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BOUNDARY ELEMENT FORMULATIONS FOR FRACTURE MECHANICS PROBLEMS

By

Wei-Liang Wu

SUBMITTED IN PARTIAL FULFILLMENT OF THE
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This thesis is submitted to the University of Wollongong, and has not been submitted for a higher degree to any other university or institution.

Wei-Liang Wu

November 2004

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To My Parents with Love and Gratitude.

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