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Solving semiring constraint satisfaction problems

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Solving Semiring Constraint Satisfaction Problems

A thesis submitted in fulfillment of the
requirements for the award of the degree

Doctor of Philosophy

from

UNIVERSITY OF WOLLONGONG

by

Louise Leenen

School of Computer Science and Software Engineering

January 2010

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by

Louise Leenen

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*Dedicated to
Tommy, Thomas, and André*

Declaration

This is to certify that the work reported in this thesis was done by the author, unless specified otherwise, and that no part of it has been submitted in a thesis to any other university or similar institution.

Louise Leenen
January 7, 2010

Abstract

The Semiring Constraint Satisfaction Problem (SCSP) framework is a popular approach for the representation of partial constraint satisfaction problems. Considerable research has been done in solving SCSPs, but limited work has been done in building general SCSP solvers. In this thesis, we present various methods to solve SCSPs.

We first consider how a SCSP might be relaxed: we relax individual constraints until an acceptable solution can be obtained. A second semiring is used to define a measure of difference between the original problem and the relaxed problem. This research was first presented at the *International Workshop on Preferences and Soft Constraints* at CP-2005 [40], and an extended version of the paper has been published in the *Information Processing Letters* journal [41].

We then show how the two semirings of a relaxed SCSP can be combined into a single semiring structure. This combined semiring structure will make it possible to use existing tools for solving SCSPs to solve Combined SCSPs. This work appears in Leenen et al. [42].

The remainder of this thesis focusses on algorithms to solve SCSPs. A significant amount of work has been devoted to solving the well-known maximum satisfiability problem (Max-SAT) [1, 63] and the related Weighted Max-SAT problem. This prompted us to modify the methods for solving Max-SAT, into methods for solving SCSPs. We show how to translate a SCSP into a variant of the Weighted Max-SAT Problem, which we call a Weighted Semiring Max-SAT problem, and then present a local search algorithm that is a modification of the GSAT algorithm for solving Max-SAT. This work appears in Leenen et al. [38].

Finally, we extend well-known algorithms for maximal constraint satisfaction into

SCSP algorithms. We present a branch and bound algorithm, a backjumping algorithm, and a forward checking algorithm. Our branch and bound algorithm performs significantly better than CONFLEX [17], a well-known fuzzy CSP solver. The branch and bound algorithm has been presented in Leenen et al. [38]. The forward checking and backjumping algorithms perform better than the branch and bound algorithm on harder problems. This work appears Leenen et al. [39].

List of Publications resulting from the research presented in this thesis:

- L. Leenen, T. Meyer, and A. Ghose. Relaxations of semiring constraint satisfaction problems. In *Proceedings of the 7th International Workshop on Preferences and Soft Constraints (SOFT-05)*, 2005.
- L. Leenen, T. Meyer, P. Harvey, and A. Ghose. A relaxation of a semiring constraint satisfaction problem using combined semirings. In *Proceedings of the 9th Pacific Rim International Conference on Artificial Intelligence (PRICAI-2006)*, pages 907–911, 2006.
- L. Leenen, T. Meyer, and A. Ghose. Relaxations of semiring constraint satisfaction problems. *Information Processing Letters*, 103(5):177–182, 2007.
- L. Leenen, A. Anbulagan, T. Meyer, and A. Ghose. Modelling and solving semiring constraint satisfaction problems by transformation to weighted semiring max-SAT. In *Proceedings of the Twentieth Australian Joint Conference on Artificial Intelligence (AI-2007)*, pages 202–212, 2007.
- L. Leenen and A. Ghose. Branch and bound algorithms to solve semiring constraint satisfaction problems. In *Proceedings of the 10th Pacific Rim International Conference on Artificial Intelligence (PRICAI-2008)*, 2008.

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