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## New results with near- Yang sequences

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## New results with near- Yang sequences

### Abstract

We construct new TW -sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new  $OD(60(2m + 1) + 4t; 13(2m + 1), 13(2m + 1), 13(2m + 1), 13(2m + 1))$  and new  $W(60(2m + 1) + 4t; 13s(2m + 1))$  for all  $t \geq 0$ ,  $m \leq 30$ ,  $s = 1, 2, 3, 4$ .

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## New Results with Near-Yang Sequences

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**Abstract.** We construct new  $TW$ -sequences, weighing matrices and orthogonal designs using near-Yang sequences. In particular we construct new  $OD(60(2m+1) + 4t; 13(2m+1), 13(2m+1), 13(2m+1), 13(2m+1))$  and new  $W(60(2m+1) + 4t; 13s(2m+1))$  for all  $t \geq 0$ ,  $m \leq 30$ ,  $s = 1, 2, 3, 4$ .

### 1. Introduction

For definitions we refer the reader to [9, Introduction] and [11, Section 2]. We give one new definition.

**Definition 1. (near-Yang sequences)** A triple  $(F, G, H)$  of sequences is said to be a set of near-Yang sequences for length  $n$  (abbreviated as  $NY(n)$ ) if the following conditions are satisfied.

- (i)  $F = (f_k)$  is a  $(0, 1, -1)$  sequence of length  $n$ .
- (ii)  $G = (g_k)$  and  $H = (h_k)$  are sequences of length  $n$  with entries  $0, 1, -1$ , such that  $G + H = (g_k + h_k)$  and  $G - H = (g_k - h_k)$  are both  $(0, 1, -1)$  sequences of length  $n$ .
- (iii)

$$\begin{aligned} g_s + g_{n-s+1} &\equiv 0 \pmod{2} \\ h_s + h_{n-s+1} &\equiv 0 \pmod{2} \end{aligned} \quad s = 1, \dots, \lfloor \frac{n}{2} \rfloor$$

- (iv)  $N_F(s) + N_G(s) + N_H(s) = 0$ ,  $s = 1, \dots, n-1$ .

where

$$N_X(s) = \sum_{i=1}^{n-s} x_i x_{i+s}.$$

### 2. Computational Results

In Koukouvinos, Kounias, Seberry, Yang and Yang [6] it is shown that if in (ii) of the definition  $G \pm H$  are both  $(1, -1)$  sequences then conditions (i), (ii) and (iv) imply condition (iii) but this is not true for near-Yang sequences. These sequences are normal sequences  $NS(\ell)$ .

We searched for normal sequences  $NS(\ell)$ .  $NS(\ell)$  do exist for the following lengths  $\ell \in \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 25, 26, 29, 32, \dots\}$  and they do not exist for  $\ell \in \{6, 14, 17, 21, 22, 23, 30, 46, 56, 62, 78,$

Table 1: Normal sequences via Simulated Annealing.

Length $\ell$	Sequences	Weight
20	$F = - - + + + + + + - + + - + - + + + - + -$	20
	$G = 0 + - 0 0 0 + 0 0 0 0 0 0 + 0 0 0 - + 0$	6
	$H = - 0 0 + + + 0 - + + - - + 0 - - - 0 0 +$	14
25	$F - - + - + - + + + - - - + + - - + - - +$	25
	$G = 0 - 0 - 0 + 0 + 0 + 0 - 0 + 0 - 0 - 0 - 0 + 0 + 0$	12
	$H = - 0 - 0 - 0 + 0 + 0 - 0 + 0 - 0 + 0 + 0 - 0 - 0 +$	13

94, ...} [3, 6, 16]. We note here that we found normal sequences of length 25 and 20 (Table 1) using simulated annealing; this is described in Gysin [3]. Sequences of length 25 can be obtained from Turyn sequences of lengths 13 and 12 and a complete search for these was carried out over 20 years ago. It is known that there are eight inequivalent sets of Turyn sequences of lengths 13 and 12 [2] and hence by the construction discussed in [6] probably at least sixteen inequivalent sets of normal sequences of length 25. It would be interesting to know if there are  $NS(25)$  which cannot be made from Turyn sequences. There exist  $NS(20)$  which cannot be made from Turyn sequences, an example is given in Table 1. In those small cases where  $NS(\ell)$  do not exist we searched for  $NY(n)$ , which contain more zeros in appropriate positions. We obtained the following new results:

$NY(n)$  with weight  $u = 12$  exist for the following lengths:  $n \in \{7, 11, 13, 15\}$ .

In Table 2 two conditions were imposed in counting the number of inequivalent triples of sequences: two triples of sequences were considered equivalent if one triple of sequences can be changed into the other triple of sequences by reversing and/or negating one or more sequences of the triple; if the three sequences  $F$ ,  $G$  and  $H$  all started or ended with '0' they were considered to be of smaller length and not counted for this length  $n$ .

This allows us to find new 4-complementary sequences of lengths  $15(2m+1)$ ,  $23(2m+1)$ ,  $27(2m+1)$ ,  $31(2m+1)$  and weights  $13(2m+1)$ ,  $m \leq 30$ .

### 3. Construction

**Definition 2. (suitable sequences)** [5, 8, 11]  $A, B, C, D$  are suitable sequences  $SS(m+p, m; w)$  with elements 0, 1, -1 of lengths  $m+p, m+p, m, m$  and total total weight  $w$  if  $A$  and  $B$  are disjoint,  $C$  and  $D$  are disjoint and  $A, B, C$ , and  $D$  have zero non-periodic autocorrelation function.

We use a modified version of Yang's [5, 8, 16] theorem

Table 2: New near-Yang sequences.

Length $n$	No of Seq.	Sequence Examples	Weight
7	2	$F = +++0--$	6
		$G = 0++0-+0$	4
		$H = +00000+$	2
		$F = +++0--$	6
		$G = 0+000+0$	2
		$H = +0+0-0+$	4
11	12	$F = -0+0+00000+$	4
		$G = 0+0+000+0-0$	4
		$H = +0+00000-0+$	4
13	24	$F = +0+0+0-0+000-$	6
		$G = 0+00000000+0$	2
		$H = +00+00000-00+$	4
15	26	$F = +0+00000+00000-$	4
		$G = 0+0000+0-0000+0$	4
		$H = +00+0000000-00+$	4

**Theorem 1.** Let  $A, B, C, D$  be  $SS(m+p, m; w)$  and  $F, G, H$  be  $NY(n)$  with total weight  $u$  and  $0', 0$  be sequences of zeros of length  $m+p$  and  $m$  respectively and  $X^*$  be the reverse sequence of  $X$  then

$$Q = \{A f_n, C g_1 - D h_1; 0', 0; A f_{n-1}, C g_2 - D h_2; 0', 0; \dots; A f_1, C g_n - D h_n; 0', 0; B^*, 0\}$$

$$R = \{B f_n, D g_n + C h_n; 0', 0; B f_{n-1}, D g_{n-1} + C h_{n-1}; 0', 0; \dots; B f_1, D g_1 + C h_1; 0', 0; -A^*; 0\}$$

$$S = \{0', 0; A g_n + B h_1, -C f_1; 0', 0; A g_{n-1} + B h_2, -C f_2; \dots; 0', 0; A g_1 + B h_n, -C f_n; 0', D^*\}$$

$$T = \{0', 0; -B g_1 + A h_n, D f_1; 0', 0; -B g_2 + A h_{n-1}, D f_2; \dots; 0', 0; -B g_n + A h_1, D f_n; 0', C^*\}$$

are  $TW$ -sequences of length  $(2m+p)(2n+1)$  and total weight  $(u+1)w$ .

This gives many new  $TW$ -sequences, weighing matrices and orthogonal designs. Many other corollaries are also possible.

**Example 1.** Let  $F = \{+++0--\}$ ,  $G = \{0+000+0\}$ ,  $H = \{+0+0-0+\}$  and  $A, B, C, D$  be suitable sequences of length  $m+p$  and  $m$  and total weight  $w$ . Then

with  $0'$  and  $0$  zero vectors of length  $m + p$  and  $m$  respectively we have

$$\begin{aligned}
Q &= \{-A, -D; 0', 0; A, C; 0', 0; -A, -D; 0', 0; 0', 0; 0', 0; \\
&\quad A, D; 0', 0; A, C; 0', 0; A, -D; 0', 0; B^*, 0\} \\
R &= \{-B, C; 0', 0; B, D; 0', 0; -B, -C; 0', 0; 0', 0; 0', 0; \\
&\quad B, C; 0', 0; B, D; 0', 0; B, C; 0', 0; -A^*, 0\} \\
S &= \{0', 0; B, -C; 0', 0; A, -C; 0', 0; B, -C; 0', 0; 0', 0; \\
&\quad 0', 0; -B, C; 0', 0; A, -C; 0', 0; B, C; 0', 0; D^*\} \\
T &= \{0', 0; A, D; 0', 0; -B, D; 0', 0; -A, D; 0', 0; 0', 0; \\
&\quad 0', 0; A, -D; 0', 0; -B, D; 0', 0; A, -D; 0', 0; C^*\}
\end{aligned}$$

are  $TW$ -sequences of length  $15(2m + p)$  and total weight  $13w$ .

**Corollary 1.** *Suppose there are suitable sequences of length  $m + p$ ,  $m + p$ ,  $m$ ,  $m$  and total weight  $w$ ,  $SS(m + p, m; w)$  and near-Yang sequences of length  $n$  and total weight  $u$ . Then there are  $TW$ -sequences of length  $(2n + 1)(2m + p)$  and total weight  $(u + 1)w$ .*

**Corollary 2.** *Suppose there exist  $SS(m + p, m; w)$ . Then since there are near-Yang sequences of length 7 and total weight 12 there are  $TW$ -sequences of length  $15(2m + p)$  and total weight  $13w$ .*

From [11] we see  $SS(m + 1, m; 2m + 1)$  exist for all  $m \leq 30$  hence there exist  $TW$ -sequences of length  $15(2m + 1)$  and weight  $13(2m + 1)$  for all  $m \leq 30$ . Using theorems 3.6, 3.7, 3.8 of [11] we have  $OD(60(2m + 1) + 4t; 13(2m + 1), 13(2m + 1), 13(2m + 1), 13(2m + 1))$  for all  $t \geq 0$  and  $m \leq 30$ . Furthermore  $Q, R, S, T$  can be used in the Goethals-Seidel array to form  $OD(4t; 13w, 13w, 13w, 13w)$  for every  $t > 15(2m + 1)$ .

Recalling that variables in an  $OD$  can be set equal or set zero to give weighing matrices, we obtain  $W(60(2m + 1) + 4t; 13s(2m + 1))$  and  $W(4t; 13sw)$ ,  $s = 1, 2, 3, 4$ . Since  $NS(n)$  are  $NY(n)$  with total weight  $2n$ .

**Corollary 3.** *Suppose there exist  $SS(m + p, m; w)$ . Then since there are normal sequences of length 25 and total weight 50 there are  $TW$ -sequences of length  $51(2m + p)$  and total weight  $51w$ . If  $w = 2m + p$  then we have  $T$ -sequences.*

Again using theorems 3.6, 3.7, 3.8 of [11] we have  $OD(104(2m + 1) + 4t; 51(2m + 1), 51(2m + 1), 51(2m + 1), 51(2m + 1))$  for all  $t \geq 0$  and  $m \leq 30$ . Furthermore  $Q, R, S, T$  can be used in the Goethals-Seidel array to form  $OD(4t; 51w, 51w, 51w, 51w)$  for every  $t > (2n + 1)(2m + 1)$ .

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