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# SOFT CLAY STABILIZATION BY MANDREL DRIVEN GEOSYNTHETIC VERTICAL DRAINS

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### ABSTRACT

Geosynthetic (prefabricated) vertical band drains are now considered as one of the most cost effective ground improvement technics in many parts of the world, where construction on soft compressible clays is inevitable. However, smear effects caused by PVD installation (eg. mandrel based), drain clogging, drain kinking and well resistance of long drains retard the excess pore pressure dissipation making these drains often less effective in the field, contrary to expectations. Consequently, the rate of settlement of the stabilised soft clay becomes significantly less than what is expected from ideal drains. This paper addresses comprehensively, the numerical modelling aspects of PVD, and the interpretation of field data taken from several case studies, which elucidate the drain performance under various boundary conditions. Theoretical and finite element modelling details are described based on various research studies, mainly through the authors' own experience. In particular, the experimental data obtained from large-scale consolidation tests are highlighted and interpreted.

### **1 INTRODUCTION**

In South and Central America, Southeast Asia and Australia, many highly populated coastal cities are dotted with very soft clays. In some regions, low-lying land is characterized by highly compressible weak organic and peaty soils of varying thickness. These soft clay deposits have low bearing capacities as well as excessive settlement characteristics. When such low-lying areas are selected for development work, it is essential to raise the ground with fill in order to keep the surface area above the groundwater table or flood level. If fill is placed without proper compaction, a differential settlement may develop due to the heterogeneity of the fill and the compressibility of underlying weak subsoils.

Usually when a strong bearing stratum is found only at large depths beyond 15m, foundation expenses can become very high and they may not commensurate with the cost of the superstructure. This is particularly so in the case of small-scale structures and low-rise buildings subject to low to moderate loads. On compressible soils, construction problems arise during construction of embankments (railways and highways). Therefore, an economical solution is often looked at to improve the engineering properties of the underlying soil rather than the use of long pile foundations. For example, preloading is one of the more commonly employed ground improvement techniques suitable for low-lying areas. In preloading, a surcharge load equal to or greater than the expected foundation loading is applied on the ground surface, which induces consolidation under the applied load, if natural drainage is available (eg. sand layers). A drawback in this method is that usually, the preload should be applied for a sufficient period of time to facilitate adequate drainage, but this may not always be viable with stringent construction schedules. Installation of geosynthetic (prefabricated) vertical drains or PVD can decrease the preloading period significantly, where the consolidation process (radial) is accelerated by shorter drainage paths (Fig. 1).



Figure 1: Potential benefit of vertical drains (modified after Hausmann, 1990)

PVDs are applicable for moderately to highly compressible soils, that are normally consolidated or lightly over consolidated, and for improving deep soft clay deposits of low permeability. These soft soils include organic and inorganic silts and clays with low to moderate sensitivity, varved cohesive soils and decomposed peat and black cotton soils.

# 2 FACTORS AFECTING THE PERFORMANCE OF PVD

The geosynthetic or prefabricated vertical drains can be installed using two different methods, which are either dynamic or static. In the dynamic method of installation, a steel mandrel is driven into the ground using either a vibrating hammer or a conventional drop hammer. In the static procedure, the mandrel is pushed into the soil by means of a static force. Static pushing is preferable for driving the mandrel into the ground. This is because, the dynamic methods seem to create a higher excess pore water pressure and a greater disturbance of the surrounding soil during installation. The installation procedure results in significant shear strain accompanied by the increase in total stress and pore water pressure, apart from the displacement of the soil surrounding the mandrel. The degree of disturbance depends on several factors as described below.

### 2.1 TYPES OF MANDREL AND SMEAR EFFECT

Generally, the soil disturbance increases with the total mandrel cross sectional area and its shape (mandrel shoe). The mandrel size should be as close as possible to that of the drain to minimise displacement. While working on the effect of mandrel driven drains on the behaviour of soft clays (Akagi, 1977, 1981), observed that when a closed-end mandrel is driven into a saturated clay, the clay can be subjected to large excess pore water pressure, and associated with ground heave and lateral displacement. The strength and coefficient of consolidation of the surrounding soil can then decrease considerably. However, the excess pore pressure dissipated rapidly, followed by the process of consolidation after installation of the mandrel or before the fill is placed. Bergado et al. (1991) reported from a case study of a Bangkok clay embankment stabilised with vertical drains, that a faster rate of settlement took place where the drains were installed using a mandrel with a smaller cross section area rather than a larger mandrel. Not surprisingly, this verifies that a smaller smear zone was developed in the former.

Barron (1948) stated that if the drain wells are installed by driving cased holes and then back filling as the casing is withdrawn, the driving and pulling of the casing would distort and remould the adjacent soil. It is known that in varved clays, the finer and more impervious layers will be dragged down and smeared over the more pervious layers, resulting in a zone of reduced permeability of the soil adjacent to the well. As remoulding retards the consolidation process of the subsoil, it has to be considered in any theoretical formulation. Barron (1948) considered the concept of reduced permeability to be equivalent to lowering the overall value of the coefficient of consolidation. Similarly, Hansbo (1979) also introduced a zone of smear in the vicinity of the drain with a reduced coefficient of permeability.

The overall change of permeability and compressibility within the smear zone brought about a different behaviour from the undisturbed soil, hence, the prediction of the behaviour of the soil stabilised with vertical drains cannot be made accurately if the effect of smear is ignored. Barron (1948) and Hansbo (1981) modelled the smear zone by dividing the soil cylinder dewatered by the central drain into two distinct zones. The smear zone is the disturbed region in the immediate vicinity of the drain and the other zone is the intact or undisturbed region outside the smear zone. More recently, Onoue et al. (1991) introduced a three zone hypothesis defined by: (a) plastic smear zone in the immediate vicinity of the drain where the soil is highly remolded during the process of installation of the drain; (b) plastic zone where the permeability is reduced moderately; and (c) the undisturbed zone where the soil is not at all affected by the process of drain installation. This three-zone analysis was supported after extensive laboratory testing by Ting et al. (1990). However, due to the complex variation of permeability in the radial direction from the drain, the solution of the three-zone approach becomes increasingly difficult. For practical purposes, the two-zone approach is generally sufficient and most convenient in analytical and numerical models.

Jamiolkowski et al. (1981) proposed that the diameter of the smear zone ( $d_s$ ) and the cross sectional area of the mandrel can be related by the expression:

$$d_s = \frac{(5to6)d_m}{2} \tag{1}$$

where,  $d_m$  is the diameter of the circle with an area equal to the mandrel cross-section. Based on the results of Holtz and Holm (1973) and Akagi (1979), Hansbo(1987) proposed a simplified relationship as follows:

$$d_s = 2d_m \tag{2}$$



Figure 2 : Large-scale consolidation apparatus (Indraratna & Redana, 1995).

Indraratna & Redana (1998) suggested that the estimated smear zone is about 3-4 times the cross-sectional area of the mandrel. The proposed relationship was verified using the specially designed large-scale consolidometer shown in Fig. 2 (Indraratna & Redana, 1995). The schematic illustration of the consolidometer and the locations of the recovered specimens are shown in Fig. 3. Figure 4 shows the variation of  $k_h/k_v$  ratio along the radial distance from the central distance. It is clear that the  $k_h/k_v$  ratio decreases within the smear zone. According to Hansbo (1987) and Bergado et al. (1991), in the smear zone, the  $k_h/k_v$  ratio was found to be close to unity, which is in agreement with the laboratory results of Indraratna & Redana (1998).

### 2.2 WELL RESISTANCE

The resistance to water flowing in the vertical drains is commonly known as the well resistance. The well resistance increases with the drain length, thereby reducing the consolidation rate. Well resistance retards the pore pressure dissipation, hence, the rate of settlement. The three main factors, which increase well resistance are the deterioration of the drain filter (reduction of drain cross section), passing of fine soil particles through the filter (i.e. clogging) and folding or kinking of the drain due to large soil movement. Often, these aspects are difficult to quantify. Hansbo (1979, 1981) presented a closed form solution, which includes the effect of well resistance on drain performance.

#### 2.3 EFFECT OF DRAIN UNSATURATION

As a result of the installation process, air can be trapped in the annulus space between the drain and the soil. Unless the soil is highly plastic with a very high water content, for example dredged mud, there is a possibility of having an annular space partially filled with trapped air (air gap) upon mandrel withdrawal. This results in a situation where the inflow of water into the drain becomes retarded. In numerical analysis, it can be assumed that the PVD and the air gap together is an unsaturated vertical interface, having a thickness equal to that of the mandrel. Figure 5 shows the variation of drain saturation with respect to time (initial degree of saturation of 50% for 1m length), and Figure 6 shows the resulting effect on consolidation curves, for varying levels of saturation.



Figure 3: (a) Sections of test equipment showing the central drain and smear zone and (b) locations of cored specimens obtained to determine the consolidation and permeability characteristics (Indraratna & Redana, 1998).



Figure 4: Ratio of  $k_h/k_v$  along the radial distance from the central drain (Indraratna & Redana, 1998).



Figure 5: Effect of variation of drain saturation with time (Indraratna et al., 2001)



Figure 6: Variation of degree of consolidation due to drain unsaturation (Indraratna et al., 2001)

### **3** VERTICAL DRAIN THEORY – BACKGROUND AND DEVELOPMENTS

The formulation of radial consolidation around a vertical sand drain system is an extension of the one-dimensional consolidation theory (Terzaghi, 1925). If the coefficient of consolidation in the horizontal direction is higher than that in the vertical direction, and because the vertical drains reduce the drainage path considerably in the radial direction, the effectiveness of vertical drains in accelerating the rate of consolidation is obviously improved. Kjellman (1948) solution based on equal vertical strain hypothesis, was developed on the assumption that horizontal sections remain horizontal throughout the consolidation process. Independently, Barron (1948) presented a more comprehensive solution to the problem of radial consolidation via drain wells, where two extreme cases of free strain and equal strain were studied to show that the average consolidation obtained in these cases is nearly the same. In the 'free strain hypothesis' the load is uniform over a circular zone of influence for each vertical drain. Also the differential settlements occurring over this zone have no effect on the redistribution of stresses by arching of the fill load. In the 'equal vertical strain hypothesis' arching is assumed to occur in the upper layer of clay during the consolidation process without any differential settlement. The arching effect implies a more or less rigid boundary at the surface of the soil layer, which means that the vertical strain is uniform in the horizontal section of the clay layer. Takagi (1957) modified Barron's solution to account for a variable rate of loading. Richart (1959) presented a comprehensive design charts for the effect of smear, where the influence of variable void ratio was included. A simplified solution to the problem of smear and well resistance was proposed by Hansbo (1979, 1981), in contrast to previous studies of Barron (1948) and Yoshikuni and Nakanodo (1974). More recently, Onoue (1988) presented a rigorous solution based on the free strain hypothesis. Details of some of the well-known concepts are summarized below.



Figure 7: Schematic illustration of soil cylinder with vertical drain (modified after Hansbo, 1979)

### 3.1 EQUAL VERTICAL STRAIN HYPOTHESIS

A convenient procedure for predicting radial consolidation was introduced by Barron (1948), extending the initial concepts proposed by Terzaghi (1925). The Fig. 7 presents a schematic illustration of a soil cylinder with a central vertical drain, where,  $r_w$ =the radius of the drain,  $r_s$ = the radius of smear zone, R =the radius of soil cylinder and l= the length of the drain installed into the soft ground. The permeability in the vertical and horizontal directions are  $k_v$  and  $k_h$ , respectively, and  $k'_h$  is the reduced coefficient of permeability in the smear zone. The three dimensional consolidation equation for radial drainage is given by (Barron, 1948):

$$\frac{\partial \overline{u}}{\partial t} = c_v \left( \frac{\partial^2 u}{\partial z^2} \right) + c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$
(3)

where, t is the time elapsed after the load is applied, u is the excess pore water pressure at radius r and at depth z. For radial flow only, the above equation simplifies to:

$$\frac{\partial \overline{u}}{\partial t} = c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$
(4)

For radial flow only, the solution of the excess pore pressure,  $(u_r)$  based on "equal strain" hypothesis is given by:

$$u_r = \frac{4\overline{u}}{D^2 F(n)} \left[ R^2 \ln\left(\frac{r}{r_w}\right) - \frac{r^2 - r_w^2}{2} \right]$$
(5)

D is the diameter of soil cylinder, and the drain spacing factor, F(n), is represented by:

$$F(n) = \frac{n^2}{n^2 - 1} ln(n) - \frac{3n^2 - 1}{4n^2}$$
(6)

where, *n* is drain spacing ratio ( $R/r_w$ ).

The average excess pore water pressure is given by:

$$\overline{u} = u_o \exp\left(\frac{-8T_h}{F(n)}\right) \tag{7}$$

Therefore, the average degree of consolidation,  $\overline{U}_h$ , is given by:

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$$\overline{U}_{h} = 1 - \exp\left(-\frac{8T_{h}}{F(n)}\right) \tag{8}$$

The time factor  $T_h$  is defined by the expression:

$$T_h = \frac{c_h t}{D^2} \tag{9}$$

The coefficient of radial drainage consolidation,  $c_h$ , is given by:

$$c_h = \frac{k_h \left(1+e\right)}{a_v \gamma_w} \tag{10}$$

where  $\gamma_w$  is unit weight of water and  $a_v$  is the coefficient of compressibility of the soil, *e* is the void ratio, and  $k_h$  is the lateral or horizontal permeability of the soil.

The solution of Eqn. (4) taking account of smear effect is given by:

$$u_{r} = \overline{u}_{r} \frac{1}{\nu} \left[ ln \frac{r}{r_{s}} - \frac{r^{2} - r_{s}^{2}}{2R^{2}} + \frac{k_{h}}{k_{h}'} \left( \frac{n^{2} - s^{2}}{n^{2}} \right) ln s \right]$$
(11)

In the above, the smear factor v is given by:

$$v = \left[\frac{n^2 - s^2}{n^2} ln \frac{n}{s} - \frac{3}{4} + \frac{s^2}{4n^2} + \frac{k_h}{k_v} \left(\frac{n^2 - s^2}{n^2}\right) ln s\right]$$
(12)

and

$$\overline{u} = u_o \exp\left(\frac{-8T_h}{v}\right) \tag{13}$$

In the above equation, s is the size of the smear zone with respect to drain, and it is given by:

$$s = r_s / r_w$$
.

The average degree of consolidation including smear is now represented by:

$$\overline{U}_{h} = 1 - exp\left(-\frac{8T_{h}}{v}\right) \tag{15}$$



Figure 8: Average consolidation rates (a) for vertical flow and (b) for radial flow (modified after Barron, 1948).

Figure 8 presents the curves of average radial excess pore water pressure,  $\overline{u}_r$ , and average degree of consolidation,  $U_h$  (purely radial flow) versus time factor  $T_h$  for various values of n. On the same plot, the average degree of consolidation,  $\overline{U}_v$  due to vertical flow versus time factor  $T_v$  is also indicated.

(14)

### 3.2 EQUAL STRAIN SOLUTION

Later, Hansbo (1981) proposed an approximate solution for vertical drain based on the 'equal strain hypothesis' by considering both smear and well resistance. The general concept of this solution is also represented by Fig. 7, and the rate of flow of internal pore water in the radial direction can be estimated using Darcy's law. The average degree of consolidation,  $\overline{U}$ , of the soil stabilised with a central vertical drain is given by:

$$\overline{U}_{h} = 1 - \exp\left(-\frac{8T_{h}}{\mu}\right) \tag{16}$$

In the above equation,

$$\mu = ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75 + \pi z(2l-z)\frac{k_h}{q_w}$$
(17)

The effect of smear only is represented by:

$$\mu = ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75$$
(18)

The effect of well resistance (without smear) is given by:

$$\mu \approx \ln(n) - 0.75 + \pi z (2l - z) \frac{k_h}{q_w}$$
<sup>(19)</sup>

If both smear and well resistance are ignored, the above equation becomes the same as Barron (1948) solution:

$$\mu = \ln(n) - 0.75 \tag{20}$$

#### 3.3 EQUIVALENT PLANE STRAIN CONSOLIDATION MODEL (INDRARATNA & REDANA, 1997)

Most finite element analyses on embankments are conducted based on the plane strain assumption, even though the actual consolidation around vertical drains is axisymmetric. Therefore, to employ a realistic 2-D (plane strain) finite element analysis for vertical drains, the equivalence between the plane strain and axisymmetric analysis needs to be established, in order to ensure the correct time- settlement relationship.

The eqivalence of axisymmetric and plane strain conditions can be executed in three ways:

- 1. geometric matching approach the spacing of the drains is made equivalent while keeping the permeability the same
- 2. permeability matching approach permeability coefficient is made equivalent while keeping the spacing of drains to be the same.
- 3. combination of permeability and geometric matching approach plane strain permeability is calculated for a convenient drain spacing.

Indraratna & Redana (1997) converted the vertical drain system (Fig. 7) into an equivalent parallel drain wall by adjusting the coefficient of permeability of the soil and by assuming the plane strain cell to have a width of 2B (Fig. 9). The half width of drains  $b_w$  and half width of smear zone  $b_s$  may be taken to be the same as their axisymmetric radii  $r_w$  and  $r_s$  respectively, hence:

$$b_w = r_w \quad \text{and} \quad b_s = r_s \tag{21}$$

The equivalent diameter  $(d_w)$  or radius  $(r_w)$  of band drains is determined by 'perimeter equivalence' (Hansbo, 1979) to make:

$$d_w = 2\frac{(a+b)}{\pi} \text{ or } r_w = \frac{(a+b)}{\pi}$$
(22)

Through numerical analysis, Rixner et al. (1986) presented the equivalent drain diameter (d) as the average of drain thickness and width by considering the shape of the drain and effective drainage area.

$$d = \frac{a+b}{2} \tag{23}$$

Where, a = the width of the PVD and b = the thickness of the PVD



Figure 9: Transformation of an axisymmetric unit cell into equivalent plane strain (Indraratna & Redana, 1997). From basic principles, Indraratna & Redana (1997) derived the average degree of consolidation in plane strain condition as

$$\overline{U}_{hp} = 1 - \frac{\overline{u}}{\overline{u}_o} = 1 - exp\left(\frac{-8T_{hp}}{\mu_p}\right)$$
(24)

In the above equation,  $\overline{u}_0$  = initial pore pressure,  $\overline{u}$  = pore pressure at time t (average values),  $T_{hp}$  = time factor in plane strain, and

$$\mu_{p} = \left[ \alpha + \left(\beta\right) \frac{k_{hp}}{k'_{hp}} + \left(\theta\right) \left(2lz - z^{2}\right) \right]$$
(25)

where,  $k_{hp}$  and  $k'_{hp}$  are the undisturbed horizontal and corresponding smear zone permeabilities, respectively. The parameters  $\alpha$ ,  $\beta$  and the flow term  $\theta$  in equation are given by:

$$\alpha = \frac{2}{3} - \frac{2b_s}{B} \left( 1 - \frac{b_s}{B} + \frac{b_s^2}{3B^2} \right)$$
(25a)

$$\beta = \frac{1}{B^2} (b_s - b_w)^2 + \frac{b_s}{3B^3} (3b_w^2 - b_s^2)$$
(25b)
$$2 = \frac{2k_{bm}^2}{2} (1 - b_w)^2$$
(25c)

$$\theta = \frac{2\kappa_{hp}}{k'_{hp}q_z B} \left(1 - \frac{b_w}{B}\right)$$
(25c)

 $\boldsymbol{q}_z$  is the equivalent plane strain discharge capacity.

At a given stress level and at each time step, the average degree of consolidation for both axisymmetric ( $\overline{U}_h$ ) and equivalent plane strain ( $\overline{U}_{hp}$ ) conditions are made equal, hence:

$$\overline{U}_{h} = \overline{U}_{hp} \tag{26}$$

Combining Equations. (24), (26) with the original Hansbo (1981) theory, the time factor ratio can be represented by following equation:

$$\frac{T_{hp}}{T_h} = \frac{k_{hp}}{k_h} \frac{R^2}{B^2} = \frac{\mu_P}{\mu}$$
(27)

By assuming the magnitudes of R and B to be the same, Indraratna and Redana (1997) presented the permeability relationship between  $k_{hp}$  and  $k'_{hp}$  as follows:

$$k_{hp} = \frac{k_h \left[ \alpha + (\beta) \frac{k_{hp}}{k'_{hp}} + (\theta) (2lz - z^2) \right]}{\left[ ln \left( \frac{n}{s} \right) + \left( \frac{k_h}{k'_h} \right) ln(s) - 0.75 + \pi \left( 2lz - z^2 \right) \frac{k_h}{q_w} \right]}$$
(28)

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If well resistance is ignored in the above equation by omitting all terms containing l and z, the influence of smear effect can be represented by the ratio of the smear zone permeability to the undisturbed permeability as given by:

$$\frac{k'_{hp}}{k_{hp}} = \frac{\beta}{\frac{k_{hp}}{k_h} \left[ ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) ln(s) - 0.75 \right] - \alpha}$$
(29)

In the above expression, if both smear and well resistance effects are ignored, then the simplified ratio of plane strain to axisymmetric permeability is obtained, as proposed earlier by Hird et al. (1992):

$$\frac{k_{hp}}{k_h} = \frac{0.67}{\left[\ln(n) - 0.75\right]}$$
(30)

The well resistance is derived independently and it yields an equivalent plane strain discharge capacity  $(q_z)$  of drains as explained by Hird et al. (1992):

$$q_z = \frac{2}{\pi B} q_w \tag{31}$$

### 4 PROPERTIES OF PREFABRICATED BAND DRAINS

### 4.1 EQUIVALENT DRAIN DIAMETER

The conventional theory of radial consolidation (axisymmetric) assumes that the vertical drains are circular in crosssection. Consequently, the thickness of band-shaped drain needs to be converted to an equivalent diameter. This implies that the equivalent diameter of a circular drain has the same theoretical drainage capacity as the relatively thin bandshaped drain. The algebraic expression for equivalent diameter for a prefabricated band-shaped drain were discussed earlier, based on Equations (22) and (23). More details are given by Hansbo (1981), Rixner et al.(1986), and Indraratna (2002).



Figure 10: Equivalent diameter of band-shaped vertical drain based on various studies (Indraratna, 2000).

More recently, Pradhan et al. (1993) suggested that the equivalent diameter of band-shaped drains could be better estimated by considering the flow net around the soil cylinder of diameter  $d_e$  (Fig. 10). The mean square distance of the flow net is determined to be:

$$\overline{s}^{2} = \frac{1}{4}d_{e}^{2} + \frac{1}{12}a^{2} - \frac{2a}{\pi^{2}}d_{e}$$
(32)  
Such that,  $d_{w} = d_{e} - 2\sqrt{(\overline{s}^{2})} + b$ 
(33)

While equation (23) was justified through numerical modeling, Equation (22) proposed by Hansbo (1981) is mathematically more appropriate, because it is based on the size of the perimeter. In the opinion of the author, Equation (32) overestimates the equivalent diameter, hence, actual smear zone.

#### 4.2 APPARENT OPENING SIZE (AOS)

Both sand drains and the geosynthetic filter (jacket) of PVD have to perform two basic functions, which are retaining the soil particles and at the same time allowing the pore water to pass through. In general, the drain permeability must exceed the soil permeability, hence,

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(34)

 $k_{filter} > 2 k_{soil}$ 

Effective filtration can minimise soil particles from moving through the filter, and the well-known filtration requirement is given by (ASTM, 1993):

$$\frac{O_{95}}{D_{85}} \le 3 \tag{35}$$

In the above inequality,  $O_{95}$  indicates the approximate largest particle that would effectively pass through the geosynthetic filter.  $D_{85}$  indicates the diameter of clay particles corresponding to 85% passing.

The retention ability of the drain filter is represented by:

$$\frac{O_{50}}{D_{50}} \le 24 \tag{36}$$

Filter material can become clogged if the soil particles become trapped within the filter fabric, without wash-out. Clogging is usually prevented by ensuring that (Christopher and Holtz, 1985):

$$\frac{O_{95}}{D_{15}} \ge 3$$
 (37)

#### 4.3 DISCHARGE CAPACITY

The discharge capacity of the prefabricated vertical drain is essential to analyse well resistance. However, the well resistance is usually less significant than the role of drain spacing and the soil disturbance (smear effect). The discharge capacity is a function of the volume of the core or the drain channel, the lateral earth pressure acting on the drains, possible kinking of the drain due to large displacement, clogging of geosynthetic filter, and the biological and chemical degradation. The influence of the ratio of discharge capacity to lateral permeability ( $q_w/k_h$ ) on the rate of consolidation was discussed by Jamiolkowski et al. (1983). Based on the above factors, the actual discharge capacity,  $q_w$ , is given by (Rixner et al., 1986):

$$q_{w} = (F_{t})(F_{c})(F_{fc})q_{req}$$
(38)

The parameters  $F_t$ ,  $F_c$  and  $F_{fc}$  are the influence factors due to time; drain deformation and clogging, respectively. The term  $q_{req}$  is the theoretical discharge capacity estimated from Barron's theory, and it is given by:

$$q_{req} = \frac{\varepsilon_f U_{10} l \pi c_h}{4T_b} \tag{39}$$

where,  $\varepsilon_f$  is the ultimate settlement of the soft soil equivalent to 25% of the length of the drain installed,  $U_{10}$  is the 10 percent degree of consolidation, l is the length of the vertical drain,  $c_h$  is horizontal coefficient of consolidation and  $T_h$  is the time factor for lateral consolidation.



Figure 11: Estimation of vertical discharge capacity (after Rixner et al., 1986).

From laboratory tests, the influence factor of time,  $F_t$ , has been estimated to be less than 1.2, and it is often assumed to be 1.25 (conservative). The reduction of the discharge capacity under unfavourable conditions of (bending, folding or twisting) has been suggested to be about 48%, which gives an influence factor of deformation,  $F_c$  of about 2. From filtration tests, the value of  $F_{fc}$  is suggested to vary between 2.8 and 4.2. After considering the worst circumstances that

may occur in the field, the discharge capacity,  $q_w$  of the PVD could be taken as high as 500-800 m<sup>3</sup>/year, but certainly can be reduced to 100-300 m<sup>3</sup>/year, under elevated lateral pressure (Rixner et al., 1986).

The discharge capacity of various PVD is shown in Figure 11, where it is clear that the discharge capacity is affected by the lateral confining pressure. In the absence of laboratory test data, the discharge capacity can be conservatively assumed to be  $100 \text{ m}^3$ /year, based on various numerical models (Indraratna, 2002). Values presented by Hansbo (1981) for high lateral pressures are in the order of 20 m<sup>3</sup>/year.

Holtz et al. (1991) pointed out that the discharge capacity of PVD could vary from 100-800 m<sup>3</sup>/year. Based on a number of experimental studies, they concluded that as long as the initial discharge capacity of PVD exceeds 100-150 m<sup>3</sup>/year, some reduction in discharge capacity due to installation should not seriously influence the consolidation rates. Nevertheless, the discharge capacity can fall below this desired value due to drain kinking and clogging (25-100 m<sup>3</sup>/year). Clearly, the 'clogged' drains are associated with  $q_w$  values approaching zero.

### 5 INFLUENCE ZONE OF DRAINS



The influence zone (D) is a controlled variable (Fig. 12), since it is a function of the drain spacing (S) as given by:

D = 1.13 S (square pattern), and

D = 1.05 S (triangular pattern)

A square pattern of drains is often easier to lay out during field installation, however, a triangular pattern is usually preferred since it provides a more uniform consolidation between drains.

### **6 OVERALL CONSOLIDATION**

#### 6.1 RATE OF CONSOLIDATION

The primary reason for using PVD is to reach the desired degree of consolidation within a specified period of time. In a vertical drain system, the combined radial and vertical consolidation should be considered, and Carillo (1942) proposed this combined effect as:

$$1 - U = (1 - U_r)(1 - U_V)$$

where U – overall degree of consolidation

 $U_r$  – average degree of consolidation due to radial drainage

 $U_v$  – average degree of consolidation due to vertical drainage

### 6.2 COEFFICIENT OF CONSOLIDATION

#### (a) log U vs t approach

Aboshi and Monden (1963) presented a curve fitting method using log U and linear t by taking 'log' on both sides of Barron's solution (Eqn. 8), hence:

$$T_h = -\frac{F(n)}{8} ln \left(1 - \bar{U}\right) \tag{43}$$





(42)

(40)

(41)

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Without any consideration of smear effects, Equation 43 represents the theoretical time factor for radial consolidation of perfect drains. The coefficient of radial consolidation  $c_h$  is evaluated by plotting log U vs  $T_h$ , where a linearised slope provides the value of  $c_h$ .

Considering the effect of smear, Equation 15, can be rewritten by taking 'log' on both sides of the equation to give:

$$T_h = -\frac{v}{8} ln \left( 1 - \bar{U} \right) \tag{44}$$

Substituting  $c_h = \frac{T_h H_d^2}{t}$  yields the following expression:

$$c_{h} = -\frac{D^{2}\nu}{8} \frac{H_{d}\ln(1-\bar{U})}{dt}$$

$$\tag{45}$$

### (b) Settlement data based approach

The method introduced by Asaoka (1978) considers a series of settlements ( $\rho_{I}$ ,  $\rho_{I-I}$ ,  $\rho_{i}$ ,  $\rho_{i+I}$  etc.) which are observed and plotted at constant time intervals. Here, the coefficient of radial drainage consolidation is derived from:

$$c_h = -\frac{D^2 \nu}{8} \frac{\ln \beta}{\Delta t} \tag{46}$$

The expression for v is given earlier in Equation 12,  $\beta$  is the slope of the line formed by the observed displacement data, and  $\Delta t$  is the time interval between observations. In order to obtain the average coefficient of consolidation, the smear factor v is replaced by the drain spacing factor, F(n) to give:

$$c_h = -\frac{D^2 F(n)}{8} \frac{\ln \beta}{\Delta t} \tag{47}$$

Where F(n) was defined earlier in Equation 5.

### 7 HORIZONTAL TO VERTICAL PERMEABILITY RATIO

The coefficient of permeability can be determined by laboratory oedometer tests using one dimensional consolidation theory. This indirect method of determining the coefficient of permeability is often subjected to error because of the assumption of constant permeability (time independent), constant coefficient of volume change and constant coefficient of consolidation  $(k, m_v \text{ and } c_v)$ , respectively, during the consolidation process. The modern (modified) triaxial and oedometer tests used in conjunction with a permeameter produce more reliable values. For example, the falling head permeability test conducted on the modified oedometer appears to be adequate (simpler and faster) for estimating permeabilities of natural clays.

Using modified laboratory equipment, the permeability characteristics of a number of intact clays have been determined by Tavenas et al. (1983). The horizontal permeability was determined using samples rotated horizontally (90°) and of intermediate inclination, 45°. For marine clays (Champlain sea formation, Canada), the anisotropy ratio ( $r_k = k_h/k_v$ ) determined by modified oedometer test varied between 0.91 and 1.42 with an average of 1.1. In triaxial tests, the same ratio was found to vary in the range of 0.81-1.16 with an average of 1.03. These results suggest that anisotropy is not a major influence factor in this soil.

For the laboratory results plotted in Fig.4 (Indraratna & Redana, 1998), the value of  $k'_h / k'_v$  in the smear zone varies between 0.9 and 1.3 with an average of 1.15. Hansbo (1987) claimed that for extensive smearing, the lateral permeability coefficient ( $k'_h$ ) would approach that of the vertical permeability ( $k'_v$ ), thus suggesting a ratio  $k'_h / k'_v$  of unity. The experimental results of Indraratna & Redana (1998) plotted in Figure 4 seem to be in agreement with Hansbo (1987). For the range of consolidation pressures, it is noted that the ratio  $k_h/k_v$  varies between 1.4 and 1.9 with an average of 1.63 in the undisturbed zone. Shogaki et al. (1995) reported that the mean value of  $k_h/k_v$  were in the range of 1.36-1.57 for a large number of undisturbed soil samples obtain from Hokkaido to Chugoku in Japan. Tavenas et al. (1983) described that for soil specimens tested in oedometer, the  $k_h/k_v$  ratio was varied between 0.91 and 1.42 for intact natural clays, and from 1.2 to 1.3 for Matagami varved clay. Bergado et al. (1991) reported that for Bangkok clay, the horizontal permeability coefficient of undisturbed zone to that of the smear zone varied between 1.5 and 2. In the smear zone, the ratio of  $k'_h / k'_v$  was found to be almost unity. Further details are given by Indraratna (2002). 8

# SOIL MODELS AND FINITE ELEMENT METHOD

The Cam-clay model through critical state soil mechanics has received wide acceptance due to its simplicity and accuracy in modelling to model soft clay behaviour. Utilising the critical state concepts and the theory of soil plasticity (Schofield & Wroth, 1968), the modified Cam-clay was developed to overcome the problems of the original Cam-clay model (Roscoe & Burland, 1968). The obvious difference between the modified Cam-clay and original Cam-clay model is the shape of the yield locus. The yield locus of the former is elliptical. Leroueil (2001) explained in detail, aspects of strain rate related limit state curves and creep effects that need to be incorporated in accurate soft clay modelling. Detailed soft clay modelling aspects (in some cases, the effects of secondary consolidation) have been discussed on various occasions by: Larsson (1986); Leroueil (1996); Leroueil et al. (1978); Tavenas and Leroueil (1980); Tavenas et al. (1994); Mesri and Choi (1985); Mesri and Lo (1991); Mesri (1991); and Mesri et al. (1994); Lacasse (2001); Samson (1985); Burland (1990); Arai et al. (1991) among others. An elaborate analysis of the theoretical pore pressure dissipation to describe the settlement trendss has been presented by Eriksson et al. (2000) based on their experience of Stockholm-Arlanda airport.

The finite element software codes CRISP, SAGE-CRISP, ABAQUS and FLAC include the modified Cam-clay model. These programs have been successfully used in the past for analysing the behaviour of soft clay embankments. The basic element types used in consolidation analysis are the linear strain triangle (LST) consisting of six displacement nodes, three-noded linear strain bar (LSB) elements with two-pore pressure nodes at either end and a sole displacement node in the middle, and the eight-noded LSQ elements also having a linear pore pressure variation (Fig. 13). More elaborate details are given by Britto and Gunn (1987), in the finite element applications employing modified Cam-clay. The vertical drain interfaces are best represented by LSB elements (Indraratna & Redana, 2000).



Figure 13. Element type employed in CRISP (Indraratna, 2002).

# 9 NUMERICAL MODELLING

Pore pressures, settlements and lateral displacements of PVD installed field site can be analysed (coupled consolidation) using sophisticated finite element software such as ABAQUS and PLAXIS. According to past experience, finite element prediction of lateral deformation has been relatively poor in contrast to settlements (Indraratna et al., 1994; Indraratna, 2002). Selected finite element models applied to prefabricated vertical drains are described below.

### 9.1 DRAIN EFFICIENCY BASED ON PORE PRESSURE DISSIPATION

In 1990's, the performance of embankments stabilised with vertical drains at Muar clay, Malaysia was analysed using the finite element approach by the first author and his co-workers. The effectiveness of the prefabricated drains was evaluated according to the rate of excess-pore pressure dissipation at the soil drain interface. Both single and multidrain (whole embankment) analyses were carried out to predict the settlement and lateral deformation beneath the embankment (Indraratna et al., 1994; 1997; 2001, 2002).

In one comprehensive study, a plane strain analysis was applied to a single drain and to the entire multi-drain PVD scheme. An axisymmetric horizontal permeability was used in the plane strain model. As described by Ratnayake (1991), the prediction of settlement (single drain analysis) over-predicts the measured settlement, even after the effect of smear is included. In the case of multi-drain analysis, the settlement was still over predicted, and therefore, it was necessary to consider more accurately, the dissipation of the excess pore pressures at the drain boundaries at a given time.



Figure 14. Undissipated excess pore pressures at drain-soil interfaces (Indraratna et al., 1994).

The average undissipated excess pore pressures could be determined by back-analysis of the settlement data at the centreline of the embankment (Fig. 14). In this analysis, 100% represents 'zero dissipation' when the drains are fully charged. Towards the end of the first stage of consolidation (ie., 2.5 m of fill at 105 days), the undissipated pore pressures decrease from 100% to 16%. At the second stage of loading, the undissipated pore pressures decrease from 100% to 18% after a period of 284 days, whereby the maximum embankment height of 4.74m has already been attained. From Figure 14, it can be deduced that 'perfect drain' conditions are observed only after a period of 400 days. Even though the finite element results and field data are in general agreement during the initial stages, the discrepancy beyond 100 days is too significant to be attributed solely to the plane strain (2D) approach. These excess pore-pressures reflect the retarded efficiency of the vertical drains, as characteristics of drain clogging or drain kinking. A better prediction was obtained for settlement, pore pressure and lateral deformation when 'non-zero' excess pore pressures were input (prescribed) at drain interface, simulating 'partially clogged' conditions. The 'non zero' excess pore pressures can also represent the smear effect and well resistance that contribute to decreased efficiency, as elaborated later.

#### 9.2 MATCHING PERMEABILITY & GEOMETRY

A modelling technique of vertical drains in two-dimensional finite element analysis using CRISP (Britto and Gunn, 1987) was presented by Hird et al. (1992,1995). Permeability and geometry matching techniques were applied to several embankments stabilised with vertical drains in Porto Tolle, Italy (Fig. 15), Harlow (UK) and Lok Ma Chau (Hongkong). Although an acceptable prediction of settlements was obtained, the pore water pressure dissipation was not well predicted. A Lok Ma Chau (Hongkong), the settlements were significantly over-predicted, indicating that mechanisms other than smear and well resistance can affect the pore pressure dissipation in PVD.



Figure 15: Comparison of average surface settlement for Porto Tolle embankment at embankment centerline (Hird et al., 1995)

#### 9.3 ROLE OF DISCHARGE CAPACITY

Chai et al. (1995) extended the method proposed by Hird et al. (1992) to properly include the effect of well resistance and clogging. Four types of analyses were presented considering: (a) no vertical drains, (b) embankment with vertical drains and discharge capacity of the drains increasing with depth, (c) drains with constant discharge capacity and (d) the same as (e) but assuming that the drains were clogged below 9 m depth. In plane strain, the discharge capacity of the drain required for matching the average degree of horizontal consolidation is given by:



Figure 16. Average degree of horizontal consolidation (Chai et al., 1995).

A single drain model of 5 m long employed, and both elastic and elasto-plastic analyses were applied to predict its performance. Very good agreement between axisymmetric and plane strain models was obtained when varied discharge capacity  $q_{wp}$  was incorporated (Fig. 16), in the elasto-plastic analysis. For elastic analysis, incorporating the well resistance leads to a more realistic excess pore water pressure variation with depth as shown in Figure 17. The varied discharge capacity gives a more uniform and closer match between the axisymmetric and plane strain methods compared to the constant discharge capacity assumption.

The above model was tested against the performance of an embankment stabilised with vertical drains founded in Muar clay, Malaysia. The analysis verifies that the vertical drains not only increase the settlement rate, but also reduce the lateral deformation. A more accurate excess pore pressure distribution was obtained when both well resistance and clogging were introduced in the finite element analysis. Further details are given by Indraratna et al. (2000) and Indraratna (2002).



Figure 17: Excess pore pressure variation with depth (Chai et al., 1995).

#### 9.4 LATERAL DISPLACEMENT AS STABILITY INDICATOR

Indraratna et al. (1997) studied the role of soft clay stabilisation by preloading with geogrid and vertical band drains. In contrast, the behaviour of sand compaction piles was also investigated for embankments constructed on Muar clay, Malaysia. The settlement and lateral displacement of the soft clay beneath the embankments were analysed, and the findings were compared to the field measurements. In this two-dimensional (plane strain) analysis, the vertical drain system was converted into an equivalent drainage wall, as explained earlier (Fig 9). In this section, the use of lateral displacement as a stability indicator will be briefly described.



Figure 18: Normalised lateral deformation with respect to maximum settlement (Indraratna et al., 2002b)

The current analysis employed critical state soil mechanics, and the deformations were calculated on the basis of the fully coupled (Biot) consolidation model in conjunction with finite element method. The soil underneath the embankment was discretised using linear strain quadrilateral (LSQ) elements. The vertical drain interfaces were simulated with LSB elements (Fig. 13).



Figure 19: Normalised lateral deformation with respect to maximum fill height (Indraratna et al., 2002b)

The PVD were modelled as ideal and non-ideal drains, where in the former, the well resistance factor was ignored. The accurate prediction of lateral displacement depends on the proper assessment of the Cam-clay parameter  $\lambda$ , the shear resistance at the embankment base (interface), and the nature of assumptions made in the plane strain modelling of vertical drains. The true soil properties are influenced by the applied stress range and the actual stress path of the subsoil at a given depth and lateral distance. The normally consolidated parameters associated with the Cam-clay theories over-estimate the lateral displacement, if the applied stresses are smaller than the pre-consolidation pressure, i.e. initial stages of embankment construction, where the behaviour is closer to a lightly overconsolidated soil.

Where:

 $\alpha$  = ratio of maximum lateral displacement at the toe to the maximum settlement at centreline of embankment.

 $\beta_1$  = ratio of maximum lateral displacement to the corresponding fill height.



 $\beta_2$  = ratio of maximum settlement to the corresponding fill height.

Figure 20: Cross-section of embankment TV1 with 15 m long PVD showing location of monitoring instruments (Bergado et al., 1998).

Table 1:Effect of ground improvement on normalised deformation factors (I	Indraratna et al.,	1997).
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Ground improvement scheme	α	$\beta_1$	β <sub>2</sub>
Sand compaction piles for pile/soil stiffness ratio 5 (h = 9.8 m, including 1 m sand layer	0.185	0.018	0.097
Geogrids + vertical band drains in square pattern at 2.0 m spacing (h=8.7 m)	0.141	0.021	0.149
Vertical band drains in triangular pattern at 1.3 m spacing (h = 4.75 m)	0.123	0.034	0.274
Embankment rapidly constructed to failure on untreated foundation (h=5.5m)	0.634	0.104	0.164

Based on normalized deformations, the performance of vertical band drains and sand compaction piles was compared (Table 1). The ratio of maximum lateral displacement to fill height ( $\beta_1$ ) and the ratio of maximum settlement to fill height ( $\beta_2$ ) can be considered as stability indicators. In comparison with an unstabilised embankment constructed to failure (Indraratna et al., 1992) where  $\alpha > 0.63$  and  $\beta_1 > 0.1$ , the stabilised foundations are characterised by much smaller values for  $\alpha$  and  $\beta_1$ . It is important to note that the normalised settlement ( $\beta_2$ ) on its own does not seem to be a distinct indicator of instability. The normalised parameters  $\alpha$  and  $\beta_1$  are plotted in Figures 18 and 19, respectively. Both these Figures show that lateral deformation is less for the embankment stabilised with PVD. This is because the SCP's were installed at 2.2 m spacing, hence they do not effectively control the lateral deformation at this increased spacing. The PVD's, on the other hand provide better lateral restrain especially when installed at a much closer spacing of 1.3 m.

#### 9.5 EFFECT OF VACUUM PRESSURE

Bergado et al. (1998) analysed the performance of embankment stabilised with vertical drains, where a combined preload and vacuum pressure were applied at the Second Bangkok International Airport site. Similar to a previous techniques demonstrated by Chai and Miura (1997), the analysis adopts the concept that PVD increase the mass permeability in the vertical direction. Therefore, it is possible to establish an overall coefficient of permeability of the natural subsoil, incorporating the radial flow towards the PVD. This equivalent vertical permeability ( $K_{ve}$ ) is derived, based on equal average degree of consolidation, and it includes both smear and well resistance.

In this method, the approximate average degree of vertical consolidation,  $U_{\nu}$  is given by:

$$U_{v} = 1 - exp(-3.54)T_{v}$$
<sup>(49)</sup>

where,  $T_v$  is the dimensionless time factor.

The equivalent vertical permeability,  $K_{ve}$  is expressed by:

$$K_{ve} = \left(1 + \frac{2.26L^2 K_h}{FD_e^2 K_v}\right) K_v \tag{50}$$

where the algebraic expression for F is given by:

$$F = ln\left(\frac{D_e}{d_w}\right) + \left(\frac{K_h}{K_s} - 1\right) ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4} + \frac{\pi 2L^2 K_h}{3q_w}$$
(51)

In the above equation,  $D_e$  is the equivalent diameter of a unit drain influence zone,  $d_s$  is the equivalent diameter of the disturbed zone,  $d_w$  is the equivalent drain diameter,  $K_h$  and  $K_s$  are the undisturbed and disturbed lateral permeability of the soil, respectively, L is the drain length and  $q_w$  is discharge capacity of drain.

At second Bangkok airport site, TV1 and TV2 are two embankments, each with base area of 40 x 40 m which were analysed by Bergado et al.(1998). The PVD were installed to a depth of 15 m and 12 m for embankment TV1 and TV2, respectively. The design parameters of PVD are given in Table 2. A typical cross section of embankment TV1 is shown in Figure 20. Vacuum preloading was modeled by fixing the pore pressure at the top surface. It can be concluded that the vacuum assisted consolidation has been very effective for both embankments, TV1 and TV2. For instance, the performance of embankment TV2 with vacuum preloading (compared to the embankment at the same site without vacuum preloading), showed an acceleration in the rate of settlement of about 60%, and a reduction in the otherwise required period of preloading by about 4 months.

In a large scale consolidometer, Indraratna et al. (2002a) analysed the settlement behaviour of clay under vacuum and surcharge loading using the finite element code ABAQUS, incorporating the modified Cam Clay model. The discritized finite element mesh (symmetric along embankment centerline) for the laboratory consolidation cell (Fig. 21), consists of linear strain quadrilateral elements with four pore pressure nodes at corners, and eight displacement nodes. The clay layer is characterized by drained conditions at the upper boundary, and the vacuum pressure is simulated by applying a linearly varying negative pore pressure at the drain boundary. The negative surcharge (-100kPa) is also distributed along the top of soft clay, which is the same as that of the drain top (maximum).

The consolidation properties of the soft clay are given by:  $C_c = 0.34$ ,  $C_r = 0.14$ ,  $k_h = 1.598 \times 10^{-10}$  m/s, and the maximum preconsolidation pressure ( $P'_c$ ) was 20 kPa. The modified Cam-clay properties of this soil are given in Table 3. The extent of smear zone was assumed to be four times the drain diameter based on previous testing (Indraratna & Redana, 1998). The laboratory based permeability parameters were converted to plane strain equivalents using Equations (3) and (4). In the smear zone, the converted permeability  $k'_{hp}$  was 0.634 x 10<sup>-10</sup> m/s; outside the smear zone,

the average converted permeability  $k_{hp}$  was 0.74 x 10<sup>-10</sup> m/s for the range of applied consolidation pressure.

![](_page_20_Figure_2.jpeg)

Figure 21:Finite element mesh of the soil cell in large scale consolidometer (Indraratna et al., 2002a)

Spacing, S	1.0 m (triangular pattern)
Diameter of drain, $d_w$	50 mm
Diameter of smear zone, $d_s$	300 mm
Ratio of $K_h/K_s$	10
Drainage length, l	15 m for TV1 and 12 m for TV2
Discharge capacity, $q_w$ (per-drain)	50 m <sup>3</sup> /year

Table 2: Design parameters of PVD for 2<sup>nd</sup> Bangkok International Airport Site (Bergado et al., 1998)

Comparison of predicted surface settlement with the measured data is shown in Figure 22. For the initial loading phase, the numerical predictions provide a good agreement with the observed data, incorporating the smear effects. In the subsequent loading step, the predicted settlement curve lies significantly below the laboratory measurements. This may be because the vacuum pressure did not propagate effectively with depth as assumed by the linear variation. Although it was not feasible to check whether the vacuum pressure had propagated below a certain depth of the drain, Figure 22 still verifies that the application of vacuum pressure can accelerate consolidation, thereby compensating for the adverse effects of smear.

Figures 23 and 24 illustrate the propagation of excess pore pressure with time throughout the cell. The lateral spread of the effect of vacuum is a time dependent process. Only after about 14 days, the centerline of the cell indicates fully dissipated zero pore pressures. Although not shown here, after several months, the right hand side of the cell (i.e. >150mm away from the drain centerline) experiences negative pore pressures. Figures 25 and 26 represent the variation of vertical effective stress, and they clearly show the effect of vacuum pressure on increasing the effective stress with time, thereby enhancing the shear strength of the soil.

Soil properties	Magnitude
λ	0.15
κ	0.05
Critical state line slope, M	1.1
Void ratio, $e_{cs}$	1.55
Poisson's ratio, v	0.25

	Table 3:	Modified	Cam-clay	properties
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 $\lambda$ ,  $\kappa$ : slope of volume vs. log pressure for consolidation and swelling, respectively

![](_page_21_Figure_5.jpeg)

![](_page_21_Figure_6.jpeg)

# 10 RELIABILITY OF EQUIVALENT PLANE STRAIN MODELLING

As described earlier, Indraratna & Redana (1997) analysed the effect of smear zone and well resistance in a vertical drain using a 2D plane strain formulation whereby, the vertical drain system was converted to equivalent parallel drain walls by adjusting the spacing of the drains and the coefficient of permeability of the soil (Equations 21-31). The transformed permeability coefficient could be incorporated in a finite element code, through appropriate subroutines (eg. SAGE-CRISP or ABAQUS). A finite element analysis was executed for both axisymmetric and equivalent plane strain models; as an example, consider a unit PVD installed to a depth of 5 m under the ground surface at spacing of 1.2m. The model parameters and soil properties are summarised in Table 4.

The size of the smear zone was taken to be 4 times the size of the mandrel based on the experimental results. A simplified permeability variation was assumed (Fig. 27), where the coefficient of permeability within the smear zone was taken to be constant (i.e.  $k'_{hp} = 5.02 \times 10^{-10}$  m/s). An increased permeability coefficient was used for the undisturbed zone ( $k_{hp} = 2.97 \times 10^{-9}$  m/s). The difference in permeability between axisymmetric and equivalent plane strain conditions is shown in Figure 27.

Figures 28 and 29 illustrate the settlement vs. time and excess pore pressure against time plots for drains with smear and well resistance (single drain analysis). It is noted that for short drains the effect of well resistance is less significant than the smear effect. The comparison between axisymmetric and equivalent plane strain models is in good agreement. The

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pore pressures are more difficult to match than the settlements as indicated by these plots. The maximum error in settlement plot is about 7 %, whereas the maximum deviation in excess pore water pressure is about 15 %. Based on the above single drain analysis, Figs. 28 - 29 provide convincing evidence that the equivalent (converted) plane strain model is a convenient and effective substitute for the axisymmetric model. In finite element modeling, the 2-D, plane strain analysis is expected to cut down the computational time considerably, in comparison with the time taken by a 3-D, axisymmetric model, especially in the case of multi-drain analysis.

![](_page_22_Figure_3.jpeg)

Figure 23: Contour plots of excess pore water pressure after 1 day (a) drain with no vacuum pressure (b) drain with linearly varied vacuum pressure: 100kPa at top, 0 at bottom (Indraratna et al., 2002a)

![](_page_22_Figure_5.jpeg)

Figure 24: Contour plots of excess pore water pressure after 14 days (a) drain with no vacuum pressure (b) drain with linearly varied vacuum pressure:100kPa at top, 0 at bottom (Indraratna et al., 2002a)

![](_page_22_Figure_7.jpeg)

Figure 25:Contour plots of effective vertical stress after 1day (a) drain with no vacuum pressure (b) drain with linearly varied vacuum pressure: 100kPa at top, 0 at bottom (Indraratna et al., 2002a)

![](_page_23_Figure_2.jpeg)

Figure 26: Contour plots of effective vertical stress after 14 days (a) drain with no vacuum pressure (b) drain with linearly varied vacuum pressure: 100kPa at top, 0 at bottom (Indraratna et al., 2002a)

![](_page_23_Figure_4.jpeg)

Figure 27: Simplified variation of permeability within and out side smear zone (Indraratna, 2000).

![](_page_23_Figure_6.jpeg)

Figure 28: Comparison of the average surface settlement for axisymmetric and equivalent plane strain analyses with smear and well resistance (Indraratna , 2000).

![](_page_24_Figure_2.jpeg)

Figure 29: Comparison of the excess pore pressure at mid depth for axisymmetric and equivalent plane strain analyses with smear and well resistance (Indraratna ,2000).

Table 4. Wodel parameters and son pro	pernes
Spacing, S	1.2m
Radius of drain, $r_w$	0.03m
Radius of mandrel	0.05m
Ratio of $K_h/K_v$ in undisturbed zone	2
Ratio of $K_h/K_v$ in smear zone	1
Plane strain permeability in undisturbed zone, $k_{hp}$	2.97 x 10 <sup>-9</sup> m/s
Plane strain permeability in smear zone, $k'_{hp}$	5.02 x 10 <sup>-10</sup> m/s
Radius of unit cell	0.6m
Gradient of volume vs log pressure relation for consolidation, $\lambda$	0.2
Gradient of volume vs log pressure relation for swelling, $\boldsymbol{\kappa}$	0.04
Slope of critical state line, M	1.0
Void ratio at unit consolidation pressure, $e_{cs}$	2
Poisson's ratio, v	0.25
Saturated unit weight of the soil, $\gamma_s$	18 kN/m <sup>3</sup>

Table 4. Model	narameters and	soil	nronerties
1 abic 4. Mouch	Darameters and	SOII	

### 11 GENERAL APPLICATION OF FINITE ELEMENT MODELLING AND COMPARISON WITH FIELD OBSERVATIONS

In this section of the paper, a comprehensive numerical analysis of the performance of several test embankments based on plane strain model will be presented. Figure 30 shows the subsoil profile, modified Cam-clay parameters and the effective stress conditions of the site of second Bangkok International Airport (SBIA). The unit weight of the weathered crust (over consolidated) is about 18 kN/m<sup>3</sup> and the unit weight of the very soft clay is about 14.3 kN/m<sup>3</sup> at a depth of 7 m. A typical finite element mesh employing the multi-drain analysis is given in Figure 31, where the soft clay is discretized into linear strain quadrilateral (LSQ) elements. For the zone stabilised with Prefabricated Vertical Drains (PVD), a finer mesh was used so that each drain element (LSB) includes the smear zone on either side. The locations of inclinometers and piezometers are conveniently placed on the mesh, so that the measurement points coincide with the mesh nodes. The embankment load is applied in stages (ie. sequential construction) as simplified in Figure 32. The equivalent plane strain parameters were calculated based on Eqns. 29 and 30 (Indraratna & Redana, 1997). A minimum discharge capacity ( $q_w$ ) of 100m<sup>3</sup>/year was chosen to model the settlements and pore water pressure dissipation in the soft clay.

Figure 33 shows the results of the plane strain finite element analysis of a typical embankment (TS1) together with the measured settlements. It is shown that the analysis based on 'perfect drain' conditions (i.e. no smear, complete pore pressure dissipation) over-predicts the measured settlement. However, the inclusion of smear effect improves the

![](_page_25_Figure_3.jpeg)

Figure 30: Sub-soil profile, Cam-clay parameters and stress condition used in numerical analysis, Second Bangkok International Airport, (modified from AIT, 1995)

![](_page_25_Figure_5.jpeg)

Figure 31: Finite element mesh of the embankment for plane strain analysis with constant drain length (Indraratna & Redana, 2000).

accuracy of the predictions. While the inclusion of both smear and well resistance underestimates the measured settlements, the role of well resistance could be regarded as less significant, in comparison with the smear effect for drains less than 20m in length.

Figure 34 illustrates the measured and calculated excess pore pressures along the centerline of the embankment, at a depth of 2 m below the ground surface. For 'smear only' analysis, the pore water pressure increase is well predicted during Stage 1 and Stage 2 loading, but beyond Stage 3 loading, the calculated pore pressure is significantly smaller than the field data. The perfect drain predictions, not surprisingly, underestimate the actual pore pressures. The inclusion of the effects of smear and well resistance improves the model accuracy.

![](_page_26_Figure_2.jpeg)

Figure 32: Loading history for construction of embankments TS1, TS2 and TS3 at Second Bangkok International Airport (AIT, 1995).

![](_page_26_Figure_4.jpeg)

Figure 33. Surface settlement at the centre-line for embankment TS1, Second Bangkok International Airport (Indraratna & Redana, 2000).

Observed and calculated lateral deformation for the inclinometer installed 23m away from the centerline of the embankment is shown in Figure 35. The effect of the compacted soil crust is clearly seen around 1–2 m depth, where the lateral displacements decrease significantly. This favourable influence of the compacted crust is analogous to placing a geogrid at the base of the embankment. Very often, in practice, the top most crusted soil is excavated and removed before building the embankment. The authors recommend that even if a geogrid is placed, the role of the compacted crust must be fully exploited to reduce the lateral displacements which otherwise may affect stability. The behaviour of the compacted crust cannot be modeled by the use of modified Cam-clay parameters. It is best to model this thin top most crusted layer using elastic properties or considering it as a very stiff clay. The lateral displacements at 44 days and 294 days are reasonably well predicted when both smear and well resistance are considered. The inclusion of smear effect on its own underestimates the lateral displacement, while the 'perfect drain' yields the smallest lateral deformation. The predicted lateral deformation for 'no drains' is plotted for comparison, which provides a numerical upper bound. It is verified that the use of PVD is expected to reduce the lateral movement of soft clay under embankment loading, but never to the extent of what is predicted by 'perfect drains'.

![](_page_26_Figure_7.jpeg)

Figure 34. Variation of excess pore water pressures at 2 m depth below ground level at the centre-line for embankment TS1 (Indraratna & Redana, 2000).

### 12 SUMMARY AND CONCLUSIONS

In many countries where coastal areas accommodate the highest population densities, improvement of compressible soft clays remains to be of paramount importance. In this paper, the use of prefabricated vertical drains as means of soft clay stabilization was described, highlighting the aspects of smear effects, well resistance, drain interface unsaturation and other mechanisms that affect the performance of PVD. The behaviour of soft clay under the influence of PVD was described on the basis of numerous case histories, where both field measurements and numerical predictions were available. The large-scale laboratory experiments provided an excellent technique for estimating the extent of smear and the resulting effect on lateral (radial) permeability. Such large equipment also facilitates the study of vacuum pressure application for accelerating consolidation.

![](_page_27_Figure_4.jpeg)

Figure 35: Lateral displacement profiles at 23 m away from centerline of embankments a) after 44 days, and b) after 294 days (Indraratna & Redana, 2000).

This paper has described in detail, the application of 2-D plane strain theory for PVD installed in soft clay, and the finite element approach has been extended to multi-drain behaviour for several embankments. The numerical results indicate that the inclusion of the smear effect and well resistance improves the reliability of the predicted settlement, excess pore pressures and lateral displacements of the soft clay foundation. For drains that are normally less than 20m, the effect of well resistance is not significant compared to the smear effect. The numerical models also show that a 'perfect drain analysis always over-predicts the settlements, by under-predicting the pore pressures. Moreover, the current numerical models indicate that the accurate prediction of lateral displacement is often difficult due to the inherent assumptions made in the 2D plane strain analysis, however, by incorporating both smear and well resistance the accuracy of prediction can be improved. In order to model the well resistance correctly in a numerical analysis, the appropriate discharge capacity of the PVD needs to be estimated, and this can often be carried out through available semi-empirical charts. Nevertheless, in the field, kinking and clogging of drains reduce the discharge capacity considerably, thereby

increasing the well resistance factor in the governing consolidation equations. Furthermore, drains with a small discharge capacity tend to increase lateral movement and retard the excess pore water pressure dissipation, while sufficiently closely spaced drains have a positive effect on controlling lateral displacement. It is clear that a 2-D plane strain model incorporated in a multi drain finite element analysis is still acceptable, in view of computational efficiency in a PC environment. A true 3-D multi-drain analysis with an individual axisymmetric zone of influence with smear for each drain, will often exceed the computational capacity when applied to an embankment project with a large number of PVD. In this context, the equivalent plane strain model proposed by the authors with further refinement will continue to offer a sufficiently accurate predictive tool for design, performance verification and back analysis.

The occurrence of a possible air-gap between the drain and surrounding soil caused due to mandrel withdrawal also affects the pore pressure dissipation. This adds to the complication of predicting the correct soil deformation. Preliminary studies verify that such an unsaturated interface can decrease the rate of consolidation significantly. Although the application of vacuum pressure in conjunction with a traditional earthfill is an effective and convenient method of accelerating consolidation particularly for the soft clay affected by smear and well resistance, in the field, the cost can become an inhibiting factor. Nevertheless, the numerical modeling aspects of vacuum pressure and its effect on overall soil consolidation via multi-drain PVD systems require further study and research developments.

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