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1 **Three-dimensional DEM investigation of the fracture behaviour of**
2 **thermally degraded rocks with consideration of material anisotropy**

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10 **Abstract**

11 A complete understanding of the fracture behaviour of anisotropic rocks under
12 elevated temperatures is fundamentally important for rock and reservoir
13 engineering applications. This paper shows a three-dimensional numerical
14 investigation of the fracture behaviour of anisotropic sandstone, with
15 consideration of the effects of temperature and material anisotropy. In the study,
16 a 3D semi-circular bend (SCB) model was established by using the Discrete
17 Element Method (DEM). The thermal responses of different minerals and the
18 strength anisotropy of incipient bedding planes were considered in the model.
19 The DEM model was calibrated against a series of laboratory experiments on
20 Midgley Grit sandstone (MGS) that exhibits intrinsic anisotropy. The pure mode
21 I, mode II, and mixed-mode (I+II) fracture characteristics of the MGS were
22 investigated under elevated temperatures (up to 600 °C) using the established
23 DEM model. The thermal degradation (i.e., fracturing) of the rock, the fracture
24 load, the evolution of micro-cracks, and the stress-strain relationship around
25 notch tips were analysed, with emphasis on enlightening the micro-
26 mechanisms underlying the fracture behaviour. The results of the study were
27 discussed and then compared with experimental observations and theoretical
28 predictions.

29 **Keywords**

30 Anisotropic rock; Temperature; Fracture behaviour; Discrete Element Method;
31 Rock discontinuity

32 **Nomenclature**

A_b	the cross-sectional area of parallel bond
a	notch length
B1-B5	five bedding planes with different strengths
C_v	specific heat
CMTSS	conventional maximum tangential stress criterion
d, d_1, d_2, d_3, d_4	different bed spacings
DEM	Discrete Element Method
EGS	enhanced geothermal system
GASED	generalized average strain energy density
GMTSN	generalized maximum tangential strain criterion
GMTSS	generalized maximum tangential stress criterion
ISRM	International Society for Rock Mechanics and Rock Engineering
K_{Ic}, K_{IIc}, K_{Eff}	mode I, mode II and mixed-mode fracture toughness
k_n	parallel bond normal stiffness
L	bond length
LR	loading rate
MGS	Midgley Grit sandstone
P_f	fracture load
PFC	Particle Flow Code
R	radius of SCB specimen
r	particle radius
s	half of the support spacing in three-point bending tests
SCB	semi-circular bend
T	temperature
T^-	T^- stress in SCB specimens
t	thickness of SCB specimens
TPB	three-point bending
Y_I, Y_{II}, T^*	dimensionless geometry factors
α	linear thermal expansion coefficient

α_b	the linear expansion coefficient of bond material
α_p	the coefficient of linear thermal expansion
β	inclination of pre-existing notch
γ	thermal resistance
∇F	thermal force acting on parallel bond
∇r	change of particle radius
∇T	temperature increment

33

34 **1. Introduction**

35 A better understanding of the fracture characteristics of anisotropic rocks in high
 36 temperature environment is fundamentally important for many rock and
 37 reservoir engineering applications such as disposal of high-level radioactive
 38 waste [1-3], and design of hydraulic fractures for EGS [4]. The fracture of a rock
 39 material takes place when the stress intensity factor at the crack tip reaches a
 40 critical value, which is termed as fracture toughness. Fracture toughness is a
 41 crucial parameter of a rock material that describes its fracture behaviour and
 42 ability to resist fracturing [5]. This parameter is often used in fracture prediction
 43 [6] and reservoir stimulation control [7-8].

44 Previous investigations on fracture behaviour of rocks showed that fracture
 45 toughness can vary with factors like temperature, pressure [9-11], material
 46 anisotropy [12-13], geometrical properties of tested samples [14-15],
 47 experimental setup and the loading conditions [16]. Therefore, the rock fracture
 48 behaviour is complex and often difficult to be fully understood. However, there
 49 is a consensus that rock fracture toughness can increase under elevated
 50 temperatures until it reaches an elasto-plastic transition phase [17], after which
 51 a decrease can be expected [12,18-20]. The transition phase is rock type-
 52 dependent, and depends on mineralogy. De Castro lima and Paraguassu [21]
 53 pointed out that the increase in thermal expansion coefficient can be up to 20%
 54 when the quartz content for granitic rock is increased.

55 The process of fracture initiation and propagation is markedly affected by rock
 56 material anisotropy, which mainly arises from: (1) complex geological formation
 57 (due to the existence of irregular pores, flaws, unequally-cemented layers, and

58 unevenly-distributed micro-cracks, etc.), and (2) thermally- or mechanically-
59 induced cracks that increase the degree of heterogeneity. For anisotropic rocks
60 with pre-existing cracks, fractures may not propagate smoothly as expected for
61 the case of isotropic rocks but instead arrest due to the pre-existing cracks [22]
62 and sometimes divert into anisotropic planes [13, 23]. Na et al. [24] presented
63 a numerical investigation on the fracture initiation and propagation of layered
64 shale under Brazilian tests, and revealed that fractures can initiate, propagate
65 and coalesce in many ways due to the presence of anisotropic layers. Lee et
66 al. [25] experimentally demonstrated how the mechanically-induced fracture
67 interacted with pre-existing discontinuities (mineral veins) in notched SCB shale
68 samples. Three typical fracture patterns were observed in their study: (1)
69 crossing the veins with a slight deflection from planar path, (2) diversion along
70 veins, and (3) fracture initiation at the side of the notch (rather than the notch
71 tip), jogging, and propagation along the veins.

72 Geological bedding planes may have different strengths due to various
73 diagenetic (various deposition and sedimentation rates) and environmental
74 (weathering, unloading) conditions. However, the fracture complexity of
75 anisotropic rocks stemming from the bedding strength anisotropy is not well
76 understood, and associated studies are rare in literature. This is mainly due to
77 the fact that the incipency of a discontinuity is extremely difficult to be fully
78 reflected in the current experimentations, and the sampling of natural rocks and
79 sample preparation are always uncontrollable (the incipency of a discontinuity
80 refers to the relative tensile strength of that of parent rock [26, 27]). It is
81 extremely difficult to obtain a group of natural rock samples containing
82 discontinuities with controllable incipency [28]. The role of discontinuity
83 incipency has only been associated with how it affects fracture behaviour in
84 only one previous study [29]. However, only limited surrogate discontinuities
85 were prepared and tested in that study. This limitation can be overcome by
86 numerical techniques.

87 The main objective of this study was to investigate the effects of temperature
88 and material anisotropy on the fracture behaviour of anisotropic sandstone
89 through the Discrete Element Method (DEM). The semi-circular bend (SCB)
90 setup suggested by ISRM [30] was used for that purpose. The micro-

91 mechanisms underlying the fracture behaviour of the anisotropic rock under
92 temperature and loading were explored. The effects of bed spacing and relative
93 bed position on the fracture load and effective fracture toughness were
94 discussed. A review of available studies of the temperature effect on fracture
95 toughness was then reported, followed by discussion, and comparison of the
96 results of the present numerical study with theoretical predictions.

97 **2. Thermo-mechanical scheme and SCB sample genesis**

98 **2.1 Thermo-mechanical scheme used in the study**

99 In this study, the Particle Flow Code (PFC 3D) was used to mimic the thermo-
100 mechanical behaviour of rocks. This section briefly describes of the thermo-
101 mechanical scheme implemented in the PFC 3D. A detailed description of the
102 code is given in [31].

103 The thermal algorithm available in the PFC 3D allows the consideration of the
104 thermal expansion of both the bonded particles and the parallel bonds by
105 assigning different timescales to the mechanical and thermal processes [31].
106 Note that in the current PFC 3D algorithm, only the parallel bond model can
107 consider thermal expansion, not the bonds defined with the smooth joint contact
108 model. The thermal force (∇F) acting on the parallel bonds under high
109 temperatures was only accounted for at the normal contact direction, without
110 considering the bond lateral expansions, such as [31]:

$$111 \quad \nabla F = -k_n A_b \alpha_b L \nabla T \quad (1)$$

112 where k_n is the parallel bond normal stiffness, A_b is the cross-sectional area of
113 the parallel bond, α_b is the linear expansion coefficient of the bond material
114 which is the average value of the expansion coefficients of the particles
115 connected by the bond, L is the bond length, and ∇T is the temperature
116 increment.

117 In the thermal process, the particles can expand under high temperatures by
118 changing the particle radius (r), which is described as:

$$119 \quad \nabla r = -\alpha_p r \nabla T \quad (2)$$

120 where α_p is the coefficient of linear thermal expansion associated with the
121 granular particles.

122 **2.2 Genesis of SCB sample considering material anisotropy**

123 Fig. 1 shows the representative anisotropic SCB sample used in the study. The
124 sample contained incipient bedding planes with different tensile strengths. The
125 genesis of the SCB sample follows the four steps given below:

- 126 • Generation of particles — A group of assembly particles with a radius of
127 1.0-1.5 mm was generated in a semi-circular vessel with a radius (R) of
128 50 mm and a thickness (t) of 30 mm. The particle radius follows a uniform
129 distribution [32]. Each particle was randomly identified as a mineral type,
130 and four mineral groups were generated with a similar mineral
131 composition to the MGS sandstone (i.e., 70% quartz, 10% feldspar, 15%
132 clay and 5% biotite [27]).
- 133 • Insertion of anisotropic layers — Five incipient bedding planes (B1-B5)
134 with different tensile strengths [27, 33] were inserted into the SCB
135 sample. Note that, short transverse-oriented layers were focused in the
136 present study, as proposed by Roy et al. [29] and Shang et al. [16].
- 137 • Supporting and loading bars — One loading bar (red cylinder in Fig. 1)
138 and two supporting bars (blue cylinders, with a spacing of 55 mm) were
139 created, following the ISRM standard (i.e., $0.5 \leq s/R \leq 0.8$ [27]). A group
140 of particles (light green particles in Fig. 1) contacting the three bars was
141 generated, and a zero thermal expansion coefficient was assigned to
142 these particles to eliminate stress concentration at the bar-particle
143 contacts during the thermal treatment.
- 144 • Generation of pre-existing notches — Within each SCB sample, a notch
145 with a length (a) of 25 mm and an inclination (β) between 0° - 46° was
146 generated (Fig. 1), allowing a full consideration of fracture modes for the
147 specific combination (i.e., for the setup $a/R = 0.5$ and $s/R = 0.55$, $\beta = 0^\circ$
148 represents mode I, $\beta = 46^\circ$ mode II, and $0^\circ < \beta < 46^\circ$ mixed mode [16, 34]).

149 The generated SCB samples were first heated to the desired temperatures (up
150 to 600°C), before performing the three-point bending (TPB) tests by moving
151 the loading bar at a constant loading rate (LR) of 0.001 m/s (Fig. 1).

152 **2.3 Selection of micro-parameters and thermal parameters**

153 The micro-parameters of the parallel bond and the smooth joint of the SCB
154 sample have been calibrated by Shang et al. [32], by performing direct tensile
155 tests on intact MGS and bedding planes (see [32] for details). Fig. 2a shows a
156 comparison of the laboratory and DEM results; while the failure patterns of the
157 rock samples are shown in Fig. 2b. The calibrated micro-parameters used in
158 this study are shown in Table 1. It should be noted that tensile tests were used
159 to calibrate the parallel bond micro-parameters, based on two reasons: (1)
160 elimination of the intrinsic limitation of the parallel bond model that tends to
161 overestimate the tensile strength of rock; (2) from a macroscopic point of view,
162 the SCB samples were expected to exhibit tensile failures in the three-point-
163 bending tests.

164 A set of thermal parameters including linear thermal expansion coefficient (α),
165 thermal resistance (γ) and specific heat (C_v) were used in the study to consider
166 thermal effects [31]. Specifically, different linear thermal expansion coefficients
167 were assigned to the each mineral of MGS (i.e., quartz, $24.3 \times 10^{-6} \text{ K}^{-1}$; feldspar,
168 $8.7 \times 10^{-6} \text{ K}^{-1}$; biotite, $1.0 \times 10^{-6} \text{ K}^{-1}$; clay, $3.6 \times 10^{-6} \text{ K}^{-1}$ [35, 36]). The specific heat
169 of the DEM sample and thermal resistance per unit length were 920 J/ (kg K)
170 and 0.43 m K/W, respectively. The micro-parameters α and C_v were set equal
171 to the macro-values of the MGS.

172 **3. Results**

173 **3.1 Thermal degradation of rock**

174 Fig. 3 shows the thermal response of the simulated MGS sample ($\beta=0^\circ$) that
175 was heated to different temperatures ($20 \text{ }^\circ\text{C} \leq T \leq 600 \text{ }^\circ\text{C}$). For clarity, the
176 particles are not shown in the figure, only thermally induced microcracks and
177 the five bedding planes (i.e., B1-B5) are presented. As can be seen, a few
178 microcracks were thermally induced under relatively low temperatures ($T \leq 200$
179 $^\circ\text{C}$). Interestingly, those microcracks clustered around the pre-existing notch.
180 This was probably related to the creation of the pre-existing notch that has led
181 to a mechanical degradation of adjacent rock materials. Hundreds of
182 microcracks were induced within the rock sample when the applied temperature
183 was increased to 350 $^\circ\text{C}$, while thousands of microcracks can be seen when

184 the temperatures were further increased to 400, 500 and 600 °C. It is noted that
185 the failure of parallel bonds dominated under different temperatures, certainly
186 due to the fact that only the parallel bonds can consider thermal expansion in
187 the current PFC scheme as mentioned in section 2.1.

188 **3.2 Thermal influence on fracture load**

189 The temperature dependency of the fracture load in three-point-bending tests
190 has been reported by many researchers [11, 17-20]. A consensus has been
191 reached that the fracture load tends to increase with temperature and then
192 decrease above a certain threshold value. As demonstrated in Fig. 4a, slight
193 increases in the fracture loads of the isotropic MGS samples were observed for
194 all cases ($\beta=0^\circ - 46^\circ$) when the temperatures were raised to ~350 °C. A further
195 increase in temperature has led to a pronounced decrease in the fracture load.
196 However, the decrease in the fracture load above 350 °C was not due to mineral
197 transition as observed in the laboratory [37].

198 A similar relationship between the fracture load and temperature was noticed
199 for the anisotropic MGS (Fig. 4b). It was noted that the fracture loads of the
200 anisotropic MGS (Fig. 4b) slightly decreased compared with that of isotropic
201 MGS (Fig. 4a). The change in fracture load between the two types of rocks was
202 due to the presence of the five weaker bedding planes. For example, in ambient
203 temperature, the measured mode I fracture load of the isotropic MGS was 882
204 kN (black dots in Fig. 4a). It increased to 1045 kN when the temperature was
205 raised to 350 °C, and then decreased to 713 kN at a temperature of 600 °C.
206 For the anisotropic MGS, the corresponding fracture loads were 667 (20 °C),
207 780 (350 °C) and 323 kN (600 °C), respectively (Fig. 4b).

208 **3.3 Fracture complexity of anisotropic rocks having been thermally** 209 **degraded**

210 The origin of the fracture complexity of an anisotropic rock mainly arises from
211 the interaction between the mechanically induced fractures and the pre-existing
212 defects (i.e., pre-existing micro-cracks, anisotropic layers, and thermal cracks).
213 A fracture deviation is often expected in anisotropic rocks. Figs. 5-7 show how
214 the deviation occurred in the anisotropic MGS samples under different
215 temperatures and fracture modes (i.e., mode I, mode II and mixed-mode I+II).

216 In these figures, the particle displacement field (in the x direction) are presented,
217 showing positive horizontal displacements on the right side of the principal
218 failure plane and negative displacements on the left side. For clarity, the
219 particles are represented by the centre point; whilst the thermally induced
220 cracks (green) and mechanically induced cracks (black) are differentiated.

221 As shown in Fig. 5a, a curved principal fracture plane (indicated by the yellow
222 dashed line) was initiated from the pre-existing notch tip and diverted into the
223 relatively weak bedding plane (i.e., B3), similarly to what was observed in a
224 laboratory observation (Fig. 5n) [38]. The unexpected fracture plane observed
225 in Fig. 5n implied that some factors were at play (probably the pre-existing
226 micro-cracks that cannot be seen by the naked eye). However, the curved
227 fracture path shown in Fig. 5a was likely due to the anisotropic layers. This
228 hypothesis was further evidenced by the similar fracture planes shown in Figs.
229 5b, 5d, and 5f. Fig. 5i shows a similar fracture pattern observed in the
230 anisotropic Mancos shale [39], where the fracture initiated from the notch tip
231 and propagated approximately along a weaker bedding plane. A second
232 deviation along another adjacent bedding plane was seen, leading to an “S”-
233 shaped macroscopic failure plane.

234 Another interesting observation was that two curved fracture planes (all initiated
235 from the notch tips but on the opposite side) were induced in the samples that
236 were previously heated to 200 (Fig. 5c) and 350 °C (Fig. 5e). The generation of
237 the twin fractures was probably related to the thermally induced micro-cracks,
238 which were just located at the opposite side of the notch tips and promoted
239 fracture initiation. This fracture behaviour is, however, presently difficult to be
240 verified by experiments as it is still hard to find comparable laboratory
241 experiments that consider both temperature and material anisotropy.

242 It is known that a single fracture often initiates from notch tips in TPB tests,
243 although a secondary fracture deviation sometimes occurs (Fig. 5i). Apart from
244 this, the specimen can also fail away from the pre-existing notch, as
245 demonstrated in Fig. 5f. Fig. 5f shows that many micro-cracks were induced
246 around the supporting bar. A similar rupture was observed by Wang and Yang
247 [40] in their fracture toughness tests on a bedded coal (Fig. 5m). The

248 mechanisms underlying the rupture are complex and there are still no
249 consensus agreements. However, it is envisaged that the rupture can be
250 associated with the micro-cracks or other defects (like micro-poles) present in
251 the rock.

252 Fig. 6 shows the fracture characteristics of the samples under different
253 temperatures and mixed-mode (I+II) loading ($\beta=30^\circ$). The curved macro-
254 fractures were initiated from the pre-existing notch tips (Figs. 6a-6e). A special
255 feature was observed when the pre-heating temperature was increased to 400
256 °C, where the induced fracture diverted to the anisotropic layer B4 (Fig. 6f). This
257 observation was similar to that found in a laboratory study performed on an
258 artificial anisotropic sandstone (Fig. 6i, [29]). The SCB samples that were
259 heated to a relatively high temperature (600 °C) did not show a clear
260 macroscopic fracture plane (Fig. 6h), which was due to the widely spread
261 thermally induced micro-cracks that markedly changed the stress condition of
262 the rock.

263 Although the fracture behaviour of the samples under mode II loading ($\beta=46^\circ$
264 in Fig. 7) show some similarities with that of samples shown in Figs. 5 and 6, a
265 clear difference in the fracture pattern was observed. As indicated in Figs. 7c
266 and 7e, the mechanically induced fractures did not initiate from the notch tip,
267 but from the intersection points of the B4 layers and the pre-existing notches.
268 Similar experimental observations are shown in Figs. 7i and 7j [39, 41]. The
269 main reason for this unexpected fracture pattern was due to the thermally
270 induced cracks on the side of the pre-existing notches, which contributed to the
271 initiation of the curved fractures.

272 **3.4 Evolution of micro-cracks**

273 As an example, the evolution of micro-cracks against computational step under
274 different fracture modes (i.e., mode I, mode II, and mixed mode) is shown in
275 Fig. 8, where the induced micro-cracks are also shown. The tensile failure of
276 the parallel bonds dominated the three modes without exception, implying that
277 the tested SCB samples failed in tension from the microscopic point of view
278 (Fig. 8).

279 The number of micro-cracks increased gradually with computational step until
280 reaching a critical point where a dramatic increase in the number of micro-
281 cracks occurs.

282 **3.5 Stress versus strain around notch tips**

283 To understand the effect of temperature on the stress - strain relationship
284 around notch tips, three measurement spheres (embedded in PFC 3D) with a
285 constant radius of 7.5 mm (more than 5 times of the maximum particle radius)
286 were used, with their centers located at the notch tips. The measured notch tip
287 stress, strain, and displacement have three components (i.e., along the x, y and
288 z directions), which are related to principal values. The x component of those
289 measured values is presented considering the opening nature of the SCB
290 samples (i.e., along the x direction).

291 Fig. 9 shows the calculated stress - strain curves (in the x direction) for different
292 temperatures ($T = 20, 300$ and 600 °C). It is noted that the initial notch tip stress
293 measured under 600 °C was not null, mainly because of the excessive
294 expansions of the particles and bonds, which caused initial stress within the
295 sample prior to the mechanical loading. Clear fluctuations in notch tip stress
296 were measured until the peak values were reached. The peak notch tip stress
297 increased slightly when the temperature was increased from 20 to 300 °C, while
298 it decreased dramatically when the temperature was further raised to 600 °C.

299 **4. Synthesis, comparison, and discussion**

300 **4.1 DEM model genesis**

301 In the model genesis, we considered different strengths for the bedding planes.
302 This feature has been largely ignored in literature and is extremely difficult to
303 be fully reflected in the current experimentations, as mentioned in the
304 introduction. The numerical study reported here considered the bedding plane
305 incipency and is expected to help us to better understand the fracture
306 behaviour of anisotropic rocks. It can be anticipated that the displacement field
307 (Figs. 5-7) and fracture load (Fig 4) are different, if only one bedding plane
308 strength is considered.

309 In our model, each particle was randomly specified as a mineral type, and four
310 different minerals were generated (see Section 2.1), following the main mineral
311 composition of the targeted rock. To accurately reflect the thermal response of
312 the rock, different thermal parameters (i.e., thermal expansion coefficient) were
313 assigned to different minerals. This means that the particles in our model can
314 expand differently under temperature, which can mimic to the maximum extent
315 realistic thermal responses of real rock materials. It is expected that the results
316 reported in the study can be affected if only one mineral type was considered.

317 Rock minerals have distinct structures, and the physical and mechanical
318 properties (such as cementation or consolidation) between mineral grains can
319 be different. For simplicity, the interparticle bond properties have not been
320 modified to match the nature of rock minerals in this study, mainly because
321 differentiating the bond properties between different minerals would require a
322 tedious calibration process of the numeric model. In addition, to the best of the
323 authors' knowledge, laboratory experiments on the quantification of the
324 intergranular forces are rare in literature, which make the calibration even more
325 difficult.

326 **4.2 Bed spacing and position**

327 Sedimentary rocks have different bed spacings due to various rates of
328 deposition. The relative position of bedding planes can also vary due to the
329 diagenesis (i.e., sediment transportation and deposition). For simplicity, a
330 constant bed spacing (17.5 mm) was used in the study, which follows a general
331 description of the tested rock [27, 32]. To understand the effects of relative bed
332 position and spacing on fracture behaviour, five additional cases (i.e., Cases 2-
333 6) were studied, considering different relative bed position and spacing (Table
334 2) following Shang et al. [32]. Fig. 10 presents the fracture load against
335 temperature for the additional five cases. Comparing the five cases with that
336 shown in Fig. 4, a similar evolution trend in the fracture load magnitude versus
337 temperature was observed, irrespective of bed spacing and relative bed
338 position. The fracture loads initially increased (slightly) until ~350 °C and then
339 decreased dramatically. It is unsurprising that, there were clear differences in
340 the magnitude of fracture load for different cases, and that the fracture loads in

341 Cases 5 and 6 (Figs. 10d and 10e) were relatively larger than those measured
342 in Cases 2 - 4 (Figs. 10a - 10c).

343 4.3 Comparison of results with experimental observations

344 The present study also demonstrated that temperature indeed significantly
345 affected the fracture behaviour of anisotropic Midgley Grit sandstone, in terms
346 of fracture load (Figs. 4 and 10) and fracture complexity (Figs. 5-7). A
347 comparison of the fracture toughness of different rocks under various
348 temperatures is shown in Fig. 11. Note that equations (3) - (5) were used to
349 calculate the fracture toughness for all different modes [16], as it has been
350 verified that the influence of anisotropy has a very little influence on the stress
351 intensity factors [42].

$$352 \quad K_{ic} = \frac{P_f \sqrt{\pi a}}{2Rt} Y_i, \quad i=I, II \quad (3)$$

$$353 \quad K_{Eff} = \sqrt{K_{Ic}^2 + K_{IIc}^2} \quad (4)$$

$$354 \quad T = \frac{P_f}{2Rt} T^* \quad (5)$$

355 where T is the T stress in SCB specimens, Y_I and Y_{II} are dimensionless
356 geometry factors for mode I and mode II loading conditions, respectively. T^* is
357 also a geometry factor. Values of these factors of SCB specimens depended
358 on the notch length ($a = 25$ mm), span length ($2s = 55$ mm) as well as notch
359 angle ($0^\circ \leq \beta \leq 46^\circ$), as reported by Ayatollahi and Aliha [34] and Shang et al.
360 [16].

361 As illustrated in Fig. 11, the fracture toughness - temperature curves obtained
362 from the DEM study showed a similar pattern with those derived from laboratory
363 experiments. However, some discrepancies exists. Many rocks such as
364 Manoharpur sandstone [17] exhibited an increase trend in fracture toughness
365 until ~ 100 °C, after which their fracture toughness values decreased. This
366 observed initial increment in fracture toughness has been attributed to thermal
367 expansion of minerals [17], leading to the closing and compaction of pre-
368 existing cracks and pores. The present DEM simulations performed on the clay-

369 rich MGS showed a clear increment in mode I fracture toughness until ~350 °C
370 (Fig. 11a), which is more close to the experimental observations made by
371 Chandler et al. [19] on Darley Dale sandstone and by Funatsu et al. [11] on
372 Kimachi sandstone, where continuous increments of mode I fracture toughness
373 values were observed until 200 °C (no further data under higher temperatures).
374 These two sandstones are also clay-rich, and the dehydration as well as
375 dihydroxylation (which often occurs around 400 °C) of the clay minerals are
376 expected [37], which can degrade the layer structures of clay minerals, leading
377 to the decrease of fracture load and toughness. The compaction of the pre-
378 existing cracks and transition of clay minerals however cannot be revealed in
379 the current DEM model. However, a clear decrease in failure load above 350 °
380 C was observed certainly due to the effect from the thermal cracking (see Fig.
381 3). A similar temperature effect (compared with laboratory observations) was
382 observed on the fracture behaviour of clay-rich rocks.

383 **4.4 Comparison of results with theoretical predictions**

384 A comparison of the DEM results (in terms of $K_{\text{Eff}}/K_{\text{Ic}}$) of the present study with
385 the theoretical predictions (shown in Shang et al. [16]) from four different
386 fracture criteria is presented in Fig. 12. The derivation of the expressions
387 ($K_{\text{Eff}}/K_{\text{Ic}}$) of the four criteria is available in Shang et al. [16], and will not be
388 repeated here. As shown in Fig. 12a, the DEM results of the isotropic MGS
389 under various temperatures matched well with the theoretical predictions.
390 However, for the anisotropic MGS, a relatively large discrepancy was observed
391 (Fig. 12b), especially when the notch angle was 10° (much smaller effective
392 fracture toughness was numerically predicted). This difference can be
393 attributed to the combined effects from temperature and anisotropy (i.e. short-
394 transverse oriented layers). A clear discrepancy between our DEM results and
395 theoretical predictions was noted, and this needs a reliable experimental study
396 to testify if the classic fracture criteria can be used to assess the fracture
397 behaviour of thermally degraded anisotropic rocks.

398 **5. Conclusion**

399 This paper reports a three-dimensional DEM investigation of the thermal
400 influence on the fracture behaviour of sandstone, with consideration of material

401 anisotropy. The thermal degradation of the rock, the fracture complexity, the
402 evolution of micro-cracks, the fracture load, as well as the stress and strain
403 around notch tips were considered. The results of the DEM study were
404 compared with experimental results and with theoretical predictions.

405 Several conclusions can be drawn from the present study: (1) the combined
406 effects of temperature and material anisotropy significantly affected the fracture
407 behaviour of Midgley Grit sandstone; (2) a slight increase in fracture load was
408 observed when the applied temperature was increased to ~ 350 °C, after which
409 the number of thermal cracks increased dramatically, and the fracture load and
410 fracture toughness reduced significantly; (3) four main fracture patterns were
411 observed, which include single curved fracture, twin curved fractures (for mode
412 I, $T=350$ °C), fracture away from the pre-existing notch tips (for mode II, $T=200$
413 °C), and rupture of rock matrix. The deviation of the fracture into anisotropic
414 layers was often observed for the transverse oriented SCB samples, leading to
415 fracture complexity; (4) The DEM results reported in the study agreed broadly
416 with experimental results and theoretical predictions; (5) Laboratory fracture
417 toughness experiments are needed to testify if the classic fracture criteria can
418 be used to assess the fracture characteristics of thermally degraded anisotropic
419 rocks.

420 **Conflict of interest**

421 We have no conflict of interests to declare

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546 **Figure captions**

547 **Fig 1** Setup of the semi-circular bend DEM model containing bedding planes
548 with different strengths.

549 **Fig 2** Calibration of the DEM model [32]. (a) Comparison of the stress-strain
550 curves obtained from laboratory experiments and the calibrated DEM model,
551 and (b) fracture patterns of rock samples in direct tension.

552 **Fig 3** DEM simulation of the degradation and fracture of Midgley Grit sandstone
553 (MGS) subjected to different temperatures (particles are not shown for clarity).

554 **Fig 4** Fracture load versus temperature in the TPB tests. (a) Isotropic MGS and
555 (b) anisotropic MGS.

556 **Fig 5** (a-h) Fracture characteristics of the thermally degraded SCB rocks under
557 mode I loading ($\beta=0^\circ$). (i-n) Experimental observations on different geo-
558 materials (bedded coal and isotropic sandstone) under similar loading condition.
559 Particle displacement field in the x direction is presented.

560 **Fig 6** (a-h) Fracture characteristics of the thermally degraded SCB rocks under
561 mixed-mode (I+II) loading ($\beta=30^\circ$). (i-j) Experimental observations on different
562 geo-materials (anisotropic sandstone and bedded coal) under similar loading
563 condition.

564 **Fig 7** (a-h) Fracture characteristics of the thermally degraded SCB rocks under
565 mode II loading ($\beta=46^\circ$). (i-j) Experimental observations on different rocks
566 (Mancos shale and sandstone) under similar loading condition.

567 **Fig 8** Evolution of micro-cracks against computational time step for the
568 simulated rock with different modes and temperatures. (a) Mode I and $T= 20$
569 $^\circ\text{C}$, (b) mixed-mode and $T=100^\circ\text{C}$, and (c) mode II and $T= 200^\circ\text{C}$.

570 **Fig 9** (a) Notch tip stress versus strain, and (b) notch tip stress versus
571 displacement ($\beta=30^\circ$; $T= 20, 300,$ and 600°C).

572 **Fig 10** Influence of bed spacing and relative bed position on the fracture load.

573 **Fig 11** Fracture toughness against temperature: comparison of DEM results
574 with experiments (from literature). (a) Mode I, (b) mode (II), and (c) mixed-mode.

575 **Fig 12** Effective fracture toughness of Midgley Grit sandstone against notch
576 angle: comparison of DEM results with theoretical predictions from four fracture
577 criteria (ambient temperature). (a) isotropic rock, and (b) anisotropic rock.