1987

FF-Pade Method of Model Reduction in Frequency Domain

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Abstract

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Disciplines

Computer Sciences | Physical Sciences and Mathematics

Publication Details

This article was originally published as Xiheng, H, FF-Pade Method of Model Reduction in Frequency Domain, IEEE Transactions on Automatic Control, AC-32(3), 1987, 243-246. Copyright Institute of Electrical and Electronics Engineers 1987. Original journal available here

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FF-Padé Method of Model Reduction in Frequency Domain

HU XIHENG

Abstract—**In this note the FF-Padé method based upon some new concepts in model reduction is presented. The new method will overcome the chief drawbacks of the current methods. Some typical examples are used to show convincingly that one has to break free from the conventional approaches in order to obtain better results in model reduction.**

INTRODUCTION

During the last two decades, much effort has been made to solve the problem of model reduction for large-scale systems. The current methods adopt either the time-domain or the frequency-domain approach and the latter can be further divided into several groups. The first group is the classical reduction method (CRM) which is based upon the classical theories of mathematical approximation or mathematical concept such as the Padé approximation, the continued-fraction method (second Cauer form), and the time-moment-matching method [1]–[3]. It can be proved that all the CRM approaches are equivalent to each other. There are, however, two serious drawbacks which limit the applications of CRM.

1) The reduced model obtained by CRM may be unstable although the original system is stable.

2) The low accuracy in the mid- and high-frequency ranges.

The second group is a development from CRM, and can be called the modified reduction method (MRM). The best known MRM is the stability-criterion method (SCM), in which the characteristic equation of the reduced model is assigned beforehand to satisfy one of some criteria of stability (such as the Routh stability criterion, Hurwitz polynomial, Mihailov criterion, the stability equation, etc.), while the parameters in the numerator are adjusted as in CRM to improve the degree of accuracy at the low-frequency range. However, the absolute stability of the SCM is achieved only at the cost of a serious loss of accuracy [5], [7]. There is another method for modeling transfer functions using basic performance specifications and frequency-response data at the dominant frequencies [4]. However, in that method one has to be faced with a set of nonlinear equations the solutions of which rely on special algorithms such as the Newton–Raphson multidimensional method. An extensive discussion for the estimation of good starting values was forwarded by [4] to ensure rapid convergence of the numerical approach.

In this note the FF-Padé method based upon some new concepts is presented. Some typical examples are used to show that one has to break free from the above conventional approaches in order to obtain better results in model reduction.

Manuscript received April 29, 1985; revised August 22, 1985, February 17, 1986, June 30, 1986, and September 19, 1986.

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IEEE Log Number 8612533.

THE NEW CONCEPTS

The new method is based on the following ideas.

1) An effective way of improving the CRM is to utilize as much mid-frequency information as is contained in the original system and fully incorporate it into the reduced model. When accurate fitting in the mid-frequency range is achieved, the stability problem will be solved as a natural consequence.

2) The technique for separate treatment of low-frequency and mid-frequency ranges.

3) The principle of an optimal allotment of the limited informational resources.

There are $(m + n + 1)$ independent parameters in an m/n-order reduced model, and our aim is to convey as much information of the original system as possible with this limited number of parameters. These parameters may be called the informational resources available to the model, which are, of course, much fewer than that of the original system.

There are two important ranges in the frequency response of a system:

1) the low-frequency range which includes the steady-state values and the long-term transients of the system; and

2) the mid-frequency range which involves the short-term transients and affects the stability of the control system.

This mid-frequency range which is sometimes called "dominant frequency" range, usually indicates a range containing a prominent peak or valley, or the cross frequency in the frequency response of the original system.

The CRM model obtained by expansion into a series about $s = 0$ can only fit the original system at the lower frequency range. Although many methods of MRM are posed to overcome the two drawbacks above, the methods, unfortunately, treat the two problems in an unrelated manner (for example, using SCM to guarantee stability and improve high frequency fitting accuracy by matching Markov parameters). The results by MRM are, therefore, usually unsatisfactory.

The two drawbacks of CRM do not emerge as two unrelated phenomena. According to the investigation conducted by the author, they are in effect two different symptoms out of one weakness [8]. The weakness is almost invariably due to the fitting inaccuracy in the dominant frequency range.

An optimal allotment of informational resources implies a prudent distribution of these resources among the prospective parameters of a reduced model in the two frequency ranges.

FF-PADÉ METHOD OF MODEL REDUCTION

In the frequency-fitting-Padé approximation method (FF-Padé method), the characteristic response of the original system at dominant mid-frequencies (usually oscillatory) is reproduced by point-fitting at two selected frequencies near each prominent peak or valley, while the low-frequency characteristic is modeled by the Padé approximation. Let the $\hat{m}/\hat{n}$-order original transfer function be:

$$G(s)=\frac{\hat{a}_0+\hat{a}_1 s+\cdots+\hat{a}_{\hat{m}} s^{\hat{m}}}{1+\hat{b}_1 s+\cdots+\hat{b}_{\hat{n}} s^{\hat{n}}}=\sum_{i=0}^{\infty} c_i s^i \tag{1}$$

$$(\hat{m} \leq \hat{n})$$

and its m/n-order reduced model be:

$$R^{FF}_{m/n}(s)=\frac{a_0+a_1 s+\cdots+a_m s^m}{1+b_1 s+\cdots+b_n s^n} \tag{2}$$

$$(m \leq n,\ n < \hat{n}).$$

The latter is obtained by the FF-Padé method in three steps as follows [5], [6].

Step 1: Build up the $((m + n + 1) - 2p) \times (m + n + 1)\, D$ equation set of Padé approximation as

$$\left[\begin{array}{c|cccc} & 0 & 0 & \cdots & 0 \\ & -c_0 & -c_{-1} & \cdots & -c_{-n+1} \\ I_{m+1} & \cdot & \cdot & & \cdot \\ & \cdot & \cdot & & \cdot \\ & -c_{m-1} & -c_{m-2} & \cdots & -c_{m-n} \\ \hline & c_m & c_{m-1} & \cdots & c_{m-n+1} \\ & c_{m-1} & c_m & \cdots & c_{m-n-2} \\ 0 & \cdot & \cdot & & \cdot \\ & \cdot & \cdot & & \cdot \\ & \cdot & \cdot & & \cdot \end{array}\right] \left[\begin{array}{c} a_0 \\ a_1 \\ \cdot \\ \cdot \\ a_m \\ \hline b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{array}\right] = \left[\begin{array}{c} c_0 \\ c_1 \\ \cdot \\ \cdot \\ c_m \\ \hline -c_{m+1} \\ -c_{m+2} \\ \cdot \\ \cdot \\ \cdot \end{array}\right] \tag{3}$$

$$c_i = \begin{cases} c_i & \\ 0 & \text{for } i<0. \end{cases}$$

Step 2: Form a $2p \times (m+n+1)$ D equation set for frequency-fitting at p points. Its typical form is as:

$$-a_0 \sin\theta_i + a_1\omega_i \cos\theta_i + a_2\omega_i^2 \sin\theta_i - a_3\omega_i^3 \cos\theta_i + \cdots - b_1\omega_i g_i + 0 + b_3\omega_i^3 g_i + 0 - \cdots = 0 \tag{4a}$$

$$a_0 \cos\theta_i + a_1\omega_i \sin\theta_i - a_2\omega_i^2 \cos\theta_i - a_3\omega_i^3 \sin\theta_i + \cdots + 0 + b_2\omega_i^2 g_i + 0 + b_4\omega_i^4 g_i + \cdots = g_i \qquad i = 1, 2, \cdots, p \tag{4b}$$

where g_i and θ_i are the amplitude and phase of the original system $G(s)$ at the frequency ω_i, which is chosen near a dominant frequency.

Step 3: Determine the parameters $a_0, \cdots, a_m, b_1, \cdots, b_n$ by solving the $(m+n+1)$ D linear equation set obtained by combining the results of Steps 1 and 2.

It can be seen that the computation in the FF-Padé method involves only the construction and solution of a set of linear equations.

It should be pointed out that the choice of frequency-fitting points of a system is quite crucial. If they are incorrectly picked, the low-order model may be unsatisfactory or even unstable. But the probability of obtaining a stable reduced model is quite high if the frequency-fitting points are picked in the neighborhood of a dominant peak or valley of the original system. The range of the neighborhood ω_i proximity may be suggested as follows:

$$\omega_i \in [\omega_L, \omega_H] \tag{5}$$

and

$$|G(\omega_H)| = |G(\omega_L)| = |G(\omega_p)| - 0.7g_0 \qquad \text{(for a peak)} \tag{6a}$$

$$|G(\omega_H)| = |G(\omega_L)| = |G(\omega_v)| + 0.7g_0 \qquad \text{(for a valley)} \tag{6b}$$

where the ω_p and ω_v are the frequencies at the peak or valley, respectively, $g_0 = |G(\omega_p)| - |G(\omega_v)|$ as in Fig. 1 (peak case).

Example 1 will show that different choice of frequency-fitting points within the proximity indicated as above would lead to a family of models with very similar frequency and time responses although the poles and zeros of one model may be situated in locations quite different from those of the corresponding poles and zeros of other models in the family.

Illustrative Examples

Example 1: The following system was given by Shamash [9]:

$$G(s) = \frac{(s+2)^2(s+5)^2(s+1000)}{(s+1)^2(s+10)^2(s+100)^2}. \tag{7}$$

The current methods of model reduction have given poor results. Application of CRM reveals that its 1/2 and 2/3 order models are unstable. Shamash proposed a stable 2/3 order reduced model by partial Padé method with a retained pole at -100 as follows:

$$R_{2/3}^{SH}(s) = \frac{5.367(s+1.4237 \pm j1.2806)}{(s+.8774)(s+2.2429)(s+100)}. \tag{8}$$

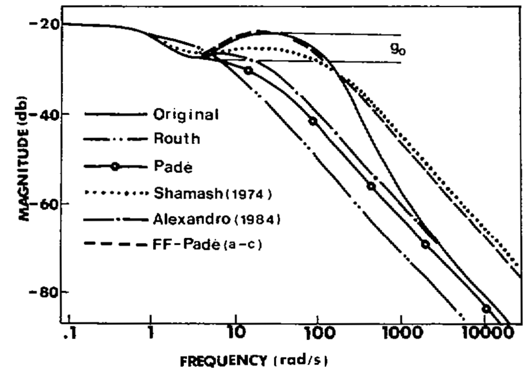

Fig. 1.

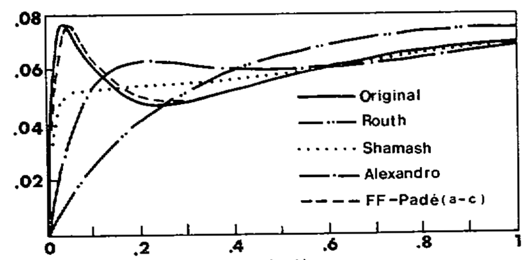

Fig. 2.

A current result by Alexandro [10] is

$$R_{2/3}^{AL}(s) = \frac{1.9(s+1.39 \pm j1.36)}{(s+.874)(s+2.46)(s+31.2)} \tag{9}$$

and the reduced model using the Routh method is

$$R_{2/3}^{RH}(s) = \frac{.303(s+1.03 \pm j.640)}{(s+.965)(s+1.04)(s+4.42)}. \tag{10}$$

From Figs. 1 and 2 it can be seen that all the above results are unsatisfactory. Letting $p = 2$ and choosing two fitting points at $\omega_1 = 6$ rad/s and $\omega_2 = 30$ rad/s, or $\omega_1 = 8$ rad/s and $\omega_2 = 35$ rad/s, or $\omega_1 = 10$ rad/s and $\omega_2 = 40$ rad/s, respectively, the 2/3 order models by FF-Padé method are obtained as:

$$R_{2/3,a}^{FF}(s) = \frac{4.797(s+3.059 \pm j1.2798)}{(s+.7735)(s+17.6245)(s+38.7)} \tag{11a}$$

$$R_{2/3,b}^{FF}(s) = \frac{4.795(s+3.2949 \pm j.6306)}{(s+.7535)(s+20.4391)(s+35.04)} \tag{11b}$$

$$R_{2/3,c}^{FF}(s) = \frac{4.7836(s+5.2374)(s+2.2131)}{(s+.7189)(s+27.339 \pm j4.8765)}. \tag{11c}$$

The frequency and time responses of the three FF-Padé models of (11a)–(11c) are very similar. They are also shown in Figs. 1 and 2 for comparison. It can be seen that they are much better than the other results.

The other three examples below are chosen out of 1000 examples investigated by the author [8].

Example 2: In the following system there is a pair of dominant complex poles close to the imaginary axis, which indicates the existence of a strong oscillation with a very low damping.

$$G(s) = \frac{(1+2.0587s)(1+2.5529s+5.4342s^2)(1+3.2648s+2.1476s^2)}{(1+3.0092s+.7970s^2)(1+6.8538s+.6965s^2)(1+.1394s+.6861s^2)}. \tag{12}$$

TABLE I
REDUCED MODELS OF EXAMPLE 2

Routh	$R^{RH}_{2/3}(s) = \dfrac{1 + 7.8764s + 27.1688s^2}{1 + 10.0024s + 23.4544s^2 + 10.1616s^3}$ $R^{RH}_{3/4}(s) = \dfrac{1 + 7.8764s + 27.3921s^2 + 52.043s^3}{1 + 10.0024s + 23.6777s^2 + 12.3952s^3 + 5.0106s^4}$
FF-Pade	$R^{FF}_{2/3} = \dfrac{1 - 1.4257s + 4.3109s^2}{1 + .7003s + .8613s^2 + .0837s^3}$

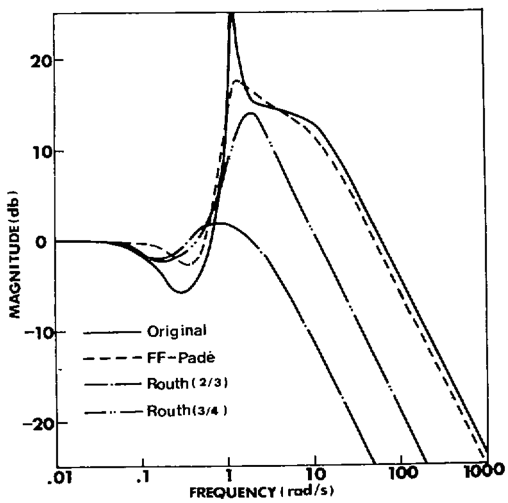

Fig. 3.

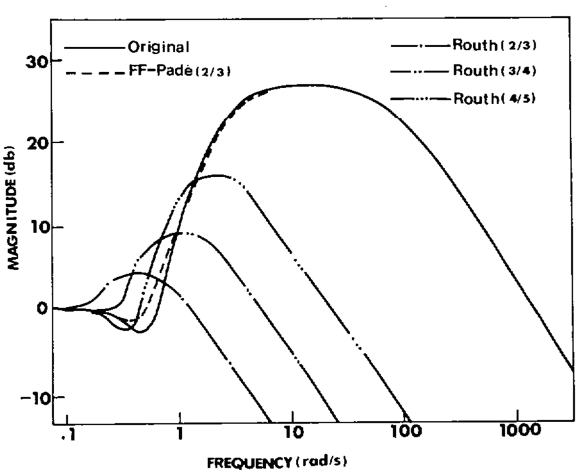

Fig. 4.

Its various reduced models are listed in Table I. The 1/2 to 4/5 order models by CRM are unstable. The reduced models by SCM are stable of course, but their dynamic responses differ very much from that of the original system (see Fig. 3). The $R^{FF}_{2/3}(s)$ is obtained by the method presented in this note with two frequency-fitting points at $\omega_1 = 1$ rad/s and $\omega_2 = 4$ rad/s. It is not only stable but also reproduces the characteristic of the original system over a wide range of frequency.

Example 3: The original system is

$$G(s) = \frac{1 + 8.8818s + 29.9339s^2 + 67.087s^3 + 80.3787s^4 + 68.6131s^5}{1 + 7.6194s + 21.7611s^2 + 28.4472s^3 + 16.5609s^4 + 3.5338s^5 + .0462s^6}. \tag{13}$$

It acts like a bandpass filter as shown in Fig. 4.

TABLE II
REDUCED MODELS OF EXAMPLE 3

Routh	$R^{RH}_{2/3}(s) = \dfrac{1 + 8.8812s + 29.0409s^2}{1 + 7.6194s + 20.8619s^2 + 21.6437s^3}$ $R^{RH}_{3/4}(s) = \dfrac{1 + 8.8818s + 29.7715s^2 + 65.6449s^3}{1 + 7.6194s + 21.5987s^2 + 27.2100s^3 + 13.1700s^4}$
FF-Pade	$R^{FF}_{2/3}(s) = \dfrac{1 + 2.0098s + 3.7169s^2}{1 + .7474s + .1898s^2 + 2.4977s^3}$

TABLE III
REDUCED MODELS OF EXAMPLE 4

Original	$G(s) = \dfrac{1 + 7.7617s + 13.5756s^2 + 67.6016s^3 + 40.2492s^4 + 144.0994s^5}{1 + 14.8243s + 75.7619s^2 + 163.2959s^3 + 139.3768s^4 + 38.6263s^5 + 3.3282s^6}$
Routh	$R^{RH}_{3/4}(s) = \dfrac{1 + 7.7617s + 13.2887s^2 + 65.3749s^3}{1 + 14.8243s + 75.4750s^2 + 159.0429s^3 + 118.1959s^4}$ $R^{RH}_{4/5}(s) = \dfrac{1 + 7.7617s + 13.5474s^2 + 67.3831s^3 + 39.8750s^4}{1 + 14.8243s + 75.7337s^2 + 162.8784s^3 + 137.2515s^4 + 34.1479s^5}$
FF-Pade	$R^{FF}_{3/4}(s) = \dfrac{1 + 1.1483s + 4.55894s^2 + 5.0011s^3}{1 + 8.2109s + 5.2508s^2 + 1.4141s^3 + .200s^4}$ with 3 frequency-fitting points at $\omega_1 = .47$, $\omega_2 = 3$, $\omega_3 = 10 rad/s$. $R^{FF}_{4/5}(s) = \dfrac{1 + .6269s + 9.1012s^2 + 2.7987s^3 + 20.2783s^4}{1 + 7.6895s + 20.3142s^2 + 18.8441s^3 + 5.3695s^4 + .4683s^5}$ with 4 frequency-fitting points at $\omega_1 = .43$, $\omega_2 = .5$, $\omega_3 = 3, \omega_4 = 10 rad/s$.

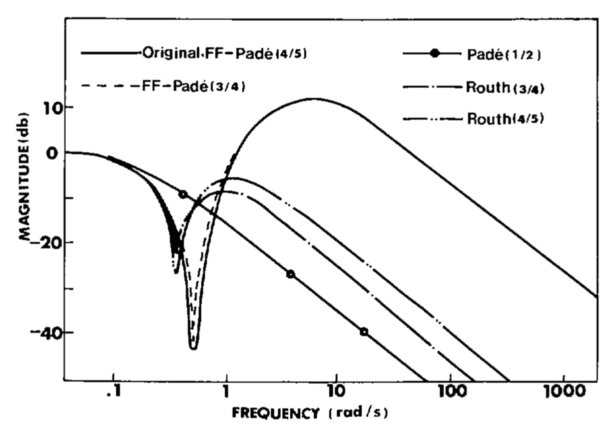

Fig. 5.

The reduced models by SCM are of course stable but the reduced models of different orders obtained by CRM are all unstable. However, none of them can reproduce the bandpass characteristic of the original system, not even with the 4/5 order model. But the 2/3 order FF-Padé model with two frequency-fitting points at $\omega_1 = 3$ rad/s and $\omega_2 = 100$ rad/s can reproduce the original well enough. Plots are shown in Fig. 4 and various reduced models are listed in Table II.

Example 4: The following system acts like a trapper (at $\omega_t = 0.47$ rad/s) which has a specific use in electronic networks. Its transfer function and frequency response are shown in Table III and Fig. 5, respectively. In fact, the 2/3 to 4/5 order reduced models by CRM are all unstable. The 1/2 order model is stable (by chance, most likely), but it cannot work as a trapper.

Two reduced models, $R^{FF}_{3/4}(s)$ and $R^{FF}_{4/5}(s)$ are obtained by using the

FF-Padé method with three and four frequency-fitting points, respectively. It can be seen very clearly that even such a complex system can be reduced by the new method with good results, while almost all other current methods fail.

CONCLUSION

To achieve good results in model reduction one should pay more attention to its methodology. The important problem is how to use the limited informational resources of a reduced model most effectively, instead of trying to guarantee absolute stability of the model at the cost of a serious loss in accuracy. It is necessary to reflect the characteristic of the original system in the dominant mid-frequency more faithfully. The FF-Padé method presented in this note can serve this purpose very well.

ACKNOWLEDGMENT

The author gratefully acknowledges his consultants, Prof. Z. H. Qian and Prof. Q. J. Zhou for their concern and guidance.

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The Observer-Based Controller Design of Discrete-Time Singularly Perturbed Systems

HOSSEIN OLOOMI AND MAHMOUD E. SAWAN

Abstract—**A class of linear shift-invariant discrete-time singularly perturbed systems with inaccessible states is considered. A design technique is formulated by which the stabilizing controller can be formed through the controllers of the slow and fast subsystems. Sufficient conditions for stability of the closed-loop system under this composite controller are given.**

I. INTRODUCTION

One of the notable features of the singular perturbation approach in the design of control systems with two-time-scale property is that the simplified controller structure is obtainable [1], [2]. With the advent and rapid development of microcomputer systems and because of the need to implement control systems using digital computers, a new motivation for simplified design exists in the case of sampled-data systems with two-time-scale property. Much of the interest has been centered around the design of feedback controllers for two-time-scale sampled-data control systems in the past few years [3]–[7]. However, most of these design methods are based on the assumption that the state variables are available for direct measurement [3]–[5]. Since this assumption is most likely to fail in practical situations, alternative design techniques such as output feedback [7] and LQG [6] designs have been proposed to overcome this difficulty.

Design of a dynamic feedback controller using a full-order observer for two-time-scale continuous-time systems was considered in [8]. Here we extend the work of [8] to discrete-time systems. The hybrid composite controller developed in this note makes it different from the discrete composite controller previously obtained in [9]. The observer-based controller design of [9] was formulated in the slow-time-scale where the asymptotic stability of the fast modes was presupposed. Therefore, unstable fast modes could not be stabilized through this discrete composite controller. Such an assumption is not made in this note since the problem is formulated in the fast-time-scale.

Results of this note are also attractive because in contrast to the output feedback design [7], where the composite control does not naturally decompose into the slow and fast subsystem controller designs, the slow and fast properties of the composite controller are preserved. Furthermore, it is preferable to the LQG design presented in [6] because the asymptotic system state reconstruction is possible without specifying noise statistics.

II. PROBLEM FORMULATION

Consider the linear shift-invariant discrete-time singularly perturbed system described by the state and output equations [4]

$$x_1(n+1)=(I+\epsilon A_{11})x_1(n)+\epsilon A_{12}x_2(n)+\epsilon B_1u(n);\; x_1(0)=x_{10} \quad (1a)$$

$$x_2(n+1)=A_{21}x_1(n)+A_{22}x_2(n)+B_2u(n);\; x_2(0)=x_{20} \quad (1b)$$

$$y(n)=C_1x_1(n)+C_2x_2(n)+Du(n) \quad (1c)$$

where $x_1 \in R^{n_1}$ are the slow states, $x_2 \in R^{n_2}$ are the fast states, $u \in R^m$ is the input, $y \in R^p$ is the output, $0 < \epsilon \ll 1$, and all matrices describing the above system are assumed to be constant matrices of appropriate dimensions. Defining

$$A=\begin{pmatrix} I+\epsilon A_{11} & \epsilon A_{12} \\ A_{21} & A_{22}\end{pmatrix},\; B=\begin{pmatrix}\epsilon B_1 \\ B_2\end{pmatrix},\; C=[C_1\; C_2],\; x(n)=\begin{pmatrix}x_1(n)\\ x_2(n)\end{pmatrix} \quad (2)$$

system (1) takes the following compact form:

$$x(n+1)=Ax(n)+Bu(n);\; x(0)=x_0 \quad (3a)$$

$$y(n)=Cx(n)+Du(n). \quad (3b)$$

Using the standard procedure for design of the observer-based controller [10], the dynamic feedback controller for (3) is defined to be

$$u(n)=-G\hat{x}(n) \quad (4)$$

where $\hat{x}(n)$, the estimate of $x(n)$, and $e(n)$, the reconstruction error, are given by

$$\begin{aligned}\hat{x}(n+1)&=A\hat{x}(n)+Bu(n)+K[y(n)-C\hat{x}(n)-Du(n)]\\ &=(A-KC)\hat{x}(n)+Ky(n)+(B-KD)u(n)\end{aligned} \quad (5)$$

$$e(n)=x(n)-\hat{x}(n) \quad (6)$$

$$e(n+1)=(A-KC)e(n);\; e(0)=x_0-\hat{x}(0). \quad (7)$$

Manuscript received October 3, 1985; revised September 10, 1986. Paper recommended by Past Associate Editor, H. Khalil.
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IEEE Log Number 8612534.

0018-9286/87/0300-0246$01.00 © 1987 IEEE