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A novel formulation for optimum conceptual design of buildings of rectangular shapes

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Abstract

In many optimum design cases, we look for a set of design variables selected from a given list which assure the optimum of objective function together with satisfaction of the constraints. That is, optimum conceptual design mostly consists of selecting the best combination of a finite number of structural elements and the available parameters. In this paper a novel technique is presented for the conceptual design optimization of framed buildings with a rectangular plan. The method supports all the buildings with grid pattern column layout. To that end, we take advantage of the knapsack problem as a basic applied combinatorial optimization problem. The objective is to find a rectangular column layout of maximum profit for the frames that also satisfy the imposed geometric constraints. The method is presented in a general form and is not confined to optimization under a certain type of action effect and/ or geometrical constraints. It gives the methodology the ability of being formulated for various objective functions and constraints and different structural systems. Civil-Comp Press, 2013.

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A Novel Formulation for Optimum Conceptual Design of Buildings of Orthogonal Shapes

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Abstract

This paper presents a description of a new methodology for the optimum preliminary layout design of reinforced concrete multi-span beams considering the relevant cost elements. First, a new objective function is presented, as an alternative to traditional cost functions for reinforced concrete beams. The ability of being easily employed in layout optimization problems, gives the new cost function a distinct advantage over its alternatives. Examples are included to illustrate the performance of the new methodology.

Keywords: cost optimization; layout optimization; reinforced concrete; multi-span beam; Ant colony optimization.

1   Introduction

The design process can be divided into four stages: the formulation of functional requirements stage, the conceptual (preliminary) design stage, the optimization stage and the detailing stage. An iterative procedure for the four stages is often required before achieving the final solution. In what is named optimal design, the required structural behaviour together with the design loads and geometrical constraints are initially specified and then, the cost or the objective function is defined. In a comprehensive structural optimization process, selecting an appropriate preliminary geometric layout design of structures is of great importance, as it influences all the subsequent stages of the design procedure. The aim of this computational effort is to

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determine the preliminary geometry so that the desired behaviour is achieved at the lowest possible cost. The outcome can be considered as an optimum starting point, in a comprehensive optimization design process of reinforced concrete continuous beams.

Preliminary design, as the earliest phase of the design process, commences with a set of initial concepts. Designers at this early stage must understand the many factors affecting the project being designed, including account for efficiency, construction cost, operation cost, quality and comfort of the built project, and the potential for revenue generation. Significant complexity comes from the need to determine the relative benefits of all of these various quantities and qualities [1]. In fact, generally there is not a single layout design solution having optimal performance with respect to all requirements because the objective criteria are often conflicting, and designers must evaluate different competing criteria with the view to achieve a good compromise design. That is, the selection of a suitable layout design involves making informed subjective compromises between conflicting objective criteria.

The phase of preliminary layout design is usually carried out according to the architectural requirements without considering the relative costs of concrete, reinforcing steel, formwork or other relevant costs. In fact, the phases of the detailed design and optimization, only deal with a predefined layout plan and the effects of the preliminary layout design on the total costs are mainly neglected. Albeit, designers often are intended to spend most of their working time on the detailed design phase, where the scope for improvement is much less [2]. On the other hand, in contrast to steel structures, where the cost optimization can, more or less, be formulated as a weight minimization problem; for concrete structures, the optimization problem needs to be formulated as a cost minimization problem; because different materials are involved. In practice, a minimum weight design may not lead to a minimum cost design in concrete structures. Ideally, the optimization problem should be considered in terms of costs of materials, fabrication, erection, maintenance, and disassembling the structure at the end of its life cycle as the cost elements.

A number of methods have been developed as general methods for the layout optimization of structures [3-5]. In literature, topology or layout optimization methods rarely consider the cost factors and the objective function is optimized regardless of the involving cost elements. Therefore the topology optimization methods may result in a sub-optimal solution.

Sharafi et al.[6-11], Govindaraj and Ramasamy [12] have attempted to make use of evolutionary methods, to deal with the problem in a discretized form. Muc and Gurba [13] described the concept of using Genetic Algorithm in layout optimization of composite structures. Wang and Liu [14] described a methodology for optimizing both weight and cost for composite structures. Hadi [15] employed a Neural Network (NN) method to deal with the cost optimization of RC beams. Recently, using new heuristics, some different methods have been employed for the layout optimization of structures by Nimtawat and Nanakorn [16, 17], Zhu et al. [18], Shaw et al. [19]. Zou [20] described a multiobjective life-cost optimization approach for topologically predefined reinforced concrete frames. Liu and Qiao [21] presented a

technique for topology optimization of structures with different tensile and compressive properties in layout design of bridges.

Nevertheless, the applications of these methods are limited to either cost optimization of structures with predefined shapes or layout optimization of structures without considering the relevant costs. Therefore, there is a need to study topology optimization methods that can take cost into account. This study describes a developed approach for optimization of preliminary layout that considers all cost elements and is applicable for multi-span RC beams.

At first, a new cost function is proposed to deal with the cost optimization problem of rectangular RC beams, which can be used in layout optimization of multi-span RC beams as well as considering the cross-sectional action effects. Then, some numerical examples demonstrate the methodology.

2   Problem Definition

When designing concrete structures, the designer is faced with four classes of design variables. The first class is material design variables such as the type of concrete to be used. The second class is topological design variables such as the number of spans in a frame. The third class is geometric layout variables such as the length of spans in a continuous beam; and finally, cross section design variables such as the dimensions of a concrete section [22]. In most occasions, in optimization process, the designer is not free to set the pre-assigned parameters and design variables, and the existing circumstances may dictate to the designer what to do. Exploiting experience, nevertheless, one may shift from one set of variables to another that causes a new definition for the objective function (cost function in this case). Moreover, any changes in choosing design variables may lead to changes in constraints and even vary their nature from design variables to behaviour ones and vice versa. In other words, depending on the nature of the optimization problem, the process of achieving an optimum feasible solution may be much quicker, shifting from one design space to another by changing design variables as the space dimensions.

The selection of the cost function can be considered the most important decision in the entire optimal design process. Therefore, it is essential to introduce a cost function that represents the most influential cost components and more importantly, is applicable for a variety of similar optimization problems. Furthermore, it must be capable of matching the explicit constraints of structures, which are often given by formulas in design codes.
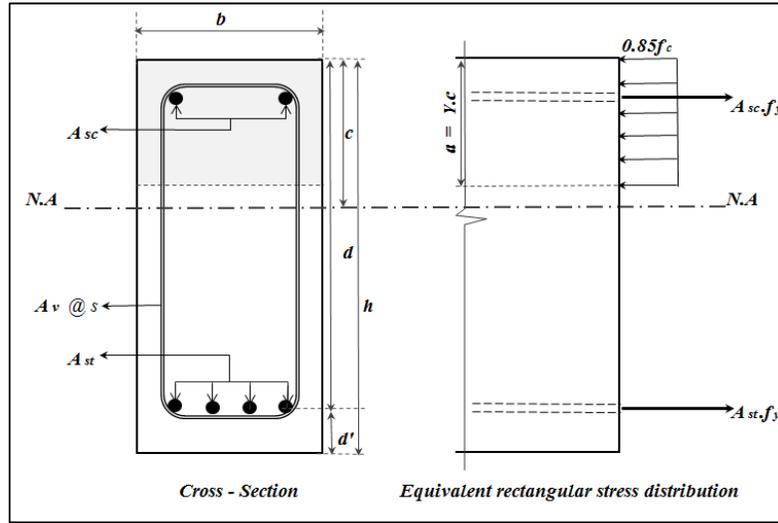
Figure 1: An arbitrary section of a rectangular RC beam [10, 9]

A cost function generally includes the cost of materials, transportation, fabrication and even maintenance costs, in addition to repair and insurance costs, which can be presented by a weighted sum of a number of properties. The effect of these factors in optimal cost can be imposed on the weighted coefficients of the cost function. In concrete structures, at least three different cost items should be considered in optimization: costs of concrete, steel, and the formwork. So, the general cost function for a reinforced concrete beam can be expressed in the following form [10, 9]:

$$C = C_c + C_s + C_f \tag{1}$$

where C, C_c, C_s and C_f are the total cost, cost of concrete, cost of steel and cost of the formwork, respectively. Obviously, for pre-stressed and fibre reinforced concrete sections the relevant costs for these two items are added to the total cost. Considering the unit costs per parameter and the fact that, unit costs of longitudinal reinforcement steel usually differ from shear steel; Eq. (1) can be written as

$$C = c_c A_c + c_{sl} A_{sl} + c_{sv} A_{sv} + c_f P_f \tag{2}$$

Where c_c, c_{sl}, c_{sv} and c_f are the unit costs of concrete, longitudinal steel, shear steel and formwork respectively and A_c, A_{sl}, A_{sv} and P_f are their corresponding quantities. In most published optimization studies [23], the costs are calculated for a member such as a beam, and coefficients and quantities are presented based on members or the unit length of each member in Eq. (2). If such a classic cost function for a beam optimization problem is used, some or all of the parameters A_c, A_{sl}, A_{sv} and P_f are considered the design variables. Then, based on explicit constraints, which are presented in codes and/or implicit behavior constraints of the problem, the optimum values of the design variables will be found using optimization techniques. In other

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words, most existing cost optimization functions deal only with cross-sectional variables.
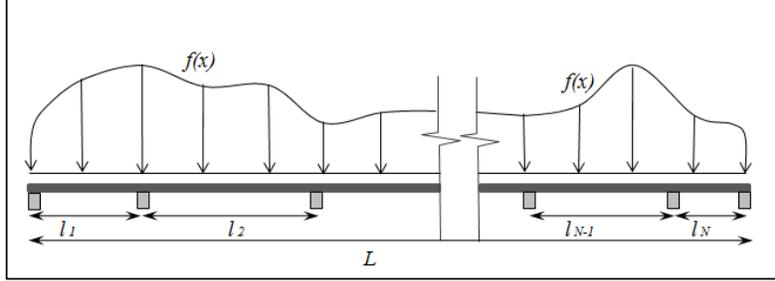
Figure 2: A multi-span rectangular RC beam

In case of the multiobjective optimization of structures including a combination of material, geometrical layout, topological and cross-sectional design variables, using currently available formulation leads to a significant number of design variables and constraints. This is because each of the stated conditions might require a complicated trial and error procedure. In fact, the design variables of this kind, even in the traditional methods of design, require sound engineering judgment to be chosen, and it can be extremely time-consuming to determine their solution for each section of each member. Therefore, unless alternative design variables are selected for the cost function, the multiobjective optimization procedure might be too unwieldy in such cases.

In the problem of preliminary layout optimization of multi-span RC beams, parameters like cross-sectional area, perimeter of concrete section or area of reinforcement steel are not truly suitable as design variables, because such variables are not obtained from an explicit mathematical procedure. These parameters are mainly calculated using suggested relations in design codes, which do not uniquely provide the exact values for these parameters. Furthermore, in order to determine such parameters, the first step is to complete the structural analysis procedure. Then, using the structural analysis outputs such as forces and moments together with design code requirements, the cross-sectional characteristics of the concrete beam are primarily determined. In such a case, each step of the layout optimization, as in an iterative procedure, includes both analysis and design processes, and besides the classic cross-sectional variables, the layout of a structure and consequently the outputs of structural analysis would be variables of the problem. Therefore, designers have to repeat the design procedure to achieve the optimal cross-sectional variables that are usually functions of other analysis outputs.

Consider Eq. (3) as a potential alternative cost function to Eq. (2) in an arbitrary beam cross-section.

$$C = c_1 M_u^+ + c_2 M_u^- + c_3 V_u \qquad (3)$$

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where M_u^+ and M_u^- are the positive and negative bending moment capacities and V_u is the shear capacity of the section. If an appropriate set of {c_1, c_2, c_3} could be found, such that Eq. (3) represents the cost of the section, the design variables would shift from A_c, A_{sl}, A_{sv} and P_f to M_u^+, M_u^- and V_u. In fact, due to the relationship between the capacity factors of the section and structural analysis outputs in design codes, the aim of using cost functions such as Eq. (3) is to use structural analysis outputs instead of structural design factors.

Using structural analysis outputs, say internal action effects of a beam, as design variables, has some advantages over using structural design outcomes such as cross-sectional characteristics of a beam. Firstly, design action effects of each section can be easily obtained from structural analysis, and in an iterative mathematical procedure, re-analyzing a structure is considerably less time-consuming and more precise than re-designing the structure. If Eq. (2) is used for finding the optimum layout of a large structure with respect to the involved cost elements, the optimization tool needs to deal with both structural analysis and structural design in each step in order to move towards an optimum solution. Besides, in the process of obtaining A_c, A_{sl}, A_{sv} and P_f, in the first step, one needs to have access to the relevant M_u^+, M_u^- and V_u. Therefore, using M_u^+, M_u^- and V_u as optimization variables seems to be a shortcut in finding the optimum cost in the layout optimization of multi-span beams.

Moreover, in Eq. (3), the cost function is considered in a section rather than a member. Such ability is an advantage, as it enables the designer to select a number of sections for each member and in the entire structure to control the cost, and there is no necessity to conduct the optimization process over the entire member. Since the cross-sectional characteristics of a member, say a RC beam, such as A_c, A_{sl}, A_{sv} and P_f varies along the member, in order to come up with an accurate cost function for a member, one needs to have a clear idea about the probable distribution of bending moments and shear forces. Such a requirement makes the formulation extremely complicated and sometimes impractical for some multi-objective problems.

The rest of this paper describes how to find an appropriate set of coefficients that make Eq. (3) a viable alternative to Eq. (2) for layout optimization of reinforced concrete continuous beams. Using the proposed cost function, a method to find the optimum lengths of spans in a multi-span beam in order to minimize the cost is formulated.

3   Mathematical Formulation

In formulating the cost optimization of a beam and the relevant constraints, the Australian design standard for reinforced concrete structures, AS-3600 [24] is used, which is based on limit states design method of concrete structures. The evaluation of the area of steel reinforcement and the area of concrete section are based on the strength limit states of the section that can occur in either concrete or steel.

Consider an arbitrary section of a rectangular reinforced concrete beam as shown in Figure 1. The dimensions of the section are *b* and *h*, the areas of tension and

compression reinforcement steel are A_{st}, A_{sc} respectively and the area of shear reinforcement steel in a unit length of beam is A_{sv}/s.

The capacity or the ultimate strength of the section in negative and positive flexure and shear are M_u^-, M_u^+ and V_u respectively, which can be obtained from Eq. (4) through Eq. (6) in a balanced section in which both tension and compression steel yield [9, 10].

$$M_u^- \cong A_{sc} f_{yl} d(1- 0.5\gamma k_u) \tag{4}$$

$$M_u^+ \cong A_{sc} f_{yl} d(1- 0.5\gamma k_u) + (A_{st}-A_{sc}) f_{yl} (d - a/2) \tag{5}$$

$$V_u \cong f_{yv} d A_{sv}/s + \beta bd (f'_c)^{(1/2)} \tag{6}$$

where f_{yl} and f_{yv} are the yield strength of the longitudinal reinforcement steel and the shear reinforcement steel respectively, f'_c is the characteristic compressive cylinder strength of concrete at 28 days, and β is a coefficient based on the standard. Parameters k_u and γ are the neutral axis parameter and the ratio of the depth of the assumed rectangular compressive stress block to $k_u d$. Other parameters are shown in Figure 1.

In order to shift from Eq. (2) to Eq. (3) and come up with the set of $\{c_1, c_2, c_3\}$, the first step is to determine how variations of A_c, A_{sl}, A_{sv} and P_f affect M_u^+, M_u^- and V_u and vice versa. That is, the reciprocal relationships between these two sets of variables need to be identified, and to be found out how increasing or decreasing the amount of each cross-sectional parameter influences the section strength capacities and how one should change the cross-sectional parameters to vary section capacities. As stated above, given the unit costs c_c, c_{sl}, c_{sv} and c_f , the cost function can be defined using Eq. (2) for each section. If any of cross-sectional parameters A_c, A_{sl}, A_{sv} or P_f changes, the cost function varies in a section as follows

$$\Delta C = c_c \Delta A_c + c_{sl} \Delta A_{sl} + c_{sv} \Delta A_{sv} + c_f \Delta P_f \tag{7}$$

On the other hand, using Eq. (3) variations in section capacities would change the cost function as follows

$$\Delta C = c_1 \Delta M_u^+ + c_2 \Delta M_u^- + c_3 \Delta V_u \tag{8}$$

Eqs. (7) and (8) show the contribution of each factor to cost changes and sensitivity of the cost to each term. For example, changing a unit of A_c causes a change of c_c units in cost. Therefore, if the effect of variations of A_c, A_{sl}, A_{sv} and P_f on variations of M_u^+, M_u^- and V_u are determined, the contribution of each section capacity to cost changes, that is the set of $\{c_1, c_2, c_3\}$, can be found out.

In order to simplify the calculations, two assumptions are made. First, since we intend the section to remain in a balanced condition, any changes for compression steel are considered equal to those of tension steel.

$$\Delta A_{st} = \Delta A_{sc} = \Delta A_{sl} \tag{9}$$

This means, in case of the layout optimization of multi-span beams in which, the aim is to find the optimum spans lengths, variation of steel quantities for compression and tension steel in a balanced section can be considered equal in each step of an iterative process. Such an assumption causes a section to remain in balanced condition in every step and results in a balanced section at the end of the optimization procedure. The second assumption is that the depth of the compressive region of concrete, the parameter a, is not considerably affected by variations of the section width b. The width of a beam section is generally constrained by a number of factors including architectural constraints, standards' limitations and limitations of columns and joints. All these constraints cause the width of the section to have a minor variation during the optimization process. It does not mean that the variation of width is ignored, but in practice, such a variation is so limited that it has a negligible effect on the depth of the compressive region.

Considering the fore mentioned assumptions and using Eqs. (4) through (7), if A_s varies:

$$\begin{cases} \frac{\Delta M_u^-}{\Delta A_{sl}} \cong f_{yl}d(1 - 0.5\gamma k_u) \rightarrow \Delta A_{sl} \cong [f_{yl}d(1 - 0.5\gamma k_u)]^{-1} \Delta M_u^- = K_1 \, \Delta M_u^- \\ \frac{\Delta M_u^+}{\Delta A_{sl}} \cong f_{yl} \, d(1 - 0.5\gamma k_u) \rightarrow \Delta A_{sl} \cong [f_{yl} \, (1 - 0.5\gamma k_u)]^{-1} \Delta M_u^+ = K_1 \, \Delta M_u^+ \quad (10) \\ \frac{\Delta V_u}{\Delta A_{sl}} \cong 0 \rightarrow \Delta V_u \text{ and } \Delta A_{sl} \text{ are indipendant of each other} \end{cases}$$

If A_v varies, the variation of the section capacity will be as follow:

$$\begin{cases} \Delta M_u^- \text{ and } \frac{\Delta A_{sv}}{s} \text{ are indipendant of each other} \\ \Delta M_u^+ \text{ and } \frac{\Delta A_{sv}}{s} \text{ are indipendant of each other} \\ \frac{\Delta V_u}{\left(\frac{\Delta A_{sv}}{s}\right)} \cong f_{yv}d \rightarrow \frac{\Delta A_{sv}}{s} \cong (f_{yv} \, d)^{-1} \Delta V_u = K_2 \, \Delta V_u \end{cases} \quad (11)$$

If the area of section, i.e. bd varies:

$$\Delta A_c = \Delta(bd) = b \, \Delta d + d \, \Delta b + \Delta b \, \Delta d \quad (12)$$

M_u^- is not a function of b and based on the second assumption we made, the dependence of $M^+{}_u$ on changes of b is neglected. Moreover, the first term of V_u is not dependent on b as well. So, the variations of the section capacities due to variation of section area are as follow:

$$\frac{\Delta M_u^{(b)}}{\Delta A_c^{(b)}} = \frac{\Delta M_u^{(b)}}{\Delta(bd)} \cong \frac{\Delta M_u^{(b)}}{b(\Delta d)} \quad (13)$$

$$\begin{cases} \frac{\Delta M_u^-}{\Delta(bd)} \cong \frac{f_{yl}A_{sc}}{b}(1-0.5\gamma k_u) \rightarrow \Delta(bd) \cong \left[f_{yl}\frac{A_{sc}}{b}(1-0.5\gamma k_u)\right]^{-1}\Delta M_u^- = K_3\,\Delta M_u^- \\[2mm] \frac{\Delta M_u^+}{\Delta(bd)} \cong \frac{f_{yl}}{b}(A_{st}-0.5\gamma k_u A_{sc}) \rightarrow \Delta(bd) \cong \left[\frac{f_{yl}}{b}(A_{st}-0.5\gamma k_u A_{sc})\right]^{-1}\Delta M_u^+ = K_4\,\Delta M_u^+ \\[2mm] \frac{\Delta V_u}{\Delta(bd)} \cong \frac{f_{yv}A_{sv}}{bs}+\beta(f'_c)^{1/2} \rightarrow \Delta(bd) \cong [\frac{f_{yv}A_{sv}}{bs}+\beta(f'_c)^{1/2}]^{-1}\Delta V_u = K_5\,\Delta V_u \end{cases} \quad (14)$$

The variation of the perimeter of a rectangular section, which determines the variation of formwork, affects the section capacity as follow:

$$\Delta P_f = \Delta b + 2\Delta d \qquad (15)$$

$$\begin{cases} \frac{\Delta M_u^-}{\Delta b+2\Delta d} \cong 0.5 f_{yl}A_{sc}(1-0.5\gamma k_u) \rightarrow \Delta P_f \cong 2\left[f_{yl}A_{sc}(1-0.5\gamma k_u)\right]^{-1}\Delta M_u^- = K_6\,\Delta M_u^- \\[2mm] \frac{\Delta M_u^+}{\Delta b+2\Delta d} \cong 0.5 f_{yl}(A_{st}-0.5\gamma k_u A_{sc}) \rightarrow \Delta P_f \cong 2\left[f_{yl}(A_{st}-0.5\gamma k_u A_{sc})\right]^{-1}\Delta M_u^+ = K_7\,\Delta M_u^+ \\[2mm] \frac{\Delta V_u}{\Delta b+2\Delta d} \cong \frac{2}{\frac{f_{yl}A_{sv}}{s}+\beta b(f'_c)^{1/2}}+\frac{1}{\beta d(f'_c)^{1/2}} \rightarrow \Delta P_f \cong \left[\frac{2}{\frac{f_{yv}A_{sv}}{s}+\beta b(f'_c)^{\frac{1}{2}}}+\frac{1}{\beta d(f'_c)^{\frac{1}{2}}}\right]\Delta V_u = K_8\,\Delta V_u \end{cases} \quad (16)$$

Multiplying both sides of Eqs. (10) by $c_{sl}/2$, Eqs. (11) by c_{sv}, Eqs. (14) by $c_c/3$ and Eqs. (16) by $c_f/3$, and adding them up will result in:

$$\begin{aligned} c_c\,\Delta A_c + c_{sl}\,\Delta A_{sl} + c_{sv}\,\Delta A_{sv} + c_f\,\Delta P_f = & \left(\tfrac{1}{3}c_c K_4 + \tfrac{1}{2}c_{sl}K_1 + \tfrac{1}{3}c_f K_7\right)\Delta M_u^+ \\ & + \left(\tfrac{1}{3}c_c K_3 + \tfrac{1}{2}c_{sl}K_1 + \tfrac{1}{3}c_f K_6\right)\Delta M_u^- \\ & + \left(\tfrac{1}{3}c_c K_5 + c_{sv}K_2 + \tfrac{1}{3}c_f K_8\right)\Delta V_u \end{aligned} \quad (17)$$

Comparing Eq. (17) with Eqs. (8) and (9) results in:

$$\begin{cases} c_1 = \tfrac{1}{3}c_c K_4 + \tfrac{1}{2}c_{sl}K_1 + \tfrac{1}{3}c_f K_7 \\[2mm] c_2 = \tfrac{1}{3}c_c K_3 + \tfrac{1}{2}c_{sl}K_1 + \tfrac{1}{3}c_f K_6 \\[2mm] c_3 = \tfrac{1}{3}c_c K_5 + c_{sv}K_2 + \tfrac{1}{3}c_f K_8 \end{cases} \quad (18)$$

The coefficients c_1, c_2 and c_3 determine how the parameters M_u^-, M_u^+ and V_u contribute to the cost function. Now, in order to re-analyze a rectangular beam to achieve the optimum criteria, one can use Eq. (3) in lieu of Eq. (2). For this purpose, the cost will be the sum of cost functions of all selected sections in the structure.

$$C_t = \sum_{i=1}^{NS} C_i = \sum_{i=1}^{NS}(c_{1i}M_{ui}^+ + c_{2i}M_{ui}^- + c_{3i}V_{ui}) \qquad (19)$$

where C_t is the total cost of the structure and NS is the number of selected sections to control the cost.

The strength constraints on each selected section i in the structure under a load case may be written as

$$\begin{cases} \emptyset_f \, M_{ui}^+ \geq M_i^{*^+} \\ \emptyset_f \, M_{ui}^- \geq M_i^{*^-} \qquad \text{for i=\{1,2,...NS\}} \\ \emptyset_s \, V_{ui} \geq |V_i^*| \end{cases} \qquad (20)$$

in which, $M_i^{*^+}$, $M_i^{*^-}$ and $|V_i^*|$ are positive and negative flexure and shear action effects of section i. $\emptyset_f$ and $\emptyset_s$ are strength reduction factors in flexure and shear respectively. The serviceability requirements limit the maximum deflection u_{max} on the entire member to Δ_{max} under the serviceability load case. So, for all sections:

$$u_{\text{max i}} \leq \Delta_{\text{max i}} \qquad \text{for i=\{1,2,...NS\}} \qquad (21)$$

Other constraints for durability, fire resistance, minimum cover and minimum flexural strength, can be easily added to the problem as well, based on the relevant design codes.

Now, consider a multi-span RC beam with N spans and a total constant length of L, under the arbitrary loading system $f(x)$ as shown in Figure 2. The aim is to re-design the beam to determine the optimum span lengths in order to minimize the cost. According to Eqs. (19) and (20), the total cost is a function of the beams' action effects under the loading system, which in turn are functions of the span lengths in a structural analysis. The general formulation of the problem is:

$$\begin{cases} \min_{l_1, l_2, ..., l_N} \;\; Cost(l_1, l_2, ..., l_N) = \sum_{i=1}^{NS}(c_{1i}M_{ui}^+ + c_{2i}M_{ui}^- + c_{3i}V_{ui}) \qquad for\ i = \{1,2,...NS\} \\ \qquad - - - - - - - - - - - - - - - - - \\ \quad s.t \begin{cases} [K]\{U\} = \{F\} \\ \emptyset_f\{M_u^+\} \geq \{M^{*^+}\} \\ \emptyset_f\{M_u^-\} \geq \{M^{*^-}\} \\ \emptyset_s\{V_u\} \geq |V^*| \\ \{u_{max}\} \leq \{\Delta_{max}\} \\ \{l_{min}\} \leq \{l_i\} \leq \{l_{max}\} \\ other\ constraints\ based\ on\ standards \end{cases} \end{cases} \qquad (22)$$

where the first constraint is the behavior constraint of the structural optimization problem determining the relation between design action effect $\{M^{*^+}\}$, $\{M^{*^-}\}$ and $\{V^{*^-}\}$, and spans lengths based on general formulation of the displacement method in which, *[K]* is the global stiffness matrix of the structure, *{U}* is the displacement vector and *{F}* is the externally load vector obtained from *f(x)*. Using the Euler-Bernoulli beam element for finite element analysis of structures, the dimension of the above vectors will be *2N*; where *N* is the number of spans. The dimension of other variable constraint vectors, say *{M_{ui}^+}*, *{M_{ui}^-}*, is *NS*; because the other constraints and cost function will be evaluated in *NS* critical sections. Furthermore, the length of each span can be constrained between l_{min} and l_{max} in order to satisfy architectural considerations [9, 10].
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4   Numerical Examples

Example 1: consider a three-span beam with geometry details, live, and dead loads shown in Figure 3. The beam supports a slab of thickness 120 mm in the interior of a residential building. The material properties and relative cost factors are: $f'_c=25$ MPa, $f_{yl}= f_{yv} =400$ MPa, $c_c=1$, $c_{sl}= c_{sv} =66$, $c_f=0.42$. The ratio of effective span to the maximum total deflection should not be less than 250. The aim is to find the optimum l_1, l_2, and l_3, so that the cost is minimized.
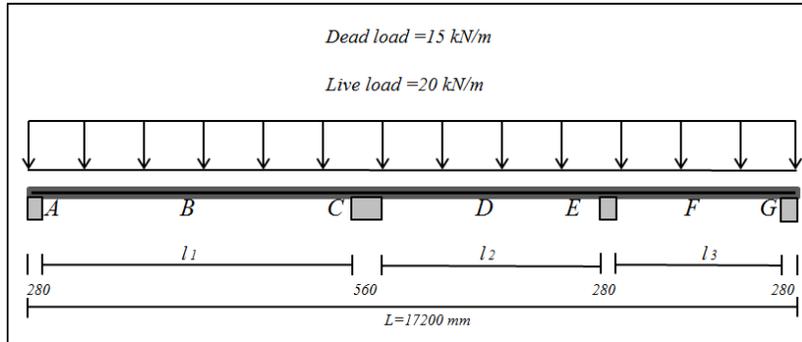
Figure 3: Example 1- A three-span beam with uniformly distributed loads

A similar problem given $l_1 = 7000$ mm, $l_2 =5000$ mm and $l_3 =4000$ mm, has been solved by Kanagasunda and Karihaloo [25]. This design is used as the initial design to start the optimization process for the spans. It should be added that every initial design based on preliminary judgment of the designer and/or using approximate charts or formulas, which meet the design code requirements, can be used as the initial design and as the starting point of optimization process. According to their solution for the seven selected sections A through G, based on AS3600 (2009), and considering the abovementioned costs, the primary cost of the structure based on Eq. (2) equals 4648 units

It should be noted that, the number of selected sections completely depends on the number of critical or control ones, and one may choose more sections to achieve much accurate results. Using Eqs. (10), (11), (14) and (16) the values of K_1 to K_8, and using Eq. (18) the values of c_1, c_2 and c_3 for sections A to G are obtained. Having the necessary coefficients, the cost function can be defined, based on Eq. (19).

The optimum lengths are $l_1 = 4740$ mm, $l_2 =5980$ mm and $l_3 =5280$ mm resulting in a total cost of 3817 units based on Eq. (2). That means the recent layout with the obtained spans lengths suggest around 18% less cost compared to the initial layout.

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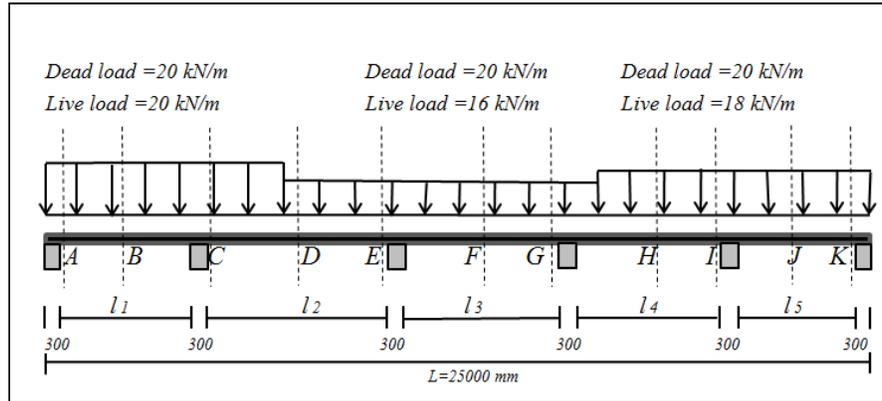
Figure 4: Example 2 - A five span beam and selected sections to control the cost

Example 2: A five-span continuous beam with a total length of 25000 mm is considered as shown in Figure 4. The material properties and relative cost factors are: f'_c=25 MPa, f_{yl}=500 MPa, f_{yv} =250 MPa, c_c=1, c_{sl}=75, c_{sv} =64, c_f=0.45. The aim is to find the optimum l_1, through l_5, so that the cost is minimized. The eleven sections A through K, are taken as the control sections.

The preliminary design of the beam comprised five equal spans, which results in a total cost of 5949 units. The optimum lengths are l_1 = 3450 mm, l_2 =4600 mm, l_3 =5400 mm, l_4 =4400 mm and l_5 =5350 mm resulting in a total cost of 5220 units.

5   Conclusion

The main objective of this study is to propose an appropriate model for cost optimization of RC beams layout design. This model considers the effects of layout design on total cost, and suggests an optimization method for layout design of multi-span beams. Using action effects instead of cross-sectional properties of structures, the proposed model simplifies the process of cost and layout optimization of multi-span beams, and is applicable to various topology optimization problems of RC beams when the cost elements are considered. The presented examples show that the proposed algorithm using the new cost optimization function provides acceptable results, and can be easily employed to optimally design the preliminary layout of RC continuous beams.

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