1-1-1996

# Texture analysis using Gabor wavelets

Golshah Naghdy
*University of Wollongong*, golshah@uow.edu.au

Jianli Wang
*University of Wollongong*, jianli@uow.edu.au

Philip Ogunbona
*University of Wollongong*, philipo@uow.edu.au

# Texture analysis using Gabor wavelets

## Abstract

Receptive field profiles of simple cells in the visual cortex have been shown to resemble even- symmetric or odd-symmetric Gabor filters. Computational models employed in the analysis of textures have been motivated by two-dimensional Gabor functions arranged in a multi-channel architecture. More recently wavelets have emerged as a powerful tool for non-stationary signal analysis capable of encoding scale-space information efficiently. A multi-resolution implementation in the form of a dyadic decomposition of the signal of interest has been popularized by many researchers. In this paper, Gabor wavelet configured in a 'rosette' fashion is used as a multi-channel filter-bank feature extractor for texture classification. The 'rosette' spans 360 degrees of orientation and covers frequencies from dc. In the proposed algorithm, the texture images are decomposed by the Gabor wavelet configuration and the feature vectors corresponding to the mean of the outputs of the multi-channel filters extracted. A minimum distance classifier is used in the classification procedure. As a comparison the Gabor filter has been used to classify the same texture images from the Brodatz album and the results indicate the superior discriminatory characteristics of the Gabor wavelet. With the test images used it can be concluded that the Gabor wavelet model is a better approximation of the cortical cell receptive field profiles.

## Keywords

texture, analysis, gabor, wavelets

## Disciplines

Physical Sciences and Mathematics

## Publication Details

# Texture Analysis Using Gabor Wavelets

G. Naghdy, J. Wang, and P. Ogunbona
Department of Electrical and Computer Engineering
University of Wollongong, Australia

## ABSTRACT

Receptive field profiles of simple cells in the visual cortex have been shown to resemble even-symmetric or odd-symmetric Gabor filters. Computational models employed in the analysis of textures have been motivated by two-dimensional Gabor functions arranged in a multi-channel architecture. More recently Wavelets have emerged as a powerful tool for non-stationary signal analysis capable of encoding scale-space information efficiently. A multi-resolution implementation in the form of a dyadic decomposition of the signal of interest has been popularised by many researchers. In this paper, Gabor wavelet configured in a 'rosette' fashion is used as a multi-channel filter-bank feature extractor for texture classification. The 'rosette' spans 360 degrees of orientation and covers frequencies from DC. In the proposed algorithm, the texture images are decomposed by the Gabor wavelet configuration and the feature vectors corresponding to the mean of the outputs of the multi-channel filters extracted. A minimum distance classifier is used in the classification procedure.

As a comparison the Gabor filter has been used to classify the same texture images from the Brodatz album and the results indicate the superior discriminatory characteristics of the Gabor wavelet. With the test images used it can be concluded that the Gabor wavelet model is a better approximation of the cortical cell receptive field profiles.

Key words - Human visual system, Gabor functions, Gabor transform, Gabor Wavelets, texture analysis, texture classification.

## 1. INTRODUCTION

Texture analysis plays an important role in vision research, and is an important feature of object recognition and classification. There have been a number of reports on texture analysis using multi-channel or multi-resolution methods with promising results in the areas of texture segmentation, description and recognition over the last decade [1], [16], [17], [18]. Texture classification, has also received much attention and some techniques of texture feature representation towards classification have been proposed [21].

One plausible approach in texture classification is to employ the known paradigms with respect to the Human Visual System (HVS) in a multi-tier configuration. The low level vision, for example, can be rooted in the multi-channel model of simple cells. The Receptive Field Profiles (RFPs) of simple cells in

the visual cortex often resemble even-symmetric or odd-symmetric Gabor filters [14]. Researchers in computational texture analysis have used 2-D Gabor functions as the channel filters [18]. In the proposed systems, the orientations, radial frequency bandwidth and center frequencies of filters are tuned according to the spatial properties of the textures. Boundaries between textures can be detected by the channel amplitude comparisons, and discontinuity in texture phase can be detected by locating large variations in the channel phase demodulation. Though the results are promising, it is obvious that filters cannot be customized to individual textures in a truly autonomous texture segmentation and classification architecture.

The problem of early research in this area lies in that the filters are not generic and are strictly designed to suit the images at hand. This total image dependency defies the premise that simple cells in visual cortex possess RFPs tuned to a prescribed range of frequencies and orientations. The extent of the elasticity of the RFP of a simple cell is not fully determined or understood at the present time. There are evidences, however, to suggest that the RFPs of a group of simple cells in the visual cortex have been fixed (hardwired) shortly after birth. These group of simple cells act as feature detectors for a range of frequency and orientation. In order to simulate their feature detection properties, the channel filters which are image independent and can detect full range frequency and orientation features are required.

Wavelets have recently been utilised as a powerful tool for non-stationary signal analysis. They have been widely applied to multi resolution image processing. The aim of this study is to develop a generic feature vector generator for texture classification. To this end, Gabor Wavelets, which are formulated as "rosette" configuration, are used as multi-channel filter banks to generate the feature vector. The "rosette" configuration spans over 360 degrees of orientation and covers frequencies from DC.

In this algorithm, the feature vector is generated by decomposing the image into several frequency and orientation bands using Gabor Wavelets (filter banks) and then extracting one feature from each band. In the subsequent stage, a classification process is completed by searching the minimum distance feature vector. The classification results using natural texture images from Brodatz album are reported, and Gabor transform functions are also tested as the comparison. From the result, it is concluded that Gabor Wavelets exhibit a better discriminatory feature detector characteristics, hence, it is an effective tool in texture classification. Furthermore, Gabor Wavelets with their feature detection capabilities and optimal spatial spatial/frequency resolution could be a good approximation of the RFPs' of the simple cells in visual cortex.

The rest of this paper is organised as follow; in section 2, fundamentals of Wavelets and Gabor transform are introduced. Detailed methods of image texture classification by Wavelets and Gabor transform are shown in section 3. Results of the classification are in section 4, and section 5 is discussion.

## 2. GABOR WAVELETS AND TRANSFORM

### 2.1 SHORT TIME FOURIER TRANSFORM

Fourier transform is used widely in signal processing. It is the foot stone of modern signal analysis. The Fourier transform and its reverse are defined as follows:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) \exp(-j\omega t) dt \qquad (1)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \exp(j\omega t) d\omega \qquad (2)$$

where $F(\omega)$ is the Fourier transform of the time basis signal $f(t)$, and

$$\exp(j\omega t) = \cos(\omega t) + j\sin(\omega t) \qquad (3)$$

The Fourier transform can provide us with the activities of the signal in the frequency domain without any reference to where/when these activities are accruing. This prevents us from further investigating the representations in both time and frequency domain. The spread of neurons on human visual cortex, however, indicates a joint spatial and spatial frequency decomposition where not only the frequency and orientation of the objects are detected but their locations are registered as well. This demands another tool which can analysis the signal in joint time-frequency domain.

Such a function can be achieved by the Short Time Fourier transform (STFT). The STFT is defined as:

$$STFT(\tau,\omega) = \int s(t)g(t-\tau)\exp(-j\omega t)dt \qquad (4)$$

The STFT can be explained as the Fourier transform of a signal that is windowed by the function $g(t)$ that shifts in the time domain. The STFT also states the contribution of the sine and cosine to the signal, but it is restricted near the position (point) $\tau$ in the time domain. The STFT with Gaussian window is called Gabor transform.

The Gabor transform can be regarded as a filter-bank, whose impulse response in time domain is Gaussian modulated by sine and cosine wave. As the frequency of the sine and cosine function (which is $\omega$ in (4)) changes, a set of filters with the same window size are constructed. Gabor transform has been used to simulate human visual system by many researchers [5], [6], [17].

The problem of STFT is that the size of the window in the time domain is fixed, and the inflexibility of the window size results in a fixed resolution in both time and frequency domain.

One dimensional Gabor transform basis functions can written as :

$$h(t) = \exp[\frac{t^2}{2}]\cos(\alpha^j \cdot 2\pi \cdot t) \qquad (5)$$

Figure 1 shows three basis functions where $\alpha = \frac{1}{\sqrt{2}}$ and $j = 0,1,2$.



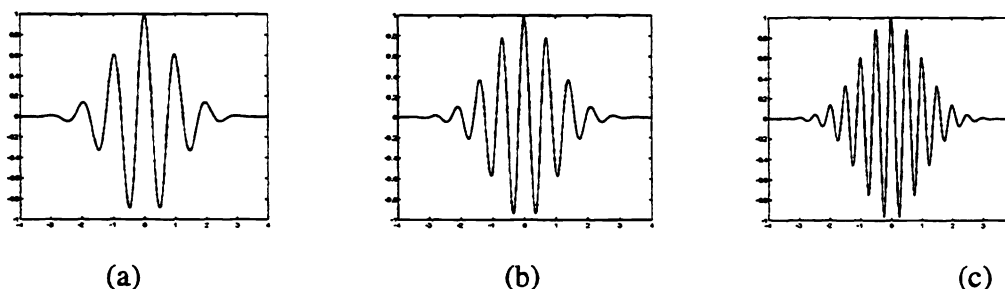| (a) | (b) | (c) |

Figure 1, one dimensional Gabor transform basis functions. a) j=0, b) j=1, and c) j=2

If Gaussian is chosen as the window function in STFT, say Gabor transform, $d_t$ and $d_f$ are the standard deviation of the Gaussian in time and frequency domain respectively. Gabor transform is often employed because it meets the bound with equality [1]. Fixed resolution makes it impossible to detect a small change in the time domain, for example, in Gabor transform, an edge can not be located with a precision better than the standard deviation of the Gaussian [1]. See Figure 2,
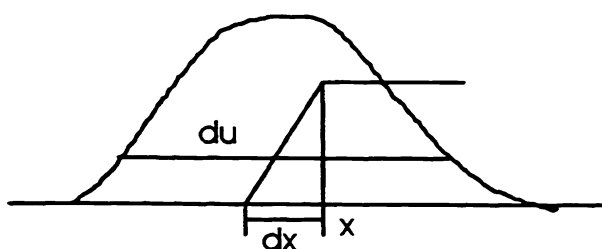


Figure 2, a window function whose standard deviation is du and an edge whose variation is dx.

In Figure 2, if the variation (dx) is small compared to du, the response of Gabor transform changes slowly. Therefore STFT (Gabor transform) is suitable for analysis of stationary signals. For non-stationary signals, for example, in most natural images or textures, where flexible components are often included, STFT can not give us effective support.

Some of the pitfalls associated with STFT such as fixed resolution can be overcome by the application of wavelets.

## 2.2 WAVELETS

Wavelets is a set of functions which are the translation and/or dilation of a "mother wavelet". It is, by no means, a new technology. One of the most famous wavelet, Haar wavelet, was developed in the beginning of this century. The idea of looking at signals in different resolution (scale) has also been realized and applied in many areas. But the real life of wavelets began as Morlet (geophysicist), Grossmann (physicist) and Meyer (mathematician), provided it with strong mathematical foundation. They called their work as "ondelettes" (wavelets). Daubechies and Mallat connected the wavelets theory with digital signal processing in its discrete form [1], [2]. Through these, wavelets became a "popular topic" in digital signal processing [3], [4], [12], image processing, especially in image coding [7] and image texture analysis [8], [9], [10], [11].

Wavelets is defined as:
$$h_{b,a}(t) = \frac{1}{\sqrt{a}} h^*(\frac{t-b}{a}) \tag{6}$$

the continuous wavelet transform is defined as:
$$CWT(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} h^*(\frac{t-b}{a}) s(t) dt \tag{7}$$

where $s(t)$ is the signal, $a$ and $b$ are the dilation and translation factors respectively and $h(t)$ is called mother wavelet. Wavelets are a set of functions which are the translation (refer to scale $b$ in (6)) and dilation (refer to scale $a$ in (6)) of the original mother wavelet. Wavelet transform is to decompose the signal ($s(t)$ in (7)) into the set of wavelet functions.

Wavelet transform obtains a flexible resolution in both time and transform domain. As $a$ is large value, the wavelet function is the dilation of the mother wavelet, which has low resolution in the time domain and high resolution in the transform domain. As $a$ becomes smaller, finer resolution in time domain and coarser resolution in the transform domain are obtained.

The basis function for Gabor wavelets is expressed as:
$$h(t) = \exp[\alpha^{2j} \cdot \frac{t^2}{2}] \cos(\alpha^j \cdot 2\pi \cdot t) \tag{8}$$

In signal analysis-synthesis process, orthonormal basis is essential, and so is in wavelet transform. An analyzing wavelet should meet two conditions:

$$\int_{-\infty}^{+\infty}|h(t)|^2\,dt < \infty \tag{9}$$

$$2\pi\int_{-\infty}^{\infty}\frac{|H(\omega)|^2}{|\omega|}d\omega < \infty \tag{10}$$

where $H(\omega)$ is the Fourier transform of $h(t)$. The first condition implies that the wavelet functions should have finite energy, and the second condition, which is often called the admissibility condition, implies that if $H(\omega)$ is smooth then $H(0) = 0$. The current focus of the research in wavelets is to find various optimal and orthogonal wavelets function for different tasks.
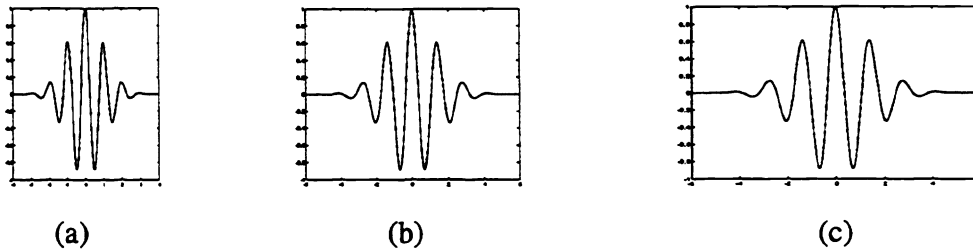


(a)             (b)             (c)

Figure 3, One dimensional Gabor wavelets basis functions where $\alpha = \dfrac{1}{\sqrt{2}}$ and a) $j$=0 b) $j$=1 and c) $j$=2

Employing continuous wavelet transform in practical applications is difficult due to its redundancies and theoretical nature. In order to resolve this problem, the scale is discretized. If the scale $a$ and $b$ are discrete values, the wavelet transform is called discrete wavelet transform. The scales are often taken as:

$$a = a_0^m \qquad b = nb_0a_0^m \tag{11}$$

then the wavelets are represented as:

$$h_{mn}(t) = a_0^{-m/2}h(a_0^{-m}t - nb_0) \tag{12}$$

The discrete wavelet transform is convenient for the applications in discrete signal processing, especially in digital image processing, because images are two dimensional discrete signals.

In this work, discrete form of wavelets is used. Similar to STFT, wavelets can be viewed as band-pass filters. Equation (6) can be rewritten as:

$$Wf(t) = f * s(a,b) \tag{13}$$

It could be interpreted that at the position $b/a$ and with a scale $a$, the signal $f$ is convoluted by the wavelets, in another words, the signal is filtered by a band-pass filter whose impulse response is $s(t)$. By way of this interpretation, wavelets are connected to the idea of filters-bank.

## 3. GABOR TRANSFORM AND WAVELETS IN IMAGE TEXTURE CLASSIFICATION

Experiments on Human Visual systems have shown that the location of intensity changes within the filed of view can be detected through the activities of localized cells tuned to certain frequency and orientation. Multiresolution decomposition enables us to obtain the texture features in joint space-frequency representation. There are numerous research works in this area, [6], [9], [10], [13]-[19]. In most of these works, Gabor elementary function is used [14]-[19].

Gabor elementary function is defined as:

$$h(x,y) = g(x',y')\exp[j2\pi(Ux+Vy)] \tag{14}$$

where $(x',y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$, $g(x,y)$ is two dimensional Gaussian function:

$$g(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\{-\frac{1}{2}[(\frac{x}{\sigma_x})^2 + (\frac{y}{\sigma_y})^2]\} \tag{15}$$

where $\sigma_x$ and $\sigma_y$ are the spread of the Gaussian in the x and y direction. Normally, $\sigma_x$ and $\sigma_y$ are assumed to be the same. So taking (15) into (14), rewrite the Gabor element function as:

$$g(x,y) = \frac{1}{2\pi}\exp[\frac{x^2+y^2}{\sigma^2}] \cdot \exp[j2\pi\omega_0(x\cos\theta + y\sin\theta)] \tag{16}$$

where $\omega_0 = \sqrt{U^2 + V^2}$ is the frequency of the sinusoid, and $\theta$ is the orientation of the sinusoid. Gabor element function actually is a two dimensional Gaussian modulated by sinusoid with the frequency $\omega_0$ and orientation $\theta$. The Fourier transform of (14) is:

$$H(u,v) = \exp\{-2\pi^2\sigma^2[([u-U]')^2 + ([v-V]')^2]\} \tag{17}$$

where

$$[(u-U)',(v-V)'] = [(u-U)\cos\theta + (v-V)\sin\theta, -(u-U)\sin\theta + (v-V)\cos\theta] \tag{18}$$

$H(u,v)$ is a Gaussian shifted in $U$ and $V$ units in frequency axes $(u,v)$ with an orientation of $\theta$.

There are a number reasons for choosing Gabor elementary functions for texture analysis:

- Joint space/spatial-frequency representation of image. Different choices of $\omega_0$ and $\theta$ make the function become a set of functions, facilitating the Multiresolution, multifrequency decomposition of the signals.

- Ability to detect orientation. Different choices of $\theta$ can make the function rotate in the $360°$ degree of orientation.

- Experiments in human visual system state that Gabor function is a good approximation of the receptive field profile of simple cells in the visual cortex [14]. Results from the psychological

experiments also suggest that the simple cells in the visual cortex have receptive field profiles that have fixed bandwidth in frequency and orientation domains.

The following Gabor elementary function is used in this work:

$$h(x,y) = \exp[-\frac{x^2+y^2}{2}] \cdot \exp[j\pi\omega_0(x\cos\theta+y\sin\theta)] \qquad (19)$$

In Gabor transform, with its fixed Gaussian function, equation (19) can be rewritten as:

$$h(x,y) = \exp[-\frac{x^2+y^2}{2}] \cdot \exp[j\pi\alpha^j(x\cos\theta+y\sin\theta)] \qquad (20)$$

where $\alpha = \frac{1}{\sqrt{2}}$, $j = 0,1,2...$ and $\theta = \frac{k\pi}{N}$, $N = 0,1,2,...$, $k = 0,1,2,...,N-1$. The different choices of frequency $j$ and orientation in spatial $\frac{k}{N}$ construct a set of filters.

The function of Gabor wavelets is:

$$h(x,y) = \exp[-\alpha^{2j}\frac{x^2+y^2}{2}] \cdot \exp[j\pi\alpha^j(x\cos\theta+y\sin\theta)] \qquad (21)$$

where the choice of $\alpha$ and $\theta$ are the same as that in Gabor transform. Comparing equation (20) and (21), the distinct difference between Gabor transform and Gabor wavelets is easily detected. As the frequency of the sinusoid changes, the window size remains the same in Gabor transform, but changes in the same way as the sinusoid in Gabor wavelets. This can be seen in Figures 1 and 3 which refer to one dimensional form of the Gabor transform and wavelets.

The elementary functions of Gabor transform and Gabor wavelets are used to construct spatial domain filters. Each filter is made of a pair of filters which are the real and imaginary part of the complex sinusoid. These pair are convoluted with texture image separately. The output of a filter is the modulation of the output of complex sinusoid. It is computed as:

$$Output = \sqrt{R^2_{output} + I^2_{output}} \qquad (22)$$

The mean of the outputs of one filter in different positions is stored as one feature of the texture. In another words, every filter is employed to capture one feature of a texture. For each texture, a multidimensional feature vector is constructed based upon the filters used.

## 4. RESULTS

The textures used here are all natural texture images from the Brodatz album in 256 gray-level and 256*256 pixels (Figure 4). Twenty samples of each of the textures is stored using a CCD camera. Ten samples of each image are used to form the feature vector templates and all are used in the classification

test. Confusion matrix is used to represent the experimental results. One hundred randomly selected pixels of each image are used in forming the feature vector templates and the classification test. Sixteen features are included which are four orientations ($0°, 45°, 90°, 135°$) and four frequencies ($j = 0,1,2,3$. in (20), (21)). The windows size of the Gabor transform is fixed as 17*17 in all frequencies of sinusoid, and that of Gabor wavelets is changed which is always four times the standard deviation of the Gaussian. The results of classification of these two methods are in table 1 and table 2. The comparison of the results of using these two methods is in table 3.

The results show that Gabor wavelets performs better than using Gabor filters in texture classification.
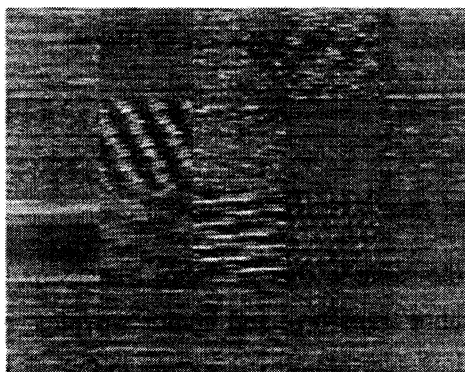


Figure 4, texture images

## 5. DISCUSSION

From the results, it can be concluded that both Gabor transform and Gabor wavelets are efficient in image texture classification. This is partly due to the joint frequency/spatial-frequency characteristic of Gabor elementary function and partly due to the fact that they both are sensitive to frequency and orientation variation. Gabor wavelets performs better because of its flexible resolution in both time and frequency domain.

Further research is needed in determining the optimal feature vector dimensionality. This could result in an object-orientated construction of the filter-bank. The degree of elasticity of the receptive field profile of the simple cells and their variance in response to changing environment is a challenging question that has major implications for the artificial vision studies based on human visual system.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 19 | | | | | | | | | | | 1 | | | | | | | | |
| 1 | | 20 | | | | | | | | | | | | | | | | | | |
| 2 | | | 19 | | | | | | | | | | | | | | | | 1 | |
| 3 | | | | 20 | | | | | | | | | | | | | | | | |
| 4 | | 1 | | | 17 | | | | 2 | | | | | | | | | | | |
| 5 | | | | | | 19 | | | | | | | | | | 1 | | | | |
| 6 | | | | | | | 20 | | | | | | | | | | | | | |
| 7 | | | | | | | | 20 | | | | | | | | | | | | |
| 8 | | | | | | | | | 20 | | | | | | | | | | | |
| 9 | | | 3 | | | | | | | 17 | | | | | | | | | | |
| 10 | | | | | | | | | | | 20 | | | | | | | | | |
| 11 | | | | | | | | | | | | 18 | | | | | 2 | | | |
| 12 | | | | | | | | | | | | | 20 | | | | | | | |
| 13 | | | | | | | | | | | | | | 20 | | | | | | |
| 14 | | | | | | | | | | | | | | | 20 | | | | | |
| 15 | | | | | | | | | | | | | | | | 20 | | | | |
| 16 | | | | | | | | | | | | | | | | | 19 | | | 1 |
| 17 | | | | | | | | | | | | | | | | | | 20 | | |
| 18 | | | | | | | | | | | | | | | | | | | 20 | |
| 19 | | | | | | | | | | | | | | | | | | | | 20 |

Table 1, confusion matrix of Gabor wavelets.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 15 | | | | | | | 3 | | | | 1 | | | 1 | | | | | |
| 1 | | 13 | | | | 5 | | | | 2 | | | | | | | | | | |
| 2 | | | 18 | | 2 | | | | | | | | | | | | | | | |
| 3 | | | | 18 | | | | | | | | | | 2 | | | | | | |
| 4 | | 1 | | | 16 | | | | 3 | | | | | | | | | | | |
| 5 | | | | | | 17 | | | | | 2 | | | | | 1 | | | | |
| 6 | | | 2 | | | | 18 | | | | | | | | | | | | | |
| 7 | | | | 5 | | | | 15 | | | | | | | | | | | | |
| 8 | | | | | | | | | 15 | | | 3 | | | | | 2 | | | |
| 9 | | | | | | | 1 | | | 16 | | 3 | | | | | | | | |
| 10 | 2 | | | | | | | | | | 18 | | | | | | | | | |
| 11 | | | | | | | | | | | | 20 | | | | | | | | |
| 12 | | 2 | | | | | | | | | | | 18 | | | | | | | |
| 13 | | | | | | | | | | | | | | 19 | | | | | 1 | |
| 14 | | | | | | | | | | | | | 3 | | 16 | | | | | 1 |
| 15 | | | 1 | | | | | | | | | | | | | 15 | 4 | | | |
| 16 | 3 | | | | | | | | | | | | | | | | 16 | | | 1 |
| 17 | | | | | 1 | | | | | | | 4 | | | | | | 15 | | |
| 18 | | | | 4 | | | | | | | | | | | | | | | 16 | |
| 19 | | | | | | | 2 | | | | | | | | | | | | | 18 |

Table 2, confusion matrix of Gabor transform functions.

| Texture | Wavelets | GT | Texture | Wavelets | GT |
|---------|----------|------|---------|----------|------|
| 0: D2   | 95%      | 75%  | 10: D49 | 100%     | 90%  |
| 1: D3   | 100%     | 65%  | 11: D50 | 90%      | 100% |
| 2: D9   | 100%     | 90%  | 12: D51 | 100%     | 90%  |
| 3: D12  | 100%     | 90%  | 13: D53 | 100%     | 95%  |
| 4: D14  | 85%      | 80%  | 14: D55 | 100%     | 80%  |
| 5: D17  | 95%      | 85%  | 15: D57 | 100%     | 75%  |
| 6: D20  | 100%     | 90%  | 16: D65 | 95%      | 80%  |
| 7: D24  | 100%     | 75%  | 17: D66 | 100%     | 75%  |
| 8: D26  | 100%     | 75%  | 18: D68 | 100%     | 80%  |
| 9: D35  | 85%      | 80%  | 19: D94 | 100%     | 90%  |

Table 3, results of the classification, where "Wavelets" is the test by Gabor wavelets and "GT" is the test by Gabor transform.

## 6. REFERENCE

[1]     S. G. Mallat, "Multifrequency channel decomposition of images and wavelet model", IEEE Transactions on ASSP, Vol. 37, No. 12, pp. 2091-2110, December 1989.

[2]     S. G. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 11, No. 7, pp. 674-693, July 1989.

[3]     O. Rioul and M. Vetterli, "Wavelets and signal processing", IEEE S. P. magazine, pp. 14-38, October 1991.

[4]     M. Vetterli and C. Herley, "Wavelets and filter banks: theory and design", IEEE Transactions on Signal Processing, Vol. 40, No. 9, pp. 2207-2232

[5]     J. Behar, M. Porat and Y. Y. Zeevi, "Image reconstruction from localized phase", IEEE Transactions on Signal Processing, Vol. 40, No. 4, pp. 736-743, April 1992.

[6]     M. Porat and Y. Y. Zeevi, "Localized texture processing in vision: analysis and synthesis in the Gaborian space", IEEE Transactions on Biomedical Engineering, Vol. 36, No. 1, pp. 115-129, January 1989.

[7]     M. Antonini, M. Berlaud, P. Mathieu and I. Daubechies, "Image coding using wavelet transform", IEEE Transactions on Image Processing, Vol. 1, No. 2, pp. 205-220, April 1992.

[8]     T. Chang and C. C. J. Kuo, "Texture analysis and classification with tree-structured wavelet transform", IEEE Transactions on Image Processing, Vol. 2, No. 4, pp. 429-440, October 1993.

[9]     A. Laine and J. Fan, "Texture classification by wavelets packet signatures", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 15, No. 11, pp. 1186-1191, March 1992.

[10]   A. C. Bovik, N. Gopal, T. Emmoth and A. Restrepo, "Localized measurement of emergent image frequencies by Gabor wavelets", IEEE Transactions on Information Theory, Vol. 38, No. 2, pp. 691-712, March 1992.

[11]   G. Wang, J. Zhang and G. Pan, "Solution of inverse problems in image processing by wavelet expansion", IEEE Transactions on Image Processing, Vol. 4, No. 5, pp. 579-593, May, 1995.

[12]   Q. Zhang and A. Benveniste, "Wavelet network", IEEE Transactions on Neural Networks, Vol. 3, No. 6, pp. 889-898, November 1992.

[13]   D. A. Pollen and S. Ronner, "Visual cortical neurons as localized spatial frequency filters", IEEE Transactions on System, Man, and Cybernetics, Vol. smc-13, No. 5, pp. 907-916, October 1983.

[14]   J. G. Daugman, "Uncertainty relation for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters", J. Opt. Soc. Am. A., Vol. 2, No. 7, pp. 1160-1169, July 1985.

[15]   S. J. Anderson and D. C. Burr, "Receptive field properties of human motion detector units inferred from spatial frequency masking", Vision Research, Vol. 29, No. 10, pp. 1343-1358, 1989.

[16]   M. Clark and A. C. Bovik, "Experiments in segmentating texton patterns using localized spatial filters", Pattern Recognition, Vol. 22, No. 6, pp. 707-717, 1989.

[17]   J. G. Daugman, "Complete discrete 2-D Gabor transforms by neural networks for image analysis and compression", IEEE Transactions on ASSP, Vol. 36, No. 7, pp. 1169-1179, July 1988.

[18]   A. C. Bovik, M. Clark and W. S. Geisler, "Multichannel texture analysis using localized spatial filters", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 12, No. 1, pp. 55-73, January 1990.

[19]   D. Dunn, W. E. Higgins and J. Wakely, "Texture segmentation using 2-D Gabor elementary functions", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 16, No. 2, pp. 130-149, February 1994.

[20]   P. Brodatz, "Textures: A Photographic Album for artist and designers", New York, Denvor, 1966.

[21]   H. Greenspan, R. Goodman, R. Chellappa and C. H. Anderson, "Learning texture discrimination rules in a multiresolution system", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 16, No. 9, pp. 894-901, September 1994.