# Defined Benefit Pension Plans: Cost of Living Adjustments and Membership Heterogeneity

by

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## Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

**Abstract**

In this paper we explore different structures for cost of living adjustments (COLA), based on Hardy *et al.* (2020)'s five significant criteria: affordability, sustainability, efficiency, adequacy, and fairness. Full COLA protects plan members' real income after retirement but leads to high costs and solvency risks. A two-tier COLA method, as an alternative, moderates the risks and protects low-paid members' benefits. We provide a new insight using a heterogeneous pension plan model, consisting of equal populations of blue-collar (lower-paid) and white-collar (higher-paid) members. We further modify the existing benefit structure of the heterogeneous plan to achieve a DB pension plan conforming to the five criteria.

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Finally, I would like to express my thanks and love to my parents for their unwavering support for my dream, and to my friends for accompanying me.

## Dedication

*To Mom and Dad*

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# Nomenclature

| | |
|---|---|
| $\alpha$ | accrual rate |
| $\beta$ | basis point spread |
| $\ddot{a}_{(x,t)}$ | expected present value of annuity due at age $x$ at time $t$ |
| ${}_tp_x^{(\tau)}$ | probability that a member at age $x$ is still in pension plan at age $x+t$ |
| ${}_tp_x$ | probability that a member at age $x$ is still alive at age $x+t$ |
| $Abf(t)$ | asset brought forward at time $t$, i.e., before all cash flow at time $t$ |
| $b(x,t)$ | benefit of a member at age $x$ at time $t$ |
| $FAS(t)$ | final average salary of a member at age $x$ at time $t$ |
| $FCAP(t)$ | cap on final average salary at time $t$ |
| $g(t)$ | valuation salary growth rate at time $t$ |
| $gs(t)$ | actual salary growth factor at time $t$ |
| $i(t)$ | valuation interest rate at time $t$ |
| $j(t)$ | valuation inflation rate at time $t$ |
| $js(t)$ | actual inflation rate at time $t$ |
| $L(x,t)$ | number of members at age $x$ at time $t$ |
| $NE(x)$ | number of new entrants at age $x$ |
| $ny(x,t)$ | average years of service of a member at age $x$ at time $t$ |

| | |
|---|---|
| $s(x,t)$ | average salary of a member at age $x$ at time $t$ |
| $w$ | proportion of assets invested in equity |
| $Y(t)$ | yield-to-maturity on risk free bonds at time $t$ |
| $yb(t)$ | return on long-term bonds at time $t$ |
| $ye(t)$ | return on equity at time $t$ |
| $YMPE(t)$ | yearly maximum pensionable earnings at time $t$ |

# Chapter 1

# Introduction

A traditional Defined Benefit (DB) plan provides guaranteed pensions to employees on their retirement. The benefit amount is commonly expressed as a multiple of an accrual rate, the average years of service, and the final average salary of the employee. Under this plan, employers, or sponsors are the ones who take the responsibility (possibly with employees if contributions are shared) to ensure that retirees can obtain promised benefit amounts.

Even though two-thirds of pension plan membership in Canada is in DB, the percentage change in DB pension plan membership is declining in the private sector compared to the change in Defined Contribution (DC) plans (Statistics Canada, 2020). The unpopularity of DB pension plans is due to various factors. One is market performance. By the end of 2008, the average funding ratio had reduced to 75% (Sheikh and Sun, 2013). Longevity improvements also accelerated the decline of DB pension plans. The underestimation of plan members' life expectancy in DB pension plans can lead to severe financial consequences and even bankruptcy. U.S. private DB pension liabilities were undervalued by $84 billion due to overestimation in the valuation mortality assumptions in 2007, while plan sponsors were responsible for making up the extra liabilities by making additional contributions (Kisser *et al.*, 2012). These factors resulted in pension underfunding. From the 2018 Report on the Funding of Defined Benefit Pension Plans in Ontario, 984 DB plans of the 1,364 plans were underfunded under a solvency valuation (Financial Services Commission of Ontario, 2019).

Despite the decline in private sector DB plans, Dobson (2017) found that DB plans are still Canadians' preferred pension plan, according to the Pension and Retirement Income Preferences Survey results, as they offer guaranteed and predictable lifetime incomes. The survey respondents want their retirement benefits to keep up with inflation and they are willing to achieve these properties by paying additional contributions.

In this paper we explore DB plan design alternatives. The plan design is based on the following criteria (Hardy *et al.*, 2020).

- **Affordable:** Although members are willing to contribute more, the plan should have an average total contribution rate acceptable to members.
- **Sustainable:** The volatility of total contribution rates is not too high.
- **Adequate:** The plan provides predictable and sufficient benefits.
- **Efficient:** The plan collects a proper amount of contributions and uses them effectively to maintain the adequacy of the plan and to avoid excessive surplus, contribution holidays, or excessive benefits.
- **Fair:** The plan treats different members equitably.

M'Lauchlan (1907) was one of the first authors who published a simulated pension fund model. The demographics in his model started with 1,000 members entering at age 20 and the same population at same age kept up entering each year. He applied factors, including retirement rates, mortality rates, withdrawal rates, salary increase rates, contribution rates, and interest rates, to generate the fund accumulated for decennial periods. Manly (1911) extended M'Lauchlan's research by exploring the factors' influences on pension liabilities.

The Pension Liability and Asset Simulation Model (PLASM) proposed by Winklevoss (1982) is similar to the design of our DB pension plan model. It also used Monte Carlo simulation to project stochastic salaries, contributions, pension liabilities and assets. Winklevoss (1982) also gave a brief introduction of some benefit structure parameters which affect the model results. One of the parameters is the cost of living adjustments (COLA), which offsets inflation. Jennings *et al.* (2016) demonstrated the effects of COLA protection on protecting retirement income. In this paper our first benefit structure is assumed to offer full COLA for the pension in payment, and we subsequently explore different benefit structures by implementing different methods of applying COLA. The incentive is to reduce cost and to pay attention to the impact on low-paid members.

Hardy *et al.* (2020) considered a heterogeneous plan membership, including salaried and non-salaried (e.g., hourly paid) members. Under the separate DB plans, non-salaried members had lower default risk and contribution rate than salaried members. However, little published research exists that combines heterogeneous groups. In this paper, we combine low-paid (blue-collar) and high-paid (white-collar) members in a DB pension plan and investigate the effects of different benefit structures on the different populations.

This paper is structured as follows. In Chapter 2 we introduce the basic framework for a traditional DB pension plan with full COLA and demonstrate the simulation results. We examine different COLA methods and consider the impact on low-paid members in Chapter 3. Chapter 4 introduces a heterogeneous plan including both blue-collar and white-collar members in the same DB pension plan and compares the influences under different methods of applying COLA. The last chapter concludes.

# Chapter 2

# Defined Benefit Pension Plan Model

## 2.1 Economic Scenario Generator

The valuation of liabilities of retirement benefits is sensitive to the valuation assumptions. By processing different valuation assumptions, subsequent variables alter along with the variation in the valuation liability. To obtain reliable results, we have used Wilkie's Economic Scenario Generator for all economic factors, fitted to U.S. 1951-2014 data (Zhang *et al.*, 2018) to simulate 1,000 paths. The generated series are

- Return on equity $ye(t)$,
- Return on long-term bond $yb(t)$,
- Actual inflation rate $js^{raw}(t)$,
- Yield-To-Maturity (YTM) on risk free bonds $Y(t)$.

We put a restriction on $js(t)$, of a 3% maximum for plan COLA. Moreover, the actual salary growth $gs(t)$ is 50 basis points above $js(t)$.

$$js(t) = \min\left(js^{raw}(t), 3\%\right), \tag{2.1}$$

$$gs(t) = js(t) + 0.005 \tag{2.2}$$

For the valuation assumptions, we have

- Valuation interest rate $i(t)$:

  We set $i(t)$ based on the YTM on bonds. We allow for a basis point spread between $i(t)$ and long term risk free bonds. The notation for the basis point spread is $\beta$, so that
$$i(t) = Y(t) + \beta. \tag{2.3}$$

  We will discuss the impact of the basis points on the pension plan in Section 2.7. We set the value of $\beta$, for all $t$, by setting the expected value of $i(t)$ to be equal to the expected value of the rate of return, based on the 1,000 paths. The expected value based on the 1,000 paths is estimated using the average throughout time $T$ of the 1,000 paths. We use notation $ye(i,t)$ as $ye(t)$ under the $i^{th}$ simulation path, and similarly for $yb(i,t)$, $Y(i,t)$, and $i(i,t)$. Then

$$\bar{ye} = \frac{\sum_{i=1}^{1000}\sum_{t=1}^{T} ye(i,t)}{1000T}, \tag{2.4}$$
$$\bar{yb} = \frac{\sum_{i=1}^{1000}\sum_{t=1}^{T} yb(i,t)}{1000T}, \tag{2.5}$$
$$\bar{Y} = \frac{\sum_{i=1}^{1000}\sum_{t=1}^{T} Y(i,t)}{1000T}, \tag{2.6}$$
$$\bar{i} = \frac{\sum_{i=1}^{1000}\sum_{t=1}^{T} i(i,t)}{1000T}. \tag{2.7}$$

  The proportion of assets invested in equities is $w$. From above, we set

$$\bar{i} = \bar{Y} + \beta, \tag{2.8}$$
$$\bar{i} = w \times \bar{ye} + (1-w)\bar{yb}, \tag{2.9}$$
$$\beta = w \times \bar{ye} + (1-w)\bar{yb} - \bar{Y}. \tag{2.10}$$

  We set $w = 0.6$ based on the rule of thumb in pension plan portfolios (Gerber and Shiu, 2000). Later we will consider other $w$ values. From Equation (2.10), the added basis points $\beta = 0.0081$. A simple moving average with a lag of 5 years is used to smooth $i(t)$.

- Valuation inflation rate $j(t)$

  We set $j(0)$ to be

$$j(t) = \min\left(\max\left(i(t) - 0.0255, 0\%\right), 3\%\right). \tag{2.11}$$

This implies, approximately, a real return of 2.55% on investment. Note that the 3% cap and 0% floor are assumed limits imposed by the plan rules. Because of the 5-year moving average need for $i(t)$, $j(t)$ would be smoothed similarly.

- Valuation salary growth rate $g(t)$

  We assume $g(t)$ is 200 basis points below $i(t)$ with a minimum of 1%,

$$g(t) = \max\left(i(t) - 0.02, 1\%\right). \tag{2.12}$$

## 2.2 Plan Design

We explore DB pension plan design and funding using the model plan described here. The time horizon is 30 years, and 65 is the retirement age. We do not allow early retirement in our plan.

- The pension plan offers an accrual rate $\alpha$ of 1.8% of final average salary $FAS(x,t)$ for each year of service.

- We apply the actual salary growth factor $gs(t)$ in the salary calculation and $s(x,0)$ represents a merit salary scale excluding inflation, so that

$$s(x,t) = s(x,t-1)(1 + gs(t)) \text{ for } 25 \leq x \leq 65. \tag{2.13}$$

- The final average salary $FAS(x,t)$ is defined as the average salary over the five years of employment up to age $x$ at time $t$, and is calculated

$$FAS(x,t) = \frac{1}{5}\sum_{i=1}^{5} s\left(\max(25, x-i), \max(0, t-i)\right)\left(1 + gs(0)\right)^{\min(0,t-i)} \tag{2.14}$$

- The pension in payment is assumed to increase by the actual inflation rate, $js(t)$ per year for COLA and $js(t)$ is capped at 3%.

- The pension is paid as a whole life annual annuity due.

- All new entrances and exits, including deaths and withdrawals, are assumed to occur midway through the year.

- Deaths and withdrawals before age 65 receive a lump sum with the value equal to deferred benefits at retirement age 65.

## 2.3 Demographics

The age group is from $x = 25$ to 105. Members are active from age 25 to 64. The initial data is given in the appendix. We show part of the table of active members' information.

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 25 | 17 | 17 | 0.5 | 32.0 | 0.8998 |
| 26 | 32 | 17 | 0.97 | 33.0 | 0.8997 |
| 63 | 97 | 0 | 22.93 | 98.2 | 0.9953 |
| 64 | 97 | 0 | 23.93 | 99.2 | 0.9947 |

Table 2.1: Active membership information at time 0, salary in \$000s.

Because age 25 is the earliest entrance age, the number of members at age 25 at any time equals the number of new entrants at age 25, i.e., $L(25,t) = NE(25)$ for all $t$. We use the probability that a member stays in the pension plan, ${}_tp_x^{(\tau)}$, to calculate $L(x+1, t+1)$. Similarly, the average years of service of a member at age 25 at any time under the assumption is $\frac{1}{2}$, i.e., $ny(25,t) = \frac{1}{2}$ for all $t$. The remaining $L(x,t)$ and $ny(x,t)$ when $t = 1, 2, ..., 30$ follow

$$L(x,t) = L(x-1,t-1)\ p_{x-1}^{(\tau)} + NE(x) \qquad \text{for } x \leq 65, \tag{2.15}$$

$$ny(x,t) = \frac{(L(x,t) - NE(x))\,(ny(x-1,t-1)+1) + \frac{1}{2}NE(x)}{L(x,t)} \qquad \text{for } x \leq 65. \tag{2.16}$$

Here is part of the initial information for retired members.

| $x$ | $L(x,0)$ | $b(x,0)$ | $p_x$ | $x$ | $L(x,0)$ | $b(x,0)$ | $p_x$ |
|---|---|---|---|---|---|---|---|
| 65 | 96 | 44.4 | 0.9941 | 104 | 0 | 14.0 | 0.5795 |
| 66 | 94 | 43.1 | 0.9934 | 105 | 0 | 13.6 | 0.0000 |

Table 2.2: Retired membership information at time 0, benefit amount in \$000s.

There are no more new entrants from age 56 to 105. After retirement, instead of using $p_x^{(\tau)}$, we apply the probability that a member is still alive, ${}_tp_x$, for $L(x+1, t+1)$. Members age 65 have one more year of average years of service than when they were 64. For retired members, average service is no longer tracked.

$$L(x,t) = L(x-1,t-1) \times p_{x-1} \text{ for } x \geq 66 \tag{2.17}$$

## 2.4 Valuation Methodology

We use the Traditional Unit Credit (TUC) funding method for the valuation of benefits in this paper, meaning there is no salary projection. An active member's accrued benefit is a multiple of the accrual rate, the final average salary, and the average years of service at age $x$. Given that COLA applies, we take the actual capped inflation rate $js(t)$ into consideration for the accrued benefit for a retired member. Therefore, the average accrued benefit for a member at age $x$ at time $t$ can be computed as follows,

$$b(x,t) = \alpha \ FAS(x,t) \ ny(x,t) \qquad \text{for } \ x \leq 65, \tag{2.18}$$

$$b(x,t) = b(x-1,t-1)(1+js(t)) \qquad \text{for } x \geq 66. \tag{2.19}$$

The total accrued benefit at time $t$ is

$$B(x,t) = L(x,t)b(x,t), \tag{2.20}$$

$$BSUM(t) = \sum_{x \geq 65} B(x,t). \tag{2.21}$$

The liability for retired members at age $x$ at time $t$ is based on the guaranteed benefits and the whole life annuity due. The expected present value of the annuity for a member at age $x$ at time $t$ with the valuation interest rate $i(t)$ and the valuation inflation rate $j(t)$ is

$$\ddot{a}_{(x,t)} = \sum_{k=0}^{105-x} {}_k p_x \left(1+i^{(*)}(t)\right)^{-k}. \tag{2.22}$$

Due to the existence of COLA, $1+i^{(*)}(t) = \frac{1+i(t)}{1+j(t)}$.

For active members, both the probability of surviving to the retirement age and the interest rate are used to discounted the liability.

$$V(x,t) = B(x,t) \ {}_{65-x}p_x^{(\tau)} \ \left(1+i(t)\right)^{-(65-x)} \ \ddot{a}_{(65,t)} \qquad \text{for } x \leq 65, \tag{2.23}$$

$$V(x,t) = B(x,t) \ \ddot{a}_{(x,t)} \qquad \text{for } x \geq 66. \tag{2.24}$$

The total liability for all members at time $t$ would be

$$VSUM(t) = \sum_{x} V(x,t). \tag{2.25}$$

The normal contributions are paid by members in service, and we assume all contributions are paid by employees. Under the TUC funding approach, the aggregate normal contributions for members at age $x$ at time $t$ is

$$NC(x,t) = V(x,t)\left(\frac{FAS(x+1,t)(1+g(t))\ (ny(x,t)+1)}{FAS(x,t)\ ny(x,t)} - 1\right) \text{ for } x \leq 64. \tag{2.26}$$

The total normal contributions at time $t$ are

$$NCSUM(t) = \sum_{x\leq 64} NC(x,t). \tag{2.27}$$

We assume new entrants' normal contributions from time $t-1$ to $t$ are a proportion of the normal contributions at $t-1$. That is

$$NCNE(t) = \sum_{x} \frac{1}{2} NE(x) \frac{NC(x-1,t-1)}{L(x-1,t-1)}. \tag{2.28}$$

The number of members who exit between time $t-1$ to $t$ is assumed to be $L(x,t)(1-p_x^{(\tau)})$, including deaths and withdrawals. By our assumption that exit happens midway through the year, $ny(x,t)$ increases by $\frac{1}{2}$ while $FAS(x,t)$ remains the same. The lump sum exit benefits paid for withdrawals and deaths is

$$Wb(x,t) = \alpha\ FAS(x,t)\left(ny(x,t) + \frac{1}{2}\right)(1+i(t))^{-\left(65-(x+\frac{1}{2})\right)}\ \ddot{a}_{(65,t)}. \tag{2.29}$$

Then the total cost of exit benefits from $t-1$ to $t$ is

$$WSUM(t) = \sum_{x\leq 64} L(x,t)(1-p_x^{(\tau)})Wb(x,t). \tag{2.30}$$

The accumulation factor for the assets, from time $t-1$ to $t$, based on the proportion of assets invested in equity $w$ is

$$R(t) = w(1+ye(t)) + (1-w)(1+yb(t)). \tag{2.31}$$

At $t=0$, we assume that the asset pool is formed equal to the value of the liability. At the beginning of each year, benefits are paid to all retired members and contributions are collected from members in service. The notation $A(t)$ is the asset value immediately after the cash flow at the beginning of year $t$. Then let

$$Abf(0) = VSUM(0), \tag{2.32}$$
$$A(0) = Abf(0) - BSUM(0) + NCSUM(0). \tag{2.33}$$

After investing the assets for one year, paying withdrawal benefits, and collecting new entrants' contributions at the middle of the year, the assets brought forward next year would be

$$Abf(t+1) = A(t)R(t) - WSUM(t)R(t)^{\frac{1}{2}} + NCNE(t)R(t)^{\frac{1}{2}}, \tag{2.34}$$
$$A(t) = Abf(t) - BSUM(t) + NCSUM(t). \tag{2.35}$$

So the asset liability ratio at time $t$ is,

$$ALR(t) = \frac{Abf(t)}{VSUM(t)}. \tag{2.36}$$

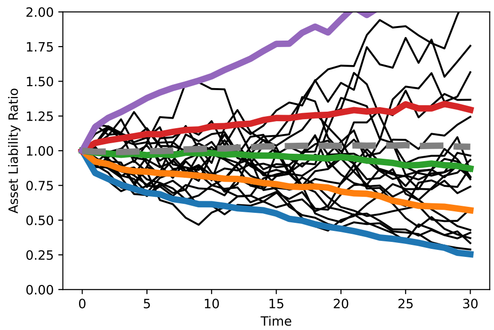
Besides asset liability ratios, we also pay attention to normal contribution rates in order to evaluate the plan. It is a scale to measure the input ratio of a participating member's normal contributions to the member's average salary. The normal contribution rate at $t$ is

$$NCR(t) = \frac{NCSUM(t)}{\sum_{x \le 64} L(x,t)s(x,t)}. \tag{2.37}$$

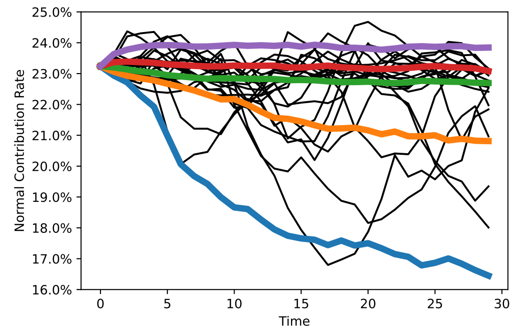
We expect normal contribution rates to lie in the range 15% to 20%.

We introduce additional contributions as an adjustment in our plan. When deficits happen, i.e., $ALR(t) < 1$, additional contributions fund part of the gap. The additional contributions would be paid by the members in service and are added to the assets. By that, both the asset liability ratio and the total contribution would rise for the next year. On the other hand, when there is a large surplus, we decrease the contribution amount from members in service. In other words, the additional contributions will be negative, though, the total contributions are bounded by 0. The total contributions drop and the asset liability ratio returns to a smaller figure. By involving additional contributions, we expect that the asset liability ratio is maintained within an acceptable range, while the total contribution rate fluctuates. As an adjustment, the amount of additional contributions needs to be defined and we will discuss this in more detail in Section 2.6.

## 2.5 Plan Results




(a) Asset Liability Ratio




(b) Normal Contribution Rate

Figure 2.1: Defined benefit plan results of 1,000 simulations; without additional contributions; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths.

| Quantiles at $t = 30$ | Asset Liability Ratio | Normal Contribution Rate |
|---|---|---|
| 5% | 0.254 (0.196, 0.289) | 0.165 (0.159, 0.173) |
| 25% | 0.571 (0.538, 0.604) | 0.208 (0.205, 0.211) |
| 50% | 0.871 (0.832, 0.918) | 0.227 (0.226, 0.228) |
| 75% | 1.294 (1,254, 1.373) | 0.231 (0.230, 0.231) |
| 95% | 2.128 (2.118, 2.441) | 0.238 (0.237, 0.240) |

Table 2.3: Defined benefit plan results of 1,000 simulations; without additional contributions; 95% confidence intervals in parentheses.

We demonstrate the result of the asset liability ratios and the normal contribution rates under the simulation of the 1,000 paths in Figure 2.1. The thick lines from bottom to top correspond to 5%, 25%, 50%, 75%, and 95% quantiles. The grey dashed line is the mean asset liability ratio over the time period. The black lines are 20 randomly chosen paths to illustrate the volatility behind the quantile plots. We use the same 20 paths for every quantile plot in this paper.

In Figure 2.1a, we see that both the 75% quantile line and the 95% quantile line of the asset liability ratio have an upward trend. The other three quantile lines decline. The 50%

quantile line is 0.871 at $t = 30$ from Table 2.3. The mean ends slightly above 1 at $t = 30$, with the 95% confidence interval $(0.986, 1.070)$. The difference between the mean and the median also indicates that some paths have significantly high asset liability ratios in the end. In Figure 2.1b, the range of the normal contribution rate is from about 16.5% to 24.0%. The median tends to be flat and levels off to 22.7% when $t = 30$. Most of the paths lie in between 22% and 24%. This plan is designed that the liability should be adequately financed by the normal contributions, but overall, the liability goes up faster than the assets. The plan is easily to be underfunded and we need additional contributions.

## 2.6 Additional Contributions

We apply additional contributions to this plan as a modifying approach. We have introduced the basic idea of additional contributions in Section 2.4 and will explain further in this section.

When a plan's assets are smaller than the liabilities at time $t$, i.e., $Abf(t) < VSUM(t)$, the fund is in deficit. At this time, additional contributions totalling $AC(t)$ are collected from the members in service. The amount is the difference between the liabilities and the assets, divided by a coefficient $k_1$, indicating a target of $k_1$ years for eliminating the deficit (ignoring interest). On the other side, when the assets exceed the liabilities by a lot, e.g., $Abf(t) > 1.2VSUM(t)$, it is inefficient to hold too many assets. Additional contributions can still be the difference between the liabilities and the assets, divided by a coefficient $k_2$. The negative additional contributions are regarded as a reduction for the plan members' total contributions. The total contributions at $t$ are the sum of the original normal contribution $NCSUM(t)$ and $AC(t)$. The total contributions will not be negative, so the floor of $AC(t)$ is $-NCSUM(t)$.

$$AC(t) = \frac{VSUM(t) - Abf(t)}{k_1} \qquad \text{when } ALR(t) \leq 1.0, \tag{2.38}$$

$$AC(t) = \max\left(\frac{1.2\,VSUM(t) - Abf(t)}{k_2}, -NCSUM(t)\right) \qquad \text{when } ALR(t) \geq 1.2. \tag{2.39}$$

After receiving the additional contributions, we have

$$A(t) = Abf(t) - BSUM(t) + NCSUM(t) + AC(t). \tag{2.40}$$

Total contributions include additional contributions and the employees cover all the cost. Compared with normal contributions, total contributions are larger when the funding is in deficit and smaller when in surplus. The total contribution rate at $t$ would be

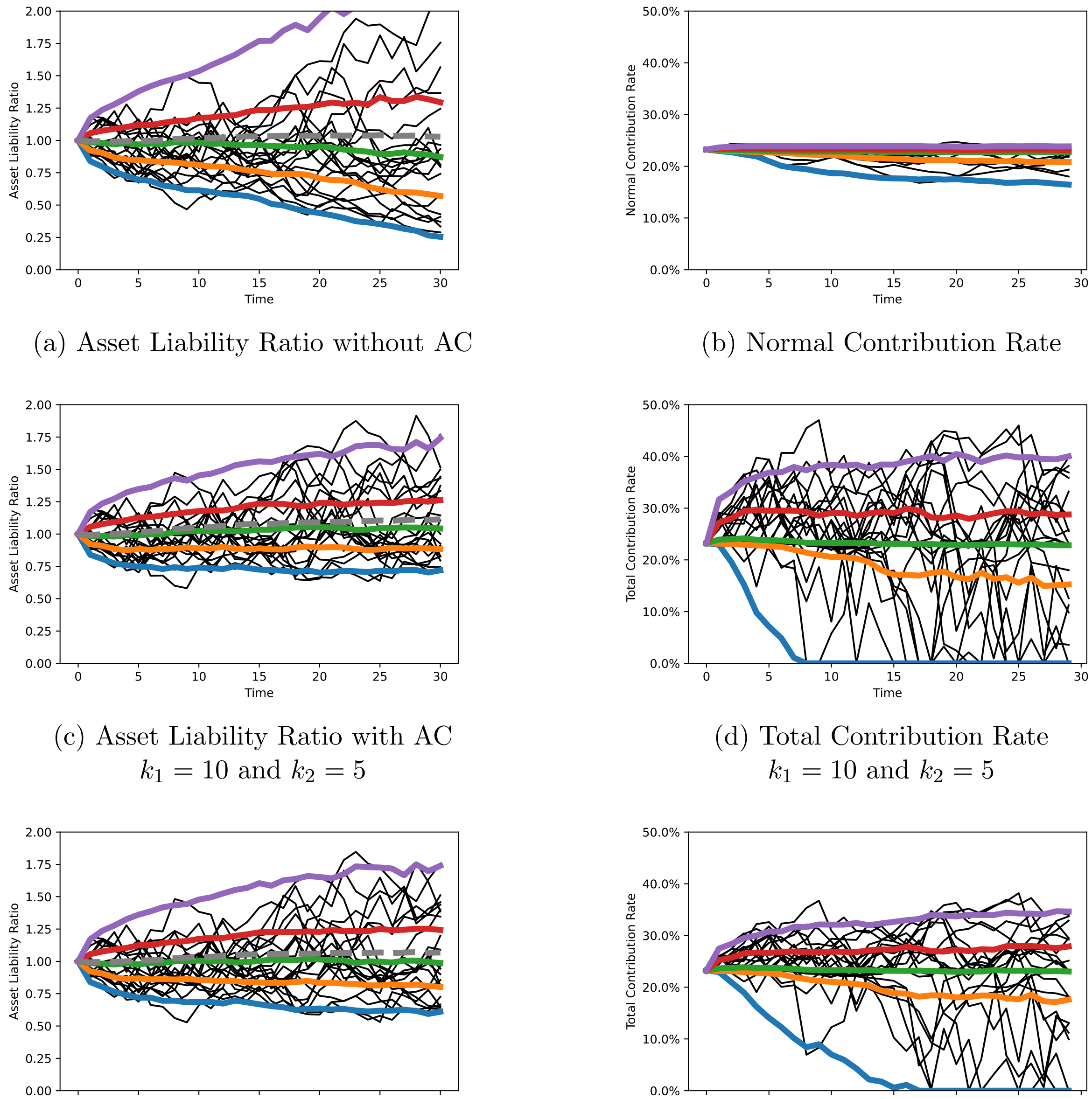
$$TCR(t) = \frac{NCSUM(t) + AC(t)}{\sum_{x \leq 64} L(x,t)s(x,t)}. \tag{2.41}$$

The degree of the asset liability ratio's rise and the volatility of the total contribution rate both depend on the amount of additional contributions, i.e., the size of the repayment relating to the coefficients $k_1$ and $k_2$. To have a clear recognition of the effects of additional contributions, we apply different coefficients, $k_1 = 10$ and $k_2 = 5$, and $k_1 = 20$ and $k_2 = 10$. We show the comparison in Figure 2.2, and specific results in Table 2.4

| Quantiles at $t = 30$ with Additional Contributions with $k_1 = 10$ and $k_2 = 5$ | | | |
|---|---|---|---|
| Quantiles | Asset Liability Ratio | Quantiles | Total Contribution Rate |
| 5% | 0.720 (0.698, 0.741) | 95% | 0.400 (0.391, 0.415) |
| 25% | 0.882 (0.861, 0.898) | 75% | 0.288 (0.279, 0.299) |
| 50% | 1.043 (1.023, 1.066) | 50% | 0.228 (0.227, 0.230) |
| 75% | 1.262 (1.234, 1.288) | 25% | 0.153 (0.128, 0.171) |
| 95% | 1.742 (1.640, 1.820) | 5% | 0.000 (0.000, 0.000) |
| Quantiles at $t = 30$ with Additional Contributions with $k_1 = 20$ and $k_2 = 10$ | | | |
| Quantiles | Asset Liability Ratio | Quantiles | Total Contribution Rate |
| 5% | 0.611 (0.581, 0.630) | 95% | 0.346 (0.340, 0.353) |
| 25% | 0.802 (0.783, 0.823) | 75% | 0.279 (0.271, 0.288) |
| 50% | 0.987 (0.966, 1.013) | 50% | 0.230 (0.229, 0.233) |
| 75% | 1.243 (1.211, 1.294) | 25% | 0.176 (0.160, 0.187) |
| 95% | 1.739 (1.684, 1.843) | 5% | 0.000 (0.000, 0.000) |

Table 2.4: Defined benefit plan results of 1,000 simulations; with different additional contributions; 95% confidence intervals in parentheses.

By adding the additional contributions with $k_1 = 10$ and $k_2 = 5$, we can see that the asset liability ratios improve significantly from Figure 2.2c. The difference between the quantile lines narrows down, especially between the 75% quantile and the 95% quantile. The median is flat with the additional contributions, and it is closer to the mean. The 20 paths have little autocorrelation. Increasing $k_1$ and $k_2$ allows recovery over longer time. From Figure 2.2e, this brings less improvements to the asset liability ratio. The 50% quantile at $t = 30$ with $k_1 = 20$ and $k_2 = 10$ is slightly below 1.0, ending at 0.987 with the 95% confidence interval $(0.966, 1.013)$, from Table 2.4.




(a) Asset Liability Ratio without AC

(b) Normal Contribution Rate

(c) Asset Liability Ratio with AC
$k_1 = 10$ and $k_2 = 5$

(d) Total Contribution Rate
$k_1 = 10$ and $k_2 = 5$

(e) Asset Liability Ratio with AC
$k_1 = 20$ and $k_2 = 10$

(f) Total Contribution Rate
$k_1 = 20$ and $k_2 = 10$

Figure 2.2: Defined benefit plan results of 1,000 simulations; with different additional contributions; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths.

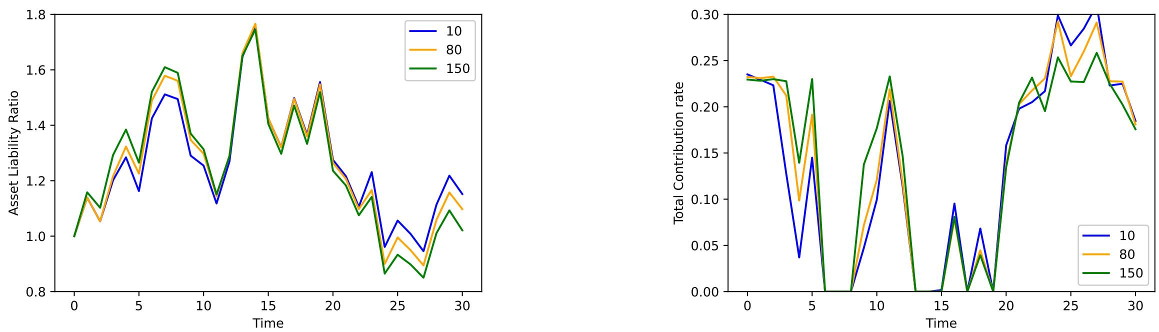
The improvement in the asset liability ratio from the additional contributions is at the expense of substantial additional volatility in the total contribution rate. From Figure 2.2a, with $k_1 = 10$ and $k_2 = 5$, we can tell that deficits are much more likely. The range of the total contribution rate explodes with the additional contributions, although the quantile lines become flat during the later time periods. In Figure 2.2d, the median stays around 23% and the 25% quantile line declines from $t = 12$. The 95% quantile line hovers around 40.0%, with the 95% confidence interval $(39.1\%, 41.5\%)$. The 5% quantile line reaches 0 at $t = 7$ and remains there to the end of the period. Increasing $k_1$ and $k_2$ to 20 and 10 respectively, the total contribution rate reduces, as we can see from Figure 2.2f. The 5% quantile line reaches 0 at $t = 16$ and the 95% quantile line ends at about 34.6%. From Table 2.4, we see that the 25% quantile and the 50% quantile rise but the 75% quantile drops compared with the smaller $k_1$ and $k_2$. The overall total contribution rate becomes less volatile with the increase in $k_1$ and $k_2$, but the asset liability risk is increased.

We note that 0% total contribution rate is inconsistent with fairness. It means that the current workers don't have to pay anything, but will still receive their pension benefit after they retire. In other words, the current active members get a free lunch.

## 2.7 Sensitivity Tests

We change the valuation interest rate margin, $\beta$, to see the effect on valuation and funding. The margin $\beta$ is 0.0081 when $w = 0.6$, so to assess the sensitivity, we set the margin to be 0.001, 0.008, and 0.015, and keep $w = 0.6$. We keep $k_1 = 10$ and $k_2 = 5$. We show the result of one path in Figure 2.3, and the result of 1,000 simulations in Table 2.5, along with the 95% confidence intervals in parentheses.

By increasing $\beta$, we observe directly the increase in $i(t)$ for each path from Equation (2.3). The assets at $t = 0$ would also decrease along with the liabilities as we assume the plan is 100% funded at the start of the projection. From Figure 2.3, both the asset liability ratio and the total contribution rate are the largest for $\beta = 0.015$ from $t = 0$ to 10, even though the plan is in surplus. Both are the smallest for $\beta = 0.015$ from $t = 20$ to the end. From Table 2.5, the increase in $\beta$ leads to the overall decrease in the asset liability ratio, except that the 95% quantile is highest when $\beta = 0.008$. For the total contribution rate, the 5% quantiles are all 0; the 25% quantile increases with the rise in $\beta$, and both the 50% and the 75% quantiles are similar for different values of $\beta$. The 95% quantile is the smallest when $\beta = 0.008$. By applying Equation (2.10) to get $\beta = 0.0081$, we get a plan for which the median of the asset liability ratio is over 1 and the total contribution rate is relatively concentrated under 40%.



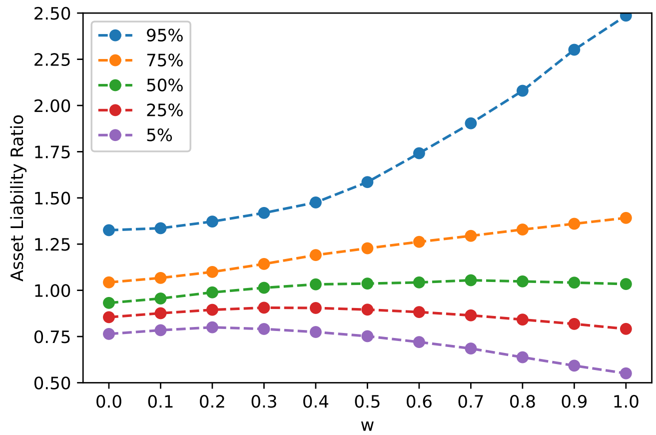

(a) Asset Liability Ratio (b) Total Contribution Rate

Figure 2.3: Sensitivity test on valuation interest rate margin, $\beta$, about defined benefit plan results of one simulation; with $k_1 = 10$ and $k_2 = 5$.

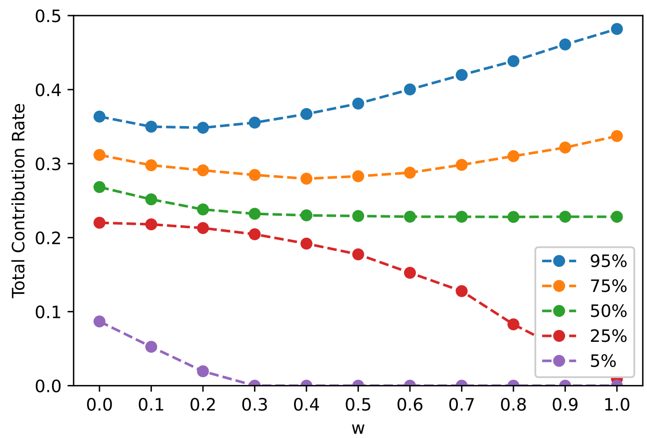
| Margin | 0.001 | 0.008 | 0.015 |
|---|---|---|---|
| Asset Liability Ratio Quantiles at $t = 30$ | | | |
| 5% | 0.726 (0.702, 0.741) | 0.720 (0.698, 0.741) | 0.708 (0.681, 0.722) |
| 25% | 0.885 (0.869, 0.901) | 0.882 (0.861, 0.898) | 0.860 (0.847, 0.882) |
| 50% | 1.047 (1.028, 1.067) | 1.043 (1.023, 1.066) | 1.025 (1.008, 1.048) |
| 75% | 1.275 (1.248, 1.301) | 1.262 (1.234, 1.289) | 1.237 (1.213, 1.270) |
| 95% | 1.728 (1.668, 1.886) | 1.741 (1.640, 1.821) | 1.689 (1.605, 1.782) |
| Total Contribution Rate Quantiles at $t = 30$ | | | |
| 5% | 0.000 (0.000, 0.000) | 0.000 (0.000, 0.000) | 0.000 (0.000, 0.000) |
| 25% | 0.152 (0.125, 0.174) | 0.153 (0.128, 0.171) | 0.163 (0.136, 0.178) |
| 50% | 0.228 (0.227, 0.231) | 0.228 (0.227, 0.230) | 0.227 (0.222, 0.229) |
| 75% | 0.288 (0.279, 0.302) | 0.288 (0.279, 0.299) | 0.289 (0.279, 0.299) |
| 95% | 0.405 (0.396, 0.418) | 0.400 (0.391, 0.415) | 0.401 (0.389, 0.414) |

Table 2.5: Sensitivity test on the valuation interest rate margin, $\beta$, defined benefit plan results of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; 95% confidence intervals in parentheses.

Even though the 60/40 equity/bond portfolio is classic, we can modify the proportion of assets invested in equity, $w$ (Gerber and Shiu, 2000). We keep $\beta$ the same to keep the same $i(t)$ under Equation (2.10), so that the liabilities would also be the same. By just changing $w$, we perform a sensitivity test to explore the relationship between $w$ and the




(a) Asset Liability Ratio
for different equity weighting, $w$




(b) Total Contribution Rate
for different equity weighting, $w$

Figure 2.4: Sensitivity test on $w$ about defined benefit plan results of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; 5%, 25%, 50%, 75%, and 95% quantiles at $t = 30$.

plan performance.

In Figure 2.4a, we observe that the range of the asset liability ratio quantiles gets larger when $w \geq 0.2$. The asset liability ratio range expands around the median in some degree as $w$ rises. It is expected that more assets invested in equity would bring more returns and risks. There is an increase in all quantiles from $w = 0$ to $w = 0.2$.

Since increasing the proportion of assets invested in equity increases the volatility of the asset liability ratio, the total contribution rate is also expected to be more volatile as $w$ increases. From Figure 2.4b, 5% quantile of the total contribution rate at $t = 30$ decreases to 0 when $w = 0.3$. The 25% quantile line is also declining. The 50% quantile decreases for low values of $w$, and keeps flat for $w \geq 0.4$. The 75% quantile displays a convex pattern with the minimums occurring at $w = 0.4$, and the 95% quantile minimum is at $w = 0.2$. Overall, our benchmark will still be $w = 0.6$, but we recognise that lower values would decrease the default risk.

# Chapter 3

# Alternative Methods of Applying Cost of Living Adjustments

In this chapter, we consider whether different COLA methods can reduce the costs and risks.

- The first method is straightforward, reducing COLA from 100% to 80%. We improve the pension plan security, but make the pension benefits less adequate.

- The second method is applying a two-tier COLA. In this method, we set a salary threshold which divides the pension benefit into two components. We apply full COLA on benefits based on final average salary up to the threshold, and partial COLA for the above-threshold part. If a member's final average salary is below the threshold, the member will still receive full COLA.

  The Yearly Maximum Pensionable Earnings (YMPE) is used for the threshold. It is the maximum amount of earnings which are used for the Canada Pension Plan, and is already embedded in most Canadian DB plans. It is set by the Canadian government. The YMPE increases every year in proportion to average earnings. The threshold at time $t$ is denoted $YMPE(t)$, and we set $YMPE(0)$ to be the current value. Then

  $$YMPE(t) = YMPE(t-1)\,(1 + gs(t))\,. \tag{3.1}$$

  We assume that benefits based on salary up to the YMPE are awarded full COLA with 3% cap, while benefits based on salary over the YMPE are only eligible for 50% COLA.

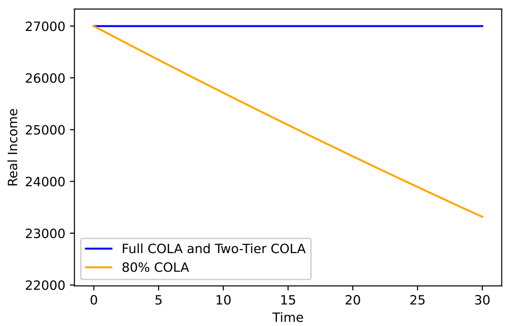
Compared with the original plan with full COLA, the two methods both reduce members' benefits and the liabilities. However, they generate different results for low-paid and high-paid members' benefits. Applying 80% COLA decreases a member's real income regardless of the salary. But there are differences between the benefits for the members with high and low salaries with the two-tier COLA method. The low-paid members who earn salaries lower than the YMPE will still receive full COLA on their benefits under the two-tier COLA method. This is appropriate for the low-paid members because they lack discretionary savings. The two-tier COLA method protects their purchasing power of their benefits. For high-paid members, who earn the salaries exceeding the YMPE, part of their benefits increases by 50% COLA. By guaranteeing the below-YMPE benefit part can be increased by full inflation, the high-paid members' purchasing power is also guaranteed up to the pension based on YMPE. Even though their real income declines, their higher income provides them with more flexibility.

For example, we assume that a member's average years of service at retirement is 30, and the accrual rate is $\alpha = 1.8\%$. We set the inflation rate $js(t) = 2.5\%$ and $YMPE(0) = 60,000$. We set the member's final average salary $FAS(65, 0)$ to be 50,000, 100,000, or 150,000 to see the effects of the different methods of applying COLA on real income with the different salaries. The results are shown in Figure 3.1. When full COLA is applied, real incomes are constant, as indicated by the blue lines in Figure 3.1. Using 80% COLA reduces real income at the same declining rate for members with different salaries, indicated by the orange lines.

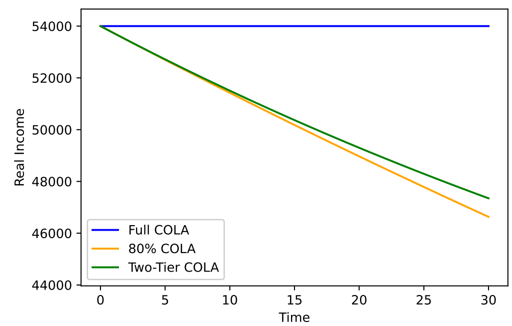
However, by using the two-tier COLA method

- The member with $FAS(65, 0) =$ 50,000 will have the same real income as applying full COLA.
- The member with $FAS(65, 0) =$ 100,000 will have a similar declining real income as when applying 80% COLA.
- The member with $FAS(65, 0) =$ 150,000 will have a faster decline in real income compared with applying 80% COLA.

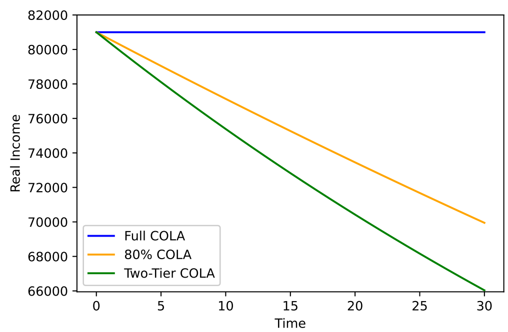
However, the starting benefit for the low-paid member is 27,000 compared with the high-paid member's 81,000. The diminishing benefits by offering reduced COLA may not be adequate to meet the minimum needs for the the low-paid members. In contrast, the two-tier COLA method provides full protection for low-paid members' real income after retirement.




(a) FAS(65,0) = 50,000




(b) FAS(65,0) = 100,000




(c) FAS(65,0) = 150,000

Figure 3.1: Real income with different benefit structures of retired members with different salary scales.

## 3.1 Plan Design

The benefit in payment using 80% COLA is

$$b(x,t) = b(x-1,t-1)(1+0.8\times js(t)) \text{ for } x \geq 66 \tag{3.2}$$

compared to Equation (2.19). The liabilities are updated to

$$V(x,t) = L(x,t)b(x,t)\ {}_{65-x}p_x^{(\tau)}\ (1+i(t))^{-(65-x)}\ \ddot{a}^{(*)}_{(65,t)} \qquad \text{for } x \leq 65, \tag{3.3}$$

$$V(x,t) = L(x,t)b(x,t)\ \ddot{a}^{(*)}_{(x,t)} \qquad \text{for } x \geq 66, \tag{3.4}$$

where

$$\ddot{a}^{(*)}_{(x,t)} = \sum_{k=0}^{105-x} {}_kp_x \left(\frac{1+i(t)}{1+0.8\times j(t)}\right)^{-k}. \tag{3.5}$$

The two-tier COLA method separates $b(x,t)$ into two elements. We set

$$b_{below}(x,0) = \frac{\alpha\times\min\left(FAS(x,0), YMPE(0)\right)\times ny(x,0)}{(1+gs(0))^{\max(x-65,0)}}, \tag{3.6}$$

$$b_{above}(x,0) = \frac{\alpha\times\max\left(FAS(x,0) - YMPE(0), 0\right)\times ny(x,0)}{(1+gs(0))^{\max(x-65,0)}}. \tag{3.7}$$

Similarly, $b(x,t)$ for $x \leq 65$ and $t \geq 1$,

$$b_{below}(x,t) = \alpha\times\min\left(FAS(x,t), YMPE(t)\right)\times ny(x,t), \tag{3.8}$$

$$b_{above}(x,t) = \alpha\times\max\left(FAS(x,t) - YMPE(t), 0\right)\times ny(x,t). \tag{3.9}$$

By having the two different benefit parts, we can apply different COLA,

$$b_{below}(x,t) = b_{below}(x-1,t-1)(1+js(t)) \qquad \text{for } x \geq 66, \tag{3.10}$$

$$b_{above}(x,t) = b_{above}(x-1,t-1)(1+0.5\times js(t)) \qquad \text{for } x \geq 66. \tag{3.11}$$

The liabilities are the sum of the two parts,

$$\begin{aligned} V(x,t) = &L(x,t)\left(b_{below}(x,t)\ {}_{65-x}p_x^{(\tau)}\ (1+i(t))^{-(65-x)}\ \ddot{a}_{(65,t)}\right) \\ &+L(x,t)\left(b_{above}(x,t)\ {}_{65-x}p_x^{(\tau)}\ (1+i(t))^{-(65-x)}\ \ddot{a}^{(**)}_{(65,t)}\right) \qquad \text{for } x \leq 65, \end{aligned} \tag{3.12}$$

$$V(x,t) = L(x,t)\left(b_{below}(x,t)\ddot{a}_{(65,t)}\right) + L(x,t)\left(b_{above}(x,t)\ddot{a}^{(**)}_{(65,t)}\right) \qquad \text{for } x \geq 66, \tag{3.13}$$

where

$$\ddot{a}^{(**)}_{(x,t)} = \sum_{k=0}^{105-x} {}_kp_x \left( \frac{1+i(t)}{1+0.5\times j(t)} \right)^{-k}. \tag{3.14}$$

The withdrawal benefits will be separated similarly to the benefits under the two-tier COLA method.

If a member's final average salary at age 65 is below the $YMPE(t)$, full benefit is always inflated with full COLA up to the 3% cap. However, if the final average salary is above the $YMPE(t)$, we would have two parts of the benefit. Part of the going benefits is inflated with 50% COLA.

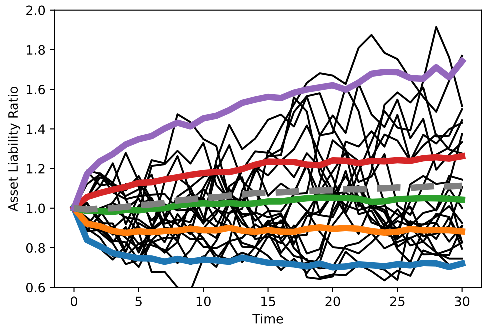
## 3.2 Plan Results

We set $k_1 = 10$ and $k_2 = 5$ for the additional contributions. In 2020, the YMPE is \$58,700 (Government of Canada, 2020). We apply $YMPE(0) = 60$ for convenience, noting that our units are \$000s. In our model demographics, the average salary exceeds the YMPE(0) by age 42 at time 0.

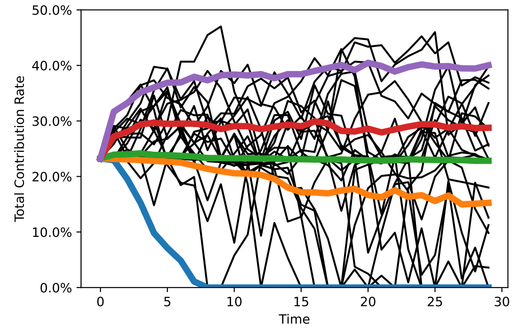
| Method | 100% COLA | 80% COLA | Two-Tier COLA |
|---|---|---|---|
| Asset Liability Ratio Quantiles | | | |
| 5% at $t = 15$ | 0.724 (0.709, 0.744) | 0.726 (0.707, 0.738) | 0.727 (0.711, 0.742) |
| 5% at $t = 30$ | 0.720 (0.698, 0.741) | 0.718 (0.693, 0.736) | 0.724 (0.702, 0.744) |
| 50% at $t = 15$ | 1.034 (1.015, 1.053) | 1.035 (1.014, 1.052) | 1.040 (1.020, 1.060) |
| 50% at $t = 30$ | 1.043 (1.023, 1.066) | 1.039 (1.014, 1.057) | 1.051 (1.025, 1.068) |
| Total Contribution Rate Quantiles | | | |
| 50% at $t = 15$ | 0.231 (0.229, 0.233) | 0.220 (0.218, 0.222) | 0.225 (0.224, 0.227) |
| 50% at $t = 30$ | 0.228 (0.227, 0.230) | 0.218 (0.216, 0.219) | 0.223 (0.222, 0.224) |
| 95% at $t = 15$ | 0.384 (0.379, 0.400) | 0.365 (0.358, 0.380) | 0.372 (0.363, 0.384) |
| 95% at $t = 30$ | 0.400 (0.391, 0.415) | 0.383 (0.368, 0.394) | 0.387 (0.374, 0.401) |

Table 3.1: Defined benefit plan results of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with different benefit structures; 95% confidence intervals in parentheses.

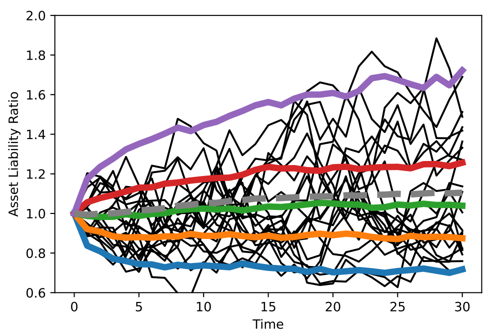
In Figure 3.2 we show the quantiles and sample paths for the asset liability ratios and the total contribution rates for the three different benefit structures. The asset liability ratios are very similar. We mainly focus on the 5% and the 50% quantiles of the asset



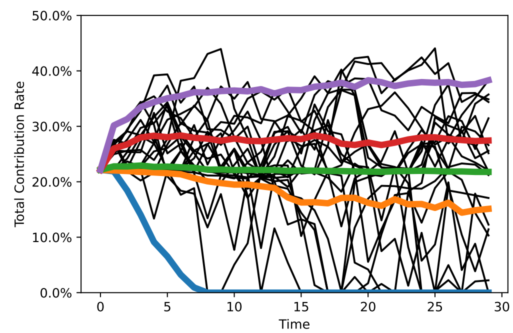

(a) Asset Liability Ratio
with 100% COLA



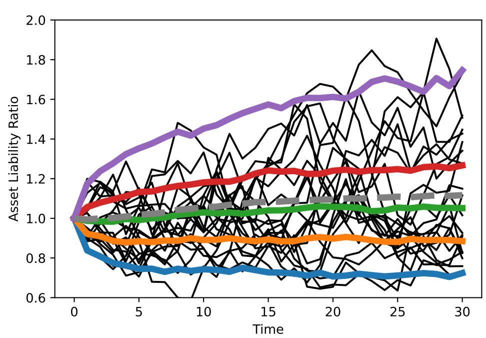

(b) Total Contribution Rate
with 100% COLA



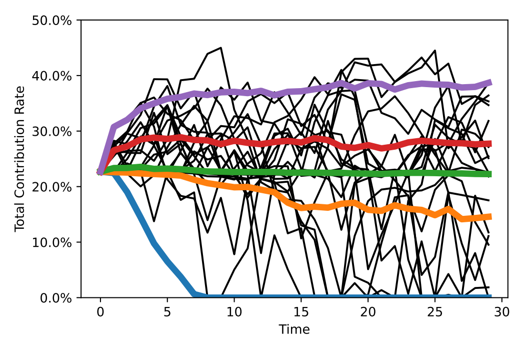

(c) Asset Liability Ratio
with 80% COLA




(d) Total Contribution Rate
with 80% COLA




(e) Asset Liability Ratio
with Two-Tier COLA




(f) Total Contribution Rate
with Two-Tier COLA

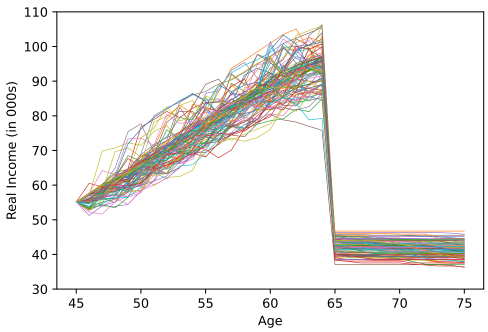
Figure 3.2: Defined benefit plan results of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with different benefit structures; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths.

liability ratio. By taking a closer look, in Table 3.1, we can see that both quantiles increase at $t = 15$ but decrease at $t = 30$ when we reduce COLA. When we use the two-tier COLA method, the asset liability ratio rises. Also, we see that reducing COLA from 100% to 80% generates slightly lower 5% and 50% quantiles than applying the two-tier COLA method at $t = 15$, though the values are close.

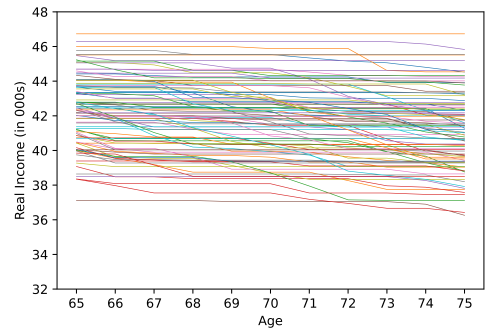
When it comes to the total contribution rate, we see in Figure 3.2 that the quantiles and the sample paths are still quite similar. The 95% quantiles under the two methods are below 40.0%. From Table 3.1, we see that the median total contribution rate also drops. Reducing COLA and using the two-tier COLA method generate similar total contribution rates, and reducing COLA has a slightly lower total contribution rate.

Even though applying either the 80% COLA or the two-tier COLA achieve similar results in terms of improving the plan management, the impacts on the members' benefits may be totally different. We investigate and track the real income of an active member age 45 at $t = 0$, who stays in the pension plan until the retirement age 65, and survives to age 75. The income before retirement would be the salary minus the total contributions, and after retirement, the income is the pension benefit.

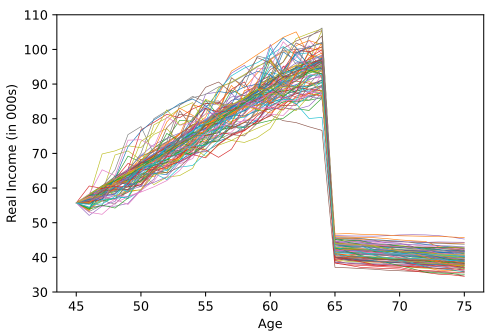
In Figure 3.3, we show 100 randomly chosen paths of the inflation-adjusted income of a member under the different benefit structures. We use the same 100 paths for every income plot in this paper. Although the incomes are inflation adjusted, the income before retirement shows an upward trend because we have the salary scale, and the actual salary growth $gs(t)$ is 50 basis points above the actual inflation rate $js(t)$ for each $t$. The incomes before retirement are similar for the three methods of applying COLA. According to the total contribution rate in Table 3.1, the 95% quantile with 100% COLA is highest among the three methods. Also, the 5% quantiles all end at 0% contribution, from Figure 3.2. When the salary is fixed, the incomes depend on the total contributions. High total contribution rates would decrease incomes. The top income before retirement in each plot is the same with 0% contribution, and the bottom has a slight increase using the reduced COLA method or the two-tier COLA method. Overall, the volatility of income before retirement is the lowest with 80% COLA, then with the two-tier COLA, and then with 100% COLA.



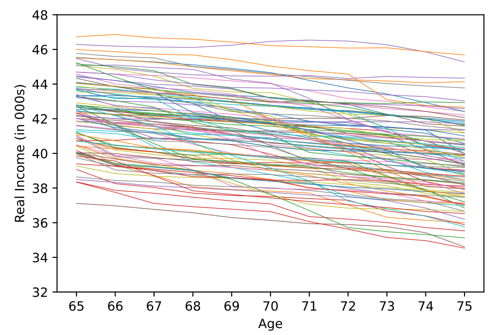

(a) 100% COLA



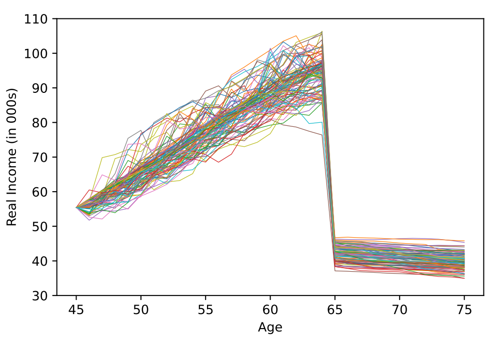

(b) 100% COLA



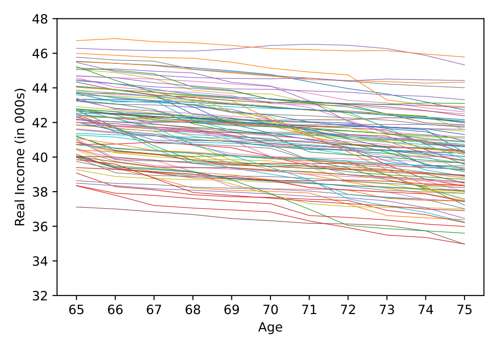

(c) 80% COLA




(d) 80% COLA




(e) Two-Tier COLA




(f) Two-Tier COLA

Figure 3.3: Real income of defined benefit plan results of 100 simulations; with $k_1 = 10$ and $k_2 = 5$; with different benefit structures; for an active member starting from $x = 45$, in \$000s, using time 0 money values.

| Method | Mean Real Income at Age 65 | Mean Real Income at Age 75 |
|---|---|---|
| 100% COLA | 42.222 (1.848%) | 41.138 (2.103%) |
| 80% COLA | 42.222 (1.848%) | 39.448 (2.154%) |
| Two-Tier COLA | 42.222 (1.848%) | 39.776 (2.112%) |

Table 3.2: Mean real income of defined benefit plan results of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with different benefit structures; for an active member starting from $x = 45$, in \$000s, using time 0 money values; estimated standard errors in parentheses.

To have a closer look at the inflation-adjusted income, we also plot the real income after retirement for each benefit structure. With the inflation adjustment, the benefit after retirement is mostly flat with full COLA in Figure 3.3b because the benefit is inflated with 3% maximum. In Figure 3.3d and Figure 3.3f, the benefits show a decline. We can see the mean real incomes at age 65 and 75 under the different benefit structures in Table 3.2. The incomes at age 65 are the same, and reducing the COLA leads to the benefit decreasing more than under the two-tier COLA method. By setting different thresholds, and COLA deduction rates, the effects would be different. For example, if the threshold is a smaller figure, more of the benefits is increased by the lower COLA. Or instead of 50% COLA, we could apply 20% COLA on the above-threshold part of the benefit. Therefore, the total benefits would be smaller.

Overall, applying 80% COLA and the two-tier COLA method have similar results on the whole plan. Both the solvency risk and the cost risk decrease slightly. The costs, in terms of median contributions, are very similar. Moreover, the two-tier COLA method protects the benefits of the members with low salaries. In the next chapter, we will consider a heterogeneous workforce and compare the different experiences of lower-paid (blue-collar) and higher-paid (white-collar) members in one plan.

# Chapter 4

# A Heterogeneous Plan

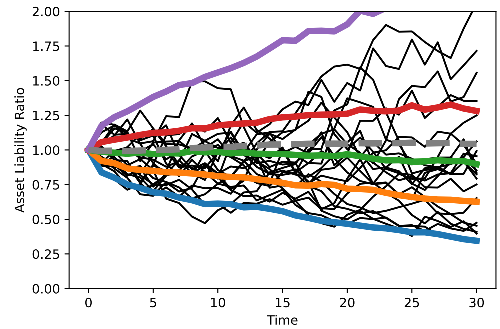
We used general aggregated membership information, summarized in Appendix A in the previous chapters. We now separate blue-collar (lower-paid) and white-collar (higher-paid) members, and assume they are in the same DB pension plan, and that blue-collar and white-collar members are treated equally within the plan. In this chapter, we follow the two-tier COLA method with $YMPE(0) = 60$ as the threshold, with 100% COLA applied below the YMPE and 50% COLA applied above the YMPE based on the final average salary. When we include additional contributions, we set $k_1 = 10$ and $k_2 = 5$.

## 4.1 Demographics

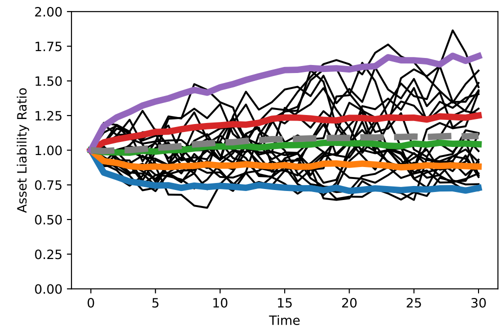
We modify the general membership information in response to the demographic of blue-collar and white-collar members and run the DB pension plan model for the two groups separately. For both groups, we keep the same number of members $L(x, 0)$, the number of new entrants $NE(x)$, and the average years of service $ny(x, 0)$. The differences are as follows, and the data is given in the appendix.

For the blue-collar members,

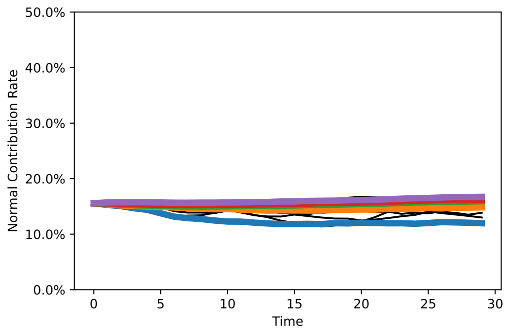
- Their average salary $s(x, 0)$ is lower than the general members', and their salary growth is flat from age 35. We set $s(25, 0) = 28.0$ and $s(65, 0) = 35.0$, compared with $s(25, 0) = 32.0$ and $s(65, 0) = 100.2$ in the general plan. Their average salary never exceeds the YMPE, which is growing at the same rate as the salary growth rate.



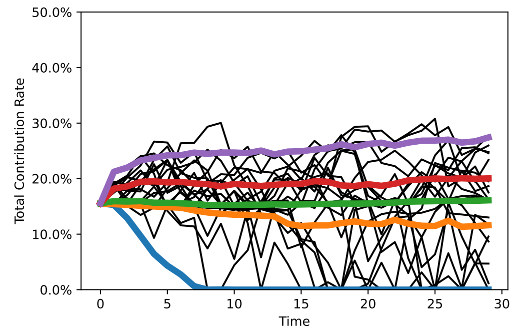

(a) Asset Liability Ratio without AC



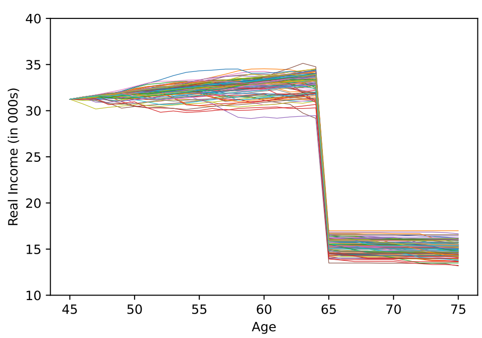

(b) Asset Liability Ratio with AC



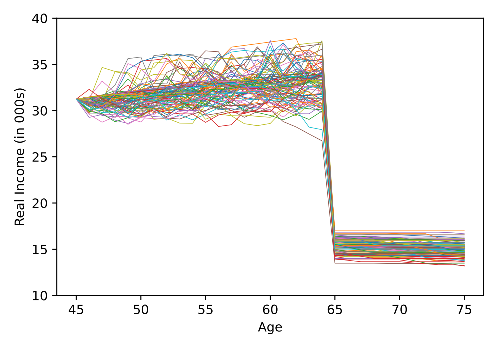

(c) Normal Contribution Rate




(d) Total Contribution Rate




(e) Income without AC




(f) Income with AC

Figure 4.1: Defined benefit plan result of 1,000 simulations; with two-tier COLA method for blue-collar; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths; real income of 100 simulations for an active blue-collar member starting from $x = 45$, in \$000s, using time 0 money values.

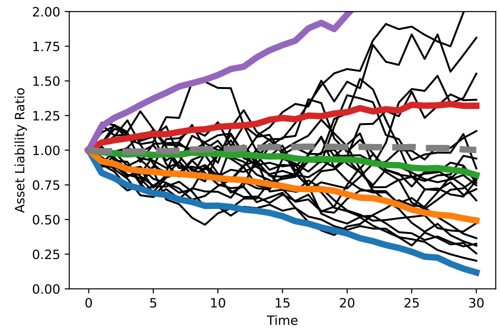
- Their initial benefit after retirement is 15.5 compared with 44.4 for the general plan. The replacement rate is 44.3% in both plans.
- We assume that the active blue-collar members have a higher probability of staying in the plan than the general plan membership.
- We assume that the blue-collar members have a higher mortality rate after retirement. The data is from RP-2014 Mortality Tables Report.

Because the blue-collar members' salaries are not high, their benefits are inflated with full COLA. The asset liability ratios for the blue collar members, assuming a seperate plan, are shown in Figure 4.1a and 4.1b. In Figure 4.1a, the 5% quantile line ends at about 0.35 and the 50% quantile line is close to 1.0. The reason why the blue-collar members demand less additional contributions is because their salary growth is low. With the additional contributions, the median is above 1.0 at $t = 30$ and almost coincides with the mean in Figure 4.1b. The 5% quantile is about 0.75. In Figure 4.1c, the quantile lines of the normal contribution rate are smaller as we expected. The median total contribution rate stays level in Figure 4.1d. The total contribution rate is less volatile than the general plan and the 95% quantile is below 30%. From Figure 4.1e and 4.1f, we see the real income with the additional contributions is volatile before retirement because of the variations in the total contributions. The income after retirement is flat because the blue-collar members' final average salary is below the YMPE. The two-tier COLA method protects the blue-collar members' benefit real value after retirement.

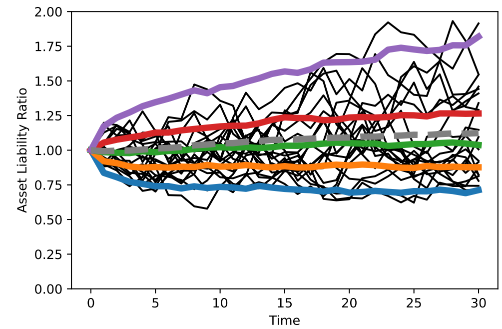
For white-collar members,

- Their average salary at age 25 is the same as the general members', where $s(25, 0) = 32.0$. The growth rate is higher and $s(65, 0) = 108.5$. Their average salary would be over the YMPE from age 41.
- Their benefit $b(65, 0)$ increases to 48.0. The replacement rate is 44.3% in both plans.
- We assume higher withdrawal rates at young ages and lower withdrawal rate at ages before the retirement age than the general plan membership.
- We assume that the white-collar members have a lower mortality rate after retirement. The data is from RP-2014 Mortality Tables Report.

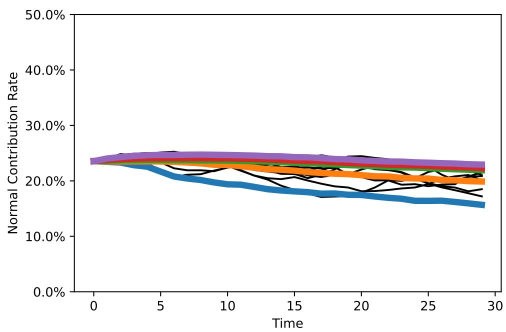
From Figure 4.2a, the white-collar members' pension plan's asset liability ratio is more volatile than the blue-collar members'. The 5% quantile line is near 0.1, and this would be



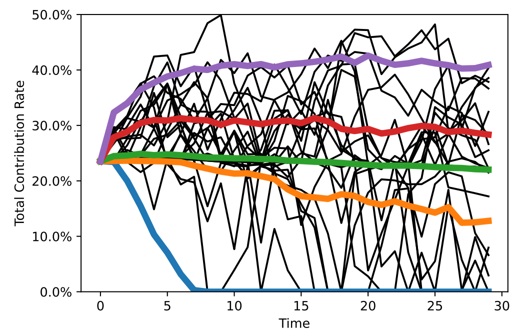

(a) Asset Liability Ratio without AC



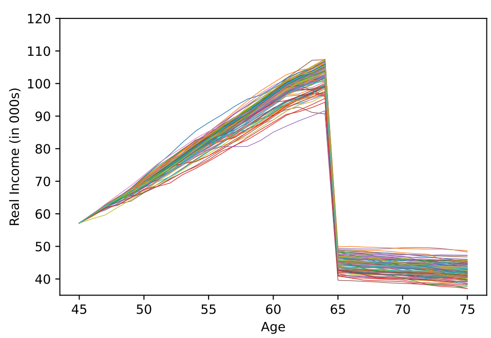

(b) Asset Liability Ratio with AC



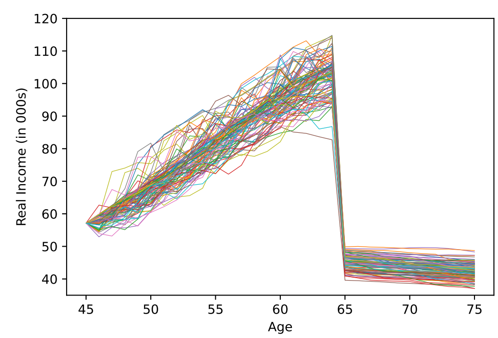

(c) Normal Contribution Rate without AC




(d) Total Contribution Rate with AC




(e) Income without AC




(f) Income with AC

Figure 4.2: Defined benefit plan result of 1,000 simulations; with two-tier COLA method for white-collar; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths; real income of 100 simulations for an active white-collar member starting from $x = 45$, in \$000s, using time 0 money values.

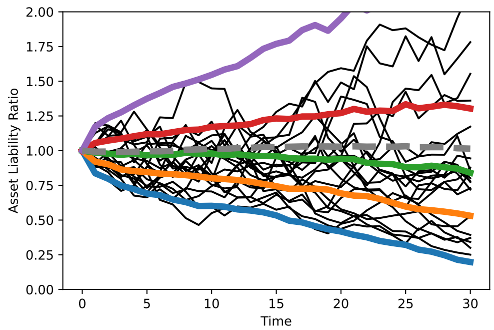
unsustainable. In Figure 4.2c, the normal contribution rate is higher than the blue-collar members'. After we apply the additional contributions and the results are shown in Figure 4.2b and 4.2d, the asset liability ratio is similar to the blue-collar members' but the total contribution rate is very volatile. The 95% quantile is over 40% and one path hits 50% in Figure 4.2f. The income after retirement in both Figure 4.2e and 4.2f shows a decline due to 50% COLA on benefits based on the final average salary above the YMPE.

## 4.2 Plan Results

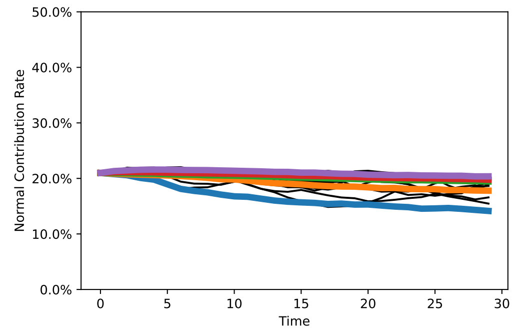
We now assume the blue-collar and white-collar members form equal numbers in one plan, and that all contributions are paid by members. The heterogeneous plan model follows the same assumptions as in Section 4.1. To first have a general observation of the heterogeneous plan, we assume there are no additional contributions.

In Figure 4.3a and Figure 4.3b we show the quantiles and sample paths of the asset liability ratio and the normal contribution rate for the heterogeneous plan. Compared with the plots for two groups having their own individual pension plans, the heterogeneous plans' plots resemble the weighted average of two individual plan's plots. By $t = 30$, the 5% quantile line is about 0.198, with the 95% confidence interval $(0.147, 0.238)$. The 50% quantile line is lower than 1.0. We still have substantial solvency risk in this plan, so additional contributions are necessary. We show the results in Figure 4.3c and Figure 4.3d. The asset liability ratio is well-controlled with additional contributions, where the median is over 1.0, and the 5% quantile is 0.717 with the 95% confidence interval $(0.694, 0.734)$. Nevertheless, the total contribution rate is volatile. The 5% quantile is 0, and the 95% quantile is 35.3%, with the 95% confidence interval $(34.2\%, 36.6\%)$. In Figure 4.3d, one path's peak is over 40% and some paths' falling to 0. Most paths lie between 20% and 35%, and the range brings the potential risk of affordability that members cannot afford the high total contributions. The overall total contribution rate for the members in the heterogeneous plan is higher than the blue-collar members' and lower than the white-collar members'.

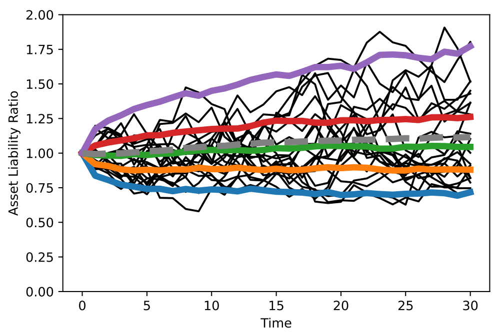
Next, we plot the real income after adding the additional contributions for the two groups and compare with the income when they are in individual plans. In Figure 4.4a and 4.4b, the red lines are the white-collar members' real income and the blue lines are the blue-collar members'. The benefit is the same for both groups since we don't change the benefit structure. But the homogeneous plan has different impacts on income before retirement.



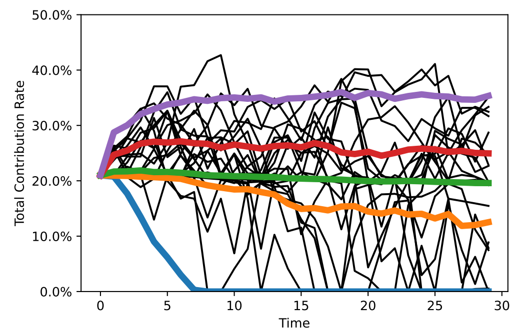

(a) Asset Liability Ratio without AC




(b) Normal Contribution Rate

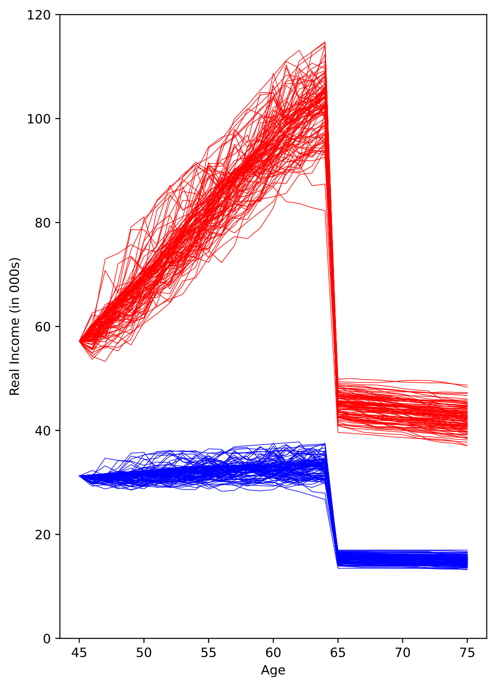



(c) Asset Liability Ratio with AC

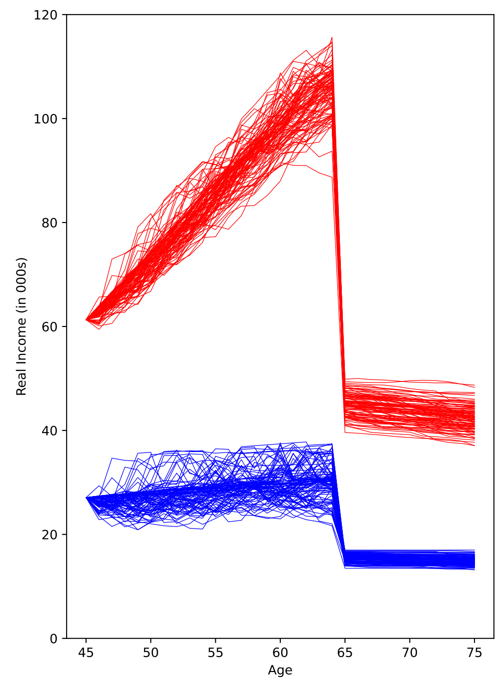



(d) Total Contribution Rate

Figure 4.3: Heterogeneous defined benefit plan result of 1,000 simulations; with two-tier COLA method; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths.




(a) Members in Individual Plan




(b) Members in Heterogeneous Plan

Figure 4.4: Real income of defined benefit plan result of 100 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; for an active member individually and heterogeneously starting from $x = 45$, in \$000s, using time 0 money values; red is white-collar, blue is blue-collar.

| | Mean Real Income | |
|---|---|---|
| | In Individual Plan | In Heterogeneous Plan |
| White Collar at age 45 | 57.144 (0.000%) | 61.298 (0.000%) |
| White Collar at age 64 | 102.418 (3.801%) | 105.686 (5.835%) |
| Blue Collar at age 45 | 31.242 (0.000%) | 27.087 (0.000%) |
| Blue Collar at age 64 | 32.939 (1.232%) | 30.223 (4.143%) |

Table 4.1: Mean real income of defined benefit plan result of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; for an active member individually and heterogeneously starting from $x = 45$, in \$000s, using time 0 money values; estimated standard errors in parentheses.

For the blue-collar members, the income risk before retirement is worse in the heterogeneous plan. Both the 50% and the 95% quantile of the total contribution rate increase from Figure 4.1d to Figure 4.3d. The high contribution rates then decrease the blue-collar members' income before retirement. The downside income is much lower in Figure 4.4b compared with Figure 4.4a. The highest income boundary is stable. The 5% quantile lines of the total contribution rate both end with 0, so there are no contributions but only salaries. The thickest cluster also falls off because of the increase in the total contribution rate's median. From Table 4.1, the incomes at age 45 and age 64 both decrease when the blue-collar members are in the heterogeneous plan. Even though the income growth rate from age 45 to 64 based on the mean income increase, the standard error for the income at age 64 also is higher in the heterogeneous plan. For the white-collar members, on the contrary, the downside of income shrinks up. The volatility of income before retirement decreases in the heterogeneous plan.

## 4.3 Benefit Structure Alternative

The conclusion of the previous section is that when blue-collar members share one pension plan with white-collar members, their income before retirement has more downside risk, compared with a separate pension plan. That is, blue-collar members would prefer to have their own pension plan rather than share one with white-collar members.

To mitigate this problem, and attempt to improve fairness in the heterogeneous plan, we add a cap on the final average salary. The cap at $t$ is denoted $FCAP(t)$, and increases in line with general earnings.

$$FCAP(t) = FCAP(t-1)\left(1 + gs(t)\right), \tag{4.1}$$

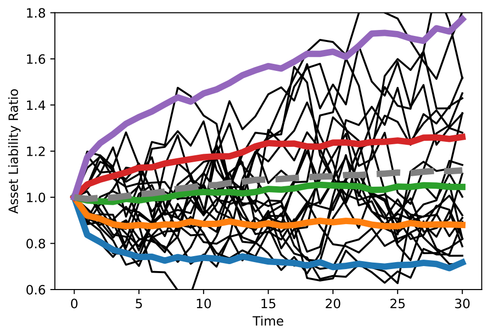
$$FAS(x,t)_{cap} = \min\left(FAS(x,t), FCAP(t)\right). \tag{4.2}$$

While the two-tier COLA method applies partial COLA on the above-threshold part, setting the cap larger than the threshold in the two-tier COLA method restricts the amount of that part. We now have two thresholds, $YMPE(t)$ and $FCAP(t)$. The pensionable salary is limited to the cap, and the contributions which are based on the pensionable earnings are also limited. The members can still invest by themselves to provide benefits on the salary above the cap. We set $FCAP(0) = 85$ initially, and will consider other values below.

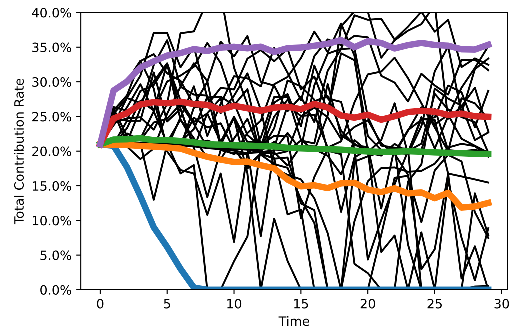
| | Quantiles at $t = 30$ | | | |
|---|---|---|---|---|
| | Asset Liability Ratio | | Total Contribution Rate | |
| No Cap | 5% | 0.717 (0.694, 0.734) | 95% | 0.353 (0.342, 0.366) |
| | 25% | 0.880 (0.857, 0.892) | 75% | 0.250 (0.242, 0.262) |
| | 50% | 1.045 (1.019, 1.062) | 50% | 0.196 (0.195, 0.197) |
| | 75% | 1.261 (1.237, 1.294) | 25% | 0.125 (0.101, 0.145) |
| | 95% | 1.772 (1.694, 1.874) | 5% | 0.000 (0.000, 0.000) |
| Have Cap | 5% | 0.710 (0.688, 0.732) | 95% | 0.330 (0.320, 0.343) |
| | 25% | 0.876 (0.851, 0.890) | 75% | 0.233 (0.224, 0.243) |
| | 50% | 1.035 (1.014, 1.058) | 50% | 0.180 (0.179, 0.180) |
| | 75% | 1.255 (1.231, 1.290) | 25% | 0.111 (0.090, 0.132) |
| | 95% | 1.791 (1.681, 1.912) | 5% | 0.000 (0.000, 0.000) |

Table 4.2: Heterogeneous defined benefit plan result of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; with or without cap on final average salary; 95% confidence intervals in parentheses.

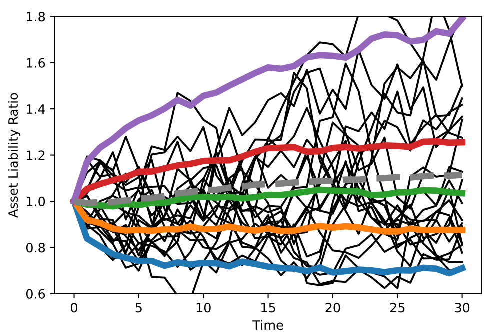
We present the asset liability ratio and the total contribution rate under this benefit structure in Figure 4.5. We also show the end points of all quantiles at $t = 30$ in Table 4.2, along with the 95% confidence intervals in parentheses. By adding the cap on final average salary, we observe that all the quantiles of the asset liability ratio slightly decrease expect the 95% quantile. From Figure 4.5d, the median decreases to 18.0% compared with Figure 4.5b. From Table 4.2, all quantiles of the total contribution rate decrease because of the decrease in the liabilities by setting the cap.



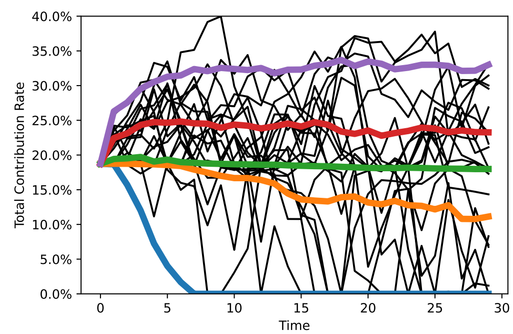

(a) Asset Liability Ratio without Cap




(b) Total Contribution Rate without Cap

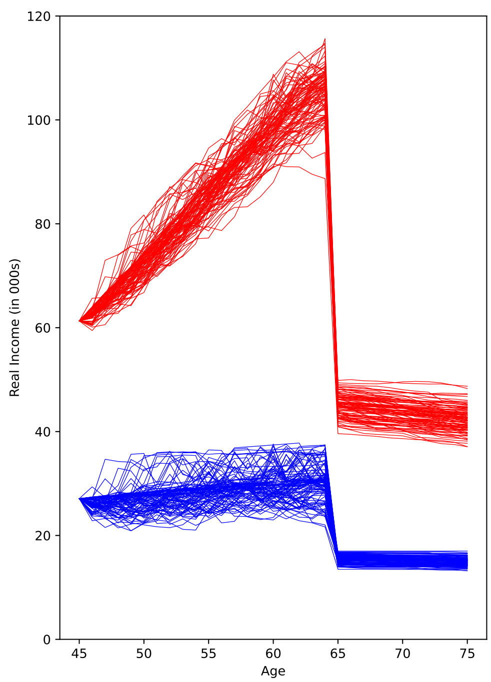



(c) Asset Liability Ratio with $FCAP(0) = 85$

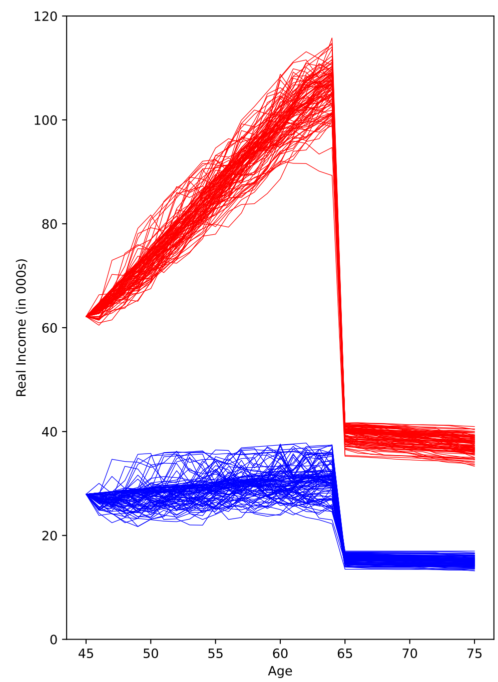



(d) Total Contribution Rate with $FCAP(0) = 85$,

Figure 4.5: Heterogeneous defined benefit plan result of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; with or without cap on final average salary; 5%, 25%, 50%, 75%, and 95% quantiles, with mean asset liability ratio and 20 sample paths.




(a) Members in Heterogeneous Plan without Cap




(b) Members in Heterogeneous Plan with $FCAP(0) = 85$

Figure 4.6: Real income of heterogeneous defined benefit plan result of 100 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; with or without cap on final average salary; for an active member in starting from $x = 45$, in 000s, using time 0 money values; red is white-collar, blue is blue-collar.

|  |  | Mean Real Income |  |
|---|---|---|---|
| Collar | Age | No Cap | Have Cap |
| White Collar | 45 | 61.298 (0.000%) | 62.148 (0.000%) |
|  | 64 | 105.686 (5.835%) | 106.218 (5.691%) |
|  | 65 | 45.047 (1.971%) | 39.565 (1.424%) |
|  | 75 | 42.228 (2.263%) | 37.415 (1.703%) |
| Blue Collar | 45 | 27.087 (0.000%) | 27.937 (0.000%) |
|  | 64 | 30.223 (4.143%) | 30.755 (3.921%) |
|  | 65 | 15.349 (0.673%) | 15.349 (0.673%) |
|  | 75 | 14.954 (0.766%) | 14.954 (0.766%) |

Table 4.3: Mean real income of heterogeneous defined benefit plan result of 100 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; with or without cap on final average salary; for an active member in starting from $x = 45$, in 000s, using time 0 money values; estimated standard errors in parentheses.

We also show the income for blue-collar and white-collar members. Because $FCAP(0) = 85$, which is larger than the blue-collar members' final average salary, it does not influence the blue-collar members' income after retirement. The overall income before retirement increases because the contributions decrease. The lower total contribution rate controls the downside risk of the income before retirement. For white-collar members, the income before retirement also increases. They already had a less volatile income before retirement in the heterogeneous plan. So we pay attention to the income after retirement. Setting the cap on the final average salary reduces white-collar members' benefit to around 40. Because less part of the benefits is inflated with half COLA, the decline of the white-collar members' income after retirement is slower in Figure 4.6b than in Figure 4.6a. Even though the white-collar members' income after retirement drops, the white-collar members can invest the non-pensionable earnings by themselves to supplement their benefits.

| | Quantiles at $t = 30$ | | | |
|---|---|---|---|---|
| $FCAP(0)$ | Asset Liability Ratio | | Total Contribution Rate | |
| 80 | 5% | 0.708 (0.686, 0.731) | 95% | 0.320 (0.309, 0.332) |
| | 25% | 0.874 (0.852, 0.891) | 75% | 0.225 (0.216, 0.236) |
| | 50% | 1.032 (1.012, 1.058) | 50% | 0.173 (0.172, 0.174) |
| | 75% | 1.254 (1.231, 1.289) | 25% | 0.109 (0.866, 0.127) |
| | 95% | 1.793 (1.686, 1.920) | 5% | 0.000 (0.000, 0.000) |
| 85 | 5% | 0.710 (0.688, 0.732) | 95% | 0.330 (0.320, 0.343) |
| | 25% | 0.876 (0.851, 0.890) | 75% | 0.233 (0.224, 0.243) |
| | 50% | 1.035 (1.014, 1.058) | 50% | 0.180 (0.179, 0.180) |
| | 75% | 1.255 (1.231, 1.290) | 25% | 0.111 (0.090, 0.132) |
| | 95% | 1.791 (1.681, 1.912) | 5% | 0.000 (0.000, 0.000) |
| 90 | 5% | 0.713 (0.691, 0.733) | 95% | 0.340 (0.330, 0.354) |
| | 25% | 0.877 (0.853, 0.891) | 75% | 0.240 (0.231, 0.251) |
| | 50% | 1.038 (1.016, 1.058) | 50% | 0.187 (0.186, 0.188) |
| | 75% | 1.257 (1.233, 1.292) | 25% | 0.117 (0.094, 0.137) |
| | 95% | 1.781 (1.687, 1.890) | 5% | 0.000 (0.000, 0.000) |

Table 4.4: Sensitivity test on cap on final average salary about defined benefit plan result of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; 95% confidence intervals in parentheses.

| | | Mean Real Income | |
|---|---|---|---|
| $FCAP(0)$ | Age | White Collar | Blue Collar |
| 80 | 45 | 62.470 (0.000%) | 28.258 (0.000%) |
| | 64 | 106.430 (5.616%) | 30.967 (3.818%) |
| | 65 | 37.238 (1.341%) | 15.349 (0.673%) |
| | 75 | 35.374 (1.597%) | 14.954 (0.766%) |
| 85 | 45 | 62.148 (0.000%) | 27.937 (0.000%) |
| | 64 | 106.218 (5.691%) | 30.755 (3.921%) |
| | 65 | 39.565 (1.424%) | 15.349 (0.673%) |
| | 75 | 37.415 (1.703%) | 14.954 (0.766%) |
| 90 | 45 | 61.813 (0.000%) | 27.602 (0.000%) |
| | 64 | 106.001 (5.765%) | 30.538 (4.023%) |
| | 65 | 41.892 (1.508%) | 15.349 (0.673%) |
| | 75 | 39.456 (1.809%) | 14.954 (0.766%) |

Table 4.5: Sensitivity test on cap on final average salary about mean real income of defined benefit plan result of 1,000 simulations; with $k_1 = 10$ and $k_2 = 5$; with two-tier COLA method; for an active member starting from $x = 45$, in \$000s, using time 0 money values; estimated standard errors in parentheses.

We test values of $FCAP(0)$ from 80 to 90 to do a sensitivity test on the $FCAP(0)$ salary. In Table 4.4, we observe that a smaller $FCAP(0)$ decreases the overall asset liability ratio except the 95% quantile. The 95 quantile of the total contribution rate drops to 32% when $FCAP(0) = 80$, with the 95% confidence interval $(30.9\%, 33.2\%)$. The required contributions decrease will lead to a further increase in the income before retirement. In Table 4.5, a smaller $FCAP(0)$ generates a higher income before retirement for both the blue-collar and white-collar members. Though the white-collar members' benefit decreases, the decline rate from age 65 to 75 is slower when $FCAP(0)$ decreases.

We set the cap on final average salary not only to incentivise blue-collar members to stay in the heterogeneous pension plan, but also to increase fairness. The income before retirement has a lower volatility and a higher mean for the blue-collar members. As long as the $FCAP(0)$ is smaller than their salary, the adjustments will not hurt their income after retirement. However, adding a relatively small $FCAP(0)$ damages the plan funding. Small $FCAP(0)$ generates both low 5% and 50% quantiles, increasing the default risk and inadequacy. Moreover, it brings a high 95% quantile but an inefficient funding. The heterogeneous plan also leaves some other unanswered questions. From Table 4.4, the problem of free lunch still exists as long as the 5% quantile line of the total contribution rate ends with 0. By increasing the recovery time for surplus, this problem could be mitigated and the asset liability ratio would increase. However, the 95% quantile when $FCAP(0) = 80$ is almost 1.8. We would end up with a larger number when we increase surplus recovery time. Based on our criteria, it is inefficient for the plan to have a large surplus. How to deal with these questions needs future research.

# Chapter 5

# Conclusion

This paper first demonstrates that the traditional DB pension plan offering full COLA has significant solvency risk under reasonable assumptions, based on the 1000 simulated paths generated by the Wilkie's Economic Scenario Generator. The high solvency risk violates the adequacy criterion because of the default risk. We include additional contributions in the plan. Even though the inclusion of additional contributions improves the asset liability ratio, it introduces cost risk. The total contribution rate is volatile, with the highest values reaching about 50% and the lowest falling to 0. It is not sustainable, affordable, or fair.

This paper provides two different methods of applying COLA. The first is reducing 100% COLA to 80%. The second is the two-tier COLA method. We use YMPE as the threshold, with below-YMPE benefit eligible for 100% COLA, and above-YMPE benefit eligible for 50% COLA. The two methods both decrease the liabilities and mitigate the solvency risk and the cost risk, but the effects are not very substantial. Even though the two methods generate similar results, they have different impacts on members' income, especially for low-paid members. The two-tier COLA method protects income after retirement for the members who earn below YMPE.

We then follow the two-tier COLA method and give a new perspective of including blue-collar (lower-paid) and white-collar (higher-paid) members in one heterogeneous pension plan. The member populations are the same for the two groups. We follow the fairness criterion by treating the two groups equally. But the heterogeneous plan brings lower and more volatile income before retirement for the blue-collar members, because they need to share the large liabilities created by the white-collar members. We further set different caps on the pensionable final average salary. A lower cap on the final average salary results in lower asset liability ratio and total contribution rate. Even though the white-collar members receive fewer benefits, because of the cap, they can invest their non-pensionable earnings individually. However, the heterogeneous plan leaves some unanswered questions such as inefficiency or inadequacy.

Although the heterogeneous plan looks more fair, it still favors the white-collar members. The blue-collar members take the cost risk brought by the white-collar members. The two-tier COLA, and setting the caps on the final average salary, help to reduce the unfairness, but maybe not enough. We could impose a lower final average salary to further reduce the risks and increase affordability and fairness, but it becomes inefficient and inadequate. Or we can lower the equity weighting in investment to pursue sustainability, but it will push up costs. The challenge remains to balance members' interests and achieve the five criteria.

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# APPENDICES

# Appendix A

# General Members' Information

## A.1 Active Members

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ (in 000s) | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 25 | 17 | 17 | 0.50 | 32.0 | 0.8998 |
| 26 | 32 | 17 | 0.97 | 33.0 | 0.8997 |
| 27 | 45 | 16 | 1.45 | 35.3 | 0.8997 |
| 28 | 55 | 15 | 1.92 | 36.7 | 0.8997 |
| 29 | 64 | 14 | 2.39 | 38.2 | 0.8997 |
| 30 | 70 | 12 | 2.89 | 39.7 | 0.8997 |
| 31 | 74 | 11 | 3.39 | 41.3 | 0.8997 |
| 32 | 77 | 10 | 3.88 | 42.9 | 0.8997 |
| 33 | 78 | 9 | 4.38 | 44.6 | 0.9097 |
| 34 | 78 | 7 | 4.94 | 46.4 | 0.9197 |
| 35 | 75 | 5 | 5.58 | 48.1 | 0.9296 |
| 36 | 76 | 5 | 6.18 | 49.7 | 0.9396 |
| 37 | 78 | 5 | 6.75 | 51.5 | 0.9496 |
| 38 | 79 | 5 | 7.29 | 53.3 | 0.9496 |
| 39 | 80 | 5 | 7.80 | 55.1 | 0.9495 |
| 40 | 81 | 5 | 8.29 | 57.1 | 0.9495 |
| 41 | 82 | 5 | 8.75 | 59.1 | 0.9495 |
| 42 | 83 | 6 | 9.09 | 60.8 | 0.9494 |
| 43 | 85 | 6 | 9.41 | 62.7 | 0.9494 |
| 44 | 87 | 6 | 9.73 | 64.5 | 0.9593 |

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ (in 000s) | $s(x,0)$ | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 45 | 89 | 6 | 10.04 | 66.5 | 0.9693 |
| 46 | 93 | 6 | 10.36 | 68.5 | 0.9792 |
| 47 | 97 | 6 | 10.69 | 70.5 | 0.9791 |
| 48 | 100 | 6 | 11.01 | 72.3 | 0.9790 |
| 49 | 104 | 6 | 11.35 | 74.1 | 0.9789 |
| 50 | 107 | 5 | 11.80 | 76.0 | 0.9788 |
| 51 | 110 | 5 | 12.24 | 77.9 | 0.9787 |
| 52 | 111 | 4 | 12.78 | 79.8 | 0.9786 |
| 53 | 113 | 4 | 13.31 | 81.8 | 0.9784 |
| 54 | 112 | 2 | 14.06 | 83.8 | 0.9782 |
| 55 | 111 | 1 | 14.93 | 85.5 | 0.9780 |
| 56 | 108 | 0 | 15.93 | 87.2 | 0.9778 |
| 57 | 106 | 0 | 16.93 | 89.0 | 0.9776 |
| 58 | 103 | 0 | 17.93 | 90.7 | 0.9773 |
| 59 | 101 | 0 | 18.93 | 92.6 | 0.9770 |
| 60 | 98 | 0 | 19.93 | 94.4 | 0.9866 |
| 61 | 98 | 0 | 20.93 | 96.3 | 0.9962 |
| 62 | 98 | 0 | 21.93 | 97.3 | 0.9958 |
| 63 | 97 | 0 | 22.93 | 98.2 | 0.9953 |
| 64 | 97 | 0 | 23.93 | 99.2 | 0.9947 |

## A.2 Retired Members

$s(65, 0) = 100.2$

| $x$ | $L(x, 0)$ | $b(x, 0)$ (in 000s) | $p_x$ | $x$ | $L(x, 0)$ | $b(x, 0)$ (in 000s) | $p_x$ |
|---|---|---|---|---|---|---|---|
| 65 | 96 | 44.4 | 0.9941 | 86 | 23 | 23.8 | 0.9354 |
| 66 | 94 | 43.1 | 0.9934 | 87 | 19 | 23.1 | 0.9278 |
| 67 | 92 | 41.8 | 0.9926 | 88 | 15 | 22.5 | 0.9192 |
| 68 | 91 | 40.6 | 0.9917 | 89 | 12 | 21.8 | 0.9097 |
| 69 | 89 | 39.4 | 0.9907 | 90 | 9 | 21.2 | 0.8991 |
| 70 | 86 | 38.3 | 0.9896 | 91 | 6 | 20.6 | 0.8873 |
| 71 | 84 | 37.1 | 0.9883 | 92 | 4 | 20.0 | 0.8743 |
| 72 | 81 | 36.1 | 0.9869 | 93 | 3 | 19.4 | 0.8599 |
| 73 | 78 | 35.0 | 0.9853 | 94 | 1 | 18.8 | 0.8439 |
| 74 | 75 | 34.0 | 0.9836 | 95 | 1 | 18.3 | 0.8264 |
| 75 | 72 | 33.0 | 0.9816 | 96 | 0 | 17.7 | 0.8071 |
| 76 | 68 | 32.0 | 0.9793 | 97 | 0 | 17.2 | 0.7860 |
| 77 | 64 | 31.1 | 0.9768 | 98 | 0 | 16.7 | 0.7629 |
| 78 | 60 | 30.2 | 0.9740 | 99 | 0 | 16.2 | 0.7377 |
| 79 | 56 | 29.3 | 0.9709 | 100 | 0 | 15.8 | 0.7104 |
| 80 | 51 | 28.5 | 0.9673 | 101 | 0 | 15.3 | 0.6809 |
| 81 | 47 | 27.6 | 0.9634 | 102 | 0 | 14.9 | 0.6493 |
| 82 | 42 | 26.8 | 0.9590 | 103 | 0 | 14.4 | 0.6154 |
| 83 | 37 | 26.1 | 0.9540 | 104 | 0 | 14.0 | 0.5795 |
| 84 | 32 | 25.3 | 0.9485 | 105 | 0 | 13.6 | 0.0000 |
| 85 | 28 | 24.6 | 0.9423 | | | | |

# Appendix B

# Blue-Collar Members' Information

## B.1 Active Members

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ (in 000s) | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 25 | 17 | 17 | 0.50 | 28.0 | 0.9326 |
| 26 | 32 | 17 | 0.97 | 29.3 | 0.9317 |
| 27 | 45 | 16 | 1.45 | 30.6 | 0.9308 |
| 28 | 55 | 15 | 1.92 | 32.0 | 0.9298 |
| 29 | 64 | 14 | 2.39 | 33.5 | 0.9289 |
| 30 | 70 | 12 | 2.89 | 35.0 | 0.9280 |
| 31 | 74 | 11 | 3.39 | 35.0 | 0.9270 |
| 32 | 77 | 10 | 3.88 | 35.0 | 0.9261 |
| 33 | 78 | 9 | 4.38 | 35.0 | 0.9354 |
| 34 | 78 | 7 | 4.94 | 35.0 | 0.9447 |
| 35 | 75 | 5 | 5.58 | 35.0 | 0.9540 |
| 36 | 76 | 5 | 6.18 | 35.0 | 0.9633 |
| 37 | 78 | 5 | 6.75 | 35.0 | 0.9725 |
| 38 | 79 | 5 | 7.29 | 35.0 | 0.9715 |
| 39 | 80 | 5 | 7.80 | 35.0 | 0.9705 |
| 40 | 81 | 5 | 8.29 | 35.0 | 0.9695 |
| 41 | 82 | 5 | 8.75 | 35.0 | 0.9685 |
| 42 | 83 | 6 | 9.09 | 35.0 | 0.9675 |
| 43 | 85 | 6 | 9.41 | 35.0 | 0.9665 |
| 44 | 87 | 6 | 9.73 | 35.0 | 0.9756 |

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ (in 000s) | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 45 | 89 | 6 | 10.04 | 35.0 | 0.9847 |
| 46 | 93 | 6 | 10.36 | 35.0 | 0.9938 |
| 47 | 97 | 6 | 10.69 | 35.0 | 0.9927 |
| 48 | 100 | 6 | 11.01 | 35.0 | 0.9916 |
| 49 | 104 | 6 | 11.35 | 35.0 | 0.9904 |
| 50 | 107 | 5 | 11.80 | 35.0 | 0.9893 |
| 51 | 110 | 5 | 12.24 | 35.0 | 0.9882 |
| 52 | 111 | 4 | 12.78 | 35.0 | 0.9870 |
| 53 | 113 | 4 | 13.31 | 35.0 | 0.9858 |
| 54 | 112 | 2 | 14.06 | 35.0 | 0.9846 |
| 55 | 111 | 1 | 14.93 | 35.0 | 0.9834 |
| 56 | 108 | 0 | 15.93 | 35.0 | 0.9822 |
| 57 | 106 | 0 | 16.93 | 35.0 | 0.9809 |
| 58 | 103 | 0 | 17.93 | 35.0 | 0.9796 |
| 59 | 101 | 0 | 18.93 | 35.0 | 0.9782 |
| 60 | 98 | 0 | 19.93 | 35.0 | 0.9868 |
| 61 | 98 | 0 | 20.93 | 35.0 | 0.9951 |
| 62 | 98 | 0 | 21.93 | 35.0 | 0.9946 |
| 63 | 97 | 0 | 22.93 | 35.0 | 0.9940 |
| 64 | 97 | 0 | 23.93 | 35.0 | 0.9933 |

## B.2 Retired Members

$s(65,0) = 35.0$

| $x$ | $L(x,0)$ | $b(x,0)$ (in 000s) | $p_x$ | $x$ | $L(x,0)$ | $b(x,0)$ (in 000s) | $p_x$ |
|---|---|---|---|---|---|---|---|
| 65 | 96 | 15.5 | 0.9893 | 86 | 23 | 8.3 | 0.9156 |
| 66 | 94 | 15.0 | 0.9883 | 87 | 19 | 8.1 | 0.9061 |
| 67 | 92 | 14.6 | 0.9872 | 88 | 15 | 7.9 | 0.8956 |
| 68 | 91 | 14.2 | 0.9860 | 89 | 12 | 7.6 | 0.8839 |
| 69 | 89 | 13.8 | 0.9847 | 90 | 9 | 7.4 | 0.8709 |
| 70 | 86 | 13.4 | 0.9832 | 91 | 6 | 7.2 | 0.8569 |
| 71 | 84 | 13.0 | 0.9815 | 92 | 4 | 7.0 | 0.8424 |
| 72 | 81 | 12.6 | 0.9797 | 93 | 3 | 6.8 | 0.8273 |
| 73 | 78 | 12.2 | 0.9776 | 94 | 1 | 6.6 | 0.8118 |
| 74 | 75 | 11.9 | 0.9753 | 95 | 1 | 6.4 | 0.7959 |
| 75 | 72 | 11.5 | 0.9728 | 96 | 0 | 6.2 | 0.7795 |
| 76 | 68 | 11.2 | 0.9699 | 97 | 0 | 6.0 | 0.7627 |
| 77 | 64 | 10.9 | 0.9668 | 98 | 0 | 5.8 | 0.7451 |
| 78 | 60 | 10.6 | 0.9633 | 99 | 0 | 5.7 | 0.7267 |
| 79 | 56 | 10.2 | 0.9594 | 100 | 0 | 5.5 | 0.6999 |
| 80 | 51 | 9.9 | 0.9550 | 101 | 0 | 5.3 | 0.6708 |
| 81 | 47 | 9.7 | 0.9501 | 102 | 0 | 5.2 | 0.6396 |
| 82 | 42 | 9.4 | 0.9446 | 103 | 0 | 5.0 | 0.6063 |
| 83 | 37 | 9.1 | 0.9385 | 104 | 0 | 4.9 | 0.5709 |
| 84 | 32 | 8.8 | 0.9317 | 105 | 0 | 4.7 | 0.0000 |
| 85 | 28 | 8.6 | | | | | |

# Appendix C

# White-Collar Members' Information

## C.1 Active Members

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ (in 000s) | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 25 | 17 | 17 | 0.50 | 32.0 | 0.8680 |
| 26 | 32 | 17 | 0.97 | 33.1 | 0.8689 |
| 27 | 45 | 16 | 1.45 | 35.4 | 0.8697 |
| 28 | 55 | 15 | 1.92 | 36.9 | 0.8706 |
| 29 | 64 | 14 | 2.39 | 38.5 | 0.8714 |
| 30 | 70 | 12 | 2.89 | 40.1 | 0.8723 |
| 31 | 74 | 11 | 3.39 | 41.8 | 0.8732 |
| 32 | 77 | 10 | 3.88 | 43.5 | 0.8740 |
| 33 | 78 | 9 | 4.38 | 45.3 | 0.8846 |
| 34 | 78 | 7 | 4.94 | 47.2 | 0.8952 |
| 35 | 75 | 5 | 5.58 | 49.1 | 0.9059 |
| 36 | 76 | 5 | 6.18 | 50.8 | 0.9165 |
| 37 | 78 | 5 | 6.75 | 52.7 | 0.9272 |
| 38 | 79 | 5 | 7.29 | 54.7 | 0.9281 |
| 39 | 80 | 5 | 7.80 | 56.7 | 0.9290 |
| 40 | 81 | 5 | 8.29 | 58.8 | 0.9299 |
| 41 | 82 | 5 | 8.75 | 61.0 | 0.9308 |
| 42 | 83 | 6 | 9.09 | 62.9 | 0.9317 |
| 43 | 85 | 6 | 9.41 | 65.0 | 0.9326 |
| 44 | 87 | 6 | 9.73 | 67.0 | 0.9433 |

## C.2 Active Members

| $x$ | $L(x,0)$ | $NE(x)$ | $ny(x,0)$ | $s(x,0)$ (in 000s) | $p_x^{(\tau)}$ |
|---|---|---|---|---|---|
| 45 | 89 | 6 | 10.04 | 69.2 | 0.9540 |
| 46 | 93 | 6 | 10.36 | 71.4 | 0.9648 |
| 47 | 97 | 6 | 10.69 | 73.7 | 0.9657 |
| 48 | 100 | 6 | 11.01 | 75.7 | 0.9666 |
| 49 | 104 | 6 | 11.35 | 77.7 | 0.9675 |
| 50 | 107 | 5 | 11.80 | 79.9 | 0.9684 |
| 51 | 110 | 5 | 12.24 | 82.1 | 0.9693 |
| 52 | 111 | 4 | 12.78 | 84.2 | 0.9702 |
| 53 | 113 | 4 | 13.31 | 86.5 | 0.9710 |
| 54 | 112 | 2 | 14.06 | 88.8 | 0.9719 |
| 55 | 111 | 1 | 14.93 | 90.8 | 0.9727 |
| 56 | 108 | 0 | 15.93 | 92.8 | 0.9735 |
| 57 | 106 | 0 | 16.93 | 94.9 | 0.9743 |
| 58 | 103 | 0 | 17.93 | 96.9 | 0.9751 |
| 59 | 101 | 0 | 18.93 | 99.1 | 0.9758 |
| 60 | 98 | 0 | 19.93 | 101.2 | 0.9865 |
| 61 | 98 | 0 | 20.93 | 103.5 | 0.9972 |
| 62 | 98 | 0 | 21.93 | 104.8 | 0.9968 |
| 63 | 97 | 0 | 22.93 | 105.9 | 0.9965 |
| 64 | 97 | 0 | 23.93 | 107.2 | 0.9960 |

## C.3 Retired Members

$s(65,0) = 108.5$

| $x$ | $L(x,0)$ | $b(x,0)$ (in 000s) | $p_x$ | $x$ | $L(x,0)$ | $b(x,0)$ (in 000s) | $p_x$ |
|---|---|---|---|---|---|---|---|
| 65 | 96 | 48.0 | 0.9959 | 86 | 23 | 25.8 | 0.9430 |
| 66 | 94 | 46.6 | 0.9953 | 87 | 19 | 25.1 | 0.9355 |
| 67 | 92 | 45.3 | 0.9947 | 88 | 15 | 24.3 | 0.9272 |
| 68 | 91 | 44.0 | 0.9940 | 89 | 12 | 23.6 | 0.9178 |
| 69 | 89 | 42.7 | 0.9932 | 90 | 9 | 22.9 | 0.9072 |
| 70 | 86 | 41.4 | 0.9923 | 91 | 6 | 22.3 | 0.8954 |
| 71 | 84 | 40.2 | 0.9912 | 92 | 4 | 21.6 | 0.8822 |
| 72 | 81 | 39.1 | 0.9900 | 93 | 3 | 21.0 | 0.8673 |
| 73 | 78 | 37.9 | 0.9887 | 94 | 1 | 20.4 | 0.8507 |
| 74 | 75 | 36.8 | 0.9872 | 95 | 1 | 19.8 | 0.8323 |
| 75 | 72 | 35.8 | 0.9855 | 96 | 0 | 19.2 | 0.8119 |
| 76 | 68 | 34.7 | 0.9836 | 97 | 0 | 18.7 | 0.7894 |
| 77 | 64 | 33.7 | 0.9814 | 98 | 0 | 18.1 | 0.7648 |
| 78 | 60 | 32.7 | 0.9789 | 99 | 0 | 17.6 | 0.7396 |
| 79 | 56 | 31.8 | 0.9761 | 100 | 0 | 17.1 | 0.7122 |
| 80 | 51 | 30.8 | 0.9730 | 101 | 0 | 16.6 | 0.6827 |
| 81 | 47 | 29.9 | 0.9694 | 102 | 0 | 16.1 | 0.6509 |
| 82 | 42 | 29.1 | 0.9653 | 103 | 0 | 15.6 | 0.6170 |
| 83 | 37 | 28.2 | 0.9607 | 104 | 0 | 15.2 | 0.5810 |
| 84 | 32 | 27.4 | 0.9555 | 105 | 0 | 14.7 | 0.0000 |
| 85 | 28 | 26.6 | 0.9496 | | | | |