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Abstract

Concentrates on estimating the spatial correlations between soil water infiltration observations, with special emphasis on resistant methods to remove nonstationarity. After this removal, robust semivariogram estimators are used to examine the spatial dependencies for various tillage treatments. There is some indication that infiltration characteristics inherit different types of spatial dependency, depending on the tillage treatment applied.-from Authors

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A Robust-Resistant Spatial Analysis of Soil Water Infiltration

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Data taken at adjacent spatial locations often exhibit correlation which must be taken into account in their analysis. Geostatistical methods, originally developed for the mining industry, have proven to be adaptable to hydrological problems. This paper concentrates on estimating the spatial correlations between soil water infiltration observations, with special emphasis on resistant methods to remove nonstationarity. After this removal, robust semivariogram estimators are used to examine the spatial dependencies for various tillage treatments. There is some indication that infiltration characteristics inherit different types of spatial dependency, depending on the tillage treatment applied.

INTRODUCTION

In this article we demonstrate how soil water infiltration data taken at spatial locations that conform to a regular grid can be analyzed sensibly; this is in spite of possible nonstationarities caused by superposition of different tillage treatments. The emphasis is on discovering the spatial structure using resistant (i.e., arithmetically stable) and robust (i.e., model stable) methods. After appropriate transformation of the data and adjustment for trend, we use (robust) geostatistical techniques to summarize the spatial structure. Robust variogram estimators are computed for each of the tillage treatments, and an interesting positive spatial dependency is observed in moldboard-plowed soil that is not seen for chiselplowed or no-till soil.

The four tillage treatments involved in this study are moldboard plow (15–20 cm), chisel plow (15–20 cm), paraplow (25–30 cm), and no-tillage (see *Bowen* [1981] for a discussion of tillage operations). These treatments were established with tillage in the fall of 1982 at the Agronomy and Agricultural Engineering Research Center near Ames, Iowa. The soil is a Webster silty clay loam (Typic Haplaquoll). All of the tillage plots were chisel-plowed in 1981 and left untilled before the fall of 1982 with continuous corn production. Consequently, a large data set (not suitable for spatial analysis) was collected and this forms the basis of the soil water infiltration research presented in the work by *Mukhtar et al.* [1986]. On five contiguous plots (a small fraction of the larger study) a spatial study of soil water infiltration was planned.

Soil water infiltration measurements were made at locations, i.e., on a 3-by-8 grid arrangement, within each plot. Two sets of infiltration measurements were obtained, one set in May and one set in July 1983; Figure 1 gives the details. Notice that no measurements were taken on the middle of the five plots, because of the necessity to do all measurements on the same day. In May, some observations were lost due to compaction of the soil by tractor wheels.

Double-ring infiltrometers [Bertrand, 1965] were used to

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Paper number 6W4746. 0043-1397/87/006W-4746\$05.00 measure ponded infiltration volumes, and water stage recorders were used to record the subsidence of water in the inner ring as a function of time (details can be found in the work by *Mukhtar et al.* [1986]). Infiltration theory developed for homogeneous isotropic systems was unable to describe with physical meaning the infiltration processes for these field conditions. Thus similar to *Gish and Starr* [1983], only the 30-min cumulative infiltration values were used in this analysis (the values are presented in Figure 1).

GEOSTATISTICS

Geostatistics is the name proposed by Matheron [1963] for a method of spatial analysis that is used to predict ore reserves from sample data whose relative spatial locations are known. The books by David [1977], Journel and Huijbregts [1978], and Clark [1979] all address problems and present case studies exclusively in the mining field. More recently, it has been realized that hydrological data which are spatial in nature can also be analyzed using geostatistics, although the questions asked of the data are usually different (see, for example, Delhomme [1979], Chirlin and Dagan [1980], Neuman [1980], Luxmore et al. [1981], Russo and Bresler [1981], Vieira et al. [1981], Russo and Bresler [1982], Kitanidis and Vomvoris [1983], and Grah et al. [1983]).

Most of the articles above present the assumptions needed to perform a geostatistical analysis, but there is often scant attention paid to whether it is reasonable to analyze the data as if they came from a process which satisfies these assumptions [Horowitz and Hillel, 1983; Hamlett et al., 1986]. Therefore we present briefly the theory of geostatistics and indicate places where it is crucial to check the assumptions.

A regionalized variable Z(x), where x denotes a spatial location that can vary continuously over some domain D in two- or three-dimensional space, is the random measurement taken at location x. Relative to another location x', Z(x) and Z(x') are assumed to depend (up to second-order moments) only on x - x'. More specifically, the intrinsic hypothesis [Matheron, 1963] makes the following stationarity assumptions: (1) $E\{Z(x) - Z(x')\} = 0$, for any x, x' in D, which is equivalent to stating that the expectation of any Z(x) is constant, and (2) $2\gamma(x - x') = E\{[Z(x) - Z(x')]^2\}$, for any x, x' in D, which defines the variogram $2\gamma(h)$ (the semivariogram is

MOLDBOARD			PARAPLOW				CHISEL					NOTILL		
31.55	9.60 27 . 90	12.50	0.64 7.54	23.87 36.64	15.24 26.47			10.24	8.93	9.32 14 . 77	4.30	1 6.28 9.75	11.43 9.49	
31.10	18.84 35.65	4.18 6.84	2.10 5.40	38.82	23.52 42.02			6.81	8.55	1.72 11 . 84	6.10	8.84 13 . 41	14.22 14 . 84	
38.05	53.25	4.69 13 . 90	3.35 13.43	28.47 10 . 67	30.98 20 . 33			3.99	1.83	3.41 7.96	4.48	14.65 15.38	13.29 10 . 41	
17.62	43.83 39.04	8.02 18.15	1 7 .4 0 26 . 49	1 8.44 30.28	20.79 35.20	_	→ N	7.10	4.65	4.14 5.32	8.67	13.46 15.29	14.48 12.10	
8.64	42.82 34.14	13.29 28.53	9.7 1 39 . 82	12.54 27 . 52	26.44 39.65			2.12	5.29	0.50 8.31	3.54	13.98 12.56	11.20 20.59	
6.65	20.64 23.30	8.43 25.97	3.82 20 . 19	28.47 25 . 15	24.28 44.42			6.02	3.52	6.2 0 5.84	2.22	10.53 15 . 21	18.09 13.12	
5.78	22.45 18.93	3.32 38.31	6.95 6.48	11.80 31.78	48.02 60.04			6.33	4.94	2.91 8.29	8.58	17 .22 8.88	18.20 18 .1 9	
22.78	22.94 31.29	2.99 10 . 00	2.79 16 . 20	7.65 63.32	31.10 38.71			8.40	2.53	1.33 5.41	10.35	9.84 15.32	13.00 11 . 11	

Fig. 1. Thirty-minute cumulative soil water infiltration data (in centimeters) and their spatial locations, together with tillage treatments. Distance between readings is 3 m in the E-W direction and 1.5 m in the N-S direction within tillage treatments and 3 m between adjacent tillage treatments moldboard and paraplow and chisel and no-till; 9 m separates the closest readings associated with paraplow and chisel treatments. Top numbers (in boldfaced type) are the May data, and the bottom numbers are the July data.

simply $\gamma(h)$). Note that x - x' is a subtraction of two vectors in space. A slightly less general set of assumptions are those of second-order stationarity, where assumption 2 becomes Cov (Z(x), Z(x')) = C(x - x'). When C(h) is defined, it is easily shown that $\gamma(h) = C(0) - C(h)$.

Assumption 1 is often overlooked by researchers who proceed directly to estimating $2\gamma(h)$. However, suppose $Z(x) = \mu(x) + \varepsilon(x)$, where $\mu(x)$ is deterministic drift and $\varepsilon(x)$ is stationary with zero mean. Then it is easily seen that $2\gamma_Z(h) = \{\mu(x + h) - \mu(x)\}^2 + 2\gamma_\varepsilon(h)$. On a transect where x is one dimensional, often $\mu(x) = ax + b$, and hence $2\gamma_Z(h) = (ah)^2 + 2\gamma_\varepsilon(h)$. We should be trying to estimate $2\gamma_\varepsilon(h)$, but its effect is masked by the presence of $(ah)^2$ when we use the raw Z data to estimate the variogram. Assumption 2, while not overlooked, is rarely checked. If the covariance function $C(\)$ exists, then at the very least C(0) should be a constant σ^2 for all x in D, which can be checked.

In the analysis of the soil water infiltration data given in the next section, we found both assumptions 1 and 2 to be violated. This was remedied by working with square-root data and performing a column-by-column removal of column medians. Note that the very nature of this study (namely, different tillage treatments in different locations) means that stationarity in the mean (assumption 1) is highly unlikely. It has been our experience [Cressie, 1985c; Cressie and Read, 1985; Cressie, 1986; Hamlett et al., 1986] that stationarity in spatial data is very much the exception rather than the rule.

For the rest of this section assume 1 and 2 hold. There remains the problem of estimating the variogram function (think of it as a parameter) defined in 2, from data $Z(x_1)$, $Z(x_2)$, \cdots , $Z(x_n)$, taken at locations x_1, x_2, \cdots, x_n . Define

$$S(h) = \{(i, j): x_i - x_j = h\}$$
(1)

where N(h) is the number of elements in the set S(h). When data are equally spaced on a transect or are on a regular grid

(as is the case in this study), it is easy to determine S(h) for h in directions of the grid axes or grid diagonals.

Matheron [1963] proposed the method-of-moments estimator of $2\gamma(h)$, namely,

$$2\hat{\gamma}(h) = \frac{1}{N(h)} \sum_{S(h)} \{Z(x_i) - Z(x_j)\}^2$$
(2)

provided N(h) > 0; it is unbiased, but possesses no other statistical optimality properties. Some have mistakenly believed (2) to be the variogram itself, but clearly it is just an estimator. Other estimators do exist, and, in fact, it was concern for the lack of robustness of (2) that led *Cressie and Hawkins* [1980] to propose

$$2\bar{y}(h) = \left[\frac{1}{N(h)} \sum_{S(h)} |Z(x_i) - Z(x_j)|^{1/2}\right]^4 / [0.457 + 0.494/N(h)] \quad (3)$$

as a robust alternative to $2\hat{\gamma}(h)$. Sampling variances of (2) and (3) are derived in the work by *Cressie* [1985a]. The denominator in (3) is a bias-correcting term which ensures $E(2\hat{\gamma}(h)) \simeq$ $2\gamma(h)$. *Cressie and Hawkins* [1980] add an additional $0.045/N(h)^2$ to the denominator, whose effect on the corrections here is negligible. Outliers in the data $\{Z(x_i)\}$ can be difficult to detect both because of the spatial aspect and because it is not feasible to inspect every datum of a large data set for outliers. The Cressie-Hawkins estimator automatically downweights contaminated data, whereas in the *Matheron* [1963] estimator the squared terms exacerbate contamination (see *Cressie* [1984], *Hawkins and Cressie* [1984], and *Cressie* [1985a] for further discussion and comparisons). Other estimators have been proposed by *Cressie* [1979], *Armstrong and Delfiner* [1980], and *Omre* [1984].

Having estimated the variogram (at various values of h), one usually wants to assimilate a theoretical model with the



Fig. 2. Stem-and-leaf diagrams of the July data for (a) all data combined, (b) the moldboard and paraplow data combined, and (c) the chisel and no-till data combined. In Figures 2a and 2b 4|0 means 40 cm, and in Figure 2c 4|0 means 4.0 cm.

estimator; e.g., the spherical model, the exponential model, the linear model, etc. [Matheron, 1971]. The spherical model

$$\gamma(h; c_0, c_s, a_s) = c_0 + c_s \{ (3/2)(h/a_s) - (1/2)(h/a_s)^3 \}$$
(4a)

$$0 < h \leq a_s$$

$$\gamma(h; c_0, c_s, a_s) = c_0 + c_s \qquad h > a_s \qquad (4b)$$

is popular because it shows the sort of strong positive correlation often seen in mining, soil science, hydrology, etc., and because the parameters c_0 (nugget effect, or small-scale structure plus experimental error), $c_0 + c_s$ (sill, or stationary variance), and a_s (range, or limit of dependence) are easily interpretable. Fitting such a model as (4) to $\{2\hat{\gamma}(h_i): i = 1, \dots, k\}$ or to $\{2\hat{\gamma}(h_i): i = 1, \dots, k\}$ requires care. By-eye or adhoc methods have been the practice in the past, but *Cressie* [1985a] has developed a generalized and weighted least squares approach that removes subjective biases from the fit. This is extremely important, since the variogram is the cornerstone of further analyses such as kriging [see *Matheron*, 1971; *Burgess and Webster*, 1980] and efficient treatment comparisons.

A ROBUST-RESISTANT ANALYSIS

An initial look at the data via a stem-and-leaf diagram [*Tukey*, 1977], shown here for July (Figure 2*a*), indicates a highly skewed distribution. That this is a misleading interpretation is apparent when we look at moldboard plow (M) and

paraplow (P) (the two southern plots) together and compare it to chisel plow (C) and no-till (N) (the two northern plots) together. Figures 2b and 2c show each to be roughly symmetric with a couple of outliers, but on completely different scales of magnitude.

If we want to combine observations over the whole spatial domain, rather than analyzing the data as a collection of subproblems, some type of scale-equilibrating transformation is needed. To make the plots comparable, at the very least we need homogeneous variances; this leaves us free to compare (eventually) the treatment levels and to obtain a meaningful estimate of standard error. For each column of eight measurements we computed the resistant quantities, median \hat{X} and interquartile-range-squared IQ^2 . The median is obtained using the convention that when n is odd, it is the middle value of n ordered observations, and when n is even, it is the average of the "lower middle" value and the "upper middle" value. The



Fig. 3. Interquartile-range-squared (IQ squared) versus median (within columns) plots of the May data for (a) measured data, (b) square-root transformed data, and (c) log-transformed data.

lower (upper) quartile is simply the median of the lower (upper) "half" of the data (which includes the median when n is odd). The interquartile range is then the difference between the upper quartile and the lower quartile and is a measure of spread much like the standard deviation. Figure 3 is based on all the May data and shows graphs of IQ^2 versus \tilde{X} for raw and for transformed data. The compromise square-root transformation seems to do the best job of straightening out the data so that variation is no longer a function of location. We decided to use resistance-based rather than classical varianceversus-mean graphs, because of the several outliers that would certainly have undue influence on parts of the graphs. A similar series of graphs for July (not shown here) shows little difference between square-root and log transformations. Therefore on balance, we chose to use the square-root transformation for both the May and July data.

Variogram estimators (see equations (2) and (3)), based on square-root data, can now be combined across plots if necessary; this was out of the question for the raw data (Figures 4b and 4c should be compared to Figures 2b and 2c). Figure 4 shows stem-and-leaf diagrams of transformed data made stationary in the mean by subtracting, column by column, the column median \sqrt{X} from the square-root data \sqrt{X} , leaving behind residuals data at the $4 \times 3 \times 8$ spatial locations (stationarity was further verified by observing no trend in plots of row medians of these residuals against row number).

For those who think the square-root scale is unnatural, we would like to make the following comments. We arrived at this transformation by noticing that the medians of columns were roughly linearly related to their respective IQ^2 . Must we stop the analysis and report that the data do not fit the assumptions? No, fortunately we were able to find a scale (the square-root scale) where the transformed data can be modeled as realizations from a process whose variance is stationary. Of course, there are still problems with stationarity in the mean, but usually this can be handled by subtracting column and/or row effects.

This good fortune is not happenstance, and it is likely to occur for most data sets. Cressie [1985b] explores this through a "universal transformation principle" which says that additivity of small effects (normality), additivity of small effects to large effects (stationarity of variances), and additivity of large effects (no interaction) all tend to occur on the same scale. Therefore it makes sense to analyze the data on this scale and to convert the answers back to the original scale when necessary. Anyone who has taken logs of their data has essentially been invoking the above principle. We have simply expanded the possible scales to include squares, square roots, cube roots, and reciprocals, as well as logs. Moreover, the conclusions of Cressie [1985b] show that parameters defined for the raw data are available from those of the transformed data. He demonstrates, using precisely the July data in Figure 1 the equivalence between estimating scaled semivariograms of the raw data and semivariograms of the transformed data.

In all that follows, we use the robust estimator of the semivariogram given by (3) on square-root data. Figure 5 shows the semivariogram estimator, combined over all plots, and individually for each plot, for May. The same selection of semivariograms for July is presented in Figure 6. At best, the number of pairs used in the individual semivariogram estimators are $21(\log h = 1)$, $18(\log h = 2)$, $15(\log h = 3)$, $12(\log h = 4)$, and $9(\log h = 5)$. For May they are considerably less in places; however, a general picture seems to emerge. Treatments no-till (N), chisel (C), and paraplow (P) exhibit no spatial structure but moldboard (M) shows strong positive spatial dependence, evidenced by the variogram estimator's (Figure 5e) rapid increase between lags h = 1 through lags h = 3 (early lags are the most reliably estimated; see *Cressie* [1985a]).

	4a
-2 -2 -1 -1 -0 0 0 1 1 2 2	13 5589 111344 55556667888889 0000111112222233333444 000001111111222333334 55666666677789 00111333 567 23 5
	4b
-2 -1 -1 -0 0 0 1 1 2 2	13 5589 11344 5689 01123334 011223334 5567 111333 567 23 5
	4c
-11 -10	1
- 9 - 8 - 7	004
- 6 - 5 - 4 - 3 - 2 - 1 - 0 0 1 2 3	06 247 137 12 224 128 3357 33357 123348 04
4 5 6 7 8 9	9 0445 24 0 299

Fig. 4. Stem-and-leaf diagrams of square-root transformed July data (made stationary by subtracting, column by column, the column median \sqrt{X} from the square-root data) for (a) all data combined, (b) the moldboard and paraplow data combined, and (c) the chisel and no-till data combined. In Figures 4a and 4b 0|1 means 0.1 cm^{1/2}, and in Figure 4c 0|1 means 0.01 cm^{1/2}.



Fig. 5. The $\bar{\gamma}$ values in the E-W direction using residuals resulting from the May data for (a) all data combined, (b) no-till data, (c) chisel data, (d) paraplow data, and (e) moldboard data. Lag distance is 3 m.

When these are combined into an overall semivariogram (Figure 5a), as one might be tempted to do to obtain more pairs at various lags, the M treatment dominates, leading one to think (wrongly) that there is a general positive spatial dependence throughout all the plots. Since there are missing observations from treatment M in May, it is interesting to see if this picture persists in July, when no observations are lost (from any treatment).

In July, N and C still show no structure, but there is evidence that P has acquired somewhat weak positive spatial dependence. Sampling fluctuations, however, could easily account for this impression (Figure 6d). The strong positive spatial dependence for M (Figure 6e) is once again apparent for July, where 50% more observations are available.

Some physical explanation of these conclusions is called for. Before we start however, we would like to emphasize that these are tentative, for the following reasons. First, without replication we cannot be sure that the behavior of M, P, C, and N is not due to their plot location. Second, the semivariogram shapes given by Figures 5 and 6 are only an impression and must be judged in the light of sampling fluctuation. Nevertheless, our conclusions are supported by a previous study [Hamlett et al., 1986]. The data in Figure 1 were not originally collected for a full-scale spatial analysis, but we have emerged with a hypothesis we would not have expected before we started our analysis. Spatial dependence as measured by the variogram seems to vary with tillage.

The soil physical disturbance levels associated with the four tillage treatments can be ranked as follows: moldboard plow is greater than paraplow is greater than chisel plow is greater than no-till. The tillage tools affect the soil surfaces differently, and thus can be expected to have differing effects on the soil water infiltration. We expected to see treatment differences show up in the mean level of soil water infiltration, but did not expect to see them in the spatial-dependence measures. We are excited by the prospect of comparing treatments via semivariograms, since this adds another dimension to the analysis of soil water infiltration for different treatments. (The



Fig. 6. The $\bar{\gamma}$ values in the E-W direction using residuals resulting from the July data for (a) all data combined, (b) no-till data, (c) chisel data, (d) paraplow data, and (e) moldboard data. Lag distance is 3 m.

use of semivariograms for treatment comparisons is also seen in the work by *Bresler et al.* [1982], who analyze crop yield under controlled line-source irrigation.) That M should be so different from C is surprising; we already had hints that M and N show different spatial relationships in a study of soil water tension [Hamlett et al., 1986].

The spatial analysis reported herein gives rise to the hypothesis that the greater the action of a tillage tool, the more likely it is that positive spatial dependence will be found. The tillage tools that provide the highest level of change in the soil surface, namely, M and P, are the two treatments that showed possible spatial correlation in soil water infiltration over the lag distances sampled. Less soil surface disturbance occurs with C and N and no spatial dependence was apparent in these treatments at the lag distances sampled. To corroborate this evidence, a well-designed study that replicates treatments say by a randomized blocks design and that measures the infiltration in each plot prior to tilling and then after should be implemented.

Summary

Measurements of soil water infiltration sampled across four areas of land receiving various tillage treatments were analyzed using geostatistical methods. Resistant data analytic approaches were used to remove identified nonstationarity in data means and variances. By construction then, the data are more likely to satisfy the stationarity assumptions needed for variogram estimation. A robust estimator [Cressie and Hawkins, 1980] was used to estimate semivariograms for the various tillage treatments. Strong spatial dependence was found consistently in the moldboard plow treatment. The paraplow treatment exhibited weak spatial structure. No spatial structure was found in the chisel plow and notill treatments. The results indicate that tillage with higher disturbance levels may provide more spatial correlation in the soil surface physical condition. Efficient estimation and testing of treatment comparisons in the presence of spatial dependence should be an area of fruitful future research.

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