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## Theoretical and numerical perspectives and field observations for the design and performance evaluation of embankments constructed on soft marine clay

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1 Chapter 2  
2  
3 **Theoretical and Numerical Perspectives and Field**  
4 **Observations for the Design and Performance**  
5 **Evaluation of Embankments Constructed on Soft**  
6 **Marine Clay**  
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11 **A. S. Balasubramaniam<sup>3</sup>**  
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20 **ABSTRACT**  
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22 In this chapter, a two-dimensional plane strain solution is adopted for the embankment analy-  
23 sis, which includes the effects of smear zone caused by mandrel driven vertical drains. The  
24 equivalent (transformed) permeability coefficients are incorporated in finite element codes,  
25 employing modified Cam-clay theory. Selected numerical studies have been carried out to  
26 study the effect of embankment slope, construction rate, and drain spacing on the failure of  
27 the soft clay foundation. Finally, the observed and predicted performances of well-instru-  
28 mented full-scale trial embankments built on soft Malaysian marine clay have been discussed  
29 in detail. The predicted results agree with the field measurements.  
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32 **1. INTRODUCTION**  
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34 The rapid development and associated urbanization have compelled engineers to construct  
35 earth structures, including major highways, over soft clay deposits of low bearing capac-  
36 ity coupled with excessive settlement characteristics. In the coastal regions of Australia  
37 and Southeast Asia, soft clays are widespread and particularly in the vicinity of capital  
38 cities. Because soft soils are weak, unreinforced embankments can only be built 4–5 m  
39 high. However, higher embankments are often needed and their rapid construction is  
40 pertinent given the usual stringent deadlines. To achieve these goals, special construction

1 measures such as light-weight embankment fill, the provision of reinforcement at the bot-  
2 tom of the embankment, and suitable ground improvement techniques and staged embank-  
3 ment construction must be considered. The application of prefabricated vertical drains  
4 (PVDs) with preloading (vacuum pressure or surcharge) has become common practice and  
5 is one of the most effective techniques for ground improvement.

6 Many improvement techniques have been developed to suit particular soil condition,  
7 with most soft clay methods based on consolidation. Preloading with vertical drains is a  
8 successful ground improvement technique, which involves the loading of the ground sur-  
9 face to induce most of the ultimate settlement of the underlying soft formation. Usually, a  
10 surcharge load equal to or greater than the expected foundation loading is applied to accel-  
11 erate consolidation with the aid of vertical drains. The application of vacuum pressure can  
12 reduce the amount of surcharge fill material required to obtain the same consolidation set-  
13 tlement because it generates suction, which increases the effective stress and accelerates  
14 consolidation.

15 Consolidation of soil is the process of decreasing the volume in saturated soils by  
16 expelling the pore water. Therefore, the consolidation rate is governed by the compress-  
17 ibility, permeability, and length of the drainage path. The settlement level is directly related  
18 to the void ratio change, which is directly proportional to the rate of dissipation of excess  
19 pore water pressure. For three decades, vertical drains with preloading have been used to  
20 accelerate the consolidation process before commencing construction.

21 Preloading on its own can reduce the total and differential settlement facilitating  
22 the choice of foundations, but when vertical drains are used with preloading, the settlement  
23 process can be accelerated considerably (Figure 1). The main advantages of vertical drains  
24 are: (i) to increase the shear strength of soil through a decreased void ratio and moisture con-  
25 tent; (ii) to decrease the time for preloading to minimize the same level of postconstruction  
26 settlement; (iii) to reduce differential settlement during primary consolidation; and (iv) to  
27 curtail the height of surcharge fill required to achieve desired precompression.

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## 30 **2. INSTALLATION AND MONITORING OF VERTICAL DRAINS**

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32 Before installing vertical drain it is essential that the site be prepared. This may involve  
33 removing surface vegetation and debris and grading the site for a sand blanket to act as a  
34 medium for expelling water from the drains and an appropriate working mat. The vertical  
35 drains can be installed by either the washing jet method, the static method or the dynamic  
36 method. The washing jet method is primarily used when installing large diameter sand  
37 drains, whereby sand is washed in through the jet pipe. PVDs are usually installed by the  
38 static or dynamic method (Figure 2). In the latter, the mandrel is driven into the ground with  
39 either a vibrating or drop hammer, but in the former, the mandrel is pushed into the soil by  
40 a static load. The static method usually causes less ground disturbances and is preferred for

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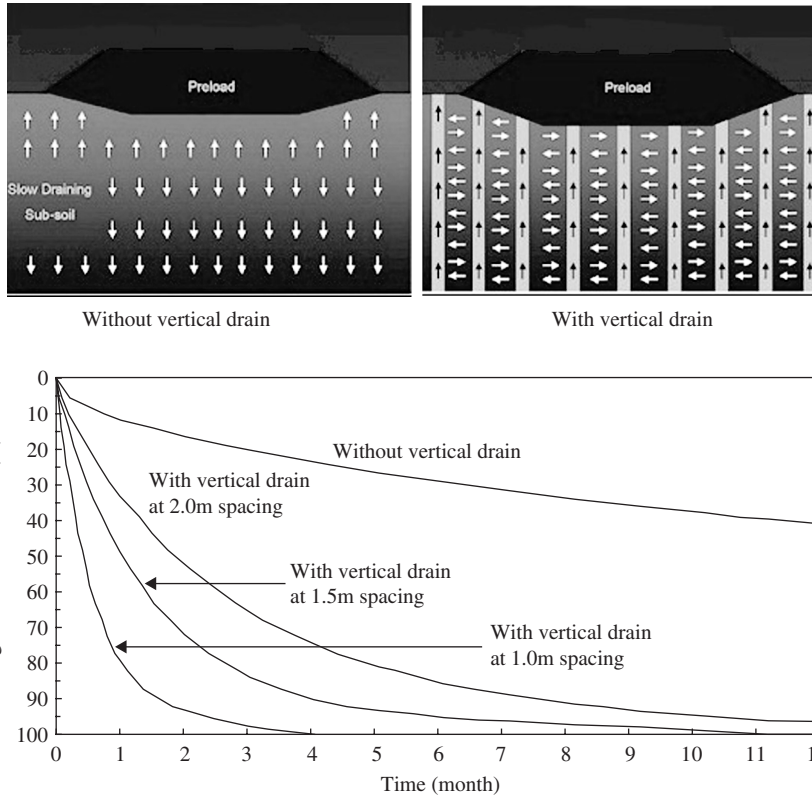


Figure 1. Potential benefit of vertical drains (adapted from Lau et al., 2000).

more sensitive soils. Although faster, the dynamic methods generate higher excess pore pressures and a greater disturbance of the soil around the mandrel during installation.

On major projects, instrumentation is essential for verifying performance and observing design amendments, as warranted, to prevent unacceptable displacement. Figure 3 shows a typical scheme of instruments required to monitor the performance of a soft clay foundation beneath an embankment containing PVD. The most commonly used instruments are inclinometers, settlement indicators, and piezometers, as described in the following section.

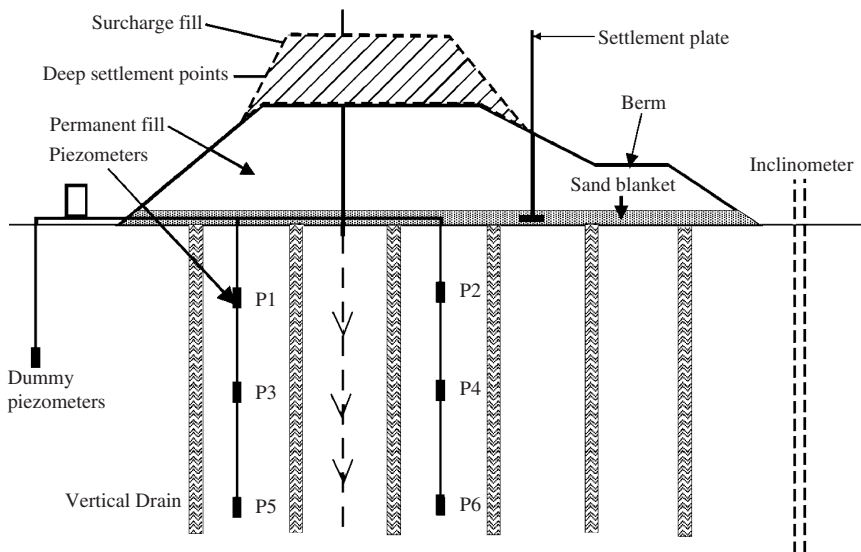
**2.1. Inclinometers**

These instruments are used to monitor the lateral (transverse) movements of natural slopes or embankments. An inclinometer casing has a grooved metal or plastic pipe that is placed into a borehole (Dunncliff, 1988). The space between the wall of the borehole and the casing is backfilled with a sand or gravel grout. The bottom of the pipe must rest on a firm base to achieve a stable point of fixity. To monitor embankment performance, inclinometers

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**Figure 2.** Typical installation rig (Source: Colbond bv, The Netherlands, <http://www.colbond-gepsynthetic.com>).



**Figure 3.** Basic instrumentation for a typical embankment (adapted from Rixner et al., 1986).

1 are normally placed at or near the toe of the embankment where excessive lateral movement  
2 is usually of some concern.

### 4 **2.2. Settlement indicators**

5 Settlement plates or points are commonly installed where significant settlement is pre-  
6 dicted (Dunnicliff, 1988) to record the magnitude and rate of settlement under a load.  
7 Therefore, they should be placed immediately after installing the vertical drains. In the  
8 simplest form, this instrument is a settlement plate consisting of a steel plate placed on the  
9 ground before construction of embankment. Surface settlement points measure vertical  
10 displacement with depth, for example, along an embankment centerline. Typically, a ref-  
11 erence rod and protecting pipe are attached to the settlement-monitoring platform.  
12 Settlement is often evaluated periodically until the surcharge embankment is completed,  
13 then at a reduced frequency, measuring the elevation of the top of the reference rod.  
14 Benchmarks used for reference datum must be stable and remote from all other possible  
15 vertical movements. Further information about settlement points is given elsewhere  
16 (Dunnicliff, 1988).

### 18 **2.3. Piezometers**

19 A detailed description and analysis of various types of piezometers to measure in situ pore  
20 water pressure are presented by Hanna (1985) and Dunnicliff (1988). Piezometers should  
21 be installed at the bottom of the sand blanket, at various intermediate depths within the  
22 compressible layer. A dummy piezometer is usually installed a sufficient distance away  
23 from the embankment to record natural groundwater level and excess pore water pressure  
24 at a given location is determined by comparison with the “dummy” level.

## 28 **3. DRAIN PROPERTIES**

### 29 **3.1. Diameter of influence zone**

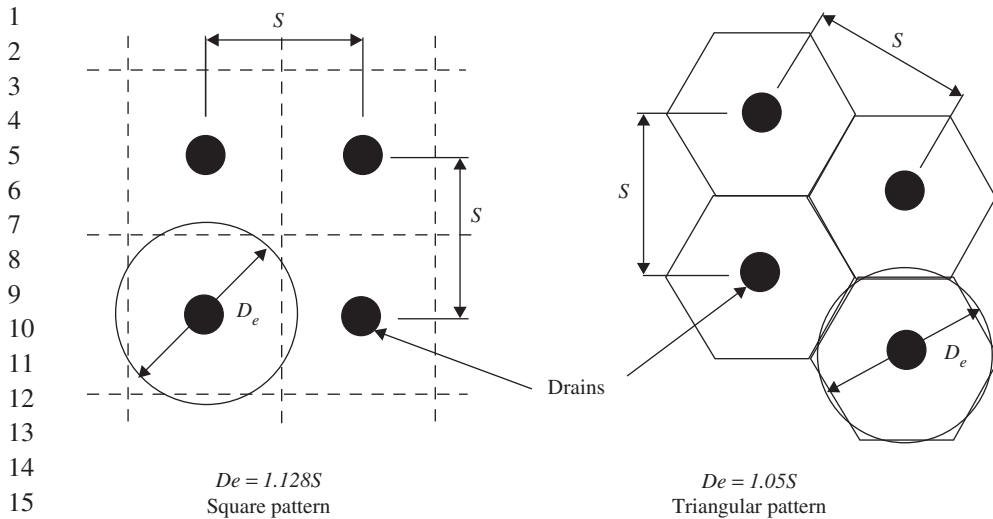
30 As shown in Figure 4, the equivalent diameter of the influence zone ( $D_e$ ) can be found in  
31 terms of the drain spacing ( $S$ ) as follows (Hansbo, 1981):

$$33 \quad D_e = 1.13S \quad \text{for drains installed in a square pattern} \quad (1)$$

35 and

$$37 \quad D_e = 1.05S \quad \text{for drains installed in a triangular pattern} \quad (2)$$

39 Drains in a square pattern may be easier to lay out and control during installation in the  
40 field but a triangular pattern usually provides a more uniform consolidation between them.



**Figure 4.** Typical drain installation patterns and the equivalent diameters (adapted from Barron, 1948 and Hansbo, 1981).

### 3.2. Equivalent drain diameter of band-shaped vertical drain

Most prefabricated drains have rectangular cross-section (band-shaped, Figure 5), but for design purposes, the rectangular (width  $a$ , thickness  $b$ ) section has to be converted into an equivalent circle with a diameter of  $d_w$ , because the conventional theory of radial consolidation assumes that drains are circular.

The following typical equation is used to determine the equivalent drain diameter:

$$d_w = 2(a + b)/\pi \quad (\text{Hansbo, 1979}) \quad (3)$$

Atkinson and Eldred (1981) proposed that a reduction factor of  $\pi/4$  should be applied to Eq. (3) to take account of the corner effect, where the flow lines rapidly converge. From the finite element studies, Rixner et al. (1986) proposed that

$$d_w = (a + b)/2 \quad (4)$$

Pradhan et al. (1993) suggested that the equivalent diameter of band-shaped drains should be estimated by considering the flow net around the soil cylinder of diameter  $d_c$  (Figure 6). The mean-square distance of their flow net is calculated as

$$s^{-2} = \frac{1}{4}d_c^2 + \frac{1}{12}a^2 - \frac{2a}{\pi^2}d_c \quad (5)$$

$$\text{Then, } d_w = d_c - 2\sqrt{(s^{-2})} + b \quad (6)$$

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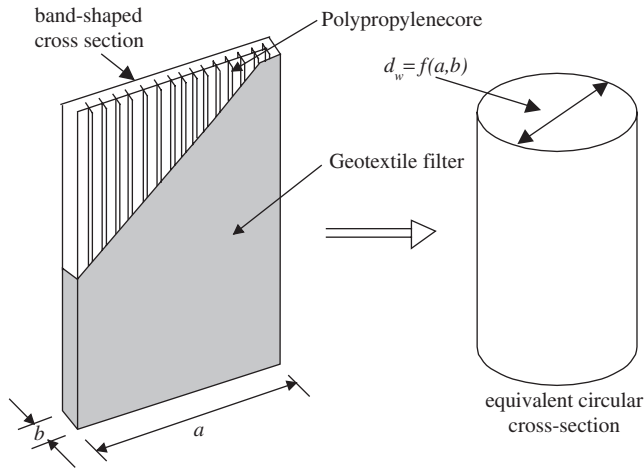


Figure 5. Conceptual drawing of a band-shaped PVD and equivalent diameter well.

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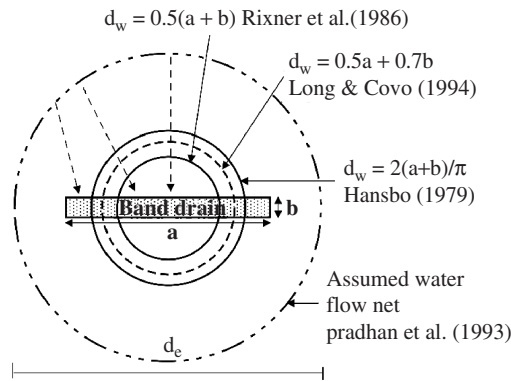


Figure 6. Equivalent diameters of band-shaped vertical drains.

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More recently, Long and Covo (1994) found that the equivalent diameter  $d_w$  could be computed using an electrical analogue field plotter:

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$$d_w = 0.5a + 0.7b \tag{7}$$

38 **3.3. Discharge capacity**

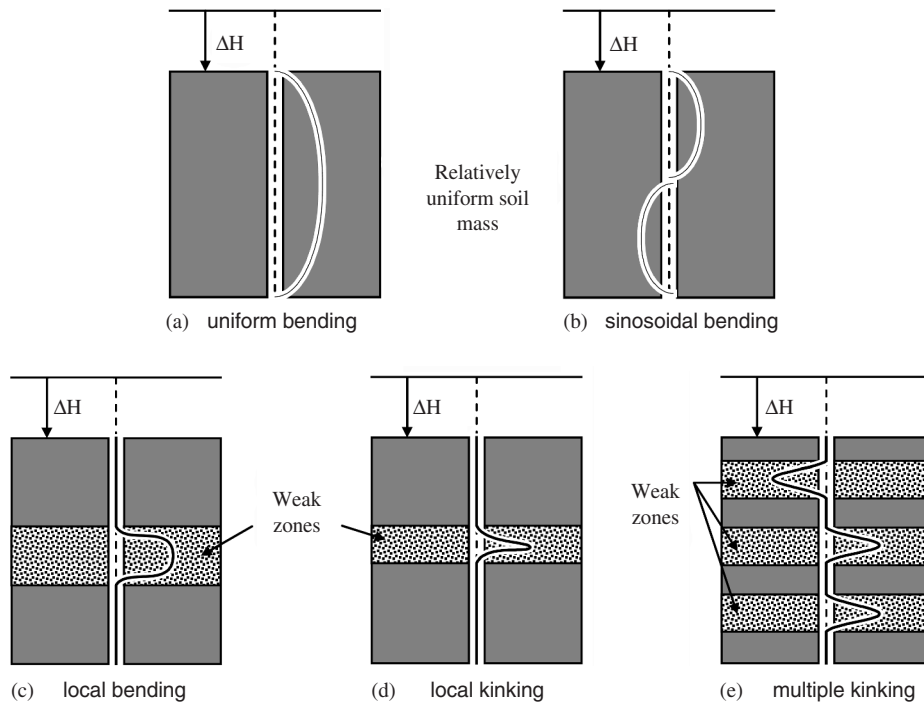
39 The discharge capacity is probably the most important parameter that controls the performance of prefabricated vertical drains. According to Holtz et al. (1991), the discharge

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1 capacity depends primarily on the following factors (Figure 7): (i) the area of the  
2 drain core available for flow; (ii) the effect of lateral Earth pressure; (iii) possible folding,  
3 bending and crimping of the drain; and (iv) infiltration of fine particles into the drain  
4 filter.

5 The current recommended values are given in Table 1 and the discharge capacities of  
6 various types of drains are shown in Figure 8 as a function of lateral confining pressure.



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**Figure 7.** Possible deformation modes of PVD (adapted after Holtz et al., 1991).

**Table 1.** Current recommended values for specification of discharge capacity

Source	Value	Lateral stress (kPa)
Kremer et al. (1982)	256	100
Kremer (1983)	790	15
Jamiolkowski et al. (1983)	10–15	300–500
Rixner et al. (1986)	100	Not given
Hansbo (1987)	50–100	Not given
Holtz et al. (1989)	100–150	300–500
de Jager and Oostveen (1990)	315–1580	150–300

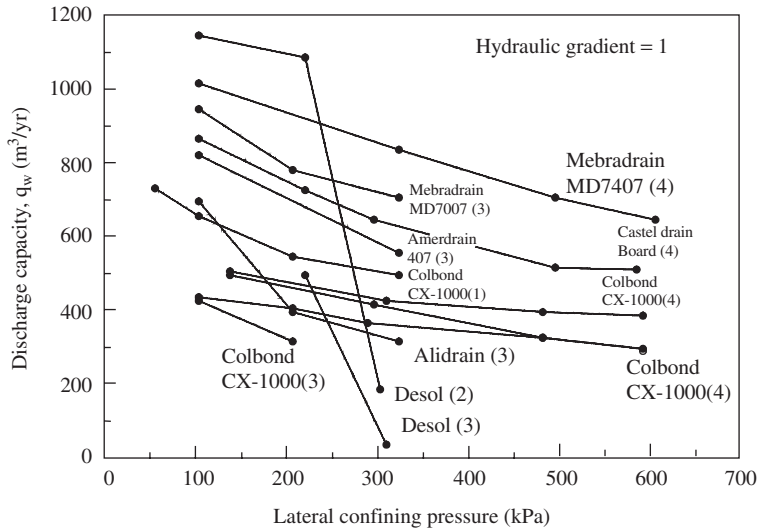


Figure 8. Typical values of vertical discharge capacity (data from Rixner et al., 1986).

#### 4. FACTORS INFLUENCING THE VERTICAL DRAIN EFFICIENCY

##### 4.1. Smear zone

In the field, vertical drains are installed using a steel mandrel, which is pushed into ground statically or dynamically then withdrawn, leaving the drain in the subsoil. This process causes significant remolding of the subsoil, especially in the immediate vicinity of the mandrel. The resulting smear zone will have reduced lateral permeability, which adversely affects consolidation process.

The combined effect of permeability and compressibility within the smear zone causes a different behavior from the undisturbed soil. Predicting soil behavior surrounding the drain requires an accurate estimation of the smear zone properties. In many classical solutions (Barron, 1948; Hansbo, 1981; Indraratna et al., 1997), the influence of the smear zone is considered with an idealized two-zone model.

Two parameters are necessary to characterize the smear effect, namely, the diameter of the smear zone ( $d_s$ ) and the permeability ratio ( $k_h/k_s$ ), i.e., the value in the undisturbed zone ( $k_h$ ) over the smear zone ( $k_s$ ). Both the diameter of the smear zone and its permeability are difficult to quantify and determine from laboratory tests, and so far, there is no comprehensive or standard method to measure them. The extent of the smear zone and its permeability vary with the installation procedure, size and shape of the mandrel, and the type and sensitivity of soil (macro fabric). Field and laboratory observations (Indraratna and Redana, 1998) indicated a continuous variation of soil permeability with the radial

1 distance away from the drain centreline. Also, the smear zone diameter ( $d_s$ ) has been the  
 2 subject of much discussion in literature dealing with PVD.

3 Investigations by Holtz and Holm (1973) and Akagi (1977) indicate that

$$4 \quad d_s = 2d_m \quad (8)$$

6 where  $d_m$  is the diameter of the circle with an area equal to the cross-sectional area of the  
 7 mandrel. Jamiolkowski et al. (1981) proposed that

$$10 \quad d_s = (2.5 - 3.0)d_m \quad (9)$$

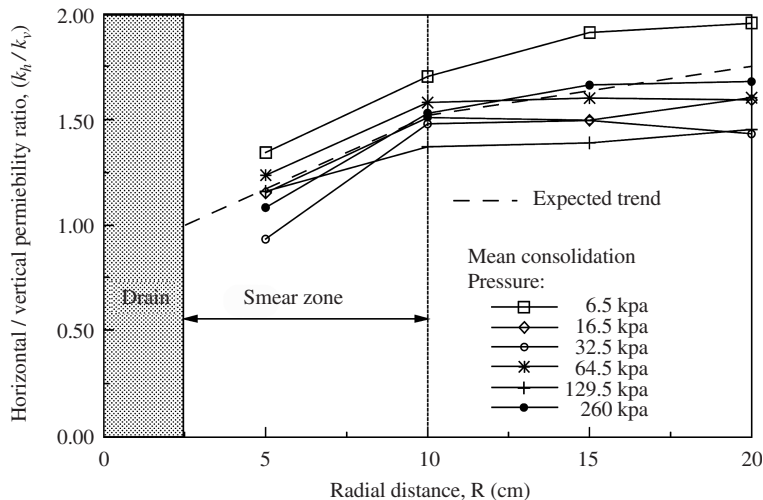
11 Hansbo (1981, 1997) proposed a different relationship as follows:

$$14 \quad d_s = (1.5 - 3.0)d_w \quad (10)$$

15 Based on laboratory study and backanalysis, Bergado et al. (1991) proposed that the fol-  
 16 lowing relation could be assumed:

$$19 \quad d_s = 2d_w \quad (11)$$

20 Indraratna and Redana (1998) proposed that the estimated smear zone could be as large as  
 21  $(4-5)d_w$ . This proposed relationship was verified using a specially designed large-scale  
 22 consolidometer (Indraratna and Redana, 1995). Figure 9 shows the variation of  $k_h/k_v$  ratio



40 **Figure 9.** Ratio of  $k_h/k_v$  along the radial distance from central drain (modified after Indraratna and Redana, 1998).

1 along the radial distance from the central drain. According to Hansbo (1987) and Bergado  
 2 et al. (1991), the  $k_h/k_v$  ratio was found to be close to unity in the smear zone. This agrees  
 3 with the study by Indraratna and Redana (1998). More recently, the primary author and his  
 4 co-workers at the University of Wollongong attempted to estimate the extent of the smear  
 5 zone caused by mandrel installation using the Cylindrical Cavity Expansion theory. They  
 6 used the modified Cam-clay (MCC) model and verified that the extent of smear zone pro-  
 7 posed by Indraratna and Redana (1998) was reasonable. Most recent results indicate that  
 8 for most soft clays the extent of the smear zone is between  $4d_w$  and  $6d_w$ . The recommended  
 9 smear zone parameters by different researchers are listed in Table 2.

#### 11 4.2. The effect of a sand mat

12 Part or all water ingress to drains will flow to the ground first and then drain out through  
 13 the sand mat. Since the hydraulic conductivity of sand is considerably higher than clay, it  
 14 can usually be assumed that there is no hydraulic resistance in the sand mat. However, in  
 15 some cases, depending on local materials, lower quality clayey sand may be used as a sand  
 16 mat. In these instances, the hydraulic resistance in the sand mat may influence the rate of  
 17 consolidation of the clay subsoil the amount which is a function of the hydraulic conduc-  
 18 tivity of the sand as well as the embankment geometry.

#### 20 4.3. Well resistance

21 Well resistance refers to the finite permeability of the vertical drain with respect to the soil.  
 22 Head loss occurs when water flows along the drain and delays radial consolidation. It  
 23 should be pointed out that well resistance is controlled not only by the discharge capacity  
 24 of the drain  $q_w$ , but also by the permeability of the soil  $k_h$ , the maximum discharge length  
 25  $l_m$  and any geometric deficiencies (bending, kinks, etc.) on the drains.

28 **Table 2.** Proposed smear zone parameters

Source	Extent	Permeability	Remarks
Barron (1948)	$r_s = 1.6r_m$	$k_h/k_s = 3$	Assumed
Hansbo (1979)	$r_s = 1.5 \sim 3r_m$	Open	Based on available literature at that time
Hansbo (1981)	$r_s = 1.5r_m$	$k_h/k_s = 3$	Assumed in case study
Bergado et al. (1991)	$r_s = 2r_m$	$k_h/k_v = 1$	Laboratory investigation and backanalysis for soft Bangkok clay
Onoue (1991)	$r_s = 1.6r_m$	$k_h/k_s = 3$	From test interpretation
Almeida et al. (1993)	$r_s = 1.5 \sim 2r_m$	$k_h/k_s = 3 \sim 6$	Based on experience
Indraratna et al. (1998)	$r_s = 4 \sim 5r_m$	$k_h/k_v = 1.15$	Laboratory investigation (for Sydney clay)
Chai and Miura (1999)	$r_s = 2 \sim 3r_m$	$k_h/k_s = C_f(k_h/k_s)$	$C_f$ the ratio between lab and field values
Hird et al. (2000)	$r_s = 1.6r_m$	$k_h/k_s = 3$	Recommend for design
Xiao (2000)	$r_s = 4r_m$	$k_h/k_s = 1.3$	Laboratory investigation (for Kaolin clay)

40  $r_s$  = radius of smear zone.

1 Mesri and Lo (1991) proposed the governing equation for vertical flow within the ver-  
 2 tical drain in terms of excess pore water pressure at soil–drain interface. Based on Mesri’s  
 3 equation, a well-resistance factor ( $R$ ) is defined as

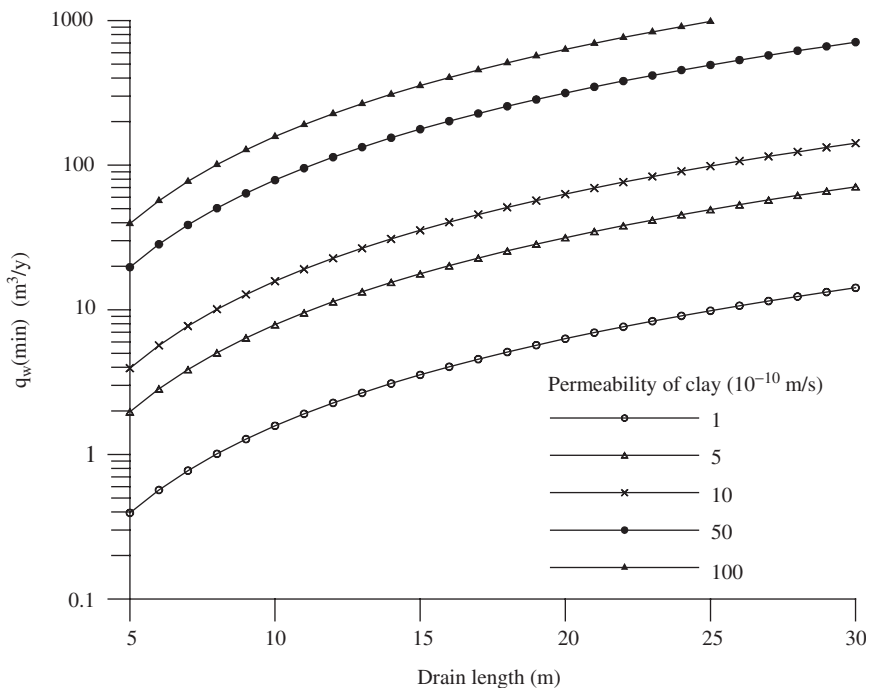
$$4 \quad R = \pi (k_w/k_h)(r_w/l_m)^2 = q_w/(k_h l_m^2) \quad (12)$$

6 Analysis of the field performance of vertical drains indicated that the well resistance is  
 7 negligible when  $R > 5$ . In other words, the minimum discharge capacity  $q_{w(\min)}$  of vertical  
 8 drains required for negligible well resistance may be determined from

$$10 \quad q_{w(\min)} = 5k_h l_m^2 \quad (13)$$

11 The above relationship is illustrated in Figure 10 for most typical values of  $k_h$  and  $l_m$ .

12 Table 3 summarizes the well-resistance indices proposed by various investigators to  
 13 evaluate the influence of finite discharge capacity of vertical drains. Note that the proposed  
 14 indices are also transformed to the well-resistance factor ( $R$ ) proposed by Mesri and Lo  
 15 (1991). It can be seen that these indices depend on  $R$ , except for the expression proposed  
 16 by Aboshi and Yoshikuni (1967) and Stamatopoulos and Kotzias (1985), in which the drain  
 17 spacing is also included.



36 **Figure 10.** Minimum discharge capacity required (based on Eq. (13)).

**Table 3.** Summary of proposed well-resistance indices

Source	Index of well resistance
Aboshi and Yoshikuni (1967)	$R_i = \frac{(n^2 - 1)}{4F(n)n^2} \frac{k_h}{k_w} \left( \frac{l_m}{r_w} \right)^2 = \frac{\pi(n^2 - 1)}{4F(n)n^2} \frac{1}{R}$
Yoshikuni and Nakanodo (1974), Onoue (1988)	$L = \frac{8}{\pi^2} \frac{k_h}{k_w} \left( \frac{l_m}{r_w} \right)^2 = \frac{8}{\pi} \frac{1}{R}$
Hansbo (1981)	$W = 2 \frac{k_h}{k_w} \left( \frac{l_m}{r_w} \right)^2 = 2\pi \frac{1}{R}$
Stamatopoulos and Kotzias (1985)	$R_i = \frac{1}{F(n)} \frac{k_h}{k_w} \left( \frac{l_m}{r_w} \right)^2 = \frac{\pi}{F(n)} \frac{1}{R}$
Zeng and Xie, (1989)	$G = \frac{1}{4} \frac{k_h}{k_w} \left( \frac{l_m}{r_w} \right)^2 = \frac{\pi}{4} \frac{1}{R}$
Mesri and Lo (1991)	$R = \pi \frac{k_w}{k_h} \left( \frac{r_w}{l_m} \right)^2 = \frac{q_w}{k_h l_m^2}$

$n = D_e/d_w$  is the spacing ratio.

In general, laboratory and field data indicate that the discharge capacities of most commercial PVDs have little influence on the consolidation rate of clay, especially for drains that are not too long (Indraratna et al., 1994). For values of  $q_w > 100\text{--}150$  m<sup>3</sup>/year (in the field) and where drains are shorter than 30 m, there should be no significant increase in the consolidation time. Given these circumstances, it may be claimed that for commercial PVDs, well resistance is usually negligible in most practical situations unless the drains are excessively long and geometric deficiencies occur during installation (bending, kinks, etc). In most soft clays, well resistance can be ignored for PVD < 15 m long.

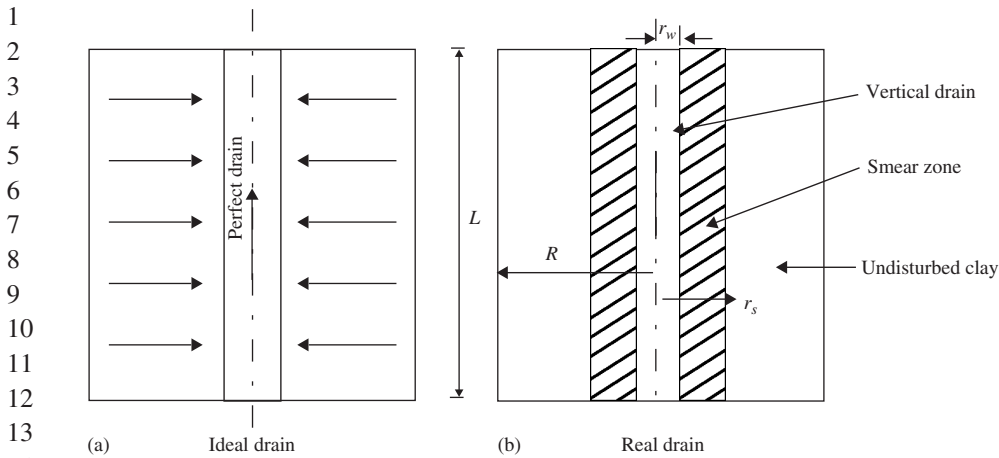
## 5. DEVELOPMENT OF VERTICAL DRAIN THEORY

Analytical solutions already developed for consolidation of ground improved with vertical drains invariably employ the “unit cell” model, as illustrated in Figure 11. The theory for radial drainage consolidation has been addressed by many researchers (Rendulic, 1936; Carrillo, 1942; Barron, 1948; Yoshikuni and Nakanode, 1974; Hansbo, 1981; Onoue, 1988; Zeng and Xie, 1989).

### 5.1. Rendulic and Carillo diffusion theory

Rendulic (1936) formulated and solved the differential equation for one-dimensional vertical compression by radial flows

$$\frac{\partial u}{\partial t} = c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (14)$$



**Figure 11.** Unit-cell model of a drain surrounding by soil cylinder (after Barron, 1948).

where  $r$  is the coordinate and  $c_h$  the horizontal coefficient of consolidation ( $k_h/\gamma_w m_v$ ).

Carillo (1942) showed that for radial drainage and associated 1-D consolidation, the excess pore water pressure  $u_{r,z}$  is given by

$$\frac{\partial u}{\partial t} = c_h \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + c_v \frac{\partial^2 u}{\partial z^2} \quad (15)$$

$$u_{r,z} = u_r u_z / u_0 \quad (16)$$

where,  $u_r$  and  $u_z$  are the excess pore water pressure due to radial flow and vertical flow only, and  $u_0$  is the initial pore water pressure. By substituting the average excess pore water pressure into Eq. (16), the average degree of consolidation of the compressible stratum can be obtained by combining  $U_z$  and  $U_r$ , hence

$$(1 - \bar{U}) = (1 - \bar{U}_z)(1 - \bar{U}_r) \quad (17)$$

where  $\bar{U}$  is the average degree of consolidation of the clay at time  $t$  for combined vertical and radial flow, and  $\bar{U}_z$  and  $\bar{U}_r$  are the average degree of consolidation at time  $t$  for vertical and radial flow, respectively. It should be noted that both Rendulic and Carillo's solutions are for "ideal" drains only (infinite discharge capacity with no smear zone).

### 5.2. Barron's (1948) 'equal strain' rigorous solution

Barron (1948) addressed the smear and well-resistance effects that can decrease the performance of vertical drains. He presented closed-form solutions for two extreme cases for

1 radial drainage-induced consolidation, namely, “free strain” and “equal strain”, and showed  
 2 that the average consolidation obtained in both cases is almost the same.

3 The “free strain” hypothesis assumes that the load is uniform over a circular zone of influ-  
 4 ence for each vertical drain. The differential settlements occurring over this zone do not affect  
 5 the redistribution of stresses caused by the fill load arching. In contrast, the “equal strain”  
 6 assumes that arching occurs in the upper layer during consolidation without any differential  
 7 settlement in the clay layer, which is what its simplicity is commonly used by researchers.

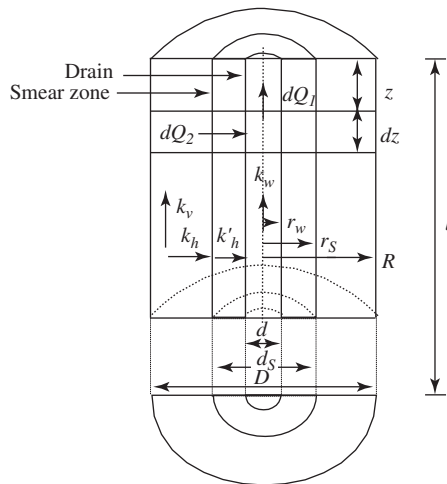
8 Figure 12 shows the schematic illustration of a soil cylinder with a central vertical  
 9 drain, where  $r_w$  is the radius of the drain,  $r_s$  is the radius of smear zone,  $R$  is the radius of  
 10 soil cylinder and  $l$  is the length of the drain installed into soft ground. The coefficient of  
 11 permeability in the vertical and horizontal directions are  $k_v$  and  $k_h$ , respectively, and  $k'_h$  is  
 12 the coefficient permeability in the smear zone. Based on “equal strain”, Barron (1948) pro-  
 13 posed a solution to Eq. (14) taking the smear effect into account as

$$u_r = \bar{u} \frac{1}{v} \left[ \ln\left(\frac{r}{r_s}\right) - \frac{(r^2 - r_s^2)}{2R^2} + \frac{k_h}{k'_h} \left(\frac{n^2 - s^2}{n^2}\right) \ln(s) \right] \quad (18)$$

14 where the smear factor  $v$  is given by

$$v = F(n, s, k_h, k'_h) = \left[ \frac{n^2}{n^2 - s^2} \ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{s^2}{4n^2} + \frac{k_h}{k'_h} \left(\frac{n^2 - s^2}{n^2}\right) \ln(s) \right] \quad (19)$$

21 and  $\bar{u} = u_0 \exp(-8T_h/v)$  (20)



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40 **Figure 12.** Schematic of soil cylinder with vertical drain (adapted from Hansbo, 1997).



1 In the above expression,  $s = r_s/r_w$ ,  $n=R/r_w$ ,  $T_h$  is the time factor given by  $T_h = c_h t/4R^2$ ,  $\bar{u}$   
 2 the average excess pore water pressure, and  $u_0$  the initial excess pore water pressure.

3 The average degree of consolidation,  $\bar{U}_r$ , in the soil body is given by

$$4 \quad \bar{U}_r = 1 - \exp\left(-\frac{8T_h}{v}\right) \quad (21)$$

### 7 5.3. Hansbo (1981) – Analysis with smear and well resistance

8 Hansbo (1981) presented an approximate solution for vertical drain based on the “equal  
 9 strain” by taking both smear and well resistance into consideration. The  $\bar{U}_r$ , presented by  
 10 Hansbo (1981), can be expressed as

$$12 \quad \bar{U}_r = 1 - \exp(-8T_h/\mu) \quad (22)$$

14 where upon ignoring the insignificance terms, gives

$$16 \quad \mu = \ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right)\ln(s) - 0.75 + \pi z(2l - z)\frac{k_h}{q_w} \quad (23)$$

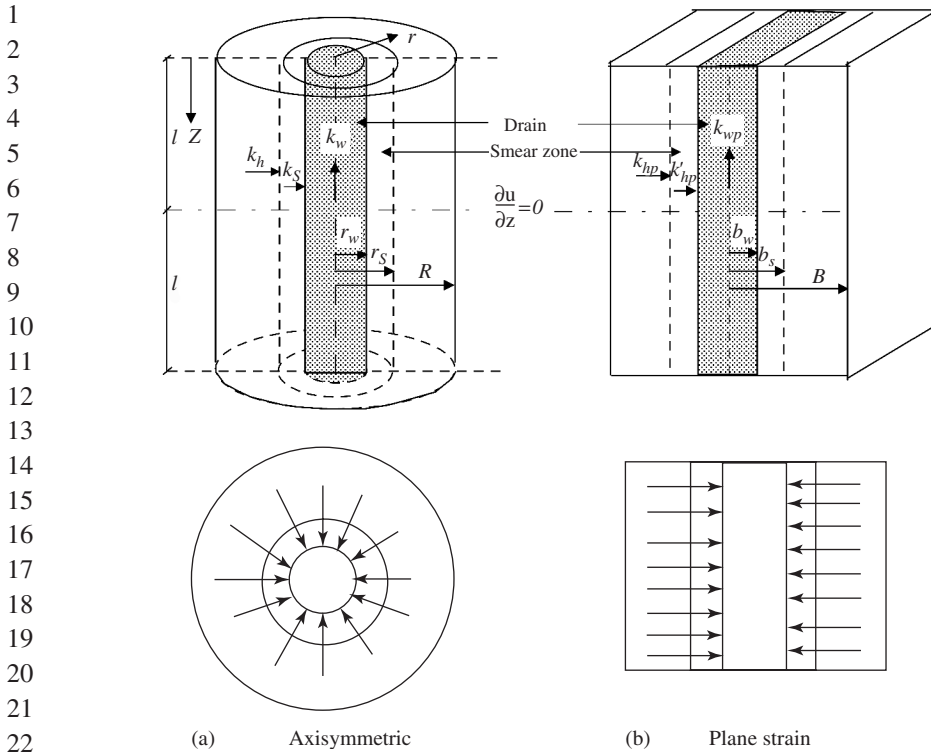
## 20 6. 2-D MODELLING OF VERTICAL DRAINS

22 Even though each vertical drain is axisymmetric, finite element analyses dealing with mul-  
 23 tidrain embankments have commonly been conducted under “plane strain” conditions for  
 24 optimizing computational efficiency. Therefore, to employ a realistic 2-D plane strain analy-  
 25 sis for vertical drains, the appropriate equivalence between the plane strain and axisymmet-  
 26 ric analysis needs to be established in terms of consolidation settlement. Figure 13 shows  
 27 the conversion of an axisymmetric vertical drain into an equivalent drain wall. This can be  
 28 achieved in several ways (Hird et al., 1992; Indraratna and Redana, 1997), for example: (i)  
 29 geometric matching – the drain spacing is matched while maintaining the same perme-  
 30 ability coefficient; (ii) permeability matching – coefficient of permeability is matched  
 31 while keeping the same drain spacing; and (iii) combination of (i) and (ii), with the plane  
 32 strain permeability calculated for a convenient drain spacing.

### 34 6.1. Shinsha et al. (1982) – permeability transformation

35 Shinsha et al. (1982) first proposed an acceptable matching criterion for converting the  
 36 permeability coefficients. The equivalent coefficient of permeability was calculated on the  
 37 assumption that the required time for a 50% degree of consolidation in both schemes was  
 38 the same, giving

$$40 \quad k_{pl}/k_{ax} = (B/D_c)^2 T_{h50}/T_{r50} \quad (24)$$



**Figure 13.** Conversion of an axisymmetric unit cell into plane strain condition (adapted from Hird et al., 1992 and Indraratna and Redana, 1997).

where  $T_{h50}=0.197$  is a dimensionless time factor at 50% consolidation of laminar flow and  $T_{r50}$  the corresponding radial flow.

**6.2. Hird et al. (1992) – geometry and permeability matching**

By adapting Hansbo’s (1981) theory for the plane strain case, Hird et al. (1992) showed that the average degrees of consolidation  $U$ , at any depth and time in the two unit cells were theoretically identical if well resistance was ignored:

$$\frac{k_{pl}}{k_{ax}} = \frac{2B^2}{3R^2 \left[ \ln\left(\frac{R}{r_s}\right) + \left(\frac{k_{ax}}{k_s}\right) \ln\left(\frac{r_s}{r_w}\right) - \frac{3}{4} \right]} \tag{25}$$

where subscripts “ax” and “pl” represent the axisymmetric and plane strain conditions, respectively. Note that the geometric matching is achieved by substituting  $k_{pl}=k_{ax}=k_h$  in

Eq. (25), whereas the permeability matching is obtained by substituting  $B=R$ . For incorporating well resistance, the following dimensionless expression can be used:

$$Q_w/q_w = 2B/\pi R^2 \quad (26)$$

### 6.3. Bergado and Long (1994) – equal discharge concept

The converted permeability, including smear effect, is introduced based on the condition of the equal discharge rate in both schemes and on the assumption that the coefficient of permeability is independent of the seepage state:

$$\frac{k_{pl}}{k_{ax}} = \frac{\pi D(1 - a_s)}{2S \left[ \ln\left(\frac{\alpha D}{d_s}\right) + \left(\frac{k_{ax}}{k_s}\right) \ln\left(\frac{d_s}{d_w}\right) \right]} \quad (27)$$

where  $a_s = t/D$ ,  $t$  is the thickness of the walls in 2-D model,  $D$  and  $S$  are the row spacing and pile spacing of the actual case, respectively,  $\alpha = D_c/D$ ,  $S = D$  and  $\alpha = 1.05$  for square pattern,  $S = 0.866D$  and  $\alpha = 1.13$  for triangular pattern.

### 6.4. Chai et al. (1995) – well resistance and clogging

Chai et al. (1995) successfully extended the analysis by Hird et al. (1992) to include the effect of well resistance and clogging. In this approach, the discharge capacity of the drain in plane strain ( $q_{wp}$ ) for matching the average degree of horizontal consolidation is given by

$$q_{wp} = \frac{4k_h l^2}{3B \left[ \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} \ln(s) - \frac{17}{12} + \frac{2l^2 \pi k_h}{3q_{wa}} \right]} \quad (28)$$

### 6.5. Kim and Lee (1997) – Time factor analysis

They assume that the time durations for the two systems (plane strain and axisymmetric) to achieve a 50% and 90% degree of consolidation are the same. Then, the following simple expression is obtained:

$$\frac{k_{pl}}{k_{ax}} = \left(\frac{B}{R}\right)^3 \frac{T_{r50}}{T_{h50}} \frac{T_{r90}}{T_{h90}} \frac{S}{\pi d_w} \quad (29)$$

### 6.6. Indraratna and Redana (1997) – Rigorous solution for parallel drain wall

Indraratna and Redana (1997) converted the vertical drain system shown in Figure 13 into an equivalent parallel drain wall by adjusting the coefficient of soil permeability. They assumed that the half-widths of unit cell  $B$ , of drains  $b_w$ , and of smear zone  $b_s$  are the same

1 as their axisymmetric radii  $R$ ,  $r_w$  and  $r_s$ , respectively. The equivalent permeability of the  
2 model is then determined by

$$3 \quad k_{hp} = \frac{k_h \left[ \alpha + (\beta) \frac{k_{hp}}{k'_{hp}} + (\theta)(2lz - z^2) \right]}{4 \quad \left[ \ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) \ln(s) - 0.75 + \pi(2lz - z^2) \frac{k_h}{q_w} \right]} \quad (30)$$

5  
6  
7  
8 The associated geometric parameters  $\alpha$ ,  $\beta$  and the flow term  $\theta$  are given by

$$9 \quad \alpha = \frac{2}{3} \frac{(n-s)^3}{(n-1)n^2} \quad (31a)$$

$$10 \quad \beta = \frac{2}{3} \frac{(s-1)}{(n-1)n^2} [3n(n-s-1) + (s^2 + s + 1)] \quad (31b)$$

11  
12 and

$$13 \quad \theta = \frac{2k_{hp}}{Bq_z} \left( 1 - \frac{1}{n} \right) \quad (31c)$$

14  
15 where  $q_z = 2q_w/\pi B$  is the equivalent plane strain discharge capacity.

16 In Eq. (30), as  $k_{hp}$  appears on both sides of the equation the solution is obtained by iteration  
17 with an initially assumed  $k_{hp}/k'_{hp}$  ratio.

18 To verify the above model a finite element analysis was undertaken for both axisymmetric  
19 and equivalent plane strain models. As an example, a unit drain was analyzed with a drain  
20 installed to a depth of 5 m below the ground surface at 1.2 m spacing. The model parameters  
21 and soil properties were  $r_w = 0.03$  m,  $r_m = 0.05$  m,  $k_h = 1 \times 10^{-8}$  m/s,  $k'_h = 5 \times$   
22  $10^{-9}$  m/s, and the corresponding equivalent coefficients of plane strain permeability were  
23  $k'_{hp} = 5.02 \times 10^{-10}$  m/s, and  $k_{hp} = 2.97 \times 10^{-9}$  m/s based on Eq. (30). The water table was  
24 assumed to be at the surface and  $r_s = 5r_m$  (based on experimental results). For the elasto-plastic  
25 finite element analysis, MCC model was used as follows:  $\lambda = 0.2$ ,  $\kappa = 0.04$ ,  $M = 1.0$ ,  
26  $e_{cs} = 2$  and Poisson's ratio  $\nu = 0.25$ , with a saturated unit weight of 18 kN/m<sup>3</sup>.

27 The results of both axisymmetric and plane strain analysis are plotted in Figure 14. The  
28 average degree of radial consolidation  $U_h$  (%) is plotted against the time factor  $T_h$  for perfect  
29 drain conditions. As illustrated, the proposed plane strain analysis agree perfectly with  
30 the axisymmetric analysis, with the maximum deviation between the two methods being  
31 less than 5%.

32 Figures 15 and 16 illustrate the settlements and excess pore pressure variations over  
33 time, including smear plus well resistance for a single drain and again the axisymmetric  
34 model and the equivalent plane strain model agreed.

35 Based on the above single drain analysis, Figures 14–16 provide concrete evidence that  
36 the equivalent (converted) plane strain model is an excellent substitute for the axisymmetric  
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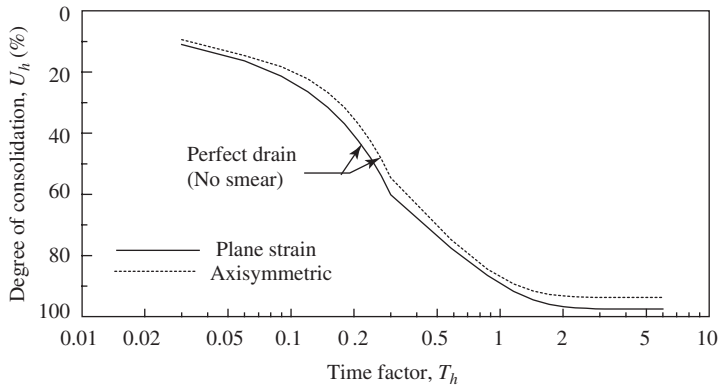


Figure 14. Average degree of consolidation versus time factor (modified after Indraratna et al., 2000).

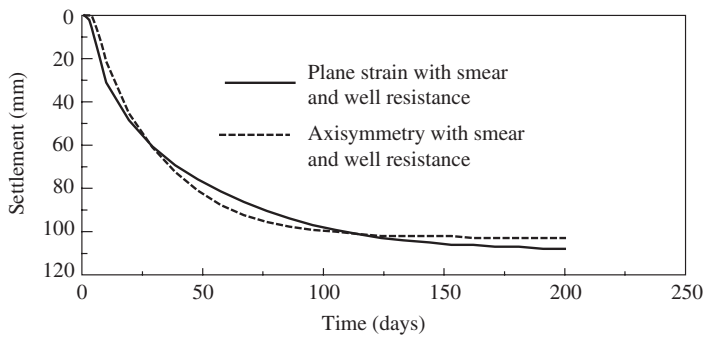


Figure 15. Comparison of the surface settlement (modified after Indraratna et al., 2000).

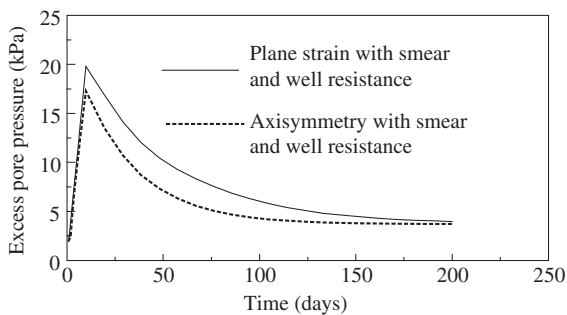


Figure 16. Comparison of the excess pore pressure (modified after Indraratna et al., 2000).

1 model. In finite element modelling, the 2-D plane strain analysis is expected to reduce com-  
 2 putational time considerably compared to the time taken by a 3-D, axisymmetric model,  
 3 especially in multidrain analysis.

## 6 7. SIMPLE 1-D MODELLING OF VERTICAL DRAINS

7  
 8 A simple approximate method for modelling the effect of PVD is proposed by Chai et al.  
 9 (2001). Because PVD increases the mass permeability of subsoil in the vertical direction,  
 10 it is logical to establish a value for vertical permeability which approximately represents  
 11 the effect of vertical drainage of natural subsoil and radial permeability toward the PVD.  
 12 This equivalent vertical permeability ( $k_{ve}$ ) was derived from an equal average degree of  
 13 consolidation under the 1-D condition. To obtain a simple expression for  $k_{ve}$ , an approxi-  
 14 mation equation for consolidation in the vertical direction was proposed:

$$15 \quad U_v = 1 - \exp(-C_d T_v) \quad (32)$$

16  
 17 where  $T_v$  is the time factor for vertical consolidation, and  $C_d = \text{constant} = 3.54$ . The  
 18 equivalent vertical permeability,  $k_{ve}$ , can be expressed as:

$$19 \quad k_{ve} = \left( 1 + \frac{2.5l^2 k_h}{\mu D_e^2 k_v} \right) k_v \quad (33)$$

20  
 21 where  $l$  is drain length,  $D_e$  the equivalent diameter of unit cell, and

$$22 \quad \mu = \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} \ln(s) - \frac{3}{4} + \frac{\pi 2l^2 k_h}{3q_w} \quad (34)$$

## 23 24 25 26 27 28 8. A FINITE ELEMENT MODEL PERSPECTIVE FOR GENERAL DESIGN

29  
 30 Finite element analysis is an important tool for current design processes (Potts and  
 31 Zdravkovic, 2000). In this section, selected numerical studies have been carried out to  
 32 study how the embankment slope, construction rate, and drain spacing affect the failure of  
 33 a soft clay foundation using the finite element code ABAQUS. The subsoil profiles are  
 34 given in Tables 4 and 5 and the plane strain permeability coefficients are calculated using  
 35 Eq. (30).  
 36

### 37 8.1. Element types for soil and soil-drain interface

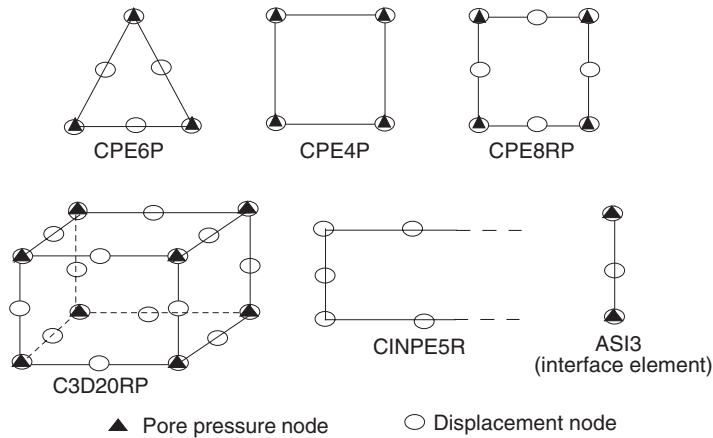
38 The types of elements used in consolidation analysis in the finite element code ABAQUS  
 39 are shown in Figure 17. The basic element type is a four-node bilinear displacement and  
 40 pore pressure element (CPE4P) consisting of four displacement and pore pressure nodes at

**Table 4.** Soil properties used in finite element analysis

Depth (m)	Soil type	$k_h$ ( $10^{-9}$ m/s)	$e_o$	$\lambda$	$\kappa$	$\nu$	$M$	$\gamma$ (kN/m <sup>3</sup> )
0–2.0	Weathered clay	30.1	1.8	0.3	0.03	0.3	1.2	16.0
2.0–8.5	Very soft clay	12.7	2.8	0.73	0.08	0.3	1.0	14.5
8.5–10.5	Soft clay	6.02	2.4	0.5	0.05	0.25	1.2	15.0
10.5–13.0	Medium clay	2.55	1.8	0.3	0.03	0.25	1.4	16.0
13.0–18.0	Stiff clay	0.60	1.2	0.1	0.01	0.25	1.4	18.0

**Table 5.** In situ stress condition used in finite element analysis

Depth (m)	$\sigma'_{ho}$ (kPa)	$\sigma'_{vo}$ (kPa)	$u$ (kPa)
0	5	5	–5
0.5	8	8	0
2	11	11	15
8.5	28	39.75	80
10.5	35	49.75	100
13.0	49	64.75	125
15.0	57	80.75	145



**Figure 17.** Types of elements used in consolidation analysis (Hibbitt, K. & Sorensen 2004).

the corners. The higher order of this element is a 20-node tri-quadratic displacement and tri-linear pore pressure nodes with reduced integration (C3D20RP), which contain 20 displacement nodes and eight pore pressure nodes. As explained by Hibbitt, K. & Sorensen (2004) reduced integration elements use a lower order of integration to form element stiffness. ABAQUS recommends using reduced integration elements because it usually gives more accurate results and is less time consuming than full integration. The common element

1 type used in the analysis presented here is the CPE8RP element, which contains eight dis-  
2 placement nodes and four pore pressure nodes.

3 Interface elements are most appropriate to simulate soil–drain interaction. Since the  
4 thickness of PVD is relatively small compared to its spacing, the interface element is  
5 envisaged as the soil element having properties similar to the adjacent soil except for per-  
6 meability. A three-node interface element (ASI3) is shown in Figure 17, where there are  
7 two pore pressure nodes at the ends.

8 In finite element analysis, the pore pressure shape function is usually one order less  
9 than the displacement shape function. In most of the elements shown in Figure 17, the pore  
10 pressure shape function is linear, while the displacement shape functions are either quad-  
11 ratic or cubic.

12 Figure 18 presents a typical discretized finite element mesh, which is used for numer-  
13 ical analysis, where only one-half of the embankment is considered by symmetry. A foun-  
14 dation depth of 15 m was considered sufficient for analysis, assuming the existence of a  
15 stiff clay layer beneath this depth. The mesh consists of more than 1000 CPE8RP elements  
16 and the vertical drains are modelled by an interface element (ASI3). A finer mesh was  
17 employed for the zone beneath the embankment, with a half-width of 20 m. The embank-  
18 ment loading is simulated by applying incremental vertical loads.

19

## 20 8.2. Embankment constructed on soft clay without any improvement

21 **8.2.1. Effect of the slope of the embankment.** To illustrate the effect of embankment  
22 slope on foundation failure, two plane strain finite element analyses were conducted using  
23 the finite element mesh shown in Figure 18. Two slopes are considered here, 2:1 and 3:1  
24 (horizontal:vertical), and the loading is simulated by a constant rate of 0.1 m/week.  
25 Failure is identified when the solution fails to converge and displacement continues to  
26 increase without any further load added.

27 Figure 19 shows the predicted heave at the toe of the embankment based on the two  
28 models. A measurable change in settlement rate close to failure is observed and, finally,  
29 settlement increases without having to increase the embankment height. The decrease in  
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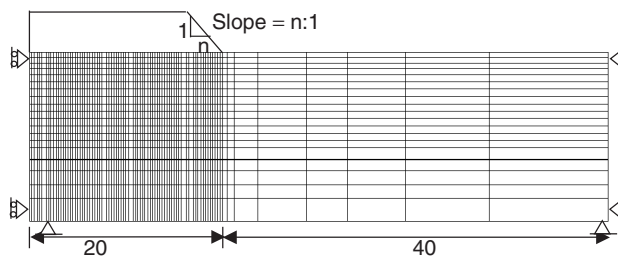


Figure 18. Finite element mesh.



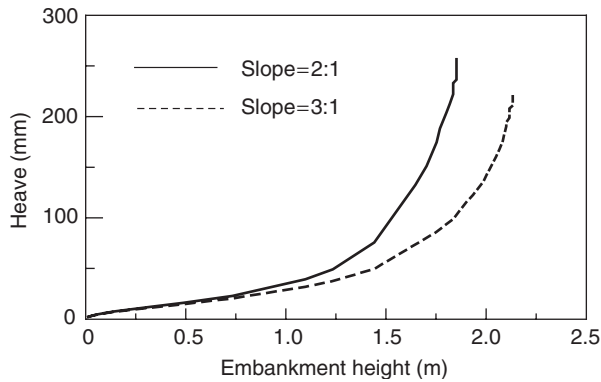


Figure 19. Heave at the toe of the embankment.

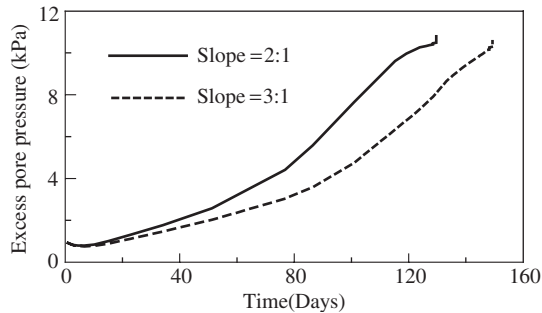


Figure 20. Excess pore pressure distribution 2 m below the embankment toe.

embankment slope has the effect of increasing the embankment height at failure from 1.8 to 2.1 m. Figure 20 presents excess pore pressure distribution at 2 m below ground level at the embankment toe. As expected, excess pore pressure increment is not gradual and a sudden increase is observed because the point considered here is located within the expected failure zone. Predicted surface settlement profile at failure based on these two models is presented in Figure 21.

**8.2.2. Effect of loading rate of the embankment.** To study the effect of construction rate of the embankment on failure height, plane strain finite element analysis was conducted for the two different construction rates, 0.1 m/week and 0.35 m/week, for an embankment slope of 3:1. The predicted heave at the toe of the embankment is shown in Figure 22. The slow rate of construction permits a greater embankment height at failure,

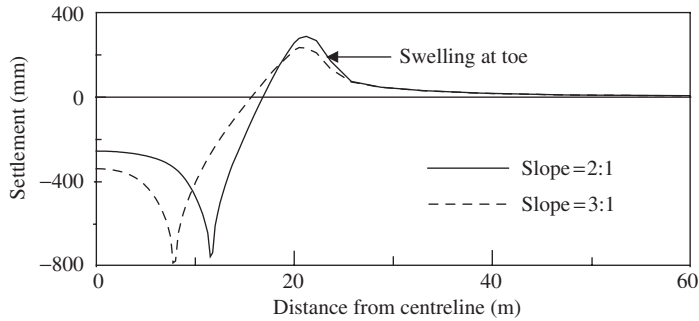


Figure 21. Surface settlement profile at failure.

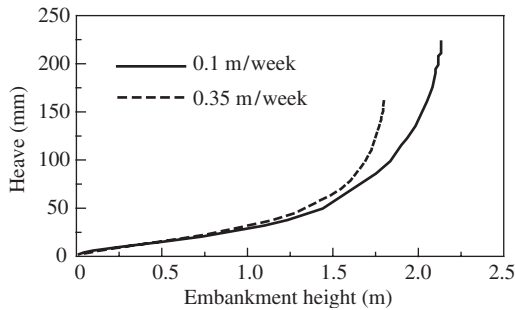


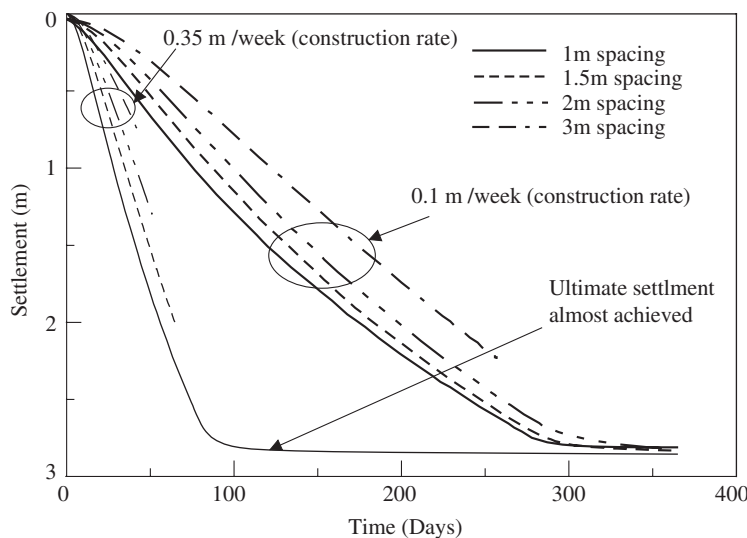
Figure 22. Heave at the toe of embankment.

because this gradual rate of construction allows the soft clay to gain shear strength upon pore pressure dissipation.

### 8.3. Influence of drain spacing

To investigate the effect of vertical drains on embankment stability, four different drain spacings were considered in the analysis; 1, 1.5, 2 and 3 m. The embankment is raised to a maximum height of 4 m, with two different construction rates, 0.1 m/week and 0.35 m/week. The slope of the embankment is assumed to be 2:1.

Figures 23 and 24 show the predicted surface settlement at the centerline and toe of the embankment, respectively. For a construction rate of 0.1 m/week, impending failure is not noticed for a small drain spacing of up to 2 m, which suggests that the higher dissipation of pore pressure and slower construction rate allow the soft clay foundation to gain sufficient strength to support a 4 m high embankment. If the construction rate is increased to 0.35 m/week, the foundation stabilized with PVD at 1 m spacing reaches its ultimate settlement within a shorter period (100 days) compared to a construction rate of 0.1 m/week



**Figure 23.** Predicted centerline surface settlement for different drain spacing.

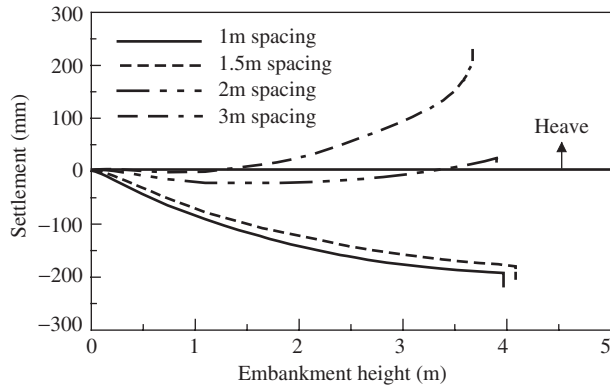
(300 days). It is not possible to reach the final embankment height of 4 m if the drain spacing is 1.5, 2 or 3 m (Figure 24b).

## 9. PERFORMANCE OF TEST EMBANKMENTS CONSTRUCTED ON SOFT MARINE CLAY IN MALAYSIA

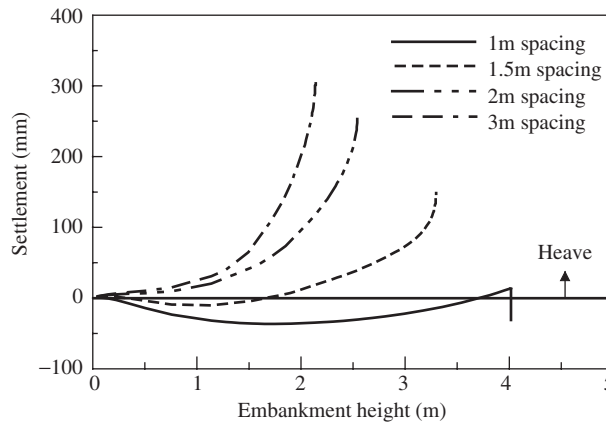
To study the performance and cost effectiveness of various ground improvement methods, in 1986 the Malaysian Highway Authority constructed a series of 15 trial embankments in Muar clay with nine different ground improvement techniques. The site of the test embankment is about 500 km east of Malacca on the southwest coast of Malaysia (Figure 25). The finite element program ABAQUS is used to predict the behavior of two of these embankments; one without any foundation improvement (i.e. north of embankment #1 in Figure 25), the other with geosynthetic vertical drains (PVD) at 1.3 m spacing installed in a triangular pattern (i.e. #14 in Figure 25).

### 9.1. Embankment constructed to failure

This embankment is shown in Figure 25, and is located just north of embankment #1. The cross-section of embankment showing the key instruments with subsoil variation, and the discretized finite element mesh, are shown in Figures 26 and 27, respectively. The piezometers P5, P6 and the inclinometers I3, I4 are used to monitor embankment failure. The



(a)

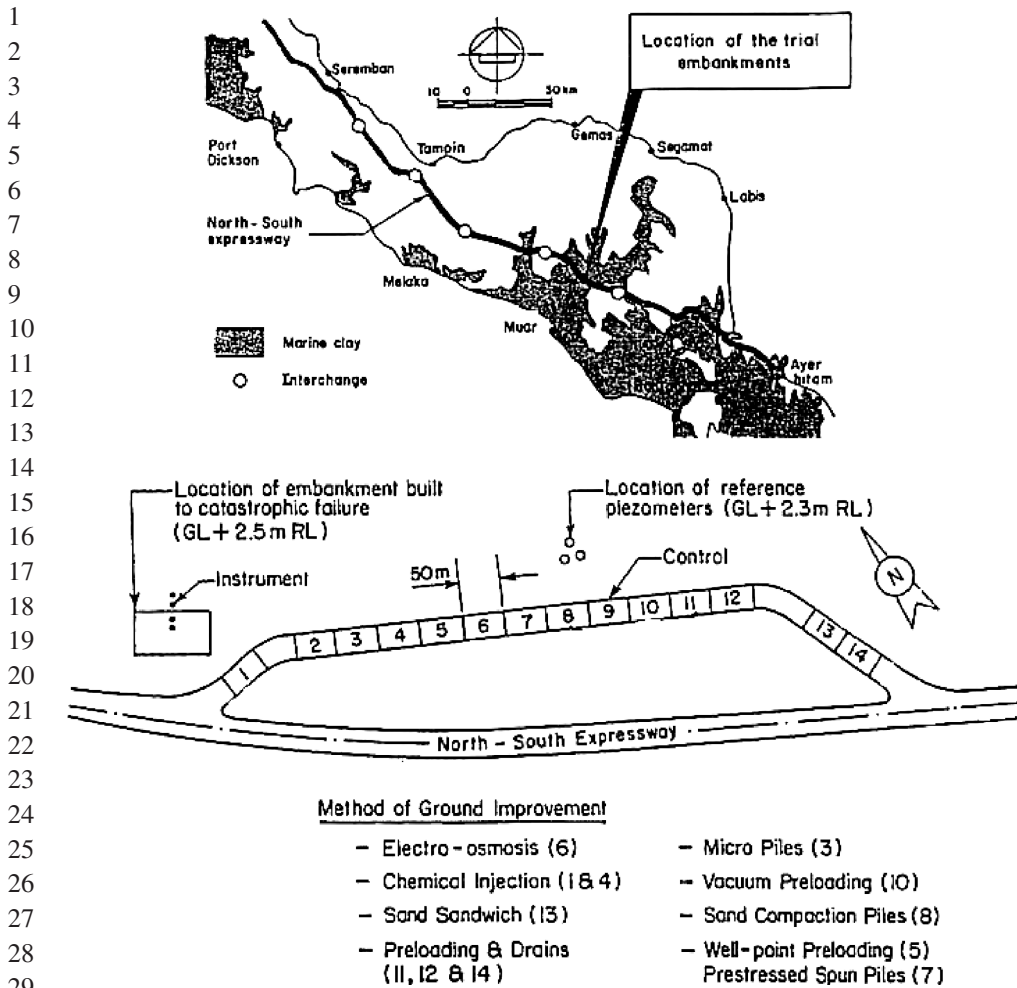


(b)

28 **Figure 24.** Predicted surface settlement at toe for different drain spacing at a construction rate of (a) 0.1 m/week,  
29 (b) 0.35 m/week.

30  
31 embankment was raised with a fill material of  $20.5 \text{ kN/m}^3$  bulk unit weight at a constant  
32 rate of 0.4 m/week (Indraratna et al., 1992). The MCC parameters and the in situ stresses  
33 are given in Tables 6 and 7, respectively.

34 The excess pore pressure variation along the embankment centerline, the surface set-  
35 tlement, and the lateral displacement at 10 m from the centerline are plotted in Figures  
36 28–30 for a fill height of 5 m. As expected, the lateral displacement is significantly  
37 reduced in the stiffer clay layer. In general, the MCC theory overestimates the lateral dis-  
38 placements. The reason for this discrepancy can be attributed to several factors, including  
39 the lateral variability of soil parameters, the use of a simplified associated flow rule, and  
40 the effect of the stiff surficial crust (Potts and Zdravkovic, 2001). It was found that lateral

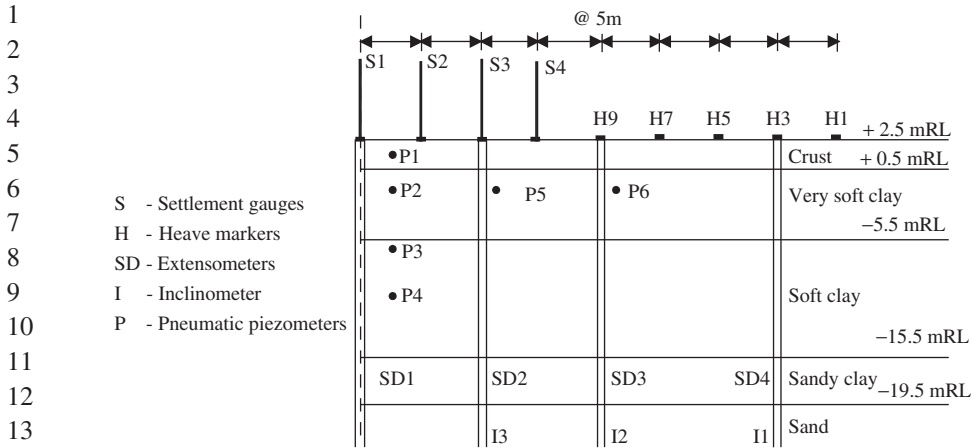


**Figure 25.** Location of Marine clay deposits and relative location of trial embankments along North-South expressway, Malaysia (modified after Indraratna et al., 1997).

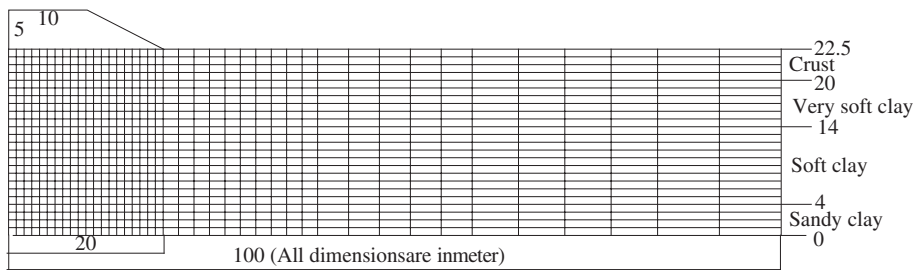
displacements are also sensitive to nominal changes of compression parameter  $\lambda$  (Indraratna et al., 1992).

**9.2. Embankment stabilized with geosynthetic vertical drain**

The location of this embankment is shown in Figure 25 as embankment # 14. The embankment cross-section with key instrumentation and the associated subsoil profile is shown in Figure 31. The equivalent drain radius based on Eq. (4) is estimated to be  $r_w=0.03$  m and the smear zone radius is taken as  $r_s=0.15$  m. The Cam-clay parameters and equivalent



**Figure 26.** Cross-section of the failed embankment showing key instrumentations (modified after Indraratna et al., 1992).



**Figure 27.** Finite element mesh for embankment constructed to failure.

**Table 6.** Soil parameters used in finite element analysis

Depth	$k$	$\lambda$	$e_{cs}$	$M$	$\nu$	$\gamma_s$ (kN/m <sup>3</sup> )	$k_h$ (m/s)	$k_v$ (m/s)
0–2.5	0.05	0.13	3.07	1.19	0.3	16.5	$1.5 \times 10^{-9}$	$0.8 \times 10^{-9}$
2.5–8.5	0.05	0.13	3.07	1.19	0.3	15.5	$1.5 \times 10^{-9}$	$0.8 \times 10^{-9}$
8.5–18.5	0.08	0.11	1.61	1.07	0.3	15.5	$1.1 \times 10^{-9}$	$0.6 \times 10^{-9}$
18.5–22.5	0.10	0.10	1.55	1.04	0.3	16.0	$1.1 \times 10^{-9}$	$0.6 \times 10^{-9}$

Source: Indraratna and Sathanathan (2003)

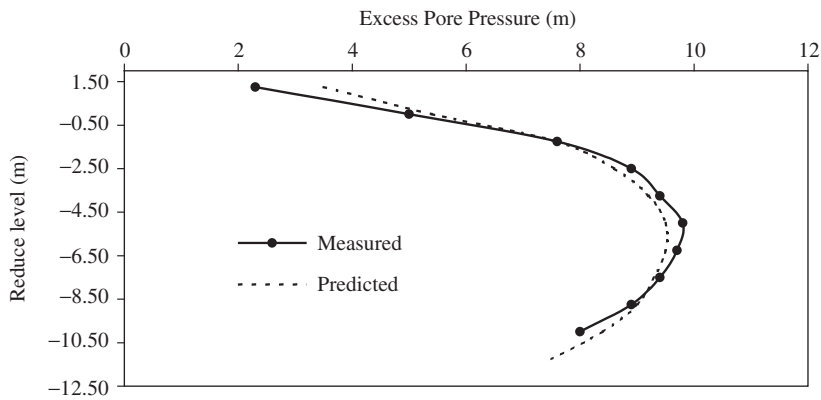
plain strain permeabilities based on Eq. (30) are given in Table 8. Table 9 tabulates the in situ stress distribution. Embankment construction was carried out in two loading stages; during the first 14 days the height was raised to 2.57 m (Stage 1), and after a 90 day rest period the height was raised to 4.74 m in 24 days (Stage 2).

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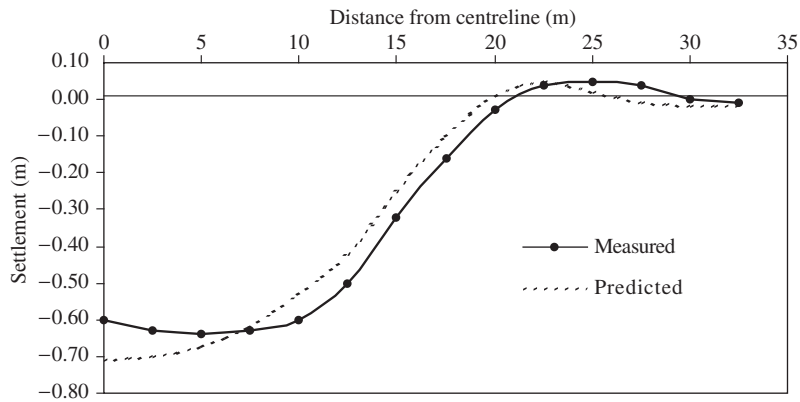
**Table 7.** In situ stress condition

Depth (m)	$\sigma_{h0}$ (kPa)	$\sigma_{v0}$ (kPa)	u (kPa)	P'c (kPa)
0	0	0	0	110
2.5	13.2	22.0	16.7	110
8.5	33.7	56.1	75.5	40
18.5	67.9	113.1	173.6	60
22.5	81.5	135.9	212.9	60

Source: Indraratna and Sathanathan (2003)



**Figure 28.** Excess pore pressure variation under embankment centerline.



**Figure 29.** Surface settlement variation.

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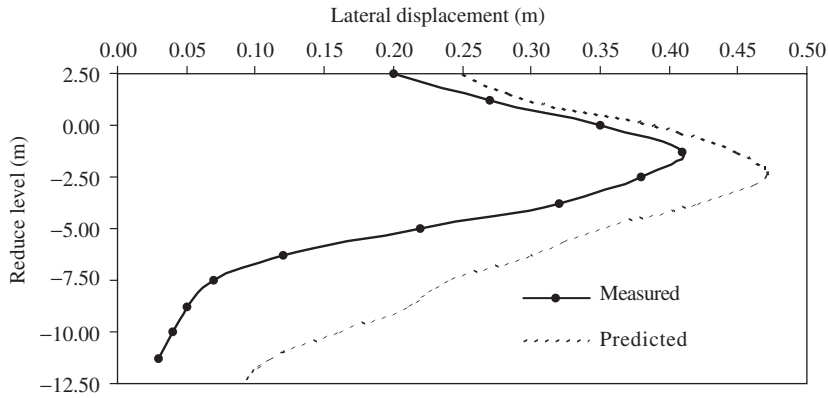


Figure 30. Lateral displacement profile at 10 m from centerline.

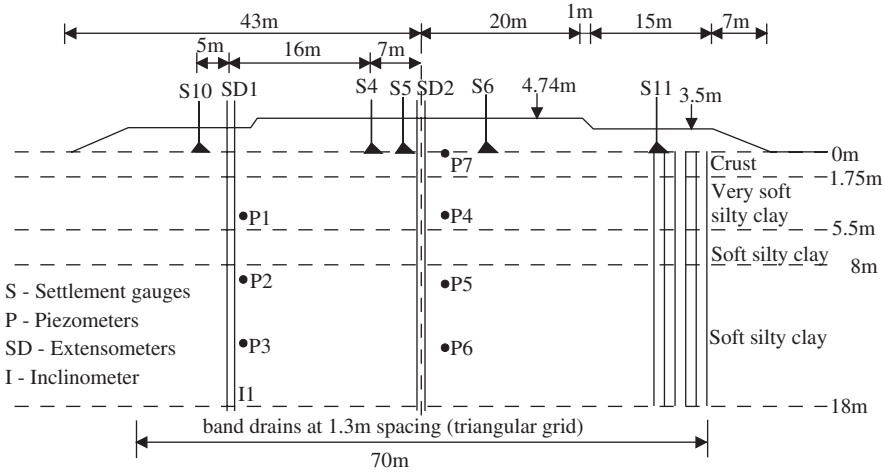


Figure 31. Cross-section of test embankment with key instrumentation (modified after Indraratna et al., 1994).

The finite element mesh of the embankment is shown in Figure 32 and the location of inclinometer (ID1-23 m away from the centerline) and piezometers are conveniently defined at mesh nodes. The well resistance of the drain was included because they were 18 m long. The well resistance was simulated by considering the vertical permeability of the transformed drain wall as previously discussed in Eq. (31c). The equivalent coefficient of permeability of drain was estimated as 0.0005 m/s by a single drain analysis.

The predicted and measured settlements at the centerline and along the surface are shown in Figures 33 and 34, respectively. Heave is also predicted beyond the toe of the

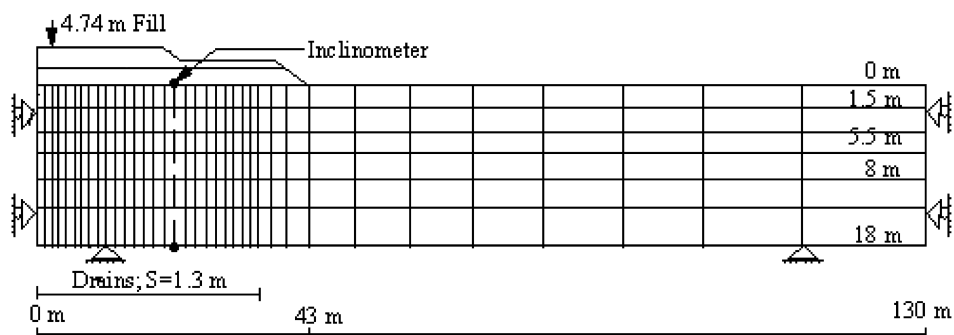


1 **Table 8.** Soil parameters used in finite element analysis

Depth (m)	$\kappa$	$\lambda$	$e_{cs}$	$M$	$\nu$	$\gamma_s$	Permeability ( $10^{-9}$ m/s)			
							$k_h$	$k'_h$	$k_{hp}$	$k'_{hp}$
0–1.75	0.06	0.30	3.10	1.19	0.29	15.0	6.4	3.0	2.45	0.60
1.75–5.50	0.06	0.60	3.10	1.19	0.31	15.0	5.2	2.7	1.36	0.58
5.50–8.0	0.05	0.30	3.06	1.12	0.29	15.5	3.1	1.4	0.81	0.29
8.0–18.0	0.04	0.35	1.61	1.07	0.26	16.0	1.3	0.6	0.34	0.13

8 *Source:* Indraratna and Sathanathan (2003)10 **Table 9.** In situ stress condition

Depth (m)	$\sigma'_{v0}$ (kPa)	$\sigma'_{h0}$ (kPa)	$u$ (kPa)	$P'_c$ (kPa)
0	0	0	0	110
1.75	28.6	17.3	0	95
5.50	48.4	29.1	36.7	44
8.0	62.6	37.6	61.3	60
18.0	124.6	74.8	159.3	135

17 *Source:* Indraratna et al. (1994)30 **Figure 32.** Finite element mesh used in plane strain analysis (adapted after Indraratna et al., 2000).

32 embankment, i.e. at about 45 m away from the centerline but regrettably, no field data  
 33 were available for comparison. The predictions acceptably agree with the limited field  
 34 measurements obtained near the centerline.

35 The evaluated and measured excess pore water pressure variations are shown in  
 36 Figure 35. The measured excess pore pressure does not indicate much dissipation during  
 37 Stage 2 due to the piezometer malfunctioning. Even though the prediction of excess pore pres-  
 38 sure is made accurately in Stage 1 by including the smear effect, the predicted postconstruc-  
 39 tion pore pressure only improved slightly by including both smear and well resistance. As  
 40 expected, the “perfect drain” underestimates the measurements. Observed and predicted lateral

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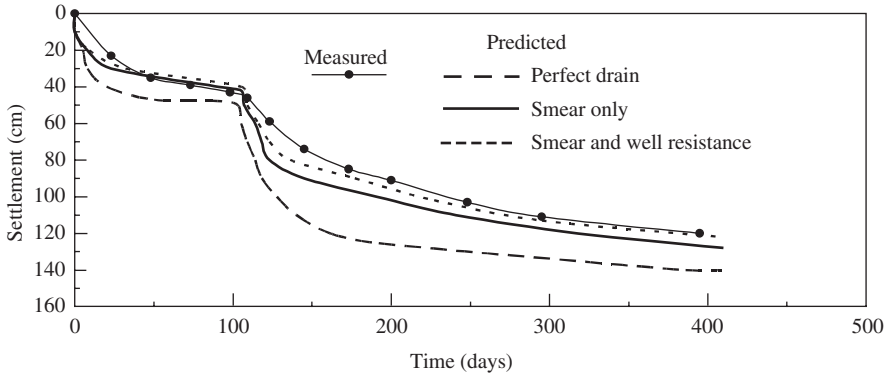


Figure 33. Settlement at embankment centerline.

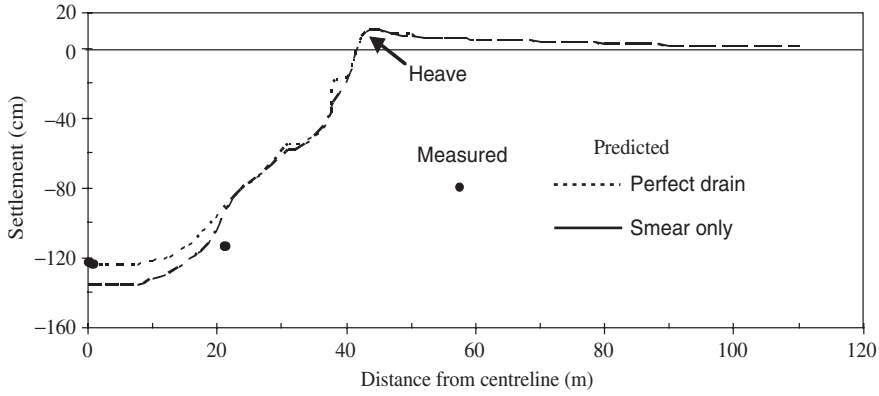


Figure 34. Surface settlement profile after 400 days.

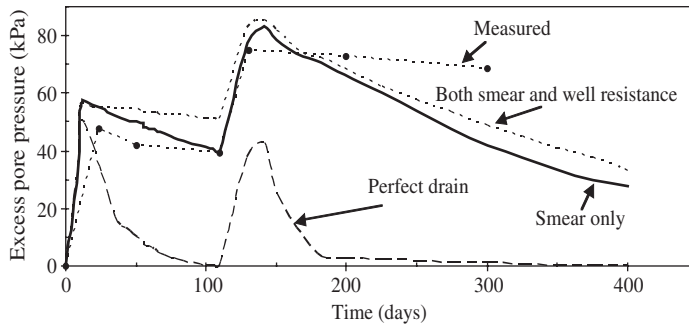


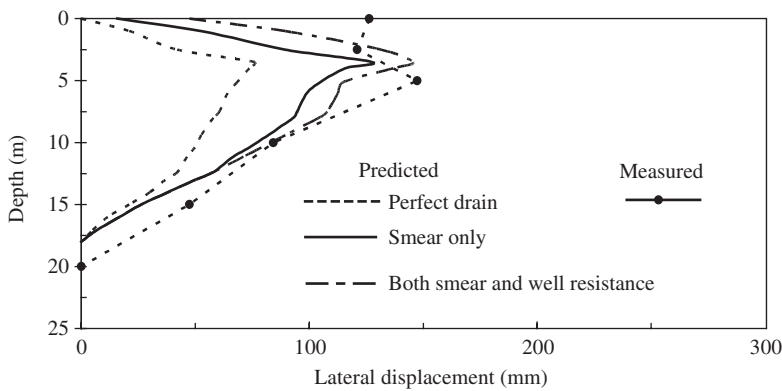
Figure 35. Excess pore water pressure variation at piezometer location, P6.

1 deformations are plotted in Figure 36. Acceptable agreement between the field data and pre-  
 2 dictions is obtained when both the smear and well resistance are considered. The perfect drain  
 3 condition gives the smallest lateral deformation while maximizing vertical deformation.  
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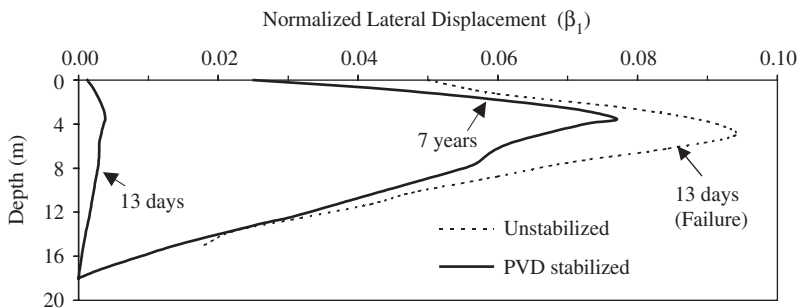
5 **9.3. Normalized deformation factors**

6 The lateral displacement and settlement can be normalized with respect to the correspon-  
 7 ding fill height to examine the effectiveness of the ground improvement techniques. Thus,  
 8 the following “stability” indicators are defined (Indraratna et al., 1997):  $\beta_1$ , the ratio  
 9 between lateral displacement and the corresponding fill height,  $\beta_2$  the ratio between settle-  
 10 ment and the corresponding fill height, and  $\alpha = \beta_1/\beta_2$ .

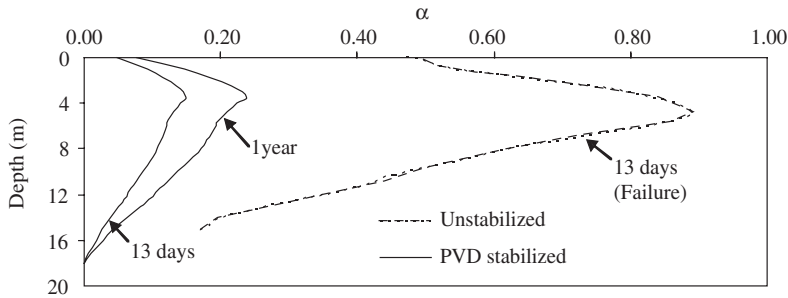
11 Figure 37 shows variation of  $\beta_1$  with depth and Figure 38 shows the variation of  $\alpha$  with  
 12 depth. The normalized displacement of PVD-stabilized embankment is considerably less  
 13 than an embankment constructed to failure. These results clearly show that vertical drains  
 14 effectively decrease lateral deformations and enable the critical height of an embankment  
 15



26 **Figure 36.** Lateral displacement profile after about 300 days at Inclinator I1.



39 **Figure 37.** Normalized lateral displacement.



**Figure 38.** The variation of lateral displacement/settlement ratio with depth.

**Table 10.** Normalized deformation factors (modified after Indraratna et. al., 1997)

Ground improvement scheme	$\alpha$	$\beta_1$	$\beta_2$
Sand compaction piles for pile/soil stiffness ratio of 5 ( $h=9.8$ m, including 1 m sand layer)	0.185	0.018	0.097
Geogrids + vertical band drains in square pattern at 2.0 m spacing ( $h=8.7$ m)	0.141	0.021	0.149
Vertical band drains in triangular pattern at 1.3 m spacing ( $h=4.75$ m)	0.127	0.035	0.275
Embankment rapidly constructed to failure on untreated foundation ( $h=5.5$ m)	0.695	0.089	0.128

to be increased. After 13 days, the untreated embankment fails with unacceptably large lateral displacement (Figure 37). The PVD-stabilized foundation takes more than 7 years before lateral displacement become similar to the failed embankment.

The normalized deformation factors for a few trial embankments are also compared in Table 10. In comparison with the unstabilized embankment constructed to failure, the stabilized foundations are characterized by considerably smaller values for  $\alpha$  and  $\beta_1$ , which elucidates their obvious implications on stability. The normalized settlement ( $\beta_2$ ) on its own is not a proper indicator of instability but is still a useful stability indicator when taken in conjunction with  $\alpha$  and  $\beta_1$ . For example, the foundation having SCP gives the lowest values of  $\beta_1$  and  $\beta_2$ , clearly suggesting the benefits of sand compaction piles over band drains.

## 10. CONCLUDING REMARKS

In this chapter, the use of prefabricated vertical drains, their properties and associated merits and demerits have been discussed. The behavior of soft clay under the influence of PVD was described on the basis of numerous case histories where both field measurements and

1 numerical predictions were available. A sophisticated 3-D multidrain analysis with an indi-  
2 vidual axisymmetric zone of influence, with smear for each and every drain, will easily  
3 exceed computational capacity when applied to a real embankment project with a large  
4 number of PVD. In this context, the equivalent plane strain models will continue to offer a  
5 sufficiently accurate predictive tool for design, performance verification, and back analysis.

6 Selected numerical studies have been carried out to study the effect of embankment  
7 slope, construction rate, and drain spacing on the failure of soft clay foundations. Finally,  
8 the observed and the predicted performances of well-instrumented full-scale trial embank-  
9 ments built on soft Malaysian marine clay have been discussed using the plane strain theo-  
10 ry. The numerical results based on ABAQUS conclude that the inclusion of both smear  
11 and well resistance improves the accuracy of the predicted settlement, excess pore pres-  
12 sures, and lateral deformation. As expected, the perfect drain analysis always overpredicts  
13 settlement and underpredicts excess pore pressures. The results presented here reaffirm  
14 that the effects of soil disturbance (smear) and well resistance are important for estimating  
15 deformation. While an accurate prediction of surface settlement is generally feasible, the  
16 acceptable prediction of lateral displacement is often difficult due to inherent assumptions  
17 made in the plane strain models. An accurate prediction of lateral displacement undoubt-  
18 edly depends on the correct assessment of the value of  $\lambda$  of the MCC model, and the dis-  
19 charge capacity of PVD, among other parameters.

## 20 21 22 **ACKNOWLEDGMENTS**

23 The authors acknowledge the Malaysian Highway Authority for providing the trial  
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25 ter have come from the authors' previous publications such as the ASCE and Canadian  
26 Geotechnical Journal papers, as cited in the text and listed in the section.  
27

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