETA*0.01744444
GR=GAMA*0.01744444
```

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

```
XGL= (COS (A) *COS (GR) -SIN (A) *COS (B) *SIN (GR) ) *X+
+ (-COS (A) *SIN (GR) -SIN (A) *COS (B) *COS (GR) ) *Y
YGL= (SIN (A) *COS (GR) +COS (A) *COS (B) *SIN (GR) ) *X+ (-SIN (A) *
+SIN (GR) +COS (A) *COS (B) *COS (GR) ) *Y
ZGL= (SIN (B) *SIN (GR) ) *X+ (SIN (B) *COS (GR) ) *Y
```

!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE  
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS  
!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME

```
R=SQRT (XGL*XGL+YGL*YGL+ZGL*ZGL)
```

```
L=1
```

```
RLONGS (I) =90+57.32498682*DATAN (YGL/XGL)
```

```
IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I)=RLONGS(I)+180
IF(RLONGS(I-1).GT.RLONGS(I).AND.RLONGS(I-1).GT.300)RLONGS(I)
==RLONGS(I)+360
```

```
IF(RLONGS(I).GT.380)RLONGS(I)-RLONGS(I)-540
```

```
! IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I-1)=RLMAX
! IF(RLONGS(I).GE.RLMAX)L=L+1
! IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I)=L*180+2*RLONGS(I)-90
```

```
!THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE
!THE VALUES WHEN RLONGS(I) DECREASES, AS IT USES
! IF (RLONGS(I).GE.RLMAX)L=L+1
```

```
! IF(I.GE.BR)RLONGS(I)=RLONGS(I)+90
```

```
! IF(I.GE.2*BR.AND.I.LE.3*BR)RLONGS(I)=RLONGS(I)+180
```

```
! IF(I.GE.3*BR)RLONGS(I)-RLONGS(I)+270
```

```
! IF(M.GT.146.AND.M.LT.873)RLONGS(I)=RLONGS(I)-180
```

```
! IF(RLONGS(I).LT.0.AND.RLONGS(I-1).GE.269)RLONGS(I)=360
! ++RLONGS(I)
```

```
! IF(RLONGS(I).GT.0.AND.RLONGS(I-1).LE.360)RLONGS(I)=360
! ++RLONGS(I)
```

```
LATS=57.32498682*DACOS(ZGL/R)-90
```

```
!THE SPHERICAL COORDINATES OF THE POINT W AT THE EARTH'S SURFACE ARE
```

```
! LONGW=XLONG+360*0.000072921152*T/6.28
! LATW=XLATT
```

```
! RELATIVE SATELLITE POSITION WILL DEPEND ON THE RATE OF EARTH
! ROTATION, THEREFORE EARTH ROTATION AT THE PARTICULAR MOMENT
! IS GIVEN BY:
```

```
! EOMEGA=360/(24*60*60)
```

```
! ANGULAR VELOCITY CALCULATED IN DEG/SEC
```

```
T=T+H
ELONG=T*360/(24*60*60)+RLONGS(1)
NUM1=ELONG/360
```

```
IF(ELONG.GT.360)ELONG=ELONG-NUM1*360
```

```
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:
```

```
!     ELGRT=RLONGS(1)-ELONG
```

```
     SFSLGRT(I)=RLONGS(I)-ELONG
     IF(ABS(ELONG-RLONGS(I)).GT.350)
+SFSLGRT(I)=RLONGS(I)-ELONG+360
```

```
!     LATREL=LATS-LATW
!     LONGREL=RLONGS(I)-LONGW
!     LONGGR=RLONGS(I)-T*0.0041801847
!     LONGREL1=RLONGS(I)-LONGGR
```

```
ALT=R-6366.2
```

```
!     WRITE (9,23) I,T,X,Y,ALT,RLONGS(I),ELONG,SFSLGRT(I),LATS
```

```
WRITE (9,23) T,X,Y,VX,VY,XE,YE,XM,YM,ALT,RLONGS(I),SFSLGRT(I),LATS
23  FORMAT (5X,13E14.6)
```

```
100  CONTINUE
```

```
RPER=AP+6366.2
RAP=AA+6366.2
```

```
ZB=RAP+RPER
ZRZ=RAP-RPER
RAZ=1-(ZRZ/ZB)*(ZRZ/ZB)
IF(ZRZ.EQ.0)GO TO 111
B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(399059.852*0.5*ZB*RAZ))
111 B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(399059.852*0.5*ZB))
SINCL=B-B1
```

```
WRITE (*,309) SINCL
309  FORMAT( 5X 'SINCL=' (F10.3))
```

```
WRITE (*,3099) XC,YC
3099  FORMAT( 5X 'CENT. OF MASS =' (2F10.3))
```

```
200  CONTINUE
```

```
M121=M121+1
```

```
STOP
END
```

```
FUNCTION F(T,X,Y,XE,YE,XM,YM)
```



```

F=(XE-X)*398785.2/(SQRT((XE-X)**2+(YE-Y)**2)**3)
++(XM-X)*4906.92/(SQRT((XM-X)**2+(YM-Y)**2)**3)

RETURN
END
FUNCTION G(T,X,Y,XE,YE,XM,YM)
G=(YE-Y)*398785.2/(SQRT((XE-X)**2+(YE-Y)**2)**3)
++(YM-Y)*4906.92/(SQRT((XM-X)**2+(YM-Y)**2)**3)
RETURN
END

FUNCTION FE(T,X,Y,XE,YE,XM,YM,SMS)
FE=(XM-XE)*4906.92/(SQRT((XE-XM)**2+(YE-YM)**2)**3)
++(X-XE)*6.673E-20*SMS/(SQRT((XE-X)**2+(YE-Y)**2)**3)
RETURN
END

FUNCTION GE(T,X,Y,XE,YE,XM,YM,SMS)
GE=(YM-YE)*4906.92/(SQRT((XE-XM)**2+(YE-YM)**2)**3)
++(Y-YE)*6.673E-20*SMS/(SQRT((XE-X)**2+(YE-Y)**2)**3)
RETURN
END

FUNCTION FM(T,X,Y,XE,YE,XM,YM,SMS)
FM=(X-XM)*6.673E-20*SMS/(SQRT((XM-X)**2+(YM-Y)**2)**3)
++(XE-XM)*398785.2/(SQRT((XM-XE)**2+(YM-YE)**2)**3)
RETURN
END

FUNCTION GM(T,X,Y,XE,YE,XM,YM,SMS)
GM=(Y-YM)*6.673E-20*SMS/(SQRT((XM-X)**2+(YM-Y)**2)**3)
++(YE-YM)*398785.2/(SQRT((XM-XE)**2+(YM-YE)**2)**3)
RETURN
END

```

□

## **APPENDIX 4.6.**

**Program GTRTGPH**

Program GTRTGPH

```
PROGRAM gtrTGPH
DIMENSION RLONGS(30000),NIM(30000),FSLGRT(30000)
```

```
! Runge Kutta method IV order used for orbit calculation
REAL K1,K2,K3,K4,L1,L2,L3,L4,LATS
REAL LONGW,LATW, LONGREL,LATREL, LONGREL1, LONGGR
RLONGS(T-1)=0
```

```
READ (5,*) T,X,Y,H,N,VX,VY
```

```
WRITE(*,'(A\)' ) ' IS APOGEE INITIAL POINT ENTER 1 FOR YES '
READ(*,*) Y
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE T '
READ(*,*) T
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE '
READ(*,*) AP
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '
READ(*,*) AA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE OF TIME STEP H '
READ(*,*) H
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE RAAN '
READ(*,*) ALFA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE INCLINATION '
READ(*,*) BETA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE ARG OF PER '
READ(*,*) GAMA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1
```

```
Y=0
```

```
X=AP+6366.2
XM=AA+6366.2
```

```
RMIN=X
RMAX=XM
```

```
N=SQRT(((X+XM)**3)*1.2411954E-5)/H
```

```
VX=0
```

```
VY=SQRT((0.00981*(6366.2**2)*XM)/((X+XM)/2*X))
VYM=SQRT((198792.3045*X)/((X+XM)*XM))
```

```
IF(Y.EQ.1)VY=VYM
```

```
FORMAT (I5,2F10.3,2I5,2F10.3)
```

```

21 WRITE (6,21)
   FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
22 WRITE (6,22)
   FORMAT(T2,'ORBIT')

```

```

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE
!TRACKS COULD BE PLOTTED

```

```

RPER=SQRT(9.87*(RMIN+RMAX)**3/397584.3246)
RNUM=RPER/H

```

```

M=0

```

```

NUM=N/RNUM

```

```

DO 200 I1=1,N1

```

```

DO 100 I=1,N

```

```

BR=RNUM/4

```

```

M=M+1

```

```

X=X+H*VX
Y=Y+H*VY
K1=H*F(T,X,Y)
L1=H*G(T,X,Y)
K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1)
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1)
K3=H*F(T+0.5*H,X+0.5*K2,Y+0.5*L2)
L3=H*G(T+0.5*H,X+0.5*K2,Y+0.5*L2)
K4=H*F(T+0.5*H,X+0.5*K3,Y+0.5*L3)
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3)
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)

```

```

A=ALFA*0.01744444
B=BETA*0.01744444
GR=GAMA*0.01744444

```

```

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

```

```

XGL=(COS(A)*COS(GR)-STN(A)*COS(B)*STN(GR))*X+
+(-COS(A)*SIN(GR)-SIN(A)*COS(B)*COS(GR))*Y
YGL=(SIN(A)*COS(GR)+COS(A)*COS(B)*SIN(GR))*X+(-SIN(A)*
*SIN(GR)+COS(A)*COS(B)*COS(GR))*Y
ZGL=(SIN(B)*SIN(GR))*X+(SIN(B)*COS(GR))*Y

```

```

!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS
!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME

```

```

R=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)

```

```

L=1

```

```

RLONGS(I)=90+57.32498682*DATAN(YGL/XGL)

```

```

IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =RLONGS (I) +180
IF (RLONGS (I-1) .GT. RLONGS (I) .AND. RLONGS (I-1) .GT. 300) RLONGS (I)
+=RLONGS (I) +360

```

```

IF (RLONGS (I) .GT. 380) RLONGS (I) =RLONGS (I) -540

```

```

C IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I-1) =RLMAX
C IF (RLONGS (I) .GE. RLMAX) L=L+1
C IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =L*180+2*RLONGS (I) -90

```

```

!THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE
!THE VALUES WHEN RLONGS (I) DECREASES, AS IT USES
! IF (RLONGS (I) .GE. RLMAX) L=L+1

```

```

C IF (I .GE. BR) RLONGS (I) =RLONGS (I) +90
C IF (I .GE. 2*BR .AND. I .LE. 3*BR) RLONGS (I) =RLONGS (I) +180
C IF (I .GE. 3*BR) RLONGS (I) =RLONGS (I) +270

```

```

C IF (M .GT. 146 .AND. M .LT. 873) RLONGS (I) =RLONGS (I) -180
C IF (RLONGS (I) .LT. 0 .AND. RLONGS (I-1) .GE. 269) RLONGS (I) =360
C ++RLONGS (I)
C IF (RLONGS (I) .GT. 0 .AND. RLONGS (I-1) .LE. 360) RLONGS (I) =360
C ++RLONGS (I)

```

```

LATS=57.32498682*DACOS (ZGL/R) -90

```

```

!THE SPHERICAL COORDINATES OF THE POINT W AT THE EARTH'S SURFACE ARE

```

```

C LONGW=XLONG+360*0.000072921152*T/6.28
CE LATW=XLATT

```

```

! RELATIVE SATELLITE POSITION WILL DEPEND ON THE RATE OF EARTH
! ROTATION, THEREFORE EARTH ROTATION AT THE PARTICULAR MOMENT
! IS GIVEN BY:

```

```

C EOMEGA=360/(24*60*60)
! ANGULAR VELOCITY CALCULATED IN DEG/SEC
T=T+H
ELONG=T*360/(24*60*60)
NUM1=ELONG/360

```

```

IF (ELONG .GT. 360) ELONG=ELONG-NUM1*360

```

! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM  
 ! ZERO, BUT WHERE THE ZERO IS?  
 ! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS  
 ! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:

C ELGRT=RLONGS (1) -ELONG

FSLGRT (I)=RLONGS (I) -ELONG-RLONGS (I)

C LATREL=LATS-LATW

C LONGREL=RLONGS (J) -LONGW

C LONGGR=RLONGS (I) -T\*0.0041801847

C LONGREL1=RLONGS (I) -LONGGR

ALT=R-6366.2

WRITE (9,23) I, T, X, Y, ALT, RLONGS (I), ELONG, FSLGRT (I), LATS

23 FORMAT (5X, I5, 8E14.6)

100 CONTINUE

C ELONG=ERTHM

200 CONTINUE

C IF(N.GT.RPER) GO TO 101

STOP

END

FUNCTION F(T,X,Y)

F=(-X)\*399059.852/(SQRT(X\*X+Y\*Y))\*\*3

RETURN

END

FUNCTION G(T,X,Y)

G=(-Y)\*399059.852/(SQRT(X\*X+Y\*Y))\*\*3

RETURN

END

□

## **APPENDIX 4.7.**

**Programs that combine Gravitational  
Anomalies with the Air Drag**

PROGRAM GRADMAINI  
! Runge Kutta method IV order used for orbit calculation

IMPLICIT REAL\*8 (A-H,K,L,O-Z)  
DIMENSION RLONGS(30000),NUM(30000),SFSLGRT(30000)  
C IMPLICIT REAL\*8 (A-H,K,L,O-Z)

REAL K1,K2,K3,K4,L1,L2,L3,L4,LATS  
REAL LONGW,LATW, LONGREL, LATREL, LONGREL1, LONGGR  
RLONGS(I-1)=0

C READ (5,\*) T,X,Y,H,N,VX,VY,ZB1,ZINCL

WRITE(\*,'(A\)' ) ' ENTER VALUE T ' '  
READ(\*,\*) T

WRITE(\*,'(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE ' '  
READ(\*,\*) AP

WRITE(\*,'(A\)' ) ' ENTER VALUE APOGEE ALTITUDE ' '  
READ(\*,\*) AA

WRITE(\*,'(A\)' ) ' ENTER VALUE H ' '  
READ(\*,\*) H

WRITE(\*,'(A\)' ) ' ENTER VALUE RAAN ' '  
READ(\*,\*) ALFA

WRITE(\*,'(A\)' ) ' ENTER VALUE INCLINATION ' '  
READ(\*,\*) BETA

WRITE(\*,'(A\)' ) ' ENTER VALUE ARG OF PER ' '  
READ(\*,\*) GAMA

WRITE(\*,'(A\)' ) ' ENTER VALUE ORBIT No. ' '  
READ(\*,\*) N1

OPEN (M1)

Y=0

X=AP+6366.2  
XM=AA+6366.2

RMIN=X  
RMAX=XM

N=SQRT(((X+XM)\*\*3)\*1.2411954E-5)/H

VX=0

VY=SQRT((0.00981\*(6366.2\*\*2)\*XM)/((X+XM)/2\*X))  
VYM=SQRT((198792.3045\*X)/((X+XM)\*XM))



```

10  FORMAT (I5,2F10.3,2I5,3F10.3)

WRITE (6,21)
21  FORMAT (/ ,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
WRITE (6,22)
22  FORMAT(T2,'ORBIT')

L=1

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE
!TRACKS COULD BE PLOTTED

RPER=SQRT(9.87*(RMIN+RMAX)**3/397584.3246)
RNUM=RPER/H

M=0

NUM=N/RNUM

DO 200 I1=1,N1

DO 100 I=1,N

R=(SQRT(X*X+Y*Y))-6366.2
IF(R.GE.105.AND.R.LE.2500)GO TO 9

BR=RNUM/4

M=M+1

X=X+H*VX
Y=Y+H*VY
K1=H*F(T,X,Y,GAN)
L1=H*G(T,X,Y,GAN)
K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1,GAN)
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1,GAN)
K3=H*F(T+0.5*H,X+0.5*K2,Y+0.5*L2,GAN)
L3=H*G(T+0.5*H,X+0.5*K2,Y+0.5*L2,GAN)
K4=H*F(T+0.5*H,X+0.5*K3,Y+0.5*L3,GAN)
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3,GAN)
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
T=T+H

A=ALFA*0.01744444
B=BETA*0.01744444
GR=GAMA*0.01744444

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

XGL=(COS(A)*COS(GR)-SIN(A)*COS(B)*SIN(GR))*X+
+(-COS(A)*SIN(GR)-SIN(A)*COS(B)*COS(GR))*Y
YGL=(SIN(A)*COS(GR)+COS(A)*COS(B)*SIN(GR))*X+(-SIN(A)*
+SIN(GR)+COS(A)*COS(B)*COS(GR))*Y
ZGL=(SIN(B)*SIN(GR))*X+(SIN(B)*COS(GR))*Y

!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS

```

!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME

RG=SQRT (XGL\*XGL+YGL\*YGL+ZGL\*ZGL)

L=1

RLONGS (I)=90+57.32498682\*DATAN (YGL/XGL)

IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =RLONGS (I)+180

IF (RLONGS (I-1) .GT. RLONGS (I) .AND. RLONGS (I-1) .GT. 300) RLONGS (I)  
==RLONGS (I)+360

IF (RLONGS (I) .GT. 380) RLONGS (I) =RLONGS (I) -540

IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I-1) =RLMAX

IF (RLONGS (I) .GE. RLMAX) L=L+1

IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =L\*180+2\*RLONGS (I) -90

!THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE

!THE VALUES WHEN RLONGS (I) DECREASES, AS IT USES

! IF (RLONGS (I) .GE. RLMAX) L=L+1

IF (I .GE. BR) RLONGS (I) =RLONGS (I) +90

IF (I .GE. 2\*BR .AND. I .LE. 3\*BR) RLONGS (I) =RLONGS (I) +180

IF (I .GE. 3\*BR) RLONGS (I) =RLONGS (I) +270

IF (M .GT. 146 .AND. M .LT. 873) RLONGS (I) =RLONGS (I) -180

IF (RLONGS (I) .LT. 0 .AND. RLONGS (I-1) .GE. 269) RLONGS (I) =360  
++RLONGS (I)

IF (RLONGS (I) .GT. 0 .AND. RLONGS (I-1) .LE. 360) RLONGS (I) =360  
++RLONGS (I)

LATS=57.32498682\*DACOS (ZGL/RG) -90

ELONG=T\*360/(24\*60\*60) +RLONGS (1)

NUM1=ELONG/360

IF (ELONG .GT. 360) ELONG=ELONG-NUM1\*360

! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM

! ZERO, BUT WHERE THE ZERO IS?

! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS

! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:

ELGRT=RLONGS (1) -ELONG

SFSLGRT (I) =RLONGS (I) -ELONG

```
IF (ABS (ELONG-RLONGS (I)) .GT. 350)
+SFSLGRT (I)=RLONGS (I) -ELONG+360
```

```
LATREL=LATS-LATW
LONGREL=RLONGS (I) -LONGW
LONGGR=RLONGS (I) -T*0.0041801847
LONGREL1=RLONGS (I) -LONGGR
```

```
ALT=RG-6366.2
```

```
IF (R.GE.2500) WRITE
(10,23) I, T, X, Y, VX, VY, ALT, RLONGS (I), ELONG, SFSLGRT (I), LATS
```

```
REWIND 9
```

```
23 FORMAT (5X, 6E14.6)
```

```
IF (R.GE.2500) GO TO 99
```

```
9 IF (R.GT.105.AND.R.LT.2500) CALL AS3I (X, Y, VX, VY, H, ZINCL)
IF (R.GE.105.AND.R.LE.2500) WRITE (*, *) ' IT IS INSIDE THE 99 LOOP '
```

```
99 L=L+1
```

```
GAN=9.78*(1+0.0053024*((SIN(LATS))**2)-(5.9E-6)*
+(SIN(2*LATS))**2)*(6367.445**2)/5.9761E+24
```

```
3098 WRITE (*, 3098) GAN
FORMAT (5X 'g=f(LATITUDE) IS ' (F10.3))
```

```
100 CONTINUE
```

```
200 CONTINUE
```

```
STOP
END
```

```
FUNCTION F(T, X, Y, GAN)
IMPLICIT REAL*8 (A-H, K, L, O-Z)

F=(-X)*GAN*40678.884/(SQRT(X*X+Y*Y))**3
RETURN
END
```

```
FUNCTION G(T, X, Y, GAN)
IMPLICIT REAL*8 (A-H, K, L, O-Z)

G=(-Y)*GAN*40678.884/(SQRT(X*X+Y*Y))**3
RETURN
END
```

SUBROUTINE AS3I (X, Y, VX, VY, H, ZINCL)

! Runge Kutta method IV order used for orbit calculation  
! INCLUDING AIRDRAG PERTURBATION FOR ALTITUDES 105-2500km  
IMPLICIT REAL\*8 (A-H, O-Z)

X=X+H\*VX  
Y=Y+H\*VY  
RK1=H\*F1 (T, X, Y, DRAGX)  
RL1=H\*G1 (T, X, Y, DRAGY)  
RK2=H\*F1 (T+0.5\*H, X+0.5\*RK1, Y+0.5\*RL1, DRAGX)  
RL2=H\*G1 (T+0.5\*H, X+0.5\*RK1, Y+0.5\*RL1, DRAGY)  
RK3=H\*F1 (T+0.5\*H, X+0.5\*RK2, Y+0.5\*RL2, DRAGX)  
RL3=H\*G1 (T+0.5\*H, X+0.5\*RK2, Y+0.5\*RL2, DRAGY)  
RK4=H\*F1 (T+0.5\*H, X+0.5\*RK3, Y+0.5\*RL3, DRAGX)  
RL4=H\*G1 (T+0.5\*H, X+0.5\*RK3, Y+0.5\*RL3, DRAGY)  
VX=VX+1./6.\*(RK1+2.\*RK2+2.\*RK3+RK4)  
VY=VY+1./6.\*(RL1+2.\*RL2+2.\*RL3+RL4)  
T=T+H

R=SQRT (X\*X+Y\*Y) - 6366.2  
R1=SQRT (X\*X+Y\*Y)  
VTOT=SQRT (VX\*VX+VY\*VY)

VXANG=VX/VTOT  
VYANG=VY/VTOT

FI=1-R1\*6.63146E-04/VTOT\*COS (ZINCL)  
F4=FI\*FI

SD=(0.818+0.25/8)/8\*10E-6

! SD IS GEOMETRICAL CHARACTERISTIC OF THE SPACECRAFT  
! IN THIS CASE TAKEN AS CYLINDAR, 1m LONG 1/8m DIA  
! CALCULATED AS THE PROJECTION OF THE MEAN AREA,  
! AS THE SPACECRAFT  
! IS ROTATING IN SPACE

! ANOTHER CHARACTERISTIC THAT DEPENDS ON THE GEOMETRY IS  
! CD - AIRDRAG COEFFICIENT, HERE 2.2 BASED ON EXPERIMENTS

CALL I2I1 (R, DENSY)  
DENSY=2.14E-7

DRAG=0.5\*DENSY\*VTOT\*VTOT\*F4\*SD\*2.2  
DRAG=0.5\*1E-9\*VTOT\*VTOT\*F4\*SD\*2.2  
DRAGX=DRAG\*VXANG  
DRAGY=DRAG\*VYANG

WRITE (10, 23) T, X, Y, VX, VY, R

FORMAT (5X, 6E14.6)  
REWIND 9

RETURN

END

```
FUNCTION F1(T,X,Y,DRAGX)
  IMPLICIT REAL*8 (A-H,O-Z)
  F1=(-X)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGX
  RETURN
END
```

```
FUNCTION G1(T,X,Y,DRAGY)
  IMPLICIT REAL*8 (A-H,O-Z)
  G1=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGY
  RETURN
END
```

□

```
SUBROUTINE I2I1(Z,DENSY)
! LINEAR INTERPOLATION USED FOR DETERMINING
! CHARACTERISTICS OF THE ATMOSPHERE
```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
DIMENSION DENS(26,2)
```

```
M1=0
```

```
OPEN(UNIT=9, FILE='DATA.DAT')
DO 304 J=1,2
DO 303 I=1,26
READ(9,10) DENS(I,J)
10  FORMAT(E16.8)
```

```
M=M+1
```

```
C   IF(Z.GE.DENS(I-1,2).AND.Z.LE.DENS(I,2))GO TO 99
```

```
303 CONTINUE
```

```
304 CONTINUE
```

```
DO 300 I=1,26
```

```
IF(Z.GT.DENS(I,2).AND.Z.LT.DENS(I+1,2))GO TO 99
300 CONTINUE
```

```
C   M=0
```

```
C   M1=0
```

```
C   DO 100 I=1,26
```

```
C   M=M+1
```

```
C   IF(Z.GE.DENS(I,2).AND.Z.LE.DENS(I+1,2))M=M1
```

```
C   100 CONTINUE
```

```
C   M=M1
```

```
99  ALTN=DENS(I,2)-DENS(I-1,2)
```

```
ALTB=Z-DENS(I,2)
```

```
DENSF=DENS(I,1)-DENS(I-1,1)
```

```
C   DENSY=ALTB*DENSF*10E6
```

```
DENSY=DENS(I-1,1)+ALTB*DENSF/ALTN
```

```
C   WRITE(10,231)DENSY,DENS(I+1,1),DENS(I,2),ALTB,DENSF,ALTN,Z,M1
```

```
C   231  FORMAT(7E14.6,I5)
```

```
C   91  IF(Z.EQ.DENS(M11,2))DENSY=DENS(M11,1)
```

```
RETURN
```

```
END
```

□

## **APPENDIX 4.8.**

**Program that combines Gravitational  
Anomalies with the Moon impact**

## Combination of Gravitational Perturbations

PROGRAM THREE BODY PROBLEM GCS

! Runge Kutta method IV order used for orbit calculation

DIMENSION RLONGS(30000), NUM(30000), SFSLGRT(30000)

REAL K1, K2, K3, K4, L1, L2, L3, L4, K1E, K2E, K3E, K4E, L1E, L2E, L3E,  
+L4E, K1M, K2M, K3M, K4M, L1M, L2M, L3M, L4M, LATS, LONGREL, LONGGR

C READ (5, \*) T, X, Y, XE, YE, XM, YM, H, N, VX, VY, VXE, VYE, VXM,  
C +VYM, SMS

WRITE(\*, '(A\)' ) ' ENTER VALUE T '  
READ(\*, \*) T

WRITE(\*, '(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE '  
READ(\*, \*) AP

WRITE(\*, '(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '  
READ(\*, \*) AA

WRITE(\*, '(A\)' ) ' ENTER VALUE XE '  
READ(\*, \*) XE  
WRITE(\*, '(A\)' ) ' ENTER VALUE YE '  
READ(\*, \*) YE

WRITE(\*, '(A\)' ) ' ENTER VALUE XMO '  
READ(\*, \*) XMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE YMO '  
READ(\*, \*) YMO

WRITE(\*, '(A\)' ) ' ENTER VALUE H '  
READ(\*, \*) H

WRITE(\*, '(A\)' ) ' ENTER VALUE VXE '  
READ(\*, \*) VXE  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYE '  
READ(\*, \*) VYE

WRITE(\*, '(A\)' ) ' ENTER VALUE VXMO '  
READ(\*, \*) VXMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYMO '  
READ(\*, \*) VYMO

WRITE(\*, '(A\)' ) ' ENTER VALUE SMS '  
READ(\*, \*) SMS

WRITE(\*, '(A\)' ) ' ENTER VALUE RAAN '  
READ(\*, \*) ALFA

WRITE(\*, '(A\)' ) ' ENTER VALUE INCLINATION '  
READ(\*, \*) BETA

WRITE(\*, '(A\)' ) ' ENTER VALUE ARG OF PER '  
READ(\*, \*) GAMA



## Combination of Gravitational Perturbations

```
WRITE(*, '(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1
```

```
C OPEN (M1)
```

```
! CALCULATE THE POSITION OF THE CENTRE OF THE MASS, COMPARE IT WITH
ASSUMED
! CENTRE OF THE SATELLITE ORBIT- CN'N: WHAT ABOUT AXES MODIFICATION?
```

```
EMS=5.9761E24
RMMS=7.3534E22
```

```
! THE PART OF THE PROGRAM THAT WILL RECALCULATE VALUES OF THE MOON
! POSITION W.R.T. THE SYSTEM IN SATELLITE ORBIT PLANE CONSISTS OF:
!
! 1. DEFINE POSITION OF THE MOON IN ITS ORBIT PLANE, IE: X=384749.9km,
! Y=0, Z=0 THERE IS ALSO A PARTICULAR VELOCITY ASSOCIATED TO THIS
! POSITION WHICH WILL BE TRANSFORMED BY THE SAME EQUATIONS.
!
! 2. LET IGNORE THE OTHER TWO ROTATIONS, AND PERFORM ONLY THAT ONE FOR THE
! INCLINATION ANGLE (6.65DEG RELATIVE TO THE EQUATOR, ACCORDING TO THE
! HANDBOOK)
!
! 3. NEXT ROTATION WOULD BRING THE MOON PLANE COORDINATES TO THE SATELLITE
! SYSTEM PERFORMED BY THE SAME SPACE TRANSFORMATIONS, BUT WITH
! TRANSPOSED EQUATIONS
!
! THEORETICAL APPROACH IS IN THE THESIS 'THREE BODY PROBLEM' SECTION
! ONCE THE INITIAL VALUE IS GIVEN, ONE COMPLETE SATELLITE ORBIT WILL BE
! COMPUTED, AND AT THE BEGINNING OF THE NEXT ONE NEW INCLINATION ANGLE
! WILL BE DETERMINED
```

```
ALFA1=0
BETA1=6.65
GAMA1=0
! MOON ORBIT ELEMENTS
```

```
AM=ALFA1*0.01744444
BM=BETA1*0.01744444
GRM=GAMA1*0.01744444
```

```
XM00=(COS(AM)*COS(GRM)-SIN(AM)*COS(BM)*SIN(GRM))*XM0+
+(-COS(AM)*SIN(GRM)-SIN(AM)*COS(BM)*COS(GRM))*YM0
YM00=(SIN(AM)*COS(GRM)+COS(AM)*COS(BM)*SIN(GRM))*XM0+(-SIN(AM)*
+SIN(GRM)+COS(AM)*COS(BM)*COS(GRM))*YM0
ZM00=(SIN(BM)*SIN(GRM))*XM0+(SIN(BM)*COS(GRM))*YM0
```

```
VXM00=(COS(AM)*COS(GRM)-SIN(AM)*COS(BM)*SIN(GRM))*VXM0+
+(-COS(AM)*SIN(GRM)-SIN(AM)*COS(BM)*COS(GRM))*VYM0
VYM00=(SIN(AM)*COS(GRM)+COS(AM)*COS(BM)*SIN(GRM))*VXM0+
+(-SIN(AM)*SIN(GRM)+COS(AM)*COS(BM)*COS(GRM))*VYM0
VZM00=(SIN(BM)*SIN(GRM))*VXM0+(SIN(BM)*COS(GRM))*VYM0
```

## Combination of Gravitational Perturbations

```
XM=(COS (GR) *COS (A) -SIN (A) *SIN (GR) *COS (B) ) *XM00+ (SIN (GR) *COS (B) *
+COS (A) +COS (GR) *SIN (A) ) *YM00+ (SIN (GR) *SIN (B) ) *ZM00
YM=(-SIN (GR) *COS (A) -SIN (A) *COS (B) *COS (GR) ) *XM00+ (-SIN (GR) *SIN (A)
++COS (B) *COS (GR) *COS (A) ) *YM00+ (COS (GR) *SIN (B) ) *ZM00
ZM=(SIN (A) *SIN (B) ) *XM00+ (-SIN (B) *COS (A) ) *YM00+COS (B) *ZM00
```

```
VXM=(COS (GR) *COS (A) -SIN (A) *SIN (GR) *COS (B) ) *VXM00+ (SIN (GR) *
+COS (B) *COS (A) +COS (GR) *SIN (A) ) *VYM00+ (SIN (GR) *SIN (B) ) *VZM00
VYM=(-SIN (GR) *COS (A) -SIN (A) *COS (B) *COS (GR) ) *VXM00+ (-SIN (GR) *
+SIN (A) +COS (B) *COS (GR) *COS (A) ) *VYM00+ (COS (GR) *SIN (B) ) *VZM00
VZM=(SIN (A) *SIN (B) ) *VXM00+ (-SIN (B) *COS (A) ) *VYM00+COS (B) *VZM00
```

Y=0

X=AP+6366.2  
XM1=AA+6366.2

RMIN=X  
RMAX=XM1

N=SQRT (( (X+XM1) \*\*3) \*1.2411954E-5) /H

VX=0

VY=SQRT ((0.00981\*(6366.2\*\*2)\*XM1) / ((X+XM1) /2\*X) )  
VYM=SQRT ((198792.3045\*X) / ((X+XM1) \*XM1) )

! try to determine the satellite position for the geosynchronised orbit  
! the initial condiditons are: T=0, (X=42205.1713km, Y=0), XE=0, YE=0,  
H=5,  
! N=17280, VX=0, VY=3.06925, VXE=0, VYE=0, VXM=0, VYM=1.024, SMS=0

! the observed orbit is of very high altitude, therefore the effect due to  
! atmospheric drag and gravitational anomalies will be ignored and only  
Lunar  
! impact will be observed

10 FORMAT (I5,6F10.3,2I5,7F10.3)  
WRITE (6,21)  
21 FORMAT (/ ,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/ )  
WRITE (6,22)  
22 FORMAT(T2,'ORBIT')

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE  
!TRACKS COULD BE PLOTTED

RPER=SQRT(9.87\*(RMIN+RMAX)\*\*3/397584.3246)

RNUM=RPER/H

M=0  
M12=0

NUM=N/RNUM

WRITE (9,33)  
FORMAT(' (' 'Setq' )

DO 200 I1=1,N1

M12=M12+1

DO 100 I=1,N

BR=RNUM/4

M=M+1

X=X+H\*VX  
Y=Y+H\*VY  
XE=XE+H\*VXE  
YE=YE+H\*VYE  
XM=XM+H\*VXM  
YM=YM+H\*VYM

K1=H\*F(T,X,Y,XE,YE,XM,YM,GAN)  
L1=H\*G(T,X,Y,XE,YE,XM,YM,GAN)

K1E=H\*FE(T,X,Y,XE,YE,XM,YM,SMS)  
L1E=H\*GE(T,X,Y,XE,YE,XM,YM,SMS)

K1M=H\*FM(T,X,Y,XE,YE,XM,YM,SMS)  
L1M=H\*GM(T,X,Y,XE,YE,XM,YM,SMS)

K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,GAN)  
L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,GAN)

K2E=H\*FE(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)  
L2E=H\*GE(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K2M=H\*FM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)  
L2M=H\*GM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E  
+,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E  
+,XM+0.5\*K2M,YM+0.5\*L2M,GAN)  
L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E  
+,XM+0.5\*K2M,YM+0.5\*L2M,GAN)

K3E=H\*FE(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E

## Combination of Gravitational Perturbations

```

+, XM+0.5*K2M, YM+0.5*L2M, SMS)
L3E=H*GE (T+0.5*H, X+0.5*K2, Y+0.5*L2, XE+0.5*K2E, YE+0.5*L2E
+, XM+0.5*K2M, YM+0.5*L2M, SMS)

```

```

K3M=H*FM (T+0.5*H, X+0.5*K2, Y+0.5*L2, XE+0.5*K2E, YE+0.5*L2E
+, XM+0.5*K2M, YM+0.5*L2M, SMS)
L3M=H*GM (T+0.5*H, X+0.5*K2, Y+0.5*L2, XE+0.5*K2E, YE+0.5*L2E
+, XM+0.5*K2M, YM+0.5*L2M, SMS)

```

```

K4=H*F (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3M, YM+0.5*L3M, GAN)
L4=H*G (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3M, YM+0.5*L3M, GAN)

```

```

K4E=H*FE (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3E, YM+0.5*L3M, SMS)
L4E=H*GE (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3M, YM+0.5*L3M, SMS)

```

```

K4M=H*FM (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3M, YM+0.5*L3M, SMS)
L4M=H*GM (T+0.5*H, X+0.5*K3, Y+0.5*L3, XE+0.5*K3E, YE+0.5*L3E
+, XM+0.5*K3M, YM+0.5*L3M, SMS)

```

```

VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)

```

```

VXE=VXE+1./6.*(K1E+2.*K2E+2.*K3E+K4E)
VYE=VYE+1./6.*(L1E+2.*L2E+2.*L3E+L4E)

```

```

VXM=VXM+1./6.*(K1M+2.*K2M+2.*K3M+K4M)
VYM=VYM+1./6.*(L1M+2.*L2M+2.*L3M+L4M)

```

```

XC= ((SMS*X+EMS*XE+RMMS*XM) / (SMS+EMS+RMMS) )
YC= ((SMS*Y+EMS*YE+RMMS*YM) / (SMS+EMS+RMMS) )

```

```

A=ALFA*0.01744444
B=BETA*0.01744444
GR=GAMA*0.01744444

```

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

```

XGL= (COS (A) *COS (GR) -SIN (A) *COS (B) *SIN (GR) ) *X+
+ (-COS (A) *SIN (GR) -SIN (A) *COS (B) *COS (GR) ) *Y
YGL= (SIN (A) *COS (GR) +COS (A) *COS (B) *SIN (GR) ) *X+ (-SIN (A) *
+SIN (GR) +COS (A) *COS (B) *COS (GR) ) *Y
ZGL= (SIN (B) *SIN (GR) ) *X+ (SIN (B) *COS (GR) ) *Y

```

!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE  
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS  
!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME

```

R=SQRT (XGL*XGL+YGL*YGL+ZGL*ZGL)

```

```

L=1

```

```

RLONGS (I) =90+57.32498682*DATAN (YGL/XGL)

```

## Combination of Gravitational Perturbations

```
IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =RLONGS (I) +180
IF (RLONGS (I-1) .GT. RLONGS (I) .AND. RLONGS (I-1) .GT. 300) RLONGS (I)
==RLONGS (I) +360
```

```
IF (RLONGS (I) .GT. 380) RLONGS (I) =RLONGS (I) -540
```

```
! IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I-1) =RLMAX
! IF (RLONGS (I) .GE. RLMAX) L=L+1
! IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =L*180+2*RLONGS (I) -90
```

```
! THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE
! THE VALUES WHEN RLONGS (I) DECREASES, AS IT USES
! IF (RLONGS (I) .GE. RLMAX) L=L+1
```

```
! IF (I .GE. BR) RLONGS (I) =RLONGS (I) +90
```

```
! IF (I .GE. 2*BR .AND. I .LE. 3*BR) RLONGS (I) =RLONGS (I) +180
```

```
! IF (I .GE. 3*BR) RLONGS (I) =RLONGS (I) +270
```

```
! IF (M .GT. 146 .AND. M .LT. 873) RLONGS (I) =RLONGS (I) -180
```

```
! IF (RLONGS (I) .LT. 0 .AND. RLONGS (I-1) .GE. 269) RLONGS (I) =360
! ++RLONGS (I)
```

```
! IF (RLONGS (I) .GT. 0 .AND. RLONGS (I-1) .LE. 360) RLONGS (I) =360
! ++RLONGS (I)
```

```
LATS=57.32498682*DACOS (ZGL/R) -90
```

```
LATS1=LATS/57.32498682
```

```
GAN=9.78*(1+0.0053024*((SIN(LATS1))**2)-(5.9E-6)*
+(SIN(2*LATS1))**2)
```

```
! THE SPHERICAL COORDINATES OF THE POINT W AT THE EARTH'S SURFACE ARE
```

```
! LONGW=XLONG+360*0.000072921152*T/6.28
! LATW=XLATT
```

```
! RELATIVE SATELLITE POSITION WILL DEPEND ON THE RATE OF EARTH
! ROTATION, THEREFORE EARTH ROTATION AT THE PARTICULAR MOMENT
! IS GIVEN BY:
```

```
! EOMEGA=360/(24*60*60)
```

```
! ANGULAR VELOCITY CALCULATED IN DEG/SEC
```

```
T=T+H
```

## Combination of Gravitational Perturbations

```
ELONG=T*360/(24*60*60)+RLONGS(1)
NUM1=ELONG/360
```

```
IF(ELONG.GT.360)ELONG=ELONG-NUM1*360
```

```
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:
```

```
! ELGRT=RLONGS(1)-ELONG
```

```
SFSLGRT(I)=RLONGS(I)-ELONG
IF(ABS(ELONG-RLONGS(I)).GT.350)
+SFSLGRT(I)=RLONGS(I)-ELONG+360
```

```
! LATREL=LATS-LATW
! LONGREL=RLONGS(I)-LONGW
! LONGGR=RLONGS(I)-T*0.0041801847
! LONGREL1=RLONGS(I)-LONGGR
```

```
ALT=R-6366.2
```

```
! WRITE (9,23) I, T, X, Y, ALT, RLONGS(I), ELONG, SFSLGRT(I), LATS
```

```
WRITE (9,23) T, X, Y, VX, VY, XE, YE, XM, YM, ALT, RLONGS(I), SFSLGRT(I), LATS
23 FORMAT (5X,13E14.6)
```

```
100 CONTINUE
```

```
RPER=AP+6378.2
RAP=AA+6378.2
```

```
ZB=RAP+RPER
ZRZ=RAP-RPER
```

```
C IF(ZRZ.EQ.0)GO TO 111
```

```
RAZ=1-(ZRZ/ZB)*(ZRZ/ZB)
```

```
C B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(40650.887*GAN*0.5*ZB*RAZ))
```

```
C 111 B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(40650.887*GAN*0.5*ZB))
```

```
SINCL=B-B1
```

```
C WRITE (*,309) SINCL
```

```
C 309 FORMAT( 5X 'SINCL=' (F10.3))
```

```
WRITE (*,3099) XC,YC
```

```
3099 FORMAT( 5X 'CENT. OF MASS =' (2F10.3))
```

```
200 CONTINUE
```

```
M121=M121+1
```

STOP  
END

FUNCTION F(T,X,Y,XE,YE,XM,YM,GAN)

F=(XE-X)\*40650.887\*GAN/(SQRT((XE-X)\*\*2+(YE-Y)\*\*2)\*\*3)  
++(XM-X)\*4906.92/(SQRT((XM-X)\*\*2+(YM-Y)\*\*2)\*\*3)

RETURN  
END

FUNCTION G(T,X,Y,XE,YE,XM,YM,GAN)

G=(YE-Y)\*40650.887\*GAN/(SQRT((XE-X)\*\*2+(YE-Y)\*\*2)\*\*3)  
++(YM-Y)\*4906.92/(SQRT((XM-X)\*\*2+(YM-Y)\*\*2)\*\*3)

RETURN  
END

FUNCTION FE(T,X,Y,XE,YE,XM,YM,SMS)

FE=(XM-XE)\*4906.92/(SQRT((XE-XM)\*\*2+(YE-YM)\*\*2)\*\*3)  
++(X-XE)\*6.673E-20\*SMS/(SQRT((XE-X)\*\*2+(YE-Y)\*\*2)\*\*3)

RETURN  
END

FUNCTION GE(T,X,Y,XE,YE,XM,YM,SMS)

GE=(YM-YE)\*4906.92/(SQRT((XE-XM)\*\*2+(YE-YM)\*\*2)\*\*3)  
++(Y-YE)\*6.673E-20\*SMS/(SQRT((XE-X)\*\*2+(YE-Y)\*\*2)\*\*3)

RETURN  
END

FUNCTION FM(T,X,Y,XE,YE,XM,YM,SMS)

FM=(X-XM)\*6.673E-20\*SMS/(SQRT((XM-X)\*\*2+(YM-Y)\*\*2)\*\*3)  
++(XE-XM)\*398785.2/(SQRT((XM-XE)\*\*2+(YM-YE)\*\*2)\*\*3)

RETURN  
END

FUNCTION GM(T,X,Y,XE,YE,XM,YM,SMS)

GM=(Y-YM)\*6.673E-20\*SMS/(SQRT((XM-X)\*\*2+(YM-Y)\*\*2)\*\*3)  
++(YE-YM)\*398785.2/(SQRT((XM-XE)\*\*2+(YM-YE)\*\*2)\*\*3)

RETURN  
END

□

## **5.0. Ground Track Programs**



# **Ground Track: Kepler's model**

```

PROGRAM gtrTGPH

c DIMENSION RLONGS(30000),NUM(30000),FSLGRT(30000)

! Runge Kutta method IV order used for orbit calculation
C IMPLICIT REAL*8(A-H, K,L-M,O-Z)
C IMPLICIT REAL*8 LONGW,LATW, LONGREL, LATREL, LONGREL1, LONGGR
C RLONGS(I-1)=0

IMPLICIT REAL*8 ( A-H,K-M,O-Z )

C READ (5,*) T,X,Y,H,N,VX,VY

C WRITE(*,'(A\)' ) ' IS APOGEE INITIAL POINT ENTER 1 FOR YES '
C READ(*,*) Y

WRITE(*,'(A\)' ) ' ENTER VALUE T '
READ(*,*) T

WRITE(*,'(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE '
READ(*,*) AP

WRITE(*,'(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '
READ(*,*) AA

WRITE(*,'(A\)' ) ' ENTER VALUE OF TIME STEP H '
READ(*,*) H

WRITE(*,'(A\)' ) ' ENTER VALUE RAAN '
READ(*,*) ALFA

WRITE(*,'(A\)' ) ' ENTER VALUE INCLINATION '
READ(*,*) BETA

WRITE(*,'(A\)' ) ' ENTER VALUE ARG OF PER '
READ(*,*) GAMA

WRITE(*,'(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1

WRITE(*,'(A\)' ) ' ENTER OUTPUT ORBIT No. '
READ(*,*) i2

OPEN (UNIT=10,FILE='KEPLER.DAT',FORM='FORMATTED',STATUS='UNKNOWN')

Y=0

X=AP+6366.2
XM=AA+6366.2

RMIN=X
RMAX=XM

N=SQRT(((X+XM)**3)*1.2411954E-5)/H

```

VX=0

VY=SQRT((0.00981\*(6366.2\*\*2)\*XM)/((X+XM)/2\*X))  
VYM=SQRT((198792.3045\*X)/((X+XM)\*XM))

A=ALFA\*0.01744444  
B=BETA\*0.01744444  
GR=GAMA\*0.01744444

XGL1=(DCOS(A)\*DCOS(GR)-DSIN(A)\*DCOS(B)\*DSIN(GR))\*X+  
+(-DCOS(A)\*DSIN(GR)-DSIN(A)\*DCOS(B)\*DCOS(GR))\*Y  
YGL1=(DSIN(A)\*DCOS(GR)+DCOS(A)\*DCOS(B)\*DSIN(GR))\*X+(-DSIN(A)\*  
\*DSIN(GR)+DCOS(A)\*DCOS(B)\*DCOS(GR))\*Y

RLONGS1=57.32498682\*DATAN(YGL1/XGL1)

C IF(Y.EQ.1)VY=VYM

10 FORMAT (I5,2F10.3,2I5,2F10.3)

WRITE (6,21)

21 FORMAT (/ ,5X, 'I' ,5X, 'T' ,5X, 'X' ,5X, 'Y' ,5X, 'VX' ,5X, 'VY' ,/)

WRITE (6,22)

22 FORMAT(T2, 'ORBIT')

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE  
!TRACKS COULD BE PLOTTED

RPER=SQRT(9.87\*(RMIN+RMAX)\*\*3/397584.3246)  
RNUM=RPER/H

M=0

NUM=N/RNUM

i3=0

DO 200 I1=1,N1

DO 100 I=1,N

BR=RNUM/4

M=M+1

X=X+H\*VX

Y=Y+H\*VY

K1=H\*F(T,X,Y)

L1=H\*G(T,X,Y)

K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)

L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1)

K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)

L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2)

K4=H\*F(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3)

```
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3)
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
```

```
!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES
```

```
XGL=(DCOS(A)*DCOS(GR)-DSIN(A)*DCOS(B)*DSIN(GR))*X+
+(-DCOS(A)*DSIN(GR)-DSIN(A)*DCOS(B)*DCOS(GR))*Y
YGL=(DSIN(A)*DCOS(GR)+DCOS(A)*DCOS(B)*DSIN(GR))*X+(-DSIN(A)*
*DSIN(GR)+DCOS(A)*DCOS(B)*DCOS(GR))*Y
ZGL=(DSIN(B)*DSIN(GR))*X+(DSIN(B)*DCOS(GR))*Y
```

```
!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS
!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME
```

```
R=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)
```

```
L=1
```

```
c RLONGS=57.32498682*DATAN(YGL/XGL)-RLONGS1
```

```
RLONGS=57.32498682*DATAN(YGL/XGL)
```

```
IF(XGL.LT.0)RLONGS=180+RLONGS
```

```
IF(RLONGS.GT.180)RLONGS=RLONGS-360
```

```
LATS=57.32498682*DACOS(ZGL/R)-90
```

```
T=T+H
```

```
ELONG=T*360/(24*60*60)
```

```
NUM1=ELONG/360
```

```
IF(ELONG.GE.360)ELONG=ELONG-NUM1*360
```

```
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:
```

```
C ELGRT=RLONGS(1)-ELONG
```

```
FSLGRT=RLONGS-ELONG
```

```
IF(FSLGRT.LT.0.AND.ABS(FSLGRT).GT.180)FSLGRT=FSLGRT+360
```

```
C LATREL=LATS-LATW
```

```
C LONGREL=RLONGS(I)-LONGW
```

```
C LONGGR=RLONGS(I)-T*0.0041801847
```

```
C LONGREL1=RLONGS(I)-LONGGR
```

```
ALT=R-6366.2
```

```
i3=i3+1
```

```
if(i1.ge.i2.and.i3.eq.10)WRITE (10,23)T,X,Y,ALT,RLONGS,ELONG,  
+                               FSLGRT,LATS  
if(i3.eq.10)i3=0
```

```
23  FORMAT (5X,8E14.6)
```

```
IF(ALT.LT.AP)ALTPER=ALT  
IF(ALT.GT.AP)ALTAP=ALT
```

```
100 CONTINUE
```

```
C  ELONG=ERTHM
```

```
WRITE (11,29)altper,altap  
if(i3.eq.10)i3=0
```

```
29  FORMAT (5X,2E14.6)
```

```
200 CONTINUE
```

```
C  IF(N.GT.RPER)GO TO 101
```

```
STOP  
END  
FUNCTION F(T,X,Y)  
IMPLICIT REAL*8 ( A-H,K-M,O-Z )
```

```
F=(-X)*399059.852/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

```
FUNCTION G(T,X,Y)  
IMPLICIT REAL*8 ( A-H,K-M,O-Z )
```

```
G=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

□

# **Ground Track: Air drag perturbed model - Plate**

```
PROGRAM orbit
! Runge Kutta method IV order used for orbit calculation
```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
C DIMENSION RLONGS(30000),NUM(30000),SFSLGRT(30000)
C IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
C REAL K1,K2,K3,K4,L1,L2,L3,L4,LATS
```

```
C RLONGS(I-1)=0
```

```
C READ (5,*) T,X,Y,H,N,VX,VY,ZB1,ZINCL
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE T '
READ(*,*) T
```

```
WRITE(*,'(A\)' ) ' FOR CIRCULAR ORBIT ENTER EQUAL '
WRITE(*,'(A\)' ) ' APOGEE AND PERIGEE ALTITUDES IN km '
```

```
899 WRITE(*,899)
FORMAT( / ' ENTER VALUE PERIGEE ALTITUDE ' )
READ(*,*) AP
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '
READ(*,*) AA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE H '
READ(*,*) H
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE RAAN '
READ(*,*) ALFA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE INCLINATION '
READ(*,*) BETA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE ARG OF PER '
READ(*,*) GAMA
```

```
WRITE(*,'(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1
```

```
WRITE(*,'(A\)' ) ' FIRST ORBIT NO. FOR OUTPUT '
READ(*,*) I3
```

```
WRITE(*,'(A\)' ) ' SECOND ORBIT NO. FOR OUTPUT '
READ(*,*) I4
```

```
WRITE(*,'(A\)' ) ' THIRD ORBIT NO. FOR OUTPUT '
READ(*,*) I5
```

```
OPEN (M1)
OPEN (UNIT=10,FILE='KAISS.DAT',FORM='FORMATTED',STATUS='UNKNOWN')
```

```

WRITE (11,299) T,AP,AA,H,ALFA,BETA,GAMA,I3,I4,I5
299  FORMAT(/,' T ',F10.1,/, ' AltPer ',F10.5,/, ' AltAp ',F10.5,/, ' H ',
+      F5.2,/, ' RAAN= ',F5.2,/, ' INCLINATION=',F5.2,/, ' AOP=',F5.2,/,
+      ' 1stOrbitNo ',I3,/, ' 2ndOrbitNo ',I3,/, ' 3rdOrbitNo ',I3,/)

Y=0

C  IF(AP.EQ.AA)GOTO 59

X=AP+6366.2
XM=AA+6366.2

RMIN=X
RMAX=XM

N=SQRT(((X+XM)**3)*1.2411954E-5)/H

VY=SQRT((0.00981*(6366.2**2)*XM)/((X+XM)/2*X))
VYM=SQRT((198792.3045*X)/((X+XM)*XM))

59  IF(AP.EQ.AA) RC=AP+6378.14

IF(AP.EQ.AA)VY=SQRT(399059.852/RC)
IF(AP.EQ.AA)VYM=VY

IF(AP.EQ.AA)X=RC
IF(AP.EQ.AA)XM=RC

IF(AP.EQ.AA)RMIN=RC
IF(AP.EQ.AA)RMAX=RC

IF(AP.EQ.AA)TC=SQRT(4*9.8696044*RC**3/399059.852)
IF(AP.EQ.AA)TC1=SQRT(4*9.8696044*RC**3/399059.852)/3600

IF(AP.EQ.AA)WRITE(*,'(A\)' ) ' CIRCULAR ORBIT PERIOD IN HOURS '
IF(AP.EQ.AA)WRITE(*,89) TC,TC1,RC,AP,N1
89  FORMAT(4F10.2,I5)

IF(AP.EQ.AA)N=TC/H

VX=0

10  FORMAT (I5,2F10.3,2I5,3F10.3)

WRITE (6,21)
21  FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
WRITE (6,22)
22  FORMAT(T2,'ORBIT')

L=1

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE
!TRACKS COULD BE PLOTTED

```



```
RPER=SQRT(9.87*(RMIN+RMAX)**3/397584.3246)
```

```
IF(AP.EQ.AA)RPER=RC
```

```
RNUM=RPER/H
```

```
M=0
```

```
NUM=N/RNUM
```

```
DO 200 I1=1,N1
```

```
WRITE (11,29) ALTPER,ALTAP
```

```
29 FORMAT (/ ,5X, 'ALT PERIGEE', F10.5, 'ALT APOGEE', F10.5, /)
```

```
C ALT=AP
```

```
DO 100 I=1,N
```

```
R=(SQRT(X*X+Y*Y))-6366.2
```

```
IF(R.GE.105.AND.R.LE.2500)GO TO 9
```

```
BR=RNUM/4
```

```
M=M+1
```

```
X=X+H*VX
```

```
Y=Y+H*VY
```

```
K1=H*F(T,X,Y,GAN)
```

```
L1=H*G(T,X,Y,GAN)
```

```
K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1,GAN)
```

```
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1,GAN)
```

```
K3=H*F(T+0.5*H,X+0.5*K2,Y+0.5*L2,GAN)
```

```
L3=H*G(T+0.5*H,X+0.5*K2,Y+0.5*L2,GAN)
```

```
K4=H*F(T+0.5*H,X+0.5*K3,Y+0.5*L3,GAN)
```

```
L4=H*G(T+0.5*H,X+0.5*K3,Y+0.5*L3,GAN)
```

```
VX=VX+1./6.*(K1+2.*K2+2.*K3+K4)
```

```
VY=VY+1./6.*(L1+2.*L2+2.*L3+L4)
```

```
T=T+H
```

```
A=ALFA*0.01744444
```

```
B=BETA*0.01744444
```

```
GR=GAMA*0.01744444
```

```
!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES
```

```
XGL=(DCOS(A)*DCOS(GR)-DSIN(A)*DCOS(B)*DSIN(GR))*X+
```

```
+(-DCOS(A)*DSIN(GR)-DSIN(A)*DCOS(B)*DCOS(GR))*Y
```

```
YGL=(DSIN(A)*DCOS(GR)+DCOS(A)*DCOS(B)*DSIN(GR))*X+(-DSIN(A)*
```

```
+DSIN(GR)+DCOS(A)*DCOS(B)*DCOS(GR))*Y
```

```
ZGL=(DSIN(B)*DSIN(GR))*X+(DSIN(B)*DCOS(GR))*Y
```

```
!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE  
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS
```

```

!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME
RG=SQRT (XGL*XGL+YGL*YGL+ZGL*ZGL)
L=1
RLONGS=57.32498682*DATAN (YGL/XGL)
IF (XGL.LT.0) RLONGS=180+RLONGS
IF (RLONGS.GT.180) RLONGS=RLONGS-360
C      IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =RLONGS (I) +180
C      IF (RLONGS (I-1) .GT. RLONGS (I) .AND. RLONGS (I-1) .GT. 300) RLONGS (I)
C      ==RLONGS (I) +360
C      IF (RLONGS (I) .GT. 380) RLONGS (I) =RLONGS (I) -540
!      IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I-1) =RLMAX
!      IF (RLONGS (I) .GE. RLMAX) L=L+1
!      IF (RLONGS (I-1) .GT. RLONGS (I) ) RLONGS (I) =L*180+2*RLONGS (I) -90
!THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE
!THE VALUES WHEN RLONGS (I) DECREASES, AS IT USES
! IF (RLONGS (I) .GE. RLMAX) L=L+1
!      IF (I .GE. BR) RLONGS (I) =RLONGS (I) +90
!      IF (I .GE. 2*BR .AND. I .LE. 3*BR) RLONGS (I) =RLONGS (I) +180
!      IF (I .GE. 3*BR) RLONGS (I) =RLONGS (I) +270
!      IF (M .GT. 146 .AND. M .LT. 873) RLONGS (I) =RLONGS (I) -180
!      IF (RLONGS (I) .LT. 0 .AND. RLONGS (I-1) .GE. 269) RLONGS (I) =360
!      ++RLONGS (I)
!      IF (RLONGS (I) .GT. 0 .AND. RLONGS (I-1) .LE. 360) RLONGS (I) =360
!      ++RLONGS (I)
LATS=57.32498682*DACOS (ZGL/RG) -90
ELONG=T*360 / (24*60*60)
NUM1=ELONG/360
IF (ELONG .GE. 360) ELONG=ELONG-NUM1*360
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:

```

```

!   ELGRT=RLONGS(1)-ELONG

      SFSLGRT=RLONGS-ELONG

      IF(SFSLGRT.LT.0.AND.ABS(SFSLGRT).GT.180)SFSLGRT=SFSLGRT+360

C     IF(ABS(ELONG-RLONGS(I)).GT.350)
C     +SFSLGRT(I)=RLONGS(I)-ELONG+360

!   LATREL=LATS-LATW
!   LONGREL=RLONGS(I)-LONGW
!   LONGGR=RLONGS(I)-T*0.0041801847
!   LONGREL1=RLONGS(I)-LONGGR

      ALT=RG-6366.2

      IF(R.GE.2500) WRITE(10,23)T,X,Y,ALT,SFSLGRT,LATS

      REWIND 9

23   FORMAT(5X,6E14.6)

      IF(R.GE.2500)GO TO 99

      i2=0

9     IF(R.GT.105.AND.R.LT.2500) CALL AS3I(I1,i2,I5,X,Y,VX,VY,H,
+     ALTPER,ALTAP,ZINCL,ALFA,BETA,GAMA)
C     IF(R.GE.105.AND.R.LE.2500) WRITE(*,*) ' IT IS INSIDE THE 99 LOOP '

99    L=L+1

      GAN=9.78*(1+0.0053024*((DSIN(LATS))**2)-(5.9E-6)*
+ (DSIN(2*LATS))**2)*(6367.445**2)/5.9761E+24

C     WRITE(*,3098) GAN
C     3098   FORMAT((E10.3))

100  CONTINUE

200  CONTINUE

      STOP
      END

      FUNCTION F(T,X,Y,GAN)

```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
F=(-X)*GAN*40678.884/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

```
FUNCTION G(T,X,Y,GAN)  
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
G=(-Y)*GAN*40678.884/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

□

```
SUBROUTINE AS3I (I1, I2, I5, X, Y, VX, VY, H,  
+             ALTPER, ALTAP, ZINCL, ALFA, BETA, GAMA)
```

```
! Runge Kutta method IV order used for orbit calculation  
! INCLUDING AIRDRAG PERTURBATION FOR ALTITUDES 105-2500km  
IMPLICIT REAL*8 (A-H, L, O-Z)
```

```
A=ALFA*0.01744444  
B=BETA*0.01744444  
GR=GAMA*0.01744444
```

```
C WRITE(*,*) ' IT IS INSIDE THE 99 LOOP '
```

```
ALTP=ALT
```

```
X=X+H*VX  
Y=Y+H*VY  
RK1=H*F1 (T, X, Y, DRAGX)  
RL1=H*G1 (T, X, Y, DRAGY)  
RK2=H*F1 (T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGX)  
RL2=H*G1 (T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGY)  
RK3=H*F1 (T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGX)  
RL3=H*G1 (T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGY)  
RK4=H*F1 (T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGX)  
RL4=H*G1 (T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGY)  
VX=VX+1./6.*(RK1+2.*RK2+2.*RK3+RK4)  
VY=VY+1./6.*(RL1+2.*RL2+2.*RL3+RL4)  
T=T+H
```

```
R=SQRT (X*X+Y*Y) -6366.2  
R1=SQRT (X*X+Y*Y)  
VTOT=SQRT (VX*VX+VY*VY)
```

```
VXANG=VX/VTOT  
VYANG=VY/VTOT
```

```
FI=1-R1*6.63146E-04/VTOT*DCOS (ZINCL)  
F4=FI*FI
```

```
SD=74.1*108.4*(0.818+0.25*74.1/108.4)*10E-6
```

```
! SD IS GEOMETRICAL CHARACTERISTIC OF THE SPACECRAFT  
! IN THIS CASE TAKEN AS CYLINDAR, 1m LONG 1/8m DIA  
! CALCULATED AS THE PROJECTION OF THE MEAN AREA,  
! AS THE SPACECRAFT  
! IS ROTATING IN SPACE
```

```
! ANOTHER CHARACTERISTIC THAT DEPENDS ON THE GEOMETRY IS  
! CD - AIRDRAG COEFFICIENT, HERE 2.2 BASED ON EXPERIMENTS
```

```
CALL I2I5 (R, DENSY)
```

```
C WRITE (12, 289) R, DENSY
```

```
289 FORMAT (5X, 2E14.6)
```

```

DRAG=0.5*DENSY*VTOT*VTOT*F4*SD*3.93
C DRAG=0.5*1E-9*VTOT*VTOT*F4*SD*2.2
  DRAGX=DRAG*VXANG
  DRAGY=DRAG*VYANG

XGL=(DCOS(A)*DCOS(GR)-DSIN(A)*DCOS(B)*DSIN(GR))*X+
+(-DCOS(A)*DSIN(GR)-DSIN(A)*DCOS(B)*DCOS(GR))*Y
YGL=(DSIN(A)*DCOS(GR)+DCOS(A)*DCOS(B)*DSIN(GR))*X+(-DSIN(A)*
*DSIN(GR)+DCOS(A)*DCOS(B)*DCOS(GR))*Y
ZGL=(DSIN(B)*DSIN(GR))*X+(DSIN(B)*DCOS(GR))*Y

RM=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)

RLONGS=57.32498682*DATAN(YGL/XGL)

IF(XGL.LT.0)RLONGS=180+RLONGS

IF(RLONGS.GT.180)RLONGS=RLONGS-360

LATS=57.32498682*DACOS(ZGL/R1)-90

ELONG=T*360/(24*60*60)
NUM1=ELONG/360

IF(ELONG.GE.360)ELONG=ELONG-NUM1*360

SFSLGRT=RLONGS-ELONG
IF(SFSLGRT.LT.0.AND.ABS(SFSLGRT).GT.180)SFSLGRT=SFSLGRT+360

ALT=R1-6366.2

c IF(I1.GE.69)WRITE (10,23)T,X,Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

C WRITE (10,23)T,X,
C + Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

IF(ALT.LT.ALTP)ALTPER=ALT
IF(ALT.GT.ALTP)ALTAP=ALT

I2=I2+1

IF(I1.ge.I5.AND.I2.EQ.10) WRITE(10,23)T,
+ X,Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

IF(I2.EQ.10)I2=0

23 FORMAT (5X,8E14.6)
REWIND 9

RETURN
END

```

```
FUNCTION F1(T,X,Y,DRAGX)
IMPLICIT REAL*8 (A-H,O-Z)
F1=(-X)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGX
RETURN
END
```

```
FUNCTION G1(T,X,Y,DRAGY)
  IMPLICIT REAL*8 (A-H,O-Z)
G1=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGY
RETURN
END
```

□

```
SUBROUTINE I2I5(Z,DENSY)
! LINEAR INTERPOLATION USED FOR DETERMINING
! CHARACTERISTICS OF THE ATMOSPHERE
```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
DIMENSION DENS(26,2)
```

```
M1=0
```

```
OPEN(UNIT=9, FILE='DATA.DAT')
DO 304 J=1,2
DO 303 I=1,26
READ(9,10) DENS(I,J)
10  FORMAT(E16.8)
```

```
M=M+1
```

```
IF(Z.GE.DENS(I-1,2).AND.Z.LE.DENS(I,2))GO TO 99
```

```
303 CONTINUE
304 CONTINUE
```

```
DO 300 I=1,26
```

```
IF(Z.GT.DENS(I,2).AND.Z.LT.DENS(I+1,2))GO TO 99
300 CONTINUE
```

```
M=0
M1=0
```

```
DO 100 I=1,26
M=M+1
IF (Z.GE.DENS(I,2).AND.Z.LE.DENS(I+1,2))M=M1
100 CONTINUE
```

```
M=M1
```

```
B=(Z-DENS(I+1,2))*DENS(I,1)
```

```
A=(DENS(I,2)-Z)*DENS(I+1,1)
```

```
C=DENS(I,2)-DENS(I+1,2)
```

```
DENSY=ALTB*DENSF*10E6
```

```
DENSY=(A+B)/C
```

```
WRITE(10,231)DENSY,DENS(I+1,1),DENS(I,2),ALTB,DENSF,ALTN,Z,M1
```

```
231  FORMAT (7E14.6,I5)
```

```
91  IF(Z.EQ.DENS(M11,2))DENSY=DENS(M11,1)
```

```
RETURN
END
```



# **Ground Track: Air drag perturbed model - Cylinder**

```
SUBROUTINE AS3I(I1, I2, I5, X, Y, VX, VY, H,  
+             ALTPER, ALTAP, ZINCL, ALFA, BETA, GAMA)
```

```
! Runge Kutta method IV order used for orbit calculation  
! INCLUDING AIRDRAG PERTURBATION FOR ALTITUDES 105-2500km  
IMPLICIT REAL*8 (A-H, L, O-Z)
```

```
A=ALFA*0.01744444  
B=BETA*0.01744444  
GR=GAMA*0.01744444
```

```
C WRITE(*,*) ' IT IS INSIDE THE 99 LOOP '
```

```
ALTP=ALT
```

```
X=X+H*VX  
Y=Y+H*VY  
RK1=H*F1(T, X, Y, DRAGX)  
RL1=H*G1(T, X, Y, DRAGY)  
RK2=H*F1(T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGX)  
RL2=H*G1(T+0.5*H, X+0.5*RK1, Y+0.5*RL1, DRAGY)  
RK3=H*F1(T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGX)  
RL3=H*G1(T+0.5*H, X+0.5*RK2, Y+0.5*RL2, DRAGY)  
RK4=H*F1(T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGX)  
RL4=H*G1(T+0.5*H, X+0.5*RK3, Y+0.5*RL3, DRAGY)  
VX=VX+1./6.*(RK1+2.*RK2+2.*RK3+RK4)  
VY=VY+1./6.*(RL1+2.*RL2+2.*RL3+RL4)  
T=T+H
```

```
R=SQRT(X*X+Y*Y)-6366.2  
R1=SQRT(X*X+Y*Y)  
VTOT=SQRT(VX*VX+VY*VY)
```

```
VXANG=VX/VTOT  
VYANG=VY/VTOT
```

```
FI=1-R1*6.63146E-04/VTOT*DCOS(ZINCL)  
F4=FI*FI
```

```
SD=89*18*(0.818+0.25*18/89)*10E-6
```

```
! SD IS GEOMETRICAL CHARACTERISTIC OF THE SPACECRAFT  
! IN THIS CASE TAKEN AS CYLINDAR, 1m LONG 1/8m DIA  
! CALCULATED AS THE PROJECTION OF THE MEAN AREA,  
! AS THE SPACECRAFT  
! IS ROTATING IN SPACE
```

```
! ANOTHER CHARACTERISTIC THAT DEPENDS ON THE GEOMETRY IS  
! CD - AIRDRAG COEFFICIENT, HERE 2.2 BASED ON EXPERIMENTS
```

```
CALL I2I5(R, DENSY)
```

```
C WRITE(12,289)R, DENSY
```

```
289 FORMAT (5X,2E14.6)
```

```

DRAG=0.5*DENSY*VTOT*VTOT*F4*SD*3.93
C DRAG=0.5*1E-9*VTOT*VTOT*F4*SD*2.2
  DRAGX=DRAG*VXANG
  DRAGY=DRAG*VYANG

XGL=(DCOS(A)*DCOS(GR)-DSIN(A)*DCOS(B)*DSIN(GR))*X+
+(-DCOS(A)*DSIN(GR)-DSIN(A)*DCOS(B)*DCOS(GR))*Y
YGL=(DSIN(A)*DCOS(GR)+DCOS(A)*DCOS(B)*DSIN(GR))*X+(-DSIN(A)*
*DSIN(GR)+DCOS(A)*DCOS(B)*DCOS(GR))*Y
ZGL=(DSIN(B)*DSIN(GR))*X+(DSIN(B)*DCOS(GR))*Y

RM=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)

RLONGS=57.32498682*DATAN(YGL/XGL)

IF(XGL.LT.0)RLONGS=180+RLONGS

IF(RLONGS.GT.180)RLONGS=RLONGS-360

LATS=57.32498682*DACOS(ZGL/R1)-90

ELONG=T*360/(24*60*60)
NUM1=ELONG/360

IF(ELONG.GE.360)ELONG=ELONG-NUM1*360

SFSLGRT=RLONGS-ELONG
IF(SFSLGRT.LT.0.AND.ABS(SFSLGRT).GT.180)SFSLGRT=SFSLGRT+360

ALT=R1-6366.2

c IF(I1.GE.69)WRITE (10,23)T,X,Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

C WRITE (10,23)T,X,
C + Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

IF(ALT.LT.ALTP)ALTPER=ALT
IF(ALT.GT.ALTP)ALTAP=ALT

I2=I2+1

IF(I1.EQ.I5.AND.I2.EQ.10) WRITE(10,23)T,
+ X,Y,ALT,RLONGS,ELONG,SFSLGRT,LATS

IF(I2.EQ.10)I2=0

23 FORMAT (5X,8E14.6)
REWIND 9

RETURN
END

```

```
FUNCTION F1 (T,X,Y, DRAGX)
  IMPLICIT REAL*8 (A-H,O-Z)
  F1=(-X)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGX
  RETURN
  END
```

```
FUNCTION G1 (T,X,Y, DRAGY)
  IMPLICIT REAL*8 (A-H,O-Z)
  G1=(-Y)*399059.852/(SQRT(X*X+Y*Y))**3+DRAGY
  RETURN
  END
```

□

# **Ground Track: Earth's oblateness**

```

PROGRAM orbit
! Runge Kutta method IV order used for orbit calculation

IMPLICIT REAL*8 (A-H,K,L,O-Z)
C   DIMENSION RLONGS(30000),NUM(30000),SFSLGRT(30000)
C   IMPLICIT REAL*8 (A-H,K,L,O-Z)

C   REAL K1,K2,K3,K4,L1,L2,L3,L4,LATS

C   RLONGS(I-1)=0

C   READ (5,*) T,X,Y,H,N,VX,VY,ZB1,ZINCL

WRITE(*,'(A\)' ) ' ENTER VALUE T '
READ(*,*) T

WRITE(*,'(A\)' ) ' FOR CIRCULAR ORBIT ENTER EQUAL '
WRITE(*,'(A\)' ) ' APOGEE AND PERIGEE ALTITUDES IN km '

899 WRITE(*,899)
   FORMAT( / ' ENTER VALUE PERIGEE ALTITUDE ' )
   READ(*,*) AP

WRITE(*,'(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '
READ(*,*) AA

WRITE(*,'(A\)' ) ' ENTER VALUE H '
READ(*,*) H

WRITE(*,'(A\)' ) ' ENTER VALUE RAAN '
READ(*,*) ALFA

WRITE(*,'(A\)' ) ' ENTER VALUE INCLINATION '
READ(*,*) BETA

WRITE(*,'(A\)' ) ' ENTER VALUE ARG OF PER '
READ(*,*) GAMA

WRITE(*,'(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1

WRITE(*,'(A\)' ) ' FIRST ORBIT NO. FOR OUTPUT '
READ(*,*) I3

WRITE(*,'(A\)' ) ' SECOND ORBIT NO. FOR OUTPUT '
READ(*,*) I4

WRITE(*,'(A\)' ) ' THIRD ORBIT NO. FOR OUTPUT '
READ(*,*) I5

OPEN (M1)
OPEN (UNIT=10,FILE='KAISS.DAT',FORM='FORMATTED',STATUS='UNKNOWN')

```

```

c WRITE (11,299) T,AP,AA,H,ALFA,BETA,GAMA,I3,I4,I5
c 299 FORMAT(/,' T ',F10.1,/, ' AltPer ',F10.5,/, ' AltAp ',F10.5,/, ' H ',
c + F5.2,/, ' RAAN= ',F5.2,/, ' INCLINATION=',F5.2,/, ' AOP=',F5.2,/,
c + ' 1stOrbitNo ',I3,/, ' 2ndOrbitNo ',I3,/, ' 3rdOrbitNo ',I3,/)

```

```
Y=0
```

```
C IF(AP.EQ.AA)GOTO 59
```

```
X=AP+6366.2
XM=AA+6366.2
```

```
RMIN=X
RMAX=XM
```

```
N=SQRT((X+XM)**3)*1.2411954E-5)/H
```

```
VY=SQRT((0.00981*(6366.2**2)*XM)/((X+XM)/2*X))
VYM=SQRT((198792.3045*X)/((X+XM)*XM))
```

```
59 IF(AP.EQ.AA) RC=AP+6378.14
```

```
IF(AP.EQ.AA)VY=SQRT(399059.852/RC)
IF(AP.EQ.AA)VYM=VY
```

```
IF(AP.EQ.AA)X=RC
IF(AP.EQ.AA)XM=RC
```

```
IF(AP.EQ.AA)RMIN=RC
IF(AP.EQ.AA)RMAX=RC
```

```
IF(AP.EQ.AA)TC=SQRT(4*9.8696044*RC**3/399059.852)
IF(AP.EQ.AA)TC1=SQRT(4*9.8696044*RC**3/399059.852)/3600
```

```

c IF(AP.EQ.AA)WRITE(*,'(A\)' ) ' CIRCULAR ORBIT PERIOD IN HOURS '
c IF(AP.EQ.AA)WRITE(*,89) TC,TC1,RC,AP,N1
c 89 FORMAT(4F10.2,I5)

```

```
IF(AP.EQ.AA)N=TC/H
```

```
VX=0
```

```
10 FORMAT (I5,2F10.3,2I5,3F10.3)
```

```

WRITE (6,21)
21 FORMAT (/ ,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
WRITE (6,22)
22 FORMAT(T2,'ORBIT')

```

```
L=1
```

```
!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE
!TRACKS COULD BE PLOTTED
```

RPER=SQRT(9.87\*(RMIN+RMAX)\*\*3/397584.3246)

IF(AP.EQ.AA)RPER=RC  
RNUM=RPER/H

M=0

NUM=N/RNUM

DO 200 I1=1,N1

29 WRITE (11,29) ALTPER,ALTAP  
FORMAT (/ ,5X,'ALT PERIGEE',F10.5,'ALT APOGEE',F10.5, /)

C ALT=AP

DO 100 I=1,N

R=(SQRT(X\*X+Y\*Y))-6366.2  
IF(R.GE.105.AND.R.LE.2500)GO TO 9

BR=RNUM/4

M=M+1

X=X+H\*VX  
Y=Y+H\*VY  
K1=H\*F(T,X,Y,GAN)  
L1=H\*G(T,X,Y,GAN)  
K2=H\*F(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,GAN)  
L2=H\*G(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,GAN)  
K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,GAN)  
L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,GAN)  
K4=H\*F(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,GAN)  
L4=H\*G(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,GAN)  
VX=VX+1./6.\*(K1+2.\*K2+2.\*K3+K4)  
VY=VY+1./6.\*(L1+2.\*L2+2.\*L3+L4)  
T=T+H

A=ALFA\*0.01744444  
B=BETA\*0.01744444  
GR=GAMA\*0.01744444

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

XGL=(DCOS(A)\*DCOS(GR)-DSIN(A)\*DCOS(B)\*DSIN(GR))\*X+  
+(-DCOS(A)\*DSIN(GR)-DSIN(A)\*DCOS(B)\*DCOS(GR))\*Y  
YGL=(DSIN(A)\*DCOS(GR)+DCOS(A)\*DCOS(B)\*DSIN(GR))\*X+(-DSIN(A)\*  
+DSIN(GR)+DCOS(A)\*DCOS(B)\*DCOS(GR))\*Y  
ZGL=(DSIN(B)\*DSIN(GR))\*X+(DSIN(B)\*DCOS(GR))\*Y

!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE  
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS



```

!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME
RG=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)
L=1
RLONGS=57.32498682*DATAN(YGL/XGL)
IF(XGL.LT.0)RLONGS=180+RLONGS
IF(RLONGS.GT.180)RLONGS=RLONGS-360
C      IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I)=RLONGS(I)+180
C      IF(RLONGS(I-1).GT.RLONGS(I).AND.RLONGS(I-1).GT.300)RLONGS(I)
C      ==RLONGS(I)+360
C      IF(RLONGS(I).GT.380)RLONGS(I)=RLONGS(I)-540
!      IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I-1)=RLMAX
!      IF(RLONGS(I).GE.RLMAX)L=L+1
!      IF(RLONGS(I-1).GT.RLONGS(I))RLONGS(I)=L*180+2*RLONGS(I)-90
!THE ABOVE SET OF THREE COMMANDS CAN NOT CALCULATE
!THE VALUES WHEN RLONGS(I) DECREASES, AS IT USES
! IF (RLONGS(I).GE.RLMAX)L=L+1
!      IF(I.GE.BR)RLONGS(I)=RLONGS(I)+90
!      IF(I.GE.2*BR.AND.I.LE.3*BR)RLONGS(I)=RLONGS(I)+180
!      IF(I.GE.3*BR)RLONGS(I)=RLONGS(I)+270
!      IF(M.GT.146.AND.M.LT.873)RLONGS(I)=RLONGS(I)-180
!      IF(RLONGS(I).LT.0.AND.RLONGS(I-1).GE.269)RLONGS(I)=360
!      ++RLONGS(I)
!      IF(RLONGS(I).GT.0.AND.RLONGS(I-1).LE.360)RLONGS(I)=360
!      ++RLONGS(I)
LATS=57.32498682*DACOS(ZGL/RG)-90
ELONG=T*360/(24*60*60)
NUM1=ELONG/360
IF(ELONG.GE.360)ELONG=ELONG-NUM1*360
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:

```

```

!      ELGRT=RLONGS(1)-ELONG

      SFSLGRT=RLONGS-ELONG

      IF(SFSLGRT.LT.0.AND.ABS(SFSLGRT).GT.180)SFSLGRT=SFSLGRT+360

C      IF(ABS(ELONG-RLONGS(I)).GT.350)
C      +SFSLGRT(I)=RLONGS(I)-ELONG+360

!      LATREL=LATS-LATW
!      LONGREL=RLONGS(I)-LONGW
!      LONGGR=RLONGS(I)-T*0.0041801847
!      LONGREL1=RLONGS(I)-LONGGR

      ALT=RG-6366.2

      IF(R.GE.2500) WRITE(10,23)T,X,Y,ALT,SFSLGRT,LATS

      REWIND 9

23     FORMAT(5X,6E14.6)

      IF(R.GE.2500)GO TO 99

      i2=0

9      IF(R.GT.105.AND.R.LT.2500) CALL AS3I(I1,i2,I5,X,Y,VX,VY,H,
+     ALTPER,ALTAP,ZINCL,ALFA,BETA,GAMA)
C      IF(R.GE.105.AND.R.LE.2500) WRITE(*,*) ' IT IS INSIDE THE 99 LOOP '

99     L=L+1

      GAN=9.78*(1+0.0053024*((DSIN(LATS))**2)-(5.9E-6)*
+     (DSIN(2*LATS))**2)*(6367.445**2)/5.9761E+24

C      WRITE(*,3098) GAN
C      3098     FORMAT((E10.3))

100    CONTINUE

200    CONTINUE

      STOP
      END

      FUNCTION F(T,X,Y,GAN)

```

```
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
F=(-X)*GAN*40678.884/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

```
FUNCTION G(T,X,Y,GAN)  
IMPLICIT REAL*8 (A-H,K,L,O-Z)
```

```
G=(-Y)*GAN*40678.884/(SQRT(X*X+Y*Y))**3  
RETURN  
END
```

□

# **Ground Track: Three Body Problem**

PROGRAM THREEBODYPROBLEMGROUNTRACK  
! Runge Kutta method IV order used for orbit calculation

IMPLICIT REAL\*8 (A-L,O-Z)

DIMENSION RLONGS(30000), NUM(30000), SFSLGRT(30000)

! REAL K1, K2, K3, K4, L1, L2, L3, L4, K1E, K2E, K3E, K4E, L1E, L2E, L3E

! REAL L4E, K1M, K2M, K3M, K4M, L1M, L2M, L3M, L4M, LATS

! REAL LATS1

WRITE(\*, '(A\)' ) ' ENTER VALUE T '  
READ(\*, \*) T

WRITE(\*, '(A\)' ) ' ENTER VALUE PERIGEE ALTITUDE '  
READ(\*, \*) AP

WRITE(\*, '(A\)' ) ' ENTER VALUE APOGEE ALTITUDE '  
READ(\*, \*) AA

WRITE(\*, '(A\)' ) ' ENTER VALUE XE '  
READ(\*, \*) XE  
WRITE(\*, '(A\)' ) ' ENTER VALUE YE '  
READ(\*, \*) YE

WRITE(\*, '(A\)' ) ' ENTER VALUE XMO '  
READ(\*, \*) XMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE YMO '  
READ(\*, \*) YMO

WRITE(\*, '(A\)' ) ' ENTER VALUE H '  
READ(\*, \*) H

WRITE(\*, '(A\)' ) ' ENTER VALUE VXE '  
READ(\*, \*) VXE  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYE '  
READ(\*, \*) VYE

WRITE(\*, '(A\)' ) ' ENTER VALUE VXMO '  
READ(\*, \*) VXMO  
WRITE(\*, '(A\)' ) ' ENTER VALUE VYMO '  
READ(\*, \*) VYMO

WRITE(\*, '(A\)' ) ' ENTER VALUE SMS '  
READ(\*, \*) SMS

WRITE(\*, '(A\)' ) ' ENTER VALUE RAAN '  
READ(\*, \*) ALFA

WRITE(\*, '(A\)' ) ' ENTER VALUE INCLINATION '  
READ(\*, \*) BETA

```

WRITE(*,'(A\)' ) ' ENTER VALUE ARG OF PER '
READ(*,*) GAMA

WRITE(*,'(A\)' ) ' ENTER VALUE ORBIT No. '
READ(*,*) N1

WRITE(*,'(A\)' ) ' ENTER OUTPUT ORBIT No. '
READ(*,*) I21

open (unit=10,
file='tbpdblprecSETgrtr.DAT',form='formatted',status='unknown')

! CALCULATE THE POSITION OF THE CENTRE OF THE MASS, COMPARE IT WITH
ASSUMED
! CENTRE OF THE SATELLITE ORBIT- CN'N: WHAT ABOUT AXES MODIFICATION?

EMS=5.9761E24
RMMS=7.3534E22

i3=0

! THE PART OF THE PROGRAM THAT WILL RECALCULATE VALUES OF THE MOON
POSITION
! W.R.T. THE SYSTEM IN SATELLITE ORBIT PLANE CONSISTS OF:
! 1. DEFINE POSITION OF THE MOON IN ITS ORBIT PLANE, IE: X=384749.9km,
Y=0, Z=0
! THERE IS ALSO A PARTICULAR VELOCITY ASSOCIATED TO THIS POSITION WHICH
WILL BE
! TRANSFORMED BY THE SAME EQUATIONS.
! 2. LET IGNORE THE OTHER TWO ROTATIONS, AND PERFORM ONLY THAT ONE FOR THE
! INCLINATION ANGLE (6.65DEG RELATIVE TO THE EQUATOR, ACCORDING TO THE
HANDBOOK)
! 3. NEXT ROTATION WOULD BRING THE MOON PLANE COORDINATES TO THE SATELLITE
SYSTEM
! PERFORMED BY THE SAME SPACE TRANSFORMATIONS, BUT WITH TRANSPOSED
EQUATIONS
! THEORETICAL APPROACH IS IN THE THESIS 'THREE BODY PROBLEM' SECTION
! ONCE THE INITIAL VALUE IS GIVEN, ONE COMPLETE SATELLITE ORBIT WILL BE
COMPUTED, AND
! ON THE BEGINNING OF THE NEXT ONE NEW INCLINATION ANGLE WILL BE
DETERMINED

ALFA1=0
BETA1=6.65
GAMA1=0
! MOON ORBIT ELEMENTS

AM=ALFA1*0.01744444
BM=BETA1*0.01744444
GRM=GAMA1*0.01744444

XM00=(DCOS(AM)*DCOS(GRM)-DSIN(AM)*DCOS(BM)*DSIN(GRM))*XM0+(-
DCOS(AM)*DSIN(GRM)-DSIN(AM)*DCOS(BM)*DCOS(GRM))*YM0

```

YM00=(DSIN(AM)\*DCOS(GRM)+DCOS(AM)\*DCOS(BM)\*DSIN(GRM))\*XM0+(-  
DSIN(AM)\*DSIN(GRM)+DCOS(AM)\*DCOS(BM)\*DCOS(GRM))\*YM0

ZM00=(DSIN(BM)\*DSIN(GRM))\*XM0+(DSIN(BM)\*DCOS(GRM))\*YM0

VXM00=(DCOS(AM)\*DCOS(GRM)-DSIN(AM)\*DCOS(BM)\*DSIN(GRM))\*VXM0+(-  
DCOS(AM)\*DSIN(GRM)-DSIN(AM)\*DCOS(BM)\*DCOS(GRM))\*VYM0

VYM00=(DSIN(AM)\*DCOS(GRM)+DCOS(AM)\*DCOS(BM)\*DSIN(GRM))\*VXM0+(-  
DSIN(AM)\*DSIN(GRM)+DCOS(AM)\*DCOS(BM)\*DCOS(GRM))\*VYM0

VZM00=(DSIN(BM)\*DSIN(GRM))\*VXM0+(DSIN(BM)\*DCOS(GRM))\*VYM0

XM=(DCOS(GR)\*DCOS(A)-  
DSIN(A)\*DSIN(GR)\*DCOS(B))\*XM00+(DSIN(GR)\*DCOS(B)\*DCOS(A)+DCOS(GR)\*DSIN(A))\*YM00+  
(DSIN(GR)\*DSIN(B))\*ZM00

YM=(-DSIN(GR)\*DCOS(A)-DSIN(A)\*DCOS(B)\*DCOS(GR))\*XM00+(-  
DSIN(GR)\*DSIN(A)+DCOS(B)\*DCOS(GR)\*DCOS(A))\*YM00+(DCOS(GR)\*DSIN(B))\*ZM00

ZM=(DSIN(A)\*DSIN(B))\*XM00+(-DSIN(B)\*DCOS(A))\*YM00+DCOS(B)\*ZM00

VXM=(DCOS(GR)\*DCOS(A)-  
DSIN(A)\*DSIN(GR)\*DCOS(B))\*VXM00+(DSIN(GR)\*DCOS(B)\*DCOS(A)+DCOS(GR)\*DSIN(A))\*VYM0  
0+(DSIN(GR)\*DSIN(B))\*VZM00

VYM=(-DSIN(GR)\*DCOS(A)-DSIN(A)\*DCOS(B)\*DCOS(GR))\*VXM00+(-  
DSIN(GR)\*DSIN(A)+DCOS(B)\*DCOS(GR)\*DCOS(A))\*VYM00+(DCOS(GR)\*DSIN(B))\*VZM00

VZM=(DSIN(A)\*DSIN(B))\*VXM00+(-DSIN(B)\*DCOS(A))\*VYM00+DCOS(B)\*VZM00

Y=0

X=AP+6366.2

XM1=AA+6366.2

RMIN=X

RMAX=XM1

N=SQRT(((X+XM1)\*\*3)\*1.2411954E-5)/H

VX=0

VY=SQRT((0.00981\*(6366.2\*\*2)\*XM1)/((X+XM1)/2\*X))

VYM=SQRT((198792.3045\*X)/((X+XM1)\*XM1))

! try to determine the satellite position for the geosynchronised orbit  
! the initial condiditons are: T=0, (X=42205.1713km, Y=0), XE=0, YE=0,

H=5,

! N=17280, VX=0, VY=3.06925, VXE=0, VYE=0, VXM=0, VYM=1.024, SMS=0

! the observed orbit is of very high altitude, therefore the effect due to  
! atmospheric drag and gravitational anomalies will be ignored and only

Lunar

! impact will be observed

```

10  FORMAT (I5,6F10.3,2I5,7F10.3)
    WRITE (6,21)
21  FORMAT (/,5X,'I',5X,'T',5X,'X',5X,'Y',5X,'VX',5X,'VY',/)
    WRITE (6,22)
22  FORMAT(T2,'ORBIT')

!CALCULATE THE PERIOD, SO THE NUMBER OF SUCCESSIVE
!TRACKS COULD BE PLOTTED

RPER=SQRT(9.87*(RMIN+RMAX)**3/397584.3246)
RNUM=RPER/H

M=0
M12=0

NUM=N/RNUM

!   WRITE (9,33)
! 33  FORMAT('(' 'Setq' )

DO 200 I1=1,N1

M12=M12+1

DO 100 I=1,N

BR=RNUM/4

M=M+1

X=X+H*VX
Y=Y+H*VY
XE=XE+H*VXE
YE=YE+H*VYE
XM=XM+H*VXM
YM=YM+H*VYM

K1=H*F(T,X,Y,XE,YE, XM, YM, GAN)
L1=H*G(T,X,Y,XE,YE, XM, YM, GAN)

K1E=H*FE(T,X,Y,XE,YE, XM, YM, SMS)
L1E=H*GE(T,X,Y,XE,YE, XM, YM, SMS)

K1M=H*FM(T,X,Y,XE,YE, XM, YM, SMS)
L1M=H*GM(T,X,Y,XE,YE, XM, YM, SMS)

K2=H*F(T+0.5*H,X+0.5*K1,Y+0.5*L1,XE+0.5*K1E,YE+0.5*L1E, XM+0.5*K1M, YM+0.5*L
1M, GAN)
L2=H*G(T+0.5*H,X+0.5*K1,Y+0.5*L1,XE+0.5*K1E,YE+0.5*L1E, XM+0.5*K1M, YM+0.5*L
1M, GAN)

K2E=H*FE(T+0.5*H,X+0.5*K1,Y+0.5*L1,XE+0.5*K1E,YE+0.5*L1E, XM+0.5*K1M, YM+0.5
*L1M, SMS)

```



L2E=H\*GE(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K2M=H\*FM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

L2M=H\*GM(T+0.5\*H,X+0.5\*K1,Y+0.5\*L1,XE+0.5\*K1E,YE+0.5\*L1E,XM+0.5\*K1M,YM+0.5\*L1M,SMS)

K3=H\*F(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,GAN)

L3=H\*G(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,GAN)

K3E=H\*FE(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,SMS)

L3E=H\*GE(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,SMS)

K3M=H\*FM(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,SMS)

L3M=H\*GM(T+0.5\*H,X+0.5\*K2,Y+0.5\*L2,XE+0.5\*K2E,YE+0.5\*L2E,XM+0.5\*K2M,YM+0.5\*L2M,SMS)

K4=H\*F(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3M,YM+0.5\*L3M,GAN)

L4=H\*G(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3M,YM+0.5\*L3M,GAN)

K4E=H\*FE(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3E,YM+0.5\*L3M,SMS)

L4E=H\*GE(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3M,YM+0.5\*L3M,SMS)

K4M=H\*FM(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3M,YM+0.5\*L3M,SMS)

L4M=H\*GM(T+0.5\*H,X+0.5\*K3,Y+0.5\*L3,XE+0.5\*K3E,YE+0.5\*L3E,XM+0.5\*K3M,YM+0.5\*L3M,SMS)

VX=VX+1./6.\*(K1+2.\*K2+2.\*K3+K4)  
VY=VY+1./6.\*(L1+2.\*L2+2.\*L3+L4)

VXE=VXE+1./6.\*(K1E+2.\*K2E+2.\*K3E+K4E)  
VYE=VYE+1./6.\*(L1E+2.\*L2E+2.\*L3E+L4E)

VXM=VXM+1./6.\*(K1M+2.\*K2M+2.\*K3M+K4M)  
VYM=VYM+1./6.\*(L1M+2.\*L2M+2.\*L3M+L4M)

XC=((SMS\*X+EMS\*XE+RMMS\*XM)/(SMS+EMS+RMMS))  
YC=((SMS\*Y+EMS\*YE+RMMS\*YM)/(SMS+EMS+RMMS))

A=ALFA\*0.01744444  
B=BETA\*0.01744444  
GR=GAMA\*0.01744444

!ROTATED ORBITAL PLANE TO THE GLOBAL SYSTEM HAS COORDINATES

XGL=(DCOS(A)\*DCOS(GR)-DSIN(A)\*DCOS(B)\*DSIN(GR))\*X+(-DCOS(A)\*DSIN(GR)-DSIN(A)\*DCOS(B)\*DCOS(GR))\*Y

```
YGL=(DSIN(A)*DCOS(GR)+DCOS(A)*DCOS(B)*DSIN(GR))*X+(-
DSIN(A)*DSIN(GR)+DCOS(A)*DCOS(B)*DCOS(GR))*Y
ZGL=(DSIN(B)*DSIN(GR))*X+(DSIN(B)*DCOS(GR))*Y
```

```
!FROM SPHERICAL SYSTEM THE LATITUDE AND LONGITUDE WILL BE
!DETERMINED, ALSO RADIUS WHICH WILL GIVE ALTITUDE VALUE AS
!A FUNCTION OF POSITION, WHICH IS A FUNCTION OF THE TIME
```

```
R=SQRT(XGL*XGL+YGL*YGL+ZGL*ZGL)
```

```
L=1
```

```
RLONGS=57.32498682*DATAN(YGL/XGL)
IF(XGL.LT.0)RLONGS=180+RLONGS
IF(RLONGS(I).GT.180)RLONGS=RLONGS-360
```

```
LATS=57.32498682*DACOS(ZGL/R)-90
```

```
LATS1=LATS/57.32498682
```

```
GAN=9.78*(1+0.0053024*((DSIN(LATS1))**2)-(5.9E-6)*(DSIN(2*LATS1))**2)
```

```
!THE SPHERICAL COORDINATES OF THE POINT W AT THE EARTH'S SURFACE ARE
```

```
! LONGW=XLONG+360*0.000072921152*T/6.28
! LATW=XLATT
```

```
! RELATIVE SATELLITE POSITION WILL DEPEND ON THE RATE OF EARTH
! ROTATION, THEREFORE EARTH ROTATION AT THE PARTICULAR MOMENT
! IS GIVEN BY:
```

```
! EOMEGA=360/(24*60*60)
```

```
! ANGULAR VELOCITY CALCULATED IN DEG/SEC
```

```
T=T+H
ELONG=T*360/(24*60*60)
NUM1=ELONG/360
```

```
IF(ELONG.GE.360)ELONG=ELONG-NUM1*360
```

```
! THIS IS THE VALUE OF EARTH POSITION, AT TIME T STARTS FROM
! ZERO, BUT WHERE THE ZERO IS?
! ZERO SHOULD BE AT PERIGEE, THE POINT WHERE THE TIME STARTS
! TO BE MEASURED, THEREFORE THE LONGITUDE DIFFERENCE IS:
```

```
! ELGRT=RLONGS(1)-ELONG
```

```
SFSLGRT(I)=RLONGS(I)-ELONG
IF(ABS(ELONG-RLONGS(I)).GT.350)SFSLGRT(I)=RLONGS(I)-ELONG+360
IF(SFSLGRT(I).LT.0.AND.ABS(SFSLGRT(I)).GT.180)SFSLGRT(I)=SFSLGRT(I)+360
```

```

!      LATREL=LATS-LATW
!      LONGREL=RLONGS(I)-LONGW
!      LONGGR=RLONGS(I)-T*0.0041801847
!      LONGREL1=RLONGS(I)-LONGGR

      ALT=R-6366.2

!      WRITE (9,23) I, T, X, Y, ALT, RLONGS(I), ELONG, SFSLGRT(I), LATS
i3=i3+1
if(i21.ge.i1.and.i3.eq.20)WRITE (10,23)ALT, SFSLGRT(I), LATS
if(i3.eq.20)i3=0

23      FORMAT (5X,3E14.6)

100     CONTINUE

      RPER=AP+6378.2
      RAP=AA+6378.2

      ZB=RAP+RPER
      ZRZ=RAP-RPER
!      IF(ZRZ.EQ.0)GO TO 111
      RAZ=1-(ZRZ/ZB)*(ZRZ/ZB)
!      B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(40650.887*GAN*0.5*ZB*RAZ))
!      111  B1=ACOS((ABS(X*VY)-ABS(Y*VX))/SQRT(40650.887*GAN*0.5*ZB))
      SINCL=B-B1

!      WRITE (*,309) SINCL
!      309  FORMAT( 5X 'SINCL=' (F10.3))

WRITE (*,3099) XC, YC
3099  FORMAT( 5X 'CENT. OF MASS =' (2F10.3))

200     CONTINUE

      M121=M121+1

      STOP
      END

      FUNCTION F(T, X, Y, XE, YE, XM, YM, GAN)

      IMPLICIT REAL*8 (A-L, O-Z)
      F=(XE-X)*40650.887*GAN/(SQRT((XE-X)**2+(YE-Y)**2)**3)+(XM-
X)*4906.92/(SQRT((XM-X)**2+(YM-Y)**2)**3)

      RETURN
      END
      FUNCTION G(T, X, Y, XE, YE, XM, YM, GAN)
      IMPLICIT REAL*8 (A-L, O-Z)

```

```
G=(YE-Y)*40650.887*GAN/(SQRT((XE-X)**2+(YE-Y)**2)**3)+(YM-
Y)*4906.92/(SQRT((XM-X)**2+(YM-Y)**2)**3)
RETURN
END
```

```
FUNCTION FE(T,X,Y,XE,YE,XM,YM,SMS)
IMPLICIT REAL*8 (A-L,O-Z)
FE=(XM-XE)*4906.92/(SQRT((XE-XM)**2+(YE-YM)**2)**3)+(X-XE)*6.673E-
20*SMS/(SQRT((XE-X)**2+(YE-Y)**2)**3)
RETURN
END
```

```
FUNCTION GE(T,X,Y,XE,YE,XM,YM,SMS)
IMPLICIT REAL*8 (A-L,O-Z)
GE=(YM-YE)*4906.92/(SQRT((XE-XM)**2+(YE-YM)**2)**3)+(Y-YE)*6.673E-
20*SMS/(SQRT((XE-X)**2+(YE-Y)**2)**3)
RETURN
END
```

```
FUNCTION FM(T,X,Y,XE,YE,XM,YM,SMS)
IMPLICIT REAL*8 (A-L,O-Z)
FM=(X-XM)*6.673E-20*SMS/(SQRT((XM-X)**2+(YM-Y)**2)**3)+(XE-
XM)*398785.2/(SQRT((XM-XE)**2+(YM-YE)**2)**3)
RETURN
END
```

```
FUNCTION GM(T,X,Y,XE,YE,XM,YM,SMS)
IMPLICIT REAL*8 (A-L,O-Z)
GM=(Y-YM)*6.673E-20*SMS/(SQRT((XM-X)**2+(YM-Y)**2)**3)+(YE-
YM)*398785.2/(SQRT((XM-XE)**2+(YM-YE)**2)**3)
RETURN
END
```

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