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Abstract

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COLOURED DESIGNS, NEW GROUP DIVISIBLE DESIGNS AND PAIRWISE BALANCED DESIGNS

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1. Introduction

For the definitions of a balanced incomplete block design (BIBD), a partially balanced incomplete block design (PBIBD) and a mutually orthogonal Latin square we refer the reader to Raghavarao (1971). A *group divisible design* (GDD) is a BIBD with S being the set of symbols and $B=G\cup X$ being the set of blocks, where G is a partition of S and where each block in X intersects each block in G in at most one point.

In a recent paper Sarvate and Seberry (1986) introduced a method for encrypting secret messages using crypto designs. These designs are often hard to find, but designs with some relaxed conditions can be used for encryption similarly to crypto

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designs. In this note we study a class of such crypto designs, which we call coloured designs (CD). CD's have an important application besides encryption: they are used to produce new group divisible designs.

A matrix is *x-coloured* if each non-zero entry is replaced with one symbol from a given set of x symbols; it is properly x-coloured if each of the x symbols occurs at most once in each row and at most once in each column. A *coloured* design CD(v, b, r, k, λ) or a CD($v, b, r, k, \lambda_1, \lambda_2, \ldots$) is a properly r-coloured incidence matrix of a BIBD(v, b, r, k, λ) or a PBIBD($v, b, r, k, \lambda_1, \lambda_2, \ldots$) respectively. (This has been called a colourable design elsewhere (Seberry (1987), de Launey and Seberry (1987)).)

Of course each symbol occurs exactly once in each row of a coloured design.

Latin squares (see Denes and Keedwell (1974)), Graeco–Latin designs (see, for example, Preece (1976), Seberry (1979), Street (1981), Stirling and Wormald (1976)) and balanced Room squares (see Wallis, Street and Wallis (1972)) can immediately be used as coloured designs.

2. Main theorem

Theorem 2.1. *The incidence matrix of any block design, (V, B), with treatment set V and set of blocks B can be coloured with R colours where*

$$R = \max_{v \in V,\, b \in B} (r_v, k_b)$$

with r_v the number of occurrences of treatment v and k_b the number of elements in block b.

Proof. Form a bipartite graph, G, with vertex sets V and B. Join $i \in V$ to $j \in B$ if and only if $i \in j$. Then, since each symbol i occurs in $r_i \leq R$ blocks, each vertex i has degree r_i and since each block contains $k_j \leq R$ symbols, each vertex j has degree k_j. We can edge-colour G with $\Delta(G) = R$ colours. This edge-colouring induces a colouring of the design of the required form (that is, colour symbol i in block j with colour c iff the edge $\{i, j\}$ is coloured with c).

Corollary 2.2. *If there exists a* BIBD(v, b, r, k, λ) *or a* PBIBD($v, b, r, k, \lambda_1, \lambda_2, \ldots$), *then there exists a* CD(v, b, r, k, λ) *or a* CD($v, b, r, k, \lambda_1, \lambda_2, \ldots$) *respectively.*

Coloured designs are used in Seberry (1987b), and de Launey and Seberry (1987) to construct new families of BIBD's and GDD's.

3. Main application

The matrices described in the following proof can also be constructed from generalized Hadamard matrices and latin squares but here we use a simpler formulation.

Theorem 3.1. *If there exists a* CD(v, b, r, k, λ) *where* $r-1=q$ *is a prime power, then there exists a group divisible design*

$$\mathrm{GDD}(vq^2,\ bq^2,\ (q+1)q,\ kq,\ \lambda_1=q,\ \lambda_2=\lambda,\ m=q^2,\ n=v).$$

Remark. We can apply the same technique as in the following proof for coloured PBIBD's to obtain families of PBIBD's with more associate classes.

Proof. Take the $q+1$ matrices of order q^2, $R_0, \ldots, R_q$, defined by Seberry (1986), (and which have appeared earlier in many forms; for example see Wallis (1971) and Glynn (1978)), which satisfy

$$\sum_{i=0}^{q} R_i R_i^{\mathrm{T}} = q^2 I + q^J, \qquad R_i R_j^{\mathrm{T}} = J, \qquad R_i J = qJ.$$

These matrices exist whenever q is a prime power. Now replace symbol i of the CD by R_i and each 0 by the zero matrix of order q^2 to obtain the result.

Example 1. Consider the CD(9, 12, 4, 3, 1) given in Table 1.

Table 1

a			b			c			d		
b				c			a			d	
c					d			a			b
	c		a				b				d
	d			b				c	a		
	a				c	d				b	
		c	d					b		a	
		a		d		b					c
		d			a		c		b		

Here $r-1=3=q$ (for notation, see Wallis (1971) and Seberry (1987b)), and hence we can define

$$T=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad J_3=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

and R_0, R_1, R_2, R_3 given by

$$R_0=\begin{bmatrix} I & I & I \\ I & I & I \\ I & I & I \end{bmatrix}, \qquad R_1=\begin{bmatrix} I & T & T^2 \\ T^2 & I & T \\ T & T^2 & I \end{bmatrix},$$

$$R_2 = \begin{bmatrix} I & T^2 & T \\ T & I & T^2 \\ T^2 & T & I \end{bmatrix}, \qquad R_3 = \begin{bmatrix} J_3 & 0 & 0 \\ 0 & J_3 & 0 \\ 0 & 0 & J_3 \end{bmatrix}.$$

Now we replace a by R_0, b by R_1, c by R_2 and d by R_3 in the CD(9, 12, 4, 3, 1) to obtain the GDD(81, 108, 12, 9, 3, 1, 9, 9) given in Table 2, which is not listed in Clatworthy (1973) (Clatworthy lists GDD's with $r \le 10$).

Table 2

R_0 R_1 R_2	R_1 R_2 R_3	R_2 R_0 R_0	R_3 R_3 R_1
R_2 R_3 R_0	R_0 R_1 R_2	R_1 R_2 R_3	R_3 R_0 R_1
R_2 R_0 R_3	R_3 R_3 R_0	R_1 R_1 R_2	R_0 R_2 R_1

Corollary 3.2. (i) *If $q = 3v - 4 \ge 5$ is a prime power then there exists a*

$$\text{GDD}(q^2(q+4)/3,\ q^2(q+1)(q+4)/9,\ q(q+1),\ 3q,\ \lambda_1 = q,\ \lambda_2 = 6,\ m = q^2,\ n = (q+4)/3).$$

(ii) *If $q = 3t - 1$ is a prime power then there exists a*

$$\text{GDD}((2t+1)(3t-1)^2,\ t(2t+1)(3t-1)^2,\ 3t(3t-1),\ 3(3t-1),\ \lambda_1 = 3t-1,$$
$$\lambda_2 = 3,\ m = (3t-1)^2,\ n = 2t+1).$$

(iii) *If $q = \lambda v - \lambda - 1$ is a prime power then there exists a*

$$\text{GDD}(vq^2,\ \lambda v(v-1)q^2/2,\ q(q+1),\ 2q,\ \lambda_1 = q,\ \lambda_2 = \lambda,\ m = q^2,\ n = v).$$

(iv) *If $q = (\lambda v - \lambda - 3)/3$ is a prime power and 4 divides $\lambda v(v-1)$ then there exists a*

$$\text{GDD}(vq^2,\ \lambda v(v-1)q^2/12,\ q(q+1),\ 4q,\ \lambda_1 = q,\ \lambda_2 = \lambda,\ m = q^2,\ n = v).$$

(v) *If $q = (\lambda v - \lambda - 4)/4$ is a prime power and 5 divides $\lambda v(v-1)$ then except when $v = 15$ and $\lambda = 2$ there exists a*

$$\text{GDD}(vq^2,\ \lambda v(v-1)q^2/20,\ q(q+1),\ 5q,\ \lambda_1 = q,\ \lambda_2 = \lambda,\ m = q^2,\ n = v).$$

Proof. (i) For all $v \ge 3$ there exists a CD(v, $v(v-1)$, $3(v-1)$, 3, 6).

(ii) For all $v = 2t + 1 \ge 3$ there exists a CD(v, tv, $3t$, 3, 3).

(iii) For all v there exists a CD(v, $\lambda v(v-1)/2$, $\lambda(v-1)$, 2, λ).

(iv) If 4 divides $\lambda v(v-1)$ and 3 divides $\lambda(v-1)$ then there exists a CD(v, $\lambda v(v-1)/12$, r, 4, λ).

(v) If 5 divides $\lambda v(v-1)$ and 4 divides $\lambda(v-1)$ then there exists a $\mathrm{CD}(v, \lambda v(v-1)/20, r, 5, \lambda)$ unless $v=15$ and $\lambda=2$.

For example, using $v=3$ in Corollary 3.2(i), there exists a

$$\mathrm{GDD}(75, 150, 30, 15, \lambda_1=5, \lambda_2=6, m=25, n=3).$$

Using $t=1$ and then 2 in Corollary 3.2(ii) shows that there exists a

$$\mathrm{GDD}(12, 12, 6, 6, \lambda_1=2, \lambda_2=3, m=4, n=3)$$

which is SR68 in Clatworthy and a

$$\mathrm{GDD}(125, 250, 30, 15, \lambda_1=5, \lambda_2=3, m=25, n=5).$$

Corollary 3.3. (i) *If $q=2v-3$ is a prime power then there exists a*

$$\mathrm{GDD}(vq^2, v(v-1)q^2/2, 2(v-3)(v-1), 4(2v-3), \lambda_1=2v-3, \lambda_2=6, m=q^2, n=(q+3)/2).$$

(ii) *If $q=4u-1$ is a prime power then there exists a*

$$\mathrm{GDD}((3u+1)q^2, u(3u+1)q^2, 4u(4u-1), \lambda_1=4u-1, \lambda_2=4, m=q^2, n=3u+1)$$

(iii) *If $q=4u-1$ is a prime power then there exists a*

$$\mathrm{GDD}((q+2)q^2, q^2(q+1)(q+2)/4, q(q+1), 4q, \lambda_1=q, \lambda_2=3, m=q^2, n=q+2).$$

Proof. Use Corollary 3.2(iv) with (i) $\lambda=6$, (ii) $\lambda=4$ writing $v=3u+1$ and (iii) $\lambda=3$ writing $v=4u+1$.

For example, using $v=4$ and then 5 in Corollary 3.3(i), there exists a

$$\mathrm{GDD}(100, 150, 30, 20, \lambda_1=5, \lambda_2=6, m=25, n=4)$$

and a

$$\mathrm{GDD}(245, 490, 56, 28, \lambda_1=7, \lambda_2=6, m=49, n=5).$$

Using $u=1$ and then 2 in Corollary 3.3(ii) there exists a

$$\mathrm{GDD}(36, 36, 12, 12, \lambda_1=3, \lambda_2=4, m=9, n=4)$$

and a

$$\mathrm{GDD}(343, 2401, 196, 28, \lambda_1=7, \lambda_2=4, m=49, n=7).$$

Using $q=3$ and then 7 in Corollary 3.3(iii) there exists a SBIBD(45, 12, 3) and a

$$\mathrm{GDD}(441, 1764, 56, 28, \lambda_1=7, \lambda_2=3, m=49, n=9).$$

Corollary 3.4. (i) *If $q=5u-2$ or $5u-1$ and is a prime power then there exists a*

$$\mathrm{GDD}(q^2(q+2), q^2(q+1)(q+2)/5, q(q+1), 5q, \lambda_1=q, \lambda_2=4, m=q^2, n=q+1).$$

(ii) *If $q=5u-1$ is a prime power then there exists a*

$$\mathrm{GDD}(q^2(4u+1), q^2u(4u+1), q(q+1), 5q, \lambda_1=q, \lambda_2=5, m=q^2, n=4u+1).$$

Proof. Use Corollary 3.2(v) with $\lambda=4$ writing v as $5u$ or $5u+1$ and with $\lambda=5$ writing $v=4u+1$ respectively.

Remark. If $q=4$ this gives a

$$\mathrm{GDD}(80, 80, 20, 20, \lambda_1=4, \lambda_2=5, m=16, n=5)$$

which can be easily extended to an SBIBD(85, 21, 5).

Remark. This method can always be used to give

$$\mathrm{SBIBD}\left(\frac{p^{n+1}-1}{p-1}, \frac{p^n-1}{p-1}, \frac{p^{n-1}-1}{p-1}\right)$$

but, as these are all known, we do not pursue this construction.

Appendix 1 gives a listing of GDD's obtained by these methods using BIBD's listed in Mathon and Rosa (1985) for $r\leq 15$. We have a computer listing for $r\leq 41$.

4. Other designs

We note that a symmetric CD(v, k, λ) always exists whenever an SBIBD(v, k, λ) exists. Thus Theorem 3.1 can be reformulated as:

Theorem 4.1. *Let q be a prime power. Suppose an* SBIBD($q(q+1)/\lambda+1, q+1, \lambda$) *exists, then there exists a regular*

$$\mathrm{GDD}(q^3(q+1)/\lambda+q^3, q(q+1), \lambda_1=q, \lambda_2=\lambda, m=q^2, n=q(q+1)\lambda+1).$$

Trivially an SBIBD($q+2, q+1, q$) always exists and so does an SBIBD($q^2(q+2), q(q+1), q$) for q a prime power.

Also, suppose that we are interested in pairwise balanced designs: we note that an SBIBD(31, 6, 1) exists and a BIBD(6, 9, 9, 6, 9) exists. These give regular

$$\mathrm{GDD}(31.25, 30, \lambda_1=5, \lambda_2=1)$$

and

$$\mathrm{GDD}(6.25, 9.25, 45, 30, \lambda_1=5, \lambda_2=9).$$

Thus we have a

$$\mathrm{PBD}(6.25,\ 40.25,\ 75,\ k_1=30,\ k_2=6,\ \lambda=10).$$

For convenience, we state the generalization as a theorem noting that a $\mathrm{BIBD}(q+1, 2q-\lambda, 2q-\lambda, q+1, 2q-\lambda)$ always exists.

Theorem 4.2. *Let q be prime power. Suppose an* $\mathrm{SBIBD}(q(q+1)\lambda+1, q+1, \lambda)$ *exists, then there exists a pairwise balanced design*

$$\mathrm{PBD}(q^2(q+1),\ q^2(\lambda(q^2+q-1)+2q+1),\ q(3q+1-\lambda),$$
$$k_1=q(q+1),\ k_2=q+1,\ \lambda'=2q).$$

Appendix 1

	BIBD parameters					GDD parameters						
No.	v	b	r	k	λ	$v1$	$b1$	$r1$	$k1$	λ_1	λ_2	m
1	7	7	3	3	1	28	28	6	6	2	1	4
2	4	4	3	3	2	16	16	6	6	2	–	–
3	9	12	4	3	1	81	108	12	9	3	1	9
4	13	13	4	4	1	117	117	12	12	3	1	9
5	7	7	4	4	2	63	63	12	12	3	2	9
6	5	5	4	4	3	45	45	12	12	3	–	–
7	6	10	5	3	2	96	160	20	12	4	2	16
8	16	20	5	4	1	256	320	20	16	4	1	16
9	21	21	5	5	1	336	336	20	20	4	1	16
10	11	11	5	5	2	176	176	20	20	4	2	16
11	6	6	5	5	4	96	96	20	20	4	–	–
12	13	26	6	3	1	325	650	30	15	5	1	25
13	7	14	6	3	2	175	350	30	15	5	2	25
14	10	15	6	4	2	250	375	30	20	5	2	25
15	25	30	6	5	1	625	750	30	25	5	1	25
16	31	31	6	6	1	775	775	30	30	5	1	25
17	16	16	6	6	2	400	400	30	30	5	2	25
18	15	35	7	3	1	540	1260	42	18	6	1	36
19	8	14	7	4	3	288	504	42	24	6	3	36
20	15	15	7	7	3	540	540	42	42	6	3	36
21	8	8	7	7	6	288	288	42	42	6	–	–
22	9	24	8	3	2	441	1176	56	21	7	2	49
23	25	50	8	4	1	1225	2450	56	28	7	1	49
24	13	26	8	4	2	637	1274	56	28	7	2	49
25	9	18	8	4	3	441	882	56	28	7	3	49
26	49	56	8	7	1	2401	2744	56	49	7	1	49
27	57	57	8	8	1	2793	2793	56	56	7	1	49
28	19	57	9	3	1	1216	3648	72	24	8	1	64
29	10	30	9	3	2	640	1920	72	24	8	2	64
30	7	21	9	3	3	448	1344	72	24	8	3	64

Note that GDD's with $r1$ greater than 10 are not listed in Clatworthy (1973).

Appendix 1 (continued)

	BIBD parameters					GDD parameters						
No.	v	b	r	k	λ	$v1$	$b1$	$r1$	$k1$	λ_1	λ_2	m
31	28	63	9	4	1	1792	4032	72	32	8	1	64
32	10	18	9	5	4	640	1152	72	40	8	4	64
33	46	69	9	6	1		BIBD unknown					
34	16	24	9	6	3	1024	1536	72	48	8	3	64
35	28	36	9	7	2	1792	2304	72	56	8	2	64
36	64	72	9	8	1	4096	4608	72	64	8	1	64
37	73	73	9	9	1	4672	4672	72	72	8	1	64
38	37	37	9	9	2	2368	2368	72	72	8	2	64
39	25	25	9	9	3	1600	1600	72	72	8	3	64
40	19	19	9	9	4	1216	1216	72	72	8	4	64
41	21	70	10	3	1	1701	5670	90	27	9	1	81
42	6	20	10	3	4	486	1620	90	27	9	4	81
43	16	40	10	4	2	1296	3240	90	36	9	2	81
44	41	82	10	5	1	3321	6642	90	45	9	1	81
45	21	42	10	5	2	1701	3402	90	45	9	2	81
46	11	22	10	5	4	891	1782	90	45	9	4	81
47	51	85	10	6	1		BIBD unknown					
48	21	30	10	7	3	1701	2430	90	63	9	3	81
49	81	90	10	9	1	6561	7290	90	81	9	1	81
50	91	91	10	10	1	7371	7371	90	90	9	1	81
51	31	31	10	10	3	2511	2511	90	90	9	3	81
52	12	44	11	3	2	1200	4400	110	30	10	2	100
53	12	33	11	4	3	1200	3300	110	40	10	3	100
54	45	99	11	5	1	4500	9900	110	50	10	1	100
55	12	22	11	6	5	1200	2200	110	60	10	5	100
56	45	55	11	9	2	4500	5500	110	90	10	2	100
57	100	110	11	10	1		BIBD unknown					
58	111	111	11	11	1		BIBD unknown					
59	56	56	11	11	2	5600	5600	110	110	10	2	100
60	23	23	11	11	5	2300	2300	110	110	10	5	100
61	25	100	12	3	1	3025	12100	132	33	11	1	121
62	13	52	12	3	2	1573	6292	132	33	11	2	121
63	9	36	12	3	3	1089	4356	132	33	11	3	121
64	7	28	12	3	4	847	3388	132	33	11	4	121
65	37	111	12	4	1	4477	13431	132	44	11	1	121
66	19	57	12	4	2	2299	6897	132	44	11	2	121
67	13	39	12	4	3	1573	4719	132	44	11	3	121
68	10	30	12	4	4	1210	3630	132	44	11	4	121
69	25	60	12	5	2	3025	7260	132	55	11	2	121
70	61	122	12	6	1		BIBD unknown					
71	31	62	12	6	2	3751	7502	132	66	11	2	121
72	21	42	12	6	3	2541	5082	132	66	11	3	121
73	16	32	12	6	4	1936	3872	132	66	11	4	121
74	13	26	12	6	5	1573	3146	132	66	11	5	121
75	22	33	12	8	4		BIBD unknown					
76	33	44	12	9	3	3993	5324	132	99	11	3	121

Appendix 1 (continued)

	BIBD parameters					GDD parameters						
No.	v	b	r	k	λ	$v1$	$b1$	$r1$	$k1$	λ_1	λ_2	m
77	121	132	12	11	1	14641	15972	132	121	11	1	121
78	133	133	12	12	1	16093	16093	132	132	11	1	121
79	45	45	12	12	3	5445	5445	132	132	11	3	121
80	27	117	13	3	1	3888	16848	156	36	12	1	144
81	40	130	13	4	1	5760	18720	156	48	12	1	144
82	66	143	12	6	1	7986	17303	132	66	11	1	121
83	14	26	13	7	6	2016	3744	156	84	12	6	144
84	27	39	13	9	4	3888	5616	156	108	12	4	144
85	40	52	13	10	3		BIBD unknown					
86	66	78	13	11	2	9504	11232	156	132	12	2	144
87	144	156	13	12	1		BIBD unknown					
88	157	157	13	13	1		BIBD unknown					
89	79	79	13	13	2	11376	11376	156	156	12	2	144
90	40	40	13	13	4	5760	5760	156	156	12	4	144
91	27	27	13	13	6	3888	3888	156	156	12	6	144
92	15	70	14	3	2	2535	11830	182	39	13	2	169
93	22	77	14	4	2	3718	13013	182	52	13	2	169
94	8	28	14	4	6	1352	4732	182	52	13	6	169
95	15	42	14	5	4	2535	7098	182	65	13	4	169
96	36	84	14	6	2	6084	14196	182	78	13	2	169
97	15	35	14	6	5	2535	5915	182	78	13	5	169
98	85	170	14	7	1		BIBD unknown					
99	43	86	14	7	2	7267	14534	182	91	13	2	169
100	29	58	14	7	3	4901	9802	182	91	13	3	169
101	22	44	14	7	4	3718	7436	182	91	13	4	169
102	15	30	14	7	6	2535	5070	182	91	13	6	169
103	169	182	14	13	1	28561	30758	182	169	13	1	169
104	183	183	14	14	1	30927	30927	182	182	13	1	169
105	31	155	15	3	1	6076	30380	210	42	14	1	196
106	16	80	15	3	2	3136	15680	210	42	14	2	196
107	11	55	15	3	3	2156	10780	210	42	14	3	196
108	7	35	15	3	5	1372	6860	210	42	14	5	196
109	6	30	15	3	6	1176	5880	210	42	14	6	196
110	16	60	15	4	3	3136	11760	210	56	14	3	196
111	61	183	15	5	1	11956	35868	210	70	14	1	196
112	31	93	15	5	2	6076	18228	210	70	14	2	196
113	21	63	15	5	3	4116	12348	210	70	14	3	196
114	16	48	15	5	4	3136	9408	210	70	14	4	196
115	13	39	15	5	5	2548	7644	210	70	14	5	196
116	11	33	15	5	6	2156	6468	210	70	14	6	196
117	76	190	15	6	1	14896	37240	210	84	14	1	196
118	26	65	15	6	3	5096	12740	210	84	14	3	196
119	16	40	15	6	5	3136	7840	210	84	14	5	196
120	91	195	15	7	1	17836	38220	210	98	14	1	196
121	16	30	15	8	7	3136	5880	210	112	14	7	196
122	21	35	15	9	6	4116	6860	210	126	14	6	196

Appendix 1 (continued)

	BIBD parameters					GDD parameters						
No.	v	b	r	k	λ	$v1$	$b1$	$r1$	$k1$	λ_1	λ_2	m
123	136	204	15	10	1		BIBD unknown					
124	46	69	15	10	3		BIBD unknown					
125	28	42	15	10	5		BIBD unknown					
126	56	70	15	12	3	10976	13720	210	168	14	3	196
127	71	71	15	15	3	13916	13916	210	210	14	3	196
128	36	36	15	15	6	7056	7056	210	210	14	6	196
129	31	31	15	15	7	6076	6076	210	210	14	7	196

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