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# PREDICTED AND OBSERVED BEHAVIOUR OF SOFT CLAY FOUNDATIONS STABILISED WITH VERTICAL DRAINS

Buddhima N. Indraratna,<sup>1</sup> I. W. Redana<sup>2</sup>, Wadud Salim<sup>3</sup>

## ABSTRACT

A novel plane strain approach is introduced to model the behaviour of embankment foundations on soft clay stabilised with vertical drains, where the classical axisymmetric solutions are converted to an equivalent plane strain model, incorporating the effects of smear and well resistance. This paper describes the behaviour of an embankment stabilised with vertical drains, where a specific case history is selected from Malaysia. The consolidation of soft clay is modelled on the basis of the modified Cam-clay. The settlement behaviour at various stages of embankment loading is analysed using the finite element technique, and the numerical results are compared with field measurements. The behaviour of drains with and without smear is also compared. Inclusion of smear effect in the mathematical model improves the prediction of settlements.

## INTRODUCTION

The deformations of soft clay foundations stabilised with vertical band drains has remained difficult to predict accurately, although a significant progress has been made in the past few years through rigorous numerical modeling. The conventional solution for vertical drains (single drain analysis) has been well documented (Barron, 1948; Hansbo, 1981). Attributed to the rapid development in computer technology and the increasing versatility of the finite element method, a thorough numerical analysis of the settlement of the soft soil stabilised with multiple vertical drains can now be conducted. Following previous developments (e.g. Cheung et al., 1991; Hird et al., 1992), Indraratna and Redana (1997) extended the analysis based on the plane strain solution to include explicitly the effects of smear and well resistance.

Although a single-drain analysis is often performed to model the soil behaviour along the embankment centerline (symmetric geometry), multi-drain analysis is essential to study the overall behaviour of a soft clay foundation underneath the embankment. This paper presents a plane strain, multi-drain analysis for a selected case history from Malaysia, along the North-South Expressway built on the Muar coastal plain.

## REVIEW OF VERTICAL DRAIN THEORIES

### Equal Vertical Strain Hypothesis

Conventional procedure for predicting radial consolidation was first introduced by Barron (1948). Barron developed the exact (rigorous) solution of vertical drain based on 'free strain hypothesis' and an approximate solution based on 'equal strain hypothesis'. The difference in the predicted pore water pressures calculated using the free strain and equal strain assumptions is shown to be small. The schematic illustration of a soil cylinder with a central vertical drain is given in Figure 1, in which the radius of the drain is  $r_w$ , the radius of smear zone is  $r_s$ , the radius of soil cylinder is  $R$ , and  $l$  is the length of the drain. The coefficient of permeability in the vertical and horizontal directions are  $k_v$  and  $k_h$ , respectively, and  $k'_h$  is the coefficient of permeability in the smear zone. The three dimensional consolidation equation for radial drainage is given by (Barron, 1948):

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$$\frac{\partial \bar{u}}{\partial t} = c_v \left( \frac{\partial^2 u}{\partial z^2} \right) + c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (1)$$

where  $t$  is the time elapsed after the load application,  $u$  is the excess pore water pressure at radius  $r$  and at a depth of  $z$ .

For radial flow only, the above equation can be re-written as:

$$\frac{\partial \bar{u}}{\partial t} = c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (2)$$

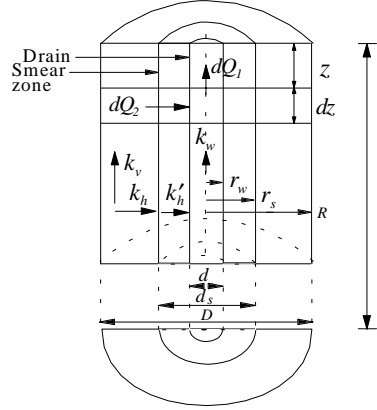


Figure 1 : Schematic of soil cylinder with vertical drain

### Plane Strain Model (Indraratna and Redana, 1997)

The vertical drain system as shown in Figure 1 may be converted into an equivalent parallel drain wall by adjusting the coefficient of permeability of the soil, but maintaining the same rate of consolidation. Figure 2 shows this conversion by assuming the plane strain cell to have a width of  $2B$ . The half width of drain  $b_w$  and half width of smear zone  $b_s$  are taken to be the same as their axisymmetric radii  $r_w$  and  $r_s$ , respectively, hence:

$$b_w = r_w \text{ and } b_s = r_s \quad (3)$$

In the case of Prefabricated Vertical Drains (PVD), Hansbo (1979) proposed that the equivalent drain diameter ( $d_w$ ) or radius ( $r_w$ ) for band drains can be determined by 'perimeter equivalence' to give:

$$d_w = \frac{2(a+b)}{\pi} \text{ or } r_w = \frac{(a+b)}{\pi} \quad (4a)$$

Subsequent numerical modelling recommends that for band drains the equivalent drain diameter could be better represented by the average of drain thickness and width (Rixner et al., 1986; Indraratna and Redana, 1999):

$$d = \frac{a+b}{2} \quad (4b)$$

where,  $a$  = the PVD width and  $b$  = the PVD thickness.

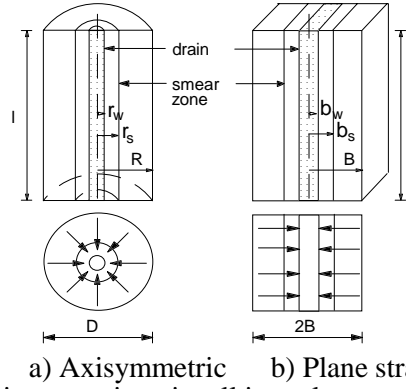


Figure 2 : Conversion of an axisymmetric unit cell into plane strain (Indraratna & Redana, 1997).

Indraratna and Redana (1997; 1999) showed that the degree of consolidation in plane strain condition can be represented by:

$$\bar{U}_{hp} = 1 - \frac{\bar{u}}{u_o} = 1 - \exp\left(\frac{-8T_{hp}}{\mu_p}\right) \quad (5)$$

Furthermore, by explicitly defining the smear zone, the  $\mu_p$  is given by:

$$\mu_p = \left[ \alpha + (\beta) \frac{k_{hp}}{k'_{hp}} + (\theta)(2lz - z^2) \right] \quad (6)$$

where,  $k_{hp}$  and  $k'_{hp}$  are the coefficient of horizontal permeability of undisturbed and disturbed soil respectively in plane strain condition. While the terms  $\alpha$  and  $\beta$  include the geometric conversion and smear zone effects, the term  $\theta$  represents the well resistance, in plain strain. Ignoring the higher order terms, the parameters  $\alpha$ ,  $\beta$  and  $\theta$  are given by:

$$\alpha = \frac{2}{3} - \frac{2b_s}{B} \left( 1 - \frac{b_s}{B} + \frac{b_s^2}{3B^2} \right) \quad (7a)$$

$$\beta = \frac{1}{B^2} (b_s - b_w)^2 + \frac{b_s}{3B^3} (3b_w^2 - b_s^2) \quad \text{and} \quad (7b)$$

$$\theta = \frac{2k_{hp}^2}{k'_{hp} B q_w} \left( 1 - \frac{b_w}{B} \right) \quad (7c)$$

The dimensions  $B$ ,  $b_s$  and  $b_w$  are defined in Figure 2 and  $q_w$  is the specific discharge capacity of drain. At each time step and at a given stress level, the average degree of consolidation for both axisymmetric ( $\bar{U}_h$ ) and equivalent plane strain ( $\bar{U}_{hp}$ ) conditions are made equal, and the time factor ratio is defined by the following equation:

$$\frac{T_{hp}}{T_h} = \frac{k_{hp}}{k_h} \cdot \frac{R^2}{B^2} = \frac{\mu_p}{\mu} \quad (8)$$

Indraratna and Redana (1997) demonstrated that if the radius of the axisymmetric influence zone around a single drain ( $R$ ) is taken to be the same as the width ( $B$ ) in plane strain (Fig. 2), then the plane strain horizontal permeability of the smear zone,  $k'_{hp}$  is solved from:

$$\frac{k_{hp}}{k_h} = \frac{\left[ \alpha + (\beta) \frac{k_{hp}}{k_h} + (\theta)(2lz - z^2) \right]}{\left[ \ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k_h'}\right) \ln(s) - 0.75 + \pi(2lz - z^2) \frac{k_h}{q_w} \right]} \quad (9)$$

Where  $n=R/r_w$  and  $s=r_s/r_w$ . If both the smear and well resistance are ignored, then the simplified ratio of the plane strain to axisymmetric horizontal permeability would be represented by:

$$\frac{k_{hp}}{k_h} = \frac{0.67}{[\ln(n) - 0.75]} \quad (10)$$

If well resistance is ignored, then the permeability in the smear zone can be isolated by the expression:

$$\frac{k'_{hp}}{k_{hp}} = \frac{\beta}{\frac{k_{hp}}{k_h} \left[ \ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k_h'}\right) \ln(s) - 0.75 \right] - \alpha} \quad (11)$$

## ANALYSIS OF MUAR CLAY EMBANKMENT (MALAYSIA)

### Sub-Soil and Embankment Condition

The Malaysian Highway Authority constructed several test embankments on the Muar Plain with vertical drains. The sub-soil consisted of a weathered crust of about 2 m above a 16.5 m thick layer of soft silty clay. This layer is underlain by a thin layer of peat (0.3-0.5 m), followed by a stiff sandy clay which extends to 22.5 m below the ground level.

Figure 3 shows the cross section of the embankment. The equivalent radius of the drain, mandrel and the smear zone are given by:  $r_w = 0.035$  m,  $r_m = 0.054$  and  $r_s = 0.27$  m, respectively. The embankment load was applied in two stages over a period of 125 days, as shown in Figure 4.

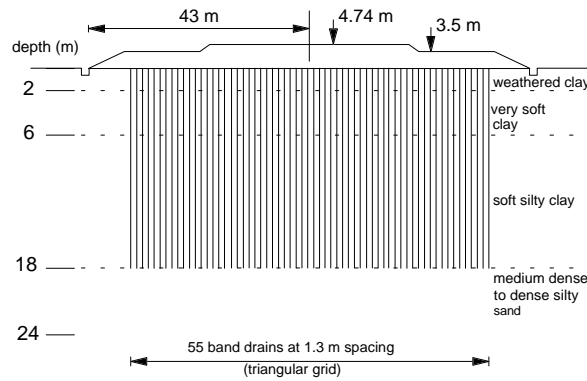


Figure 3 : Cross section through centerline of embankment with sub-soil profile (Indraratna & Redana, 1999).

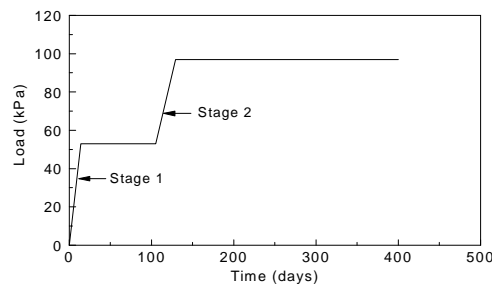
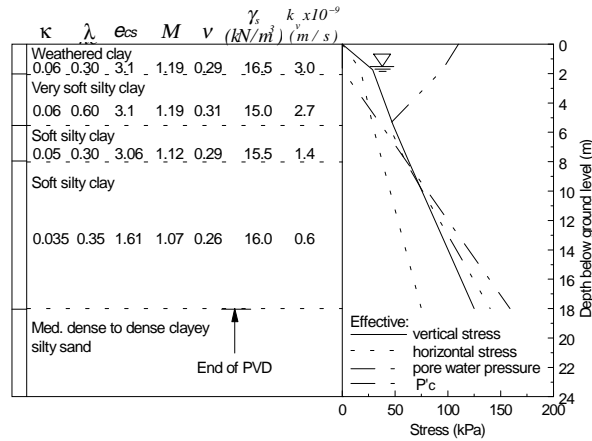


Figure 4 : Approximate construction loading history of Muar clay embankment.

## Numerical Analysis

Figure 5 shows the Cam-clay parameters based on  $CK_0U$  triaxial tests, the sub-soil and the in-situ stress distribution for the Muar clay. The finite element mesh of the embankment is shown in Figure 6, where the foundation is discretized into linear strain quadrilateral (LSQ) elements. In the analysis, the effects of both smear and well resistance were considered.



$\kappa$ ,  $\lambda$ ,  $e_{cs}$  and  $M$  are the critical state parameters (Britto & Gunn, 1987).  
 $\nu$  is the poisson's ratio.  
 $k_v$  is the vertical permeability.

Figure 5 : Cam-clay parameters and stress conditions of Muar Clay, Malaysia (Indraratna & Redana, 1999).

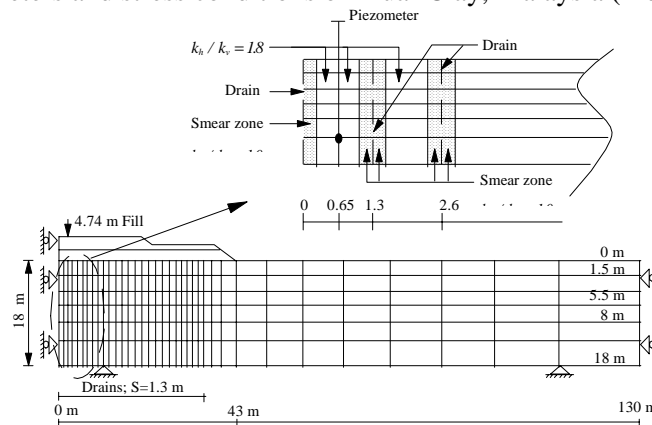


Figure 6 : Finite element mesh of the embankment for plane strain analysis, Muar clay, Malaysia (Indraratna and Redana, 1999).

The results of the plane strain, multi-drain analysis are plotted in Figure 7, together with the field data. The perfect drain analysis (no smear, complete pore pressure dissipation) overpredicts the settlement substantially. The inclusion of smear effect improves the accuracy of the predictions, and the consideration of both smear and well resistance provides a very good match with field measurements. The prediction of settlement along the ground surface from the centerline of embankment is shown in Figure 8. The limited available data agree well with the settlement profile, near the embankment centerline. Heave is also predicted near the toe of the embankment.

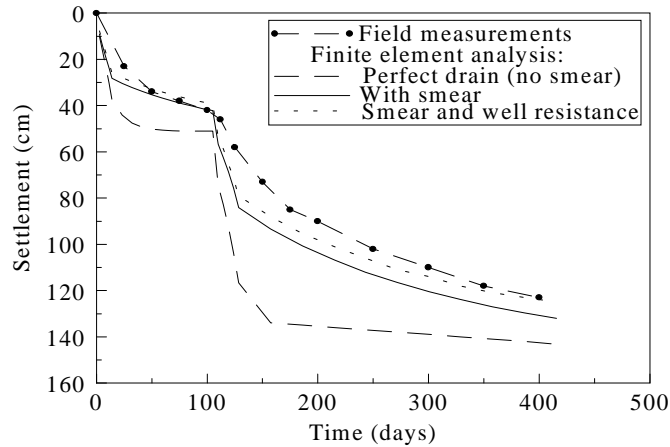


Figure 7 : Total settlements at ground along embankment centerline, Muar Clay (Indraratna and Redana, 1999).

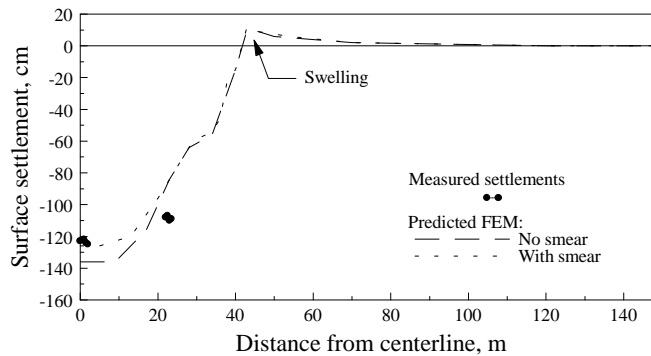


Figure 8 : Surface settlement profiles after 400 days, Muar clay (Indraratna and Redana, 1999).

## CONCLUSIONS

An equivalent plane strain, mathematical model incorporating the effects of both smear and well resistance, was introduced to evaluate the performance of soft clay foundations beneath embankments, stabilised with prefabricated vertical drains. This model was employed in the finite element analysis of a soft clay foundation stabilised with vertical drains, and the results are compared with the field measurement. The analysed results verify that the inclusion of smear effect in the mathematical model improves the accuracy of the predictions, and the consideration of both smear and well resistance provides a very good match with field measurements. Therefore, it can be concluded that the inclusion of both smear effect and well resistance in the plane strain model provides an excellent predictive tool, which can be used in conjunction with finite element analysis. Conventional analysis using 'perfect drains' substantially overestimates the settlements, hence, cannot be used in confidence for band drains. For the length of drains considered here, the well resistance is not as significant as the smear effects. Nevertheless, it is possible that well resistance becomes increasingly pronounced for much longer PVD's.

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## REFERENCES

Barron, R. A. 1948. *Consolidation of fine-grained soils by drain wells*. *Trans., ASCE*, 113, 718-742.

- Bergado, D. T., Asakami, H., Alfaro, M. C., and Balasubramaniam, A. S. 1991. *Smear effects of vertical drains on soft Bangkok clay*. J. Geotech. Eng., ASCE, 117(10), 1509-1530.
- Britto, A.M. and Gunn, M.J. 1987. *Critical state soil mechanics via finite elements*. Ellis Horwood, Ltd., Chichester, England.
- Chai, J. C., Miura, N., Sakajo, S., and Bergado, D. 1995. *Behavior of vertical drain improved subsoil under embankment loading*. J. Soil and Foundations, Japanese Geotechnical Society, 35 (4), 49-61.
- Cheung, Y. K., Lee, P. K. K., and Xie, K. H. 1991. *Some remarks on two and three dimensional consolidation analysis of sand-drained ground*. J. Computer and Geotechnics, Elsevier Science Publishers Ltd, England (GB), 12, 73-87.
- Hansbo, S. 1979. *Consolidation of clay by band-shaped prefabricated drains*. Ground Engineering, Vol 12, No. 5, pp. 16-25.
- Hansbo, S. 1981. *Consolidation of fine-grained soils by prefabricated drains*. Proc. 10th Int. Conf. on Soil Mechanics and Foundation Engineering, Stockholm, Sweden, Publications Committee of ICSMFE (ed.), A. A. Balkema, Rotterdam, The Netherlands, 3, 677-682.
- Hird, C.C., Pyrah, I.C., and Russell, D. 1992. *Finite element modelling of vertical drains beneath embankments on soft ground*. Geotechnique, 42 (3), 499-511.
- Holtz, R.D., Jamiolkowski, M., Lancellotta, R., and Pedroni, S. 1988. *Behaviour of bent prefabricated vertical drains*. Proc. 12th ICSMFE, Rio De Janeiro, Vol. 3. pp. 1657-1660.
- Indraratna, B. and Redana, I W. 1997. *Plane strain modeling of smear effects associated with vertical drains*. J. Geotech. Eng., ASCE, 123 (5), 474-478
- Indraratna, B. and Redana, I W. 1999. *Numerical modeling of vertical drains with smear and well resistance installed in soft clay*. J. Canadian Geotech. (in press).
- Ratnayake, A.M.P. 1991. *Performance of test embankments with and without vertical drains at Muar Flats site, Malaysia*. MS thesis, Asian Institute of Technology, Thailand.
- Redana, I W. 1999. *Efficiency of vertical drains installed in soft clay with special reference to smear effect*. University of Wollongong, Australia, PhD thesis.
- Rixner, J.J., Kraemer, S.R., and Smith, A.D. 1986. *Prefabricated vertical drains, Vol. II: Summary of Research Report-Final Report*. Federal Highway Administration, Report No. FHWA-RD-86/169, Washington D.C.