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# **Modelling Solids Friction Factor for Dense-Phase Pneumatic Conveying of Powders**

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## **ABSTRACT**

This paper presents results from an investigation into power-function modelling of solids friction factor for the dilute-phase and fluidised dense-phase (FDP) conveying of powders. Three different diameters/lengths of pipeline were used to generate a wide range of steady-state data and also explore important scale-up issues. The effect of pressure tapping locations on the data and derived models was also investigated. Different sets of power-function model solutions were used for comparison purposes and also to check scale-up stability and accuracy. Comparisons with predictions from recent models developed by other researchers are included.

It is concluded that certain forms of the power function model are more stable (in terms of scale-up) than others. The paper also demonstrates how existing models can go unreliable or unstable under certain scale-up conditions and discusses possible causes of such problems.

**KEYWORDS:** Pneumatic Conveying, Friction Factor, Power-function, Scale-up

## I. INTRODUCTION

The power function approach to modelling particle-wall friction factor for the prediction of pressure drop in pneumatic conveying has been used widely by researchers and designers for many years. More fundamental methods based on powder mechanics have been developed for certain products and modes of flows, such as the low-velocity slug-flow of granular products. However, the pneumatic conveying of powders, especially under dense-phase conditions, has been far more difficult to model at a similar level of detail. For this reason, the more empirical power function approach has been employed widely to avoid the need to develop fundamental relationships between friction factor and the relevant particle and bulk properties. Despite the apparent accuracy of the developed power functions, these empirical relationships occasionally and unexpectedly become unreliable or even unstable under certain scale-up conditions.

The information presented in this paper is not intended as the be-all and end-all with regard to power-function modelling, rather an investigation into its scale-up potential for a specific application.

Some researchers determine “straight-pipe” frictional data by subtracting bend/vertical effects from “total” pipeline data. Bend/vertical effects are difficult to measure experimentally and are quite sensitive to the accuracy of measured data [1]. Hence, such “back-calculation” methods can distort the subsequent straight-pipe data and models. Also, different power-function based models for particle-wall friction factor have been proposed/used/recommended over the years by various researchers (e.g. from Stegmaier [2] in 1978 to Williams and Jones [3] in 2004). These models: have used different parameter groupings; have shown good results when applied to the researcher’s own data; have not been tested properly against important criteria, such as scale-up accuracy and stability. To investigate such issues properly and also evaluate the robustness of the power function approach, it is essential to deal with straight-pipe data only (and avoid any bend/vertical effect uncertainties and complications).

The general format provided in equation 1 was employed throughout this work. The power function “extension” parameter groupings, such as air-to-solids density ratio and particle-to-pipe diameter ratio were not pursued, refer to equation 2. It has been reported that the addition of such parameter groupings may in fact cause instability or solution convergence problems [5]. It should be noted that this relatively simple format of the power function model also has been used by other researchers, such as [3] and [4]. Two methods have been used to solve the general form of  $\lambda_s$  as given in equation 1: a manually calculated solution using Excel; and a commercial software package, SigmaPlot.

$$\lambda_s = K (m^*)^a (Fr_m)^b \quad (1)$$

$$\lambda_s = K (m^*)^a (Fr_m)^b (\rho_{fm} / \rho_s)^c (d/D)^d \quad (2)$$

## II. EXPERIMENTAL

The test program consisted of pneumatically conveying fly ash through different diameters and lengths of pipeline to investigate scale-up issues. Two straight sections of pipeline were selected for analysis and comparison: P9-P10; and P11-P12, see figure 1. For each pipeline configuration, pneumatic conveying trials were performed to generate pneumatic conveying characteristic (PCC) curves. The results from the  $D = 69$  mm,  $L = 168$  m mild steel pipeline will be presented.

## III. THEORETICAL

The next step involved determining and comparing straight-pipe data and models at different locations along the pipeline (to see whether similar models are determined for the data sets having the same  $m_s$  and  $m_f$ , but different  $\rho_{fm}$  and  $Fr_m$ ). Predicting and comparing straight-pipe data for longer and/or larger diameter pipelines and establishing in the end the “best” power function format based on the above analyses was also investigated.

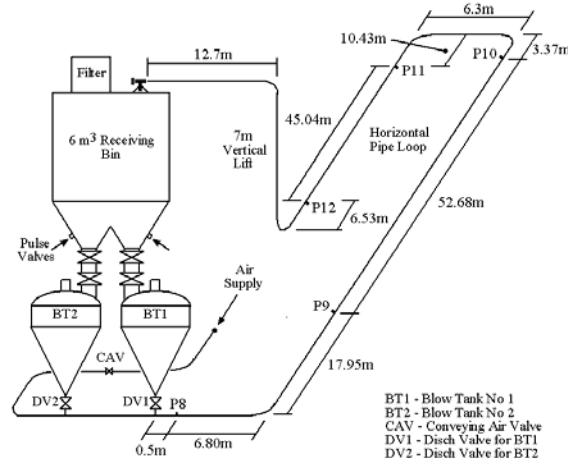


Fig 1: Fluidised dense-phase pipeline used in testing. Three pipeline geometries were investigated experimentally;  $D = 69$  mm,  $L = 168$  m mild steel pipeline (above),  $D = 105$  mm,  $L = 168$  m mild steel pipeline and  $D = 69$  mm,  $L = 554$  m mild steel pipeline.

The manually calculated solution is based on the work of Wypych [5]. Using the experimental data for a given pipeline and pipeline, a graph is produced with X and Y axes represented by equations 3 and 4. Using curve fitting, a power function is fitted to the data, equation 5, along with the corresponding statistical  $R^2$  value. By substitution, the result is equation 6.

In equation 4,  $\tau$  is unknown and the value is varied to obtain either; (a) an optimal value of  $R^2$  ( $K \neq 1$ ) or (b) a situation where  $K=1$  (where  $R^2$  is not optimized). During this process, the K and z values of equation 5 continually change.

$$X = Fr_m \quad (3)$$

$$Y = \lambda_s (m^*)^\tau \quad (4)$$

$$y = K x^z \quad (5)$$

$$\lambda_s = K (m^*)^{-\tau} Fr_m^z \quad (\text{same as eqn 1}) \quad (6)$$

The SigmaPlot software package was used as the second method of determining the lambda power function. Equation 7 was programmed into the package, where  $y = \lambda_s$ ,  $x = m^*$ ,  $z = Fr_m$  and K, a and b are variables. Using an iterative approach to solving the equation, SigmaPlot produces a summary sheet outlining among other values, the indice values (a and b) and  $R^2$ .

$$y = K x^a z^b \quad (7)$$

#### IV. RESULTS AND DISCUSSION

The focus of this work was to evaluate the different processes and formats of the power function approach, with the aim of determining as many “do’s and don’ts” as possible. However, some strategic comparisons were made between the experimental data, corresponding power function models and selected models, such as [2], [3] and [4].

The results of the power function modeling of the experimental data are displayed in Table 1 below.

The resulting models for this pipeline show a noticeable difference between the indice values for the “ $K \neq 1$ ” and the “ $K=1$ ” condition. For any given pipeline section and for each of model, there was generally quite a good agreement, however in some cases some drift was apparent.

Table 1: Power-function solutions for the D = 69 mm, L = 168 m mild steel pipeline using (a) the manually calculated method and (b) SigmaPlot

		(a)		(b)	
Pipeline section	K	Lambda Equation	R <sup>2</sup>	Lambda Equation	R <sup>2</sup>
P9 – P10	≠1	$\lambda_s = 22.2071 m^{*-0.734} Fr_m^{-2.1059}$	0.998 (8)	$\lambda_s = 21.8197 m^{*-0.7377} Fr_m^{-2.0901}$	0.999 (12)
P9 – P10	1	$\lambda_s = m^{*-0.2770} Fr_m^{-1.5306}$	0.977 (9)	$\lambda_s = m^{*-0.2142} Fr_m^{-1.6557}$	0.973 (13)
P11 – P12	≠1	$\lambda_s = 2.5234 m^{*-0.397} Fr_m^{-1.7141}$	0.977 (10)	$\lambda_s = 3.0001 m^{*-0.4483} Fr_m^{-1.6983}$	0.991 (14)
P11 – P12	1	$\lambda_s = m^{*-0.2683} Fr_m^{-1.5454}$	0.976 (11)	$\lambda_s = m^{*-0.2746} Fr_m^{-1.5329}$	0.986 (15)

Some general observations of these comparisons are:

- by forcing K = 1 generally decreases the value of R<sup>2</sup> slightly, but also appears to provide more consistent (stable) exponents for m\* and Fr<sub>m</sub> (e.g. a ≈ -0.3 and b ≈ -1.6);
- researchers that determine power function models directly from straight sections of pipe usually do so with one pair of pressure tapings. Two such pairs of pressure tapings were selected deliberately for each pipeline (viz. P9-P10 and P11-P12) to investigate the effect of “location” on the resulting models, which strictly speaking should be similar. Comparing the same power function format obtained at these two different pipeline locations revealed some interesting results: when K≠1, the values of K and the two exponents vary significantly depending on whether P9-P10 or P11-P12 data were selected (even for the same pipeline). For example, compare equations 12 and 14; and when K=1, the values of K and the two exponents are far more consistent, irrespective of whether P9-P10 or P11-P12 data were selected. For example, compare equations 13 and 15.

In 1978, Stegmaier [2] developed a power function model for data obtained on fine powders and a range of pipe sizes. It is believed that the data used by Stegmaier [2] was obtained directly from straight pipes. This model, as presented by Weber [6], is provided in equation 16.

In 2003, Jones and Williams [4] tested four different powders (pulverised fly ash, iron powder, copper ore, flour) on one pipeline (viz. D = 53 mm × L = 50 m) and determined the following power function model. They employed the “back-calculation” method of determining λ<sub>s</sub> and found that using Fr<sub>i</sub> instead of Fr<sub>m</sub> provided a better fit of the data, see equation 17. Unfortunately, the model based on Fr<sub>m</sub> was not presented. More recently, Williams and Jones [3] tested cement meal (d = 22 μm, ρ<sub>bl</sub> = 1300 kg m<sup>-3</sup>) through one pipeline (viz. 50 mm N.B. × L = 176 m) and determined the model show in equation 18, again determined by the “back-calculation” method.

$$\lambda_s = 2.1 m^{*-0.3} Fr_m^{-2} Fr_s^{0.25} (d/D)^{-0.1} \quad (16)$$

$$\lambda_s = 83 m^{*-0.9} Fr_i^{-2} \quad (17)$$

$$\lambda_e = 104 m^{*-0.9} Fr_m^{-2.2} \quad (18)$$

It is interesting to note: the values of K for the latter two models are relatively large; the corresponding exponents for m\* also are relatively large; the value of K for the Stegmaier [2] model is relatively low and the m\* exponent also is relatively low; and these differences and trends are similar to what were found experimentally on the fly ash, noting though that the values of K from [3] and [4] are significantly larger;

The true test of any model is when it is subjected to scale-up conditions (i.e. predictions compared with data obtained on longer/larger-diameter pipelines). Figure 2 shows a typical set of results for the models generated from this experimental work and the models of equation 16 to 18.

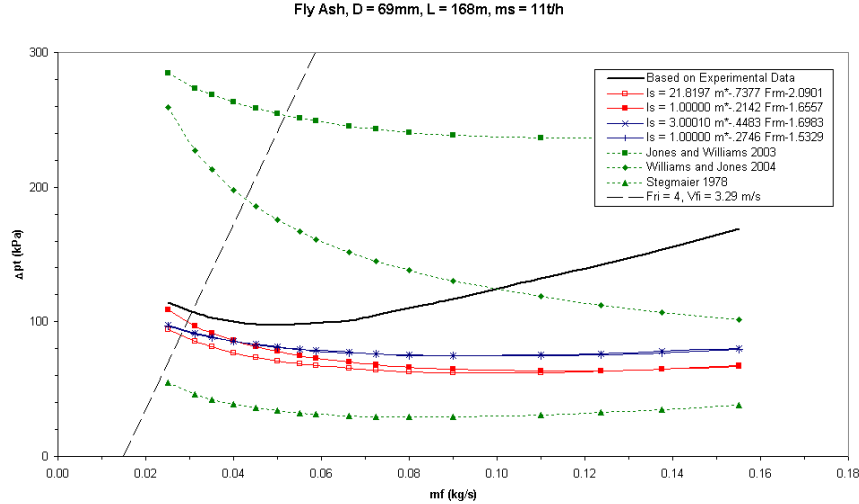


Fig 2: Predicted values of total pipeline (frictional) pressure loss, fly ash, D = 69 mm, L = L<sub>h</sub> = 168 m, m<sub>s</sub> = 11 t h<sup>-1</sup>.

The following comments are based on the results and trends shown in figure 2:

- The distance between pressure tappings P9 and P10 was 52.68 m and the distance between P11 and P12 was 45.04 m. Hence: the prediction of  $\Delta p_{tF}$  for the D = 69 mm, L = 168 m pipeline represented a “length” scale-up factor of around 3.1 to 3.7;
- The predicted  $\Delta p_{tF}$  curves should be lower than the “experimental” curve, which is based on  $\Delta p_t$  and includes acceleration, bend and vertical lift losses;
- The Stegmaier [2] model consistently under-predicts the values of  $\Delta p_{tF}$ , whereas both [3] and [4] provide gross over-predictions;
- all four models, equations 12 to 15, predict similar values of  $\Delta p_{tF}$  (i.e. within an envelope of  $\approx 20$  kPa). In the dense-phase regime (e.g.  $m_f < \approx 0.05$  kg s<sup>-1</sup>), the slopes of the curves predicted by equations 12 and 13, which are based on the P9-P10 data, are slightly steeper than those predicted by equations 14 and 15, which are based on the P11-P12 data. This “slope difference” trend in the dense-phase regime indicates that in equation 1,  $Fr_m$  or  $V_{fm}$  alone is inadequate to cope with the data sets obtained at different locations along the same pipeline. Perhaps air density is a contributing factor and the inclusion of an air-to-particle density ratio (viz.  $\rho_{fm}/\rho_s$ ), similar to equation 2, may be able to improve this situation.

## V. CONCLUSIONS

The data obtained on fly ash was selected to investigate FDP modelling. The following conclusions are based on the predictions/comparisons made by models from other researchers, [2], [3] and [4], and the experimentally determined PCC curves.

The relatively simple power function model given in equation 1 provides fairly consistent exponents for  $m^*$  and  $Fr_m$  only when  $K = 1$  (e.g.  $a \approx -0.3$  and  $b \approx -1.6$ ). When  $K \neq 1$ , the values of  $K$ ,  $a$  and  $b$  vary significantly and depend on pipeline location (and even solution method). Even though the fit to experimental data can be quite good (e.g.  $R^2 > 0.99$ ), a power function model with a large value of  $K$  (say,  $> 5$  or  $10$ ) results in dramatic over-predictions of total pipeline (frictional) pressure loss. When  $K = 1$ , both Sigmaplot and the Excel-based manual method provide similar values of  $a$  and  $b$ .

Equation 15 provides the best predictions in terms of both pipe diameter and length scale-up. Also, it appears that improvements in prediction may be possible by including air density and pipe diameter based parameter groupings.

Although it under-predicts  $\Delta p_{tF}$ , the Stegmaier [2] model appears quite stable and

consistent during scale-up. It may be possible to modify this model to improve scale-up predictions based on actual experimental data.

The power function models provided by both Jones and Williams [3] and Williams and Jones [4] incorporate large values of  $K = 104$  and  $83$ , respectively, and provide dramatic over-predictions even for moderate scale-up factors. Also, the trends predicted by Williams and Jones [4] are fundamentally incorrect, where  $\Delta p_{TF}$  decreases continually and dramatically with increasing air flow.

## VI. ACKNOWLEDGEMENTS

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## VII. NOMENCLATURE

$d$	particle diameter, m	$V_{fm}$	mean air velocity, $m\ s^{-1}$
$D$	internal diameter of pipe, m	$V_{\infty}$	particle free-settling velocity, $m\ s^{-1}$
$Fr_i$	initial Froude Number, $Fr_i = V_{fi}/(gD)^{0.5}$	$\Delta p_t$	total pipeline pressure drop, Pa
$Fr_m$	mean Froude Number, $Fr_m = V_{fm}/(gD)^{0.5}$	$\Delta p_{TF}$	total pipeline (frictional) pressure drop, Pa
$Fr_s$	Froude Number, $Fr_s = V_{\infty}/(gD)^{0.5}$	$\lambda_e$	“total” or “equivalent” friction factor
$g$	gravitational acceleration, $m\ s^{-2}$	$\lambda_s$	particle friction factor
$K$	constant	$\rho_{fm}$	mean air density, $kg\ m^{-3}$
$L$	total length of conveying pipeline, m	$\rho_s$	particle density, $kg\ m^{-3}$
$L_h$	horizontal pipeline length, m		superscript
$m^*$	solids to air mass flow rate ratio, $m^* = m_s\ m_f^{-1}$	a	model indice
$m_f$	air mass flow rate, $kg\ s^{-1}$	b	model indice
$m_s$	product mass flow rate, $kg\ s^{-1}$	c	model indice
$V_{fi}$	initial air velocity, $m\ s^{-1}$	d	model indice
		$\tau$	model variable

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