Street, Ross

Polynomials as spans. (English. French summary) Zbl 07238806

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Polynomials in a locally cartesian closed category $\mathcal{E}$ were shown to be the morphisms of a bicategory [*N. Gambino* and *J. Kock*, Math. Proc. Camb. Philos. Soc. 154, No. 1, 153–192 (2013; Zbl 1278.18013)]. Polynomials were defined in any category $\mathcal{C}$ with pullbacks and shown to form a bicategory [*M. Weber*, Theory Appl. Categ. 30, 533–598 (2015; Zbl 1330.18002)]. The author seeks to better understand the composition of polynomials.

The meaning of polynomial in a bicategory in this paper is different from that in §4 of Weber's [loc. cit.] which is about polynomials in 2-categories. Weber dealt with the 2-category as a **Cat**-enriched category, taking the polynomials to be diagrams of the same shape as in the case of ordinary categories, and accommodating the presence of 2-cells, so that, if a category is put down as a 2-category with only identity 2-cells, then his polynomials in the 2-category are just polynomials in the category.

The author introduces the term *calibration* for a class of morphisms, called *neat*, in a bicategory after [*J. Benabou*, C. R. Acad. Sci., Paris, Sér. A 281, 831–834 (1975; Zbl 0349.18005)]. A morphism in a bicategory is defined to be a *right lifter* when every morphism into its codomain has a right lifting through it. *Polynomials* in a calibrated bicategory $\mathcal{M}$ are spans with one leg a right lifter and the other leg neat. The bicategory Poly $\mathcal{M}$ is obtained by taking isomorphism classes of 2-morphisms. A *polynomic category* $\mathcal{M}$ is one in which the neat morphisms are all the groupoid fibrations in $\mathcal{M}$. It is shown that the bicategory Spn $\mathcal{C}$ of spans is polynomic for any finitely complete $\mathcal{C}$, in which the polynomials are the polynomials in $\mathcal{C}$ in the sense of Weber's [loc. cit.].

The bicategory Rel $\mathcal{E}$ of relations in a regular category $\mathcal{E}$ is calibrated by morphisms which are isomorphic to graphs of monoorphisms in $\mathcal{E}$. The author gives, for $\mathcal{E}$ a topos, a reinterpretation of the bicategory of polynomials in Rel $\mathcal{E}$ as a Kleisli construction.

By providing a calibration for the bicategory Mod of two-sided modules between categories, the author gives a reinterpretation of the bicategory of polynomials in Mod as a Kleisli construction.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18C15 Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads
18C20 Eilenberg-Moore and Kleisli constructions for monads
18F20 Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)

Keywords:

span; partial map; powerful morphism; polynomial functor; exponentiable morphism; calibrated bicategory; right lifting

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