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## Extended adaptive neuro-fuzzy inference systems

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# **Extended Adaptive Neuro-Fuzzy Inference Systems**

A thesis submitted in fulfillment of the requirements for the  
award of the degree:

**Doctor of Philosophy**

UNIVERSITY OF WOLLONGONG

Lau Chun Yin Dip. CompSc. B.CompSc., M.CompSc.

Faculty of Informatics

School of Information Technology & Computer Science

2006

# Certification

I, Lau Chun Yin, declare that this thesis, submitted in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, in the School of Information Technology & Computer Science, Faculty of Informatics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. This document has not been submitted for qualifications at any other academic institutions.

Lau Chun Yin

3rd October 2006

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# Abstract

This thesis presents a novel extension to the Adaptive Neuro-Fuzzy Inference System (ANFIS) which we call extended ANFIS (EANFIS). The extension includes the introduction of an output class based membership function architecture, in which each output class in a discrete output situation has its own membership function and in the case of a continuous output, only one class; the possibility of determining the structure of the rule base from the underlying structure of the input variables; the determination of a possibly non-symmetric membership function the parameters of which can be determined automatically from the given input variables; the possibility of incorporating global information on the input variables through a Linear Discriminant Analysis in combination with the local input variable structure as represented by the membership functions. The possibility of determining the structure of the rule section before the training process commences means that the proposed EANFIS architecture can be applied to possibility large scale practical problems, as it does not require the formation of all possible combination of rules before the training process commences. In other words, the EANFIS architecture together with its structure determining procedures overcomes the current limitation facing ANFIS architecture when applied to systems with large number of inputs. The possibility of determining a membership function from the input variables means

the user no longer needs to select a membership function from a set of candidate membership functions. The possibility of incorporating global information on the input variables in addition to the local information on input variables means that the EANFIS architecture can take advantage when such global information might be useful in improving the performance of the Neuro-Fuzzy system. The new EANFIS architecture is evaluated on a number of standard benchmark problems, and have been found to have superior performance. In addition, as this is an EANFIS, rules can be extracted from the trained system, thus providing information on the way in which the underlying system is operating. The proposed EANFIS recommends itself readily for applications in practical systems.

# Abbreviations

**ANFIS** Adaptive Network Based Fuzzy Inference System

**ANN** Artificial Neural Network

**COA** Centroid Of Area

**FIS** Fuzzy Inference System

**FLD** Fisher Linear Discriminant

**LDA** Linear Discriminant Analysis

**MLP** Multilayer Perceptrons

**N2Lmap** Nonlinear to Linear mapping

**NOAA** National Oceanic and Atmospheric Administration

**PCA** Principal Component Analysis

**QRcp** QR factorization with column pivoting

**RBFN** Radial Basis Function Network

**RMS** Root Mean Square



**SOM** Self Organizing Map

**SVD** Singular Value Decomposition

**SVM** Support Vector Machines

**TSK** Takagi-Sugeuno-Kang Fuzzy Model

# Notation

[ ]: continuous set

{ }: discrete set

*plaintext* : a plain text indicates a variable

**bold** : bold face indicates a vector

**BOLD** : bold face and capital letter indicates a matrix

*CAPITAL* : capital letter indicates the upper bound of a variable

$\alpha_{rd}$ : weight in Takagi-Sugeuno-Kang (TSK) Fuzzy Model of rule  $r$  in  $d$  dimension

$\delta$ : is a desire value

$\Delta$ : is a small constant

$\eta$ : is a learning constant

$\theta_r$ : Output from  $r$  Linear Discriminant Analysis (LDA) node

$\pi_r$ : Output form the probability layer of rule  $r$

$\bar{\pi}_r$ : Normalized output from the probability layer in rule  $r$

$\phi_r$ : Output from rule layer of rule  $r$

$\bar{\phi}_r$ : Normalized output from rule layer of rule  $r$

$\tau$ : output cluster type where  $\tau \in \{1, T\}$ , T is the upper bound of  $\tau$

$\sigma$ : is the spread of a Gaussian function

$\varphi$ : is a activation function

$B_r$ :  $r$ th basis function of Radial Basis Function Network (RBFN)

$\mathbf{c}_r$ : membership function center of rule  $r$  where  $\mathbf{c} \in \mathfrak{R}^D$

$d$ : data dimension where  $d \in \{1, D\}$ , D is the upper bound of  $d$

$e$ : indicate an error value

$g$ : index number of grid point where  $g \in \{1, G\}$ , G is the upper bound of  $g$

$i$ : index number of input vector where  $i \in \{1, I\}$ , I is the upper bound of  $i$

$r$ : index number of fuzzy rule where  $r \in \{1, R\}$ , R is the upper bound of  $r$

$\mathbf{x}_i$ :  $i$ th input vector where  $\mathbf{x}_i \in \mathfrak{R}^D$

$w_r$ : is a weight attached to rule  $r$

$y_i$ : is a system output value of  $i$ th input pair

$z_{gd}$ :  $g$ th non-linear grid center in  $d$  dimension

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