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S. Vafi
University of Wollongong, sina@uow.edu.au

Tadeusz A. Wysocki
University of Nebraska-Lincoln, wysocki@uow.edu.au

Mehran Abolhasan
University of Technology Sydney

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Serially Concatenated Turbo Codes

Sina Vafi
School of Engineering and IT
Charles Darwin University
Darwin, Australia

Tadeusz Wysocki
School of Computer and Electronics Engineering
University of Nebraska-Lincoln
Omaha, USA

Mehran Abolhasan
ICT
University of Wollongong
Wollongong, Australia

Abstract**—The paper presents a new scheme of concatenated codes, referred to as Serially Concatenated Turbo (SCT) codes. The code is constructed as the serial combinations of two turbo codes, i.e. turbo Recursive Systematic Convolutional (RSC) codes and turbo Bose Ray Chaudhuri Hocquenghem (BCH) codes, linked by a pseudo-random interleaver. In comparison with the conventional turbo RSC codes, SCT codes have higher minimum distance values. Based on conducted simulations, it is found that SCT codes outperform turbo RSC codes at the waterfall and error floor regions, while they require reasonable number of iterations at their iterative decoding structure to achieve good performance.**

I. INTRODUCTION

Turbo codes are amongst the most effective Forward Error Correction (FEC) codes, which have been considered in many applications since the last decade. They are regularly designed by a parallel concatenation of two identical codes, either two Recursive Systematic Convolutional (RSC) codes or two block codes as linear Bose Ray Chaudhuri Hocquenghem (BCH) and Reed-Solomon (RS) codes, which are linked by an interleaver.

In turbo codes, the existence of error floor occurred at the medium to high Signal to Noise Ratios (SNRs), is due to the relatively low minimum weight value of the code. The error floor phenomenon makes a serious problem for those applications, whose data are sensitive to the channel noise. Hence, it is vital to employ stronger FEC codes than currently applied turbo codes to mitigate this drawback.

Another type of concatenated code can be constructed by the serial concatenation of two component codes, which are linked by an interleaver. Benedetto et.al proposed a model of serially concatenated codes designed by an outer convolutional code and an inner RSC code with the code rate one (1) [1]. The analysis shows that the minimum distance value of these codes are greater than that of turbo RSC codes and consequently they provide better protection for data against the channel noise. An improvement to this structure can be achieved utilizing turbo RSC codes instead of convolutional codes to construct a concatenated code with an event greater minimum distance value.

Codes with the higher performance than conventional concatenated codes can be constructed by the multiple combination of convolutional or block codes [12]. Huebner et.al proposed a code designed by the serial concatenation of simple repetition block codes and a turbo RSC code [2]. Shea introduced serial concatenation of a high-rate block code such as Rectangular Parity Check Code (RCPC) with a turbo RSC code [3]. Like the conventional parallel and serially concatenated codes, the proposed code is decoded by an iterative decoding technique. In this case, both RCPC and turbo RSC decoders participate in the iterative decoding process. Indeed, an internal iterative decoding related to the turbo RSC code and an external iterative decoding for the serial concatenation of RCPC and turbo RSC codes is employed. The proposed code will have a better performance than a turbo RSC code at the expense of decoding complexity, since two distinct iterative decodings are performed at its decoder. The improvement is obtained by applying very high number of iterations. Simulation results indicate that the performance of the code is significantly decreased when the iterative decoding is being accomplished with a low number of iterations [4]. Alternatively, Li proposed turbo accumulated codes formed by the combination of single parity check codes with RSC codes having the rate one (1). Codes with the high rate create similar or even better performance than turbo RSC codes, while at the low rates, they do not provide significant performance [5]. New scheme of serially concatenated codes were proposed by combination of BCH and RSC codes [6], [7]. Codes apply dual-error correcting BCH codes concatenated with a simple RSC code. Advantages of these codes are simplicity of their constituent codes and their flexibility in the rate compatibility. Recently, new schemes of hybrid codes were proposed, which are constructed on the basis of parallel and serial combinations of simple repeat-accumulate-accumulate codes [8]. In comparison with the multiple serially concatenated codes, such codes have better performance and higher flexibility to produce a high-performance code with a high minimum weight. Analysis and simulation results show that the codes performance is being improved at the expense of higher number of iterations, which consequently increases their complexity design.

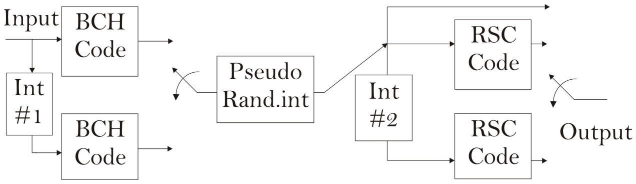

Fig. 1. Structure of Serially Concatenated Turbo (SCT) codes.

The main similarity between the abovementioned codes is that they utilize a block code with a simple structure. At the decoding process, high number of iterations is required gaining a reliable performance from the code. This increases the complexity of design, particularly for the code implemented in [6], which apply a double error-correcting BCH code. In order to construct a code with a higher performance, constituent codes with the high performance are suggested. As a suggestion, a turbo block code can be replaced by those simple block codes proposed by different authors. In this role, two different types of concatenated codes are combined with each other forming new schemes of concatenated codes. Concatenated codes formed by the serial concatenation of two turbo codes, i.e. turbo RSC and turbo block codes, are named Serially Concatenated Turbo (SCT) codes.

This paper investigates the performance of SCT codes constructed by different types of turbo BCH and RSC codes. The mathematical analysis demonstrates that the code with the higher minimum distance have better performance in reduction of the noise. The analysis results are confirmed by the simulations. It is also proved that with the similar codeword lengths and applying reasonable number of iterations, SCT codes outperform turbo RSC codes. The organization of the paper is as follows:

Section II reviews the structure and analysis of SCT codes. In section III, simulation results of different SCT codes and their comparison with the corresponding turbo RSC codes are presented. Finally, section IV, concludes the paper.

II. ANALYSIS OF SERIALLY CONCATENATED TURBO CODES

A Serially Concatenated Turbo (SCT) code is constructed by the serial combination of two constituent outer turbo BCH and inner turbo RSC encoders, which are linked by an interleaver. Figure 1 shows the structure of SCT codes.

Due to the parallel concatenation of two basic BCH and RSC codes, turbo BCH and RSC codes have the minimum weight (free distance) greater than their constituent codes. Consequently, it is concluded that concatenation of these two turbo codes will generate a code with a minimum weight higher than its constituent turbo codes.

In this paper, turbo BCH codes are designed by two identical BCH codes (n, k), where n and k represent input bitstream and codeword length, respectively. BCH codes are linked together by a deterministic row-column interleaver, whose size is the multiple of input bitstream, i.e. $M * k$, where M is an integer ($M > 0$). For every bitstream with the length k, turbo BCH code generates codewords with the length $(2n-k)$. Hence its code rate is $R_{C_1} = \frac{k}{(2*n-k)}$. Data with the length $L = (2 * n - k) * M$ are permuted by the pseudo-random interleaver and then entered to the turbo RSC encoder. In turbo RSC codes, only the first RSC encoder is terminated. This adds m tail bits to the data received from the pseudo random interleaver, where m is equal to the number of memories applied for each RSC encoder. For turbo RSC codes, row-column interleavers matched with the codeword length of turbo BCH codes are considered. Considering a turbo RSC code with the rate $R_{C_2} = \frac{1}{3}$, the SCT codes' rate is calculated by $R_T = R_{C_1} * R_{C_2} = \frac{k}{(2*n-k)*3}$ value.

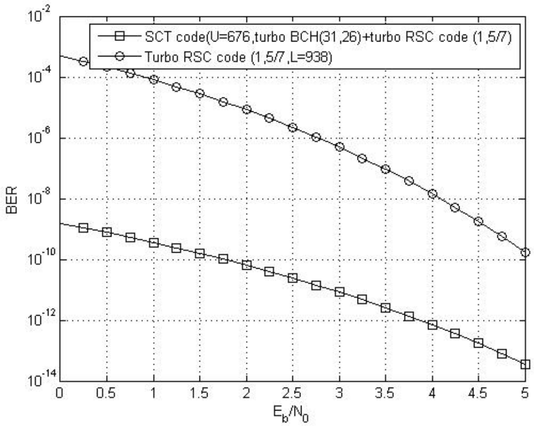

Fig. 2. Upper bound values of SCT and turbo RSC codes.

Analysis of SCT codes can be accomplished by determining Input-Output Weight Enumerating Function (IOWEF). Applying a random interleaver between two constituent turbo codes, IOWEF of the overall SCT code is given by [1]:

$$A^{C_{SCT}}(W, Z) = \sum_{l=0}^{L} \frac{A^{C_{TBCH}}(W, l) \times A^{C_{TRSC}}(l, Z)}{\left(\begin{array}{c} L \\ l \end{array} \right)} \tag{1}$$

Where $A^{C_{TBCH}}(W, l)$ and $A^{C_{TRSC}}(l, Z)$ represent IOWEF of turbo BCH and RSC codes, on the basis of input bitstreams with the weight W and l to produce output bitstreams with the weights l and Z, respectively. IOWEF parameter of each turbo code is independently determined by their weight distributions. The obtained weights are used to determine the effective IOWEF of the code, which only considers the minimum weight specifications of the code. Considering square block interleavers, whose row and column lengths are equal, the weight distributions of turbo BCH codes are determined by the algorithm proposed in [9]. The algorithm applies low-weight input bitstreams with the length k and reflects their effect on the interleaved data for the second BCH code.

For the turbo BCH (31,26) code with the length $U = 26 \times 26 = 676$, the minimum weight 5 with 100 multiplicities is obtained from bitstreams having weight 1. Hence, the effective IOWEF of this code is given by:

$$A_{eff}^{C_{TBCH}}(W, l) = 1 + W(100l^4) \tag{2}$$

For the turbo RSC code, the algorithm proposed in [10] is utilized, which calculates low weight of the code based on self-terminating patterns of the first and the second RSC encoders. For the 4-state turbo RSC code $(1, \frac{5}{7})$ implemented by a row-column interleaver with the length $L = 938 = (14 \times 67)$, the minimum weight of 20 with 912 multiplicities is obtained from input bitstreams having weight of 4. Therefore, the effective IOWEF of this code is given by:

TABLE I
SPECIFICATIONS OF CONSTITUENT TURBO BCH AND TURBO RSC FOR DIFFERENT SERIALLY CONCATENATED TURBO CODES.

SCT	Turbo BCH code			Turbo RSC code(Rate=$\frac{1}{3}$)	
code	(n,k)	Int.	Rate	State	Int.
1	(31,26)	936(26×36)	0.73	$4(1,\frac{5}{7})$	1298(22×59)
2	(31,26)	234(13×18)	0.73	$16(1,\frac{35}{23})$	328(41×8)
3	(63,57)	513(19×27)	0.83	$16(1,\frac{35}{23})$	625(25×25)
4	(63,57)	3306(58×57)	0.83	$4(1,\frac{5}{7})$	4004(143×28)

$$A_{eff}^{C_{TRSC}}(l,Z) = 1 + l^4(912Z^{16}) \tag{3}$$

The computed effective IOWEFs are used for determining the effective IOWEF of the SCT code having input bitstream length $U = 676$ and the overall rate $R_T = 0.73 \times 0.33 \approx 0.24$. In the above example, the effective IOWEF of the SCT code is obtained by:

$$A_{eff}^{C_{SCT}}(W,Z) \approx 3.15 \times 10^{-9} W Z^{16} \tag{4}$$

Consequently, for the Additive White Gaussian Noise (AWGN) channel, the upper bound of probability of error of the SCT code is determined by [1]:

$$P_b(e)_{SCT} \leq \sum_{w=w_m}^{LR_{C_1}} \frac{w}{LR_{C_1}} A_{eff}^{C_{SCT}}(W,Z)|_{W=Z=e^{-R_T E_b/N_0}} \tag{5}$$

Where w_m represents the minimum weight of input bitstreams generating an error for the outer code.

Figure 2 shows the upperbound of probability of error for the SCT code based on the effective IOWEF achieved in Equation 5. In order to verify the performance of the SCT code, its upperbound is compared with the upperbound of the corresponding turbo RSC code. In this comparison, it is assumed that both codes generate the same codeword length. Considering the minimum weight specifications, the probability of error of the turbo RSC code is upperbounded by [10]:

$$P_b(e)_{TRSC} \approx \frac{N_{free}\tilde{w}_{free}}{L} Q\left(\sqrt{d_{free}\frac{2RE_b}{N_0}}\right) \tag{6}$$

Where R, $\frac{E_b}{N_0}$, N_{free} and $\tilde{w}_{free}$ denote the turbo RSC code rate, the signal to noise ratio per information bit, number of multiplicities and the average weight of information of the minimum weight (free distance value, i.e. $d_{free} = 20$), respectively. In this specific example, $\tilde{w}_{free} = 4$.

Figure 2 also shows the upperbound of probability of error for the turbo RSC code. The illustrated upperbounds represent that SCT code has higher capability to reduce the error. This is an expected result since the minimum weight of SCT code, $((w_{min})_{SCT} = (w_{min})_{TBCH} \times (w_{min})_{TRSC} = 5 \times 20 = 100)$ is greater than the minimum weight of that turbo RSC code, i.e. 20.

III. SIMULATION RESULTS

In simulations, the performance of SCT codes with different bitstream lengths modulated by Binary Phase Shift Keying (BPSK) modulation over AWGN channel is verified. Table I shows specifications of constituent turbo BCH and RSC codes. At the decoder, the iterative decoding of the turbo encoded data is accomplished by Soft Output Viterbi Algorithm (SOVA). The decoder is implemented by two internal iterative decoders related to turbo BCH and RSC codes, and an external iterative decoder for the serially combined two turbo codes. The iterative decoding is initially started by turbo RSC codes. The extrinsic information obtained at the last iteration of the turbo RSC decoder is being used as an input signal for the iterative turbo BCH decoder. The extrinsic information of iterative turbo BCH decoder is used as a-priori information for the next iteration of turbo RSC decoding.

In internal iterative decodings, 8 iterations are considered for the iterative decoding of turbo RSC codes, while the technique proposed in [11] is being utilized to optimize the number of iterations for the iterative turbo BCH decoding. For external iterative decoding, 4 and 8 iterations are considered. The performance of SCT codes are compared with the regular turbo RSC codes. Turbo RSC codes apply the code rate $\frac{1}{3}$ and the suitable interleaver length so as to produce similar number of encoded bits with SCT codes. Therefore, in comparison with turbo RSC codes, SCT codes have shorter bitstream lengths and the higher code rates.

Figure 3 shows the performance of the SCT code formed by turbo BCH (31,26) and turbo RSC $(1, \frac{5}{7})$ codes. Although error floor of SCT code occurs at the relatively high Bit Error Rates (BERs), it outperforms turbo RSC codes by 0.5 dB. A slight improvement at the error floor region can be obtained when the external iterative decoding is being increased from 4 to 8 iterations. This is being observed by more than 0.25 dB at $BER \approx 5 \times 10^{-6}$.

Figure 4 illustrates the performance of SCT codes constructed by turbo BCH (31,26) and turbo RSC $(1, \frac{35}{23})$ codes. Simulations confirm that the SCT code has better performance than turbo RSC code at the medium to high SNRs. The graphs express that the SCT code implemented by 8 external iterations has 0.5 dB better performance at $BER \approx 10^{-6}$.

Figure 5 shows the performance of turbo BCH (63,57) code combined with different turbo RSC codes. SCT code with the bitstream length $U = 513$, outperforms the corresponding 16-state turbo RSC code by 0.25 dB at $E_b \approx 10^{-7}$. Similarly, better error protection is achieved for SCT code implemented by 4-state turbo RSC code. Again, although the error floor of the SCT code occurs at the high BERs, it still has better performance than that of turbo RSC code. The improvement is obtained by 0.75 dB at $BER \approx 10^{-4}$. Applying algorithm proposed in [11], simulation results indicate that for $BER \leq 10^{-4}$, the iterative decoding of turbo BCH code is stopped after 1 iteration. This demonstrates the interactive effect of high-performance turbo codes in the reduction of overall number of iterations. SCT codes implemented by BCH

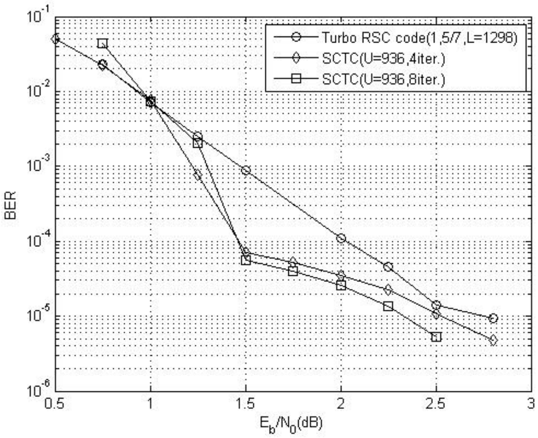

Fig. 3. Performance of SCT codes constructed with turbo BCH code (31,26) and 4-state turbo RSC code $(1, \frac{5}{7})$.

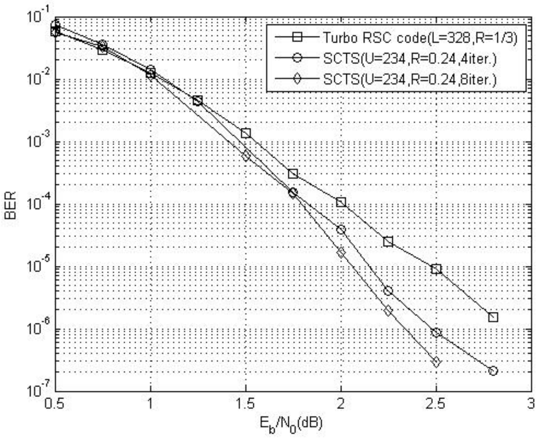

Fig. 4. Performance of SCT codes constructed with turbo BCH code (31,26) and 4-state turbo RSC code $(1, \frac{35}{23})$.

(63,57) code have higher rates. This is because of utilizing turbo BCH (63, 57) code with the rate higher than turbo BCH (31,26) code. It is concluded that applying turbo BCH codes with the higher rate will construct SCT codes with rates closer to the rate of the conventional turbo RSC codes, while they can outperform them.

IV. CONCLUSIONS AND FUTURE WORK

In this paper, a new type of FEC code was proposed. This was implemented by the serial concatenation of turbo RSC and turbo BCH codes. For different codes, it is confirmed that codes outperform turbo RSC codes at the waterfall region. At the error floor region, codes provided similar performance with the turbo RSC codes, while their constituent codes applied low number of iterations. Gaining this progress requires relatively high complexity on the design of codes, specially at the iterative decoding structure. Optimization on the iterative decoding performance will be continued to reduce the iteration number of internal and external decoders, while it maintains the code capability to reduce the error. Like other types of concatenated codes, the results express the effect of interleaver on the performance of SCT codes. Another difficulty is utilizing the reliable deterministic and simple interleavers for the given length of input bitstream. This issue is particularly noticeable in the interleaver design of turbo RSC code. Future work will also investigate designing high-performance interleavers matched with the specifications of constituent turbo codes.

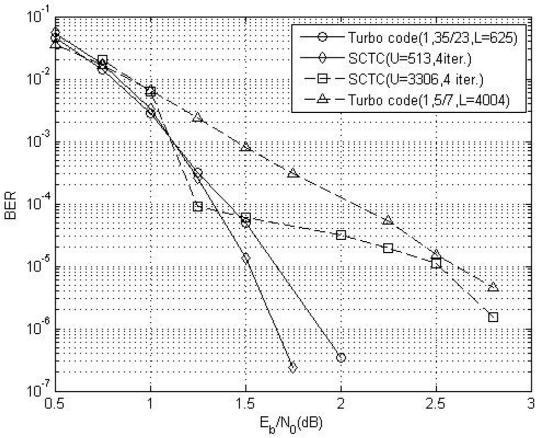

Fig. 5. Performance of SCT codes constructed with the turbo BCH code (63,57) and different turbo RSC codes.

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