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Effect of DC Offset on OFDM System with Zero-Padded Suffix

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***Abstract*—The effect of direct current (DC) offset on orthogonal frequency division multiplexing (OFDM) system with zero-padded (ZP) suffix is analysed in this paper. It is found that the ZP appended OFDM system suffers more from DC offset than a cyclic prefix inserted OFDM system and even demonstrates bit error flooring if the DC power exceeds a maximum allowed threshold. Simple algorithms are also proposed for DC offset estimation and compensation. The analysis is confirmed by simulation using multiband OFDM specification for ultra-wideband application.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) data transmission has been widely used in today's digital communication systems such as wireless local area networks (WLANs) and digital audio/video broadcasting (DAB/DVB) [1-3]. Through inverse fast Fourier transform (IFFT), an OFDM transmitter modulates information on a number of orthogonal subcarriers to eliminate intersymbol interference (ISI). A cyclic prefix (CP) or zero-padded (ZP) suffix of length no less than the maximum channel multipath delay is also inserted or appended per transmitted OFDM symbol to facilitate easy frequency domain equalization at the receiver via fast Fourier transform (FFT). The disadvantages of an OFDM system have been known as the large peak-to-average power ratio and the sensitivity to frequency synchronization error. The adverse impact of the direct current (DC) offset on its performance seems overlooked sometimes.

In a practical OFDM receiver, a DC component is often present at the amplification and down mixing stages of the radio frequency (RF) front-end before analog-to-digital conversion (A/D). It has been commonly understood that the DC offset is undesirable because it takes up part of the A/D's dynamic range and also makes the automatic gain control difficult. Recent study has shown that the DC offset also degrades performance of a CP inserted OFDM system provided that the receiver's RF front-end introduces significant carrier frequency offset [4].

A CP inserted OFDM transmitter uses more than necessary transmitted power since the CP does not carry additional information and it is often simply discarded at the receiver. For low power applications such as wireless personal area networks (WPANs), a ZP appended OFDM system will be preferable. The multiband (MB) OFDM standard [5] developed for ultra-wideband application is such an example.

In this paper the effect of DC offset on an OFDM system with ZP suffix is analysed. It is revealed that the DC offset problem in a ZP appended OFDM receiver is more serious than that in a CP inserted OFDM receiver due to the overlap-add operation. No matter there is a carrier frequency offset present or not, the DC offset always appears as additional noise on subcarriers and even leads to bit error flooring on the receiver performance. The implication is that effective DC offset compensation is necessary not only for a better RF front-end design as conventionally believed but also for achieving the expected benefits offered by the ZP appended OFDM system.

The rest of the paper is organized as follows. In Section II, a ZP appended OFDM system model is described in general and the received OFDM signal with DC offset after overlap-add operation is formulated. The effect of DC offset on subcarriers in an OFDM symbol is analysed in Section III and the normalized maximum allowed DC power which causes bit error flooring is also determined. In Section IV, simple algorithms for DC offset estimation and compensation are proposed. Simulation results using MB-OFDM specification are provided in Section V. Finally, conclusions are drawn in Section VI.

II. RECEIVED SIGNAL MODEL

A. System model

The block diagram of an OFDM system with ZP suffix is depicted in Fig. 1. At the transmitter side, a data symbol block of size N is first produced by serial-to-parallel conversion (S/P) from the input data symbol sequence $a[k]$ (QAM mapped from information bits). Then an N_{FFT}-point IFFT ($N_{FFT} > N$) is performed to generate an OFDM signal block. After parallel-to-serial conversion (P/S) and ZP suffix appending, an OFDM symbol is formed. Finally, the signal is transmitted after digital-to-analog conversion (D/A) and passing through the transmitter RF front-end. At the receiver side, the OFDM signal is first received by the receiver RF front-end and sampled by the A/D to produce the discrete received signal $y[n]$. After performing an overlap-add operation, the received OFDM signal $r[n]$ is obtained, which is then processed through FFT, equalization, and detection to retrieve the transmitted data symbols.

The transmitted OFDM symbol before ZP appending can be expressed as

$$x[n] = \sum_{k=1}^{N} a[k] e^{j\frac{2\pi}{N_{FFT}}kn}, \quad 0 \le n \le N_{FFT} - 1, \qquad (1)$$

where $a[k]$, $1 \le k \le N$, is modulated on subcarrier k and the subcarrier numbered zero (DC component) is not used for data transmission. After zero-padding N_{ZP} zero samples, the total number of signal samples for the transmitted OFDM symbol is $N_{FFT} + N_{ZP}$, which corresponds to the OFDM symbol duration $(N_{FFT}+N_{ZP})T$, where T is the sampling period of the D/A .

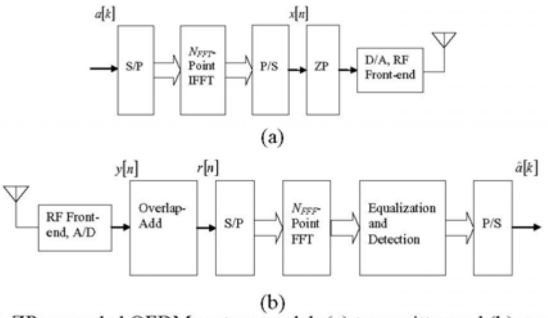

Fig. 1. ZP appended OFDM system model: (a) transmitter and (b) receiver.

The OFDM signal arrived at the receiver digital baseband, where a DC offset is present as a complex constant C, can be expressed as

$$y[n]=x[n]*h[n]+C+z[n],\quad 0\le n\le N_{FFT}+N_{ZP}-1, \tag{2}$$

where $h[n]$ is the discrete channel impulse response with maximum path delay L ($N_{ZP}\ge L$ to avoid ISI), $*$ denotes the linear convolution operation, and $z[n]$ represents the additive white Gaussian noise (AWGN).

B. Overlap-add operation

The overlap-add operation at the receiver is performed to turn the linear convolution of the transmitted signal with the channel to a circular one, so that after FFT the relationship between the transmitted data symbols $a[k]$ and the discrete channel frequency response $H[k]$, which is the N_{FFT}-point discrete Fourier transform (DFT) of $h[n]$, becomes multiplicative and thus simple frequency domain equalization can be applied.

As illustrated in Fig. 2, to perform the overlap-add operation, the last N_{ZP} samples of $y[n]$ are added to the beginning of $y[n]$. The resulting received signal sequence becomes

$$r[n]=x[n]\otimes_{N_{FFT}} h[n]+d[n]+v[n], \tag{3}$$

where $\otimes_{N_{FFT}}$ denotes the circular convolution of length N_{FFT}, $d[n]$ is the interference component due to the DC offset, which can be expressed as

$$d[n]=\begin{cases}2C, & 0\le n\le N_{ZP}-1\\ C, & N_{ZP}\le n\le N_{FFT}-1\end{cases}, \tag{4}$$

and $v[n]$ is the noise component after overlap-add operation, which is

$$v[n]=\begin{cases}z[n]+z[n+N_{FFT}], & 0\le n\le N_{ZP}-1\\ z[n], & N_{ZP}\le n\le N_{FFT}-1\end{cases}. \tag{5}$$

From (4) and (5) we see that the overlap-add operation not only introduces additional DC component for the first N_{ZP} samples of the received OFDM signal $r[n]$ but also increases noise power for these samples. Both will degrade the system performance significantly.

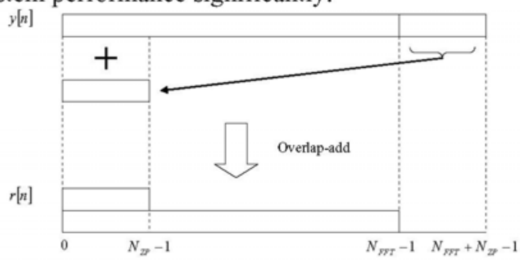

Fig. 2. Illustration of overlap-add operation.

III. DC Offset Analysis

A. Effect of DC offset

We only focus on analyzing the effect of the DC offset in this paper. First, the Fourier transform of $d[n]$ is found from (4) as

$$D(e^{j\omega})=N_{ZP}C\Phi_{N_{ZP}}(\omega)+N_{FFT}C\Phi_{N_{FFT}}(\omega), \tag{6}$$

where

$$\Phi_M(\omega)=\frac{\sin\left(\frac{\omega M}{2}\right)}{M\sin\left(\frac{\omega}{2}\right)}e^{-j\frac{\omega(M-1)}{2}} \tag{7}$$

is the Fourier transform of the following sequence

$$\phi_M[n]=\begin{cases}\frac{1}{M}, & 0\le n\le M-1\\ 0, & otherwise\end{cases}. \tag{8}$$

To show the impact of the DC offset on each subcarrier in the received OFDM signal, the N_{FFT}-point DFT of $d[n]$ is further evaluated by sampling $D(e^{j\omega})$ at each subcarrier frequency $\omega=\frac{2\pi}{N_{FFT}}k$ for $k=1,2,\ldots,N_{FFT}-1$. After normalized by a factor $\frac{1}{N_{FFT}}$ (so that the value for subcarrier zero is equal to the DC component of $d[n]$), the DFT becomes

$$D[k]=\frac{1}{N_{FFT}}D\left(e^{j\frac{2\pi}{N_{FFT}}k}\right)=\frac{N_{ZP}}{N_{FFT}}C\Phi_{N_{ZP}}\left(\frac{2\pi}{N_{FFT}}k\right)+C\Phi_{N_{FFT}}\left(\frac{2\pi}{N_{FFT}}k\right). \tag{9}$$

As shown in Fig. 3, the first term in (9) is not always zero thus it will appear as additional noise for most subcarriers. The strength of this additional noise on each subcarrier depends on its distance from the DC. Subcarriers near DC are affected the most, such as subcarrier 1 and $N_{FFT}-1$. The second term in $D[k]$ is always zero for every subcarrier (except subcarrier zero), thus it does not affect any data subcarrier.

Note that for simplicity the above analysis does not take into account any possible carrier frequency offset. If there exists a carrier frequency offset Δf at the receiver, all subcarriers in Fig. 3 will be shifted a distance $2\pi\Delta fT$ in ω axis. Then, the effect of DC offset becomes more serious since the second term in (9) is no longer zero. It is interesting to know that for a CP inserted OFDM system the DC offset only produces the second term in $D[k]$[4]. We see that the ZP appended OFDM system suffers more from DC offset.

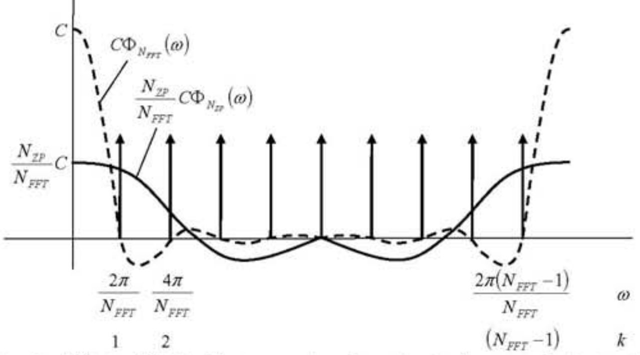

Fig. 3. Effect of DC offset on subcarriers (vertical arrows indicate the positions of data subcarriers).

B. Bit error flooring

When the DC offset becomes significant, the energy leakage to a subcarrier can even cause the receiver make wrong decision in absence of any other noise, leading to the effect of bit error flooring on the receiver performance.

The DC power necessary to cause a bit error floor is determined by the data symbol mapping constellation used in the OFDM symbol. For a QPSK constellation as shown in Fig. 4, where the amplitude of a subcarrier is assumed to be A, the DC level needs to be larger than $A/\sqrt{2}$ in order to change a constellation point into a different quadrant. Since subcarrier 1 or $N_{FFT}-1$ is affected by the DC offset the most, the maximum allowed DC level C_0 can be determined by

$$\frac{N_{ZP}}{N_{FFT}}C_0\left|\Phi_{N_{ZP}}\left(\frac{2\pi}{N_{FFT}}\right)\right| = \frac{A}{\sqrt{2}}. \tag{10}$$

For an OFDM system with $\frac{N_{ZP}}{N_{FFT}} \le \frac{1}{4}$, we have

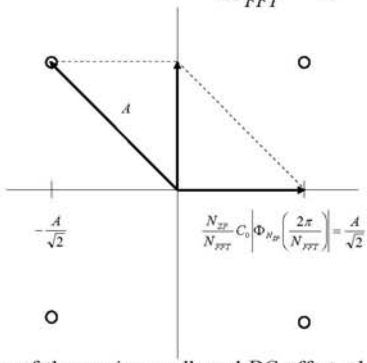

Fig. 4. Determination of the maximum allowed DC offset which causes bit error flooring for QPSK constellation.

$\left|\Phi_{N_{ZP}}\left(\frac{2\pi}{N_{FFT}}\right)\right| \approx 1$. Thus, from (10) we have

$$C_0 \approx \frac{A}{\sqrt{2}} \cdot \frac{N_{FFT}}{N_{ZP}}. \tag{11}$$

For an OFDM symbol containing N subcarriers, the total signal power is NA^2. The DC-to-signal power ratio, or, normalized DC power, is then defined as

$$\bar{c}^2 = \frac{C^2}{NA^2}. \tag{12}$$

This gives the normalized maximum allowed DC power

$$\bar{c}_0^2 = \frac{C_0^2}{NA^2} = \frac{1}{2N}\left(\frac{N_{FFT}}{N_{ZP}}\right)^2, \tag{13}$$

at which the bit error flooring occurs.

IV. DC Offset Estimation

From above analysis we have seen that the DC offset significantly degrades the ZP appended OFDM system performance and therefore the DC offset compensation is absolutely necessary. In this section, we present simple algorithms for DC offset estimation and compensation used in an MB-OFDM [5] receiver.

In addition to ZP appending, MB-OFDM also alternately transmits OFDM symbols in different subbands according to a time-frequency code (i.e., frequency hopping) to spread the signal spectrum. Fig. 5 shows a typical MB-OFDM signal waveform and its packet format. To help describe the algorithms clearly, we denote $y[n]$ as the received preamble sequence and $R_i[k]$ (i denotes a symbol index) the FFT of a received OFDM symbol in the header and payload segments.

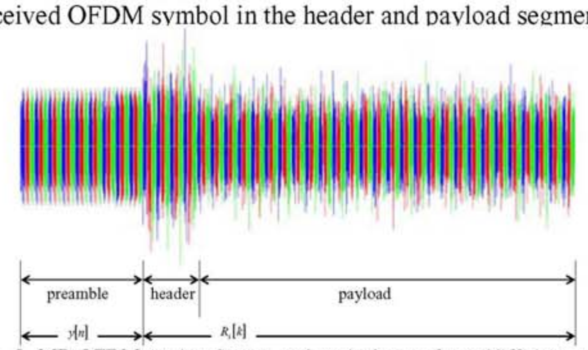

Fig. 5. MB-OFDM packet format and typical waveform (different colours indicate OFDM symbols transmitted in different subbands).

The DC offset estimation during reception of a signal packet proceeds in two stages. Stage 1 is a coarse estimation, which is performed in the preamble period. Since the OFDM symbol and the packet haven't been synchronized in this period, the DC offset is estimated *sample by sample* as each $y[n]$ is received with the iteration

$$C^{(n+1)} = C^{(n)} + \alpha\left(y[n] - C^{(n)}\right), \tag{14}$$

where $C^{(n)}$ is the estimated DC offset at the nth iteration (the initial value $C^{(0)}$ is set to zero) and $0 < \alpha < 1$ is an updating coefficient.

After synchronization and channel estimation, the receiver starts to receive OFDM symbols in the header and payload segments. The DC offset is then updated *symbol by symbol* (stage 2 fine estimation). The iteration is switched to

$$C^{(i+1)} = C^{(i)} + \beta\left(R_i[0] - C^{(i)}\right), \qquad (15)$$

where $C^{(i)}$ is the estimated DC offset for the i th OFDM symbol, $0 < \beta < 1$ is a different updating coefficient, and $R_i[0]$ is the DC component calculated through FFT. Making use of $R_i[0]$ avoids extra complexity for DC offset estimation since otherwise this information would be wasted.

For DC offset compensation, the estimated $C^{(i)}$ is simply removed from the received OFDM symbol before performing overlap-add and FFT.

Note that, since for different subbands the receiver may produce different DC offsets, the above described DC offset estimation and compensation have to be performed separately for each subband.

V. Simulation Results

Simulations are carried out to confirm the effect of the DC offset on the ZP appended OFDM system as analysed in above sections and demonstrate the performance of the proposed DC offset estimation and compensation algorithms for MB-OFDM with parameters $N_{FFT} = 128$, $N_{ZP} = 32$, and $N = 112$. Only QPSK constellation mapping is used and the channel coding is also turned off.

From (13) we can easily calculate the normalized maximum allowed DC power to be -11.46 dB for this system. Fig. 6 shows an example waveform of a received OFDM symbol with -10 dB DC offset, which would obviously not cause any signal clipping by the A/D in normal operation. However, this small DC offset is already serious enough to cause bit errors even without any other noise.

The simulated bit error rate (BER) versus normalized signal-to-noise ratio E_b/N_0 curves also confirm this analysis. As shown in Fig. 7, the bit error floors are observed for DC offsets -5 dB and -10 dB. Only when the DC offset is below -20 dB can the adverse effect be ignored. Fig. 8 shows the performance after DC offset compensation using the proposed algorithms. The parameters used are $\alpha = \frac{1}{64}$ and $\beta = \frac{1}{2}$. We see that the DC offset compensation is very effective. Even with -5 dB DC offset, the performance is as good as that without any DC offset.

VI. Conclusions

A ZP appended OFDM system enjoys higher power efficiency than its CP inserted counterpart, but it is more seriously hampered by the DC offset as analysed and demonstrated in this paper. Even without any carrier frequency offset or additional noise, the DC offset alone could cause certain bit errors, resulting in the effect of bit error flooring. This effect can be described by the maximum allowed DC power defined in this paper. Therefore, a DC offset compensation is necessary not only for the sake of fully utilizing the dynamic rang of the A/D in the ODFM receiver but also for realizing the higher power efficiency provided by ZP appending without additional penalty.

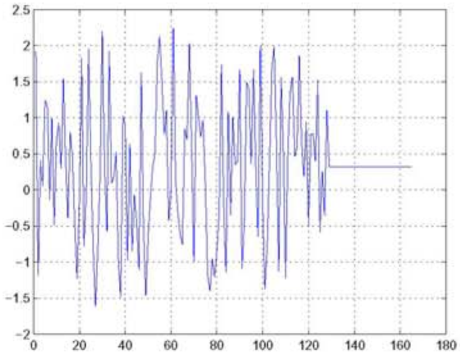

Fig. 6 Example OFDM symbol waveform with -10dB DC offset.

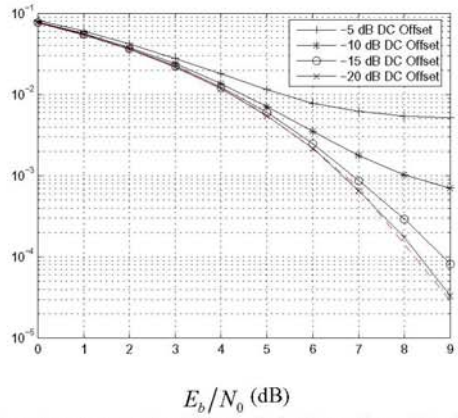

Fig. 7. BER performance under different DC offsets (dotted line indicates the performance without DC offset).

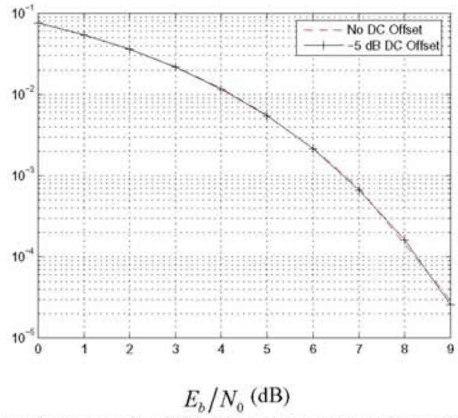

Fig. 8. BER performance after DC offset compensation (dotted line indicates the performance without DC offset).

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