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Theoretical and experimental research on designer surface plasmons in a metamaterial with double sets of circular holes

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Abstract: The designer surface plasmons (DSPs) are studied by the use of a kind of metamaterial with a structure of double sets of circular holes: subwavelength apertures, and indentations. The diameter and spacing of the indentations are smaller at least one order of magnitude than those of the apertures. A theoretical model is built to analyze the DSPs sustained by the indentations by using effective dipoles method. The influence of the DSPs on the transmission property is revealed for electromagnetic waves passing through the apertures. In order to verify the theoretical predication, a set of the metamaterial samples is made and the transmission spectra are measured in microwave regime. Our results provide a new proof for the existence of DSPs and are promising for proposing some techniques for optoelectronic devices in terahertz and microwave regime.

OCIS codes: (240.6680) Surface plasmons; (240.6690) Surface waves; (040.2235) Far infrared or terahertz; (260.2110) Electromagnetic theory; (310.2790) Guided waves; (120.7000) Transmission.

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1. Introduction

Since the concept of the plasmon polariton-like bound surface states, namely mimicking or spoof or designer surface plasmons (DSPs), on structured surfaces of a perfect conductor was presented by Pendry and his coauthors [1,2], many theoretical and experimental researches have been done in this topic [3–16]. A rigorous and systematic theoretical analysis for DSPs was made in Refs. 3 and 11. The existence of DSPs was directly verified in a microwave reflectivity measurement [4] and indirectly verified in a terahertz enhanced transmission test [5]. DSP propagation on structured metal surfaces was directly observed [6].

As we know, the real surface plasmons (SPs) at a metal–dielectric interface [17] can affect the transmission property of electromagnetic waves through subwavelength apertures in metal films or foils in high frequency regime [18–20]. While in low frequency regime, e.g. terahertz and microwave regime, metal can be regarded as perfect conductors according the traditional electromagnetic theory. Thus excitation of the SPs cannot be expected, and consequently there should be no any SP states to affect the transmission property. Can the DSP sates affect the transmission property at low frequency regime if the metal–dielectric interface is structured? To answer this question, we recently proposed a kind of metamaterial [16], which is a metallic foil with a structure of double sets of holes: subwavelength square apertures and small rectangle indentations. The sizes and spacing of the indentations are smaller at least one order of magnitude than those of the apertures. Following the approach in [1,2], we built a theoretical model, the results of which showed that the DSPs sustained by the rectangle indentations could affect the transmission property of the incident wave passing through the subwavelength apertures. If let λ_{peck}^{0} denote the peak transmission wavelength relative to the foil with the apertures alone and λ_{peck} denote that relative to the foil with the double sets of holes, one get $\lambda_{peck} > \lambda_{peck}^{0}$ from our model. Such a conclusion, tentatively called red-shift effect, could be experimentally checked easily. Unfortunately, this theoretical model has not been checked experimentally so far.

In this paper, we theoretically and experimentally investigate a metamaterial with a structure of double sets of circular holes: subwavelength apertures, and indentations. The diameter and spacing of the indentations are smaller at least one order of magnitude than those of the apertures. By using the effective dipoles theory [3,11] instead of former effective-medium method [1,2], we build a theoretical model to analyze the DSPs sustained by the circular indentations, and obtain an analytical form of the dispersion relationship for the anisotropic DSP states. Note that the reason why we chose the effective dipoles theory is that this method can be used under the condition that the depth of perfect conductor surface is finite, which is correspond to the real experiments. Basing on these results, we further theoretically analyze the transmission property of radiation through the subwavelength apertures. An analytical formula for the peak transmission wavelengths, which predicts a red-shift effect similar with that of rectangle indentations [16], is obtained. In order to verify our validity of our model, a set of the proposed metamaterials are fabricated and the transmission

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spectra of the samples are measured at microwave regime. The predicted red-shift effect does exist from the experimental results. However it should be noted that the shift amount of the measured one is much larger than the calculated one from the analytical formula. A rigorous numerical modeling, in order to fully investigate the transmission properties of proposed metamaterials, is also performed by using the Finite-difference time domain (FDTD) method to directly solve Maxwell equations. The simulation results are in good agreement with the measured ones. Our results provide a new evidence for the existence of DSPs and have some potential applications in techniques for optoelectronic devices, e.g. adjustable frequency selector, in microwave and terahertz regime.

2. Theoretical model

Our model system is shown in Fig. 1. We suppose that there is a perfect conductor surface (PCS) pierced by a square array of circular through holes. The hole radius and their spacing are *a* and *d*, respectively. The Cartesian coordinate system is constructed as shown in Fig. 1(a) and the *z* axis is taken to be the surface normal. Meanwhile, the depth of the PCS is assumed to be infinite ($h \to \infty$). For such a structure, if $a \ll d \ll \lambda$ (λ is the wavelength of radiation), the basic physics involved DSP states on the structured surface can be described exactly by the view of effective dipoles [3,11]. Therefore, the scattered field of each small hole shown in Fig. 1(a) is equivalent to that generated by an electric dipole perpendicular to the surface and a parallel magnetic dipole. These dipoles are proportional to the external perpendicular electric field and parallel magnetic field via the polarizabilities α_{E} and α_{M}, respectively [21].

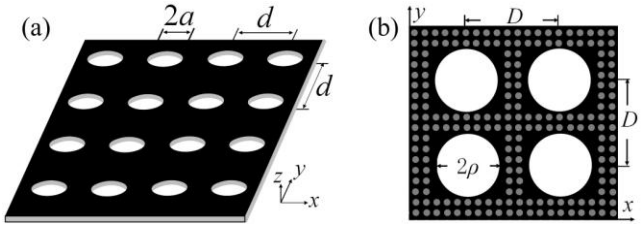

Fig. 1. (a) A set of circular through holes with diameter 2*a* arranged on a *d* × *d* lattice is pierced into a PCS with infinite depth ($h \to \infty$). (b) Schematic structure of the designed plasmonic metamaterial with double sets of circular holes. A set of additional subwavelength circular apertures with larger diameter ρ arranged on *D* × *D* lattices are structured on the hole-array surface shown in Fig. 1(a). Note that *a* and *d* are smaller than ρ and *D*, respectively, at least one order of magnitude.

We assume that a unit *p*-polarized plane wave with wave vector $\boldsymbol{k}$ in the *x*-*z* plane is incident upon the surface. The electric and magnetic dipoles can be presented as $\mathbf{p} = p\hat{\mathbf{z}} = \alpha_{\mathrm{E}}\mathbf{E}^{\mathrm{ext}}$ and $\mathbf{m} = m\hat{\mathbf{y}} = \alpha_{\mathrm{M}}\mathbf{H}^{\mathrm{ext}}$, where $\mathbf{E}^{\mathrm{ext}}$ and $\mathbf{H}^{\mathrm{ext}}$ are the total field (incident plus reflected) in the absence of any surface structure. Through the effective dipole model [3,11], DSP states can be sustained by the structured surface shown in Fig. 1(a) under the condition $1/\mathrm{Re}\{\alpha_{\mathrm{E}}^{-1}\} + 1/\mathrm{Re}\{\alpha_{\mathrm{M}}^{-1}\} > 0$. The DSP momentum component parallel to the surface can be obtained as

$$k_{\parallel}^{2} = k^{2} + P\frac{A^{3}k^{4}}{d^{4}}, \tag{1}$$

where $k = 2\pi/\lambda$ is the light momentum in vacuum, $A = \pi a^2$ is the hole cross section, and *P* is expressed as

$$P = \frac{4\pi^{2}}{A^{3}}\left(\frac{1}{\mathrm{Re}\{\alpha_{\mathrm{E}}^{-1}\}} + \frac{1}{\mathrm{Re}\{\alpha_{\mathrm{M}}^{-1}\}}\right)^{2}. \tag{2}$$

Moreover, the DSP states described by Eq. (1) can provide interaction between additional surface features like holes of larger dimensions, which is similar to the behavior associated

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with real surface plasmons (SPs). To investigate the effect of additional hole arrays with larger dimensions on DSP states, we consider a designed plasmonic metamaterial with double sets of circular holes, as shown in Fig. 1(b). A set of additional subwavelength circular apertures with larger diameter ρ arranged on $D \times D$ lattices are structured on the hole-array surface shown in Fig. 1(a). Note that a and d are smaller than ρ and D at least one order of magnitude, respectively. Since the additional array of larger apertures acts as a grating, the momentum of the grating allows for coupling between external radiation and DSP states. Then, by using an approximation in [20], the conservation of momentum is given by

$$\mathbf{k}_{\parallel} = \mathbf{k}_t^{(\mathrm{rad})} + j\mathbf{G}_x + l\mathbf{G}_y, \tag{3}$$

in which $\mathbf{k}_{\parallel}$ is the parallel component of DSP momentum, $\mathbf{k}_t^{(\mathrm{rad})}$ is that of external radiation momentum. The reciprocal lattice vectors of the additional array are $\mathbf{G}_x$ and $\mathbf{G}_y$, and j and l are integers. One can obtain from Fig. 1(b) that $|\mathbf{G}_x| = |\mathbf{G}_y| = 2\pi/D$. Thus the parallel component of DSP momentum $\mathbf{k}_{\parallel}$, from Eq. (1), 2 and 3, is dependent on geometric parameters of bigger and smaller circular holes simultaneously. For simplicity, we consider radiation normally impinging at the interface, which is $\mathbf{k}_t^{(\mathrm{rad})} = 0$. Combining the equations above, we find that the locations of transmission (or resonance) peak wavelengths are given by

$$\lambda_{jl} = \lambda_{jl}^0 \sqrt{1 + P\frac{4\pi^2 n_{\mathrm{in}}^2 A^3}{d^4 \lambda_{jl}^2}}, \tag{4}$$

where n_{in} is the refractive index of the incident medium, and $\lambda_{jl}^0 = n_{\mathrm{in}} D \Big/ \sqrt{j^2 + l^2}$ denotes the peak wavelengths for the case only with additional aperture array of larger dimensions. Note that the peak wavelengths depend on the geometrical and optical parameters of the small holes, thus allowing us to design the peak wavelengths by adjusting these parameters. The corresponding detailed analysis is just like that for rectangle holes [16], and no necessary to repeat here. One can see from Eq. (4) that $\lambda_{jl} > \lambda_{jl}^0$ for a given set of j and l because $P > 0$. This is that so-called red-shift effect.

In the long-wavelength limit, it should be noted that α_{E} or α_{M} can be obtained from the electrostatic or magnetostatic far field induced by an external field. Specially, when the small holes with $d \times d$ lattice are perforated in a thin PSC ($h \ll 2a$), the equations discussed above are still valid, but the parameter P must be replaced by $P_{\mathrm{thin}} = 4P$ in this case due to cooperative interaction between both sides of the film [3,22].

3. Experimental and simulated results

An experiment is performed at microwave regime to verify the red-shift effect. A set of sample pair is made via using well-developed printed circuit board (PCB) technology. The two samples in a pair have the same structural parameters, but one sample in a pair is only pierced with arrays of subwavelength apertures with larger geometrical size, denoted by S0, while another sample in the pair is simultaneously made a square array of smaller circular holes, denoted by S1, as shown in Fig. 2. The metal film is made of copper with thickness h = 0.035mm, printed on a plastic board (Flame Resistant) with thickness 0.3mm and refractive index n_{p} = 2.049 at 10 GHz. The refractive index of the medium in both small holes and larger apertures are n_{h} = 1. Note that the copper film can be treated as perfect conductor film in microwave domain.

There are two interfaces for the sample: One is metal-air and the other is metal-plastic. They should have the DSP states and will give rise to different red-shift amounts, which can be calculated via Eq. (4). Since the thickness of the copper film is very thin ($h \ll 2a$), the parameter P must be replaced by $P_{\mathrm{thin}} = 4P$ as mentioned in section 2, and it leads to $P_{\mathrm{thin}} = 16/9\pi^3$. For a pair of sample with the following structural parameters, S0: ρ = 11 mm, D = 18 mm, and S1: a = 1 mm, d = 3 mm, ρ = 11 mm, D = 18 mm, the calculated red-shift amounts

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between S0 and S1 are $\Delta\lambda$ = 0.021 mm (Δf = 0.014 GHz) for the metal-air interface with n_{in} = 1, j = 1 and l = 0, while $\Delta\lambda$ = 0.0832 mm (Δf = 0.054 GHz) for the metal-plastic interface with $n_{in} = n_p$ = 2.049, j = 2 and l = 0.

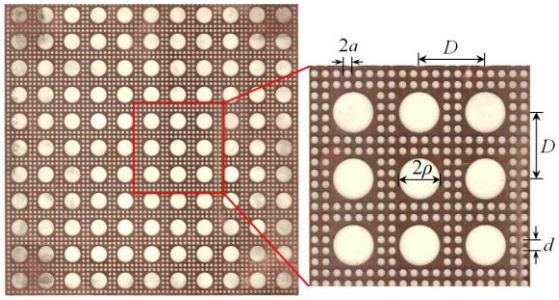

Fig. 2. One of the fabricated samples used in the experiment. The left one is the photograph and the right one used to mark symbols.

The transmission properties of the samples are measured by using two low sidelobe lens antennas with gain 25dB. The incident microwave generated by one antenna, whose frequency covers from 7 to 16GHz, is nearly vertically impinged at the surface of samples, where the polarization direction of the electric field of the incident microwave is normal to the y axis. The transmission microwave is detected by another antenna. Using Network Analyzer (N5230C, 10MHz-20GHz, Agilent Tech.), we visually observe the distribution of the transmittance of electromagnetic wave in frequency domain. Note that the distance between each antenna and the samples falls into the Fresnel zone of the lens antennas. The measured results for the samples S0 and S1 with the structural parameters above are shown in Fig. 3 (a). As can be seen the peak frequency for S1 is smaller than that for S0, indicating that the red shift of S1 relating to S0 is existent. The measured peak wavelengths are λ_{S0} = 21.15 mm and λ_{S1} = 21.46 mm, corresponding to a shift $\Delta\lambda$ = 0.31 mm (Δf = 0.205GHz), to be nearly 4 to 10 times greater than the calculated ones via Eq. (4) for the both interfaces. This is because the model above is very general and abstract when the condition $a<<d<<\lambda$ does not get well satisfied.

In order to give to a more accurate theoretical red-shift amount, we take a rigorous numerical modeling by using the FDTD method to directly solve Maxwell equations for the samples S0 and S1. The polarization direction of the electric field of the incident electromagnetic is taken normal to the y axis. The simulated transmission spectra for the metal-plastic interface are shown in Fig. 3 (b), for which the zero-order diffraction and the positive and negative first orders are considered. The simulated peak wavelengths are λ_{s0} = 21.80 mm and λ_{s1} = 22.33 mm, corresponding to a shift $\Delta\lambda$ = 0.43mm (Δf = 0.26GHz), very closer to the measured ones. The metal-air interface is also simulated, having Δf = 0.005 GHz, indicating that the DSP states in the metal-plastic interface are dominant for affecting the transmission property. It should be noticed that only the position of transmission peak is

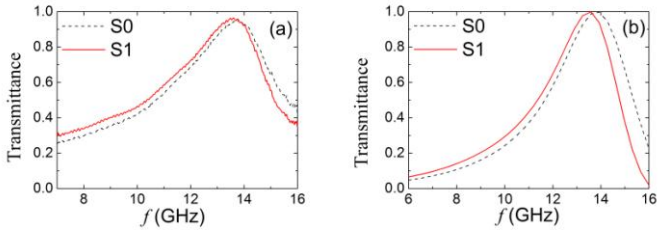

Fig. 3. The measured (a) and simulated (b) transmission spectra for one pair of samples. S0: ρ = 11 mm, and D = 18 mm; S1: a = 1 mm, d = 3 mm, ρ = 11 mm, and D = 18 mm. The simulated ones are related to the metal-plastic interface.

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changed due to the introduction of smaller circular holes, while the transmission profiles, especially the peak amplitude and full width at half maximum(FWHM), are not influenced significantly from the measured and simulated results. The former researches showed that the peak position of EOT can be changed by adding some dielectric or magnetic materials into the subwavlength structures. Such method, however, could influence the profiles of the EOT simultaneously, such as the peak amplitude and FWHM. So our proposed subwavelength structure could be used for adjustable frequency selector without damaging the profiles of EOT.

4. Discussion and conclusion

We have made a set of sample pairs with different structural parameters a , d, A, and D , including those of circular holes. The transmission properties for all the pairs are measured and simulated. All the results demonstrate that the red-shift effect is existent for each sample pair, indicating that the DSP states sustained by a structured surface with small holes can affect the transmission properties of microwave passing through the subwavelength apertures. Our results give a new powerful proof for the existence of DSPs.

At present, DSPs have many applications. For example, it was recently demonstrate that by directly sculpting DSP structures that tailor the dispersion of terahertz surface plasmon polaritons on the highly doped semiconductor facets of terahertz quantum cascade lasers, the performance of the lasers could be markedly enhanced [14]. The theoretical model and the results presented in this paper are promising for exciting some new ideas for the design and improvement of optoelectronic devices in terahertz and microwave regime, such as a frequency modulator.

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