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### A method to design transmission resonances through subwavelength apertures based on designed surface plasmons

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Abstract: A plasmonic metamaterial is proposed for which an array of subwavelength apertures is pierced into a metallic foil whose one flat surface has been made with periodic rectangle holes of finite depth. Designed surface plasmons sustained by the holes are explored when the size and spacing of the holes are much smaller than those of the apertures. The transmission property of electromagnetic waves through the metamaterials is analyzed. Results show that the designed surface plasmons characterized by the holes could support the transmission resonances of the incident wave passing through the subwavelength apertures, and that the peak transmission wavelengths could be designed by controlling the geometrical and optical parameters of the holes. Example is taken at THz regime. Our work proposes a method to design the peak wavelengths, and may affect further engineering of surface plasmon optics, especially in THz to microwave regimes.

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#### 1. Introduction

Coupling between light and the collective oscillations of the electron plasma at a metaldielectric interface gives rise to surface plasmons (SPs) [1]. These are the SP states, responsible for a host of phenomena unique to metals, for which the phenomenon of enhanced transmission of electromagnetic waves through subwavelength apertures in metal films or foils has stimulated significant interest in recent years [2] since the pioneering work by Ebbesen and his coauthors [3]. It was believed that the interaction of the incident radiation with the SPs leads to an enhancement of the transmission [4]. The resonant excitation of SPs creates huge electric fields at the surface that force light through the apertures, giving very high transmission coefficients. Such a viewpoint meets challenge at low frequency regime, in which metals can be regarded as perfect conductors, and excitation of the SPs cannot be expected, and consequently the SP excitation does not contribute to the enhanced transmission property. To overcome this problem, Pendry and his coauthors presented a new idea that there should be surface plasmon polariton-like bound surface states, namely mimicking surface plasmons (MSPs), on structured surfaces of a perfect conductor [5]. The existence of MSPs was indirectly verified using microwave reflectivity measurements [6]. MSPs promise the ability to engineer a SP at almost any frequency, thus MSP has a name designer surface plasmon (DSP) [6]. For a semi-infinite perfect conductor perforated with a two-dimensional (2D) square array of square holes whose size and spacing are much smaller than the incident wavelength, by taking a method in which a perfect conductor was optically described in the long wavelength limit as an effective medium, Pendry and his coauthors demonstrated this symmetric structure to be of a dielectric function of plasmon form characterized by its structural parameters [5]. By using the effective-medium method and the well-known method of modal expansion, they repeated their results and analyzed the other two structures made by 1D arrays of grooves and 2D arrays of square holes of infinite or finite depth [7]. In fact, the effective-medium method had been used to consider 1D periodic slits for designing metallic metamaterials with a high index of refraction [8]. Besides, the field-averaging method could provide a useful means of characterizing the effective-medium parameters of an arbitrary metamaterial [9]. It was believed that a measured enhanced transmission phenomenon at THz regime should originate from the crucial contribution of DSPs [10]. DSP propagation on structured metal surfaces was directly observed and the results were quantitatively consistent with theoretical predictions [11]. Based on a modal expansion of electromagnetic fields, a rigorous method for analyzing DSPs on a periodically corrugated metal surface has been formulated [12]. The existence of DSPs has been confirmed by rigorous numerical modeling (such as using finite element [13] and finite time domain [14] numerical methods) and by means of both analytical techniques and rigorous numerical solution of Maxwell's equations [15]. The explicit analytical expressions for determining the dispersion of DSPs sustained by a two-dimensional array of square holes were derived by employing a modified modal matching approach [16]. The DSP theory was used to study the slow-light characteristics of structured metal surfaces with a one-dimensional periodic rectangle [17] or with a graded grating of varying depth in THz domain [18].

DSPs promise the ability to engineer a SP on structured metal surfaces. It is the character distinguishes DSPs from SPs. This character allows us to actively design some relevant experiments, such as microwave reflectivity measurements [6] and highly confined guiding THz surface wave [11]. However, based on the DSP theory to actively design the transmission property of electromagnetic waves through subwavelength apertures have not been considered so far. This mainly originates from the fact that the analytical form of the dispersion relationship for DSPs has not been found when the DSPs are sustained by subwavelength apertures whose size and spacing are comparing to the incident wavelength. As we know that the analytical form is available for DSPs sustained by arrays of holes whose size and spacing are much smaller than the incident wavelength, we naturally address an idea as the following: Can we construct a kind of new plasmonic metamaterials by which we can use the DSP theory to actively design an experiment of the transmission resonances of electromagnetic waves through subwavelength apertures in THz domain? Such metamaterials are made of metallic foils. At first, some rectangle holes of finite depth are made on one flat surface of the foil with the size and spacing of the holes being much smaller than the incident wavelength. And then arrays of subwavelength apertures are pierced on the foil. We imagine that the DSP states sustained by the holes of finite depth could support the transmission resonances of incident radiation through the subwavelength apertures. In this paper, based on the DSP's theory [5,7], we obtain an analytical form of the dispersion relationship for the anisotropic DSP states sustained by the holes, and then analyze theoretically the transmission property of radiation through the subwavelength apertures. Results indicate that the DSP states characterized by the holes could support the transmission resonances of THz radiation through the subwavelength apertures. The formula of the peak transmission wavelengths is obtained, which involves the geometrical and optical parameters of the metamaterials, including those of the holes, indicating that the peak wavelengths could be designed by controlling the parameters of the holes. The shape of holes is chosen as rectangle because it involves three geometrical parameters, i.e., long side, short side and depth, more than that of square or circle shape, thus allowing us to adjust more parameters to design the peak wavelengths. Numerical examples are taken at THz regime. Our research propose a method to use the DSP theory to design experiments of transmission resonances, and may affect further engineering of surface plasmon optics, especially in THz to microwave regimes.

#### 2. Theoretical model

At first, we pay our attention on the structure of a metallic foil pierced with periodic rectangle holes of infinite depth and are devoted to find the effective medium of this structure. A set of  $a \times b$  rectangle holes arranged on a  $d \times d$  lattice is made into a square metallic foil, as shown in Fig. 1 (a). A cartesian coordinate is built for which the *x*- and *y*-axis are parallel to the sides *a* and *b*, respectively, and the *z*-axis is normal to the surface of the foil. For such a structure, if  $a, b < d << \lambda$ , where  $\lambda$  is the wavelength of radiation, externally incident radiation is insensitive to the details of the holes, which it can see only an average response that can be described by an effective permeability  $\mu_x$ ,  $\mu_y$ , and  $\mu_z$ , for which  $\varepsilon_z$  and  $\mu_z$  were determined as  $\varepsilon_z = \infty$  and  $\mu_z = \infty$  by observing that the dispersion of the waveguide mode in the holes is unaffected by parallel momentum [5,7]. In what follows, we will endeavor to find the expressions of  $\varepsilon_x$ ,  $\varepsilon_y$ ,  $\mu_y$ , and  $\mu_z$  for the asymmetric structure.



Fig. 1. (a) A set of  $a \times b$  rectangle holes of infinite depth arranged on a  $d \times d$  lattice is pierced into a square metallic foil. (b) A set of  $a \times b$  rectangle holes with finite depth h arranged on a  $d \times d$  lattice is made on one surface of a square metallic foil. (c) In the effective medium approximation the structure displayed in (b) behaves as a homogeneous but anisotropic layer of thickness h on top of a perfect conductor.

For the waveguide modes excited by an incident wave, both electric and magnetic fields are zero inside the conductor but in the rectangle holes the electric fields of transverse electric (TE) waveguide modes  $TE_{mn}$  have the form

$$\begin{cases} E_x = E_{x0}^{(mn)} \cos(\frac{m\pi}{a} x) \sin(\frac{n\pi}{b} y) \exp(ik_z z - i\omega t - i\pi/2) \\ E_y = E_{y0}^{(mn)} \sin(\frac{m\pi}{a} x) \cos(\frac{n\pi}{b} y) \exp(ik_z z - i\omega t + i\pi/2), x < a, y < b, \qquad (1) \\ E_z = 0 \end{cases}$$

where  $E_{x0}^{(mn)} = \frac{n\pi}{b} E^{(mn)}$ ,  $E_{y0}^{(mn)} = \frac{m\pi}{a} E^{(mn)}$ ,  $E^{(mn)} = \omega \mu_h \mu_0 H_0 / \left[ \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right]$ ,  $m, n = 0, 1, 2, 3, \cdots, \mu_0$  is vacuum permeability,  $H_0$  is constant, x, y and z are cartesian coordinates,  $\omega$  is the angular frequency, t is time,  $i = \sqrt{-1}$  and

$$k_{z} = k_{z}^{(mn)} = i \sqrt{\left(\frac{m\pi}{a}\right)^{2} + \left(\frac{n\pi}{b}\right)^{2} - \left(n_{h}k_{0}\right)^{2}},$$
(2)

where  $k_0$  is the free space wave vector,  $n_h = \sqrt{\varepsilon_h \mu_h}$ ,  $\varepsilon_h$  and  $\mu_h$  are the permittivity and permeability, respectively, of any material that may be filling the rectangle holes. The magnetic fields  $\mathbf{H} = (H_x, H_y, H_z)^T$  follow from Eq. (1), where the superscript *T* denotes the operation of transposition.

For these waveguide modes, only those modes  $\text{TE}_{m0}$  and  $\text{TE}_{0n}$   $(m, n \ge 1)$  can sustain DSP states because the average fields at the surface are equal to zero for the modes TE <sub>mn</sub> with  $m \ne 0$  and  $n \ne 0$ . As we are interested in the limit  $a, b << \lambda$ , we assume that the fundamental modes will dominate because they are the least strongly decaying. We thereby only consider two modes: TE<sub>10</sub> and TE<sub>01</sub>. For the mode TE<sub>10</sub>, we have  $\mathbf{H} = (H_x, 0, H_z)^T$  and  $\mathbf{E} = (0, E_y, 0)^T$  with

$$E_{y} = E_{y0} \sin(\frac{\pi}{a}x) \exp(ik_{z}z - i\omega t + i\frac{\pi}{2}), \quad 0 < x < a, 0 < y < b,$$
(3)

where  $E_{y0} = (\pi/a)E^{(10)}$ . We then suppose that there exist effective homogeneous fields in the effective medium as  $\mathbf{H}' = (H'_x, 0, H'_z)^T$  and  $\mathbf{E}' = (0, E'_y, 0)^T$  with

$$E'_{y} = E'_{y0} \exp(ik'_{z}z + ik'_{x}x - i\omega t),$$
(4)

in which  $k'_z$  is the same as in the waveguide, *i.e.*,  $k'_z = k_z$ , and  $k'_x$  is defined by the incident direction. If this effective field is to match to the incident and reflected fields external to the surface,  $E_y$  and  $E'_y$  must give the same average fields at the surface. Hence, by the use of the two equations above, after matching to incident and reflected waves, we have

$$\overline{E}_{y} = \frac{\int_{0}^{b} dy \int_{0}^{a} E_{y} dx}{d^{2}} = \frac{E_{y0}b}{d^{2}} \int_{0}^{a} \sin(\frac{\pi}{a}x) dx = \frac{2E_{y0}ab}{\pi d^{2}} = E'_{y0}.$$
(5)

By taking the same approach, we have  $H'_z = 0$  because  $\overline{H}_z = 0$  and  $H'_x \neq 0$ , indicating that the effective homogeneous fields only have two components  $E'_y$  and  $H'_x$ , thus we have  $k_z = \sqrt{\varepsilon_y \mu_x} k_0$  and  $(\mathbf{E}' \times \mathbf{H}')_z = -E'^2_y k_z / (\omega \mu_0 \mu_x)$ . We then consider the instantaneous flows of energy across the surface,  $(\mathbf{S})_z = (\mathbf{E} \times \mathbf{H})_z$  and  $(\mathbf{S}')_z = (\mathbf{E}' \times \mathbf{H}')_z$ . They must be the same inside and outside the surface, both for the real and effective media, thus we have

$$\overline{(\mathbf{E} \times \mathbf{H})_{z}} = -\frac{k_{z}}{\omega \mu_{h} \mu_{0}} \frac{E_{y0}^{2}}{d^{2}} \int_{0}^{b} dy \int_{0}^{a} \sin^{2}(\frac{\pi}{a}x) dx$$

$$= -\frac{k_{z} E_{y0}^{2} ab}{2\omega \mu_{h} \mu_{0} d^{2}} = \overline{(\mathbf{E} \times \mathbf{H}')_{z}} = -\frac{k_{z} E_{y0}'^{2}}{\omega \mu_{x} \mu_{0}}.$$
(6)

From the two equations above, we have

$$\mu_x = \frac{8ab}{\pi^2 d^2} \,\mu_h,\tag{7}$$

Substitute  $k_z = \sqrt{\varepsilon_y \mu_x} k_0$  and Eq. (7) into Eq. (2) with m = 1 and n = 0, we have

$$\varepsilon_{y} = \frac{\pi^{2} d^{2}}{8ab} \varepsilon_{h} (1 - \frac{\omega_{a}^{2}}{\omega^{2}}).$$
(8)

$$\omega_a = \frac{\pi c}{n_b a},\tag{9}$$

where c is the velocity of wave in a vacuo.

For the mode TE<sub>01</sub>, we have  $\mathbf{H} = (0, H_y, H_z)^T$  and  $\mathbf{E} = (E_x, 0, 0)^T$  with

$$E_{x} = E_{x0} \sin(\frac{\pi}{b} y) \exp(ik_{z}z - i\omega t - i\frac{\pi}{2}), \quad 0 < x < a, 0 < y < b,$$
(10)

where  $E_{x0} = (\pi/b)E^{(01)}$ . We then suppose that the effective homogeneous fields in the medium are  $\mathbf{H}' = (0, H'_y, H'_z)^T$  and  $\mathbf{E}' = (E'_x, 0, 0)^T$  with

$$E'_{x} = E'_{x0} \exp(ik'_{z}z + ik'_{y}y - i\omega t),$$
(11)

in which  $k'_z = k_z$ , too, and  $k'_y$  is defined by the incident direction. For this case, similarly, we have  $k_z = \sqrt{\varepsilon_x \mu_y} k_0$ . By the use of the same approach as above, we obtain

$$\mu_y = \frac{8ab}{\pi^2 d^2} \mu_h,\tag{12}$$

$$\varepsilon_x = \frac{\pi^2 d^2}{8ab} \varepsilon_h (1 - \frac{\omega_b^2}{\omega^2}). \tag{13}$$

$$\omega_b = \frac{\pi c}{n_b b}.\tag{14}$$

The results above show that  $\varepsilon_x \neq \varepsilon_y$ . This is easy understood for the asymmetric structure. Taking a = b, we can get the results for the case of square holes [5,7].

We then consider the DSP states sustained by a structured surface with rectangle holes of finite depth. As shown Fig. 1 (b), a set of  $a \times b$  rectangle holes with depth h arranged on a  $d \times d$  lattice is made into one surface of a square metallic foil. In the effective medium approximation the structure displayed in Fig. 1 (b) behaves as an homogeneous but anisotropic layer of thickness h on top of a perfect conductor, as shown in Fig. 1 (c). For an interface between the effective medium layer and an isotropic homogeneous dielectric medium with  $\varepsilon_{in}$ , there should be an anisotropic bound surface state, *i.e.*, DSP state. Let  $\beta$  be the wave vector of the DSP state, which has two components  $\beta_x$  and  $\beta_y$  in the x and y directions, respectively, thus

$$\boldsymbol{\beta} = \beta_x \mathbf{e}_x + \beta_y \mathbf{e}_y. \tag{15}$$

where  $\mathbf{e}_{x}$  and  $\mathbf{e}_{y}$  are the unit vectors in the x and y directions, respectively.

After some straightforward algebra the specular reflection coefficient,  $R_y$ , for a plane wave whose electric and magnetic fields have the form  $\mathbf{E}^{in} = (0, E_y^{in}, E_z^{in})^T$  and  $\mathbf{H}^{in} = (H_x^{in}, 0, 0)^T$  impinging at the surface of the homogeneous but anisotropic layer of thickness *h* with  $\mu_x$  and  $\varepsilon_y$  given by Eqs. (7) and (8) can be written as

$$R_{y} = \frac{\varepsilon_{in}^{-1}k_{z}^{in} - \varepsilon_{y}^{-1}k_{z} + (\varepsilon_{in}^{-1}k_{z}^{in} + \varepsilon_{y}^{-1}k_{z})\exp(2ik_{z}h)}{\varepsilon_{in}^{-1}k_{z}^{in} + \varepsilon_{y}^{-1}k_{z} - (\varepsilon_{y}^{-1}k_{z} - \varepsilon_{in}^{-1}k_{z}^{in})\exp(2ik_{z}h)},$$
(16)

here  $k_z^{in} = -i\sqrt{\beta_y^2 - (n_{in}k_0)^2}$  with  $n_{in} = \sqrt{\varepsilon_{in}}$  being the refractive index of the dielectric medium. By extending this formula to the case  $\beta_y > n_{in}k_0$  and looking at the zeros of the denominator of  $R_y$  and note that  $k_z = \sqrt{\varepsilon_y \mu_x} k_0$  for this case, we can obtain

$$\beta_{y} = n_{in} \frac{\omega}{c} \sqrt{1 + P_{a} \frac{\omega^{2}}{\omega_{a}^{2} - \omega^{2}}},$$
(17)

where  $P_a = PH_a^2$  for  $\mu_h = 1$ ,  $P = \frac{64a^2b^2n_{in}^2}{\pi^4d^4n_h^2}$  and  $H_a \approx \frac{1 - \exp(-2\pi h/a)}{1 + \exp(-2\pi h/a)}$  for  $\lambda >> n_h a$ . For the plane wave whose electric and magnetic fields have the form  $\mathbf{E}^{in} = (E_x^{in}, 0, E_z^{in})^T$  and  $\mathbf{H}^{in} = (0, H_y^{in}, 0)^T$  impinging at the surface of the homogeneous but anisotropic layer of thickness *h* with  $\mu_y$  and  $\varepsilon_x$  given by Eqs. (12) and (13), the specular reflection coefficient,  $R_x$ , can be obtained as

$$R_{x} = \frac{\varepsilon_{in}^{-1}k_{z}^{in} - \varepsilon_{x}^{-1}k_{z} + (\varepsilon_{in}^{-1}k_{z}^{in} + \varepsilon_{x}^{-1}k_{z})\exp(2ik_{z}h)}{\varepsilon_{in}^{-1}k_{z}^{in} + \varepsilon_{x}^{-1}k_{z} - (\varepsilon_{x}^{-1}k_{z} - \varepsilon_{in}^{-1}k_{z}^{in})\exp(2ik_{z}h)},$$
(18)

here  $k_z^{in} = -i\sqrt{\beta_x^2 - (n_{in}k_0)^2}$ . Note that  $k_z = \sqrt{\varepsilon_x \mu_y} k_0$  for this case, similarly, we have

$$\beta_x = n_{in} \frac{\omega}{c} \sqrt{1 + P_b \frac{\omega^2}{\omega_b^2 - \omega^2}}.$$
(19)

where  $P_b = PH_b^2$  for  $\mu_h = 1$ , and  $H_b \approx \frac{1 - \exp(-2\pi h/b)}{1 + \exp(-2\pi h/b)}$  for  $\lambda >> n_h b$ .

From Eqs. (17) and (19), we can obtain the dispersion curves of the anisotropic DSP waves, as shown in Fig. 2. Notably, the asymptotic frequencies corresponding to  $\beta_x$  and  $\beta_y$  are different, in contrast to an array of square holes where such two asymptotic frequencies are the same [5].



Fig. 2. Dispersion curves obtained from Eqs. (17) and (19) for  $\omega$  (in units of  $\omega_a = \pi c/a$ ) varying with  $\beta_y$  and  $\beta_x$ , respectively, with  $n_{in} = 1$ ,  $n_h = 1$ ,  $a = 45 \mu m$ ,  $b = 45 \mu m$ ,  $h = 45 \mu m$ , and  $d = 60 \mu m$ .

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Fig. 3. Schematic structure of the proposed plasmonic metamaterial. A set of subwavelength square apertures with size *A* arranged on a  $D \times D$  lattice is cut into a metallic foil whose one surface has been made with rectangle holes whose sizes *a* and *b*, depth *h*, and spacing *d* are smaller than *A* and *D* at least one order of magnitude.

For a foil with arrays of holes of finite depth made on its one surface, when an incident wave with  $\lambda \gg d > a, b$  impinges at the surface, this wave will see only an average response that can be described by the effective anisotropic homogeneous medium layer as described above. If the foil is pierced with arrays of subwavelength apertures with size *A* and spacing *D*, as shown in Fig. 3, the incident wave will passes through the apertures. The DSP states sustained by the holes could support the transmission resonances of incident wave through the subwavelength apertures. The dispersion relation for DSPs associated with this case is not

well established. Nevertheless, it has been shown experimentally that the SP dispersion relation for a plane metal-dielectric interface without subwavelength apertures allows one to determine the locations of the transmission peaks when the interface is perforated at optical frequencies to good approximation. Therefore, we assume that such an approximation is equally valid here, thus the conservation of momentum is then given by [4]

$$\boldsymbol{\beta} = \pm j \mathbf{G}_{y} \pm l \mathbf{G}_{y}, \tag{20}$$

for radiation normally incident of the interface, where *j* and *l* are zero and integers,  $\mathbf{G}_x$  and  $\mathbf{G}_y$  are the reciprocal lattice vectors for a square lattice with  $G_x = G_y = 2\pi/D$ . From the two equations above, we find that the locations of transmission peak wavelengths are given by

$$\lambda = \lambda_{jl} = \frac{n_{in}D}{\sqrt{j^2 + l^2}} \sqrt{2 + P_a \frac{\lambda_a^2}{\lambda^2 - \lambda_a^2} + P_b \frac{\lambda_b^2}{\lambda^2 - \lambda_b^2}}.$$
(21)

where  $\lambda_a = 2n_h a$ , and  $\lambda_b = 2n_h b$ . Note that the peak wavelengths depend on the geometrical and optical parameters of the pierced foils, especially on those of the holes, thus allowing us to design the peak wavelengths by adjusting the parameters of the holes.

It is necessary to point out that Eq. (21) is obtained following the approach presented in Ref. 4, in which only the momentum-matching condition was considered. This equation hence does not take into account the boundary condition between the transmission region and inside the aperture. As a consequence, it neglects some physical processes [19], such as the interference that gives rise to a resonance shift [20].

#### 3. Numerical results

In order to establish a concept of magnitude, we carried out the calculations of  $\lambda_{10}$ ,  $\lambda_{11}$  and  $\lambda_{12}$  for  $n_h$ , a, b, h, and d with  $n_{in} = 1$  and  $D = 500 \ \mu m$ . Figures 4 and 5 show the parameter dependence of these peak wavelengths. In Fig. 4, we use  $\lambda_{10}^0 = 707.10 \ \mu\text{m}$ ,  $\lambda_{11}^0 = 500.0 \ \mu\text{m}$ and  $\lambda_{12}^0 = 316.22 \ \mu\text{m}$  to normalize the values of  $\lambda_{10}$ ,  $\lambda_{11}$  and  $\lambda_{12}$ , respectively. In fact, these parameters are those ones of  $\lambda_{10}$ ,  $\lambda_{11}$  and  $\lambda_{12}$  at  $P_a = P_b = 0$ , corresponding to the peak wavelengths of the subwavelength square apertures without the rectangle holes. As can be seen from Fig. 4 (a),  $\lambda_{10}$  will increase from 708.30 µm to 708.48 µm for  $n_h$  increasing from 1 to 3, having a wavelength shift  $\Delta \lambda_{10} = 0.18 \,\mu\text{m}$ , while the corresponding values for  $\lambda_{11}$  and  $\lambda_{12}$  being  $\Delta \lambda_{11} = 0.61 \,\mu\text{m}$  and  $\Delta \lambda_{12} = 5.27 \,\mu\text{m}$ , respectively, meaning that the higher the peak wavelength's order is, the more sensitively the peak wavelength varies with  $n_b$ . If we took a nematic liquid crystal as the material to fill the holes, we could adjust the peak wavelength via controlling the refractive index of the crystal by applying a magnetic field [21]. The biggest advantage lies in here is that there is no any materials filling the subwavelength square apertures for which the THz wave will pass through. For the other parameters (that is, a, b, h, and d), the higher the order of the peak wavelength is, the more sensitively the peak wavelength varies with these parameters, which is the same as that of the parameter  $n_{h}$ . These numerical examples indicate that the peak wavelengths could be designed in a certain extent by adjusting the geometrical and optical parameters of the array of the rectangle holes, indicating that the DPS states sustained by the rectangle holes would affect the transmission property of electromagnetic wave passing through the subwavelength apertures.



Fig. 4. The normalized peak wavelengths  $\lambda_{10}/\lambda_{10}^0$ ,  $\lambda_{11}/\lambda_{11}^0$  and  $\lambda_{12}/\lambda_{12}^0$  vary with the parameters  $n_h$ , d and h. (a)  $a = b = h = 45 \mu m$ , and  $d = 60 \mu m$ . (b)  $a = b = h = 45 \mu m$ , and  $n_h = 1$ . (c)  $a = b = 45 \mu m$ ,  $d = 60 \mu m$ , and  $n_h = 1$ . The normalized parameters are taken as  $\lambda_{10}^0 = 707.10 \ \mu m$ ,  $\lambda_{11}^0 = 500.0 \ \mu m$  and  $\lambda_{12}^0 = 316.22 \ \mu m$ .



Fig. 5. The peak wavelengths vary with the parameters a and b for  $h = 45 \mu m$ ,  $d = 60 \mu m$ , and  $n_b = 1$ . (a)  $\lambda_{10}$ . (b)  $\lambda_{11}$ . (c)  $\lambda_{12}$ 

#### 4. Discussion and conclusion

The present experiments at THz regime cannot be used to check the theoretical results above because only those subwavelength apertures with the size and spacing being hundred-micron dimension were cut in freestanding 75  $\mu$ m thick stainless steel foils [22]. Some suitable experiments are expected for which the samples should be pierced with subwavelength apertures and drilled with holes whose size and spacing should be taken as ten-micron dimension or less. Such experiments not only can be used to check the results above, but also would provide evidences for the existence of DSP states at THz regime. Our research proposes a way to connect the DSP theory to the engineering of surface plasmon optics, for which the creation of designer SPs with expectant resonant peaks could readily be realized by controlling the parameters of the holes, especially in THz to microwave regimes.

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