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On the (10, 5, λ)-Family of Bhaskar Rao Designs

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Abstract

We prove a theorem for BRD(10,5,λ)s and give thirteen (13) inequivalent BRD(10,5,4)s.

1 Introduction

A *balanced incomplete block (BIBD) design* is an arrangement of v symbols in b blocks each containing ($k < v$) symbols, satisfying the following conditions : (i) every symbol occurs at most once in a block, (ii) every symbol occurs in exactly r blocks, (iii) every pair of treatments or symbols occur together in exactly λ blocks.

A *Bhaskar Rao design* [BRD(v, b, r, k, λ)] is a matrix of order $v \times b$ with (0, $\pm$ 1 entries), satisfying the following conditions : (i) it has k non-zero [$(v-k)$ zero] entries/column, (ii) it has r non-zero [$(b-r)$ zero] entries/row, (iii) λ non-zero entries are in the same column as λ other non-zero entries for every pair of rows, (iv) the inner product of any pair of distinct rows is zero, (v) when it's -1s are changed to $+1$s, the resulting matrix becomes the incidence matrix of a BIBD. The necessary conditions for the existence of a BRD(v, b, r, k, λ) are :

(i) $vr = bk$,

(ii) $\lambda(v-1) = r(k-1)$,

(iii) the inner product of distinct rows is zero,

(iv) $2|\lambda$ and $2|b$.

Theorem 1 (Chaudhry and Seberry [2]) *The conditions*

i) $\lambda(v-1) \equiv 0 \pmod 4$

ii) $\lambda v(v-1) \equiv 0 \pmod{20}$

iii) $b \equiv 0 \pmod 2$

are necessary for the existence of $BRD(v, 5, 2\lambda)$ *where* λ *may take values 1, 2, 3, ...*

Lemma 1 (Chaudhry and Seberry [2]) *Suppose in the signing of the rows of a BIBD, it happens that the rows* $\{1, ..., j\} \in A$ *are mutually orthogonal and the rows* $\{j+1, ..., v\} \in B$ *are mutually orthogonal but no row of* A *is orthogonal to any row of* B *and vice versa. Then if we take another copy of the BIBD and negate the orthogonal rows in* B*, we obtain a* $BRD(v, k, 2\lambda)$.

2 BRDs(10,5, λ)

There exist twenty one (21) inequivalent BIBDs with parameters (10,5,4), these are given in [3], appendix B. In this paper, we have constructed thirteen (13) BRD(10, 5, 4)s from the BIBD(10,5,4)s: call this set A, these are given in the appendix in the same order as in [3]. Gibbons BIB designs numbers 9, 12, 13, 14, 15, 16, 19 and 21 were found, by an exhaustive computer search, not to give BRDs: call this set B. Hence thirteen of the twenty one inequivalent BIBD(10,5,4)s can be signed to BRD(10,5,4)s and eight cannot.

We write $D_i||D_j = [D_iD_j]$ for the matrix of order $v \times 2b$ with parameters $(v, 2b, 2r, k, 2\lambda)$ when D_i is the matrix of order v with parameters (v, b, r, k, λ). We note though that the eight BIBD(10, 5, 4)s, which can not be signed to BRD(10, 5, 4)s, satisfy Lemma 1. We use $D_i||\overline{D_i}$ for the BRDs constructed using Lemma 1. Now we construct the BRD(10,5,8)s in two ways :

i) if D_i, $D_j \in A$, there are 78 different BIBD(10,5,8)s for $D_i||D_j$, so we obtain 78 inequivalent BRD(10,5,8)s;

ii) if $E_k \in B$, then $E_k||E_k$ is a BIBD(10,5,8), and so we obtain 8 BRD(10,5,8)s.

Thus we have constructed 86 BRD(10,5,8)s of the $\geq$ 135922 possible cases.

Theorem 2 *The conditions*

i) $\lambda(v-1) \equiv 0 \pmod 4$

ii) $\lambda v(v-1) \equiv 0 \pmod{20}$

iii) $b \equiv 0 \pmod 2$

iv) $\lambda \equiv 0 \pmod 4$

are necessary and sufficient for the existence of $BRD(10, 5, \lambda)$.

Remark The inequivalent BRD(10,5,4)s are given in the appendix.

References

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Appendix Thirteen inequivalent BRD(10,18,9,5,4)s are given below :

BRD #	I	II	III	IV
Gibbons #	I	II	III	IV
1	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
2	1 2 $\overline{3}$ 6 7	1 2 $\overline{3}$ 6 7	1 $\overline{2}$ 3 6 7	1 2 $\overline{3}$ 6 7
3	1 $\overline{2}$ $\overline{3}$ 8 9	1 $\overline{2}$ 3 8 9	1 2 $\overline{3}$ $\overline{8}$ 9	1 $\overline{2}$ 3 8 9
4	1 $\overline{2}$ $\overline{4}$ $\overline{5}$ 10	1 $\overline{2}$ 4 5 10	1 $\overline{2}$ $\overline{4}$ 5 10	1 $\overline{2}$ 4 $\overline{5}$ 10
5	1 3 6 7 10	1 $\overline{3}$ $\overline{6}$ $\overline{7}$ 10	1 $\overline{3}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	1 $\overline{3}$ $\overline{6}$ $\overline{8}$ 10
6	1 4 $\overline{6}$ 8 $\overline{9}$	1 $\overline{4}$ 6 8 $\overline{9}$	1 4 $\overline{6}$ 7 8	1 $\overline{4}$ $\overline{6}$ 7 $\overline{9}$
7	1 $\overline{4}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	1 $\overline{4}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	1 $\overline{4}$ 6 $\overline{9}$ $\overline{10}$	1 $\overline{4}$ 6 $\overline{9}$ $\overline{10}$
8	1 5 $\overline{7}$ $\overline{8}$ 9	1 $\overline{5}$ 7 $\overline{8}$ 9	1 $\overline{5}$ $\overline{7}$ $\overline{8}$ 9	1 5 $\overline{7}$ $\overline{8}$ 9
9	1 $\overline{5}$ 7 $\overline{9}$ $\overline{10}$	1 $\overline{5}$ 7 $\overline{9}$ $\overline{10}$	1 $\overline{5}$ 7 $\overline{9}$ 10	1 $\overline{5}$ 7 8 $\overline{10}$
10	2 $\overline{3}$ 8 9 $\overline{10}$	2 3 8 9 $\overline{10}$	2 3 $\overline{7}$ $\overline{9}$ $\overline{10}$	2 3 $\overline{7}$ $\overline{9}$ 10
11	2 $\overline{4}$ $\overline{6}$ 7 $\overline{9}$	2 4 $\overline{6}$ 7 $\overline{9}$	2 $\overline{4}$ $\overline{6}$ 7 $\overline{9}$	2 4 $\overline{6}$ 7 8
12	2 $\overline{4}$ $\overline{7}$ 8 10	2 $\overline{4}$ $\overline{7}$ 9 10	2 $\overline{4}$ 7 $\overline{8}$ 10	2 $\overline{4}$ $\overline{7}$ 8 10
13	2 $\overline{5}$ 6 $\overline{7}$ $\overline{8}$	2 5 6 $\overline{7}$ $\overline{8}$	2 5 6 $\overline{8}$ 9	2 $\overline{5}$ 6 $\overline{8}$ 9
14	2 $\overline{5}$ $\overline{6}$ 9 10	2 $\overline{5}$ $\overline{6}$ 8 10	2 $\overline{5}$ 6 8 10	2 $\overline{5}$ $\overline{6}$ 9 $\overline{10}$
15	3 4 $\overline{5}$ $\overline{6}$ 9	3 $\overline{4}$ 5 $\overline{6}$ $\overline{9}$	3 $\overline{4}$ $\overline{5}$ $\overline{6}$ 9	3 $\overline{4}$ $\overline{5}$ $\overline{6}$ $\overline{8}$
16	3 $\overline{4}$ 5 $\overline{7}$ 8	3 4 $\overline{5}$ $\overline{7}$ $\overline{8}$	3 4 $\overline{5}$ 7 $\overline{8}$	3 $\overline{4}$ 5 7 9
17	3 $\overline{4}$ 7 9 $\overline{10}$	3 $\overline{4}$ 7 $\overline{8}$ 10	3 $\overline{4}$ 8 9 $\overline{10}$	3 4 $\overline{8}$ $\overline{9}$ $\overline{10}$
18	3 $\overline{5}$ 6 8 $\overline{10}$	3 $\overline{5}$ 6 $\overline{9}$ 10	3 5 $\overline{6}$ $\overline{7}$ 10	3 $\overline{5}$ 6 7 10

BRD #	V	VI	VII	VIII
Gibbons #	V	VI	VII	VIII
1	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
2	1 2 $\overline{3}$ 6 7	1 $\overline{2}$ 3 6 7	1 $\overline{2}$ 3 6 7	1 2 $\overline{3}$ 6 7
3	1 $\overline{2}$ 3 8 9	1 $\overline{2}$ $\overline{3}$ 8 9	1 $\overline{2}$ $\overline{3}$ 8 9	1 $\overline{2}$ 3 8 9
4	1 $\overline{2}$ 4 5 10	1 2 $\overline{4}$ 5 10	1 2 $\overline{4}$ $\overline{5}$ 10	1 $\overline{2}$ $\overline{4}$ 5 10
5	1 $\overline{3}$ $\overline{6}$ $\overline{7}$ 10	1 $\overline{3}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	1 $\overline{3}$ 6 $\overline{8}$ 10	1 $\overline{3}$ $\overline{6}$ $\overline{8}$ $\overline{10}$
6	1 $\overline{4}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	1 4 6 $\overline{7}$ $\overline{8}$	1 4 $\overline{6}$ 7 9	1 4 6 $\overline{7}$ 10
7	1 $\overline{4}$ 7 8 $\overline{9}$	1 $\overline{4}$ $\overline{7}$ $\overline{9}$ $\overline{10}$	1 $\overline{4}$ $\overline{7}$ $\overline{8}$ $\overline{10}$	1 $\overline{4}$ 8 $\overline{9}$ $\overline{10}$
8	1 $\overline{5}$ 6 $\overline{8}$ 9	1 $\overline{5}$ $\overline{6}$ 9 10	1 $\overline{5}$ $\overline{6}$ $\overline{9}$ $\overline{10}$	1 $\overline{5}$ $\overline{6}$ $\overline{7}$ $\overline{9}$
9	1 $\overline{5}$ $\overline{7}$ $\overline{9}$ $\overline{10}$	1 $\overline{5}$ 7 8 $\overline{9}$	1 5 $\overline{7}$ 8 $\overline{9}$	1 $\overline{5}$ 7 $\overline{8}$ 9
10	2 3 8 9 $\overline{10}$	2 $\overline{3}$ 7 9 $\overline{10}$	2 $\overline{3}$ 7 $\overline{9}$ 10	2 3 $\overline{7}$ 9 $\overline{10}$
11	2 4 $\overline{6}$ 7 $\overline{8}$	2 4 $\overline{6}$ 7 $\overline{9}$	2 4 6 $\overline{7}$ 8	2 $\overline{4}$ 6 $\overline{7}$ $\overline{8}$
12	2 $\overline{4}$ $\overline{6}$ 9 10	2 $\overline{4}$ 6 8 $\overline{10}$	2 $\overline{4}$ 6 9 $\overline{10}$	2 $\overline{4}$ $\overline{6}$ 8 9
13	2 5 6 $\overline{7}$ $\overline{9}$	2 $\overline{5}$ 6 $\overline{8}$ 9	2 5 $\overline{6}$ $\overline{8}$ 9	2 5 $\overline{6}$ $\overline{9}$ 10
14	2 $\overline{5}$ $\overline{7}$ 8 10	2 $\overline{5}$ $\overline{7}$ 8 10	2 $\overline{5}$ 7 8 $\overline{10}$	2 $\overline{5}$ 7 8 10
15	3 4 $\overline{5}$ $\overline{6}$ $\overline{9}$	3 $\overline{4}$ 5 $\overline{6}$ 9	3 4 $\overline{5}$ 6 $\overline{8}$	3 $\overline{4}$ $\overline{5}$ 6 $\overline{9}$
16	3 $\overline{4}$ 5 $\overline{7}$ $\overline{8}$	3 $\overline{4}$ $\overline{5}$ 7 $\overline{8}$	3 $\overline{4}$ 5 7 $\overline{9}$	3 $\overline{4}$ 5 7 $\overline{8}$
17	3 $\overline{4}$ 7 $\overline{9}$ 10	3 4 8 9 $\overline{10}$	3 $\overline{4}$ 8 9 10	3 4 7 $\overline{9}$ $\overline{10}$
18	3 $\overline{5}$ 6 $\overline{8}$ 10	3 $\overline{5}$ $\overline{6}$ $\overline{7}$ $\overline{10}$	3 $\overline{5}$ $\overline{6}$ $\overline{7}$ 10	3 $\overline{5}$ 6 $\overline{8}$ 10

BRD #	X	XI	XVII	XVIII
Gibbons #	X	XI	XVII	XVIII
1	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
2	1 2 $\overline{3}$ 6 7	1 $\overline{2}$ $\overline{3}$ 6 7	1 $\overline{2}$ 3 6 7	1 $\overline{2}$ $\overline{3}$ 6 7
3	1 $\overline{2}$ 3 8 9	1 $\overline{2}$ 3 8 9	1 $\overline{2}$ $\overline{3}$ 8 9	1 2 $\overline{5}$ 7 8
4	1 2 $\overline{4}$ 5 10	1 2 $\overline{4}$ $\overline{6}$ 10	1 2 $\overline{4}$ $\overline{6}$ 10	1 $\overline{2}$ $\overline{6}$ $\overline{7}$ 9
5	1 $\overline{3}$ $\overline{6}$ $\overline{8}$ 10	1 $\overline{3}$ 5 $\overline{8}$ 10	1 $\overline{3}$ $\overline{5}$ $\overline{8}$ 10	1 3 $\overline{5}$ 9 10
6	1 4 6 $\overline{9}$ $\overline{10}$	1 4 $\overline{5}$ 6 9	1 $\overline{4}$ 5 $\overline{9}$ $\overline{10}$	1 $\overline{3}$ $\overline{6}$ 8 $\overline{10}$
7	1 $\overline{4}$ $\overline{7}$ 8 $\overline{10}$	1 $\overline{4}$ $\overline{7}$ $\overline{8}$ $\overline{10}$	1 4 $\overline{6}$ 7 $\overline{9}$	1 $\overline{4}$ 7 $\overline{8}$ 10
8	1 $\overline{5}$ $\overline{6}$ 7 $\overline{9}$	1 $\overline{5}$ $\overline{7}$ 8 $\overline{9}$	1 $\overline{5}$ 6 $\overline{7}$ $\overline{8}$	1 $\overline{4}$ $\overline{7}$ $\overline{9}$ $\overline{10}$
9	1 $\overline{5}$ $\overline{7}$ $\overline{8}$ 9	1 $\overline{6}$ 7 $\overline{9}$ $\overline{10}$	1 $\overline{7}$ 8 9 $\overline{10}$	1 5 6 $\overline{8}$ $\overline{9}$
10	2 3 7 9 10	2 $\overline{3}$ 7 9 $\overline{10}$	2 $\overline{3}$ 7 9 $\overline{10}$	2 $\overline{3}$ $\overline{7}$ $\overline{8}$ 9
11	2 $\overline{4}$ 6 $\overline{7}$ $\overline{8}$	2 4 $\overline{5}$ 7 $\overline{9}$	2 $\overline{4}$ $\overline{5}$ 7 8	2 $\overline{3}$ 8 $\overline{9}$ 10
12	2 $\overline{4}$ $\overline{6}$ 8 $\overline{9}$	2 $\overline{4}$ 8 9 $\overline{10}$	2 4 $\overline{7}$ 8 10	2 4 $\overline{5}$ $\overline{8}$ $\overline{10}$
13	2 5 $\overline{6}$ 9 $\overline{10}$	2 $\overline{5}$ 6 $\overline{7}$ $\overline{8}$	2 5 6 $\overline{8}$ 9	2 $\overline{4}$ 6 9 $\overline{10}$
14	2 $\overline{5}$ $\overline{7}$ 8 10	2 5 6 8 10	2 $\overline{5}$ 6 $\overline{9}$ $\overline{10}$	2 5 $\overline{6}$ $\overline{7}$ 10
15	3 $\overline{4}$ $\overline{5}$ 6 7	3 $\overline{4}$ $\overline{5}$ 7 10	3 4 $\overline{5}$ 7 9	3 4 $\overline{5}$ $\overline{7}$ $\overline{9}$
16	3 $\overline{4}$ 5 $\overline{8}$ $\overline{9}$	3 $\overline{4}$ 6 7 $\overline{8}$	3 $\overline{4}$ $\overline{6}$ $\overline{8}$ 9	3 $\overline{4}$ 6 $\overline{7}$ 8
17	3 4 $\overline{7}$ $\overline{9}$ 10	3 4 $\overline{6}$ $\overline{8}$ 9	3 $\overline{4}$ 6 8 10	3 5 $\overline{6}$ 7 $\overline{10}$
18	3 $\overline{5}$ $\overline{6}$ $\overline{8}$ $\overline{10}$	3 5 6 $\overline{9}$ $\overline{10}$	3 $\overline{5}$ $\overline{6}$ $\overline{7}$ $\overline{10}$	4 5 6 8 9

BRD #	XX
Gibbons #	XX
1	1 2 3 4 5
2	1 2 $\overline{3}$ $\overline{4}$ 6
3	1 $\overline{2}$ 5 7 8
4	1 $\overline{2}$ $\overline{6}$ $\overline{7}$ 9
5	1 3 $\overline{5}$ $\overline{9}$ 10
6	1 $\overline{3}$ 7 9 10
7	1 4 6 $\overline{8}$ $\overline{9}$
8	1 $\overline{4}$ $\overline{7}$ 8 $\overline{10}$
9	1 $\overline{5}$ $\overline{6}$ $\overline{8}$ $\overline{10}$
10	2 3 $\overline{6}$ 8 10
11	2 $\overline{3}$ $\overline{7}$ 8 $\overline{9}$
12	2 4 5 9 $\overline{10}$
13	2 $\overline{4}$ $\overline{8}$ 9 10
14	2 $\overline{5}$ $\overline{6}$ 7 $\overline{10}$
15	3 $\overline{4}$ 5 $\overline{7}$ $\overline{8}$
16	3 $\overline{4}$ 6 7 $\overline{10}$
17	3 $\overline{5}$ 6 8 9
18	4 $\overline{5}$ 6 $\overline{7}$ 9