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Schedule design for liner shipping networks with port time windows

Abdurahim Fares M Alharbi

University of Wollongong

Schedule Design for Liner Shipping Networks with Port Time Windows

Abdurahim Fares M Alharbi

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Supervisors: Dr. Pam Davy and Dr. Shuaian Wang

Abstract

Containers are transported by global liner companies on regularly scheduled ship routes. A large variety of general cargos are containerized, such as manufactured products, food, and garment. Liner shipping services have fixed sequences of ports of call and fixed schedules, i.e., arrival and departure times at each port of call. Liner services are announced in advance to attract potential customers. Customers can arrange the delivery of their cargo based on the available date of the cargo at the origin port and the expected arrival date at the destination port. Therefore, container liner shipping is of significant importance to the global supply chain network.

Different schedules mean different sailing times between ports, which dictate different sailing speeds. It is known in the shipping industry that the daily fuel consumption of ships increases approximately proportional to the sailing speed cubed. Therefore, schedule design affects the bunker fuel consumption and thereby air pollutant emission. Reducing the fuel consumption will also improve the sustainability of the global container transportation network.

Container shipping lines provide weekly services for transporting containers, which means that the rotation time in terms of weeks for visiting all ports of call on a ship route is equal to the number of ships deployed. As a consequence, each port of call has a ship departure on the same day every week. When the speed of ships is higher, the rotation time is shorter, and hence fewer ships are required to maintain the weekly frequency.

The objective of this thesis is to develop mathematical models and solution algorithms for designing the schedules of container liner shipping services. The aim is to minimize the sum of ship cost, fuel cost and inventory cost, while ensuring that ports are available to serve the ships on the planned days.

First, a single ship route is investigated on which each port is visited only once a week. The arrival time of containerships at each port of call on the ship route is determined while considering the available berth time windows at ports. The objective minimizes the sum of ship cost, bunker cost and container inventory cost. This problem is formulated as a nonlinear non-convex optimization model. In view of the problem structure, we develop an efficient dynamic-programming based holistic solution approach, which includes a space-time network model and a bounding technique for the total cost with given number of ships. The proposed solution method is applied to a real ship route operated by a global shipping line.

Second, we generalize the ship route such that a port on it can be visited more than once a week. As a result, more realism is captured but the resulting model is more complex. Taking into account the problem structure, we develop a holistic solution approach. In this approach, at first the port time window constraints are relaxed to obtain a mixed-integer nonlinear programming model, which is subsequently transformed to a mixed-integer linear programming model. This mixed-integer linear model is repeatedly solved by adding the violated port time window constraints until a feasible solution is obtained. This feasible solution is proved to be the global optimal solution to the problem. We have conducted extensive numerical experiments based on a real ship route in operation.

Third, we extend the schedule design for a single ship route to that for a liner shipping network that consists of many ship routes. The inventory cost is assumed to be 0. Hence, the objective minimizes the sum of bunker cost and ship operating cost. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. To address the problem, we reformulate the problem as an integer linear optimization model and propose an iterative optimization approach. The proposed solution method was applied to two networks, consisting of six ports and 21 ports, operated by a global liner shipping company.

Finally, we examine a liner shipping network schedule design while including the inventory cost in the model. As a consequence, the container transshipment and the relevant connection time must be incorporated. An elegant mathematical model is developed. The proposed model is applied to a liner shipping network consisting of 18 ports.

In sum, new mathematical models and solution algorithms have been developed to address the practical liner shipping service schedule design problems. The models and algorithms are helpful decision-support tools for liner shipping companies. The numerical examples demonstrate the applicability of the models and produce a number of useful managerial insights for liner planners.

Certification

I, Abdurahim Fares M Alharbi, declare that this thesis, submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Abdurahim Fares M Alharbi

2015

Dedications

This study is dedicated to

My father, Fares

May Allah show mercy upon him.

My mother, Suhief

Thank you for your prayers and allowed me to complete my study.

My Wife, Asma

Thank you for your love, support and being so patient with me.

My Children, Ammar, Sereen, Mohammed

I am very proud ,for their love and support.

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"In the name of Allah (God), the Most Gracious, the Most Merciful."

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I will not forget to thanks my friends who usually support and encourage me over these years, especially Dr Mohammed Aba Oud, Dr Muteb, Kalied and Abdulrazak.

I am thankful to the King Abdullah for the awarding of a PhD scholarship in his foreign scholarship program.

Finally and most importantly, this thesis is especially dedicated to my beloved family. To my mother, wife, sons, daughter, brothers and sisters for their support and long patience.

List of Publications

[**1**] Wang, S., Alharbi, A., Davy, P., 2014. Liner ship route schedule design with port time windows. Transportation Research Part C 41, 1–17.

[**2**] Alharbi, A., Wang, S., Davy, P., 2015a. Schedule design for sustainable container supply chain networks with port time windows. Advanced Engineering Informatics, doi: 10.1016/j.aei.2014.12.001.

[**3**] Wang, S., Alharbi, A., Davy, P., 2015b. Ship route schedule based interactions between container shipping lines and port operators. In: Lee, C.-Y. and Meng, Q. (Eds.), Handbook of Ocean Container Transport Logistics, International Series in Operations Research & Management Science 220, pp. 279–313. Elsevier, doi: 10.1007/978-3-319-11891-8_10.

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Chapter 1

Introduction

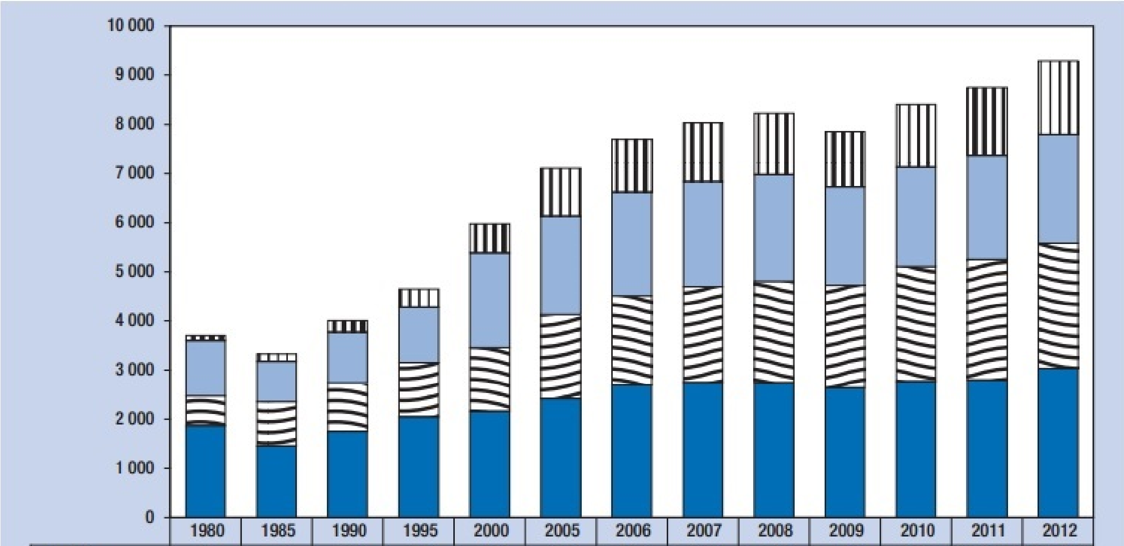
1.1 Preamble

During the last decade, It is observed that there is an increasing in the amount of goods transported on maritime transportation, as well as the number of vessels used. There are many reasons for that. Compared with other modes of transportation, it is safe, cheap and clean (Agarwal and Ergun, 2008; IMO, 2012). According to Agarwal and Ergun (2008), the services are provided at nearly one tenth of the air freight rates. It is also reported that there are fewer accidents in sea cargo transportation, and fewer pollution incidents. The amount of global trade that transported by the sea is more than 70 per cent by value (UNCTAD, 2012). In addition, IMO (2012) indicated that the per cent of global trade transported by sea is more than 90 by volume. Fig. 1.1 shows the international seaborne trade loaded by million tons from 1980, 1985, $\cdots$, 2005 then to 2012. The number was 9.6 billion tons for 2013 (UNCTAD, 2014).

1.2 Container Liner Shipping

Industrial, tramp and liner are three general modes of operation in shipping (Lawrence, 1972) (as cited in Christiansen et al., 2004; Ronen, 1983). In industrial shipping,

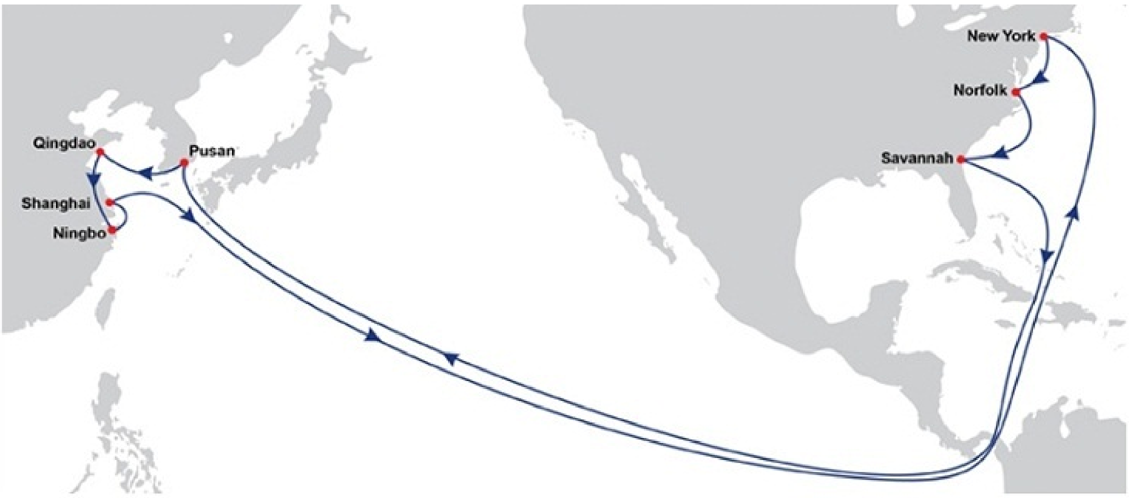
Figure 1.2. International seaborne trade, by cargo type, selected years (Millions of tons loaded)



	1980	1985	1990	1995	2000	2005	2006	2007	2008	2009	2010	2011	2012
Container	102	152	234	371	598	969	1 076	1 193	1 249	1 127	1 275	1 385	1 498
Other dry cargo	1 123	819	1 031	1 125	1 928	2 009	2 112	2 141	2 173	2 004	2 027	2 090	2 219
Five major bulks	608	900	988	1 105	1 295	1 709	1 814	1 953	2 065	2 085	2 335	2 477	2 547
Oil and gas	1 871	1 459	1 755	2 050	2 163	2 422	2 698	2 747	2 742	2 642	2 772	2 796	3 033

Figure 1.1: Source: UNCTAD Review of Maritime Transport, various issues. For 2006-2012, the breakdown by type of dry cargo is based on Clarkson Research Services Shipping Review & Outlook, various issues. Data for 2012 are based on a forecast by Clarkson Research Services in Shipping Review & Outlook, spring 2012.

the ships are under control from the shipper or the cargo owner and it likes private truck fleet shipping. The aim of industrial shipping is to minimize the cost. In contrast, tramp shipping is looking for maximum profit which likes a taxi. In tramp shipping, the ships sail to the available cargoes (Meng et al., 2015). Liner shipping likes a bus line and it mainly involves the transportation of containerized cargo (containers) such as manufactured products, food, and garment (Meng et al., 2014). Unlike tramp shipping, liner shipping services have fixed sequences of ports of call and fixed schedules, i.e., arrival and departure times at each port of call. Liner services are announced in advance to attract potential customers. For example, Fig. 1.2 and Fig. 1.3 show a liner service named North & Central China East Coast Express (NCE) provided by Orient Overseas Container Line (OOCL, 2013). The ports of call and schedule are published in the website of OOCL. Customers can arrange the delivery of their cargo based on the available date of the cargo at the origin port and the expected arrival date at the destination port. For instance, a customer that has 20 containers to be transported from Pusan to New York may contact OOCL to transport the containers. As ships visit Pusan on Sunday, the customer has to make sure that the containers are stacked in the container yard of Pusan before Saturday, so that containers can be loaded to a ship when the ship arrives. The ship will not directly transport the containers from Pusan to New York. It will transport the containers via Qingdao, Ningbo, Shanghai, and finally to New York. At the port of New York, the containers will be unloaded from the ship.



Port Rotation
Eastbound: Pusan - Qingdao - Ningbo - Shanghai - New York - Norfolk - Savannah
Westbound: New York - Norfolk – Savannah - Pusan - Qingdao - Ningbo - Shanghai

Transit Time (Days)			
Asia to North America (Eastbound)			
From \ To	New York (Thu)	Norfolk (Sat)	Savannah (Mon)
Shanghai (Sat)	26	28	30
Ningbo (Thu)	28	30	32
Qingdao (Wed)	29	31	33
Pusan (Mon)	31	33	35

Transit Time (Days)				
North America to Asia (Westbound)				
From \ To	Pusan (Sun)	Qingdao (Tue)	Ningbo (Thu)	Shanghai (Fri)
Savannah (Tue)	26	28	30	31
Norfolk (Sat)	29	31	33	34
New York (Fri)	30	32	34	35

Figure 1.2: NCE service provided by (OOCL, 2013)

In the process of container transportation, the main role of container port is to load and unload containers. First, a containership informs a port operator the estimated arrival time, and then the port operator makes a plan for servicing the ship. When the ship arrives, tug boats will tow the ship to the berth. Then the ship will be moored, and quay cranes will start to load and unload containers for the ship. At the same time, yard trucks will transport containers from and to the

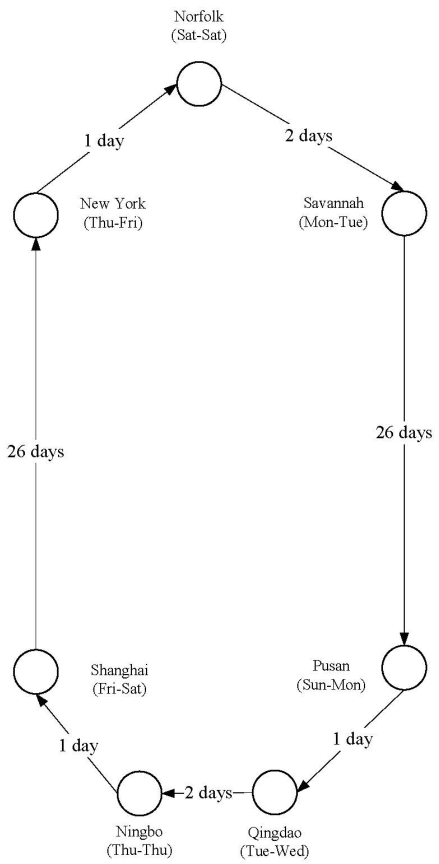


Figure 1.3: NCE service provided by (OOCL, 2013)

quay side. The container handling operation may take up to two days in general. After that, the ship is unmoored, and tug boats tow the ship out of the port.

1.3 Liner Ship Route Schedule Design

Schedule design for a liner service (ship route) is a tactical-level planning decision that is made every three to six months. To design the schedule of a ship route, the first factor to be considered is the availability of the ports. Since a port needs to provide services for a number of liner shipping companies and a number of ships, it cannot guarantee the availability of services whenever a ship arrives. For instance, a port may be able to provide services on Monday, Tuesday, and Friday, and is fully occupied on Wednesday, Thursday, Saturday, and Sunday. We use the term "port time window" to refer to the time in a week that a port can provide services to ships. Hence, schedule design is subject to the constraint of port time windows. Moreover, because of the fast growth of container trade and the long time required for the construction/expansion of port capacity, ports tend to be more congested. As a result, it is important to consider the availability of ports in schedule design. Otherwise the designed schedule may be infeasible in reality.

It should be noted that "port time window" here is different from the "time window" in other problems, e.g., the vehicle routing problems (VPRs), as shown in Fig. 1.4. In fact, in most other problems, time window is an interval that defines a convex set (Wang and Lo, 2008). However, in liner ship route schedule design, port time window defines a set of available times in a week that the port can provide berthing services, and more often than not, the set is disconnected and non-convex. Moreover, because of the weekly frequency of liner shipping services, the port time window should be considered from the viewpoint of a loop rather than a line or line segment. Take Fig. 1.5 as an example. The port time window in Fig. 1.5a is equivalent to that in Fig. 1.5b.

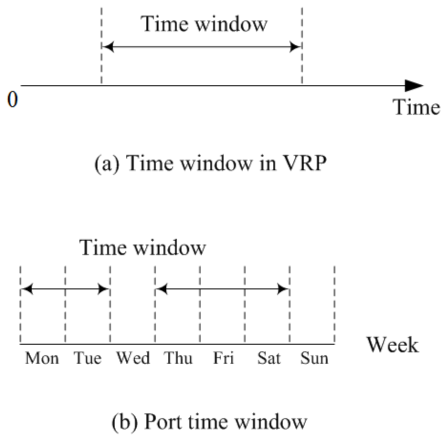


Figure 1.4: Difference of time windows

The design of schedule is also influenced by other factors because different schedules imply different ship costs, bunker costs, and inventory costs. Løfstedt et al. (2010) stated that the cost of crew, maintenance and insurance are the operation

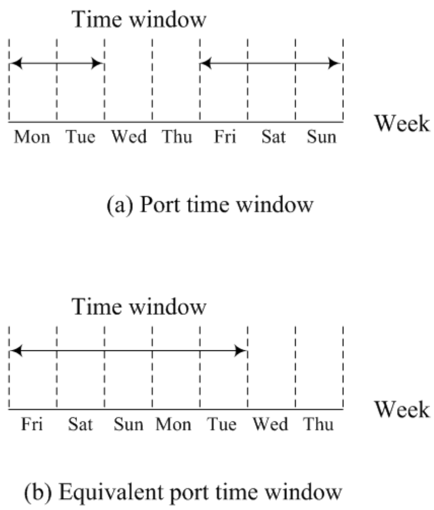


Figure 1.5: Weekly property of port time window

cost (ship cost) of a vessel and they defined the bunker cost as "the cost of bunker, which is the fuel deployed in container vessels". In addition, Hsu and Hsieh (2005) defined the inventory costs as the "opportunity cost or loss of value due to cargos cannot be used or sold in the shipping process, and are positively correlated with the cargo volume, the value of cargo, and the length of transit and storage time". Liner services are usually weekly, which means that the round-trip journey time (weeks) of a ship route is equal to the number of ships deployed on it (Alvarez, 2009; Wang and Meng, 2012a; Wang et al., 2011). As a result, sailing at a higher speed will reduce the round-trip journey time, thereby the number of ships required and the ship cost. However, a higher speed implies a higher bunker cost: the daily fuel consumption of ships increases approximately proportional to the sailing speed (knot) cubed (Ronen, 2011; Wang et al., 2013c). At the same time, a higher speed leads to a shorter transit time of containers from origin to destination, and thereby a lower inventory cost (Notteboom, 2006). Consequently, in schedule design a liner shipping company must balance the trade-off between ship cost, bunker cost, and inventory cost, while considering port time windows.

The liner networks are extremely crucial for international trade purposes (Meng and Wang, 2011a; Meng et al., 2012a; Wang et al., 2013a). These networks provide low transportation cost and optimize trade. The main objective of these ship route schedule designs is to deliver capacity under various constraints and pressure. These route schedule designs are also meant to provide cost effective, as well as, environmental friendly solutions to ship transport problems (Stanley, 2007). In shipping companies, managers try their best to cut the costs by utilizing all the available resources effectively and efficiently (Kim and Moon, 2003) (as cited in (Gao et al., 2010)). These resources include diverse types of resources e.g. human resource, container equipment, and ships. Some of the crucial decision making variables play a very significant role in shaping up the right way to increase profitability and effectiveness.

1.4 Objectives and Contributions

The objective of this thesis is to address the liner shipping service schedule design problem with port time windows (SDPTW). We design the arrival time at each port of call on each ship route that satisfies the port time window constraints while minimizing the sum of ship cost, bunker cost, and inventory cost. The designed schedule is feasible in that it takes into account port time windows. The designed schedule is also optimal because the total cost of ships, bunker, and inventory is minimized. Therefore, this problem is of significant value for liner shipping companies.

The contributions of the thesis to the state-of-the-art literature and practice are three-folds: first, it takes the initiative to address the practical liner shipping service schedule design problem with port time windows; second, it formulates models and develops efficient solution approaches by taking advantage of problems structures. Third, a number of interesting managerial insights from case studies are obtained and these managerial insights provide guidelines for liner shipping companies to make planning decisions.

1.5 Organization of the Thesis

The thesis is organized into seven chapters. Chapter 1 includes a general introduction to shipping and then explains the container liner shipping and liner ship route schedule design. In addition, it highlights the objectives and contributions of the research.

Chapter 2 reviews related literature on schedule design for liner shipping networks with port time windows. It includes four parts. The first part reviews previous research on speed optimization. The second part is about berth allocation. The third part focuses on schedule design. The final part summarizes the gap in the literature.

In chapter 3, the arrival time of containerships at each port of call on a ship route is determined while considering the available berth time windows at ports. A single liner ship route is considered, on which a port is visited only once a week. The objective minimizes the sum of ship cost, bunker cost and cargo inventory cost. The model will be a mixed-integer nonlinear programming model that can be linearized (linearly approximated), and solved by CPLEX.

Chapter 4 is generalized of the model of chapter 3. Ships on a route can visit a port more than once a week. The model will be a mixed-integer nonlinear programming model, and a holistic solution approach to obtain the global optimal solution will be proposed.

Chapter 5 extends the previous one by considering a liner shipping network that consists of many ship routes. Unlike Chapter 3 and Chapter 4, inventory costs are not considered here for simplicity. The model will be a mixed-integer nonlinear programming model, and an iterative optimization approach will be proposed to solve the model.

Chapter 6 extends the previous one by considering inventory cost in a liner shipping network. In addition to the fact that the resulting problem is larger, the container transshipment and the relevant connection time are also incorporated. An elegant mixed-integer nonlinear programming model will be proposed.

Finally, chapter 7 summarizes the works and points out future research directions.

Chapter 2

Literature review

This chapter focuses on a critical review of the existing literature related to schedule design for liner shipping services with port time windows. It is divided into four sections: speed optimization in section 2.1, berth allocation in section 2.2, schedule design in section 2.3 and finally, the literature gap in section 2.4.

2.1 Speed Optimization

Speed is a very important factor in liner ship route schedule design. Sometimes, speed is considered to be a constant during the schedule design, but it plays an important role and it is possible to make it a variable (Saldanha et al., 2006). Speed has a direct or an indirect effect on the ship fleet and costs pertaining to cargo inventory (Psaraftis and Kontovas, 2013). Optimizing or changing the speed can lead to more efficiency and profitability and this way it becomes possible for shipping to attract cargos from other modes of transport. In the real world scenario, speed is not a constant and the fuel consumption per unit time can be calculated as a function of speed. Meng et al. (2014) show that some studies keep the speed as one of the decision variables and cost effectiveness and efficiency are achieved by optimizing speed, e.g., Alvarez (2009), Gelareh and Meng (2010), Golias et al. (2010a), Lang and Veenstra (2010), Meng and Wang (2011b), Cheaitou and Cariouz (2012), Yao et al. (2012), Wang et al. (2013e) and Wang et al. (2013d). In addition, the liner shipping business is negatively affected by increasing bunker price (Notteboom and Vernimmen (2009) as cited in (Brouer et al., 2013)).

The sailing speed is one of the reasons leading to a high total operation cost of container ships. As a result of increasing the sailing speed, more bunker will be consumed (Notteboom and Vernimmen, 2009). Ronen (2011) indicates that more than 75 percent of the total operating costs of a container ship are from bunker cost. Meng et al. (2014) state that at the strategic level, the sailing speed depends on the size of the fleet and the flexibility of port times while it is influenced by weather and currents at operational level. Several researchers emphasized the importance of speed sailing and its impact on the consumption of bunker (Meng et al., 2014). For example, Kontovas (2011) studies the relationship and suggests that using at least of 4 of an exponent used if the sailing speed is more than 20 knots, while Du et al. (2011) study the relationship based on different vessel classes and they used an exponent of 4.5 for jumbo vessel, 4 for medium-sized vessel and 3.5 for feeder vessel. In addition, Wang and Meng (2012d) study the relationship based on historical data and they found that the exponential relationship between the sailing speed and the bunker consumption between 2.7 and 3.3.

Ships move at a very low speed as compared to other modes of transporting cargo like airplanes and trains. Speed is a very important decision variable in liner ship route schedule designs. Speed and the transit time force a major challenge for the companies involved in liner shipping business. The transit time is short in most of the cases where time sensitive goods are involved. Time sensitive goods have to be transported within a short span of time otherwise they cannot survive. In case of transporting these sensitive goods, speed plays a very crucial role. Examples of these sensitive goods include consumer goods with a short span of life and goods like computers, fashion related goods (Notteboom, 2006).

Keeping in mind this long duration resulting from low speed, a higher speed might play a very crucial role. It is possible that the high speed entails added economic value of quicker distribution of goods, less portfolio cost and more trade overall (Notteboom, 2006). During the last couple of years, the trade within companies has gone up tremendously, and this has led to a higher speed requirement. Moreover, there are various speed models which provide a solution by minimizing all the adverse effects of high speed and by optimizing speed giving cost effective and efficient solutions. There are tradeoffs and correlations between high speed, fuel price, impact on the environment, and operational costs (Saldanha et al., 2006).

2.2 Berth Allocation

Quay-side and yard-side are two main components of ports (Li et al., 2012b, 2009; Yip et al., 2014; Zhuang et al., 2014). Berth is “the space allotted to a vessel at anchor or at a wharf”. Berth allocation is a very important decision making resolution, and a lot of research is done on this topic. Research indicates that amongst all resources (e.g. human resource, container equipment, berths, and container gantries), the most important resource is berths (Gao et al., 2010). Allocation of berths plays a very vital role in the shipping business because well designed schedules of berths help in increasing the port during the course of business, and they also help in increasing the customer satisfaction. Both these factors lead to better results with high profitability at the ports (Imai et al., 2007).

Berth scheduling or berth allocation can be done using various mathematical formulations. The main objective of these allocation processes is to minimize all costs incurred from late departure of vessels, and also to minimize cost incurred from waiting time at the terminals (Golias and Haralambides, 2011). At the same time, the objective is to maximize revenues and efficiency by ensuring early departures of the vessels. A good solution of the berth allocation problem (BAP) can shorten the unproductive port time of ships, enabling liner shipping companies to make a

Figure 2.1: Jabal Ali Port (AllportGargoServices)

higher profit.

The BAP can be classified according to different criteria. First, there are discrete BAP (DBAP) where each berth can serve one ship at a time, and continuous BAP (CBAP) with a long straight quay and how many ships can be accommodated at the same time depends on the sizes of the ships. Second, BAP can be classified as being either static (SBAP) or dynamic (DynBAP). In SBAP, all ships are already in the port when the berth allocation is planned, whereas in DynBAP some ships are still on the voyage to the port when the port operator allocates berths. The SBAP is applicable when the port is highly congested. Third, BAP can occur at the operational level (OBAP), or tactical level (TBAP). The OBAP covers a planning horizon of usually at most one week and the TBAP aims to support port operators to negotiate with shipping lines. If TBAP accounts for the periodicity of vessel schedules, e.g., weekly arrival patterns of containerships, then if a vessel is serviced at a berth on day 7 and day 8, other vessels cannot use the berth on day 1, because day 8 and day 1 correspond to the same day in a week. The time horizon of this type of TBAP is a cylinder whose circumference equals 1 week. Hence, the resulting models (Moorthy and Teo, 2006; Zhen et al., 2011b) are significantly different from

OBAP models. If in the TBAP vessels do not arrive periodically, the time horizon is simply a rectangle with an open end and the models are very similar to OBAP models.

Besides determining the berthing time and location, some studies on DynBAP (either DBAP or CBAP and either TBAP or OBAP) also integrate other decision issues such as quay crane assignment, quay crane scheduling, container storage planning at yard, and yard truck scheduling. The models on DynBAP all aim at providing berthing and other related services at minimum cost (cost associated with quay cranes and yard trucks). However, different models have different definitions for service. Most studies assume that each ship has a preferred arrival time. Giallombardo et al. (2010) is an exception in that it examined a TBAP and assumed that there was no difference for shipping lines when their ships were scheduled to arrive. The objective was to minimize the container handling time of ships by choosing quay crane assignment profiles.

The studies considering the preference of ship arrival times can be classified into four different lines, which are briefly summarized as follows. The first line aims to minimize the total service time (turnaround time) of all ships, including waiting time for berths and container handling time, or total weighted service time where different ships have different weights, for example, Imai et al. (2008a, 2001, 2003, 2005), Cordeau et al. (2005), Moorthy and Teo (2006), Golias et al. (2010b, 2009b), and Lee et al. (2010). Note that if the handling time is constant, minimizing the service time is equivalent to minimizing the waiting time. Similarly, Imai et al. (2008b) required that if a ship's waiting time exceeded a certain limit, the ship must be served at an external terminal, and the target is to minimize the total service time of ships at the external terminal. Golias et al. (2009a) considered two objectives: minimizing the total service time of preferential customers, and minimizing the total service time of all vessels. The second line minimizes the total tardiness cost, which is the finish operation time (real departure time) minus the expected departure time if the former is larger, and 0 otherwise, for instance, Kim and Moon (2003), Chang et al. (2010), and Zhen et al. (2011b). In addition, Han et al. (2010) proposed a proactive approach for a BAP with quay crane scheduling and stochastic arrival and handling time. They took into account the expected value and standard deviation of the total service time and weighted tardiness of all ships. Chen et al. (2012) minimized the maximum relative tardiness of all ships. The third line formulates the penalty for earliness and tardiness in greater details. Meisel and Bierwirth (2009) investigated a CBAP with quay crane allocation. They assumed that each ship has an expected arrival time, an earliest start operation time, expected finish operation time, and latest allowed finish operation time. All of these time components were included in the objective function. Zhen et al. (2011a) developed an integrated model for the TBAP with yard operations planning. The model minimized the weighted sum of deviation from vessels' expected turnaround time intervals and the operations cost associated with transshipment containers. The fourth line incorporates the bunker cost of the vessels in the models. Golias et al. (2010a) considered the following elements in the objective function: (i) the total service time, (ii) the tardiness, and (iii) the emissions and fuel cost for all vessels while in transit to their next port of call. By contrast, Du et al. (2011) incorporated the tardiness and the fuel cost for all vessels while in transit from their current positions to the focal port of the BAP. These berth allocation studies are all from the points of view of port operators.

2.3 Schedule Design

The primary purpose of the scheduling models is to assign limited resources to the given tasks over the period of time efficiently (Cai et al., 1998). Basically, schedule design or model is a decision making process which is quite evident and popular in many of the service systems, manufacturing systems, and information handling systems. The schedule design of the service line means determining the speeds of the containerships to transporting containers from source to the destination (Meng et al., 2014). During the last couple of years, the topic of liner ship route schedule design has gained significant importance though. There has been a downward pressure on organizations involved in the shipping industry because of the fluctuating oil prices and the recent financial crisis (Besbes and Savin, 2009; Kjeldsen, 2012; UNCTAD, 2008; Wang and Meng, 2012b). In these difficult times, scheduling has become more important because efficient and cost effective scheduling is a possible way to earn reasonable profits. Nowadays profitability of all kinds of liner shipping companies is heavily dependent on scheduling and tactical planning which enables efficiency along with cost effectiveness (Wang et al., 2012).

The schedule designed for liner ship routes remains unchanged for three to six months (Kjeldsen, 2012; Meng et al., 2014; Song and Dong, 2011; Wang and Meng, 2012b). The schedule design is extremely important because it has a direct impact on ship cost, as well as, on the bunker cost (Wang and Meng, 2012b). The bunker cost plays a very crucial part in this system. It contributes around twenty to sixty (Ronen, 1993) or even more than seventy five percent of the total operating cost when the bunker price is high (Ronen, 2011). Ship cost is also very crucial which depends on the number of ships to deploy, determined by the length of a round trip; the length of a round trip depends on the time spent during the sailing on the sea and the time spent at ports. If one or both of them increases, the journey time becomes longer and this means the companies need more ships used in this route to achieve the weekly service. Thus, there is a trade-off between these two costs i.e. the bunker cost and the ship cost (Meng et al., 2014; Psaraftis and Kontovas, 2013).

The route scheduling also has an effect on transit time of the containers. This impact on transit time is from the starting port to the destination or ending port. This transit time forces a challenge for the companies involved in this liner shipping business (Wang and Meng, 2012b). Due to tight competition between these shipping companies, the transit time offered is very short (Bell and Bichou, 2008). The transit time is short in most of the cases where time sensitive goods are involved. Time sensitive goods are the ones which have to be transported within a short span of time otherwise they cannot survive. Examples include consumer goods with a short span of life and goods like computers which are time sensitive (Notteboom, 2006). Sometimes transshipment is also involved during this process. Transshipment operation includes transference of a container from one ship to the other ship during the time of the trip, from start to the destination port (Lachner and Boskamp, 2011). In most of the cases, it is cheaper to ship goods, sent from the source, through intermediate or transient nodes before reaching the final destination. Transshipment problems are a more general form of transportation problem, where only direct shipments are allowed from source to the destination. Petering (2011) reports that at Port of Singapore, the transshipment containers are the most of the containers handled.

There are some papers review on ship routing and scheduling, for instance, Ronen (1983), Ronen (1993), Christiansen et al. (2004) and Meng et al. (2014). Ronen (1983), Ronen (1993), and Christiansen et al. (2004) mainly concentrate on industrial and tramp shipping. While Meng et al. (2014) is dedicated to liner shipping. According to our extensive literature search, there are five articles most relevant to schedule design: Mourão et al. (2001); Wang and Meng (2011); Qi and Song (2012); Wang and Meng (2012b) and Wang and Meng (2012c).

Mourão et al. (2001) analyzed a small hub-and-spoke network at the tactical level. The network consisted of two routes – a feeder route and a main route – and one pair of ports. It assumed that all containers must be transshipped at the hub port in the feeder route. The main route had two possible schedules: Monday roster and Thursday roster. Two integer programming models were developed. The decision variables in the first model include: number of mainline ships in each type assigned to the Monday roster, number of mainline ships in each type assigned to the Thursday roster, and number of feeder ships assigned to the feeder route. In the second model, the decision variables were: number of voyages per year of the mainline ships in each type assigned to the Monday roster, number of voyages per year of the mainline ships in each type assigned to the Thursday roster, and number of voyages per year of feeder ships. The inventory costs of the containers to be shipped were considered in the objective function. These two models were solved by Excel.

Wang and Meng (2011) investigated the schedule design and container routing problem in liner shipping. They considered a general liner shipping network with many ports, many ship routes, and many origin-destination (OD) pairs. Containers in each OD pair had more than one path to be transported from origin to destination, and these paths were assumed to be given a priori. Containers in each OD pair had a market-level transit time to ensure that the container delivery service was competitive. In particular, if the real transit time was longer than the market-level transit time, a penalty was incurred; if the real transit time was shorter than the market-level transit time, a bonus was given. It was assumed that the sailing speed of ships and the time spent at each port of call were all fixed. Hence, the main decision variables were when to arrive at the first port of call on each ship route. At the same time, container routing with transshipment were incorporated. In fact, how containers were transported affected the schedule design. Hence, schedule design and container routing were studied in a holistic manner. The formulation for the schedule design and container routing problem was nonlinear, non-continuous and non-convex. An efficient genetic local search heuristic was developed. Computational results showed that the genetic local search heuristic could efficiently find good quality solutions. Moreover, the model for the container routing sub-problem could be separately used to optimize the day-to-day container routing decisions for the realized container shipment demand after the schedules have been designed.

Qi and Song (2012) designed an optimal containership schedule for a liner ship route to minimize the total expected fuel consumption. They considered uncertain port time and weekly frequency. They defined the level of service as the probability that the containership would arrive at a port no later than the published arrival time. They analytically studied the special case of 100% service levels. By proving the convexity and differentiability of the objective function, it was shown that the optimal schedule could be obtained by solving a nonlinear programming problem. With further assumption of identical distribution of the uncertain parts of port times, they analytically derived some properties of an optimal schedule, which led to useful managerial insights. For example, the shortest leg was the most problematic leg when designing the optimal schedule to achieve 100% service level and to minimize the emissions within the speed constraints, and therefore a liner shipping company should plan relatively longer time for a short leg. A general optimal ship scheduling problem was formulated, and the formulation was solved by simulation-based stochastic approximation methods. They validated the model and the properties by numerical studies. Based on a real liner case study with various scenarios analysis, they found significant fuel savings could be achieved from their model compared to the company's original schedule or to the schedule based on deterministic data, especially for the cases with larger degree of uncertainties. They also found that the total fuel consumption could be reduced by sacrificing the service levels starting from the shortest legs; whereas as the vessel lateness penalty increased, higher service levels tended to be maintained and they became evener among all port-of-calls. This would help liner companies better understand the tradeoff between the fuel consumption and the service level.

Wang and Meng (2012c) examined the design of liner ship route schedules that could hedge against the uncertainties in port operations, which included the uncertain wait time due to port congestion and uncertain container handling time. They assumed that if a ship arrived at a port later than planned, then the penalty cost first increased linearly with the delay, and when the delay exceeded a particular threshold, the penalty cost did not change any more because the customers already resorted to other approaches to handle the delay. They further assumed that if a ship was delayed, it would try to catch up with the planned schedule as early as possible by sailing at the fastest speed. The designed schedule was robust in that uncertainties in port operations and schedule recovery by fast steaming were captured endogenously. The number of ships required to maintain a weekly frequency was considered as a decision variable. The objective function minimized the ship operating cost, expected bunker cost, and the penalty cost for delay. This problem was formulated as a mixed-integer nonlinear stochastic programming model. A solution algorithm which incorporated a sample average approximation method, linearization techniques, and a decomposition scheme, was proposed. Numerical experiments based on a long-haul ship route of Maersk Line were carried out. The ship route covered two trade lanes: trans-Pacific and trans-Atlantic, and three regions: Asia, America, and Europe, and had the sequence of ports of call as follows: Tokyo (1) → Kobe (2) → Chiwan (3) → Hong Kong (4) → Kaohsiung (5) → Busan (6) → Kobe (7) → Tokyo (8) → Balboa (9) → Manzanillo (10) → Miami (11) → Jacksonville (12) → Savannah (13) → Charleston (14) → New York (15) → Antwerp (16) → Felixstowe (17) → Bremerhaven (18) → Rotterdam (19) → Le Havre (20) → New York (21) → Norfolk (22) → Charleston (23) → Manzanillo (24) → Balboa (25) → San Pedro (26) → Oakland (27) → Tokyo (1). The numerical experiments demonstrated that the algorithm obtained near-optimal solutions with the stochastic optimality gap less than 1.5% within reasonable time.

Wang and Meng (2012b) extended the work of Wang and Meng (2011). Both works have studied a liner shipping network, which contrasted Qi and Song (2012) and Wang and Meng (2012c), and both works have required a certain level of service in terms of OD transit time. Wang and Meng (2012b) was the first attempt to examine the optimal sailing speed function in view of sea contingency to minimize bunker consumption. The optimality condition for the sailing speed and the optimal sailing speed function with time were derived. They also contributed to the line of literature on optimization of sailing speed to control bunker consumption by providing an efficient and exact cutting-plane based solution algorithm. Moreover, they addressed the practical schedule design problem arising in liner shipping industry while considering port-to-port transit time with transshipment and sea contingency and uncertain port time. The port-to-port transit time with transshipment issue was solved with a mixed-integer programming model; sea contingency was investigated in the optimality condition of sailing speed; and the uncertain port time was addressed by proving the convexity of the expected bunker cost on each voyage leg with regard to the inter-arrival time between the two consecutive portcalls of the leg. The novel holistic solution algorithm exploited the special structure of the decision problem and integrated several techniques in a nice manner. The proposed model and algorithm were applied to an Asia-Europe-Oceania shipping network provided by a global liner shipping company. The network had a total of 46 ports in Asia, Europe, and Oceania. These 46 ports were served by 11 ship routes with three types of ships. There were a total of 100 container routes in the shipping network. The computational results demonstrated that the proposed model provided a useful planning tool for liner shipping companies.

2.4 Literature Gap

By summarizing the most relevant articles to schedule design in Table. 2.1, we can now identify the gap in the literature. Only two papers consider a general liner shipping network. Some papers do not consider the speed optimization or do not consider the inventory cost of cargos. Moreover, none of the studies have considered the availability of ports in their schedule design. Hence, the liner service schedule design with port time windows is a new research topic. It incorporates both shipping operations and port operations in the planning decision and hence has practical significance for liner shipping companies. Addressing the problem requires sophisticated mathematical models techniques. Although mathematical tools have been used extensively in transportation and scheduling problems (Wu and Kumar, 2012; Wu et al., 2010; Zheng et al., 2014; Zheng and Su, 2014), most research on liner shipping is qualitative. Therefore, new mathematical tools have to be developed to address the liner service schedule design problem with port time windows, which is the aim of this research.

Table 2.1: Most relevant articles to schedule design

Author	One Ship route or network	Speed optimization	Inventory cost	Port availability
Mourão et al. (2001)	only a feeder and a main route	N	Y	N
Wang and Meng (2011)	network	N	Y	N
Qi and Song (2012)	one route	Y	N	N
Wang and Meng (2012b)	one route	Y	Y	N
Wang and Meng (2012c)	network	Y	N, but transit time is considered	N

Chapter 3

Simple Single Ship Route Schedule Design

3.1 Introduction

Liner shipping likes a bus line and it mainly involves the transportation of containerized cargo (containers) such as manufactured products, food, and garment (Øvstebø et al., 2011 and Meng et al., 2014). Liner shipping services have fixed sequences of ports of call and fixed schedules, i.e., arrival and departure times at each port of call, similar to public transport operations. Liner services are announced in advance to attract potential customers. For example, Fig. 3.1 shows a liner service named North & Central China East Coast Express (NCE) provided by Orient Overseas Container Line (OOCL, 2013).

This chapter examines the interaction between shipping lines and port operators on schedule design from the viewpoint of shipping lines. Schedule design for a liner service (ship route) is a tactical-level planning decision that is made every three to six months. To design the schedule of a ship route, the first factor to be considered is the availability of the ports.

The objective of this chapter is to address the liner ship route schedule design problem with port time windows (SDPTW). We assume that each port on the ship route is visited only once in a round-trip journey. We design the arrival time at each port of call on the ship route that satisfies the port time window constraint while minimizing the sum of ship cost, bunker cost, and inventory cost. The designed schedule is feasible in that it takes into account port time windows. The designed schedule is also optimal because the total cost of ships, bunker, and inventory is minimized. Therefore, this problem is of significant value for liner shipping companies.

The rest of the chapter is organized as follows. Section 3.2 describes the problem. Section 3.3 formulates a mathematical model for the problem. Section 3.4 proposes a dynamic programming based holistic solution approach to address the problem. Section 3.5 reports a case study based on the NCE service of OOCL. Section 3.6 presents the conclusion.

3.2 Problem Description

Port Rotation
Eastbound: Pusan - Qingdao - Ningbo - Shanghai - New York - Norfolk - Savannah
Westbound: New York - Norfolk – Savannah - Pusan - Qingdao - Ningbo - Shanghai

Transit Time (Days) Asia to North America (Eastbound)			
From \ To	New York (Thu)	Norfolk (Sat)	Savannah (Mon)
Shanghai (Sat)	26	28	30
Ningbo (Thu)	28	30	32
Qingdao (Wed)	29	31	33
Pusan (Mon)	31	33	35

Transit Time (Days) North America to Asia (Westbound)				
From \ To	Pusan (Sun)	Qingdao (Tue)	Ningbo (Thu)	Shanghai (Fri)
Savannah (Tue)	26	28	30	31
Norfolk (Sat)	29	31	33	34
New York (Fri)	30	32	34	35

Figure 3.1: NCE service provided by (OOCL, 2013)

Consider a ship route such as the NCE service in Fig. 3.1. The ship route has a weekly service frequency which means each port of call is visited on the same day every week (Meng and Wang, 2012; Wang et al., 2013b). The port rotation of the ship route has a total of N ports. Define $I := \{1, 2, \cdots, N\}$, which is a set representing all the ports of call for simplifying the notation. Since the ports of call on a ship route form a loop, we can arbitrarily choose one port as the first port of call (Wang, 2013; Wang et al., 2014b). For instance, if we let New York be the first port of call, the NCE service can be coded as follows: 1 (New York) → 2 (Norfolk) → 3 (Savannah) → 4 (Pusan) → 5 (Qingdao) → 6 (Ningbo) → 7 (Shanghai) → 1 (New York). If we let Norfolk be the first port of call, the NCE service can be coded as follows: 1 (Norfolk) → 2 (Savannah) → 3 (Pusan) → 4 (Qingdao) → 5 (Ningbo) → 6 (Shanghai) → 7 (New York) → 1 (Norfolk). We let p_i represent the physical port of the ith port of call, $i \in I$. We further define the voyage from the ith port to the $(i+1)$th as leg i; leg N is the voyage from the Nth port of call to the first one. For instance, if we define New York to be the first port of call, then the first leg is the journey from New York to Norfolk, the second leg is the journey from Norfolk to Savannah, the third leg is the journey from Savannah to Pusan, the fourth leg is the journey from Pusan to Qingdao, the fifth leg is the journey from Qingdao to Ningbo, the sixth leg is the journey from Ningbo to Shanghai, the seventh leg is the journey from Shanghai to New York.

We assume that $p_i \neq p_j, i \neq j$. In other words, we assume that each physical port is visited only once during a round-trip journey. It should be noted that in reality there are many ship routes that visit a port twice in a round-trip journey, and in extreme cases, three times. The methods proposed in the chapter could be used for designing schedules for these ship routes, too, but need considerable modification. For better readability, we only consider the case that each physical port is visited only once during a round-trip journey.

3.2.1 Ship cost, bunker cost and inventory cost

We assume that a string of m homogeneous containerships are deployed on the ship route to maintain a weekly service frequency. Ships are homogeneous means that they have the same capacity, age, designed speed, and other ship specific characteristics. In reality, two ships cannot be the same because even if they were the same when constructed, different past operating conditions would make them different (e.g., fuel efficiency). However, in mathematical modeling, it is convenient to model ships with similar characteristics as identical without losing much precision. That is why we also adopt such an approach. The highest possible sailing speed of the ships is denoted by $V^{\max}$ (knot). Represent by t_i^{port} the time (h) a ship spends at port i, and L_i (n mile) the distance of leg i. The maximum speed of containerships is usually higher than that of bulk cargo ships and tankers. This is mainly because containerships transport containerized cargos with higher unit value, and hence faster delivery is more desirable. The time a ship spends at a port consists of the time for towage, mooring and unmooring, possible wait time, and container handling. The most significant time component is container handling. For instance, if the average container handling efficiency is 100 containers/h, and a total of 2000 containers are loaded or unloaded, then the container handling time is 20 hours. We assume that the container handling time is fixed. In reality, this time cannot be exactly predicted, and hence here we can consider t_i^{port} as already including some buffer time.

3.2.1.1 Ship cost

Let v_i be the sailing speed (knot) of ships on leg i. To maintain a weekly service frequency, we have the relation:

$$\sum_{j \in I} L_j/v_j + \sum_{j \in I} t_j^{\text{port}} = 168m \tag{3.1}$$

In Eq. (3.1), the left-hand side is the round-trip journey time (h), and the right-hand side is the number of ships times 168 hours/week. Eq. (3.1) is the fundamental equation defining the number of ships required to maintain a weekly frequency. For instance, if the round-trip journey time is 336 hours (two weeks), two ships are needed to maintain a weekly frequency. If the round-trip journey time is 8 weeks, 8 ships must be deployed to maintain a weekly frequency. If we can reduce the round-trip journey time from 8 weeks to 7 weeks by sailing faster, skipping ports, or shortening port time, we can save one ship. Denote by C^{ship} (USD/week) the fixed operating cost of a ship, which is the ship chartering cost but does not include bunker fuel cost. Hence, the weekly operating cost of the ships deployed on the ship route is $C^{\text{ship}}m$.

3.2.1.2 Bunker cost

As aforementioned, Eq. (3.1) implies that when the speed is higher, fewer ships need to be deployed to maintain the same weekly service frequency. However, a higher speed implies a larger amount of bunker consumed. To take into consideration the bunker cost, we let $g_i(v_i)$ (tons/n mile) be the bunker consumption function at the speed v_i on leg i. Based on the results in existing studies (Kontovas, 2011; Psaraftis and Kontovas, 2010, 2013; Ronen, 2011; Wang and Meng, 2012d), we assume that $g_i(v_i)$ is a power function of the form:

$$g_i(v_i) = a_i(v_i)^{b_i}, i \in I \tag{3.2}$$

where a_i and b_i are two coefficients calibrated from historical operating data and satisfy $a_i > 0$ and $b_i > 1$. Denote by α (USD/ton) the bunker fuel price. The weekly bunker cost is $\alpha \sum_{i\in I} L_i g_i(v_i) = \alpha \sum_{i\in I} L_i a_i(v_i)^{b_i}$. It should be noted that although we assume that the bunker consumption function has the form of Eq. (3.2), the solution method that will be elaborated later is applicable to other forms of bunker consumption functions, too.

3.2.1.3 Inventory cost

Besides the ship cost and bunker cost, the inventory cost of containers should also be incorporated. In fact, a lower speed (slow-steaming) would increase the transit time of containers, and thereby the inventory cost. We let $\bar{V}_i$ be the number of containers (twenty-foot equivalent units, or TEUs) on leg i, and β be the unit inventory cost (USD per TEU per h). Since the time spent at each port is constant, we only consider the inventory cost associated with sailing time at sea (sea time). Therefore, the total inventory cost is $\sum_{i\in I} \beta\bar{V}_i L_i/v_i$. It should be noted that $\bar{V}_i$ is actually a predicted value based on historical data. The inventory cost is included to reflect the quality of the liner shipping company's transport services[1]. Note further that β is also predicted and our model allows β to vary with different voyage legs.

3.2.2 Liner ship route schedule

We define the time 00:00 of a certain Sunday as time 0 (h), and hence 10:00 on Monday is time 24+10=34, and 10:00 next Tuesday is time 168+24*2+10=226. Since we assume that the port time (t_i^{port}) is fixed, the time of departure (t_i^{dep}) at port i is determined by the time of arrival (t_i^{arr}) and the port time (t_i^{port}), that is:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{3.3}$$

Because of the weekly service frequency, without loss of generality, we let

$$0 \leq t_1^{\text{arr}} < 168 \tag{3.4}$$

Note that the above equation is important to eliminate symmetric solutions. Because of the weekly frequency, there is no difference whether the first port of call is visited at time 20 (i.e., $t_1^{\text{arr}} = 20$) or 20 + 168 (i.e., $t_1^{\text{arr}} = 188$). Hence, we only need to

[1]In reality the liner shipping company will not pay the customers for their inventory cost.

consider the case where the arrival at the first port of call is between time 0 and 168.

We define the time when the ship returns to the first port of call as t_{N+1}^{arr}, that is:

$$t_{N+1}^{\text{arr}} := t_1^{\text{arr}} + 168m \tag{3.5}$$

This equation implies that a ship needs $t_{N+1}^{\text{arr}} - t_1^{\text{arr}} = 168m$ hours to complete a round-trip journey. This is consistent with the weekly frequency.

The *schedule* of a liner ship route is the vector defined below:

$$(t_i^{\text{arr}}, i \in I; m) \tag{3.6}$$

We stress that the schedule of a liner ship route cannot be represented by $(t_i^{\text{arr}}, i \in I)$. This is because, given $(t_i^{\text{arr}}, i \in I)$, we do not know the inter-arrival time from the last port of call to the first. The number of ships m together with $(t_i^{\text{arr}}, i \in I)$ can define the inter-arrival time from the last port of call to the first. Of course, the schedule can also be uniquely determined by $(t_i^{\text{arr}}, i \in I; t_{N+1}^{\text{arr}})$.

3.2.3 Port time window

A ship cannot arrive at a port at any time because the port may be busy during some periods of a week. Hence, we let $\Omega_i \subseteq [0, 168)$ be the time in a week during which port i is available for serving ships on the ship route, i.e., port time window. For example, $\Omega_i = [10, 20] \cup [96, 120]$ means that port i is available from 10:00 Sunday to 20:00 Sunday, and 00:00 Thursday to 00:00 Friday. $\Omega_j = [0, 24] \cup [144, 168)$ means that port j is available from 00:00 Sunday to 00:00 Monday, and 00:00 Saturday to 00:00 Sunday. In other words, the port is available from 00:00 Saturday to 00:00 Monday next week.

We assume that the port time window at each port (which corresponds to each port of call because we assume that each port is visited once in a round-trip journey) is known. In reality, a liner shipping company can obtain this port time window from port operators, because port operators have to tell it whether it is possible to arrive at a particular time.

A ship needs to stay at port i for t_i^{port} hours. Therefore, we could easily compute the feasible arrival times at port i based on Ω_i. For instance, $\Omega_i = [10, 20] \cup [96, 120]$ and $t_i^{\text{port}} = 5$ imply that t_i^{arr} could be any value in $[10, 15] \cup [96, 115]$. We let $\hat{\Omega}_i \subseteq [0, 168)$ be the set of feasible arrival times at port i in a week. It should be mentioned that because of the weekly service frequency, when $\hat{\Omega}_i = [10, 15] \cup [96, 115]$, the arrival time $t_i^{\text{arr}} = 180$ (which corresponds to time 12 of the next week) is also feasible. In fact, an arrival time is feasible if and only if $(t_i^{\text{arr}} \mod 168) \in \hat{\Omega}_i$, where the "mod" operator obtains the modulus of two integer numbers.

Therefore, the ship route schedule design problem with port time window aims to determine the optimal arrival time at each port of call on a ship route that satisfies the port time window to minimize the total cost including ship cost, bunker cost, and inventory cost.

3.3 Mathematical model

Before presenting the model, we list the notation below.

Variables

m	Number of ships deployed on the ship route
t_i^{arr}	Arrival time (h) at the ith port of call
t_{N+1}^{arr}	The time (h) when the ship returns to the 1st port of call
t_i^{dep}	Departure time (h) from the ith port of call
v_i	Sailing speed (knot) on leg i

Parameters

3.3.1 Model

The SDPTW can be formulated as:

α	The bunker fuel price (USD/ton)
β	The unit inventory cost (USD per TEU per h)
$\hat{\Omega}_i$	The set of feasible arrival times at the ith port of call
C^{ship}	The weekly operating cost of a ship (USD/week)
$g_i(v_i)$	Bunker consumption per nautical mile at the speed v_i on leg i (tons/n mile)
I	Set of legs, $I = \{1, 2, \cdots, N\}$
L_i	Oceanic distance (n mile) of the leg i
N	Number of ports on the ship route
p_i	The port i on the ship route
t_i^{port}	Time (h) a ship spends at port i
$\bar{V}_i$	Number of containers (TEUs) on leg i
$V^{\max}$	Maximum speed of the ships (knot)

[SDPTW]

$$\min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \tag{3.7}$$

subject to:

$$\sum_{j \in I} L_j / v_j + \sum_{j \in I} t_j^{\text{port}} = 168m \tag{3.8}$$

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{3.9}$$

$$0 \leq t_1^{\text{arr}} < 168 \tag{3.10}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 168m \tag{3.11}$$

$$v_i = \frac{L_i}{t_{i+1}^{\text{arr}} - t_i^{\text{dep}}}, i \in I \tag{3.12}$$

$$(t_i^{\text{arr}} \mod 168) \in \hat{\Omega}_i, i \in I \tag{3.13}$$

$$0 \leq v_i \leq V^{\max}, i \in I \tag{3.14}$$

$$m \in \{1, 2, 3, \cdots\} \tag{3.15}$$

The objective function (3.7) minimizes the sum of ship cost, bunker cost, and inventory cost. The first term is the ship cost, which is proportional to the number of ships deployed. The second term is the bunker cost, which varies nonlinearly with speed. The third term is inventory cost, which is summed over all legs. Constraint (3.8) requires that the service on this ship route is weekly. Constraint (3.9) defines the departure time from each port of call. Constraint (3.10) eliminates symmetric solutions. Constraint (3.11) defines the time when the ship returns to the first port of call after one round trip. Constraint (3.12) calculates the sailing speed on each leg. Constraint (3.13) imposes the port time window restrictions. Constraint (3.14) enforces the lower and upper limits on the sailing speed. Constraint (3.15) indicates that the number of ships is a positive integer.

3.4 Solution Method

The model [SDPTW] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has both continuous (sailing speed) and discrete variables (number of ships); (ii) it has nonlinear objective function (3.7) and nonlinear constraints (3.8) and (3.12); (iii) the set $\hat{\Omega}_i$ in Eq. (3.13) may consist of disjoint intervals, as shown in Fig. 1.4. This will lead to a non-convex domain even without considering the discrete decision variables; moreover, even if $\hat{\Omega}_i$ is convex, the "mod" operator still leads to a non-convex domain. These difficulties make the model challenging and hard to be solved by existing commercial solvers. To address the model, we have to develop our own solution algorithm. After carefully examining the properties of the problem, we develop a dynamic programming based solution method that overcomes these difficulties.

3.4.1 Space-time network for a given number of ships

Given the number of ships m, say, $\bar{m}$, the ship cost $C^{\text{ship}}\bar{m}$ is fixed. Moreover, the round-trip journey time is also fixed, that is, $168\bar{m}$ hours. The model can be reformulated as

[SDPTW-$\bar{m}$]

$$\min \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \tag{3.16}$$

subject to:

$$\sum_{j \in I} L_j / v_j + \sum_{j \in I} t_j^{\text{port}} = 168\bar{m} \tag{3.17}$$

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{3.18}$$

$$0 \leq t_1^{\text{arr}} < 168 \tag{3.19}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 168\bar{m} \tag{3.20}$$

$$v_i = \frac{L_i}{t_{i+1}^{\text{arr}} - t_i^{\text{dep}}}, i \in I \tag{3.21}$$

$$(t_i^{\text{arr}} \mod 168) \in \hat{\Omega}_i, i \in I \tag{3.22}$$

$$0 \leq v_i \leq V^{\max}, i \in I \tag{3.23}$$

Note that model [SDPTW-$\bar{m}$] no longer has discrete variables.

3.4.1.1 Property of the problem

As t_1^{arr} is between 0 and 168, we can discretize it and enumerate all possible discretized values. Given m and t_1^{arr} (say, $\bar{m}$ and $\bar{t}_1^{\mathrm{arr}}$), if the arrival time at a particular port of call is known, then the bunker cost and inventory cost associated with the voyage legs after the port of call depend only on its arrival time and are independent of the arrival times at ports of call prior to it. For instance, if we know that the arrival time at the $\bar{i}$th port of call is $\bar{t}_{\bar{i}}^{\mathrm{arr}}$, the problem can be split into two subproblem: subproblem 1 determines the arrival time at each port of call 2, 3, $\cdots$, $\bar{i}-1$; subproblem 2 determines the arrival time at each port of call $\bar{i}+1$, $\bar{i}+2$, $\cdots$, N. To be clear, we formulate the two subproblems below:

[SDPTW-$\bar{m}$-subproblem 1]

$$\min \alpha \sum_{i=1}^{\bar{i}-1} L_i g_i(v_i) + \sum_{i=1}^{\bar{i}-1} \beta \bar{V}_i \frac{L_i}{v_i} \tag{3.24}$$

subject to:

$$t_i^{\mathrm{dep}} = t_i^{\mathrm{arr}} + t_i^{\mathrm{port}}, i = 1, 2, 3, \cdots, \bar{i}-1 \tag{3.25}$$

$$v_i = \frac{L_i}{t_{i+1}^{\mathrm{arr}} - t_i^{\mathrm{dep}}}, i = 1, 2, 3, \cdots, \bar{i}-1 \tag{3.26}$$

$$(t_i^{\mathrm{arr}} \mod 168) \in \hat{\Omega}_i, i = 2, 3, \cdots, \bar{i}-1 \tag{3.27}$$

$$0 \le v_i \le V^{\max}, i = 1, 2, 3, \cdots, \bar{i}-1 \tag{3.28}$$

$$t_1^{\mathrm{arr}} = \bar{t}_1^{\mathrm{arr}} \tag{3.29}$$

$$t_{\bar{i}}^{\mathrm{arr}} = \bar{t}_{\bar{i}}^{\mathrm{arr}} \tag{3.30}$$

[SDPTW-$\bar{m}$-subproblem 2]

$$\min \alpha \sum_{i \in \bar{i}}^{N} L_i g_i(v_i) + \sum_{i \in \bar{i}}^{N} \beta \bar{V}_i \frac{L_i}{v_i} \tag{3.31}$$

subject to:

$$t_i^{\mathrm{dep}} = t_i^{\mathrm{arr}} + t_i^{\mathrm{port}}, i = \bar{i}, \bar{i}+1, \cdots, N \tag{3.32}$$

$$v_i = \frac{L_i}{t_{i+1}^{\mathrm{arr}} - t_i^{\mathrm{dep}}}, i = \bar{i}, \bar{i}+1, \cdots, N \tag{3.33}$$

$$(t_i^{\mathrm{arr}} \mod 168) \in \hat{\Omega}_i, i = \bar{i}+1, \bar{i}+2, \cdots, N \tag{3.34}$$

$$0 \leq v_i \leq V^{\max}, i = \bar{i}, \bar{i}+1, \cdots, N \tag{3.35}$$

$$t_{\bar{i}}^{\mathrm{arr}} = \bar{t}_{\bar{i}}^{\mathrm{arr}} \tag{3.36}$$

$$t_{N+1}^{\mathrm{arr}} = \bar{t}_1^{\mathrm{arr}} + 168\bar{m} \tag{3.37}$$

Hence, the decisions about the arrival time at each port of call could be made in a sequential manner, that is, the optimal arrival time at the next port only depends on the arrival time at the current port (and of course $\bar{m}$ and $\bar{t}_1^{\mathrm{arr}}$). Exploiting this property, we construct a space-time network and thereby develop a dynamic programming based solution approach.

3.4.1.2 Space-time network construction method

To construct a space-time network, in view of Eq. (3.10), we only need to consider a time horizon of $168(m+1)$ hours [2]. In other words, the time horizon is $m+1$ weeks. We discretize the time horizon into intervals, the length of each interval being 1 hour [3]. To take into account the voyage from the Nth port of call to the first one, we consider $N+1$ ports in the space-axis, where the $(N+1)$th port corresponds to the returning to the first one. Each of the $N+1$ ports is copied $168(m+1)$ times. Hence, each node (t, i) in the space-time network corresponds to a port i at a particular time t. We define the time t as the arrival time at the port i. Therefore, node (t, i) in the space-time network means that port i is visited at time t. For each port i, if $(t \mod 168) \notin \hat{\Omega}_i$, then the port is busy at the time t. Hence, it is impossible to visit port i at time t. Consequently, for each port i, if $(t \mod 168) \notin \hat{\Omega}_i$, then we remove the node (or mark it as inactive as it will not be visited).

Moreover, from each active node (t, i), the ship may visit any active node $(t', i+1)$ satisfying

$$t' \geq t + t_i^{\text{port}} + \frac{L_i}{V^{\max}} \tag{3.38}$$

In other words, from port i, a ship can only visit port $i+1$ and the sailing speed cannot exceed $V^{\max}$. Of course, the port time window at port $i+1$ is already implicitly considered by removing the nodes that cannot be visited.

We formally state the method for constructing the space-time network below:

Algorithm 1: Construction of space-time network $G(m)$[4] for a given ship number m

Step 1. (Construct nodes): Construct a space-time network with the horizontal axis being the time (hours, starting from 0 which represents 00:00 of a particular

[2] If, for example, $t_1^{\text{arr}} = 167$, then the ship will return to the first port of call at time $168m+167$. Therefore, the time horizon is $168(m+1)$ hours rather than $168m$ hours.

[3] The precision of 1 hour is more than sufficient for liner shipping applications.

[4] G means "graph".

Sunday), and the vertical axis being the space (ports). The length of the time axis is $168(m+1)$ with the discrete time points being 0, 1, 2,$\cdots$,$168(m+1)-1$. The vertical axis has $N+1$ ports, that is, the 1st port of call, the 2nd port of call,$\cdots$, the Nth port of call, and the $(N+1)$th port of call. Note that the $(N+1)$th port of call actually represents that the ship returns to the first port of call after a round-trip journey of $168m$ hours. Each of the $N+1$ ports is copied $168(m+1)$ times. Now, in the space time network, there are $168(m+1)(N+1)$ nodes. A node can be represented by an ordered pair (time unit, port ID), or (t,i), which means that port i is visited at time t.

Step 2. (Deactivate nodes):

Step 2.1. (Deactivate nodes that violate port time windows) For each node (t,i) in the space-time network, if $(t \mod 168) \notin \hat{\Omega}_i$, the ship cannot visit the node and hence we mark it as inactive;

Step 2.2. (Deactivate nodes that violate Eq. (3.10)) For each node $(t,1)$ that corresponds to the first port of call in the space-time network, if $t \geq 168$, the ship cannot visit the node and hence we mark it as inactive;

Step 2.3. (Deactivate nodes that violate Eq. (3.11)) For each node $(t,N+1)$ that corresponds to the return to the first port of call in the space-time network, if $t \leq 168m-1$, the ship cannot visit the node and hence we mark it as inactive (note that here the number of ships m is given).

Step 3. (Construct arcs):

Step 3.0. Set $i=0$;

Step 3.1. Set $i := i+1$. For each active node (t,i), $t \in \{0,1,2,\cdots,168(m+1)-1\}$, construct an arc from it to any of the active nodes $(t',i+1)$ satisfying $t' \geq t + t_i^{\text{port}} + \frac{L_i}{V^{\max}}$. Hence, the sailing time of the arc is $t'-t-t_i^{\text{port}}$. Moreover, the sailing speed is also determined, which is $v_i = L_i/(t' - t - t_i^{\text{port}})$. Therefore, the corresponding bunker cost is $\alpha L_i \mathrm{g}_i(v_i)$ and the inventory cost of the containers is $(t' - t - t_i^{\text{port}})\beta\bar{V}_i$ (as aforementioned, the inventory cost associated with port time is constant, and hence is not modeled). The cost (sum of bunker and inventory cost) of the arc is $\alpha L_i \mathrm{g}_i(L_i/(t' - t - t_i^{\text{port}})) + (t' - t - t_i^{\text{port}})\beta\bar{V}_i$.

Step 3.2. If $i = N$, Stop. Otherwise, go to Step 3.1. $\square$

3.4.1.3 An example of space-time network construction

We use an example to demonstrate the space-time network construction method. For the ease of presentation, we use "day" rather than "hour" in the discretization. That is, 0 represents Sunday, 1 represents Monday, etc. If we do not use days but use hours, there would be too many nodes in the space-time network and it would be difficult to understand it. Suppose that there are three ports of call on the ship route. The feasible arrival days are $\hat{\Omega}_1 = \{2, 3, 6\}$, $\hat{\Omega}_2 = \{0, 1, 5, 6\}$, and $\hat{\Omega}_3 = \{4, 5\}$. In addition, suppose that $t_1^{\text{port}} + \frac{L_1}{V^{\max}} = 4$ days, $t_2^{\text{port}} + \frac{L_2}{V^{\max}} = 5$ days, and $t_3^{\text{port}} + \frac{L_3}{V^{\max}} = 1$ day. The number of ships $m = 2$. The three steps in Algorithm 1 are shown in Fig. 3.2, Fig. 3.3, and Fig. 3.4, respectively.

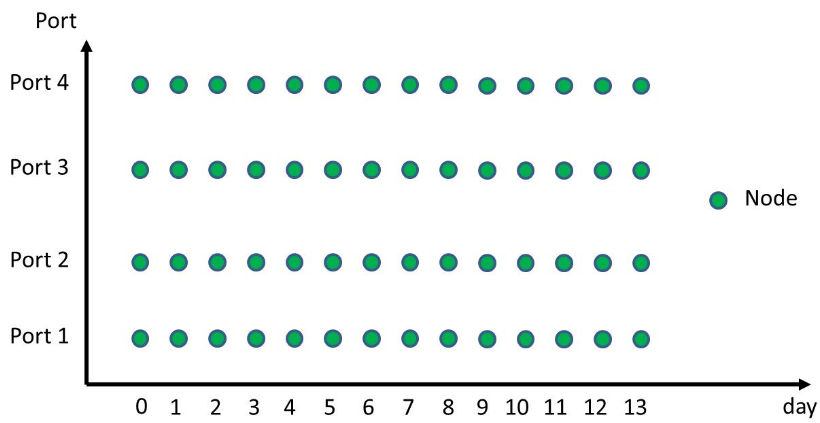


Figure 3.2: Construct nodes

Let us look at Fig. 3.2 first. As $m = 2$ and we use "day" to discretize the time, there are a total of 14 days in the time axis. Hence, each port should be copied

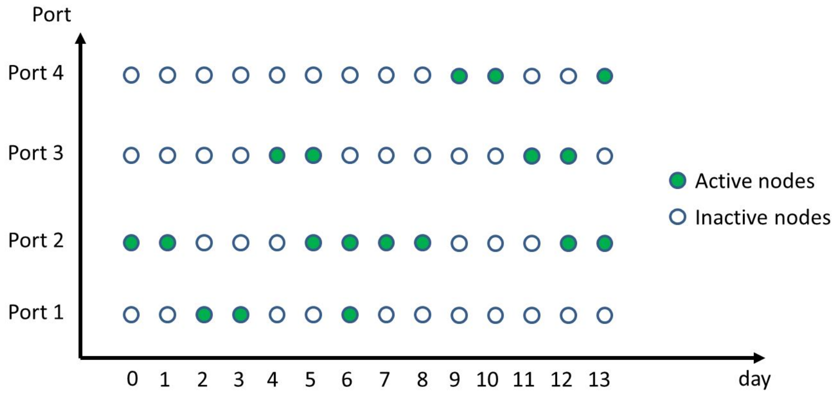


Figure 3.3: Deactivate nodes

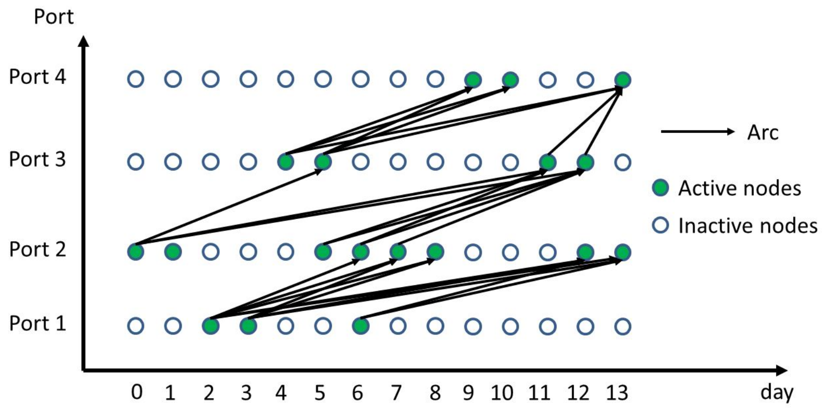


Figure 3.4: Construct arcs

14 times. Since there are three ports of call on the ship route, considering the loop property of the ship route, we need to consider 4 ports, where the fourth port is actually the return to the first port. As a result, there are a total of $4 \times 14 = 56$ nodes in the space-time network.

In Fig. 3.3, we deactivate nodes. In step 2.1, since $\hat{\Omega}_2 = \{0, 1, 5, 6\}$, nodes corresponding to port 2 are active only if $t = 0, 1, 5, 6, 7, 8, 12, 13$. In other words, only 8 nodes corresponding to port 2 are active. Since $\hat{\Omega}_3 = \{4, 5\}$, nodes corresponding to port 3 are active only if $t = 4, 5, 11, 12$. In other words, only 4 nodes corresponding to port 3 are active. Since $\hat{\Omega}_1 = \{2, 3, 6\}$ and port 1 must be visited in the first week, nodes corresponding to port 3 are active only if $t = 2, 3, 6$. In other words, only 3 nodes corresponding to port 1 are active. Since $\hat{\Omega}_1 = \{2, 3, 6\}$ and port of call 4 (which is the same port as port of call 1) must be visited in the second week (as $m = 2$), nodes corresponding to port 4 are active only if $t = 9, 10, 13$. In other words, only 3 nodes corresponding to port 4 are active.

In Fig. 3.4, we add arcs connecting the nodes. Note that the arcs connect only active nodes, and must respect the maximum speed of ships. For instance, both nodes $(2, 1)$ and $(5, 2)$ are active. However, their time difference $5 - 2 = 3$ is smaller than $t_1^{\text{port}} + \frac{L_1}{V^{\max}} = 4$. Hence, ships cannot visit node $(5, 2)$ from node $(2, 1)$.

3.4.1.4 Loop property of ship route in the space-time network

It should be noted that in the space-time network, if a ship visits port 1 at time t_1^{arr}, it must return to port 1 at time $t_1^{\text{arr}} + 168m$. This constraint poses difficulties for finding the schedule with the minimum cost. Nevertheless, we identify that the total number of possible t_1^{arr} is at most 168. Therefore, we could enumerate all possible t_1^{arr}. For each fixed t_1^{arr}, we can apply the dynamic programming approach to find the shortest path (minimum-cost path) from node $(t_1^{\text{arr}}, 1)$ to node $(t_1^{\text{arr}} + 168m, N + 1)$, denoted by $c(m, t_1^{\text{arr}})$[5]. Hence, the minimum total cost with given m is $C^{\text{ship}}m + \min_{t_1^{\text{arr}} \in \{0,1,2,\cdots,167\}} c(m, t_1^{\text{arr}})$.

3.4.2 Lower bound of the number of ships

The previous sub-section provides an approach for finding the optimal schedule with a given m. However, as m is a positive integer, we cannot enumerate all possible values of m. To overcome this difficult, we investigate how to confine the range of possible values of m.

According to Eq. (3.1), the minimum number of ships can be computed by:

$$m^{\min} = \left\lceil \left(\sum_{j \in I} L_j / V^{\max} + \sum_{j \in I} t_j^{\text{port}} \right) / 168 \right\rceil \tag{3.39}$$

[5] $c(m, t_1^{\text{arr}}) = \infty$ if $(t_1^{\text{arr}}, 1)$ is inactive or if there is no path from node $(t_1^{\text{arr}}, 1)$ to node $(t_1^{\text{arr}} + 168m, N + 1)$.

where $\lceil x \rceil$ is the smallest integer greater than or equal to x.

3.4.3 Lower bound of the total cost with given number of ships

When the number of ships is m, a lower bound on the total cost, denoted by $LB(m)$, can be computed as follows. As the ship cost in Eq. (3.7) is fixed, we minimize the sum of bunker cost and inventory cost by optimizing the speed. To facilitate the computation of the lower bound, we relax relevant constraints and only require that the speed is nonnegative. Using the bunker consumption function (3.2), we have:

$$\min_{v_i} \sum_{i \in I} \alpha L_i a_i (v_i)^{b_i} + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \tag{3.40}$$

subject to:

$$-v_i \leq 0, i \in I \tag{3.41}$$

It is easy to see that the speed on different legs can be optimized independently. Let $\lambda_i \geq 0$ be the Lagrangian multiplier associated with constraint $-v_i \leq 0$. The Karush-Kuhn-Tucker (KKT) condition of the above optimization problem is:

$$\alpha L_i a_i b_i (v_i)^{b_i - 1} - \beta \bar{V}_i L_i \frac{1}{(v_i)^2} - \lambda_i = 0 \tag{3.42}$$

$$\lambda_i(-v_i) = 0 \tag{3.43}$$

$$-v_i \leq 0 \tag{3.44}$$

$$\lambda_i \geq 0 \tag{3.45}$$

Apparently $-v_i < 0$, and therefore $\lambda_i = 0$. Hence, we can compute the optimal speed in the model, denoted by $\tilde{v}_i$:

$$\tilde{v}_i = \left(\frac{\beta \bar{V}_i}{\alpha a_i b_i} \right)^{\frac{1}{b_i + 1}} \tag{3.46}$$

Consequently, a lower bound of the total cost with m ships is:

$$LB(m) = C^{\text{ship}}m + \alpha \sum_{i \in I} L_i g_i(\tilde{v}_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{\tilde{v}_i} \tag{3.47}$$

3.4.4 Overall Algorithm

Sub-section 3.4.1 develops a space-time network model that can find the optimal schedule for a given number of ships using dynamic programming approach. Sub-section 3.4.2 obtains a lower bound on the number of ships that are needed. Sub-section 3.4.3 proposes a lower bound on the total cost for a given number of ships, and this lower bound increases with m as shown in Eq. (3.47). Based on these results, we now present the overall solution algorithm:

Algorithm 2: Solution method for the SDPTW

Step 0. Set $m = m^{\min} - 1$. Denoted by $C^* := \infty$ the minimum total cost obtained (upper bound).

Step 1. Set $m := m + 1$. If $LB(m) \geq C^*$, we have obtained the optimal solution and hence stop. Otherwise, construct the space-time network $G(m)$.

Step 2. For each $t_1^{\text{arr}} \in \{0, 1, 2, \cdots, 167\}$, find the shortest path from node $(t_1^{\text{arr}}, 1)$ to node $(t_1^{\text{arr}} + 168m, N + 1)$ and its cost $c(m, t_1^{\text{arr}})$. If $C^{\text{ship}}m + c(m, t_1^{\text{arr}}) < C^*$, set $C^* := C^{\text{ship}}m + c(m, t_1^{\text{arr}})$ and record the current solution. When all the t_1^{arr} have been examined, go to Step 1. □

Algorithm 2 terminates in a finite number of iterations. This is because once a finite upper bound C^* is found, the algorithm will stop before or when $m = \lceil C^*/C^{\text{ship}} \rceil$.

3.5 Case Study

We choose a case study of the NCE ship route in Fig. 3.1 to evaluate the proposed model and solution method. We assume that 5000-TEU ships are deployed on it. We choose 5000-TEU ships because larger ships cannot transit the Panama Canal. The operating cost C^{ship} =500,000 USD/week, the maximum speed $V^{\max}$=30 knots, the bunker price $\alpha = 400$ USD/ton and the unit inventory cost $\beta = 1$ USD per TEU per hour. The port time (h), distance (n mile), bunker consumption function $g_i(v_i)$, and volume of containers on each leg (TEUs) are shown in Table 3.1. In Table 3.1 we assume that the port time is either 1 day or 1.5 days, the bunker consumption functions may be different for different legs, and the number of containers on each leg implies that the ship load factor is between $\frac{2200}{5000} = 44\%$ and $\frac{4500}{5000} = 90\%$. The port time window at each port, i.e., Ω_i, is shown in Table 3.2, which indicates that no port is available seven days a week.

Table 3.1: Parameters in the case study

ID	Port	Port time	Distance	Bunker Function	# containers
1	New York	36	261	$0.001(v_1)^2$	2200
2	Norfolk	24	436	$0.001(v_2)^{2.1}$	3000
3	Savannah	24	9678	$0.001(v_3)^{2.3}$	3500
4	Pusan	24	467	$0.001(v_4)^2$	4200
5	Qingdao	24	386	$0.001(v_5)^2$	4000
6	Ningbo	24	101	$0.001(v_3)^2$	4300
7	Shanghai	36	10553	$0.001(v_7)^2$	4500

Table 3.2: Port time windows

ID	Port	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	New York	free	free	busy	busy	free	free	busy
2	Norfolk	busy	busy	free	busy	busy	busy	busy
3	Savannah	busy	busy	busy	free	busy	busy	free
4	Pusan	free	busy	free	busy	busy	busy	busy
5	Qingdao	busy	busy	busy	busy	busy	busy	free
6	Ningbo	busy	free	busy	busy	free	busy	busy
7	Shanghai	free	free	busy	free	free	busy	busy

3.5.1 Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We assume that currently the port of Norfolk is only available on Tuesday, as shown in Table 3.2. Both Norfolk and the liner shipping company are interested in looking at the result if more available time is provided at Norfolk. We hence examine the cases of one day available for service each week (Tuesday), two days (plus Friday), three days (plus Monday), four days (plus Saturday), five days (plus Thursday), six days (plus Sunday), and seven days (which means that Norfolk is ready to serve ships at any time). The results of the total cost and the optimal number of ships deployed are shown in Fig. 3.5.

It can be seen that more available days at Norfolk leads to a lower total cost: when the number of available days is increased from 2 to 6, the total cost is reduced by 214,639 USD per week Table 3.3. Fig. 3.5 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. The optimal ship schedule, i.e., arrival time at each port of call, is shown in Table 3.4, where e.g. "Cases 1,2" means that Norfolk is available only one or two days in a week. We observe that when the availability of Norfolk is changed, the optimal arrival times at it and its neighboring ports may also change. However, there is no impact on the optimal arrival times at ports that are a few voyage legs away from Norfolk.

Table 3.3: Impact of port time windows on each type of cost

Case	Ship cost	Bunker cost	Inventory cost
1	5000000	3108469	5802000
2	5000000	3108469	5802000
3	4500000	4098651	5131200
4	4500000	4098651	5131200
5	4500000	4098651	5131200
6	4500000	4051029	5144800
7	4500000	4051029	5144800

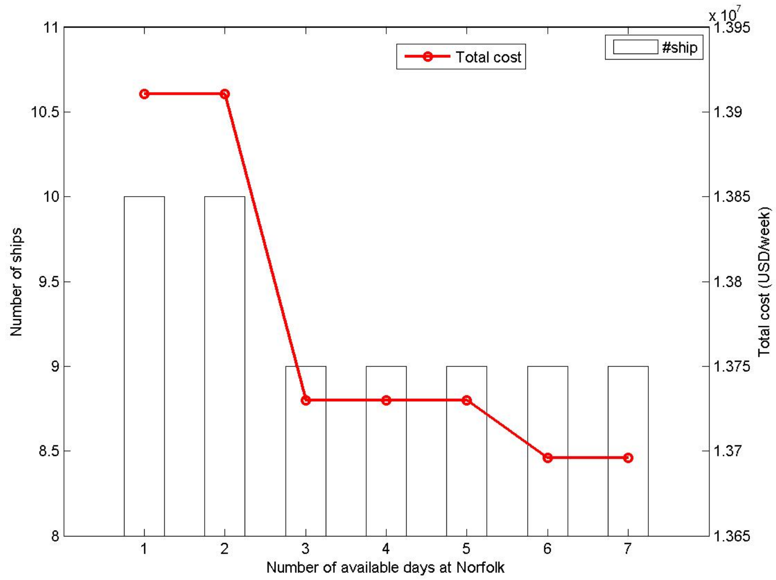


Figure 3.5: Impact of port time windows on the total cost and the number of ships

Table 3.4: Impact of port time windows on the optimal schedule

ID	Port	Cases 1,2	Cases 3,4,5	Cases 6,7
1	New York	0	108	108
2	Norfolk	48	192	175
3	Savannah	144	240	240
4	Pusan	888	888	888
5	Qingdao	984	984	984
6	Ningbo	1032	1032	1032
7	Shanghai	1080	1080	1080
1	New York	1680	1620	1620

3.5.2 Consequence of port efficiency

The port time t_i^{port} to a large extent depends on the container handling efficiency. Therefore, port operators seek to improve efficiency by optimizing quay-side and yard-side operations. To investigate the effect of port handling efficiency, we change the port time at Shanghai from 12h, 18h, 24h, 30h, to 36h, and compute the optimal solution. We find that the optimal number of ships is always 10. The total cost increases with the time spent at Shanghai, as shown in Fig. 3.6. In fact, a ship creates value when it is moving cargo, whereas standing still at ports does not create value. Moreover, when the number of ships is given, a longer port time means a shorter sailing time, which leads to higher bunker consumption. Therefore, improving port efficiency will reduce the total cost for liner shipping companies.

We then fix the port time at Shanghai at 36h, and change the port time at New York from 12h, 18h, 24h, 30h, to 36h, and compute the optimal solution. The result is shown in Fig. 3.7. It clearly shows that when the port time is increased, not only the total cost increases, but also the optimal number of ships to deploy may increase.

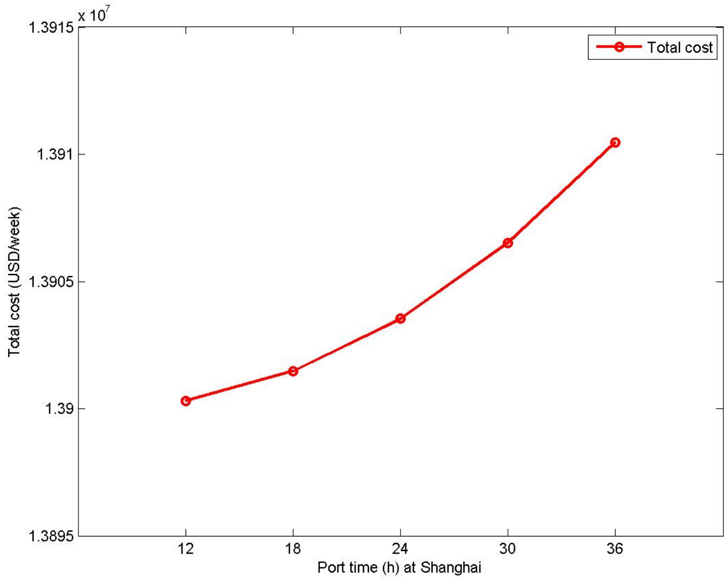


Figure 3.6: Impact of port time at Shanghai on the total cost

3.5.3 Result of bunker prices

The bunker price is volatile and hence we examine the sensitivity of the solution with different bunker prices from 300, 400, 500, 600, 700, to 800 USD/ton. The result is shown in Fig. 3.8 . We observe that the total cost increases almost linearly

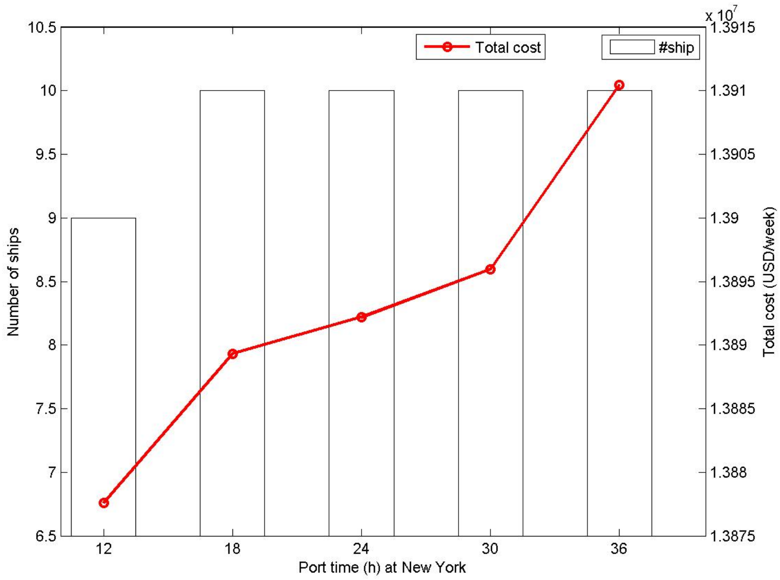


Figure 3.7: Impact of port time at New York on the total cost and the number of ships

(not strictly linearly) with the bunker price. Consequently, a higher bunker price always leads to a higher cost for liner shipping companies. In addition, Fig. 3.8 clearly shows that there is a rise in the number of vessels used when the bunker price becomes higher. This is because when more ships are deployed, the sailing speed can be reduced, resulting in a lower bunker consumption. A reduction in bunker consumption is more significant when the bunker price is higher.

3.5.4 Effect of inventory cost

Finally, we investigate the effect of the unit inventory cost β on the total cost and the optimal number of ships to deploy by changing β from 1, 1.25 through to 2. The result is shown in Fig. 3.9, which indicates that the rise of unit inventory cost leads to a decreasing in the number of ships and an increasing of the total cost. This is because when the unit inventory cost is higher, containerships have to sail at a higher speed to shorten the transit time. Therefore, the number of ships is reduced. At the same time, the total cost inevitably becomes higher.

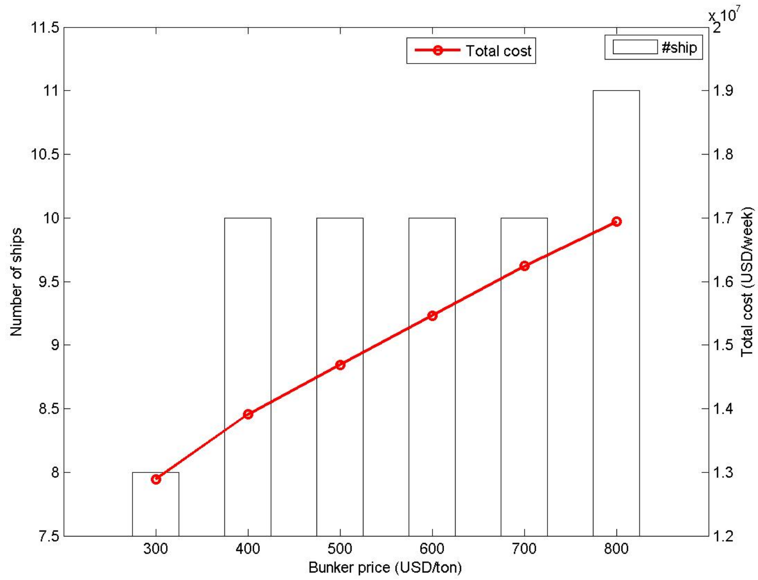


Figure 3.8: Result of bunker prices on the total cost and the number of ships

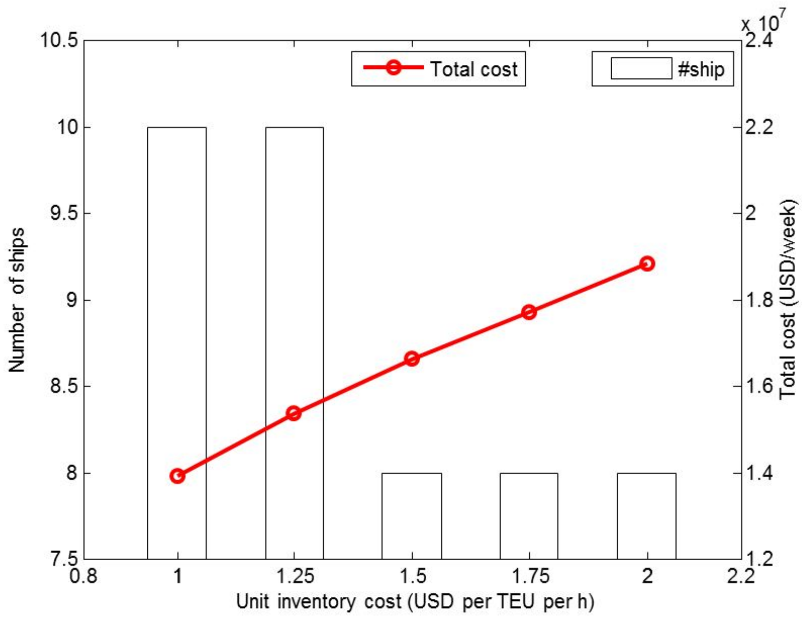


Figure 3.9: Effect of unit inventory cost on the total cost and the number of ships

3.6 Conclusions

This chapter has studied the practical liner ship route schedule design problem with port time windows. This is a significant tactical planning decision problem because it considers the availability of ports when planning liner shipping services. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a nonlinear non-convex optimization model. In view of the problem structure, we have developed an efficient dynamic-programming based holistic solution approach, which includes a space-time network model and a bounding technique for the total cost with give number of ships.

The proposed solution method is applied to the NCE service provided by OOCL. The results demonstrate that the port time windows, port handling efficiency, bunker price and unit inventory cost all affect the total cost, the optimal number of ships to deploy, and also the optimal schedule. A higher availability at ports, shorter port time, lower bunker price and larger unit inventory cost result in a lower total cost. Moreover, shorter port time, lower bunker price and smaller unit inventory cost lead to a smaller number of ships to deploy. Therefore, port operators can apply the proposed method to quantify the benefits to their customers, i.e., liner shipping companies, gained by expanding the ports' capacity and improving the ports' efficiency. Liner shipping companies may need to charter in more ships if they predict that the future bunker price will increase, or if they predict that a particular season is coming during which the value of the cargo is generally low.

Chapter 4

General single Ship Route Schedule Design

4.1 Introduction

In the previous chapter we have assumed that each port of call is visited only once in a round-trip journey time. If some ports are visited twice in a round-trip journey like the one shown in Fig. 4.1, i.e., some ports of call correspond to the same physical port, then the port time window should be dealt with more carefully. First, in the dynamic programming approach proposed in the previous chapter, when we analyze the second arrival time at a port, we have to take into account the first arrival time at the port. As a result, in each step of the dynamic programming method, i.e., at port of call $\bar{i}$, we have to record information on the arrival time at all the ports that have been visited and are to be visited again. This, in theory, may lead to the "curse-of-dimensionality", because if there are n ports that are visited twice, in the worst-case we have to record the arrival times at n ports of call, and if the possible arrival times is e.g. 168, then the state space is 168^n (without even considering the possible arrival times at $\bar{i}$), which increases exponentially with n.

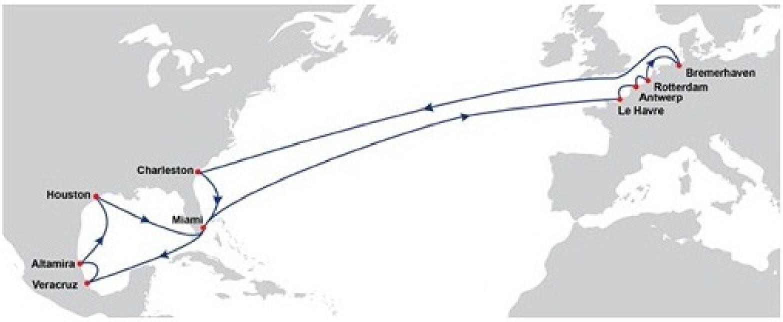
In reality, this problem may not be that serious. This is because on one side, the number of ports of call in a round-trip journey is not very large. On the other side, the number of ports that are visited twice in a round-trip journey is even smaller. In addition, some ports may be always available, especially those major transshipment hubs such as Singapore and Hong Kong that attract transshipment containers based on their quality of service.

Another minor issue that is worth mentioning is that when a ship route has ports that are visited twice in a round-trip journey, the definition of port time window at these ports should be changed. For instance, if a port is visited only once, we only need to record the possible arrival times in a week at the port, i.e., $\hat{\Omega}_i$, with regard to all berths at the port. We consider a port with two berths, assuming that both berths are available on Sunday and Monday, and the port time is 24h, then $\hat{\Omega}_i = [0, 24)$. However, if the port is visited twice, we have to record the the time window of each *berth* at the port, because different arrivals may use different berths. Of course, this is only a minor issue in the dynamic programming algorithm, because we actually do not need to record which berth the first arrival has used.

The rest of the chapter is organized as follows. Section 4.2 describes the problem. Section 4.3 formulates a mathematical model for the problem. Section 4.4 proposes a developing holistic solution approach to address the problem. Section 4.5 reports a case study based on the AGM service of OOCL. Section 4.6 presents the conclusion.

4.2 Problem description

Consider a ship route such as the AGM service in Fig. 4.1. The port rotation of the ship route has a total of N ports of call. Define a set $I := \{1, 2, \cdots, N\}$. We can arbitrarily choose one port of call as the first, and let p_i represent the physical port of the ith port of call, $i \in I$. For instance, if we let Le Havre be the first port of call, the AGM service can be coded as follows: 1 (Le Havre) $\rightarrow$ 2 (Antwerp) $\rightarrow$ 3 (Rotterdam) $\rightarrow$ 4 (Bremerhaven) $\rightarrow$ 5 (Charleston) $\rightarrow$ 6 (Miami) $\rightarrow$ 7 (Veracruz)



Port Rotation

Westbound: Le Havre - Antwerp - Rotterdam - Bremerhaven - Charleston - Miami - Veracruz - Altamira
Eastbound: Veracruz - Altamira - Houston - Miami - Le Havre - Antwerp - Rotterdam - Bremerhaven

Transit Time (Days)				
Europe to USA/Mexico (Westbound)				
From \ To	Charleston (Mon)	Miami (Wed)	Veracruz (Sun)	Altamira (Tue)
Le Havre (Mon)	15	17	21	23
Antwerp (Tue)	13	15	19	21
Rotterdam (Thu)	11	13	17	19
Bremerhaven(Fri)	10	12	16	18

Transit Time (Days)				
Mexico/USA to Europe (Eastbound)				
From \ To	Le Havre (Sun)	Antwerp (Tue)	Rotterdam (Wed)	Bremerhaven (Fri)
Veracruz (Mon)	20	22	23	25
Altamira (Wed)	18	20	21	23
Houston (Fri)	16	18	19	21
Miami (Mon)	13	15	16	18

Figure 4.1: AGM service provided by OOCL (2013)

$\rightarrow$ 8 (Altamira) $\rightarrow$ 9 (Houston) $\rightarrow$ 10 (Miami) $\rightarrow$ 1 (Le Havre). $p_6 = p_{10} =$ Miami. We define the voyage from the ith port of call to the $(i+1)$th as leg i; leg N is the voyage from the Nth port of call to the first one. Ships may also transit canals on a voyage leg (Li et al., 2012a, Qu and Meng, 2012).

4.2.1 Ship cost, bunker cost and inventory cost

We assume that a string of m homogeneous containerships are deployed on the ship route to maintain a weekly service frequency, where m is a decision variable. The highest possible sailing speed of the ships is denoted by $V^{\max}$ (knot). Represent by t_i^{port} the fixed time (day) a ship spends at port of call i (we change the unit time from "hour" to "day" because of computational complexity (Limited computer memory)), and L_i (n mile) the length of leg i. Let v_i be the sailing speed (knot) of ships on leg i. v_i is a decision variable. To maintain a weekly service frequency, we have the relation:

$$\sum_{i \in I} \frac{L_i}{24 v_i} + \sum_{i \in I} t_i^{\text{port}} = 7m \tag{4.1}$$

In Eq. (4.1), the left-hand side is the round-trip journey time (day), and the right-hand side is the number of ships times 7 days/week. Denote by C^{ship} (USD/week) the fixed operating cost of a ship, including capital cost, manning cost and consumable but not bunker cost. Hence, the weekly operating cost of ships is $C^{\text{ship}}m$.

Eq. (4.1) implies that when the speed is higher, fewer ships need to be deployed to maintain the same weekly service frequency. However, a higher speed implies a larger amount of bunker consumed. To take into consideration the bunker cost, we let $g_i(v_i)$ (tons/n mile) be the bunker consumption per nautical mile at the speed v_i on leg i. Based on the results in existing studies (Bell and Bichou, 2008; Kontovas, 2011; Psaraftis and Kontovas, 2010, 2013; Ronen, 2011), we assume that $g_i(v_i)$ is a power function of the form:

$$g_i(v_i) = a_i(v_i)^{b_i}, i \in I \tag{4.2}$$

where a_i and b_i are two coefficients calibrated from operating data and satisfy $a_i > 0$ and $b_i > 1$. Denote by α (USD/ton) the bunker fuel price. The weekly bunker cost is $\alpha \sum_{i\in I} L_i g_i(v_i) = \alpha \sum_{i\in I} L_i a_i(v_i)^{b_i}$.

Besides the ship cost and bunker cost, the inventory cost of containers should also be incorporated. In fact, a lower speed (slow-steaming) would increase the transit time of containers, and thereby the inventory cost. We let $\bar{V}_i$ be the number of containers (twenty-foot equivalent units, or TEUs) transported on leg i, and β be the unit inventory cost (USD per TEU per h). Since the time spent at each port of call is constant, we only consider the inventory cost associated with sailing time at sea (sea time). Therefore, the total inventory cost is $\sum_{i\in I} \beta \bar{V}_i L_i / v_i$.

4.2.2 Liner ship route schedule

We use "day" as the unit for liner ship route schedule design as liner shipping companies publish their schedules in terms of days, see Fig. 4.1. We define the time

00:00 of a certain Sunday as time 0 (day), and hence 00:00 on Monday is time 1, and 00:00 next Tuesday is time 7+2=9. The time of departure t_i^{dep} at port i is determined by the time of arrival t_i^{arr} and the fixed port time t_i^{port}, that is:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{4.3}$$

Because of the weekly service frequency, without loss of generality, we let

$$0 \leqslant t_1^{\text{arr}} \leqslant 6 \tag{4.4}$$

Moreover, we define the time when the ship returns to the 1st port of call as t_{N+1}^{arr}, that is:

$$t_{N+1}^{\text{arr}} := t_1^{\text{arr}} + 7m \tag{4.5}$$

The *schedule* of a liner ship route is the vector defined below:

$$(t_i^{\text{arr}}, i \in I; m) \tag{4.6}$$

In the above schedule, there is the number of ships m because $(t_i^{\text{arr}}, i \in I)$ cannot define the inter-arrival time from the last port of call to the first. Of course, the schedule can also be uniquely determined by vector $(t_i^{\text{arr}}, i \in I; t_{N+1}^{\text{arr}})$.

Because liner ship routes provide weekly services, to simplify the notation, we define W to be a set that contains all days in a week, that is,

$$W := \{0, 1, 2, 3, 4, 5, 6\}$$

where 0 represents Sunday, 1 represents Monday, etc.

4.2.3 Port time windows

To account for the availability of ports, we must consider the availability of each berth at each port. This is because some ports are visited twice a week on a ship route, such as the port of Miami on AGM, and a port usually has more than one berth.

To formulate the availability of ports, first, we let I_1 be the set of ports of call, that correspond to ports that are visited only once. If a port is visited twice, supposing that the first visit is the jth port of call, we use j' to represent the second visit. We further let I_2 represent all the ports of call that correspond to the first call at a port that is visited twice and I_2' represent all the ports of call that correspond to the second call at a port that is visited twice. Take the AGM service as an example. We have $I = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $I_1 = \{1, 2, 3, 4, 5, 7, 8, 9\}$, $I_2 = \{6\}$, and $I_2' = \{10\}$. Mathematically, the following relations hold:

$$I_2' = \{j' \in I | j \in I_2\}$$

$$I = I_1 \cup I_2 \cup I_2'$$

4.2.3.1 Berth time windows

A port may have several berths, and each berth has its own time window. Hence, we let B_i be the set of berths at the physical port p_i (the ith port of call) and the available days in a week at berth $b \in B_i$ (berth time window) is represented by Ω_i^b, $\Omega_i^b \subseteq W$. For instance, $\Omega_i^b = \{1, 2, 4\}$ means that berth b at the ith port of call is free on Monday, Tuesday and Thursday. We further define a parameter $\delta_{it_i^{\text{arr}}}^{t'}$ that equals 1 if the ship that arrives at port of call i on day t_i^{arr} needs to be served on day t', $t' \in W$. For example, if $t_i^{\text{port}} = 2$, then we have $\delta_{i0}^0 = \delta_{i0}^1 = 1$, $\delta_{i0}^2 = 0$ (a ship that arrives on day 0, i.e., Sunday, needs to be berthed on day 0 and day 1); $\delta_{i8}^0 = 0, \delta_{i8}^1 = 1$, $\delta_{i8}^2 = 1$ (a ship that arrives on day 8, i.e., the next Monday, needs to be berthed on Monday and Tuesday). Evidently, if the ship arrives on Sunday and is served by berth $b \in B_i$, the berth must be available on both Sunday and Monday. Mathematically, if a ship arrives at port of call i on day t, the following is the set of days in a week that the ship needs to be served:

$$\Pi_{it} := \{t' \in W | \delta_{it}^{t'} = 1\}$$

In the above example, we have $\Pi_{i0} = \{0, 1\}$ and $\Pi_{i8} = \{1, 2\}$. Not every berth can serve ships at any time because of limited berth time windows. A berth $b \in B_i$ whose time window is $\Omega_i^b = \{1, 2, 4\}$ cannot serve the ship if it arrives on day 0 because the berth is not available on Sunday, or $\Pi_{i0} \not\subseteq \Omega_i^b$. In sum, a berth $b \in B_i$ can serve a ship that arrives on day t only if the following relation holds:

$$\Pi_{it} \subseteq \Omega_i^b$$

Therefore, the set of possible arrival days in a week at berth b of port of call i can be written as:

$$\hat{\Omega}_i^b = \{t \in W | \Pi_{it} \subseteq \Omega_i^b\}, i \in I, b \in B_i$$

Apparently, if the port time $t_i^{\mathrm{arr}} = 1$, there will be more possible arrival days than $t_i^{\mathrm{arr}} = 2$.

4.2.3.2 Feasible arrival days at ports and berths

For ports of call in I_1, we simply let $\hat{\Omega}_i$ be the set of possible arrival days in a week at the port of call considering all the berths. We have:

$$\hat{\Omega}_i = \bigcup_{b \in B_i} \hat{\Omega}_i^b, i \in I_1$$

For example, suppose that the ith port of call has five berths and berths 1, 3 and 5 are busy all the time (there is no time windows) and berth 2 has the time window

$\Omega_i^2 = \{1, 2\} \cup \{5\}$ and berth 4 has $\Omega_i^4 = \{4\}$. Suppose further that the port time $t_i^{\text{port}} = 2$. Hence, the set of possible arrival days in a week at each berth is:

$$\hat{\Omega}_i^1 = \varnothing, \hat{\Omega}_i^2 = \{1\}, \hat{\Omega}_i^3 = \varnothing, \hat{\Omega}_i^4 = \varnothing, \hat{\Omega}_i^5 = \varnothing$$

Therefore, the set of possible arrival days in a week at the port is:

$$\hat{\Omega}_i = \bigcup_{b \in B_i} \hat{\Omega}_i^b = \{1\}$$

We can let $\Omega_i := \bigcup_{b \in B_i} \Omega_i^b$ be the time window at port of call i. However, we cannot use Ω_i to calculate the set of possible arrival days $\hat{\Omega}_i$. For instance, in the above example if we use the combined port time window $\Omega_i = \{1, 2\} \cup \{4, 5\}$, we will reach the wrong conclusion that the ship can arrive either on Monday or on Thursday. In fact, if the ship arrives on Thursday, it has to be moved from berth 4 to berth 2 on Friday. This involves considerable cost and time that prohibit such an operation in practice.

For ports of call $j \in I_2$, B_j is the set of berths at port of call j and $B_{j'}$ is the set of berths that correspond to the second call at the port. $B_j \equiv B_{j'}$ as $p_j = p_{j'}$. Because the port is visited twice, it may be of little value to come up with a set of feasible arrival days similar to $\hat{\Omega}_i, i \in I_1$. We have to directly consider the sets $\hat{\Omega}_j^b$ and $\hat{\Omega}_{j'}^b$, $b \in B_j$.

A berth cannot serve more than one ship at the same time. Suppose that port of call $j \in I_2$ has only one berth $b \in B_j$ with $\Omega_j^b = \{1, 2, 3\}$. Assume that $t_j^{\text{port}} = t_{j'}^{\text{port}} = 2$. Suppose further that the ship uses the berth b when it arrives at port of call $j \in I_2$ at time $t_j^{\text{arr}} = 1$ (Monday) and still uses the berth when it arrives at port of call $j' \in I_2'$ at time $t_{j'}^{\text{arr}} = 2 + 7 = 9$ (next Tuesday). Evidently, both arrivals are feasible, because $t_j^{\text{arr}} \in \hat{\Omega}_j^b$ and $t_{j'}^{\text{arr}} \in \hat{\Omega}_{j'}^b$. However, their combination is infeasible because the berth cannot serve two ships on day 2. Mathematically, the combination is infeasible because $\delta_{j1}^2 = 1$ and $\delta_{j'9}^2 = 1$. In sum, if two arrivals j and j' use the same berth $b \in B_j$, the following relation must hold: $\delta^{t'}_{jt^{\mathrm{arr}}_j} + \delta^{t'}_{j't^{\mathrm{arr}}_{j'}} \leqslant 1, \forall t' \in W$.

4.3 Mathematical model

The ship route schedule design problem with port time windows aims to determine the optimal arrival time and the berth to use at each port of call on a ship route that satisfies the berth time window constraints to minimize the total cost including ship cost, bunker cost, and inventory cost. Before presenting the model, we list the notation below.

Variables

m	Number of ships deployed on the ship route
t_i^{arr}	Arrival time (day) at the ith port of call
t_{N+1}^{arr}	The time (day) when the ship returns to the 1st port of call
t_i^{dep}	Departure time (day) from the ith port of call
v_i	Sailing speed (knot) on leg i
z_j^b	A binary variable that equals 1 if and only if the ship uses berth b when it arrives at port of call $j \in I_2$, $b \in B_j$
$z_{j'}^b$	A binary variable that equals 1 if and only if the ship uses berth b when it arrives at port of call $j' \in I_2'$, $b \in B_{j'}$

Parameters

α	The bunker fuel price (USD/ton)
β	The unit inventory cost of containers (USD per TEU per h)
$\hat{\Omega}_i$	The set of feasible arrival days in a week at the ith port of call, $i \in I$
$\hat{\Omega}_j^b$	The set of feasible arrival days in a week at berth b at the jth port of call, $j \in I_2$

$\hat{\Omega}^b_{j'}$	The set of feasible arrival days in a week at berth b at the second call at the port p_j
B_j	The set of berths at port of call j
$B_{j'}$	The set of berths that correspond to the second call at the port p_j
C^{ship}	The weekly operating cost of a ship (USD/week)
$g_i(v_i)$	The bunker consumption per nautical mile at the speed v_i on leg i (tons/n mile)
I	Set of ports of call, $I := \{1, 2, \cdots, N\}$
I_1	The set of ports of call that correspond to ports that are visited only once
I_2	The set of ports of call that correspond to the first call at a port that is visited twice
I'_2	The set of ports of call that correspond to the second call at a port that is visited twice
L_i	Length (n mile) of the leg i
N	Number of ports of call on the ship route, $N = \lvert I \rvert$
p_i	The physical port that corresponds to the ith port of call on the ship route
t_i^{port}	Time (day) a ship spends at port of call i
$\bar{V}_i$	Number of containers (TEUs/week) transported on leg i
$V^{\max}$	Maximum speed of the ships (knots)
$m^{\max}$	Maximum number of ships deployed on the ship route
$\mathbb{Z}^+$	Set of nonnegative integers

The SDPTW can be formulated as:

$$\text{[SDPTW]} \qquad \min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \tag{4.7}$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{4.8}$$

$$0 \leqslant t_1^{\text{arr}} \leqslant 6 \tag{4.9}$$

$$t_{i+1}^{\text{arr}} \geqslant t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\max}} \right\rceil, i \in I \tag{4.10}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \tag{4.11}$$

$$v_i = \frac{L_i}{24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})}, i \in I \tag{4.12}$$

$$0 \leqslant v_i \leqslant V^{\max}, i \in I \tag{4.13}$$

$$m \in \{1, 2, 3, \cdots, m^{\max}\} \tag{4.14}$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \tag{4.15}$$

$$(t_i^{\text{arr}} \mod 7) \in \hat{\Omega}_i, i \in I_1 \tag{4.16}$$

$$z_j^b = 1 \Rightarrow (t_j^{\text{arr}} \mod 7) \in \hat{\Omega}_j^b, \forall j \in I_2, \forall b \in B_j \tag{4.17}$$

$$z_{j'}^b = 1 \Rightarrow (t_{j'}^{\text{arr}} \mod 7) \in \hat{\Omega}_{j'}^b, \forall j' \in I_2', \forall b \in B_{j'} \tag{4.18}$$

$$z_j^b z_{j'}^b = 1 \Rightarrow \delta_{jt_j^{\text{arr}}}^{t'} + \delta_{j't_{j'}^{\text{arr}}}^{t'} \leqslant 1, \forall j \in I_2, \forall b \in B_j, \forall t' \in W \tag{4.19}$$

$$\sum_{b \in B_j} z_j^b = 1, \forall j \in I_2 \tag{4.20}$$

$$\sum_{b \in B_{j'}} z_{j'}^b = 1, \forall j' \in I_2' \tag{4.21}$$

$$z_j^b \in \{0, 1\}, \forall j \in I_2, \forall b \in B_j \tag{4.22}$$

$$z_{j'}^b \in \{0, 1\}, \forall j' \in I_2', \forall b \in B_{j'} \tag{4.23}$$

The objective function (4.7) minimizes the sum of ship cost, bunker cost, and inventory cost. Constraint (4.8) defines the departure time from each port of call. Constraint (4.9) eliminates symmetric solutions. Constraint (4.10) confirms that the sailing speed cannot exceed $V^{\max}$. Constraint (4.11) defines the time when the ship returns to the 1st port of call after one round-trip. Constraint (4.12) calculates the sailing speed on each leg. Constraint (4.13) enforces the lower and upper limits on the sailing speed. Constraint (4.14) indicates that the number of ships is a positive integer. Constraint (4.15) indicates that the arrival time at each port of call is a nonnegative integer. Constraint (4.16) imposes the port time window constraints at ports that are visited once. Constraints (4.17) and (4.18) are berth time window constraints at the ports that are visited twice. Constraint (4.19) imposes that a berth cannot serve two ships at the same time. Constraints (4.20) and (4.21) require that a ship uses exactly one berth each time it visits a port. Constraints (4.22) and (4.23) define z_j^b and $z_{j'}^b$ as binary variables, respectively.

4.4 Solution method

The model [SDPTW] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has both continuous and discrete variables; (ii) it has nonlinear objective function (4.7) and constraint (4.12); (iii) the "mod" operator leads to a disjoint domain. After carefully examining the properties of the problem, we develop a holistic solution approach. We first relax the port time window constraints in Subsection 4.4.1.1. The relaxed mixed-integer nonlinear programming model is transformed to a mixed-integer linear programming model in Subsection 4.4.1.2. We solve the mixed-integer linear programming model to obtain the optimal solution. If the port time window constraints are violated, we add constraints to exclude such a solution. The above process is repeated until a feasible solution, which is optimal, is found, as elaborated in Subsection 4.4.2.

4.4.1 Relaxed models

4.4.1.1 Relaxing port time window constraints

First, we relax the port time window constraints and obtain a relaxed problem (RP):

$$\text{[RP]} \qquad \min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i g_i(v_i) + \sum_{i \in I} \beta \bar{V}_i \frac{L_i}{v_i} \tag{4.24}$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{4.25}$$

$$0 \leqslant t_1^{\text{arr}} \leqslant 6 \tag{4.26}$$

$$t_{i+1}^{\text{arr}} \geqslant t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24 V^{\max}} \right\rceil, i \in I \tag{4.27}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \tag{4.28}$$

$$v_i = \frac{L_i}{24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})}, i \in I \tag{4.29}$$

$$0 \leqslant v_i \leqslant V^{\max}, i \in I \tag{4.30}$$

$$m \in \{1, 2, 3, \cdots, m^{\max}\} \tag{4.31}$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \tag{4.32}$$

Note that the difficult "mod" operator is relaxed. In other words, we assume that a berth is always available whenever a ship visits a port.

4.4.1.2 An equivalent mixed-integer linear programming model

[RP] is a mixed-integer nonlinear programming (MINLP) model. In view of its special structure, we transform it to an equivalent mixed-integer linear programming (MILP) model and the MILP model can be solved by off-the-shelf MILP solvers. To this end, we first define the reciprocal of the speed as a new variable:

$$u_i := \frac{1}{v_i}, i \in I \tag{4.33}$$

Hence, [RP] is transformed to another MINLP model:

[MINLP]
$$\min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i g_i(1/u_i) + \sum_{i \in I} \beta \bar{V}_i L_i u_i \tag{4.34}$$

subject to:

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{4.35}$$

$$0 \leqslant t_1^{\text{arr}} \leqslant 6 \tag{4.36}$$

$$t_{i+1}^{\text{arr}} \geqslant t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\max}} \right\rceil, i \in I \tag{4.37}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \tag{4.38}$$

$$u_i = 24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})/L_i, i \in I \tag{4.39}$$

$$u_i \geqslant 1/V^{\max}, i \in I \tag{4.40}$$

$$m \in \{1, 2, 3, \cdots, m^{\max}\} \tag{4.41}$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \tag{4.42}$$

Now the only nonlinear term is $g_i(1/u_i)$ in Eq. (4.34), which has the following form:

$$g_i(1/u_i) = a_i(u_i)^{-b_i} \tag{4.43}$$

$g_i(1/u_i)$ is a convex function shown in Fig. 4.2a. Eq. (4.39) indicates that u_i can only take a limited number of values because $t_{i+1}^{\text{arr}} - t_i^{\text{dep}}$ is a positive integer and is not greater than $7m^{\max}$. Hence, we obtain a tangent line at each of the possible values of u_i. In particular, as shown in Fig. 4.2b, we let u_i^κ denote the possible values of u_i:

$$u_i^\kappa = 24\kappa/L_i, \kappa = 1, 2, \cdots, 7m^{\max}$$

The tangent lines at the points are:

$$a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1}(u_i - u_i^\kappa), \kappa = 1, 2, \cdots, 7m^{\max}$$

We use variable $\bar{g}_i$ to represent the bunker consume on leg i for formulating the tangent lines, and we have :

$$\bar{g}_i \geqslant a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1}(u_i - u_i^\kappa), \kappa = 1, 2, \cdots, 7m^{\max}$$

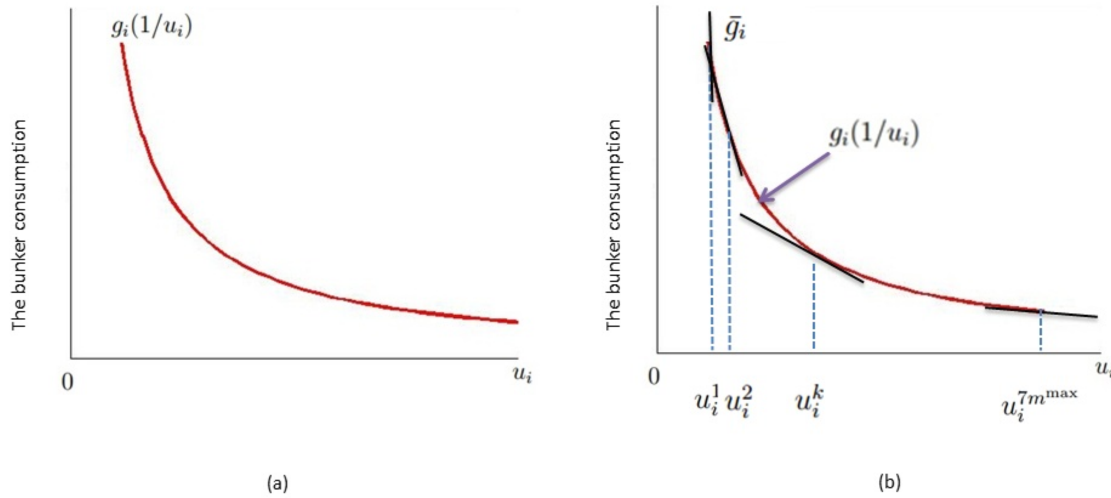


Figure 4.2: Linerization

The model [MINLP] can be transformed to a MILP model after introducing the intermediate variable $\bar{g}_i$, which is an auxilliary variable that is not smaller than the bunker consumption per nautical mile on leg i:

[MILP] $$\min C^{\text{ship}} m + \alpha \sum_{i \in I} L_i \bar{g}_i + \sum_{i \in I} \beta \bar{V}_i L_i u_i \qquad (4.44)$$

subject to:

$$\bar{g}_i \geqslant a_i(u_i^\kappa)^{-b_i} - a_i b_i (u_i^\kappa)^{-b_i-1}(u_i - u_i^\kappa), \kappa = 1, 2, \cdots, 7m^{\max}, i \in I \tag{4.45}$$

$$t_i^{\text{dep}} = t_i^{\text{arr}} + t_i^{\text{port}}, i \in I \tag{4.46}$$

$$0 \leqslant t_1^{\text{arr}} \leqslant 6 \tag{4.47}$$

$$t_{i+1}^{\text{arr}} \geqslant t_i^{\text{arr}} + t_i^{\text{port}} + \left\lceil \frac{L_i}{24V^{\max}} \right\rceil, i \in I \tag{4.48}$$

$$t_{N+1}^{\text{arr}} = t_1^{\text{arr}} + 7m \tag{4.49}$$

$$u_i = 24(t_{i+1}^{\text{arr}} - t_i^{\text{dep}})/L_i, i \in I \tag{4.50}$$

$$u_i \geqslant 1/V^{\max}, i \in I \tag{4.51}$$

$$m \in \{1, 2, 3, \cdots, m^{\max}\} \tag{4.52}$$

$$t_i^{\text{arr}} \in \mathbb{Z}^+, i \in I \tag{4.53}$$

Theorem 4.4.1 Model [RP] and model [MILP] are equivalent. In other words, if $(m^*, v_i^*, t_i^{\text{arr}*})$ is an optimal solution to [RP] and the optimal objective value is C_{RP}, then $(\hat{m} = m^*, \hat{u}_i = 1/v_i^*, \hat{t}_i^{\text{arr}} = t_i^{\text{arr}*}, \hat{g}_i = g_i(v_i^*))$ is a feasible solution to [MILP] and the resulting objective value is equal to C_{RP}. If $(\hat{m}, \hat{u}_i, \hat{t}_i^{\text{arr}}, \hat{g}_i)$ is an optimal solution to [MILP] and the optimal objective value is C_{MILP}, then $(m^* = \hat{m}, v_i^* = 1/\hat{u}_i, t_i^{\text{arr}*} = \hat{t}_i^{\text{arr}})$ is a feasible solution to [RP] and the resulting objective value is equal to C_{MILP}.

Proof: The difference between model [RP] and model [MILP] is the linearization of $g_i(v_i)$ to $\bar{g}_i$. As we use tangent lines to approximate the nonlinear function $g_i(1/u_i)$, and the tangent lines are not above the nonlinear function, Eq. (4.45) may underestimate the bunker consumption but will not overestimate it. Therefore [MILP] may underestimate the total cost, but will not overestimate it. That is, $C_{MILP} \leqslant C_{RP}$.

Now we prove that $C_{MILP} \geqslant C_{RP}$. If $(\hat{m}, \hat{u}_i, \hat{t}_i^{\text{arr}}, \hat{g}_i)$ is an optimal solution to [MILP], the integrality of $\hat{t}_i^{\text{arr}}$, the integrality of the departure times, and Eq. (4.50) imply that $\hat{u}_i$ is the same as one u_i^κ, $\kappa = 1, 2, \cdots, 7m^{\max}$. Note that there is no approximation error caused by the tangent lines at the points u_i^κ, $\kappa = 1, 2, \cdots, 7m^{\max}$. In other words, at these points $\bar{g}_i$ does not underestimate $g_i(v_i)$. Hence, the resulting objective value to [RP] of the solution $(m^* = \hat{m}, v_i^* = 1/\hat{u}_i, t_i^{\text{arr}*} = \hat{t}_i^{\text{arr}})$ is equal to C_{MILP}. This means that $C_{MILP} \geqslant C_{RP}$. Consequently, model [RP] and model [MILP] are equivalent.

4.4.2 Global optimization method

4.4.2.1 Reformulation

Still, [RP] or [MILP] is not the original model. Suppose that the optimal solution to [RP] is denoted by $(t_i^{\text{arr}*}, i \in I)$. If $(t_i^{\text{arr}*}, i \in I)$ satisfies berth time windows at all ports, then this optimal solution is also optimal to the original [SDPTW]. Otherwise, it is infeasible. Enlightened by this observation, we develop a solution method that excludes infeasible solutions from [RP] by adding linear constraints.

First, we add to [RP] the following constraints:

$$\bar{t}_i^{\text{arr}} = t_i^{\text{arr}} - 7k_i, \forall i \in I \tag{4.54}$$

$$0 \leqslant \bar{t}_i^{\text{arr}} \leqslant 6, \forall i \in I \tag{4.55}$$

$$k_i \in \{0, 1, 2, \cdots, m-1\}, \forall i \in I \tag{4.56}$$

It is easy to see that the above constraints are equivalent to:

$$\bar{t}_i^{\text{arr}} = t_i^{\text{arr}} \mod 7, \forall i \in I \tag{4.57}$$

Note that because t_i^{arr} and k_i are defined to be integers, $\bar{t}_i^{\text{arr}}$ is automatically an integer.

Next, we rewrite $\bar{t}_i^{\text{arr}}$ using binary variables:

$$\bar{t}_i^{\text{arr}} = k_i^0 + 2k_i^1 + 4k_i^2, \forall i \in I \tag{4.58}$$

$$k_i^0, k_i^1, k_i^2 \in \{0, 1\}, \forall i \in I \tag{4.59}$$

It is clear that Eqs. (4.58)–(4.59) imply that given $\bar{t}_i^{\text{arr}}$, there is a unique binary vector (k_i^0, k_i^1, k_i^2) and vice versa. For example, if $\bar{t}_i^{\text{arr}} = 5$, we have $k_i^0 = 1$, $k_i^1 = 0$, and $k_i^2 = 1$. If $k_i^0 = 0, k_i^1 = 0, k_i^2 = 1$, we have $\bar{t}_i^{\text{arr}} = 4$. We now define a new model: reformulated MILP problem (RMILP):

[RMILP]:

[MILP] with constraints (4.54)-(4.56) and (4.58)-(4.59).

Theorem 4.4.2 The two models [RMILP] and [MILP] are equivalent.

Proof: First, the only difference of the two models is that [RMILP] has more constraints than [MILP]. Therefore, [RMILP] is at least as tight as [MILP]. Hence, we only need to prove that the additional constraints in [RMILP] does not confine the domain of [MILP]. In other words, we need to prove that for any feasible solution

$(t_i^{\text{arr}}, i \in I; m)$ to [MILP], we can find a vector $(k_i, k_i^0, k_i^1, k_i^2, i \in I)$ such that all the constraints (4.54)-(4.56) and (4.58)-(4.59) are satisfied.

As $(t_i^{\text{arr}}, i \in I; m)$ is feasible to [MILP], we have $0 \leqslant t_i^{\text{arr}} \leqslant 7m - 1$, $i \in I$. Now it is easy to see that there always exists a vector $(k_i, k_i^0, k_i^1, k_i^2, i \in I)$ such that all the constraints (4.54)-(4.56) and (4.58)-(4.59) are satisfied.

4.4.2.2 Linear constraints excluding infeasible solutions

Since [RMILP] is a mixed-integer linear optimization model, we can apply off-the-shelf MILP solvers to solve it. If the resulting solution is infeasible (that is, incompatible with the available port time windows or berth time windows), we add a linear constraint that excludes this solution while keeping all other solutions, and solve [RMILP] with the added linear constraints. This process is repeated until a feasible solution is found, and this solution is also optimal.

Suppose that $(t_i^{\text{arr}*}, i \in I)$ is the optimal solution to [RMILP], $\bar{t}_i^{\text{arr}*} = t_i^{\text{arr}*} \mod 7$, and it corresponds to $(k_i^{0*}, k_i^{1*}, k_i^{2*}, i \in I)$. We now elaborate on how to check the feasibility of $(t_i^{\text{arr}*}, i \in I)$.

At a port of call $i \in I_1$, if $\bar{t}_i^{\text{arr}*} \in \hat{\Omega}_i$, then $t_i^{\text{arr}*}$ is feasible at the port of call; otherwise it is infeasible. If $t_i^{\text{arr}*}$ is infeasible, to exclude it as well as other infeasible arrival times t_i^{arr} satisfying $t_i^{\text{arr}} \mod 7 = \bar{t}_i^{\text{arr}*}$ at port of call $i \in I_1$ from model [RMILP], we add the following constraint:

$$k_i^0(1 - k_i^{0*}) + (1 - k_i^0)k_i^{0*} + k_i^1(1 - k_i^{1*}) + (1 - k_i^1)k_i^{1*} + \\ k_i^2(1 - k_i^{2*}) + (1 - k_i^2)k_i^{2*} \geqslant 1 \qquad (4.60)$$

This constraint will exclude not only solution $t_i^{\text{arr}*}$ but also all t_i^{arr} satisfying $t_i^{\text{arr}} \mod 7 = \bar{t}_i^{\text{arr}*}$. For example, if the ship cannot arrive on Tuesday, then $t_i^{\text{arr}} = 2$ is infeasible, and $t_i^{\text{arr}} = 2 + 7 = 9$, $t_i^{\text{arr}} = 2 + 2 \times 7 = 16$ and $t_i^{\text{arr}} = 2 + 3 \times 7 = 23$ are all infeasible. All these infeasible t_i^{arr} correspond to the same $\bar{t}_i^{\text{arr}*} = 2$, and correspond to the same $(k_i^{0*}, k_i^{1*}, k_i^{2*}) = (0, 1, 0)$. Hence, Eq. (4.60) becomes:

$$k_i^0 + (1 - k_i^1) + k_i^2 \geqslant 1 \tag{4.61}$$

Evidently, $(k_i^0, k_i^1, k_i^2) = (0, 1, 0)$ is the only solution that violates this constraint.

At $i \in I_2 \cup I_2'$, if $\bar{t}_i^{\text{arr}*} \in \bigcup_{b \in B_i} \hat{\Omega}_i^b$, then this arrival time $t_i^{\text{arr}*}$ is feasible; otherwise it is infeasible and we can use a constraint similar to Eq. (4.60) to exclude it from [RMILP]. However, for $j \in I_2$, even if both $t_j^{\text{arr}*}$ and $t_{j'}^{\text{arr}*}$ are feasible, their combination may not be feasible. We need to check whether there exists a berth allocation plan such that the combination $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$ is feasible. Similar to the above analysis, checking the feasibility of $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$ is actually checking the feasiblity of $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$.

Algorithm 1: Check feasibility of $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$

Step 1. Set $b_1 = 1$.

Step 2. If $b_1 > |B_j|$, $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ is infeasible, return. If $\bar{t}_j^{\text{arr}*} \notin \hat{\Omega}_j^{b_1}$, set $b_1 = b_1 + 1$ and go to Step 2.

Step 3. Set $b_2 = 1$.

Step 4. If $b_2 > |B_j|$, set $b_1 = b_1 + 1$ and go to Step 2. Else if $\bar{t}_{j'}^{\text{arr}*} \notin \hat{\Omega}_j^{b_2}$, set $b_2 = b_2 + 1$ and go to Step 4. Else go to Step 5.

Step 5. If $b_1 \neq b_2$, $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ is feasible, return. Else

Step 5.0. Set $t' = 1$;

Step 5.1. If $t' \geqslant 7$, $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ is feasible, return. Else if $\delta_{jt_j^{\text{arr}*}}^{t'} + \delta_{j't_{j'}^{\text{arr}*}}^{t'} = 2$, the berth cannot serve two ships on day t', and hence set $b_2 = b_2 + 1$ and go to Step 4. Else set $t' = t' + 1$ and go to Step 5.1. □

Note that in Algorithm 1, we have at most $|B_j|^2$ possible berth allocation plans. Hence it is not difficult to check the feasibility of $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$.

If we cannot find a berth allocation plan such that $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ is feasible, then we exclude the solution $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ from [RMILP] by adding the following constraint:

$$\begin{aligned} &k_j^0(1-k_j^{0*}) + (1-k_j^0)k_j^{0*} + k_j^1(1-k_j^{1*}) + (1-k_j^1)k_j^{1*} + \\ &k_j^2(1-k_j^{2*}) + (1-k_j^2)k_j^{2*} + k_{j'}^0(1-k_{j'}^{0*}) + (1-k_{j'}^0)k_{j'}^{0*} + \\ &k_{j'}^1(1-k_{j'}^{1*}) + (1-k_{j'}^1)k_{j'}^{1*} + k_{j'}^2(1-k_{j'}^{2*}) + (1-k_{j'}^2)k_{j'}^{2*} \geqslant 1 \end{aligned} \tag{4.62}$$

Similar to constraint (4.60), this constraint excludes all the arrival times $(t_j^{\text{arr}}, t_{j'}^{\text{arr}})$ satisfying $t_j^{\text{arr}} \mod 7 = \bar{t}_j^{\text{arr}*}$ and $t_{j'}^{\text{arr}} \mod 7 = \bar{t}_{j'}^{\text{arr}*}$.

4.4.2.3 Overall algorithm

We now elaborate on the overall solution algorithm that obtains the global optimal solution to model [SDPTW].

Algorithm 2: Overall global optimization algorithm

Step 0. Define set $\Psi_1 := \varnothing$ that will contain constraints (4.60) and set $\Psi_2 := \varnothing$ that will contain constraints (4.62).

Step 1. Solve [RMILP] with constraints (4.60) defined by set Ψ_1 and constraints (4.62) defined by set Ψ_2. The optimal solution is denoted by $(m^*, v_i^*, t_i^{\text{arr}*}, \bar{t}_i^{\text{arr}*}, k_i^*, k_i^{0*}, k_i^{1*}, k_i^{2*}, i \in I)$.

Step 2. Check each port $i \in I = I_1 \cup I_2 \cup I_2'$. If $t_i^{\text{arr}*}$ is infeasible, add to set Ψ_1 the constraint (4.60), go to Step 1;

Step 3. Check each port $j \in I_2$. If the combination of arrival times $(t_j^{\text{arr}*}, t_{j'}^{\text{arr}*})$ is infeasible, add to set Ψ_2 the constraint (4.62), go to Step 1;

Step 4. The solution $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$ is a feasible and optimal solution to [SDPTW]. Stop. □

Theorem 4.4.3 Algorithm 2 terminates in a finite number of iterations.

Proof: In each iteration of Algorithm 2, we either exclude one $\bar{t}_i^{\text{arr}*}$, or a combination of $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$. The total number of $\bar{t}_i^{\text{arr}*}$ and combinations of $(\bar{t}_j^{\text{arr}*}, \bar{t}_{j'}^{\text{arr}*})$ does not exceed $7|I| + 7^2|I_2|$, which is finite. Hence, Algorithm 2 terminates in a finite number of iterations.

Theorem 4.4.4 The feasible solution $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$ obtained in Step 4 of Algorithm 2 is optimal to [SDPTW].

Proof: Model [RMILP] is a relaxed problem of the original model [SDPTW] in that some constraints (i.e. port time window constraints) are removed but the objective function does not change. We add more and more constraints (4.60) and (4.62) in each iteration of Algorithm 2. However, these constraints do not exclude any feasible solution. Hence, the feasible solution $(m^*, v_i^*, t_i^{\text{arr}*}, i \in I)$ obtained in Step 4 of Algorithm 2, which by definition is optimal to [RMILP] with the generated constraints (4.60) and (4.62), and by definition is feasible to port time window constraints, is also optimal to [SDPTW].

4.5 Case study

We carry out case studies based on the AGM ship route in Fig. 4.1 to evaluate the applicability of the proposed models and algorithms. The AGM ship route consists of 10 ports of call in a round trip. The port of Miami is visited twice and hence there are a total of 9 physical ports. We assume that these 9 ports have a total of 30 berths, whose available times are shown in Table 4.1.

We assume that 5000-TEU ships are deployed on the ship route. The operating cost C^{ship} =500,000 USD/week. The max speed $V^{\max}$=30 knots, the bunker price $\alpha = 400$ USD/ton, the unit inventory cost $\beta = 1$ USD per TEU per hour and the maximum number of ships $m^{\max} = 20$ ships. The port time, length, bunker consumption function $g_i(v_i)$, and container number on each leg are shown in Table 4.2.

Table 4.1: Available time at each port

Port ID	Port	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	Le Havre	1	free	free			free	free	
		2			free				
		3				free			free
2	Antwerp	1	free	free	free				
		2							free
3	Rotterdam	1		free		free	free		
		2	free	free		free	free		
4	Bremerhaven	1	free	free			free	free	
		2			free				
		3				free			free
5	Charleston	1	free		free				
		2			free	free	free		free
		3		free			free		
		4	free	free		free	free		
6	Miami	1	free	free			free	free	
		2			free				
		3				free			free
		4	free		free				
		5				free			
		6		free			free		
7	Veracruz	1	free	free		free	free		
		2	free		free				
		3	free		free				
8	Altamira	1		free			free		
		2	free	free			free	free	
		3			free				
		4	free			free	free		free
9	Houston	1		free			free		
		2	free	free			free	free	
		3			free				

4.5.1 Performance of the solution algorithm

We apply the proposed global optimization algorithm (Algorithm 2) to design the schedule of the AGM ship route of OOCL. The models are all solved by matlab calling CPLEX 12.2 on a 3.2 GHz Dual Core laptop with 4 GB of RAM. The algorithm finds the optimal solution after 26 iterations in about 1 minute. Hence, the algorithm is efficient for addressing problems of realistic scales.

The number of ships and the total cost in each iteration are shown in Fig. 4.3. As

Table 4.2: Parameters in the case study

Port of call	Port	Port time	Length	Bunker function	# containers
1	Le Havre	2 days	252	$0.001(v_1)^2$	3500
2	Antwerp	1 day	149	$0.001(v_2)^{2.1}$	3600
3	Rotterdam	1 day	225	$0.001(v_3)^{2.3}$	3900
4	Bremerhaven	1 day	4014	$0.001(v_4)^2$	5000
5	Charleston	2 days	435	$0.001(v_5)^2$	4900
6	Miami	2 days	1012	$0.001(v_6)^2$	4000
7	Veracruz	1 day	233	$0.001(v_7)^2$	3500
8	Altamira	1 day	512	$0.001(v_8)^{2.1}$	3800
9	Houston	1 day	970	$0.001(v_9)^{2.3}$	4100
10	Miami	2 days	3922	$0.001(v_{10})^2$	4950

more and more constraints are added, the optimal objective value (the total cost) is non-decreasing. It is interesting to notice that the total cost does not change in the first seven iterations. This is because of "symmetrical solutions" as follows. In the first iteration, the optimal solution of $(\bar{t}_i^{\text{arr}}, i \in I)$ is $(3,6,1,3,6,2,6,1,3,0)$ (of course, it is infeasible to the original problem). Since $\bar{t}_1^{\text{arr}} = 3$ is infeasible, constraint (4.60) excludes it and solution $(4,0,2,4,0,3,0,2,4,1)$ is obtained in the second iteration (of course, it is still infeasible to the original problem). Comparing these two solutions, we find that the second solution differs from the first one in that the arrival times at all ports of call are postponed by one day. Hence, the ship cost, bunker cost, and inventory cost do not change. Similarly, the solution obtained in the third iteration simply postpones the arrival times at all ports of call by two days. Repeating in a similar manner, the optimal number of ships and the total cost in the first seven iterations are the same.

In the eighth iteration, the solution of $(\bar{t}_i^{\text{arr}}, i \in I)$ is $(0,4,6,1,3,6,3,5,0,4)$. We observe that in this solution, the inter-arrival times between two adjacent ports of call are different from the previous seven solutions. For example, in the eighth solution, $\bar{t}_2^{\text{arr}} - \bar{t}_1^{\text{arr}} = 4$, which means that $t_2^{\text{arr}} - t_1^{\text{arr}}$ is equal to 4 plus an integer number of weeks. However, in the first seven solutions, $\bar{t}_2^{\text{arr}} - \bar{t}_1^{\text{arr}} = 3$. Consequently, in the eighth solution, the sailing speed and the inventory cost on leg 1 are changed.

In the first 22 iterations, exactly one constraint (4.60) is added to the model

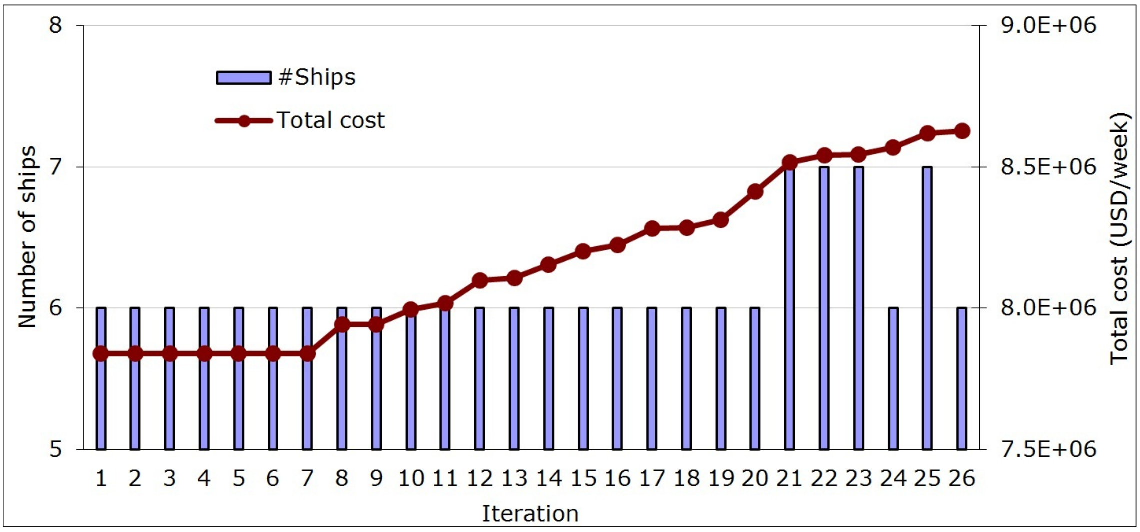


Figure 4.3: Number of ships and total cost in each iteration

in Step 2 of Algorithm 2. The solution of $(\bar{t}_i^{\text{arr}}, i \in I)$ in the 23rd iteration is $(4, 1, 3, 5, 3, 0, 4, 6, 2, 0)$. The arrival time at each single port of call all satisfies the port time windows. However, the arrival times at the 6th and 10th ports of call, which correspond to the same physical port, i.e., Miami, are both Sunday. As $t_6^{\text{port}} = t_{10}^{\text{port}} = 2$, both calls must use berth 1 of the port according to Table 4.1. As a result, their combination is infeasible, and hence in Step 3 of Algorithm 2, one constraint (4.62) is added to exclude such a combination. The solution in the 24th iteration is $(0, 6, 1, 3, 3, 0, 4, 6, 1, 5)$. Now $\bar{t}_{10}^{\text{arr}} = 5$ is infeasible. Hence, one constraint (4.60) in Step 2 of Algorithm 2 is added to exclude the solution. In the 25th iteration, the solution is $(4, 1, 3, 5, 0, 4, 2, 4, 0, 4)$, and the combination $(\bar{t}_6^{\text{arr}} = 4, \bar{t}_{10}^{\text{arr}} = 4)$ is again infeasible and therefore one constraint (4.62) is added. In the 26th iteration, the solution of $(\bar{t}_i^{\text{arr}}, i \in I)$ is $(0, 6, 1, 3, 3, 0, 4, 6, 1, 4)$, which does not violate any port time window constraint. Hence, this solution is optimal to the original problem. The optimal solution of $(t_i^{\text{arr}}, i \in I \cup \{N + 1\})$ is $(0, 6, 8, 10, 17, 21, 25, 27, 29, 32, 42)$.

The number of ships in each iteration does not show any trend in Fig. 4.3. The optimal number of ships to deploy is 6.

4.5.2 Impact of port time windows

In this section, we examine the impact of the availability of a port on the optimal schedule and the total cost. We take the example of the port of Miami, which is visited twice a week with $t_6^{\text{port}} = t_{10}^{\text{port}} = 2$. Both Miami and the liner shipping company are interested in looking at the result if more available time is provided at Miami. We hence examine 7 berth availability cases of Miami, as shown in Table 4.3. Note that a berth is not included in a case of Table 4.3 if it is busy the whole week. From case 1 to case 7, more and more available times are provided. For example, in case 1 we must have $\bar{t}_6^{\text{arr}} = 0, \bar{t}_{10}^{\text{arr}} = 0$; in case 2 either $\bar{t}_6^{\text{arr}}$ or $\bar{t}_{10}^{\text{arr}}$ can be 5; in case 3 it is further possible that either $\bar{t}_6^{\text{arr}}$ or $\bar{t}_{10}^{\text{arr}}$ is 6, etc.

Table 4.3: Different cases of available time at Miami

Case	Berth	Sun	Mon	Tue	Wed	Thur	Fri	Sat
1	1	free	free					
	4	free	free					
2	1	free	free					
	3						free	free
	4	free	free					
3	1	free	free					
	3						free	free
	4	free	free					free
4	1	free	free					
	2					free	free	
	3						free	free
	4	free	free					free
5	1	free	free					
	2					free	free	
	3						free	free
	4	free	free	free				free
6	1	free	free		free	free		
	2					free	free	
	3						free	free
	4	free	free	free				free
7	1	free	free		free	free		
	2					free	free	
	3						free	free
	4	free	free	free				free
	6			free	free			

The results of the total cost and optimal number of ships deployed for the 7 berth availability cases are shown in Fig. 4.4. More available days at Miami leads to a lower total cost: the total cost is reduced by 603,738 USD/week from case 2 to case 6 Table 4.4. Fig. 4.4 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. The optimal ship route schedule is shown in Table 4.5. We observe that the optimal arrival time at Miami and its neighboring ports may change if the time windows at the port of Miami change.

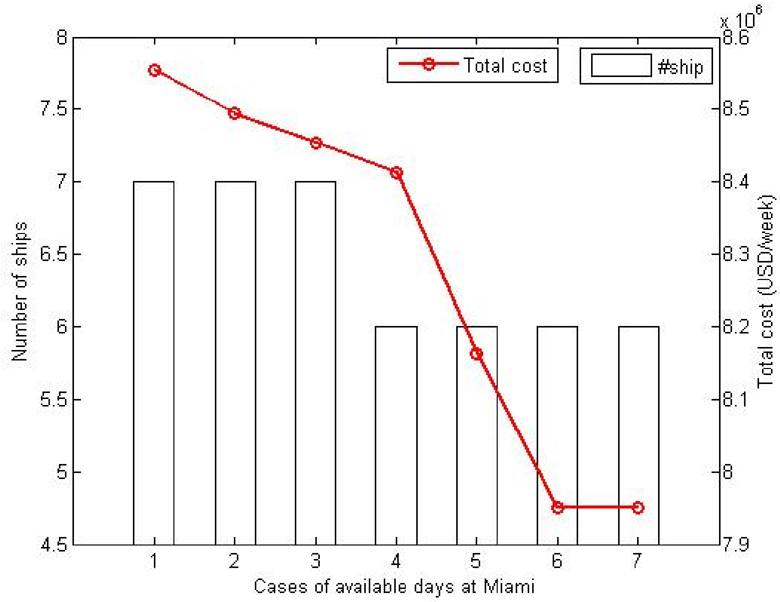


Figure 4.4: Impact of port time windows

Table 4.4: Impact of port time windows on each type of cost

Case	Ship cost	Bunker cost	Inventory cost
1	3500000	3826800	1227456
2	3500000	3781200	1213322
3	3500000	3804000	1150038
4	3000000	2966400	2447227
5	3000000	3060000	2103630
6	3000000	3040800	1909718
7	3000000	3040800	1909718

Table 4.5: Impact of port time window on the optimal schedule

Port of call	Port	Cases 1	Case 2	Cases 3	Case 4	Case 5	Cases 6,7
1	Le Havre	4	4	4	0	4	4
2	Antwerp	7	7	7	6	7	7
3	Rotterdam	10	10	10	8	10	10
4	Bremerhaven	12	12	12	10	12	12
5	Charleston	24	23	23	17	21	21
6	Miami	28	26	26	20	25	24
7	Veracruz	32	32	31	24	29	28
8	Altamira	34	34	33	26	31	30
9	Houston	37	37	36	28	33	32
10	Miami	42	42	41	32	36	36
1	Le Havre	53	53	53	42	46	46

4.5.3 Consequence of port efficiency

The port time t_i^{port} to a large extent depends on the container handling efficiency. Therefore, port operators seek to improve efficiency by optimizing quay-side and yard-side operations. To investigate the effect of port handling efficiency, we change the port time at Miami to generate four scenarios. In the first scenario, we assume both two visits use the port for only one day. The second scenario assumes the port time for the first visit (coming from Charleston) is one day and the second visit (coming from Houston) is two days. In the third scenario, the first visit needs two days to serve and one day is needed for the second visit. The last scenario is generated by assuming both visits need two days to serve which is the same as subsections 4.5.1 and 4.5.2. Table 4.6 shows the four port time scenarios.

Table 4.6: The scenarios of port times of the two calls at Miami

	Port time (day)	
Scenario	1st visit	2nd visit
1-1	1	1
1-2	1	2
2-1	2	1
2-2	2	2

We carry out numerical experiments for each combination of the 7 berth availability cases of Miami in Table 4.3 and the 4 scenarios of port time in Table 4.6. Hence, we have a total of 28 experiments, and the total costs and the number of ships to deploy are shown in Fig. 4.5 and Fig. 4.6, respectively. Consistent with the previous subsection, Fig. 4.5 indicates that the total cost always decreases when the port has more available days. Fig. 4.6 shows that the number of ships is reduced with more available days at Miami. Generally, by comparing the 4 port time scenarios, we find that improving port efficiency may reduce the total cost for liner shipping companies. However, because we do not include the inventory cost of containers associated with port time in objective functions of the models, a higher port efficiency may lead to a higher total cost if the reduced port time does not help to reduce the round-trip time of the ship route (at least by one week). For example, in berth availability case 2, the total cost of scenario 1-2 (the first visit is one day and the second visit is two days) is larger than the total cost of scenario 2-2 (both visits are two days). The reduced one day port time moves from the port time to sailing time on leg 4, which leads to more inventory cost at sea without reducing the round-trip time of the ship route. This result can be seen in Table 4.7, which reports the optimal schedules of the 4 scenarios under berth availability case 2 and the sailing time on each leg i, i.e., $t_{i+1}^{\mathrm{arr}} - t_i^{\mathrm{dep}}$. It should be noted that if we include the inventory cost of containers associated with port time, then the total cost of scenario 1-2 is always lower than that of scenario 2-2.

Finally, we note that the reduction of the round-trip time of the ship route is always an integer number of weeks, which corresponds to the reduction in the number of ships to deploy. In some cases, reducing port time leads to a smaller number of ships deployed, for example, under berth availability case 2, the optimal number of ships is 6 in scenario 1-1 and the number is 7 in scenario 2-2, as shown in Fig. 4.6.

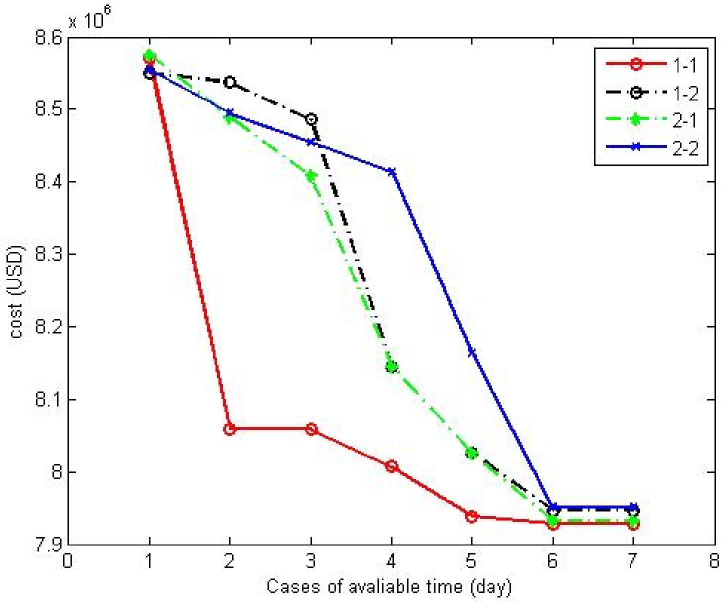


Figure 4.5: Impact of port time at Miami on the total cost

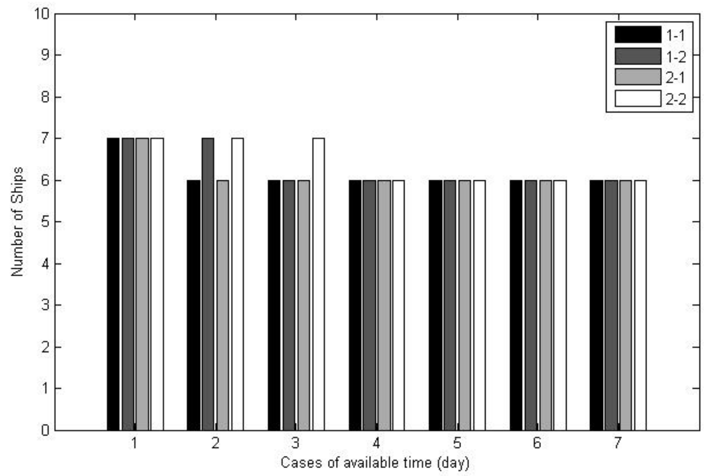


Figure 4.6: Impact of port time at Miami on the number of ships

4.5.4 Result of bunker prices

The bunker price is volatile and hence we examine the sensitivity of the solution with different bunker prices (USD/ton) from 300, 400, 500, 600, 700, to 800. The parameters are the same as Table 4.1 and Table 4.2. The results are shown in Fig. 4.7 . We observe that a higher bunker price always leads to a higher total cost for liner shipping companies. In addition, Fig. 4.7 shows that there is a rise in the number of ships used when the bunker price becomes higher. This is because when

Table 4.7: Optimal schedules of the 4 scenarios under berth availability case 2

	Arrival time t_i^{arr}				Sailing days on leg i			
Port of call	1-1	1-2	2-1	2-2	1-1	1-2	2-1	2-2
1	4	4	0	4	1	1	4	1
2	7	7	6	7	2	2	1	2
3	10	10	8	10	1	1	1	1
4	12	12	10	12	10	11	6	10
5	23	24	17	23	1	1	2	1
6	26	27	21	26	2	4	2	4
7	29	32	25	32	1	1	1	1
8	31	34	27	34	1	2	1	2
9	33	37	29	37	2	4	3	4
10	36	42	33	42	9	9	8	9
1	46	53	42	53				
Total					30	36	29	35

more ships are deployed, the sailing speed can be lower, resulting in lower bunker consumption, which is more significant when the bunker price is higher.

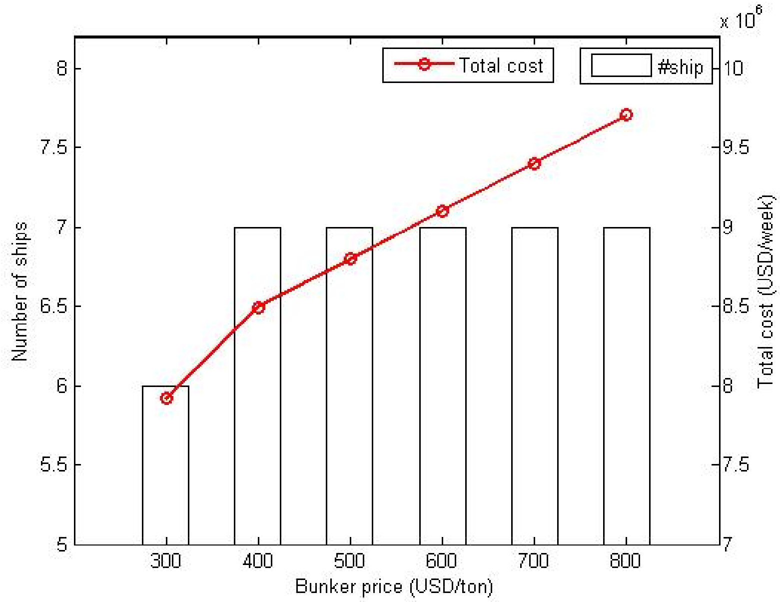


Figure 4.7: Result of bunker prices on the total cost and the number of ships

4.5.5 Effect of inventory cost

Finally, the unit inventory cost β may affect the ship route schedule and then the total cost. We change β from 1, 1.2 through to 2 and the results of 6 experiments are shown in Fig. 4.8. The number of ships decreases when the unit inventory cost rises. This shows that when the cargos in the containers are more valuable, ships should sail at a higher speed. The total cost increases almost linearly (not strictly linearly) when the unit inventory cost rises. This indicates that as a result of increase in the unit inventory cost, the total cost of a liner shipping company is higher.

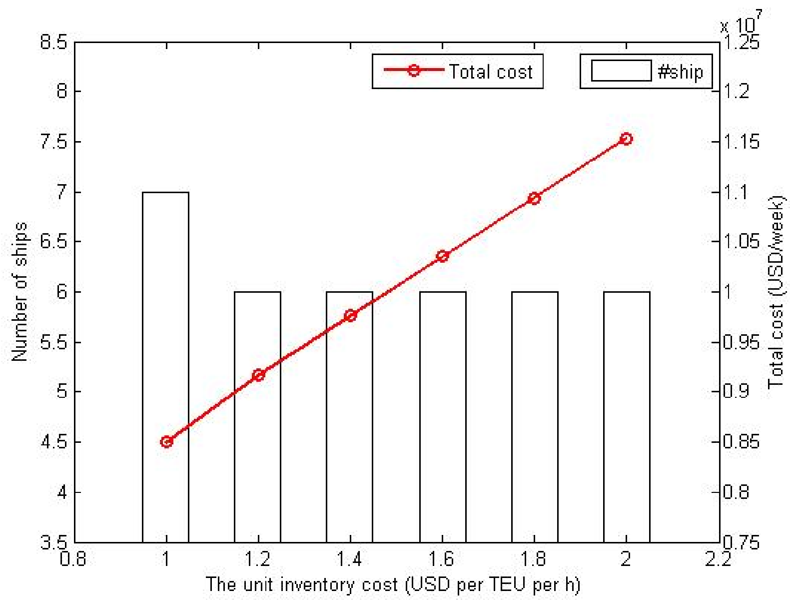


Figure 4.8: Effect of unit inventory cost on the total cost and the number of ships

4.6 Conclusions

This chapter has generalized the model of chapter 3, which is the practical liner ship route schedule design problem with port time windows, by allowing a port to be visited more than once in a round trip. This is a significant tactical planning decision problem for liner shipping companies because it considers the availability of ports when planning liner shipping services. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we developed a holistic solution approach. In this approach, at first the port time window constraints are relaxed to obtain a mixed-integer nonlinear programming model, which is subsequently transformed to a mixed-integer linear programming model. This mixed-integer linear model is repeatedly solved by adding the violated port time window constraints until a feasible solution is obtained. This feasible solution is proved to be the global optimal solution to the problem.

We have conducted extensive numerical experiments based on the AGM ship route of OOCL. According to the results, the solution approach could efficiently find the optimal solution, which demonstrates its applicability to realistic problems. The availability of ports affects the the total cost of liner shipping companies, the optimal number of ships deployed, and the ship route schedule. Therefore, it is important for liner shipping companies to consider port time windows in liner ship route schedule design. In addition, due to the importance of some of the parameters on the sailing schedule, we conducted sensitivity analysis of port efficiency, bunker price and unit inventory cost. Useful managerial insights are obtained. Firstly, improving port efficiency generally will reduce the total cost for liner shipping companies. However, because of the weekly service of a ship route, improving port efficiency may but does not necessarily lead to a reduction of the round-trip time of a ship route and thereby a smaller number of ships deployed. Secondly, increasing bunker price leads to a higher total cost and a rise in the number of ships deployed. Third, the number of ships to deploy drops and the total cost increases with a rise in the unit inventory cost.

Chapter 5

Schedule Design for Liner Shipping Networks without Considering Inventory Cost

5.1 Introduction

Container liner shipping is a significant importance to the global supply chain network. A container liner shipping network consists of many ship routes, and a shipping line has to determine the schedule for each ship route. In this chapter, we extend the previous schedule design problems by considering a liner shipping network that consists of many ship routes.

The main contribution of the chapter is that it takes the initiative to address a practical liner shipping network schedule design problem with port time windows (NSDPTW). The designed schedules are preferable as the sum of ship cost and fuel cost is minimized. The schedules are feasible because the availability of berths at each port on each day is explicitly considered in the model. The results from the model need no or very little modification before put to use. Hence, this study provides a useful decision-support tool for liner shipping companies to plan their services.

The remainder of the chapter is organized as follows. Section 5.2 describes the problem and formulates a mathematical model. Section 5.3 proposes a tailored solution approach to address the problem. Section 5.4 reports two case studies based on two networks consisting of three and five service routes. Section 5.5 presents the conclusion.

5.2 Problem Description

Port Rotation

Port	City	Transit Day	Arrive	Transit Day	Depart
SIN	Singapore	00	Tue	02	Thu
KHI	Karachi	09	Thu	11	Sat
NS9	Nhava Sheva	12	Sun	13	Mon
CMB	Colombo	16	Thu	17	Fri
SIN	Singapore	21	Tue	–	–

Figure 5.1: SWX service operated by APL (2014)

Consider a liner container shipping company that operates a number of ship routes, denoted by the set $\mathcal{R}$, regularly serving a group of ports denoted by the set $\mathcal{P}$ Meng and Wang (2011a) and Meng et al. (2012a). The port rotation of a ship route $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \to p_{r2} \to \ldots \to p_{rN_r} \to p_{r1} \tag{5.1}$$

where N_r is the number of ports of call on the ship route and $p_{ri} \in \mathcal{P}$ is the physical port corresponding to ith port of call. Let I_r be the set of ports of call of ship route $r \in \mathcal{R}$, i.e., $I_r = \{1, 2, \cdots, N_r\}$. For brevity, we define $< r, i >$ as the port of call i on ship route r. Defining $p_{r,N_r+1} = p_{r1}$, the voyage from p_{ri} to $p_{r,i+1}$ is called *leg* i, $i \in I_r$. Fig. 5.2 shows a liner shipping network with three ship routes operated by APL (2014), where ship route 1 is SWX in Fig. 5.1, ship route 2 is Surabaya Feeder Service (SUR) and ship route 3 is Semarang Feeder Service (SEM):

$$\begin{aligned}
r = 1, N_r = 4 : &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{KR}) \to p_{r3}(\text{NS}) \to p_{r4}(\text{CB}) \to p_{r1}(\text{SG}) \\
r = 2, N_r = 2 : &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{SB}) \to p_{r1}(\text{SG}) \\
r = 3, N_r = 2 : &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{SR}) \to p_{r1}(\text{SG}) \to p_{r1}(\text{CB})
\end{aligned}$$

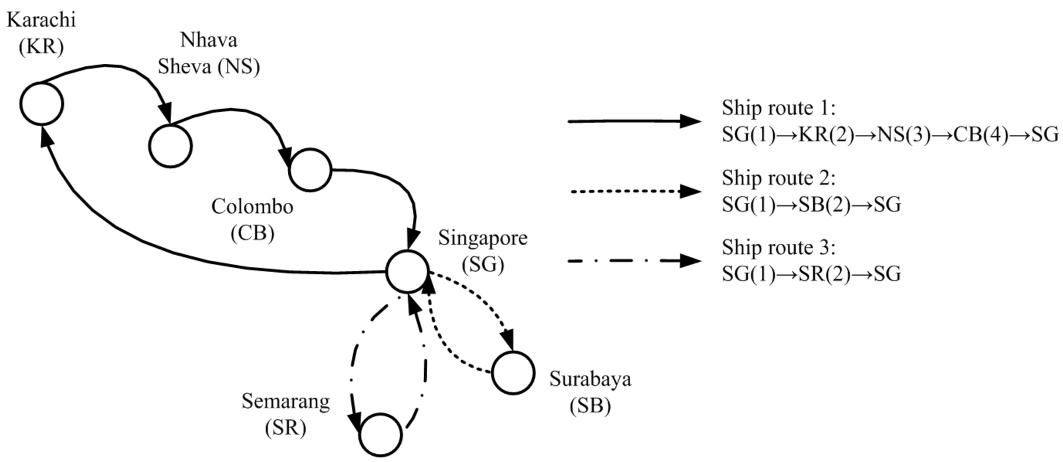


Figure 5.2: A liner shipping network with three ship routes

We further define $\mathcal{R}_p$ as the set of ship routes that visit port $p \in \mathcal{P}$, and define I_{rp} as the set of ports of call on ship route $r \in \mathcal{R}_p$ that correspond to port p. In the above example, $\mathcal{R}_{\text{SG}} = \{1, 2, 3\}$, $I_{1,\text{CB}} = \{4\}$, $I_{2,\text{SG}} = \{1\}$, and $I_{3,\text{CB}} = \varnothing$.

5.2.1 Weekly service and schedules

A string of homogeneous ships are deployed on each ship route $r \in \mathcal{R}$ to maintain a weekly service frequency Wang et al. (2013f) and Wang et al. (2013a). Let L_{ri} (n mile) be the voyage distance of the ith leg of route $r \in \mathcal{R}$, $t_{ri}^{\text{port}} \in \{1, 2\}$ be the fixed time (day) a ship spends at port of call i on ship route $r \in \mathcal{R}$ for container handling, m_r be the number of ships deployed on ship route $r \in \mathcal{R}$, and t_{ri} be the sailing time (day) on the ith leg of ship route $r \in \mathcal{R}$. We then have the relation:

$$\sum_{i \in I_r} \left(t_{ri} + t_{ri}^{\text{port}}\right) = 7m_r, \forall r \in \mathcal{R}$$

where 7 is the number of days in a week. Represent by C_r^{ship} the fixed cost (USD/week) associated with a ship on route $r \in \mathcal{R}$. C_r^{ship} includes the capital cost and the operating cost. The fixed cost associated with all the ships is:

$$\sum_{r \in \mathcal{R}} C_r^{\text{ship}} m_r$$

Define the beginning of a particular Sunday as day 0, and let t_{ri}^{arr} be the arrival time (day) at port of call i on ship route r. For instance, $t_{ri}^{\text{arr}} = 13$ means that ships arrive at $< r, i >$ at the beginning of next Saturday. The time components of a ship route have the relation

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, r \in \mathcal{R}, i \in I_r$$

We assume that the sailing time on a leg has a minimum required value, denoted by $t_{ri}^{\min}$. Hence,

$$t_{ri} \geqslant t_{ri}^{\min}, r \in \mathcal{R}, i \in I_r$$

Note that $t_{ri}^{\min}$ can be used to incorporate potential buffer time. For instance, if a leg is short, and the container handling time at port of call i is unreliable, then $t_{ri}^{\min}$

should be much larger than $\frac{L_{ri}}{24V_r^{\max}}$ so as to hedge against potential delay by fast steaming, where $V_r^{\max}$ is the maximum speed (knots) of ships deployed on ship route r.

Because liner ship routes provide weekly services, to simplify the notation, we define W to be a set of all days in a week, that is,

$$W := \{0, 1, 2, 3, 4, 5, 6\}$$

where 0 represents Sunday, 1 represents Monday, etc. Without loss of generality, we require

$$t_{r1}^{\text{arr}} \in W, r \in \mathcal{R}$$

In fact, there is no difference between $t_{r1}^{\text{arr}} = 2$ and $t_{r1}^{\text{arr}} = 9$ due to the weekly frequency of liner services. Hence, the schedule of ship routes in the liner shipping network can be defined by the following vector

$$(t_{ri}^{\text{arr}}, r \in \mathcal{R}, i \in I_r; m_r, r \in \mathcal{R}) \tag{5.2}$$

The number of ships m_r is essential in the above vector, because without it the sailing time on leg N_r cannot be determined. The weekly service implies that

$$t_{r,N_r+1}^{\text{arr}} = t_{r1}^{\text{arr}} + 7m_r, r \in \mathcal{R}$$

Therefore, we can also define the schedule as

$$(t_{ri}^{\text{arr}}, r \in \mathcal{R}, i \in I_r \cup \{N_r + 1\}) \tag{5.3}$$

which is equivalent to the schedule defined by (5.2).

The arrival time t_{ri}^{arr} at a port of call corresponds to day $t_{ri}^{\text{arr}} \mod 7$ of a week. To be more specific, we let z_{ri}^{w} be a binary variable which equals 1 if and only if ships arrive on day $w \in W$ of a week at $< r, i >$. Mathematically

$$t_{ri}^{\text{arr}} \mod 7 = w' \Longleftrightarrow \begin{cases} z_{ri}^{w'} = 1 \\ z_{ri}^{w} = 0, w \in W \backslash \{w'\} \end{cases}$$

5.2.2 Port time windows

A port may have more than one berth. Hence, we let B_p be the set of berths at the port $p \in \mathcal{P}$. A berth may not always be available in a week because a port needs to serve more than one liner shipping company. Therefore, we define the parameter δ_b^w which equals 1 if berth $b \in \cup_{p \in \mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise.

A ship uses exactly one berth when it visits a port of call and a berth cannot serve more than one ship at the same time. To formulate this constraint, let z_{ri}^{bw} be a binary variable which equals 1 if and only if ships use berth $b \in B_{p_{ri}}$ when visiting $< r, i >$ on day w. For notational convenience, we define $z_{ri}^{b,-1} := z_{ri}^{b,6}$. We have

$$\sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=1} z_{ri}^{bw} + \sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=2} (z_{ri}^{b,w-1} + z_{ri}^{b,w}) \leq \delta_b^w, p \in \mathcal{P}, b \in B_p, w \in W \quad (5.4)$$

The first term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{\text{port}} = 1$ and the arrival day is w. The second term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{\text{port}} = 2$ and the arrival day is $w - 1$ or w. The overall constraint indicates that an available berth cannot serve more than one ship on the same day.

We assume further that each port has a premium berth $\bar{b}$, which is always available and can accommodate any number of ships. However, the liner shipping company needs to pay a high penalty cost $C_{\bar{b}}$ each time the berth is used.

5.2.3 Bunker consumption

Represent by $g_{ri}(v)$ the bunker consumption (tons/n mile) function with regard to the speed v on leg i of ship route r. Based on the results in existing studies, we assume that $g_{ri}(v)$ is a power function of the form:

$$g_{ri}(v) = a_{ri}v^{b_{ri}}, a_{ri} > 0, b_{ri} > 2 \tag{5.5}$$

Hence, we let $Q_{ri}(t_{ri})$ be the bunker consumption on leg i of ship route r, and it can calculated by:

$$Q_{ri}(t_{ri}) = L_{ri}g_{ri}(L_{ri}/t_{ri})$$

5.2.4 Model

Before presenting the model, we list the notation below.

Variables

m_r	Number of ships deployed on the ship route $r \in \mathcal{R}$
t_{ri}^{arr}	Arrival time (day) at port of call i on ship route r
t_{r,N_r+1}^{arr}	The time (day) when the ship returns to the 1st port of call on ship route r
t_{ri}	Sailing time (day) on the ith leg of ship route $r \in \mathcal{R}$
k_{ri}	An integer that is associated with the arrivall time at the port of call i on ship route r, $r \in \mathcal{R}, i \in I_r$
z_{ri}^{b}	A binary variable which equals 1 if and only if ships use berth b when visiting the port of call i on ship route r including the premium berth $\bar{b}$
$z_{ri}^{\bar{b}}$	A binary variable which equals 1 if and only if ships use a premium berth $\bar{b}$ when visiting the port of call i on ship route r

z_{ri}^{w} A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the port of call i on ship route r

z_{ri}^{bw} A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the berth b at the port of call i on ship route r

Parameters

α The bunker fuel price (USD/ton)

δ_b^w A parameter that equals 1 if berth $b \in \cup_{p \in \mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise

a_{ri} A coefficient calibrated from operating data and satisfy $a_{ri} > 0$

b_{ri} A coefficient calibrated from operating data and satisfy $b_{ri} > 1$

B_p The set of berths at the port $p \in \mathcal{P}$

C_r^{ship} The weekly operating cost of a ship deployed on ship route r

$C_{\bar{b}}$ The penalty cost of using a premium berth $\bar{b}$

I_r The set of ports of call on ship route r

L_{ri} Oceanic distance (n mile) of the ith leg of route r

N_r Number of ports on the ship route r

p_{ri} The physical port that corresponds to the ith port of call on the ship route r

t_{ri}^{port} Time (day) a ship spends at port of call i on the ship route r

$t_{ri}^{\min}$ A minimum required value of the sailing time on the ith leg of route r

$\bar{V}_{ri}$ Number of containers (TEUs) on leg i on the ship route r

$V_r^{\max}$ Maximum speed of the ships on the ship route r

$m_r^{\max}$ Maximum number of ships deployed on the ship route r

$\mathbb{Z}^+$ Set of nonnegative integers

The NSDPTW can be formulated as an optimization model below. The objective function is:

[NSDPTW]

$$\min \quad \sum_{r\in\mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r\in\mathcal{R}} \sum_{i\in I_r} L_{ri} a_{ri} (L_{ri}/(24t_{ri}))^{b_{ri}} + C_{\bar{b}} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} z_{ri}^{\bar{b}} \tag{5.6}$$

which aims to minimize the sum of ship cost and fuel cost while penalizing the violation of berth time windows.

The NSDPTW is subject to a number of constraints. The first set is the basic logical constraints in the schedule:

$$0 \leqslant t_{r1}^{\text{arr}} \leqslant 6, r \in \mathcal{R} \tag{5.7}$$

$$t_{ri} \geqslant \left\{ t_{ri}^{\min}, \left\lceil \frac{L_{ri}}{24V_r^{\max}} \right\rceil \right\}, r \in \mathcal{R}, i \in I_r \tag{5.8}$$

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, r \in \mathcal{R}, i \in I_r \tag{5.9}$$

$$t_{r,N_r+1}^{\text{arr}} = t_{r1}^{\text{arr}} + 7m_r, r \in \mathcal{R} \tag{5.10}$$

$$m_r \in \{1, 2, 3, \cdots, m_r^{\max}\}, r \in \mathcal{R} \tag{5.11}$$

$$t_{ri}^{\text{arr}} \in \mathbb{Z}^+, r \in \mathcal{R}, i \in I_r \tag{5.12}$$

The objective function (5.6) minimizes the sum of ship cost, bunker cost and penalty cost. Constraint (5.7) eliminates symmetric solutions. Constraint (5.8) confirms that the sailing time on a leg is not less than a minimum required value and ships cannot sail at a speed that exceeds $V_r^{\max}$. Constraint (5.9) defines the relation of different time components in a round-trip journey. Constraint (5.10) defines the time when the ship returns to the 1st port of call after one round-trip. Constraint (5.11) indicates that the number of ships is a positive integer that does not exceed a pre-specified maximum value. Constraint (5.12) indicates that the arrival time at each port of call is a nonnegative integer.

The second set of constraints formulates the day of a week for arrival at each port of call in the network:

$$\sum_{w\in W} wz_{ri}^{w} = t_{ri}^{\text{arr}} - 7k_{ri}, r \in \mathcal{R}, i \in I_r \tag{5.13}$$

$$\sum_{w\in W} z_{ri}^{w} = 1, r \in \mathcal{R}, i \in I_r \tag{5.14}$$

$$z_{ri}^{w} \in \{0,1\}, r \in \mathcal{R}, i \in I_r, w \in W \tag{5.15}$$

$$k_{ri} \in \{0,1,2,\cdots,m_r-1\}, r \in \mathcal{R}, i \in I_r \tag{5.16}$$

Constraint (5.13) defines the arrival day of a week at each port call on the route. Constraint (5.14) requires that a ship arrives exactly once a week at each port of call. Constraint (5.15) defines z_{ri}^{w} as a binary variable. Constraint (5.16) defines the auxiliary variable k_{ri} as a nonnegative integer.

The third set of constraints considers the availability of berths:

$$\sum_{r\in\mathcal{R}_p}\sum_{i\in I_{rp},t_{ri}^{\text{port}}=1} z_{ri}^{bw} + \sum_{r\in\mathcal{R}_p}\sum_{i\in I_{rp},t_{ri}^{\text{port}}=2} (z_{ri}^{b,w-1} + z_{ri}^{bw}) \leqslant \delta_b^w, p \in \mathcal{P}, b \in B_p, w \in W \tag{5.17}$$

$$\sum_{b\in B_{p_{ri}}\cup\{\bar{b}\}} z_{ri}^{b} = 1, r \in \mathcal{R}, i \in I_r \tag{5.18}$$

$$z_{ri}^{bw} \leqslant z_{ri}^{b}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.19}$$

$$z_{ri}^{bw} \leqslant z_{ri}^{w}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.20}$$

$$z_{ri}^{bw} \geqslant z_{ri}^{b} + z_{ri}^{w} - 1, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.21}$$

$$z_{ri}^{bw} \in \{0, 1\}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.22}$$

$$z_{ri}^{b} \in \{0, 1\}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\} \tag{5.23}$$

Constraint (5.17) indicates that an available berth cannot serve more than one ship on the same day. Constraint (5.18) requires that a ship uses exactly one berth each time it visits a port including the premium berth $\bar{b}$. Constraints (5.19) , (5.20) and (5.21) impose that a ship uses a berth at a port of call once a week. Constraints (5.22) and (5.23) define z_{ri}^{bw} and z_{ri}^{b} as binary variables.

5.3 Solution Method

The model [NSDPTW] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has a large number of discrete variables; and (ii) it has nonlinear objective function. After carefully examining the properties of the problem, we develop a tailored solution method that overcomes these difficulties.

5.3.1 Linearization of the objective function

The nonlinear term $t_{ri}^{-b_{ri}}$ in the objective function (5.6) can be linearized due to the following property.

Proposition 5.3.1 The optimal t_{ri} can be determined by the optimal z_{ri}^{w} and $z_{r,i+1}^{w}$, $w \in W$.

Proof: Given z_{ri}^w and $z_{r,i+1}^w$, $w \in W$, t_{ri} can only take values as follows: $\sum_{w\in W} wz_{r,i+1}^w - \sum_{w\in W} wz_{ri}^w - t_{ri}^{\text{port}}$, $\sum_{w\in W} wz_{r,i+1}^w - \sum_{w\in W} wz_{ri}^w - t_{ri}^{\text{port}} + 7$, $\sum_{w\in W} wz_{r,i+1}^w - \sum_{w\in W} wz_{ri}^w - t_{ri}^{\text{port}} + 14$, etc., while satisfying constraint (5.8). The possible values of t_{ri} differ from each other by an integer number of weeks. As a result, different t_{ri} impact the bunker cost on the leg and the number of ships to deploy on the ship route, and do not affect other components of the model. Hence, given $\sum_{w\in W} wz_{r,i+1}^w - \sum_{w\in W} wz_{ri}^w$, we can find the best t_{ri}. □

We use an example to demonstrate the proposition. Suppose that $z_{ri}^2 = 1$ (arrival on Tuesday), $z_{r,i+1}^3 = 1$ (arrival on Wednesday), $t_{ri}^{\text{port}} = 2$, $\alpha = 500$, $L_{ri} = 12,000$, $a_{ri} = 0.001$, $b_{ri} = 2$, $C_r^{\text{ship}} = 500,000$. Suppose further that the minimum value of t_{ri} determined by constraint (5.8) is 15. Then t_{ri} can take the value of 20, 27, 34, etc. Table 5.1 reports the bunker cost on the leg, the additional ship cost compared with the minimum possible $t_{ri} = 20$, and the total cost. Hence, the optimal value of t_{ri} is 34.

Table 5.1: Impact of different t_{ri} on the costs (million USD/week)

t_{ri}	20	27	34	41	48	55	62
Bunker cost	3.75	2.06	1.30	0.89	0.65	0.50	0.39
Ship cost	0.00	0.50	1.00	1.50	2.00	2.50	3.00
Total cost	3.75	2.56	2.30	2.39	2.65	3.00	3.39

Based on Proposition 5.3.1, we can linearize the objective function (5.6). We define binary variables y_{ri}^w to be 1 if and only if the difference of the arrival times at $< r, i+1 >$ and $< r, i >$ is w days, $w \in W$. We represent by t_{ri}^{w*} the optimal value of t_{ri} when the difference of the arrival times at $< r, i+1 >$ and $< r, i >$ is w days. We stress here that t_{ri}^{w*} can be computed a priori and is not a decision variable. We further define auxiliary binary variables $\bar{k}_{ri}$. The model [NSDPTW] can be transformed to the following integer linear programming (ILP) model:

[NSDPTW-ILP]

$$\min \sum_{r\in\mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r\in\mathcal{R}} \sum_{i\in I_r} L_{ri} a_{ri} (L_{ri}/24)^{b_{ri}} \sum_{w\in W} (t_{ri}^{w*})^{-b_{ri}} y_{ri}^w$$
$$+ C_{\bar{b}} \textstyle\sum_{r\in\mathcal{R}} \sum_{i\in I_r} z_{ri}^{\bar{b}} \tag{5.24}$$

with constraints

$$t_{ri} = \sum_{w\in W} t_{ri}^{w*} y_{ri}^w, r\in\mathcal{R}, i\in I_r \tag{5.25}$$

$$\sum_{w\in W} w y_{ri}^w = \sum_{w\in W} w z_{r,i+1}^w - \sum_{w\in W} w z_{ri}^w - t_{ri}^{\text{port}} + 7\bar{k}_{ri}, r\in\mathcal{R}, i\in I_r \tag{5.26}$$

$$\sum_{w\in W} y_{ri}^w = 1, r\in\mathcal{R}, i\in I_r \tag{5.27}$$

$$y_{ri}^w \in \{0,1\}, r\in\mathcal{R}, i\in I_r, w\in W \tag{5.28}$$

$$\bar{k}_{ri} \in \{0,1,2\}, r\in\mathcal{R}, i\in I_r \tag{5.29}$$

and constraints from (5.7) to (5.23).

Constraint (5.25) defines t_{ri}^{w*} which is the optimal value of t_{ri} when the difference of the arrival times at $< r, i+1 >$ and $< r, i >$ is w days. Constraint (5.26) defines the difference of the arrival times between ports. Constraint (5.27) requires that the difference of the arrival times is a fix number between 0 and 6 at each port of call. Constraint (5.28) defines y_{ri}^w as a binary variable. Constraint (5.29) defines the auxiliary variable $\bar{k}_{ri}$ as a nonnegative integer.

5.3.2 Iterative optimization approach

Model [NSDPTW-ILP] is an integer linear programming formulation. Small-scale instances can be solved by off-the-shelf solvers. To solve large-scale instances, we propose an iterative optimization approach below:

Algorithm 1: Iterative optimization approach

Step 0. (Initialization): We define vector ($m_r = m_r^*, r \in \mathcal{R}; t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R}, i \in I_r; z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W$) as the best solution obtained (for brevity, we use ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$) to represent the vector). Note that we do not need to record the values of the variables t_{ri}, z_{ri}^w, k_{ri}, z_{ri}^b, y_{ri}^w, or $\bar{k}_{ri}$, because they can be derived from ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$). Find a feasible ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$). Since there is a premium berth at each port, such a feasible schedule always exists. The total cost can be calculated and is represented by C^*.

Step 1. (Ship route schedule optimization): Define $C_0 = C^*$.

Step 1.0. Set $\bar{r} = 1$. Define $C_1 = C^*$.

Step 1.1. (Optimize the schedule for ship route $\bar{r}$) Fix the schedule of all ship routes $r \in \mathcal{R} \setminus \{\bar{r}\}$ and optimize schedule for ship route $\bar{r}$. That is, we solve model [NSDPTW-ILP] with the following constraints:

$$m_r = m_r^*, r \in \mathcal{R} \setminus \{\bar{r}\} \tag{5.30}$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r \tag{5.31}$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.32}$$

Update ($m_{\bar{r}}^*, t_{\bar{r}i}^{\text{arr}*}, z_{\bar{r}i}^{bw*}$) using the corresponding optimal solution obtained. The resulting total cost is denoted by $\hat{C}^*$. Note that $\hat{C}^* \leqslant C^*$. Set $C^* = \hat{C}^*$.

Step 1.2. If $\bar{r} < |R|$, set $\bar{r} = \bar{r} + 1$ and go to Step 1.1.

Step 1.3. If $C_1 > C^*$, go to Step 1.0.

Step 1.4. Go to Step 2.0.

Step 2. (Port arrival time optimization):

Step 2.0. Set $\bar{p} = 1$. Define $C_2 = C^*$.

Step 2.1. (Optimize the arrival times at port $\bar{p}$) Fix the schedule of all ports $p \in \mathcal{P} \backslash \{\bar{p}\}$ and optimize schedule at port $\bar{p}$. That is, we solve model [NSDPTW-ILP] with the following constraints:

$$m_r = m_r^*, r \in \mathcal{R} \tag{5.33}$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}} \tag{5.34}$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}}, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{5.35}$$

Update $(t_{ri}^{\text{arr}*}, z_{ri}^{bw*}, r \in \mathcal{R}_{\bar{p}}, i \in I_{r\bar{p}})$ using the corresponding optimal solution obtained. The resulting total cost is denoted by $\hat{C}^*$. Note that $\hat{C}^* \leqslant C^*$. Set $C^* = \hat{C}^*$.

Step 2.2. If $\bar{p} < |\mathcal{P}|$, set $\bar{p} = \bar{p} + 1$ and go to Step 2.1.

Step 2.3. If $C_2 > C^*$, go to Step 2.0.

Step 2.4. If $C_0 > C^*$, go to Step 1.

Step 2.5. Stop (It means we cannot improve the solution by optimizing the schedule for one route or optimizing the arrival times at one port). $\square$

5.4 Case Studies

5.4.1 A six-port case study

We conduct a case study based on the ocean carrier APL to evaluate the applicability of the proposed models and methods. The network has a total of 6 ports, as shown in Fig. 5.2. There are 3 types of ship and 3 ship routes, as shown in Table 5.2, which also shows the daily bunker consumption functions related to the sailing speed v (knot). The port time at each port of call (day) and distance on each leg (n mile) are shown in Table 5.3. We assume that these 6 ports have a total of 10 berths, whose available times are shown in Table 5.4 and the bunker price $\alpha = 400$ USD/ton.

Table 5.2: Ship fleet

Ship type (TEUs)	1200	2600	6500
Weekly cost (USD)	94829	159621	267393
Max speed (knot)	18.3	20.9	20.9
Bunker consumption (ton/day)	$0.000287v^2$	$0.000358v^2$	$0.000559v^2$

Table 5.3: Parameters of the three ship routes

Route	Port of call	Port	Port ID	Port time	Distance
1	1	Singapore	1	2	2895
	2	Karachi	2	2	509
	3	Nhava Sheva	3	1	891
	4	Colombo	4	1	1575
2	1	Singapore	1	1	767
	1	Surabaya	5	2	767
3	1	Singapore	1	1	668
	2	Semarang	6	1	668

5.4.1.1 Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We consider the example of the port of Singapore, which is visited three times a week on all the ship routes in the network. Both the operator of the port of

Table 5.4: Available time at each port

Port ID	Port	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	Singapore	1	free	free		free	free		
		2	free	free					
2	Karachi	1	free	free	free	free	free	free	free
		2			free				
3	Nhava Sheva	1		free		free	free		
		2	free	free		free	free		
4	Colombo	1	free	free			free	free	
5	Surabaya	1	free	free		free	free		
6	Semarang	1	free	free				free	free
		2			free				

Singapore and the liner shipping company that operates the network are interested in looking at the result if more available berth time is provided at Singapore. We hence examine 7 berth availability cases of Singapore, as shown in Table 5.5. The two berths at Singapore have more and more available days from case 1 to case 7.

Table 5.5: Different cases of available time at Singapore

Case	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	1					free		
	2	free	free			free		
2	1					free		free
	2	free	free			free		
3	1			free		free		free
	2	free	free			free		
4	1			free		free		free
	2	free	free			free	free	
5	1		free	free		free		free
	2	free	free			free	free	
6	1		free	free		free		free
	2	free	free	free		free	free	
7	1		free	free		free	free	free
	2	free	free	free		free	free	

The results of the 7 berth availability cases are shown in Fig. 5.3. It can be seen that more available days at Singapore leads to a lower total cost: from case 1 to case 7, the total cost is reduced by 194,969 USD/week due to the increase of the number of available days at berth Table 5.6. Fig. 5.3 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed.

In particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

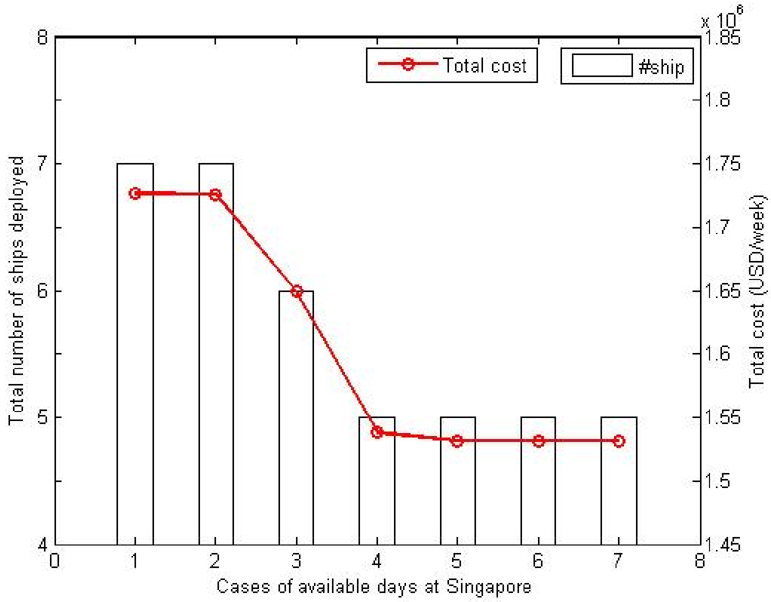


Figure 5.3: Impact of port time windows

Table 5.6: Impact of port time windows on each type of cost

Case	Ship cost	Bunker cost
1	1311079	415193
2	1311079	414905
3	1216250	433058
4	1056629	481543
5	1056629	474674
6	1056629	474674
7	1056629	474674

The optimal ship route schedules for the 7 berth availability cases are shown in Table 5.7, which reports the day of arrival at each port of call. We observe that the time windows at the port of Singapore affect the optimal arrival times at all the ports of call in the liner shipping network.

Table 5.7: Impact of port time window on the optimal schedule

Route	Port of call	Port	Cases 1	Case 2	Cases 3	Case 4	Case 5	Cases 6	Case 7
1	1	Singapore	0	0	0	4	1	1	1
	2	Karachi	8	8	8	12	11	11	11
	3	Nhava Sheva	11	11	11	15	15	15	15
	4	Colombo	15	15	15	19	18	18	18
2	1	Singapore	4	4	4	0	0	0	0
	5	Surabaya	10	10	10	3	3	3	3
3	1	Singapore	4	6	2	2	1	1	2
	6	Semarang	12	13	6	5	5	5	5

5.4.1.2 Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing t_i^{port}. To investigate the effect of port handling efficiency, we compare the default parameter settings with the situation of reducing the port time at Singapore on ship route 1 from two days to one day. We find that the optimal number of ships and the total cost increase with the time spent at Singapore, as shown in Fig. 5.4 and Fig. 5.5, in all the 7 cases of available days at Singapore. These results demonstrate that it is of significant importance for port operators to improve the container handling efficiency.

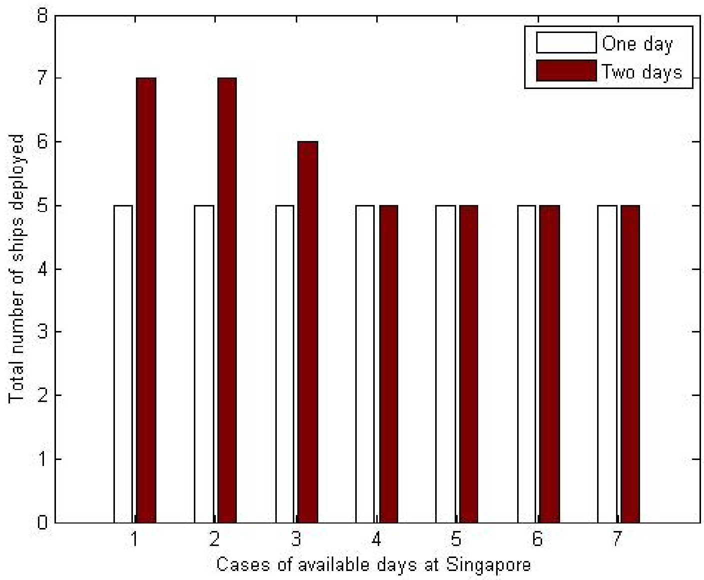


Figure 5.4: Impact of port time at Singapore on ship route 1 on the number of ships deployed

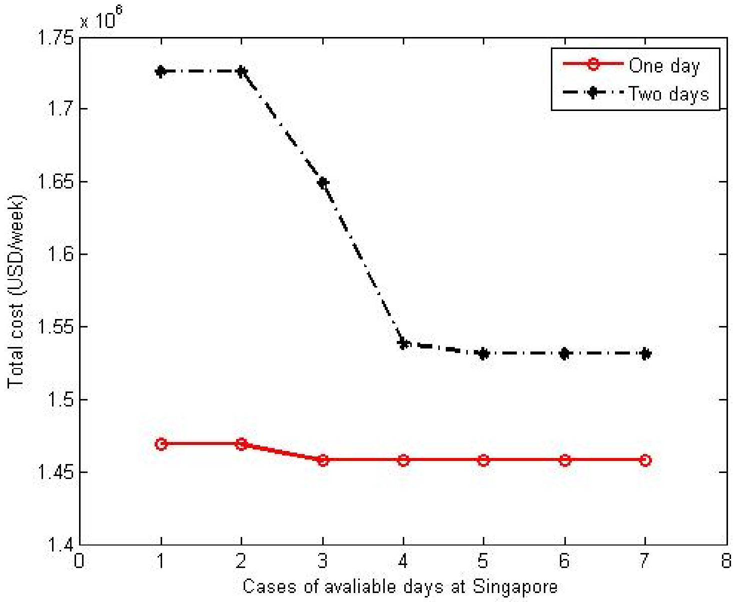


Figure 5.5: Impact of port time at Singapore on ship route 1 on the total cost

5.4.1.3 Result of bunker prices

In this section, we study the impact of the bunker price on the total cost and the number of ships deployed in the liner shipping network. We increase the bunker price from 300, 400, 500, 600, 700, to 800 and the other parameters are the same as Table 5.2, Table 5.3 and Table 5.4. The results are shown in Fig. 5.6 . We observe that a higher bunker price always leads to a higher total cost for liner shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price.

5.4.2 A 21-port case study

We examine another case study based on the ocean carrier APL to evaluate the applicability of the proposed models and methods. The network has a total of 21 ports, as shown in Fig. 5.7. There are 3 types of ship and 3 ship routes, as shown in Table 5.2, which also shows the daily bunker consumption functions related to

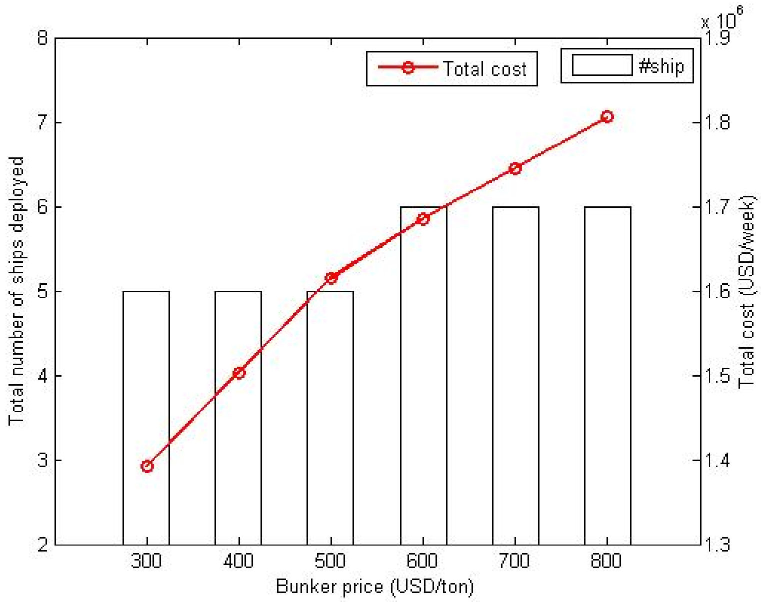


Figure 5.6: Result of bunker prices on the total cost and the number of ships deployed

the sailing speed v (knot). The port time at each port of call (day) and distance on each leg (n mile) are shown in Table 5.8. We assume that these 21 ports have a total of 44 berths, whose available times are shown in Table 5.9 and the bunker price $\alpha = 400$ USD/ton.

5.4.2.1 Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We consider the example of the port of Shanghai, which is visited one time a week on in this network. We hence examine 7 berth availability cases of Shanghai as shown in Table 5.10. The berths at Shanghai have more and more available days from case 1 to case 7.

The results of the 7 berth availability cases are shown in Fig. 5.8. It can be seen that more available days at Shanghai leads to a lower total cost: from case 1 to case 7, the total cost is reduced by 222,675 USD/week due to the increase of the number of available days at berth Table 5.11. Fig. 5.8 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. In

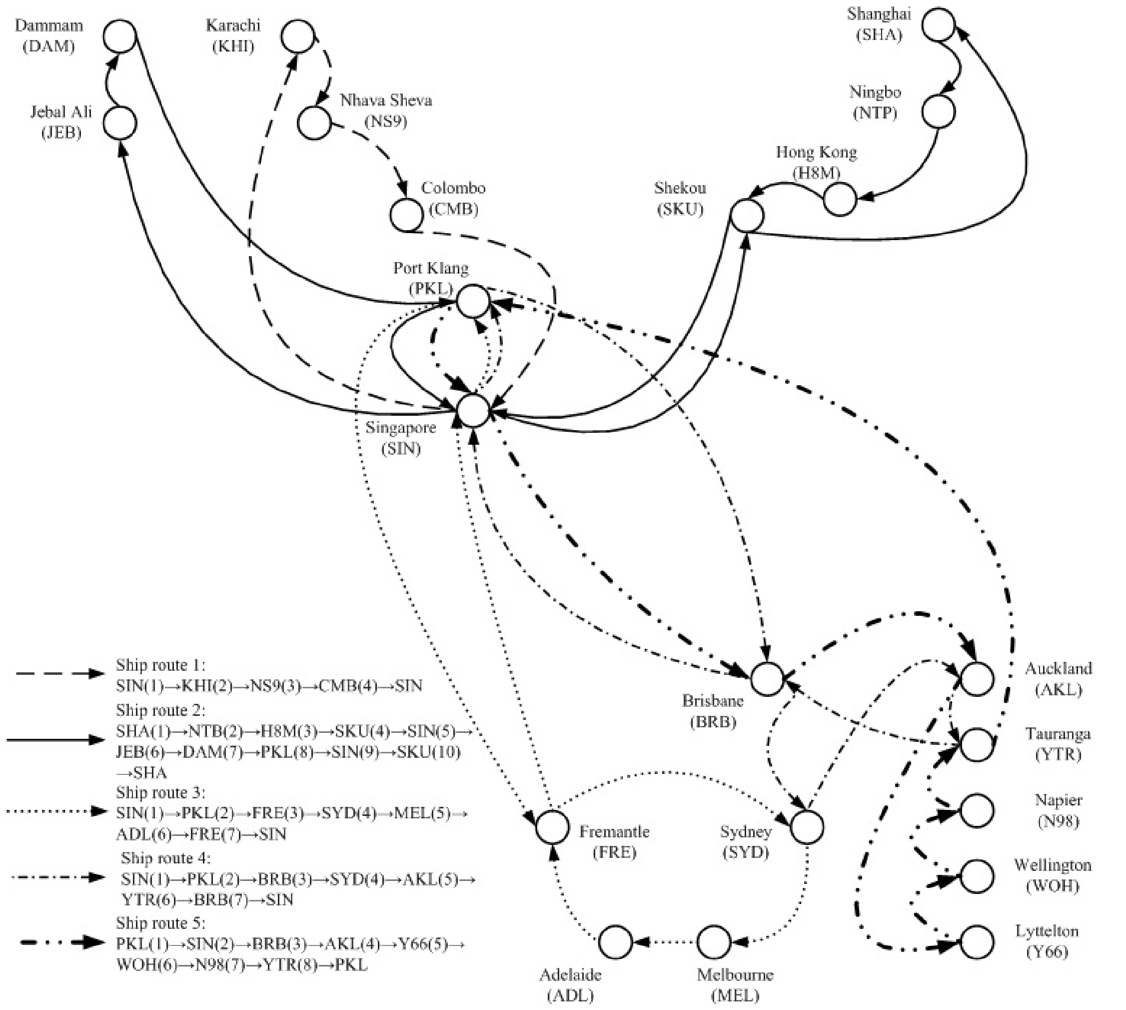


Figure 5.7: A liner shipping network with five ship routes

particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

5.4.2.2 Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing t_i^{port}. To investigate the effect of port handling efficiency, we compare the default parameter settings with the situation of reducing the port time at Shanghai on ship route 1 from two days to one day. We find that the optimal number of

Table 5.8: Parameters of the Five ship routes

Route	Port of call	Port	Port time	Distance
1	1	Singapore	2	2895
	2	Karachi	1	509
	3	Nhava Sheva	1	891
	4	Colombo	1	1575
2	5	Shanghai	2	101
	6	Ningbo	1	693
	7	Hong Kong	1	25
	8	Shekou	1	1422
	1	Singapore	1	3428
	9	Jebal Ali	2	280
	10	Dammam	2	2480
	11	Port Klang	2	193
	1	Singapore	1	1422
	8	Shekou	2	791
3	1	Singapore	1	193
	11	Port Klang	1	2359
	12	Fremantle	1	2115
	13	Sydney	1	547
	14	Melbourne	1	448
	15	Adelaide	1	1327
	12	Fremantle	1	2183
4	1	Singapore	2	193
	11	Port Klang	1	3830
	16	Brisbane	1	419
	13	Sydney	1	1253
	17	Auckland	1	107
	18	Tauranga	1	1351
	16	Brisbane	1	3642
5	11	Port Klang	1	193
	1	Singapore	2	3642
	16	Brisbane	1	1283
	17	Auckland	1	635
	19	Lyttelton	1	169
	20	Wellington	1	196
	21	Napier	1	237
	18	Tauranga	1	5110

ships and the total cost increase with the time spent at Shanghai, as shown in Fig. 5.9 and Fig. 5.10, in all the 7 cases of available days at Shanghai. These results demonstrate that it is of significant importance for port operators to improve the container handling efficiency.

Table 5.9: Available time at each port

Port ID	Port	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	Singapore	1	free	free		free	free		
		2	free	free					
		3	free	free	free	free	free	free	free
		4			free				
2	Karachi	1		free		free	free		
		2	free	free		free	free		
3	Nhava Sheva	1	free	free			free	free	
4	Colombo	1	free	free		free	free		
		2	free	free				free	free
5	Shanghai	1			free				
		2	free	free		free	free		
6	Ningbo	1	free	free					
		2	free	free	free	free	free	free	free
7	Hong Kong	1			free				
		2		free		free	free		
8	Shekou	1	free	free		free	free		
		2	free	free			free	free	
		3	free	free		free	free		
9	Jebal Ali	1	free	free				free	free
10	Dammam	1			free				
		2	free	free		free	free		
11	Port Klang	1	free	free					
		2	free	free	free	free	free	free	free
		3			free				
12	Fremantle	1		free		free	free		
		2	free	free		free	free		
		3	free	free			free	free	
13	Sydney	1	free	free		free	free		
		2	free	free				free	free
14	Melbourne	1			free				
		2	free	free		free	free		
15	Adelaide	1	free	free					
16	Brisbane	1	free	free	free	free	free	free	free
		2			free				
		3		free		free	free		
17	Auckland	1	free	free		free	free		
		2	free	free			free	free	
18	Tauranga	1	free	free		free	free		
		2	free	free				free	free
19	Lyttelton	1			free				
		2	free	free				free	free
20	Wellington	1			free				
		2	free	free		free	free		
21	Napier	1	free	free	free	free	free	free	free

Table 5.10: Different cases of available time at Shanghai

Case	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	1							free
	2						free	free
2	1	free						free
	2						free	free
3	1	free						free
	2					free	free	free
4	1	free			free	free		free
	2					free	free	free
5	1	free			free	free		free
	2		free	free		free	free	free
6	1	free	free		free	free		free
	2		free	free		free	free	free
7	1	free	free	free	free	free		free
	2		free	free		free	free	free

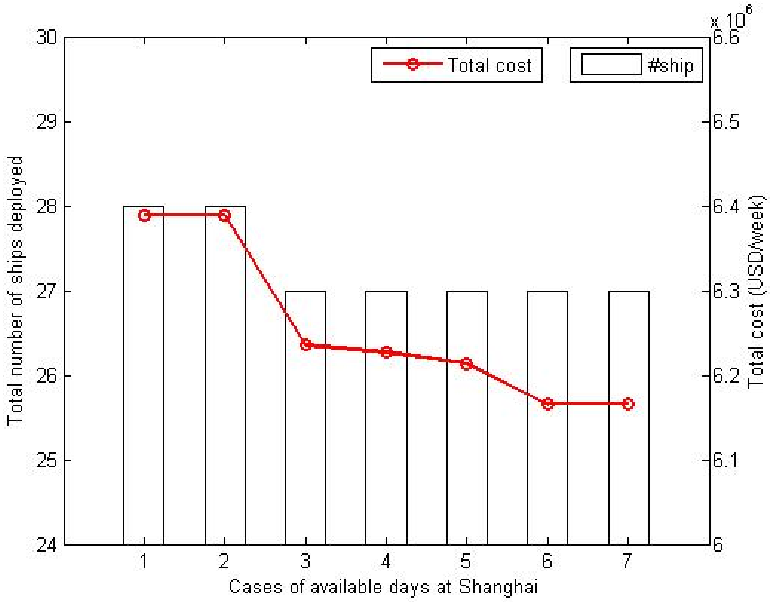


Figure 5.8: Impact of port time windows

5.4.2.3 Result of bunker prices

In this section, we study the impact of the bunker price on the total cost and the number of ships deployed in the liner shipping network. We increase the bunker price from 300, 400, 500, 600, 700, to 800. The results are shown in Fig. 5.11 . We observe that a higher bunker price always leads to a higher total cost for liner

Table 5.11: Impact of port time windows on each type of cost

Case	Ship cost	Bunker cost
1	5007604	1380569
2	5007604	1380569
3	4740211	1495628
4	4740211	1486908
5	4740211	1473271
6	4740211	1425287
7	4740211	1425287

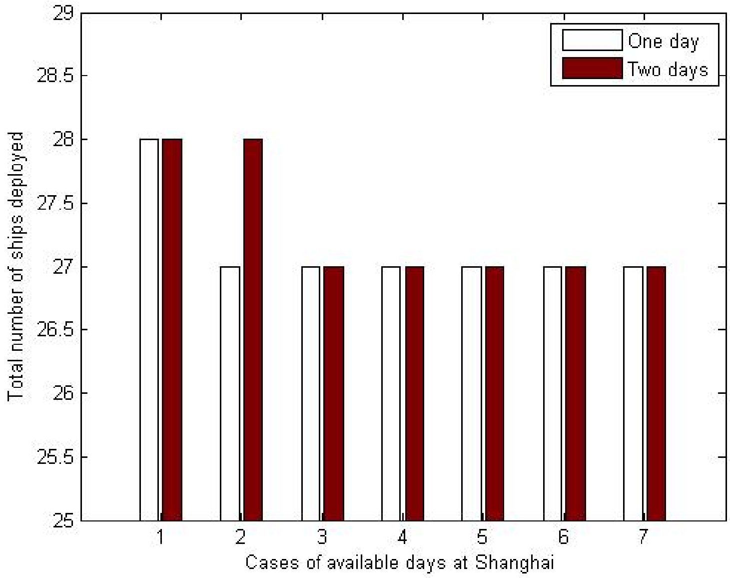


Figure 5.9: Impact of port time at Shanghai on ship route 2 on the number of ships deployed

shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price.

5.5 Conclusions

This chapter has studied the practical liner shipping network schedule design problem with port time windows. This is a significant tactical planning decision problem because it considers the availability of ports when planning liner shipping services.

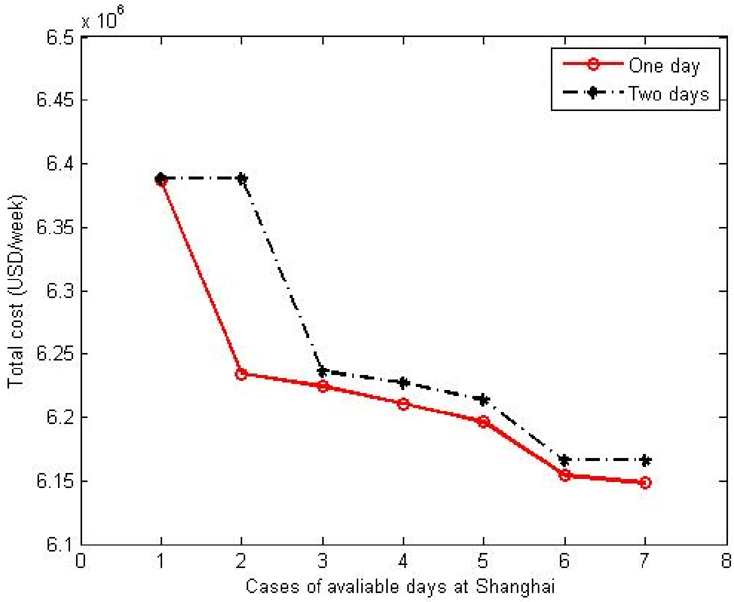


Figure 5.10: Impact of port time at Shanghai on ship route 2 on the total cost

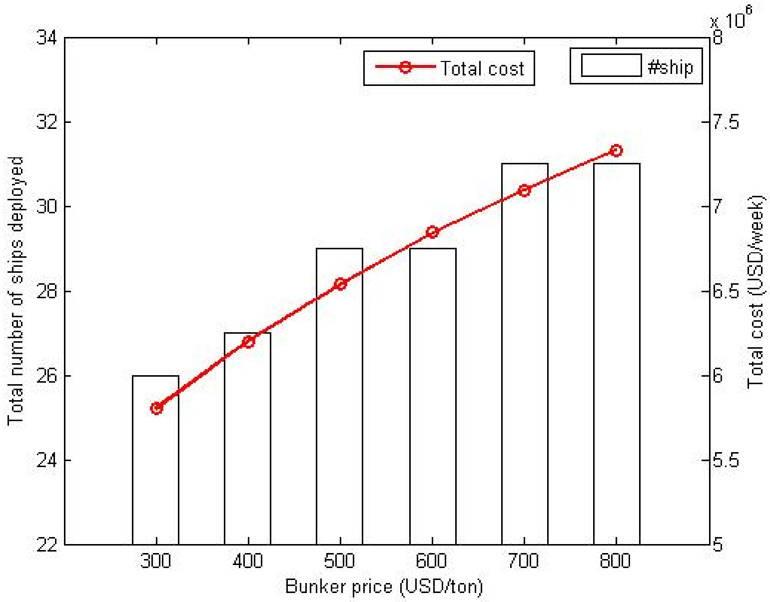


Figure 5.11: Result of bunker prices on the total cost and the number of ships deployed

As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we reformulated the problem as an integer linear optimization model and proposed an iterative optimization approach.

The proposed solution method was applied to two networks, consisting of six ports and 21 ports, operated by APL. The results demonstrate that the port time windows, port handling efficiency, and bunker price all affect the total cost, the optimal number of ships to deploy, and the optimal schedule. Higher availability at ports, shorter port time, and lower bunker price result in a lower total cost and a smaller number of ships to deploy. Therefore, port operators can apply the proposed method to quantify whether the benefits to liner shipping companies are worthwhile compared to the cost of expanding the ports' capacity and improving their efficiency. Liner shipping companies may need to charter in more ships if they predict that the future bunker price will increase.

Chapter 6

Schedule Design for Liner Shipping Networks Considering Inventory Cost

6.1 Introduction

In this chapter, we extend the previous schedule design for liner shipping networks with port time windows by considering the inventory cost. The main contribution of the chapter is to address a practical liner shipping network schedule design problem with port time windows considering inventory cost (NSDPTW2). The designed schedules are preferable as the sum of ship cost, fuel cost and inventory cost is minimized. The schedules are feasible because the availability of berths at each port on each day is explicitly considered in the model. Moreover, considering the inventory cost makes the schedule more reasonable.

The remainder of the chapter is organized as follows. Section 6.2 describes the problem and formulates a mathematical model. Section 6.4 proposes a solution approach to address the problem. Section 6.5 reports a case study based on a network consisting of four service routes. Section 6.6 presents the conclusion. To keep this chapter self-contained, we repeat some description from chapter 5.

6.2 Problem Description

Consider a liner container shipping company that operates a number of ship routes, denoted by the set $\mathcal{R}$, regularly serving a group of ports denoted by the set $\mathcal{P}$. The port rotation of a ship route $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \to p_{r2} \to \ldots \to p_{rN_r} \to p_{r1} \tag{6.1}$$

where N_r is the number of ports of call on the ship route and $p_{ri} \in \mathcal{P}$ is the physical port corresponding to ith port of call. Let I_r be the set of ports of call of ship route $r \in \mathcal{R}$, i.e., $I_r = \{1, 2, \cdots, N_r\}$. For brevity, we define $< r, i >$ as the port of call i on ship route r. Defining $p_{r,N_r+1} = p_{r1}$, the voyage from p_{ri} to $p_{r,i+1}$ is called *leg* i, $i \in I_r$. Fig. 5.2 shows a liner shipping network with three ship routes operated by APL (2014), where ship route 1 is SWX in Fig. 5.1, ship route 2 is Surabaya Feeder Service (SUR) and ship route 3 is Semarang Feeder Service (SEM):

$$\begin{aligned}
r = 1, N_r = 4: &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{KR}) \to p_{r3}(\text{NS}) \to p_{r4}(\text{CB}) \to p_{r1}(\text{SG}) \\
r = 2, N_r = 2: &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{SB}) \to p_{r1}(\text{SG}) \\
r = 3, N_r = 2: &\quad p_{r1}(\text{SG}) \to p_{r2}(\text{SR}) \to p_{r1}(\text{SG}) \to p_{r1}(\text{CB})
\end{aligned}$$

We further define $\mathcal{R}_p$ as the set of ship routes that visit port $p \in \mathcal{P}$, and define I_{rp} as the set of ports of call on ship route $r \in \mathcal{R}_p$ that correspond to port p. In the above example, $\mathcal{R}_{\text{SG}} = \{1, 2, 3\}$, $I_{1,\text{CB}} = \{4\}$, $I_{2,\text{SG}} = \{1\}$, and $I_{3,\text{CB}} = \varnothing$.

6.2.1 Weekly service and schedules

A string of homogeneous ships are deployed on each ship route $r \in \mathcal{R}$ to maintain a weekly service frequency. Let L_{ri} (n mile) be the voyage distance of the ith leg of route $r \in \mathcal{R}$, $t_{ri}^{\text{port}} \in \{1, 2\}$ be the fixed time (day) a ship spends at port of call i on ship route $r \in \mathcal{R}$ for container handling, m_r be the number of ships deployed on ship route $r \in \mathcal{R}$, and t_{ri} be the sailing time (day) on the ith leg of ship route $r \in \mathcal{R}$. We then have the relation:

$$\sum_{i \in I_r} \left(t_{ri} + t_{ri}^{\text{port}}\right) = 7m_r, \forall r \in \mathcal{R}$$

where 7 is the number of days in a week. Represent by C_r^{ship} the fixed cost (USD/week) associated with a ship on route $r \in \mathcal{R}$. C_r^{ship} includes the capital cost and the operating cost. The fixed cost associated with all the ships is:

$$\sum_{r \in \mathcal{R}} C_r^{\text{ship}} m_r$$

Define the beginning of a particular Sunday as day 0, and let t_{ri}^{arr} be the arrival time (day) at port of call i on ship route r. For instance, $t_{ri}^{\text{arr}} = 13$ means that ships arrive at $< r, i >$ at the beginning of next Saturday. The time components of a ship route have the relation

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, r \in \mathcal{R}, i \in I_r$$

We assume that the sailing time on a leg has a minimum required value, denoted by t_{ri}^{min}. Hence,

$$t_{ri} \geqslant t_{ri}^{\text{min}}, r \in \mathcal{R}, i \in I_r$$

Note that t_{ri}^{min} can be used to incorporate potential buffer time. For instance, if a leg is short, and the container handling time at port of call i is unreliable, then t_{ri}^{min} should be much larger than $\frac{L_{ri}}{24V_r^{\text{max}}}$ so as to hedge against potential delay by fast steaming, where V_r^{max} is the maximum speed (knots) of ships deployed on ship route r.

Because liner ship routes provide weekly services (Wang and Meng (2013), Wang and Meng (2014), Liu et al. (2014), Wang (2014), Wang et al. (2014a), Wang et al. (2015a) and Wang et al. (2015b)), to simplify the notation, we define W to be a set of all days in a week, that is,

$$W := \{0, 1, 2, 3, 4, 5, 6\}$$

where 0 represents Sunday, 1 represents Monday, etc. Without loss of generality, we require

$$t_{r1}^{\text{arr}} \in W, r \in \mathcal{R}$$

In fact, there is no difference between $t_{r1}^{\text{arr}} = 2$ and $t_{r1}^{\text{arr}} = 9$ due to the weekly frequency of liner services. Hence, the schedule of ship routes in the liner shipping network can be defined by the following vector

$$(t_{ri}^{\text{arr}}, r \in \mathcal{R}, i \in I_r; m_r, r \in \mathcal{R}) \tag{6.2}$$

The number of ships m_r is essential in the above vector, because without it the sailing time on leg N_r cannot be determined. The weekly service implies that

$$t_{r,N_r+1}^{\text{arr}} = t_{r1}^{\text{arr}} + 7m_r, r \in \mathcal{R}$$

Therefore, we can also define the schedule as

$$(t_{ri}^{\text{arr}}, r \in \mathcal{R}, i \in I_r \cup \{N_r + 1\}) \tag{6.3}$$

which is equivalent to the schedule defined by (6.2).

The arrival time t_{ri}^{arr} at a port of call corresponds to day $t_{ri}^{\text{arr}} \mod 7$ of a week. To be more specific, we let z_{ri}^{w} be a binary variable which equals 1 if and only if ships arrive on day $w \in W$ of a week at $< r, i >$. Mathematically

$$t_{ri}^{\text{arr}} \mod 7 = w' \Longleftrightarrow \begin{cases} z_{ri}^{w'} = 1 \\ z_{ri}^{w} = 0, w \in W \backslash \{w'\} \end{cases}$$

6.2.2 Port time windows

A port may have more than one berth. Hence, we let B_p be the set of berths at the port $p \in \mathcal{P}$. A berth may not always be available in a week because a port needs to serve more than one liner shipping company. Therefore, we define the parameter δ_b^w which equals 1 if berth $b \in \cup_{p \in \mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise.

A ship uses exactly one berth when it visits a port of call and a berth cannot serve more than one ship at the same time. To formulate this constraint, let z_{ri}^{bw} be a binary variable which equals 1 if and only if ships use berth $b \in B_{p_{ri}}$ when visiting $< r, i >$ on day w. For notational convenience, we define $z_{ri}^{b,-1} := z_{ri}^{b,6}$. We have

$$\sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=1} z_{ri}^{bw} + \sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=2} (z_{ri}^{b,w-1} + z_{ri}^{b,w}) \leq \delta_b^w, p \in \mathcal{P}, b \in B_p, w \in W \quad (6.4)$$

The first term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{\text{port}} = 1$ and the arrival day is w. The second term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{\text{port}} = 2$ and the arrival day is $w - 1$ or w. The overall constraint indicates that an available berth cannot serve more than one ship on the same day.

We assume further that each port has a premium berth $\bar{b}$, which is always available and can accommodate any number of ships. However, the liner shipping company needs to pay a high penalty cost $C_{\bar{b}}$ each time the berth is used.

6.2.3 Container inventory cost

A higher sailing speed implies a shorter transit time of containers, which leads to a lower inventory cost. Therefore we let $\bar{\beta}$ be the unit inventory cost of containers (USD per TEU per day), and $\bar{V}_{ri}$ be the volume of containers (TEUs) transported on leg i of ship route r. Since the inventory cost of containers at ports is constant (excluding transshipment containers), we are concerned about the inventory cost at sea, the sum of which can be calculated as:

$$\sum_{r\in\mathcal{R}}\sum_{i\in I_r}\bar{\beta}\bar{V}_{ri}t_{ri}$$

6.2.4 Container transshipment

Container transshipment operations at a particular port can occur only when this port is visited by ships at least twice a week. Given the set of ship routes $\mathcal{R}$, all the possible container transshipment operations can be represented by the following set:

$$\begin{aligned}\mathcal{Q} := \{< r, s, i, j > | \quad & r \in \mathcal{R}, s \in \mathcal{R}, i \in I_r, j \in I_s | p_{ri} = p_{sj}, \\ & \text{and at least one of the two inequalities is true: } r \neq s, i \neq j\}\end{aligned}$$

A quadruplet $< r, s, i, j > \in \mathcal{Q}$ represents a container transshipment operation from one ship on ship route r to another ship on ship route s at their common calling port $p_{ri} = p_{sj}$. The set $\mathcal{Q}$ can be easily identified.

For example, the set of transshipment quadruplets for the network shown in Fig. 5.2 is:

$$\mathcal{Q} := \left\{ \begin{array}{c} < 1,2,1,1 >, < 1,3,1,1 >, \\ < 2,3,1,1 >, < 3,2,1,1 >, \\ < 2,1,1,1 >, < 3,1,1,1 > \end{array} \right\}$$

where all of transshipments are at Singapore.

Represent by $\bar{V}_{rsij}$ the volume of containers transshipped from $< r, i >$ to $< s, j >$. We assume that $\bar{V}_{rsij}$ is determined by a separate container routing model and hence is fixed for the schedule design problem. The inventory cost of the containers with volume $\bar{V}_{rsij}$ at the transshipment port p_{ri} is related to the *difference* of the arrival day of a week at $< r, i >$ and the arrival day at $< s, j >$ (the latter minus the former). Let $x^w_{rsij} \in \{0, 1\}$ equal 1 if and only if the difference of the arrival days of a week at $< r, i >$ and $< s, j >$ is w, $w \in W$. For instance, if the former is Wednesday (day 3 or equivalently $z^3_{ri} = 1$) and the latter is Thursday (day 4 or equivalently $z^4_{sj} = 1$), $x^1_{rsij} = 1$. If the former is Thursday and the latter is Wednesday, then their difference is $3 - 4 + 7 = 6$ (because of weekly services) and $x^6_{rsij} = 1$.

At a port $p \in \mathcal{P}$, the minimum connection time is denoted by $\bar{t}^{\min}_p$ days. If the arrival day of ship A at $< r, i >$ is at least $\bar{t}^{\min}_p$ days earlier than the departure day of ship B at $< s, j >$, $< r, s, i, j > \in \mathcal{Q}$, then the containers can be transshipped from ship A to ship B. Otherwise, the containers have to wait for the ship (ship C) that arrives at $< s, j >$ one week later than ship B. We define $\bar{t}^w_{rsij}$ as the number of days the transshipment containers from $< r, i >$ to $< s, j >$ spend at the port if the arrival time difference is w days. $\bar{t}^w_{rsij}$ is the time interval from the arrival of ship A to the departure of ship B (or ship C). Therefore, the total inventory cost associated with container transshipment is

$$\bar{\beta} \sum_{<r,s,i,j> \in \mathcal{Q}} \bar{V}_{rsij} \sum_{w \in W} \bar{t}^w_{rsij} x^w_{rsij}$$

Note that $\bar{t}^w_{rsij}$ is a known parameter. For instance, if $w = 6$ (e.g., $z^4_{ri} = 1, z^3_{sj} = 1$), $t^{\text{port}}_{ri} = 1$, $t^{\text{port}}_{sj} = 1$, then we have $\bar{t}^1_{rsij} = 7$, which is from the arrival at the beginning Thursday to the departure at the end of Wednesday. If $w = 1$ (e.g., $z^3_{ri} = 1, z^4_{sj} = 1$), $t^{\text{port}}_{ri} = 1$, $t^{\text{port}}_{sj} = 1$, we have $\bar{t}^1_{rsij} = 2$, which is from the arrival at the beginning Wednesday to the departure at the end of Thursday. $\bar{t}^w_{rsij}$ depends on w, $\bar{t}^{\min}_p$ and t^{port}_{sj}, as summarized in Table 6.1.

Table 6.1: Connection time at a transshipment port

$\bar{t}_p^{\min}$	t_{sj}^{port}	$w=0$	$w=1$	$w=2$	$w=3$	$w=4$	$w=5$	$w=6$
1	1	1	2	3	4	5	6	7
1	2	2	3	4	5	6	7	1
2	1	8	2	3	4	5	6	7
2	2	2	3	4	5	6	7	8

6.2.5 Bunker consumption

Represent by $g_{ri}(v)$ the bunker consumption (tons/n mile) function with regard to the speed v on leg i of ship route r. Based on the results in existing studies, we assume that $g_{ri}(v)$ is a power function of the form:

$$g_{ri}(v) = a_{ri}v^{b_{ri}}, a_{ri} > 0, b_{ri} > 2 \tag{6.5}$$

Hence, we let $Q_{ri}(t_{ri})$ be the bunker consumption on leg i of ship route r, and it can calculated by:

$$Q_{ri}(t_{ri}) = L_{ri}g_{ri}(L_{ri}/t_{ri})$$

6.3 Mathematical Model

Before presenting the model, we list the notation below.

Variables

m_r	Number of ships deployed on the ship route $r \in \mathcal{R}$
t_{ri}^{arr}	Arrival time (day) at port of call i on ship route r
t_{r,N_r+1}^{arr}	The time (day) when the ship returns to the 1st port of call on ship route r
t_{ri}	Sailing time (day) on the ith leg of ship route $r \in \mathcal{R}$
k_{ri}	An integer that is associated with the arrival time at the port of call i on ship route r, $r \in \mathcal{R}, i \in I_r$

k_{rsij} An integer that is associated with the arrival time at the port of call i on ship route r and at the port of call j on ship route s, r and $s \in \mathcal{R}, i \in I_r$ and $j \in I_s$

x_{rsij}^{w} A binary variable which equals 1 if and only if the difference of the arrival days of a week at $< r, i >$ and $< s, j >$ is w, $w \in W$

z_{ri}^{b} A binary variable which equals 1 if and only if ships use berth b when visiting the port of call i on ship route r including the premium berth $\bar{b}$

$z_{ri}^{\bar{b}}$ A binary variable which equals 1 if and only if ships use a premium berth $\bar{b}$ when visiting the port of call i on ship route r

z_{ri}^{w} A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the port of call i on ship route r

z_{ri}^{bw} A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the berth b at the port of call i on ship route r

Parameters

α The bunker fuel price (USD/ton)

$\bar{\beta}$ The unit inventory cost of containers (USD per TEU per day)

δ_{b}^{w} A parameter that equals 1 if berth $b \in \cup_{p\in\mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise

a_{ri} A coefficient calibrated from operating data and satisfy $a_{ri} > 0$

b_{ri} A coefficient calibrated from operating data and satisfy $b_{ri} > 1$

B_p The set of berths at the port $p \in \mathcal{P}$

C_r^{ship} The weekly operating cost of a ship deployed on ship route r

$C_{\bar{b}}$ The penalty cost of using a premium berth $\bar{b}$

I_r The set of ports of call on ship route r

L_{ri} Oceanic distance (n mile) of the ith leg of route r

N_r Number of ports on the ship route r

p_{ri}	The physical port that corresponds to the ith port of call on the ship route r
t_{ri}^{port}	Time (day) a ship spends at port of call i on the ship route r
$t_{ri}^{\min}$	A minimum required value of the sailing time on the ith leg of route r
$\bar{V}_{ri}$	Number of containers (TEUs) on leg i on the ship route r
$\bar{V}_{rsij}$	The volume of containers transshipped from $< r, i >$ to $< s, j >$
$V_r^{\max}$	Maximum speed of the ships on the ship route r
$m_r^{\max}$	Maximum number of ships deployed on the ship route r
$\mathbb{Z}^+$	Set of nonnegative integers

The NSDPTW2 can be formulated as an optimization model below. The objective function is:

[NSDPTW2]

$$\begin{aligned} \min \quad & \sum_{r\in\mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r\in\mathcal{R}} \sum_{i\in I_r} L_{ri} a_{ri} (L_{ri}/(24t_{ri}))^{b_{ri}} + \bar{\beta} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} \bar{V}_{ri} t_{ri} \\ & + \bar{\beta} \sum_{<r,s,i,j>\in\mathcal{Q}} \bar{V}_{rsij} \sum_{w\in W} \bar{t}_{rsij}^{w} x_{rsij}^{w} + C_{\bar{b}} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} z_{ri}^{\bar{b}} \end{aligned} \tag{6.6}$$

which aims to minimize the sum of ship cost, inventory cost and fuel cost while penalizing the violation of berth time windows.

The NSDPTW2 is subject to a number of constraints. The first set is the basic logical constraints in the schedule:

$$0 \leqslant t_{r1}^{\text{arr}} \leqslant 6, r \in \mathcal{R} \tag{6.7}$$

$$t_{ri} \geqslant \left\{ t_{ri}^{\min}, \left\lceil \frac{L_{ri}}{24V_r^{\max}} \right\rceil \right\}, r \in \mathcal{R}, i \in I_r \tag{6.8}$$

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, r \in \mathcal{R}, i \in I_r \tag{6.9}$$

$$t^{\text{arr}}_{r,N_r+1} = t^{\text{arr}}_{r1} + 7m_r, r \in \mathcal{R} \tag{6.10}$$

$$m_r \in \{1, 2, 3, \cdots, m^{\max}_r\}, r \in \mathcal{R} \tag{6.11}$$

$$t^{\text{arr}}_{ri} \in \mathbb{Z}^+, r \in \mathcal{R}, i \in I_r \tag{6.12}$$

The objective function (6.6) minimizes the sum of ship cost, bunker cost and penalty cost. Constraint (6.7) eliminates symmetric solutions. Constraint (6.8) confirms that the sailing time on a leg is not less than a minimum required value and ships cannot sail at a speed that exceeds $V^{\max}_r$. Constraint (6.9) defines the relation of different time components in a round-trip journey. Constraint (6.10) defines the time when the ship returns to the 1st port of call after one round-trip. Constraint (6.11) indicates that the number of ships is a positive integer that does not exceed a pre-specified maximum value. Constraint (6.12) indicates that the arrival time at each port of call is a nonnegative integer.

The second set of constraints formulates the day of a week for arrival at each port of call in the network:

$$\sum_{w \in W} w z^w_{ri} = t^{\text{arr}}_{ri} - 7k_{ri}, r \in \mathcal{R}, i \in I_r \tag{6.13}$$

$$\sum_{w \in W} z^w_{ri} = 1, r \in \mathcal{R}, i \in I_r \tag{6.14}$$

$$z^w_{ri} \in \{0, 1\}, r \in \mathcal{R}, i \in I_r, w \in W \tag{6.15}$$

$$k_{ri} \in \{0, 1, 2, \cdots, m_r - 1\}, r \in \mathcal{R}, i \in I_r \tag{6.16}$$

Constraint (6.13) defines the arrival day of a week at each port call on the route. Constraint (6.14) requires that a ship arrives exactly once a week at each port of call. Constraint (6.15) defines z_{ri}^w as a binary variable. Constraint (6.16) defines the auxiliary variable k_{ri} as a nonnegative integer.

The third set of constraints formulates x_{rsij}^w:

$$\sum_{w \in W} w z_{ri}^w - \sum_{w \in W} w z_{sj}^w + 7k_{rsij} = \sum_{w \in W} w x_{rsij}^w, < r, s, i, j > \in \mathcal{Q} \tag{6.17}$$

$$\sum_{w \in W} x_{rsij}^w = 1, < r, s, i, j > \in \mathcal{Q} \tag{6.18}$$

$$x_{rsij}^w \in \{0, 1\}, < r, s, i, j > \in \mathcal{Q}, w \in W \tag{6.19}$$

$$k_{rsij} \in \{0, 1\}, < r, s, i, j > \in \mathcal{Q} \tag{6.20}$$

Constraint (6.17) defines the different of the arrival days of a week at each port call between two routes. Constraint (6.18) requires that a transshipment happens exactly once a week at each port of call. Constraint (6.19) defines x_{rsij}^w as a binary variable. Constraint (6.20) defines the auxiliary variable k_{rsij} as a nonnegative integer.

The fourth set of constraints considers the availability of berths:

$$\sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=1} z_{ri}^{bw} + \sum_{r \in \mathcal{R}_p} \sum_{i \in I_{rp}, t_{ri}^{\text{port}}=2} (z_{ri}^{b,w-1} + z_{ri}^{bw}) \leqslant \delta_b^w, p \in \mathcal{P}, b \in B_p, w \in W \tag{6.21}$$

$$\sum_{b \in B_{p_{ri}} \cup \{\bar{b}\}} z_{ri}^b = 1, r \in \mathcal{R}, i \in I_r \tag{6.22}$$

$$z_{ri}^{bw} \leqslant z_{ri}^b, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.23}$$

$$z_{ri}^{bw} \leqslant z_{ri}^{w}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.24}$$

$$z_{ri}^{bw} \geqslant z_{ri}^{b} + z_{ri}^{w} - 1, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.25}$$

$$z_{ri}^{bw} \in \{0,1\}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.26}$$

$$z_{ri}^{b} \in \{0,1\}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\} \tag{6.27}$$

Constraint (6.21) indicates that an available berth cannot serve more than one ship on the same day. Constraint (6.22) requires that a ship uses exactly one berth each time it visits a port including the premium berth $\bar{b}$. Constraints (6.23) , (6.24) and (6.25) impose that a ship uses a berth at a port of call once a week. Constraints (6.26) and (6.27) define z_{ri}^{bw} and z_{ri}^{b} as binary variables.

6.4 Solution Method

The model [NSDPTW2] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has a large number of discrete variables; and (ii) it has nonlinear objective function. We use the solution method developed in the previous chapter with some modification to address the problem.

6.4.1 Linearization of the objective function

Similar to proposition (5.3.1) nonlinear term $t_{ri}^{-b_{ri}}$ in the objective function (6.6) can be linearized due to the following property.

Proposition 6.4.1 The optimal t_{ri} can be determined by the optimal z_{ri}^{w} and $z_{r,i+1}^{w}$, $w \in W$.

Based on Proposition 6.4.1, we can linearize the objective function (6.6). We define binary variables y_{ri}^w to be 1 if and only if the difference of the arrival times at $< r, i+1 >$ and $< r, i >$ is w days, $w \in W$. We represent by t_{ri}^{w*} the optimal value of t_{ri} when the difference of the arrival times at $< r, i+1 >$ and $< r, i >$ is w days. We stress here that t_{ri}^{w*} can be computed a priori and is not a decision variable. We further define auxiliary binary variables $\bar{k}_{ri}$. The model [NSDPTW2] can be transformed to the following integer linear programming (ILP) model:

[NSDPTW2-ILP]

$$\begin{aligned} \min \quad & \sum_{r\in\mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r\in\mathcal{R}} \sum_{i\in I_r} L_{ri} a_{ri} (L_{ri}/24)^{b_{ri}} \sum_{w\in W} (t_{ri}^{w*})^{-b_{ri}} y_{ri}^w + \bar{\beta} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} \bar{V}_{ri} t_{ri} \\ & + \bar{\beta} \sum_{<r,s,i,j>\in\mathcal{Q}} \bar{V}_{rsij} \sum_{w\in W} \bar{t}_{rsij}^w x_{rsij}^w + C_{\bar{b}} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} z_{ri}^{\bar{b}} \end{aligned} \tag{6.28}$$

with constraints

$$t_{ri} = \sum_{w\in W} t_{ri}^{w*} y_{ri}^w, r \in \mathcal{R}, i \in I_r \tag{6.29}$$

$$\sum_{w\in W} w y_{ri}^w = \sum_{w\in W} w z_{r,i+1}^w - \sum_{w\in W} w z_{ri}^w - t_{ri}^{\text{port}} + 7\bar{k}_{ri}, r \in \mathcal{R}, i \in I_r \tag{6.30}$$

$$\sum_{w\in W} y_{ri}^w = 1, r \in \mathcal{R}, i \in I_r \tag{6.31}$$

$$y_{ri}^w \in \{0, 1\}, r \in \mathcal{R}, i \in I_r, w \in W \tag{6.32}$$

$$\bar{k}_{ri} \in \{0, 1, 2\}, r \in \mathcal{R}, i \in I_r \tag{6.33}$$

and constraints from (6.7) to (6.27).

6.4.2 Iterative optimization approach

Model [NSDPTW2-ILP] is an integer linear programming formulation. Small-scale instances can be solved by off-the-shelf solvers. To solve large-scale instances, we use an iterative optimization approach similar to the one in chapter 5:

Algorithm 2: Iterative optimization approach

Step 0. (Initialization): We define vector ($m_r = m_r^*, r \in \mathcal{R}; t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R}, i \in I_r; z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W$) as the best solution obtained (for brevity, we use ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$) to represent the vector). Note that we do not need to record the values of the variables t_{ri}, z_{ri}^{w}, k_{ri}, z_{ri}^{b}, y_{ri}^{w}, or $\bar{k}_{ri}$, because they can be derived from ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$). Find a feasible ($m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*}$). Since there is a premium berth at each port, such a feasible schedule always exists. The total cost can be calculated and is represented by C^*.

Step 1. (Ship route schedule optimization): Define $C_0 = C^*$.

Step 1.0. Set $\bar{r} = 1$. Define $C_1 = C^*$.

Step 1.1. (Optimize the schedule for ship route $\bar{r}$) Fix the schedule of all ship routes $r \in \mathcal{R} \setminus \{\bar{r}\}$ and optimize schedule for ship route $\bar{r}$. That is, we solve model [NSDPTW2-ILP] with the following constraints:

$$m_r = m_r^*, r \in \mathcal{R} \setminus \{\bar{r}\} \tag{6.34}$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r \tag{6.35}$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.36}$$

Update ($m_{\bar{r}}^*, t_{\bar{r}i}^{\text{arr}*}, z_{\bar{r}i}^{bw*}$) using the corresponding optimal solution obtained. The resulting total cost is denoted by $\hat{C}^*$. Note that $\hat{C}^* \leqslant C^*$. Set $C^* = \hat{C}^*$.

Step 1.2. If $\bar{r} < |R|$, set $\bar{r} = \bar{r} + 1$ and go to Step 1.1.

Step 1.3. If $C_1 > C^*$, go to Step 1.0.

Step 1.4. Go to Step 2.0.

Step 2. (Port arrival time optimization):

Step 2.0. Set $\bar{p} = 1$. Define $C_2 = C^*$.

Step 2.1. (Optimize the arrival times at port $\bar{p}$) Fix the schedule of at all ports $p \in \mathcal{P} \backslash \{\bar{p}\}$ and optimize schedule at port $\bar{p}$. That is, we solve model [NSDPTW2-ILP] with the following constraints:

$$m_r = m_r^*, r \in \mathcal{R} \tag{6.37}$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}} \tag{6.38}$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}}, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \tag{6.39}$$

Update $(t_{ri}^{\text{arr}*}, z_{ri}^{bw*}, r \in \mathcal{R}_{\bar{p}}, i \in I_{r\bar{p}})$ using the corresponding optimal solution obtained. The resulting total cost is denoted by $\hat{C}^*$. Note that $\hat{C}^* \leqslant C^*$. Set $C^* = \hat{C}^*$.

Step 2.2. If $\bar{p} < |\mathcal{P}|$, set $\bar{p} = \bar{p} + 1$ and go to Step 2.1.

Step 2.3. If $C_2 > C^*$, go to Step 2.0.

Step 2.4. If $C_0 > C^*$, go to Step 1.

Step 2.5. Stop. □

6.5 Case Study

We conduct a case study based on the ocean carrier APL to evaluate the applicability of the proposed models and methods. The network has a total of 18 ports, as shown in Fig. 6.1. There are 3 types of ship and 4 ship routes, as shown in Table 6.2, which also shows the daily bunker consumption functions related to the sailing speed v (knot). The port time at each port of call (day) and distance on each leg (n mile) are shown in Table 6.3. We assume that these 18 ports have a total of 39 berths, whose available times are shown in Table 6.4, the bunker price $\alpha = 400$ USD/ton and the unit inventory cost $\beta = 1$ USD per TEU per hour.

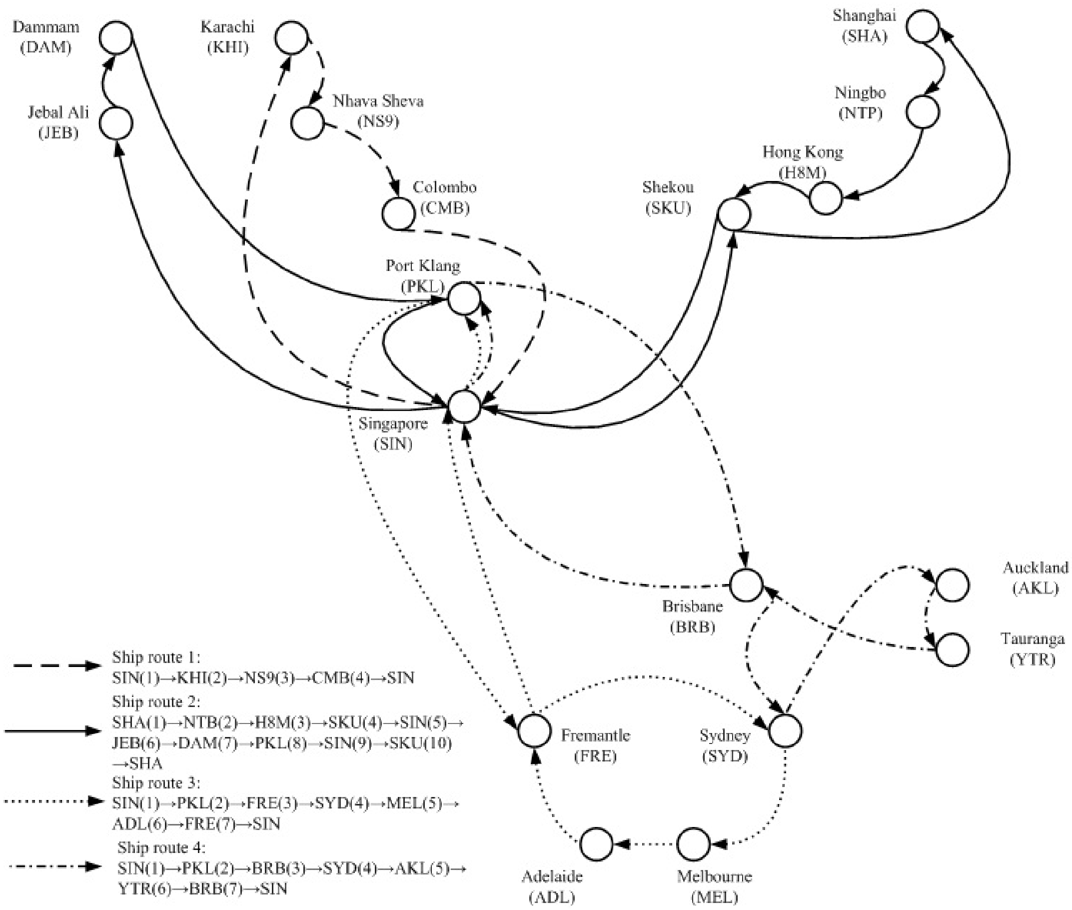


Figure 6.1: A liner shipping network with eighteen ports

Table 6.2: Ship fleet

Ship type (TEUs)	1200	2600	6500
Weekly cost (USD)	94829	159621	267393
Max speed (knot)	18.3	20.9	20.9
Bunker consumption (ton/day)	$0.000287v^2$	$0.000358v^2$	$0.000559v^2$

Table 6.3: Parameters of the four ship routes

Route	Port of call	Port	Port time	Distance
1	1	Singapore	2	2895
	2	Karachi	1	509
	3	Nhava Sheva	1	891
	4	Colombo	1	1575
2	5	Shanghai	2	101
	6	Ningbo	1	693
	7	Hong Kong	1	25
	8	Shekou	1	1422
	1	Singapore	1	3428
	9	Jebal Ali	2	280
	10	Dammam	2	2480
	11	Port Klang	2	193
	1	Singapore	1	1422
	8	Shekou	2	791
3	1	Singapore	1	193
	11	Port Klang	1	2359
	12	Fremantle	1	2115
	13	Sydney	1	547
	14	Melbourne	1	448
	15	Adelaide	1	1327
	12	Fremantle	1	2183
4	1	Singapore	2	193
	11	Port Klang	1	3830
	16	Brisbane	1	419
	13	Sydney	1	1253
	17	Auckland	1	107
	18	Tauranga	1	1351
	16	Brisbane	1	3642

6.5.1 Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We consider the example of the port of Singapore, which is visited five times a week on all the ship routes in the network. Both the operator of the port of Singapore and the liner shipping company that operates the network are interested

Table 6.4: Available time at each port

Port ID	Port	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	Singapore	1	free	free		free	free		
		2	free	free					
		3	free	free	free	free	free	free	free
		4			free				
2	Karachi	1		free		free	free		
		2	free	free		free	free		
3	Nhava Sheva	1	free	free			free	free	
4	Colombo	1	free	free		free	free		
		2	free	free				free	free
5	Shanghai	1			free				
		2	free	free		free	free		
6	Ningbo	1	free	free					
		2	free	free	free	free	free	free	free
7	Hong Kong	1			free				
		2		free		free	free		
8	Shekou	1	free	free		free	free		
		2	free	free			free	free	
		3	free	free		free	free		
9	Jebal Ali	1	free	free				free	free
10	Dammam	1			free				
		2	free	free		free	free		
11	Port Klang	1	free	free					
		2	free	free	free	free	free	free	free
		3			free				
12	Fremantle	1		free		free	free		
		2	free	free		free	free		
		3	free	free			free	free	
13	Sydney	1	free	free		free	free		
		2	free	free				free	free
14	Melbourne	1			free				
		2	free	free		free	free		
15	Adelaide	1	free	free					
16	Brisbane	1	free	free	free	free	free	free	free
		2			free				
		3		free		free	free		
17	Auckland	1	free	free		free	free		
		2	free	free			free	free	
18	Tauranga	1	free	free		free	free		
		2	free	free				free	free

in looking at the result if more available berth time is provided at Singapore. We hence examine 7 berth availability cases of Singapore, as shown in Table 6.5. The four berths at Singapore have more and more available days from case 1 to case 7.

Table 6.5: Different cases of available time at Singapore

Case	Berth	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	1				free	free		
	2		free		free	free		
	3				free	free		
	4	free						
2	1				free	free		free
	2		free		free	free		
	3				free	free		
	4	free						
3	1				free	free		free
	2		free		free	free	free	
	3				free	free		
	4	free						
4	1				free	free		free
	2		free		free	free	free	free
	3				free	free		
	4	free						
5	1				free	free		free
	2		free		free	free	free	free
	3		free	free	free	free		
	4	free						
6	1				free	free		free
	2		free		free	free	free	free
	3		free	free	free	free		
	4	free	free					
7	1	free			free	free		free
	2		free		free	free	free	free
	3		free	free	free	free		
	4	free	free					

The results of the 7 berth availability cases are shown in Fig. 6.2. It can be seen that more available days at Singapore leads to a lower total cost: from case 1 to case 7, the total cost is reduced by 1,367,497 USD/week due to the increase of the number of available days at berth Table 6.6. Fig. 6.2 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. In particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

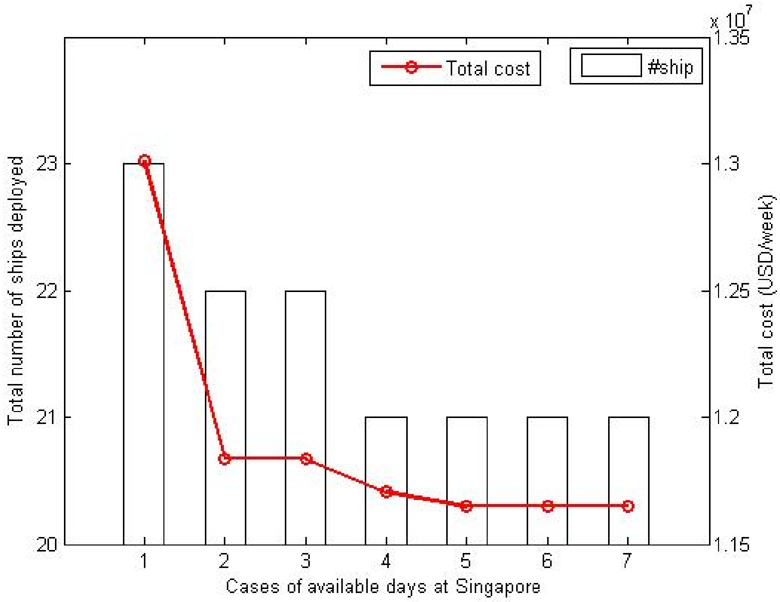


Figure 6.2: Impact of port time windows

Table 6.6: Impact of port time windows on each type of cost

Case	Ship cost	Bunker cost	Inventory cost
1	3648183	994469	8371200
2	3488562	1167070	7178400
3	3488562	1167070	7178400
4	3393733	1202230	7111200
5	3393733	1225422	7027200
6	3393733	1225422	7027200
7	3393733	1225422	7027200

6.5.2 Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing t_i^{port}. To investigate the effect of port handling efficiency, we compare the default parameter settings with the situation of reducing the port time at Singapore on all ship routes to be one day. We find that the optimal number of ships and the total cost increase with the time spent at Singapore, as shown in Fig. 6.3 and Fig. 6.4, in all the 7 cases of available days at Singapore. These results demonstrate that it is of significant importance for port operators to improve the container handling efficiency.

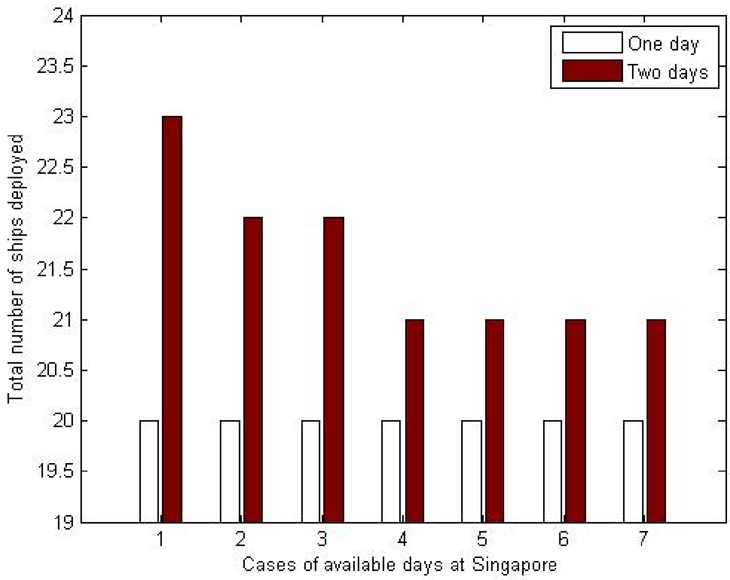


Figure 6.3: Impact of port time at Singapore on ship route 1 on the number of ships deployed

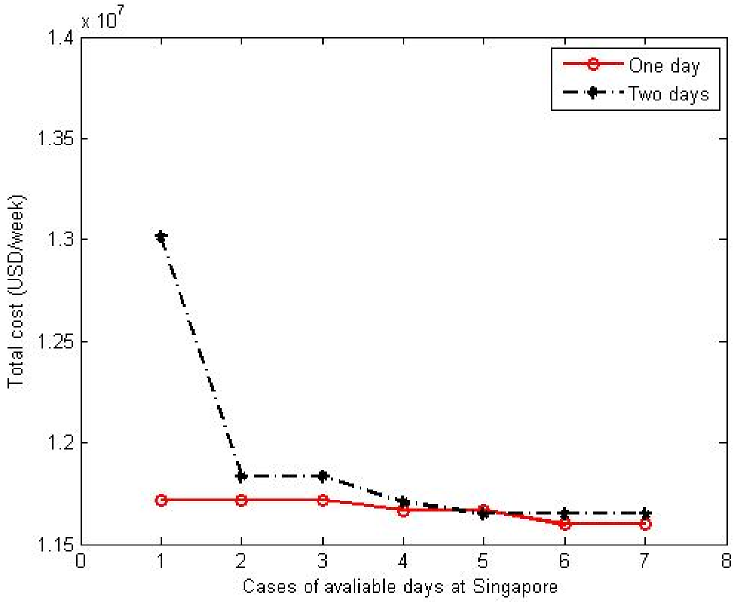


Figure 6.4: Impact of port time at Singapore on ship route 1 on the total cost

6.5.3 Result of bunker prices

In this section, we study the impact of the bunker price on the total cost and the number of ships deployed in the liner shipping network. We increase the bunker price from 300, 400, 500, 600, 700, to 800 and the other parameters are the same as Table 6.2, Table 6.3 and Table 6.4. The results are shown in Fig. 6.5 . We observe that a higher bunker price always leads to a higher total cost for liner shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price.

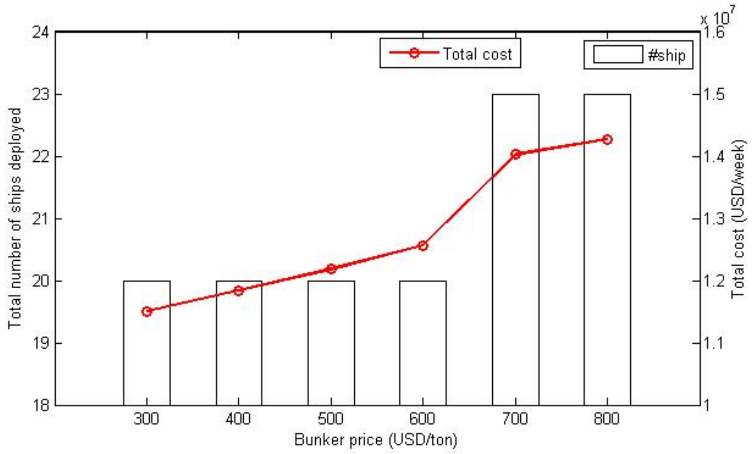


Figure 6.5: Result of bunker prices on the total cost and the number of ships deployed

6.5.4 Effect of inventory cost

Finally, the unit inventory cost β may affect the ship route schedule and then the total cost. We change β from 1, 1.2 through to 2 and the results of 6 experiments are shown in Fig. 6.6. The number of ships may not change when the unit inventory cost rises. The total cost increases almost linearly (not strictly linearly) when the unit inventory cost rises. This indicates that as a result of increase in the unit inventory cost, the total cost of a liner shipping company is higher. This shows that when the cargos in the containers are more valuable, ships should sail at a higher speed.

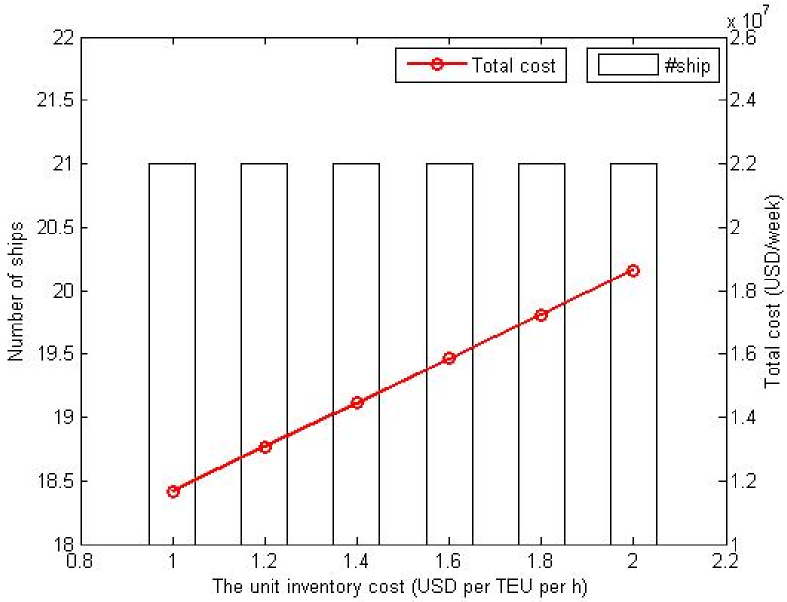


Figure 6.6: Effect of unit inventory cost on the total cost and the number of ships

6.6 Conclusions

This chapter has studied the practical liner shipping network schedule design problem with port time windows while considering the inventory cost. This is a significant tactical planning decision problem because it considers the availability of ports when planning liner shipping services. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we reformulated the problem as an integer linear optimization model and used an iterative optimization approach.

The solution method was applied to a network, consisting of 18 ports, operated by APL. The results demonstrate that the port time windows, port handling efficiency, bunker price, and inventory cost all affect the total cost, the optimal number of ships to deploy, and the optimal schedule. Higher availability at ports, shorter port time, lower bunker price, and lower the inventory cost result in a lower total cost and a smaller number of ships to deploy. Therefore, port operators can apply the proposed method to quantify whether the benefits to liner shipping companies are worthwhile compared to the cost of expanding the ports' capacity and improving their efficiency. Liner shipping companies may need to charter in more ships if they predict that the future bunker price will increase. Moreover, the higher inventory cost leads to an increase in the total cost without any change of the number of ships used in the case study.

Chapter 7

Conclusions and Future Research

7.1 Conclusions

In this thesis we have addressed liner shipping service schedule design problems. The aim is to minimize the sum of ship cost, fuel cost and inventory cost, while ensuring that ports are available to serve the ships on the planned days. These problems are realistic decisions frequently encountered by liner planners. The main contributions of the thesis are: (i) identifying the practical problems and formulating them as mathematical models; (ii) developing efficient solution algorithms to address the problems by taking advantage of the problem structures; and (iii) conducting extensive numerical experiments to test the models and algorithms and obtain managerial insights.

In chapter 3, a single ship route is investigated on which each port is visited only once a week. The arrival time of containerships at each port of call on the ship route is determined while considering the available berth time windows at ports. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a nonlinear non-convex optimization model. In view of the problem structure, we have developed an efficient dynamic-programming based holistic solution approach, which includes a space-time network model and a bounding technique for the total cost with give number of ships. The proposed solution method is applied to the NCE service provided by OOCL.

Chapter 4 is generalization of Chapter 3 in that a port on a ship route can be visited more than once a week. As a result, more realism is captured but the resulting model is more complex. In view of the problem structure, we developed a holistic solution approach. In this approach, at first the port time window constraints are relaxed to obtain a mixed-integer nonlinear programming model, which is subsequently transformed to a mixed-integer linear programming model. This mixed-integer linear model is repeatedly solved by adding the violated port time window constraints until a feasible solution is obtained. This feasible solution is proved to be the global optimal solution to the problem. We have conducted extensive numerical experiments based on the AGM ship route of OOCL.

Chapter 5 extends the previous two chapters by considering a liner shipping network that consists of many ship routes. The inventory cost is assumed to be 0. Hence, the objective minimizes the sum of bunker cost and ship operating cost. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we reformulated the problem as an integer linear optimization model and proposed an iterative optimization approach. The proposed solution method was applied to two networks, consisting of six ports and 21 ports, operated by APL.

Chapter 6 also examines a liner shipping network, but the inventory cost is included in the model. As a consequence, the container transshipment and the relevant connection time must be incorporated. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we reformulated the problem as an integer linear optimization model and proposed an iterative optimization approach. The proposed solution method was applied to a network, consisting of 18 ports, operated by APL.

7.1.1 Impact of port time windows

We examine the effect of port time windows on the total cost and the optimal schedule. Both the operator of the port and the liner shipping company that operates the service are interested in looking at the result if more available berth time is provided at the port. We examined several cases in which the berths at the port have more and more available days. The results show that more available days at the port leads to a lower total cost. Moreover, the results demonstrate that the number of available days at a port may affect the optimal number of ships deployed. In particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

7.1.2 Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing the time a ship spends at port. To investigate the effect of port handling efficiency, we looked at the results when the port time is reduced. We find that the optimal number of ships and the total cost increase with the time spent at the port. Hence, it is of significant importance for port operators to improve the container handling efficiency.

7.1.3 Result of bunker prices

We studied the impact of the bunker price on the total cost and the number of ships deployed on the liner shipping service. The results show that a higher bunker price always leads to a higher total cost for liner shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price. Therefore, liner shipping companies may need to charter in more ships if they predict that the future bunker price will increase.

7.1.4 Effect of inventory cost

The unit inventory cost of cargos may affect the ship route schedule and then the total cost. The results show that the number of ships may not change when the unit inventory cost rises and the total cost increases almost linearly (not strictly linearly) when the unit inventory cost rises. This indicates that as a result of increase in the unit inventory cost, the total cost for a liner shipping company is higher. This shows that when the cargos in the containers are more valuable, ships should sail at a higher speed. Another insight is, liner shipping companies may need to charter in more ships if they predict that a particular season is coming during which the value of the cargo is generally low.

7.2 Future Research

Uncertainty is a major challenge in liner shipping Meng et al. (2012b) and Wang et al. (2013g). In this thesis the port time and sea time are assumed to be deterministic, and possible uncertainty is incorporated by adding some "buffer" time. Such an engineering-based approach may not lead to optimal decisions. A worthwhile avenue is to capture port time and sea time uncertainty endogenously. This problem is complex because a natural problem that cannot be circumvented is: what should a ship do if it is delayed? In reality, the ship may speed up (Qi and Song, 2012; Wang and Meng, 2012b,c), may skip ports of call (Brouer et al., 2013), may swap ports of call (Brouer et al., 2013) and may leave a port early without loading all containers. The handling of delay itself is a challenging topic, even when the planned schedule is given. To deal with uncertainty, we could develop two-stage stochastic programs with recourse. In the first stage, the schedules are designed; and in the second stage, the additional fuel cost in view of uncertainty should be calculated.

To solve such a two-stage stochastic program, we can employ the sample average approximation method. The challenge is that the scale (number of decision variables and constraints) may be too large. Hence, some tailored heuristic algorithms should be developed.

A second worthwhile research direction is to plan the network and design the schedule in a holistic manner. This thesis has assumed that the port rotations are designed and the type of ships to deploy on each ship route is also given. In other words, it is assumed that planners first design the port rotations and ship deployment, and then determine the speeds and schedules. From the optimization point of view, a single model that optimizes all of the decisions could lead to better solutions in terms of the total costs. To address this problem, new formulations and solution techniques have to be proposed.

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