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Abstract

This paper evaluates the performance of the Value-at-Risk models of the foreign exchange rates at different time horizons. It starts with generating different returns at different time horizons from the USD-EUR 5-minute returns and simulates six parametric models (Normal-GARCH, Student-GARCH, Normal-IGARCH, Student-IGARCH, Ornstein-Uhlenbeck volatility, and jump) to assess the possible trading losses on 30 minutes, 6 hours, 12 hours and daily horizons. Using the variance method, Hill estimator, and the Generalized Pareto Distribution, VaR forecasts are obtained. The performance of the selected VaR models along with each VaR technique are evaluated at 1% and 5% confidence level by calculating the violation ratio. The results show that, at both high and low frequencies, the predictive power of the VaR methods display wide variation in assessing the foreign exchange risk. Furthermore, if a return generating process considers the tail fatness and the stochastic volatility structure of the exchange rate returns, the Extreme Value Theory methods will not be superior to the standard variance method.

Keywords: Value-at-Risk; Variance method; Hill estimator; Extreme Value Theory
JEL: C22, C41, G10

1. Introduction

The increasing popularity of Value-at-Risk (VaR) in recent years has played an important role in financial risk management for banks and fund managers as well as financial market regulators. The VaR represents a unique quantity that gives the expected maximum loss over a given planning horizon with a given confidence level. It serves as a tool to manage and control risk for any financial institution that have a trading portfolio. However, the choice of a planning horizon and a confidence level for VaR estimates is largely arbitrary. The time horizon used to calculate VaR should normally depend on the liquidity of the securities in the portfolios and how frequently they are traded.[1] The confidence levels or the quantiles that are useful for capital requirements range between 95% and 99%. Inside this range, many VaR values can be produced not only by using a given risk management system, such as RiskMetrics, but also by relying on different asset price generating processes.

There is a general consensus in the risk management literature (e.g. Embrechts et al., 1997) that any question concerning financial market risk management in finance involves quantile (VaR) estimation. Within the tool kit on VaR estimation, we find a plethora of methods to assess the VaR of a distribution of losses and profits. In all these methods, the typical VaR calculation involves assessing the possible extreme loss resulting from holding a portfolio for a fixed period using the volatility as a measure of risk over a given period of time. In doing so, J.P. Morgan has introduced the RiskMetrics analysis in October 1994, and most companies and banks found it simple to implement empirically with minimum computational burden. Since then, a vast body of literature developed by comparing some early VaR methods (e.g. Variance-Covariance method and Historical Simulation) with each other or with the benchmark model of RiskMetrics (e.g. Allen, 1994). Recently, the growing need to evaluate extreme risk in financial markets has shifted interest to EVT (see Embrechts et al. (1997) as example). In fact, EVT provides a useful tool for measuring the market risk and gives information about the extreme outcomes. Moreover, it provides guidance on the type of distribution one should select. The most popular EVT models are the tail index estimator of Hill (1975) and the generalized

[1] For an active trader who is likely to get a margin call, the current convention of 10-day horizon is hardly of any use.

Pareto distribution (GPD).[2] For example, McNeil and Frey (2000) found that GPD is preferable to other methods such as standard GARCH model with normal and Student-t innovations, in the sense that it can incorporate asymmetries in the tails and therefore better estimate the tail of the distribution. Their approach consisted of fitting a GARCH model to various return series and using historical simulation and threshold methods from EVT to estimate the distribution of the residuals. Furthermore, with different time horizons, they used the square-root-of-time scaling of one-day VaR estimates to obtain estimates for longer time horizons of 5 or 10 days, and find that this procedure does not perform well in practice. As an alternative, they proposed using a Monte Carlo method based on the fitted models to obtain better results at different time intervals. According to their results, EVT-based methods along with two basic stylized facts, namely *stochastic volatility* and *fat-tails*, play an important role in the estimation of VaR.

In the VaR literature, there are few studies that dealt with the performance of VaR models at the intraday level (e.g. Beltratti and Morana, 1999, and Giot, 2005) and rarely at different time scales. In fact, using intraday returns for risk management purposes can be significant for both internal and external observers of what a satisfactory market risk measure is. Internally, bank managers need a measure that allows efficient management of the bank's risk position. Bank regulators, on the other hand, want to be sure that a bank's net worth loss is accurately measured and that the bank's capital is sufficient to survive a loss. Therefore, both bank managers and bank regulators want up-to-date measures of risk. In this context, this paper tests VaR models of currency returns at different time horizons starting from high frequencies. Our study incorporates not only VaR methodologies that are prominent in the literature (Extreme Value Theory and the Variance method, in conjunction with a variety of volatility models) but also specific features of the currency markets, in particular the increasing fat-tails with the time interval of exchange rate returns and the existence of discontinuities in the price process.[3]

[2] This is also called the exceedances over threshold model of Pickands (1975). The reader can also refer to a thorough analysis of the extremes of data by Davison and Smith (1990).

[3] Although we consider risk management of currency trade in this study, the methodology applies to the trade of other financial assets

Therefore, it is an important exercise to look at the effect of various return generating processes and VaR calculation techniques at various time horizons for the quantification of foreign exchange market risk, in particular the USD-EUR market which is addressed in this study. We test the performance of selected VaR models. We start with simulating a random walk return model with different volatility specifications with both normal and Student-t distribution, and a jump model for the returns. We proceed by using the VaR computation methods, and testing the performance of these models at 1% and 5% confidence level. Finally, a reality check based on minimizing a loss function that compares the actual loss and profit with the forecasted VaR is performed.

3. Market risk models

In this section we present four basic stochastic processes that are used in the literature representing the exchange rate returns process. We consider $r_t, t = 1...T$ as the return series where $r_t = (FX_t - FX_{t-1})/FX_{t-1}$ or logged difference of FX, i.e. FX stands for exchange rate.

3.1 GARCH(1,1)

The GARCH(1,1) specification of Bollerslev (1986) for the exchange rate returns considers the volatility clusters observed in financial time series. It is presented as follows,

$$
\begin{aligned}
r_t &= \sigma_t \in_t, \\
\sigma_t^2 &= \alpha_0 + \alpha_1 \in_{t-1}^2 + \alpha_2 \alpha_{t-1}^2,
\end{aligned}
\tag{1}
$$

where ε is assumed to follow some probability distribution with zero mean and unit variance. We assume two possible distributions for the error terms, namely ε_t is IID N(0,1) and IID Student-t(v), where v represents the degrees of freedom. In most of the early papers on the intraday exchange rate returns, the mean has not been taken into consideration (see for example Andersen et al., 2001). In Nekhili et al. (2002), there are no dynamics in the mean of the intraday returns at 30 minute, 6 hours, 12 hours, and daily horizon. The drift of the continuous time process of the returns is highly insignificant confirming the assumption that the expected returns are equal to zero for all time horizons. For this reason, we do not consider a drift in the return process and consequently in the calculation of the VaR.

3.2 IGARCH(1,1)

There is now evidence that volatility has a notable degree of persistence in the returns (see for example Baillie and Bollerslev, 1989). This persistence has been interpreted within the context of an Integrated GARCH model as in Baillie et al. (1996) for the exchange rate returns. The case where α_1 in equation (1) is set equal to $1-\zeta$ and α_2 is equal to ζ, constitutes the random walk IGRACH(1,1) which is presented as follows[4],

$$\begin{aligned} r_t &= \sigma_t \in_t, \\ \sigma_t^2 &= a + \zeta \in_{t-1}^2 + (1-\zeta)\sigma_{t-1}^2, \end{aligned} \tag{2}$$

where ε_t is IID N(0,1). To test for the possibility of fat tails of the return distribution, we also consider ε_t as IID t(0,1,v) with v degrees of freedom. This model is as important for practitioners as most companies and banks choose to implement for risk quantification. The IGARCH model can be equivalent to the exponentially weighted moving average model (RiskMetrics) of J.P. Morgan (1995). This model is in fact a simple form of the IGARCH model where the pre-specified parameter $\zeta = 0.94$ and the intercept a set to zero.

3.3 Ornstein-Uhlenbeck

Stochastic volatility model is another important return generating process used by Heston (1993). The volatility of the returns is assumed to follow a mean-reverting Ornstein-Uhlenbeck process defined as follows,

$$\begin{aligned} r_t &= \sigma_t \varepsilon_t, \\ \sigma_t &= \sigma_0 e^{-a(t-t_0)} + b(1-e^{-a(t-t_0)}) + \sigma_v \int_{t_0}^{t} e^{-a(\tau-t)} dZ_\tau, \\ \sigma_{t_0} &= \sigma_0, \end{aligned} \tag{3}$$

where σ_v is the standard deviation of the volatility, a>0, b>0. The Wiener processes, ε_t and Z_t, are assumed to be independent. In his empirical work, Heston (1993) allow for a correlation between ε_t and Z_t. We may consider this for future research.

3.4 Jump process

[4] We consider the estimation of the parameters of the IGARCH(1,1) process for different time horizons, and do not consider a fixed parameter of 0.94 as it is usually done by J.P. Morgan.

To estimate the impact of news on traders, such as the fundamental macro-economic information or the intervention of domestic and foreign central banks, we consider a jump process of the exchange rate returns as follows,

$$r_t = (-\lambda\theta)(t - t_0) + \sigma(\varepsilon_t - \varepsilon_{t_0}) + \sum_{t=1}^{n_t} Ln\kappa_t \,, \tag{4}$$

where κ_t is the jump and assumed to be i.i.d and lognormal with mean θ and variance δ^2, and n_t is the actual number of jumps during the interval $t - t_o$ following a Poisson arrival process with parameter λ as a mean number of information arrivals per unit time. It is assumed that upon the arrival of "abnormal" information there is an instantaneous jump in the exchange rate of size κ_t, independent of ε_t. The noise and the Poisson process are infinitely divisible in time, appropriately scaled, and have independent increments.

5. VaR Calculations

In this section, we show the commonly used VaR estimation techniques. In the following, we use the variance method and the Extreme Value Theory based methods, namely the Hill estimator and the Generalized Pareto Distribution.

5.1 Variance Method

Variance method is the variance-covariance method used in portfolio analysis, and in the case of one single asset, as in this case, the USD-EUR exchange rate return, it becomes the variance method. The variance method relates the VaR with the variance or the standard deviation of the returns and, by intuition, the larger the variance of the returns, the larger the VaR. For instance, in the case of assuming that the returns follow a martingale process without drift with $r_t = \sigma_t \varepsilon_t$ where ε_t follow a distribution $\Phi(.)$ that is either assumed as IID N(0, σ_t^2) or as IID Student-t(0, σ_t^2 ,v) with v degrees of freedom, the VaR for the variance method is estimated by

$$VaR_t^H(\alpha) = \Phi^{-1}(\alpha)\sigma_t \,, \tag{5}$$

where $\Phi^{-1}(\alpha)$ is the $(1-\alpha)$ th quantile value of the distribution Φ. VaR_t^H denotes the Value-at-Risk at time t within a certain time horizon H. For instance, if we use a Gaussian distribution for a confidence level α of 5% $\Phi^{-1}(0.05)=1.645$. For a confidence level of 1%, $\Phi^{-1}(0.01)=2.326$.[5]

5.2 Extreme Value Theory (EVT)

Extreme value theory has become an essential and robust framework to evaluate extreme risks in financial markets. The variance method displayed above shows that the extreme risk is related to the variance, but in the case of fat-tailed distributions variance is no longer sufficient. Since the tail fatness of the exchange rate return distribution is one of its attributes that characterize the extent of the risk, we use the tail index as a determining factor in the computation of the Value-at-Risk and contrast its values with those that are based on different stochastic processes.

Generally speaking, in a model of risk, this approach consists of selecting a particular probability distribution for the data and estimating its parameters using empirical data. This way, the EVT acts in favor of providing the best tool for estimating the tail of the distribution. EVT states that the tail distribution of any ordered data must belong to just three possible general families, for which the return process r is presented as follows:

Gumbel: $\Lambda(r)=\exp\left(-\exp^{-r}\right), r\in\Re$

Fréchet: $\Omega_\omega(r)=\begin{cases}0, & r\le 0\\ \exp\left(-r^{-\omega}\right), & r>0, \omega>0\end{cases}$

Weibull: $\psi_\omega(r)=\begin{cases}\exp\left[-\left(-r^{-\omega}\right)\right], & r\le 0, \omega<0\\ 1, & r>0\end{cases}$

Fréchet and Weibull distributions have only one parameter to estimate, ω, which is called the tail index. The Student-t model and the unconditional distribution of ARCH-process both fall in the domain of this type of distributions. As in Gencay et al. (2003), if we set $\omega=1$, the density of the

[5] Suppose we have a hypothetical portfolio consisting of $100 million position in USD-EUR market, and consider that the daily volatility of USD-EUR returns is 0.5%. Assuming that in the VaR calculation the error terms are normally distributed, the 1-day 99% VaR is $100,000,000 times 0.005 times 2.326=$1,163,000.

Weibull distribution is a thin-tailed distribution relative to the normal distribution. Whereas the density of Fréchet distribution also starts from zero but it has a heavy-tail relative to the normal distribution. Finally, the Gumbel distribution has a tail behavior that lies between a thin-tail and a heavy-tail relative to the normal distribution.

A more general representation of these distributions is obtained by reparameterizing the tail index ω to $\varsigma = 1/\omega$. Therefore, a unified representation with a single parameter is well known as the generalized extreme value distribution (GEV)

$$H_{\varsigma}(r) = \begin{cases} \exp\left[-(1+\varsigma r)^{-1/\varsigma}\right], & \text{if } \varsigma \neq 0, \\ \exp\left[-\exp(\mathrm{r})\right], & \text{if } \varsigma = 0, \end{cases} \tag{6}$$

where ς is also known as the shape parameter. The case where $\varsigma = 0$ has to be interpreted as $\varsigma \to 0$ (ς tends to zero), resulting in the Gumbel distribution. When $\varsigma < 0$, we obtain the Weibull distribution, and for $\varsigma > 0$ the Fréchet distribution.

For application in insurance and finance, the Gumbel and the Fréchet family turn out to be the most important models for extremal events. In fact, the domain of attraction of the Weibull distribution are the thin-tailed distributions such as uniform and beta distribution which do not have much power in explaining financial time series. For the Gumbel distribution, the domain of attraction include the normal, exponential, gamma and lognormal distributions where only the lognormal distribution has a moderately heavy-tail.

A modification of the GEV distribution, which considers the behavior of large observations that exceed a high threshold, is now attracting interest in the finance literature, for example McNeil (1999) and Bassi et al. (1998). This new class of distributions is the generalized Pareto distribution (GPD). The GPD is a two parameter distribution that relies on the exceedances of observations over a high threshold $% u$ with the following distribution function,

$$G_{\varsigma,\beta}(r) = \begin{cases} 1-(1+\varsigma r/\beta)^{-1/\varsigma}, & \text{if } \varsigma \neq 0, \\ 1-\exp(-r/\beta), & \text{if } \varsigma = 0, \end{cases} \tag{7}$$

where ς is the shape parameter of the distribution, $\beta > 0$ represents an additional scaling parameter, and $r > 0$ when $\varsigma \geq 0$ and $0 \leq r \leq -\beta/\varsigma$ when $\varsigma < 0$. In case where the scale parameter $\beta = 1$, the distribution in Equation 7 is equivalent to $G_{\varsigma,1}(r) = 1 + \log H_{\varsigma}(r)$, when $\log H_{\varsigma}(r) > -1$. The GPD nests a number of other distributions. When $\varsigma > 0$, it becomes the ordinary Pareto distribution that is more relevant for financial time series analysis since it is heavy tailed. If $\varsigma = 0$, the GPD corresponds to the exponential distribution. For $\varsigma < 0$, it is well known as Pareto II type distribution. A more common case happens when $\varsigma < 0.5$, which is valid for high-frequency foreign exchange returns (Embrechts et al., 1997).

In this paper, the quantification of the USD-EUR foreign exchange market risk will be conducted using two approaches to calculate the VaR. The first one is the Hill estimator and the second one is the GPD. Both of these approaches estimate first the shape parameter ς and then find the VaR.

5.2.1 Hill Estimator

Let $r_t, t = 1...T$ be the realizations on exchange rate returns. By ordering the data in a descending order, where $r_1 > r_2 > ...r_T$, we use the Hill (1975) estimator for the shape parameter ς and is given by,

$$\hat{\varsigma} = \left[\frac{1}{m-1} \sum_{t=1}^{m-1} \ln|r_t| \right] - \ln|r_m| \tag{8}$$

where m is the number of order statistics, and $\hat{\varsigma}$ is the estimated value of ς. The Hill estimator is proven to be a consistent estimator of $\varsigma = 1/\omega$ for fat-tailed distributions.

Different tail estimators work well when the sample size is large. However, the estimation of the tail index is dependent on the choice of the number of order statistics m. In fact, the choice of m represents a problem in that we do not know how far we can go to select the order statistic $r(m)$ that is in the tails.

In some ad-hoc methods, the threshold level m is obtained by arbitrarily considering a confidence level and taking the corresponding percentile. Another tool in threshold determination is the Hill-plot. A Hill-plot is constructed such that estimated $\hat{\varsigma}$ is plotted as a function of m upper order statistics or the threshold. A threshold is selected from the plot where the parameter $\hat{\varsigma}$ is fairly stable. However, we find difficulty in searching for the stable portion of the shape parameters for all our simulated returns at different time horizons considered. Therefore, we looked for more efficient technique to get optimal estimates of the order statistics m.

Our estimations are conducted for optimal values of m obtained by the bootstrap procedure of Danielsson and de Vries (1997). In this technique, the fact that the estimator $\hat{\varsigma}$ is asymptotically normal allows us to minimize its mean squared error to determine the optimal number of order statistics m. The optimality is in the sense that the bias and variance of the shape parameter estimate vanish at the same rate. The idea is to construct the bootstrap expectation of $(\hat{\varsigma}-\varsigma)^2$, and to minimize it with respect to m. We employ subsample bootstrap method and take bootstrap resamples of size $T_1=T^{1-s}$, for some 1>s>0, where T is the sample size. The reason for this sample size reduction is that it guarantees convergence in probability.

Kearns and Pagan (1997), show that the convergence rate is fixed by m and that a resample size with s=1/4 works well. We take the same resample size to perform our estimations. The Hill estimators are calculated from the optimal number of order statistics obtained through the subsample bootstrap procedure, and using GAUSS programming language. It follows that the Hill estimator for Value-at-Risk for a given confidence level α is then defined as,

$$Var_t^H(\alpha) = r_m \left[\frac{m}{T\alpha} \right]^{\hat{\varsigma}}, \tag{9}$$

where T is the sample size, m is the optimal number of order statistics, α is the confidence level, and $\hat{\varsigma}$ is the estimated shape parameter.[6]

5.2.2 Generalized Pareto Distribution (GPD)

The shape parameter ς can also be estimated from the GPD distribution by maximum likelihood method. The density of the GPD distribution is

$$g(r) = \frac{1}{\beta}\left(1+\varsigma\frac{r}{\beta}\right)^{-\frac{1}{\varsigma}-1}, \tag{10}$$

[6] For example, suppose that in daily exchange rate returns, the sample size T=1000, the optimal number of order statistics $m = 900$ that corresponds to a return $r(m) = 0.01$, and the estimated shape parameter $\hat{\varsigma} = 0.50$. At 5% confidence level, the Value-at-Risk is $VaR_t(0.05) = 0.0154$. That is the exchange rate return will not exceed 1.54 percent in one day 95 percent of the time.

and the corresponding log-likelihood function is

$$L = -T\ln(\beta) - \left(\frac{1}{\varsigma}+1\right)\sum_{t=1}^{T}\ln\left(1+\frac{\varsigma}{\beta}r_t\right), \tag{11}$$

where T is the sample size.

For a given confidence level α, the VaR is calculated as follows,

$$Var_t^H(\alpha) = u + \frac{\hat{\beta}}{\hat{\varsigma}}\left((\alpha T / N_u)^{-\hat{\varsigma}} - 1\right), \tag{12}$$

where u is the threshold positioned at the $(1-\alpha)$ th sample percentile, T is the number of observations, N_u is the number of exceedances over the threshold u, and $\hat{\beta}$ and $\hat{\varsigma}$ are the parameter estimated from the GPD distribution. McNeil and Frey (2000) argue that the issue of choosing an optimal threshold does not seem critical for the GPD method as it is more important for the Hill estimator method. In fact, the GPD quantile estimator is more stable, in terms of mean squared error, with respect to the choice of the threshold.

The GPD VaR estimations are calculated using EVIS (Extreme Values in S-Plus) software of McNeil (1999).[7] The appropriate threshold value for each return process and VaR method are chosen according to the confidence levels studied. For each time horizon, we take the upper 1% of the sample for 1% confidence level, and the upper 5% of the sample for 5% confidence level.[8]

In our VaR calculations, we divide the returns data, with T observations, in two sub-samples. The first sub-sample, S_E: from $1 \ldots T_k$ with k the length of window used for estimating the VaR from each return generating process, constitutes the estimation sub-sample. The second one, S_F: from $T_{k+1} \ldots T$, represents the forecast sub-sample. Having obtained the simulated distributions from different return

[7] EVIS is downloaded over the internet at http://www.math.eth.ch/~mcneil.

[8] As an example, suppose that in daily exchange rate returns, the threshold is determined as 7 percent and estimated parameters are $\hat{\beta} = 0.05$ and $\hat{\varsigma} = 0.5$. Further suppose that T=10000 and $N_u = 500$. The VaR at 1% confidence level is $VaR_t(0.01) = 0.07 + \frac{0.05}{0.5}[(\frac{10000}{500}0.01)^{-0.5} - 1] = 0.194$. That is the exchange rate return will not exceed 19.4 percent in one day 99 percent of the time

generating processes at different time horizons, the VaRs are computed using a rolling window procedure.

In the case of the variance method, VaR are calculated by finding first the standard deviation of the simulated returns Y_i with $i=1\ldots T_k$ and then rolling until the last T-1 observation. As T_k increases, new simulated returns are included but older ones are removed. Once VaR estimates are obtained, a performance test is run on the forecast sample of the empirical USD-EUR returns.

6. VaR Performances

To assess the VaR model performances, we use the standard procedure that is based on the violation ratio or the failure rate (see Jorion, 2000). The violation ratio is the number of times returns exceed the forecasted VaR divided by the number of one-period ahead forecasts of VaR. It follows that, for each time horizon, an indicator variable can be defined as $I_t = 1(r_t <> VaR_t^H(\alpha))$ and $I_t = 0$ otherwise. The fact that the actual loss does or does not exceed VaR is a sequence of successes or failures with probability $p = \Pr[r_t < VaR_t^H(\alpha)]$. Since the return observations are independent, the indicator $[I_t]_{t=1}^T$, where T is the total number of forecasted VaR, is a Bernoulli process that follows a Binomial distribution. Therefore, we can test the null hypothesis $H_0 : p = \alpha$ against $H_1 : p \neq \alpha$, where p is the failure rate, and also construct a confidence interval for p at the confidence level α. For instance, at the 5% level, the confidence interval is $[p - 1.96\sqrt{p(1-p)/T}, p + 1.96\sqrt{p(1-p)/T}]$.

A high violation ratio corresponds to an underestimation of the risk by the VaR model. If the violation ratio is less than α %, the VaR model excessively overpredict the risk. If the violation ratio is high than α %, it implies an excessive underprediction of risk by the VaR model.

To compare the predictive capability of the performing VaR models, we employ a reality check procedure by minimizing a loss function. The loss function is calculated by weighing the difference between the actual loss or profit and the VaR forecast. It represents the objective of different agents in evaluating risk.

In fact, some agents rely on the difference between the risk forecasts and their actual profits and losses. Whereas, some others, like Bank of International Settlements (BIS) regulators, focus more on the coverage probabilities. Let's denote the loss function LF with respect to the confidence level α , $LF(\alpha)$ as follows;

$$LF(\alpha) = l^{-1} \sum_{t=T-l}^{T} \left| r_t - Var_t^H(\alpha) \right| . (\alpha I_t + (1-\alpha)(1 - I_t)), \tag{13}$$

where r_t is the return, l is the length of the window, α is the confidence level, and I_t is the indicator variable. $LF(\alpha)$ represents the observed deviation from the VaR with the occurrence probability α . Therefore, for an allocated capital of C dollar, we can evaluate the expected losses at a certain trading horizon by multiplying the loss value $LF(\alpha)$ with C. Hence, smaller loss values will indicate the best choice of the return process and the VaR method.

4. Empirical Application

As empirical returns, we use the Olsen forex data for the USD-EUR exchange rate. The sample consists of continuously recorded 5-minute bid and asks prices from January 2, 2003 through November 27, 2004 for a total of 138,816 observations. Each quote consists of a bid and an ask price with a time stamp to the nearest even second. The prices at each 5-minute interval are obtained by linearly interpolating from the logarithmic average of the bid and ask for the two closest ticks as in Muller et al. (1990) and Dacorogna et al. (1993). The continuously compounded prices are the average of the logarithm of the bid and ask prices,

$$P_{t_5} = \frac{1}{2}\left[\ln P(bid)_{t_5} + \ln P(ask)_{t_5}\right], \qquad \text{for } t_5 = 1,\ldots,138815 \tag{14}$$

Not to confound the evidence of slow trading patterns over weekends (see Bollerslev and Domowitz, 1993), we removed the weekend quotes from Friday 22:00 GMT to Sunday 22:00 GMT. The continuously compounded 5-minute returns are calculated as the log difference of the prices,

$$r_{t_5} = P_{t_5} - P_{t_5 - 1}, \qquad \text{for } t_5 = 1,\ldots,138815 \tag{15}$$

To eliminate the seasonality, we filtered the raw 5-minute returns by removing holidays as in Andersen et al. (2001). Moreover and to avoid the bias that can be caused by the buying and selling intentions of the quoting institutions on the price changes observed at high frequencies (see Dacorogna et al., 2001), we opt to work with 30-minute aggregated returns and aggregate for other frequencies, namely 6 hour, 12 hour and daily returns. The number of observations is respectively, 23135 for 30 minute interval, 1927 for 6 hour interval, 963 for 12 hour interval, and 481 for daily interval. For each data set, we take off k=8 months from the sample size T, which constitute the forecast sample S_F, and we perform maximum likelihood estimations of the different return models on the remaining data, which is the estimation subsample S_E.[9] For comparison purposes, we bootstrapped the same USD-EUR returns at different time horizons in the same way as it is used in historical simulation. In addition, we simulated the unconditional distributions of the USD-EUR returns from the proposed stochastic processes. The simulation experiments are implemented in GAUSS, where the parameters of each return process are fitted with the estimated ones, to obtain unconditional return distributions. For each simulation, we generate S=T-k independent samples with T-k observations each, and we take the last observation from each sample to obtain, in fact, T-k number of simulated returns. The number of observations T corresponds to the number of observations for the 30 minute interval, and the simulated 30 minute returns are aggregated for other time interval.[10]

7. Results

The results of the violation ratios at 1% and 5% confidence levels for the estimated VaR using the variance method (Equation 5), the Hill estimator (Equation 8), and the GPD method (Equation 12) are reported in Tables 1-4. The simulated returns obtained from the proposed stochastic processes are used at 30 minutes, 6 hours, 12 hours, and daily time horizons. The number of observations is 23,135 for 30-minute interval, 1,927 for 6-hour interval, 963 for 12-hour interval, and 481 for daily interval. For each data set, we take off more than 8 months from the sample size T (for instance k=180 days for daily intervals), which constitute the forecast sample S_F. We perform maximum likelihood estimations

[9] The GARCH estimations show that taking a forecast sample size of more than 8 months from the whole distribution lead to a decrease in the significance of the ARCH and the GARCH terms. The estimations are performed using GAUSS programming language along with TSM and MAXLIK routines. In some return models, we used S-Plus software.

[10] We estimated the parameters of the selected return models by rolling over the estimation sample S_E and we found that there is no big change in the estimations. The simulation could be performed each time we estimate a return model but, since we aggregated to obtain empirical returns for different time horizons, we are doing the same technique to construct the simulated returns for each time interval.

of the three VaR models on the remaining data, which is the estimation subsample S_E. After investigating the performance of various VaR models, we further present the results of the reality check based on the loss function $LF(\alpha)$ (Equation 13) at 1% and 5% confidence levels.

Table 1 displays the violation ratios of the daily returns for 1% and 5% confidence level. If the return generating process is a GARCH(1,1) with normal errors, the Hill estimator performs at 1% confidence level with a violation ratio of 0.5% which amounts to 0.5% risk underprediction. Some institutions may then not prefer such model because they would have to allocate more than necessary capital although they can meet their regulatory requirements. At 5% confidence level, the variance method performs better with a violation ratio of 3.8%, which amounts to 1.2% risk underprediction. In the case of assuming a GARCH(1,1) with Student-t errors as a return generating process, at both 1% and 5% confidence level, the variance method looks better than the other VaR methods. However, it overestimates the risk at 1% level and underestimates at 5% level. In using a jump process for the returns, at both confidence levels, GPD method is considered as the best alternative in quantifying the exchange rate return's risk. However, using the empirical USD-EUR daily returns, the GPD method performs only at 5% confidence level. In fact, at 1% confidence level and for a relatively small sample size as for the daily returns, it seems that there aren't enough observations at the tail of the distribution for extreme events. Nevertheless, with bootstrapped returns, the GPD performs well at both 1% and 5% confidence level.

Table 2 displays the VaR results using the 12-hour returns. By assuming a GARCH(1,1) with normal errors for the return process, both the variance method and the Hill estimator at 1% confidence level have good performances. Although these two methods underestimate the risk, the Hill estimator provides the best violation ratio with 1.1%. At 5% confidence level, only the variance method performs with 4.1% violation ratio. In assuming a GARCH(1,1) with Student-t errors, again as with the daily returns the variance method performs better at both confidence levels. If the returns are governed by a IGARCH(1,1) with normally distributed errors, at the 1% level, the GPD performs with a violation ratio of 0.8%, while at 5% level, the variance method has the best performance with a violation ratio of 7.2%. Assuming a jump return model, the GPD performs at 5% level with a violation ratio of 6.3% which amounts to 1.3% risk underestimation. For an institution, this means that less capital allocation is needed to meet its capital requirements. On the other hand, the GPD method does not perform for the empirical USD-EUR 12-hour returns as it does with the bootstrapped returns at 1% and 5% confidence level. Therefore, it turns out that the variance method works better when we take into consideration the stochastic structure of the return volatility and the tail fatness of the return distribution.

Table 3 shows the violation ratios of the 6-hour returns. We notice that only the variance method performs the best among the other VaR methods. At 1% confidence level and assuming a GARCH(1,1) with normal errors for the USD-EUR returns, the variance method has a violation ratio of 0.8% and hence overestimates the risk by only 0.2%. Assuming a GARCH(1,1) with Student-t errors for the USD-EUR returns, the variance method has a violation ratio of 0.8% at 1% level, and 5.5% at 5% level, which amounts of 0.5% of underestimation of the risk. Using the empirical USD-EUR 6-hour returns, none of the methods perform to quantify the USD-EUR market risk at the considered sample period. At 5% confidence level, the violation ratios of the empirical returns and the simulated returns from a jump process are close to each other but both underestimate the USD-EUR market risk.

In Table 4, we display the violation ratios of the 30-minute returns. The only best predictive performance comes from the GPD VaR method at 5% level and with assuming jump return model. The corresponding violation ratio is 5.8% which amounts of 0.8% underestimation of risk. It seems that at the highest frequency, taking the possible discontinuity in the return process, by modeling jumps, and the fat-tails of the distribution of the returns, by using tail estimation, plays an important role in evaluating the exchange rate return risk. However, using other return generating processes along with various VaR methods leads to an overly conservative way to estimate the risk. This fact comes to support the findings of Beltratti and Morana (1999) where the high-frequency data used for quantifying the USD-EUR market risk has led to an overestimation of risk at the 5% confidence level. In addition, the difference in the risk measurement obtained with daily data and high-frequency data is due to the difference in the return distribution at these frequencies. However, in contrast to our findings, they report that for the 30-minute time horizon, the GARCH(1,1) model performs better although it still leads to a conservative risk measurement.

Having investigated the VaR models, we will next look at possible extreme losses at the considered time horizons. This is well-known as a reality check and where a loss function is minimized. This loss function is based on weighing the difference between the actual loss or profit and the VaR forecast. Such function represents an objective for different agents in evaluating risk. Tables 5 and 6 give the reality check respectively at 1% and 5% confidence levels for the daily, 12-hour, and 6-hour simulated returns. In these tables, the return generating processes along with their VaR methods are chosen according to their previous performances. For example, at daily time horizon, if the return generating process is a GARCH(1,1) with normally distributed errors then the Hill estimator is the best VaR model to use as reported in Table 16. Whereas, if the return generating process is GARCH(1,1) with Student-t distributed errors then the variance method is the best VaR model to use. These VaR models are then compared according to their corresponding loss function as in Equation 13. For an allocated capital of C dollar, we can evaluate the expected losses at any trading horizon by multiplying the loss value $LF(\alpha)$ with C. Therefore, smaller loss values will indicate the best alternative in assuming the return process and in using the VaR method.

For all the horizons considered and at both the 1% and 5% confidence levels, the bootstrapped USD-EUR returns present the highest loss function among the other returns. At daily horizon, at both 1% and 5% confidence level, a jump return model along with the GPD method presents the lowest loss function, which is 0.0079 for 1% level and 0.0052 for 5% level. In other words, for a capital allocation of $10,000, the estimated trading loss is of $78.89 at 1% level, and $52.44 at 5% level. At 12-hour horizon, assuming a GARCH(1,1) with normal errors for the returns along with the variance method provides the lowest trading loss of $75.58 at 1% level. The same method, assuming a IGARCH(1,1) with normal errors for return process, provides the least trading loss of $35.82 at 5% level. At 6-hour horizon, the lowest trading loss, at 1% level, is amounted to $53.46 by assuming a GARCH(1,1) with normally distributed errors for the USD-EUR returns and using the variance method. At 30-minute horizon, at 5% level, the least loss amount comes with assuming a jump process for the returns.

These results present certain facts. The usual VaR results on daily data do not extend to intradaily returns in the sense that the predictive performance of some VaR models is unstable at different trading horizons. At least in our case, the EVT-based methods are highly dependent on the trading horizon. There is in fact a degradation of the previous performances of some VaR models at daily and 6 hour trading horizons by going to lower frequencies. This joins the difficulty of relying on a scaling law relationship between different time horizon VaRs such as the square-root-of-time scaling law. Moreover, we would expect that the tail index techniques that focus on the prediction of extreme events to perform better but it is clear that even at lower tail probabilities the results are not so convincing. In addition, despite the fact that the EVT-based methods has proven to be performing with stock market data, it seems that dealing with the foreign exchange market at high frequency represent a difficult empirical task to confirm some of the empirical findings obtained with stock return VaR models.

8. Conclusions

Our results document that the application of the VaR techniques on the foreign exchange market is highly dependent on the time horizon and the chosen tail probabilities. For instance, the variance technique leads to good predictive power at 6-hour horizon and less so at 12-hour horizon whereas the EVT-based methods are better in shorter horizons. At daily horizon, we are confronted with a variety of choices according to the tail probability and the loss function. In fact, with respect to the loss function, the GPD have the best predictive power at $\alpha = 1\%$ and 5%, and hence is the best method to use in the VaR estimations. At the highest frequency, the 30-minute horizon, most of the VaR techniques have proven to perform poorly in predicting the risk, with the notable exception of the GPD when assuming a jump return process.

The VaR results obtained on daily data do not extend to intradaily returns in the sense that the predictive performance of some VaR models is unstable at different trading horizons. At least in our case, the EVT-based methods are highly sensitive to the trading horizon. There is in fact a degradation of the previous performances of some VaR models at daily and 6-hour trading horizons by going to lower frequencies. This also tells that computing daily VaR from high-frequency USD-EUR data using a certain scaling law relationship may be misleading. In fact, there is a debate on using a scaling law between different time horizon VaRs such as the square-root-of-time to generate longer horizon VaRs (see McNeil and Frey, 2000). Such scaling law could not work because the return distribution behaves differently at different time scales.

Moreover, we would expect that the tail index techniques that focus on the prediction of extreme events to perform better but it is clear that even at lower tail probabilities the results are not so convincing. In addition, despite the fact that the EVT-based methods has shown to be the best VaR methods with stock market data, it seems that dealing with the foreign exchange market at high-frequency represent a difficult empirical task to confirm some of the empirical findings obtained with stock return VaR models.

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Table 1: VaR violation ratios for daily returns

1% confidence level				5% confidence level			
Returns	Variance Method	Hill Estimator	GPD	Returns	Variance Method	Hill Estimator	GPD
Empirical	7.2	8.8	0.0	Empirical	13.3	0.0	2.7*
Normal GARCH	0.0	0.5*	0.0	Normal GARCH	3.8*	0.0	0.0
Student GARCH	0.5*	13.3	0.0	Student GARCH	7.7*	0.0	1.1
Normal IGARCH	3.8	0.0	0.0	Normal IGARCH	9.4	0.0	1.6
Student IGARCH	0.0	0.0	0.0	Student IGARCH	0.0	0.0	0.0
Ornstein-Uhlenbeck	38.8	38.3	30.0	Ornstein-Uhlenbeck	42.2	11.6	34.4
JUMP	13.8	6.1	1.7*	JUMP	21.1	0.0	3.3*

* Good performance (the theoretical ratio, α , is within the confidence interval).

Table 2: VaR violation ratios for 12 hour returns

1% confidence level				5% confidence level			
Returns	Variance Method	Hill Estimator	GPD	Returns	Variance Method	Hill Estimator	GPD
Empirical	7.7	12.7	0.2	Empirical	13.8	0.0	2.5
Normal GARCH	1.3*	1.1*	0.0	Normal GARCH	4.1*	0.0	0.0
Student GARCH	0.8*	7.5	0.0	Student GARCH	6.3*	0.0	0.0
Normal IGARCH	3.3	4.7	0.8*	Normal IGARCH	7.2*	0.0	1.6
Student IGARCH	0.0	0.0	0.0	Student IGARCH	0.0	0.0	0.0
Ornstein-Uhlenbeck	38.6	43.6	12.2	Ornstein-Uhlenbeck	43.0	11.6	34.7
JUMP	12.2	8.0	2.7	JUMP	19.7	0.0	6.3*

* Good performance (the theoretical ratio, α , is within the confidence interval).

Table 3: VaR violation ratios for 6 hour returns

1% confidence level				5% confidence level			
Returns	Variance Method	Hill Estimator	GPD	Returns	Variance Method	Hill Estimator	GPD
Empirical	7.5	9.1	0.0	Empirical	12.9	0.0	20.8
Normal GARCH	0.8*	3.8	0.0	Normal GARCH	3.0	0.0	0.0
Student GARCH	0.8*	14.1	0.0	Student GARCH	5.5	0.0	1.6
Normal IGARCH	2.9	3.3	0.0	Normal IGARCH	7.2	0.0	1.5
Student IGARCH	0.0	0.0	0.0	Student IGARCH	0.0	0.0	0.0
Ornstein-Uhlenbeck	35.6	34.5	24.3	Ornstein-Uhlenbeck	40.0	10.2	32.3
JUMP	10.2	9.3	8.1	JUMP	17.9	0.0	22.9

* Good performance (the theoretical ratio, α , is within the confidence interval).

Table 4: VaR violation ratios for 30 minute returns

1% confidence level				5% confidence level			
Returns	Variance Method	Hill Estimator	GPD	Returns	Variance Method	Hill Estimator	GPD
Empirical	7.1	20.8	2.7	Empirical	12.4	0.1	22.5
Normal GARCH	1.4	2.1	0.0	Normal GARCH	3.5	0.0	0.2
Student GARCH	1.7	9.5	0.0	Student GARCH	7.1	0.1	1.9
Normal IGARCH	6.8	29.0	0.1	Normal IGARCH	12.3	1.7	1.4
Student IGARCH	0.1	23.5	0.0	Student IGARCH	1.7	0.3	0.0
Ornstein-Uhlenbeck	37.5	38.1	30.0	Ornstein-Uhlenbeck	41.0	9.5	24.4
JUMP	10.0	11.7	2.7	JUMP	16.5	0.1	5.8*

* Good performance (the theoretical ratio, α , is within the confidence interval).

Table 5: Reality Check at 1% confidence level

Returns	VaR Method	Loss Function*
Daily Horizon		
Normal GARCH	Hill	109.90
Student GARCH	Variance	115.00
JUMP	GPD	78.89
12 Hour Horizon		
Normal GARCH	Hill	75.58
Normal GARCH	Variance	87.06
Student GARCH	Variance	91.71
Normal IGARCH	GPD	90.08
6 Hour Horizon		
Normal GARCH	Variance	53.46
Student GARCH	Variance	60.86

* This value is multiplied by 10^4.

Table 6: Reality Check at 5% confidence level

Returns	VaR Method	Loss Function*
Daily Horizon		
Normal GARCH	Variance	75.00
Student GARCH	Variance	63.00
JUMP	GPD	52.44
12 Hour Horizon		
Normal GARCH	Variance	50.90
Student GARCH	Variance	44.90
Normal IGARCH	Variance	35.82
JUMP	GPD	37.74
6 Hour Horizon		
Student GARCH	Variance	30.24
30 Minute Horizon		
JUMP	GPD	7.87