

# Quantum Computing at the Frontiers of Biological Sciences

Prashant S. Emani<sup>1,2\*</sup>, Jonathan Warrell<sup>1,2\*</sup>, Alan Anticevic<sup>3</sup>, Stefan Bekiranov<sup>4</sup>, Michael Gandal<sup>5</sup>, Michael J. McConnell<sup>4,6</sup>, Guillermo Sapiro<sup>7</sup>, Alán Aspuru-Guzik<sup>8,9,10,11</sup>, Justin T. Baker<sup>12,13</sup>, Matteo Bastiani<sup>14</sup>, John Murray<sup>15,16</sup>, Stamatios N Sotiropoulos<sup>14</sup>, Jacob Taylor<sup>17,18</sup>, Geetha Senthil<sup>19</sup>, Thomas Lehner<sup>19,20,#,†</sup>, Mark B. Gerstein<sup>1,2,21,22#</sup>, Aram W. Harrow<sup>23#</sup>

<sup>1</sup>Program in Computational Biology and Bioinformatics, Yale University, New Haven, Connecticut, USA.

<sup>2</sup>Department of Molecular Biophysics and Biochemistry, Yale University, New Haven, Connecticut, USA.

<sup>3</sup>Yale School of Medicine, Department of Psychiatry, New Haven, Connecticut, USA.

<sup>4</sup>Department of Biochemistry and Molecular Genetics, University of Virginia School of Medicine, Charlottesville, Virginia, USA.

<sup>5</sup>Department of Psychiatry, Semel Institute, David Geffen School of Medicine, University of California–Los Angeles, Los Angeles, California, USA.

<sup>6</sup>Department of Neuroscience, Univ. of Virginia School of Medicine, Charlottesville, Virginia, USA.

<sup>7</sup>Department of Electrical and Computer Engineering, Duke University, Durham, North Carolina, USA.

<sup>8</sup>Department of Chemistry, University of Toronto, Toronto, Ontario, Canada.

<sup>9</sup>Canadian Institute for Advanced Research (CIFAR), Toronto, Ontario, Canada.

<sup>10</sup>CIFAR Artificial Intelligence Research Chair, Vector Institute, Toronto, Ontario, Canada

<sup>11</sup>Department of Computer Science, University of Toronto, Toronto, Ontario, Canada.

<sup>12</sup>Schizophrenia and Bipolar Disorder Program, McLean Hospital, Belmont, Massachusetts, USA.

<sup>13</sup>Department of Psychiatry, Harvard Medical School, Boston, Massachusetts, USA.

<sup>14</sup>Sir Peter Mansfield Imaging Centre, School of Medicine, University of Nottingham, Nottingham, UK.

<sup>15</sup>Department of Psychiatry, Yale University School of Medicine, New Haven, Connecticut, USA

<sup>16</sup>Department of Physics, Yale University, New Haven, Connecticut, USA.

<sup>17</sup>Joint Center for Quantum Information and Computer Science, University of Maryland, College Park, Maryland, USA.

<sup>18</sup>National Institute of Standards and Technology, Gaithersburg, Maryland, USA

<sup>19</sup>New York Genome Center, New York, New York, USA

35 <sup>20</sup> National Institute of Mental Health, Bethesda, Maryland, USA.

36 <sup>21</sup>Department of Computer Science, Yale University, New Haven, Connecticut, USA.

37 <sup>22</sup>Department of Statistics and Data Science, Yale University, New Haven, Connecticut, USA

38 <sup>23</sup>Center for Theoretical Physics, Department of Physics, Massachusetts Institute of Technology,  
39 Cambridge, Massachusetts, USA.

40 † Present Address: Neuropsychiatric Disease Genomics, New York Genome Center, New York,  
41 New York, USA.

42 \* These authors contributed equally.

43 # Corresponding authors.

44

## 45 **Abstract**

46

47 Computing plays a critical role in the biological sciences but faces increasing challenges of scale  
48 and complexity. We evaluate the potential for quantum computing algorithms to aid in the  
49 merging of insights from genetics to neuroscience.

50

## 51 **Introduction**

52 In an era of increasingly collaborative efforts towards unravelling the complexities of biology,  
53 one may posit the existence of two broad tendencies: first, an approach towards greater depth  
54 in particular fields, whether relying on intensive technological, theoretical or computational  
55 development, that aims to comprehensively explore a specific aspect of biology; and second, a  
56 recognition of the need to knit together the disparate experimental and conceptual threads  
57 across the vast spectrum of length, time and system-size scales inherent in biology into a  
58 coherent framework. Addressing both sources of complexity necessarily requires research-area-  
59 specific experimental and theoretical advances, but there is also the possibility of outsourcing  
60 some of the analytical burden to high-throughput computing resources. The significant interest  
61 in large-scale computing infrastructure evinced by governmental and private entities  
62 underscores the importance of the scientific community exploring new ways of interfacing with  
63 cutting-edge computing technologies. These include expansions of current super-computing  
64 and other massively parallel computing facilities, but also considerations of entirely new  
65 computing paradigms. Here, we consider the potential of quantum computing (QC) to address  
66 complex biological questions. Recent technological developments have carried QC capabilities  
67 from the realm of academic exploration to commercial opportunities<sup>1,2</sup>. While the scale is not  
68 currently competitive with classical technologies, there is substantial excitement in its eventual  
69 promise, and we hope to provide an entry point for biologists to certain aspects of the  
70 discussion surrounding QC. This effort is especially timely given recent policy efforts at a  
71 national or international level, such as the U.S. National Quantum Initiative Act 2018<sup>3</sup>

72 (implementation of a National Quantum Initiative for quantum information science and  
73 technology<sup>4</sup>), the European Quantum Technologies Flagship, and efforts in the UK and China<sup>5</sup>.

74 We first present a primer on quantum computation to familiarize the reader with the basic  
75 concepts and language of QC. The remainder is focused on the study of the human brain  
76 through genetics, genomics, neuroimaging, and deep behavioral phenotyping, a  
77 multidisciplinary effort that falls under the term ‘convergent neuroscience’. We highlight these  
78 areas as they exemplify the two aforementioned sources of complexity: separately, each field  
79 presents an incredibly rich set of problems that often push the limits of classical computational  
80 capability; in combination, they offer a multi-scale challenge leading from the molecular scale  
81 through the cellular and tissue levels, to brain architecture and, eventually, to complex human  
82 behaviors and disorders. The study of the emergent properties of the brain, such as cognition  
83 and behavior, is a uniquely challenging multi-level endeavor that demands pioneering  
84 approaches in computation. Accordingly, we discuss how quantum algorithms that map onto  
85 methodological issues in neuroscience may provide much needed improvements in  
86 computational efficiency, and posit open questions for eventual development of new  
87 computational solutions.

## 88 **Classical versus Quantum Circuits: State of the Art**

89 Quantum computers (QCs) promise a new form of computing that would be qualitatively  
90 different from any previous ("classical") form of computation<sup>10</sup>. While QCs are technically more  
91 difficult to build, and the best current general-purpose quantum computers have only 50-100  
92 qubits, they can solve some problems with a time that grows more slowly as a function of the  
93 input size. The term "qubit" refers to a quantum two-level system, such as the spin of a spin-1/2  
94 particle. Qubits can be thought of as a generalization of classical bits (cbits), in that cbits can be  
95 in states 0 or 1, while the state of a single qubit is described by complex numbers  $\alpha_0$  and  $\alpha_1$   
96 satisfying  $|\alpha_0|^2 + |\alpha_1|^2 = 1$ . The power of quantum computers comes from scaling. A system  
97 of  $n$  cbits can be in one of  $2^n$  possible states at any time, while the state of  $n$  qubits is described  
98 by a complex unit vector of dimension  $2^n$  (Fig. 1A and B). These vectors (also called  
99 wavevectors or wavefunctions) can be transformed by multiplying them by unitary matrices,  
100 and in many cases this can be done efficiently. For example, the wavevector can be Fourier  
101 transformed using  $O(n^2)$  elementary quantum gates. However, not all transformations can be  
102 done efficiently. The laws of quantum measurement also limit the amount of information that  
103 can be extracted from a quantum state. A full measurement of the state yields outcome  $x$  with  
104 probability  $|\alpha_x|^2$ , destroying the state in the process. Thus, even though describing the  
105 quantum state of  $n$  qubits requires an amount of information that scales exponentially with  $n$ ,  
106 measurement can only extract  $n$  bits of information. Finding a way to benefit from the  
107 exponential state space of quantum computers despite this and other limitations is the central  
108 challenge of quantum algorithm design<sup>11</sup>.

109  
110 The challenges in building quantum hardware and mitigating noise are considerable and are not  
111 addressed in this paper, since our focus is principally on algorithm development. Large-scale  
112 quantum computers are likely to rely on error-correcting codes and other error mitigation

113 strategies which will result in additional overheads, e.g. needing to use many physical qubits to  
114 store one logical qubit. However, quantum algorithms can be built out of a universal set of  
115 quantum gates in a way that does not depend on the underlying hardware, just like classical  
116 algorithms.

117  
118 Given the ubiquity of classical computers, the natural way to understand the strengths of  
119 quantum computers is by comparing their run-time scaling with the best-known classical  
120 algorithms. In some cases, these speedups are exponential: a QC with a few thousand error-  
121 corrected qubits could factor numbers that could not be factored using existing classical  
122 computers and currently known algorithms in time less than the age of the universe. In other  
123 cases, provable polynomial speedups are known: for example, given the ability to compute a  
124 function  $f(x)$  where  $x$  takes on  $N$  values, a QC can find the minimum value of  $f(x)$  in only  
125  $O(\sqrt{N})$  evaluations of  $f(x)$  while a classical computer would require  $O(N)$  steps (assuming that  
126  $f(x)$  has no other structure we can exploit)<sup>13</sup>. On the other hand, for some problems, QCs are  
127 known to be no stronger than classical computers. And in many other cases, plausible heuristic  
128 algorithms have been proposed for QCs, whose performance is only incompletely understood.

129  
130 **The source of quantum speedup.** There is not a simple description of what accounts for  
131 speedups, although the most plausible explanation is the difference between interference of  
132 amplitudes and addition of probabilities. For example, a qubit can have states  $|0\rangle$  and  $|1\rangle$ ,  
133 which correspond to cbit values 0 and 1, and, in the representation of Fig 1A, are the north and  
134 south poles. Qubits can also be in superpositions (see Box 1) such as  $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$  and  $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ , which lie  
135 on the equator in the figure. To see that these differ from each other, and also from a random  
136 mixture of  $|0\rangle$  and  $|1\rangle$ , consider the  $\sqrt{NOT}$  gate, which maps  $|0\rangle$  and  $|1\rangle$  to  $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$  and  $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ ,  
137 respectively. Starting with the  $|0\rangle$  state, applying  $\sqrt{NOT}$  once yields  $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ . This state could be  
138 thought of as analogous to a random mixture of 0 and 1, as we would expect if  $\sqrt{NOT}$  means  
139 applying  $NOT$  with probability  $\frac{1}{2}$ . However, applying  $\sqrt{NOT}$  twice yields  $|1\rangle$ , just as we would  
140 expect from a  $NOT$  gate, whereas applying the randomized version twice would yield the same  
141 uniform mixture of 0 and 1. More generally, quantum computers and randomized computers  
142 can both be thought of as taking different paths through the  $2^n$  possible bit strings, but for  
143 randomized computers we sum the nonnegative-valued probabilities of these paths to get the  
144 final output distribution, while for quantum computers we sum the complex-valued amplitudes  
145 of these paths. Adding complex numbers of roughly the same phase corresponds to  
146 constructive interference while opposite phases correspond to destructive interference,  
147 analogous to the way that light and other waves can exhibit interference.

148 While we often do not know how to take advantage of the rich possibilities offered by quantum  
149 interference, in some cases we can use them to achieve asymptotic speedups. Algorithms like  
150 Grover's are simple examples of this, making use largely of the fact that probabilities are  
151 obtained by taking the square of quantum amplitudes, so that a subroutine with a small success  
152 probability  $p$  needs to be repeated only  $O(1/\sqrt{p})$  times instead of  $O(1/p)$  times<sup>14</sup>. The  
153 quantum Fourier transform (used in period finding and Shor's factoring algorithm (Fig. 1C)) is a  
154 more sophisticated example of how complex-weighted transitions can be useful, and in some

155 cases this can give rise to exponential speedups. On the other hand, some problems are known  
156 to not admit any quantum speedup, e.g. taking the parity of  $N$  numbers requires time  $O(N)$  on  
157 either a quantum or classical computer<sup>15</sup>. It is a major open research problem to determine  
158 when quantum speedup does or does not exist, and it is unlikely to ever be fully resolved, just  
159 as there is still no single theorem describing which problems can be solved by efficient classical  
160 algorithms. We next discuss some examples of potential quantum speedups.

161 **Exponential speedup.** The main exponential speedups known are for cryptanalysis (dramatic  
162 but unlikely to be relevant here) and quantum simulation of molecules or other large quantum  
163 systems. If the properties of a molecule are not well captured by simple classical  
164 approximations then there is a good case to be made for using a quantum computer to make a  
165 better-quality approximation computationally tractable. The advantage of a QC here arises  
166 from the exponentially growing dimension of quantum states. As a result, some promising cases  
167 for quantum advantage involve molecules with large numbers of active electrons, such as  
168 organometallic compounds<sup>16</sup>.

169 **Polynomial speedup.** Typical polynomial speedups can be thought of as direct improvements of  
170 some classical algorithms. The best known of these is Grover's square-root search speedup<sup>17</sup>,  
171 which is a quadratic improvement of classical brute-force search: given a search space of size  $N$ ,  
172 brute-force search requires evaluating  $N$  points, while Grover search requires the equivalent of  
173 evaluating  $O(\sqrt{N})$  points on a quantum computer. Other, more sophisticated, algorithms also  
174 admit provably quadratic improvements. For example, a classical algorithm might search over a  
175 tree of possibilities in a manner that can improve over brute-force search by sometimes being  
176 able to quickly prune entire subtrees. Such searches can also be quadratically improved  
177 quantumly, i.e. if the classical search process explores  $N$  nodes, then the quantum algorithm  
178 requires effort roughly equal to  $\sqrt{N}$  times the effort to evaluate one node<sup>18</sup>. The strength of  
179 these algorithms is that they apply under extremely general conditions, such as needing to  
180 minimize an easily computable function. They also do not usually need more qubits than are  
181 already needed to compute the function.

182 **Heuristic speedups.** Many of the most important algorithms for classical computers either lack  
183 formal proofs of correctness or are often run outside of the regime in which these proofs of  
184 correctness apply. These include Markov chain Monte Carlo (where rigorous upper bounds on  
185 mixing time are usually not known) and gradient descent applied to non-convex problems such  
186 as deep neural networks. For quantum computers, heuristic algorithms include adiabatic  
187 optimization<sup>19</sup>, or more generally, quantum annealing (QA)<sup>20</sup>, and the quantum approximate  
188 optimization algorithm (QAOA)<sup>21</sup>. The level of speedup provided by these algorithms over  
189 classical algorithms is in general unknown, and may be anywhere from an exponential  
190 improvement to no speedup. It is expected that as quantum computers are built, our  
191 understanding of the performance of these heuristics will improve, just as much of our  
192 understanding of the performance of classical heuristics comes from empirical evidence and  
193 not only theory. In the following sections, we refer to this class of methods as “quantum  
194 heuristics”.

195 **Interfacing with classical algorithms.** There is an important caveat about quantum algorithms.  
196 Suppose for concreteness that we are minimizing a function  $f(x)$ . Then a quantum computer  
197 would need to compute  $f(x)$  in superposition over many different values of  $x$ , i.e. the  
198 computation could not leak any information about  $x$  to any outside system. This would limit its

199 ability to share the computation with a classical computer. Suppose, for example, that the  
200 evaluation of  $f(x)$  were a memory- and time-intensive calculation for which quantum  
201 speedups were not known. Then using quantum computers to improve the minimization of  $f$   
202 would need to use qubits to perform this evaluation and could not offload the computation to a  
203 classical computer. This means that the overall speedup would be less than quadratic.

204 **Big data and quantum RAM.** A related limitation of current models of quantum computers is  
205 that they cannot access large classical datasets in superposition. This means that they may be  
206 able to speed up complicated calculations on small datasets (e.g. finding the best Bayesian  
207 network) but have less advantage in solving problems on large datasets. One way to address  
208 this is with filtering or data reduction techniques, which select a small but hopefully  
209 representative sample of the data and use that as input to the optimization problem<sup>22</sup>. Or the  
210 quantum computer could be used for "small data" problems where the difficulty comes from  
211 the complexity of the analysis. A more speculative possibility is a quantum hardware solution  
212 known as a qRAM (quantum RAM)<sup>8</sup>, which would give a quantum computer the ability to  
213 coherently query a large classical dataset as a superposition of qubits: a superposition of input  
214 memory addresses would yield an output consisting of a superposition of memory cell contents  
215 (see Box 2). A qRAM would enable powerful quantum algorithmic primitives<sup>8</sup> but there are no  
216 proposals for scalable error-corrected qRAM, and it is not clear if it would ultimately be easier  
217 than making a large quantum computer<sup>23</sup>.

## 218 **Potential applications for Quantum Computing in Biology**

### 219 **Genetics and sequence analysis**

220 We first consider QC algorithms implementable on near-term quantum processors. An essential  
221 initial step in genetics and genomics is the matching of sequences of nucleotides and amino  
222 acids to organism databases, and, more specifically, the mapping of sequencing reads from  
223 experimental assays to reference genomes. Any approach needs to contend with both memory  
224 (holding a representation of the reference, and information on the mapping) and speed  
225 concerns. Dynamic programming methods, such as the Smith-Waterman algorithm<sup>30</sup>, enable  
226 queries of sequence strings against immense databases, and could be cast as Hidden Markov  
227 Models (HMMs). The recent development of Hidden Quantum Markov Models (HQMMs)<sup>9,31</sup>  
228 opens the possibility of simulating classical HMMs on currently available quantum circuits<sup>31</sup>, as  
229 well as extending model space beyond classical HMMs<sup>9</sup>. Hybrid approaches are attractive  
230 prospects: the iteration through hyperparameter space in HMMs could be classical, with  
231 quantum optimization of the maximal trajectory through state space. Given that dynamic  
232 programming methods have mostly been supplanted by the approximate but faster k-mer-  
233 based BLAST algorithm<sup>30</sup> for database searches, a QC-based improvement in efficiency could  
234 reopen the case for their utility. A similar problem occurs in the *imputation* of individual-  
235 specific mutations, especially single-nucleotide polymorphisms (SNPs): given shared sets of  
236 haplotypes across subpopulations, a relatively sparse set of SNPs can be expanded by inferring  
237 additional SNPs that co-occur with the original set with high probability. This imputation usually  
238 involves an HMM-based likelihood maximization<sup>32</sup>, which could be cast as HQMMs.

239  
240 While imputation depends on inherited SNPs within populations (germline mutations), cells  
241 also contain post-conception *de novo* variants, called “somatic variants”. Every neuron in the  
242 human brain is likely to contain private somatic variants, including single nucleotide variants,  
243 and large structural variants that alter allelic diversity for dozens of genes. Identifying their  
244 functional impact is essential. Machine-learning classifiers have been trained on case/control  
245 datasets to identify psychiatric disorder-associated variants<sup>33</sup>. However, given the large-  
246 dimensional parameter search space for the classification problem, classical computation  
247 frequently runs into search efficiency issues. These issues could possibly be ameliorated using  
248 near-term implementable QC machine learning methods<sup>34</sup>, discussed in subsequent  
249 subsections.

250  
251 Another important category of genetic analyses is the construction of optimal trees that  
252 describe the relative proximity of genetic sequences, including: ancestral recombination graphs  
253 (ARGs)<sup>35</sup>, depicting ancestral relationships between individual genomes while accounting for  
254 genetic recombination; pathogen evolutionary trees in epidemiological studies; tumor cell  
255 mutational lineages, as could be relevant to malignancy and medical response. Tree  
256 reconstruction algorithms optimize across the similarity constraints between genomic  
257 segments, mainly involving sampling from the space of possible genealogies with heuristics and  
258 simplifications<sup>36</sup>. For smaller input sequence sets, the massive tree-search space makes this an  
259 open candidate problem for speed-up using available quantum heuristic optimization  
260 methods<sup>19–21</sup>.

261  
262 We next explore problems whose QC solutions may depend on the availability and storage in  
263 memory of superpositions of qubits (qRAM). For genomic read mapping, state-of-the-art  
264 classical algorithms include the exploitation of the Burrows-Wheeler transform to efficiently  
265 perform DNA sequence alignments<sup>37</sup>, and seed-based approaches to map RNA reads to exon  
266 boundaries separated by large genomic distances<sup>38</sup>. Both methods rely on lexicographically  
267 sorted suffixes constructed from the reference genome, followed by scanning for matches of  
268 the query read. The classical complexity of sequence-matching depends on whether exact  
269 ( $O(n + m)$ ;  $n$  = number of reads,  $m$  = query read length) or inexact matches ( $O(nm)$ ), including  
270 gaps, are considered. Grover’s algorithm-based improvements in string-matching speeds<sup>39</sup>  
271 could be exploited ( $O(\sqrt{n} + \sqrt{m})$  for exact matches) to aid the scanning process. Recent work  
272 has demonstrated the potential for even further QC speed gains under the assumption of  
273 unique membership of a query string within a reference database<sup>40</sup>. The scaling of the problem  
274 is such that a reduction in complexity of even simpler mapping problems would be highly  
275 beneficial, although the need to generate superpositions of the entire reference string also  
276 creates potential problems: given the need for storing a large reference database in  
277 superposition, the current lack of qRAM is an issue. Furthermore, speed gains from Grover’s  
278 algorithm-based methods could be reduced by the cost of evaluating the function being  
279 searched, if done classically.

280  
281 SNP association and heritability analyses are problematic for near-term quantum approaches,  
282 given the need to manipulate large matrices to solve systems of linear equations. In association

283 studies, SNPs can be statistically associated with individual-level phenotypes (genome-wide  
284 association studies (GWAS)) or to quantitative molecular traits (cell/tissue gene expression,  
285 methylation, epigenetic markers, cell fractions (QTL studies)). The evaluation of total SNP  
286 heritability often involves linear mixed effects models, with genetic variance estimations carried  
287 out through techniques such as the restricted maximum likelihood (REML) method<sup>41</sup>. With  
288 qRAM, algorithms such as Quantum Least Squares<sup>42,43</sup> could offer up to exponential speed-ups  
289 through the ability to perform fast linear-algebraic operations, under certain assumptions of  
290 sparseness and condition number, although it is unclear how any advantages would be  
291 undercut by the time cost of querying the qRAM. For lower-dimensional regression problems,  
292 there is some potential for near-term quantum heuristic optimizers to tackle these tasks<sup>26</sup>.  
293

## 294 **Functional Genomics**

295 The causal chain by which genetic variation leads to expression in higher-level behaviors such as  
296 cognitive traits involves multiple intermediate molecular-to-cellular-to-system steps, governed  
297 by complex developmental processes and gene-environment interactions. Despite this  
298 complexity, recent studies have shown that genetic risk for particular traits can be partitioned  
299 across ‘intermediate’ phenotypes, such as gene expression or chromatin binding profiles; a  
300 direct approach to such analysis is to impute intermediate molecular phenotypes first, and link  
301 the imputed phenotypes to high-level traits<sup>44</sup>. However, intermediate molecular phenotypes  
302 are typically high dimensional and interdependent, such as bulk transcriptome expression  
303 profiles ( $\approx 22K$  dimensional). Possible models which can learn joint probability distributions over  
304 such levels of analyses include Bayesian Networks, undirected models such as Boltzmann  
305 Machines<sup>45</sup>, and recent deep-learning approaches such as Variational Autoencoder (VAEs).  
306 Exact optimization of such models however is intractable: structure learning in Bayesian  
307 Networks requires optimization over a search space of all directed acyclic graphs, which is  
308 super-exponential ( $O\left(n! 2^{\frac{n!}{(2!(n-2)!)} }\right)$ ), where  $n$  is the dimensionality<sup>46</sup>). On the other hand,  
309 inference in Boltzmann machines requires a search over  $O(2^n)$  states after binarization to  
310 calculate a gradient, and training VAEs requires the optimization of a non-convex objective  
311 function. Such problems may be potential candidates for quantum approaches: for smaller  
312 input sizes, near-term approaches without qRAM may be developed to perform exact searches  
313 across the space of Bayesian networks, while for moderate-sized problems, approximate  
314 quantum analogues of Boltzmann machines and VAEs have been tested in simulation and  
315 experimentally<sup>6,7</sup>, with the optimization being conducted through QA. We note also that for all  
316 these models, prior knowledge of molecular interactions may be used during training to suggest  
317 causal network interpretations.

318

319 In contrast to direct imputation of molecular phenotypes, intermediate phenotypes may be  
320 derived at the level of sets of genes (such as functional pathways), and cell-type proportions.  
321 For instance, Weighted Gene Correlation Network Analysis (WGCNA) performs a version of  
322 hierarchical clustering to derive co-expression modules, which are enriched in gene pathways<sup>47</sup>,



323 and non-negative matrix factorization (NMF) based on ‘marker-gene’ profiles can be used to  
324 decompose bulk transcriptome data into components corresponding to cell-type fractions<sup>45</sup>.  
325 Exact optimization of these models is again intractable, where exact hierarchical clustering  
326 would require a search over a large space of trees, and NMF is a non-convex optimization  
327 problem. The former may be a candidate for an exact quantum solution for small-scale  
328 problems, while both may benefit from quantum heuristic approaches (a QA approach to NMF  
329 is found in ref.<sup>48</sup> and quantum speedups for approximate clustering are described in ref.<sup>22</sup>).  
330 While clustering ~1000 - ~20,000 features is common in genomics, there are a number of  
331 applications where a relatively small number of features, ~100, are clustered across samples  
332 (e.g., protein-array data). Clustering associated with global minimization of objective functions  
333 is of great interest in these small feature number cases. More generally, comparison of clusters  
334 (and solutions to other genomic algorithms) derived from exact and approximate greedy  
335 minimization would inform the nature of the errors associated with applying greedy algorithms  
336 to large numbers of features and samples, as well as suggest possible approaches to improving  
337 the greedy algorithms in the short term. Application of these methods at full genomic scale,  
338 however, would require further technical developments in qRAM or quantum processor size.

### 339 **Mapping Neuro-Behavioral Variation via Neuroimaging and Deep Phenotyping**

340 The overarching goal of ‘convergent’ neuroscience is to link cellular-level mechanisms to  
341 system-level observations and ultimately behavior. Multi-modal neuroimaging provides rich  
342 high-dimensional data that can map neural and behavioral mechanisms in humans. While many  
343 quantitative optimizations remain to be done, one of the core challenges is accurate  
344 identification and alignment of brain anatomy across people to reference atlases. For instance,  
345 one widespread approach implemented in FreeSurfer software<sup>49</sup> employs a sequence of  
346 registration steps involving the minimization of an energy functional over the spatial  
347 transformation field. Here, potential quantum heuristic approaches could be brought to bear  
348 for images of moderate resolution if the corresponding energy function (Hamiltonian) can be  
349 mapped to an Ising-type model. A related challenge involves training statistical models to  
350 rapidly and accurately quantify neuro-behavioral variation. For instance, the presence of active  
351 psychotic symptoms in previously unseen individuals diagnosed with schizophrenia and bipolar  
352 illness can be predicted using dynamic functional connectome features derived from fMRI<sup>50</sup>.  
353 Quantum analogues (such as HQMMs<sup>9,31</sup>, see ‘Genetics and Sequence Analysis’) may help train  
354 such predictive models more efficiently.

355 Computational neuroscience has used circuit models to inform and constrain experimental  
356 observations. Dynamical neural models operate at the local circuit or global level, and use  
357 parameterizations based on known constraints (e.g. biophysical parameters) or learned *de*  
358 *novo*. Local and global neural dynamics are typically highly nonlinear, producing difficult  
359 optimization problems in the case of parametric model fitting<sup>51</sup>, and requiring a rich model-  
360 class for *de novo* learning methods. Fluctuations at equilibrium exhibit complex inter-  
361 dependencies. Additionally, the hierarchical relationships between genetics, anatomy, function  
362 and the equilibrium connectivity neural state are, in general, highly nonlinear, and only partially  
363 captured by available computational models. Current classical models relate such simulations  
364 to equilibrium distribution features (or to resting state characteristics): for instance, Ising

365 models and second-order mean-field regional models of resting-state fMRI observations<sup>52,53</sup>.  
366 These differential equation-based analyses of global brain dynamics represent regional firing  
367 rates using a mean-field approximation<sup>52</sup>. Such models can be fitted to functional neuroimaging  
368 data, by linearizing the initial stochastic nonlinear system of differential equations around a  
369 fixed point using the method of moments<sup>52</sup>, and using methods such as Approximate Bayesian  
370 Computation to fit parameters<sup>51</sup>. In the QC domain, quantum algorithms have been developed  
371 which have the potential to offer exponential speed-ups in the solution of linear differential  
372 equations<sup>54,55</sup>. Furthermore, models such as the Quantum Boltzmann machine (QBM)<sup>6</sup> and  
373 Quantum VAE<sup>7</sup>, as discussed in the previous subsection, may be naturally applied to model  
374 complex distributions as found in neurodynamics datasets.

375 General-purpose quantum solvers for nonlinear systems of differential equations have also  
376 been proposed<sup>56</sup>, although currently these seem unlikely to offer speed-ups over classical  
377 methods. Efficient general-purpose solvers would eliminate the need for linear approximations,  
378 and allow more accurate fitting of neural dynamical models, particularly out of steady state (for  
379 example, transitions between resting-state and task-based fMRI). This application may help  
380 motivate finding better quantum algorithms for nonlinear differential equations.

381 The computational challenge in human neuroscience is particularly acute in the case of ‘deep’  
382 behavioral phenotyping (e.g. digital ‘real time’ measures), which can generate massive amounts  
383 of continuously measured dynamical behavioral variables with varied granularity. In this  
384 situation, there is clear potential for ‘very deep’ optimization and the opportunity for massive  
385 state-space exploration. Relevant use-case scenarios include ‘in-the-moment’ clinical decisions  
386 that may require rapid computation. This becomes challenging for longitudinal real-time digital  
387 phenotyping, which may require rapid and precise data reduction. For instance, rich  
388 individualized phenotypic characterization using high-resolution video and audio datasets have  
389 yet to be leveraged since they are identifiable in raw form and present operational challenges  
390 to data reduction and protection of participant privacy.

391 Collectively, the complexity of human neuro-behavioral data tests the boundaries of learning  
392 algorithms, which have to deal with the high-dimensionality of data needed to robustly link  
393 nonlinear dynamics of brain states (e.g. fMRI) and the influence of time-related variables  
394 relevant to behavioral mapping. Recent deep learning approaches using interpretable recurrent  
395 networks have provided a powerful means of learning such brain-state/behavior associations  
396 de novo by jointly modeling fMRI and behavioral data<sup>57</sup>. Quantum analogs of neural network  
397 frameworks (such as QBMs<sup>6</sup> and QVAEs<sup>7</sup>) have the potential to discover novel structure in these  
398 datasets. Models such as HQMMs provide alternative dynamical models with intrinsically  
399 quantum representations<sup>31</sup>, which have been shown to have comparable or possibly improved  
400 performance relative to classical methods on small-scale problems through classical  
401 simulations. Further, there is evidence that HQMMs allow complex dynamics to be modelled  
402 with a reduced state space<sup>9</sup> compared to classical models. The application of such methods to  
403 behavioral data, though, is a long-term goal, since reliable qRAM appears necessary to handle  
404 large dataset sizes.

## 405 **Integration across disciplines**

406 Stitching together insights across fields and levels of analyses, to yield a complete picture of  
407 brain function, is an ongoing challenge. While the extent to which quantum processes are  
408 relevant across these levels is unclear, quantum machine learning may help elucidate the  
409 interdependencies between levels through its ability to learn and simulate nonlinear,  
410 potentially classically intractable, models. One promising avenue involves mechanism-agnostic  
411 machine learning methods like deep neural networks, where biological insights are gained by  
412 interpreting the model *a posteriori*. Such an interpretable framework would involve  
413 connections between modules such as gene regulatory networks on the one hand, and  
414 structural/functional neuroimaging parameters (e.g. cortical thickness, white matter integrity,  
415 dynamic functional connectivity, etc.) on the other. The exact nature of these connections  
416 could be altered in competing hypotheses. One could imagine a hierarchical network with  
417 molecular phenotypes at the base, emergent neuroimaging-based parameters at a higher layer,  
418 and behavioral phenotypes as prediction targets. An alternative framework would treat the  
419 molecular and neural system-level components as parallel factors in determining behavior, with  
420 the latter having been influenced at a developmental stage, and not directly emerging from the  
421 molecular phenotypes per se but rather operating in dependent lock-step. Thus, different  
422 architectures of relationships between levels of analysis may be constructed. The NIMH has  
423 recently supported efforts at building such multi-scale, convergent neuroscience approaches  
424 (<https://grants.nih.gov/grants/guide/pa-files/par-17-176.html>). Such an analysis could be aided  
425 by quantum neural networks (QNNs)<sup>58</sup> and quantum variational classifiers<sup>59</sup>, designed for use  
426 on non-qRAM, gate-based quantum computers. Quantum variational classifiers have been  
427 shown to be able to successfully classify states that were designed to be hard to simulate  
428 classically<sup>59</sup>. This hints at the greater generality of such circuits than their classical counterparts.  
429 Here the challenge lies in scaling up the available number of qubits.

430

## 431 **Epilogue**

432

433 While the field of QC is undergoing notable development and progress in both hardware and  
434 software, a number of significant knowledge gaps and challenges remain. To surpass classical  
435 computers, quantum computer architectures will need to improve numbers of and connectivity  
436 between qubits, reduce error rates both for operations and storage, as well as expand  
437 algorithmic development into all areas where classical computing faces inherent bottlenecks.  
438 These challenges are all significant and are partially conflicting; indeed, the central  
439 experimental QC challenge is to create quantum systems that are both highly decoupled from  
440 unwanted environmental degrees of freedom yet subject to fast and precise control and  
441 measurement. While there has been steady experimental progress over the past two decades,  
442 it is not easy to predict the rate of future improvements in QC. A recent consensus study on the  
443 progress and prospects of QC from the National Academies of Sciences, Engineering and  
444 Medicine estimates that to find a private key in a 1024-bit RSA encrypted message using Shor's  
445 algorithm requires building a quantum computer that is five orders of magnitude larger and has

446 error rates that are two orders of magnitude better than existing machines<sup>60</sup>. More than 100  
447 academic and government laboratories around the world are working to address these  
448 challenges with a variety of hardware solutions<sup>60</sup>. These include ion-trap quantum computers  
449 with 20-100 qubits that are likely to become available by the early 2020s<sup>60</sup>. Leveraging the  
450 power of lithographic technology, superconducting quantum computers hold great promise,  
451 and 5-, 16- and 20-qubit machines are currently available to users via the web. Other promising  
452 approaches include developing quantum computers based on photonic, neutral-atom and  
453 semiconductor qubits<sup>60</sup>.

454  
455 As mentioned above, many algorithmic quantum speedups depend on qRAM, but there is no  
456 practical implementation of this technology. In fact, this reliance on qRAM, in part, stems from  
457 attempts to arrive at algorithms that are essentially quantum versions of classical algorithms.  
458 An alternative approach is to design intrinsically quantum algorithms which take advantage of  
459 quantum features such as interference. This alternative approach offers the additional benefit  
460 that small-scale versions of problems are readily implementable on existing hardware. Indeed,  
461 recent advances in “near-term” quantum machine learning algorithm development exploit the  
462 exponentially large quantum state space to estimate kernel functions<sup>59,61</sup> as well as the natural  
463 ability of quantum computers to execute kernel-based classification<sup>62,63</sup>. Generalizations of  
464 these algorithms for genomics applications hold great promise and will allow assessment of the  
465 current capabilities of publicly available quantum computers<sup>34</sup>. Given the potential of quantum  
466 computers to efficiently explore a vast state space, the natural applications to neuroscience  
467 problems are largely associated with optimization and machine learning as detailed above.  
468 However, yet another path is to identify computational problems that can be naturally cast into  
469 a quantum framework. For example, the minimum free energy among all possible protein folds  
470 is an important problem with an exponentially large search space and thus a compelling target.  
471 Another natural set of problems are those associated with quantum biology – the study of  
472 chemical processes including formation of excited electron states within molecules (e.g.,  
473 proteins) in living cells, and their functional effects<sup>64</sup>. These processes are inherently quantum  
474 mechanical and may involve an exponentially vast set of excitation states, which can only be  
475 efficiently modeled by applying transformations to an exponentially large state-space afforded  
476 by a quantum computer. It is unclear whether such processes can be relevant to higher-levels  
477 of brain function (and consciousness<sup>65</sup>); the algorithms used by the brain at David Marr’s  
478 algorithmic/representational level may ultimately be classical<sup>66</sup>, although the advent of  
479 quantum machine learning means that increasingly this need not be the case for artificial  
480 agents. While a cautious albeit optimistic estimation associated with steady progress of  
481 quantum hardware development (e.g., applying Moore’s law) puts the availability of sufficiently  
482 powerful, universal quantum computers years in the future, sudden, orders-of-magnitude  
483 breakthroughs in resolution, noise reduction, etc. are not unprecedented in experimental  
484 physics. Such unforeseen breakthroughs would unleash the power of quantum computing to  
485 address pressing computational challenges in biology.

## 486 **Acknowledgements**

487 This work is a product of discussions initiated during a NIMH-convened virtual workshop,  
488 addressing computational challenges in genomics and neuroscience via massively parallel  
489 computing and QC ([https://www.nimh.nih.gov/news/events/2018/virtual-workshop-solving-  
490 computational-challenges-in-genomics-and-neuroscience-via-parallel-and-quantum-  
491 computing.shtml](https://www.nimh.nih.gov/news/events/2018/virtual-workshop-solving-computational-challenges-in-genomics-and-neuroscience-via-parallel-and-quantum-computing.shtml)). We would also like to acknowledge the help and support of Lora Bingaman  
492 of the NIMH in overseeing the administration of this collaboration.  
493

## 494 **Author Contributions**

495 All authors contributed to discussions on the design of the manuscript. G.S. and T.L. led the  
496 NIMH workshop and subsequent discussions. P.S.E., J.W., A.A., S.B., M.G., M.J.M., J.T.B., M.B.G.  
497 and A.W.H wrote the manuscript. P.S.E., J.W., A.A., S.B., M.G., M.J.M., G.S., J.M., J.T.B., G.S.,  
498 T.L., M.B.G. and A.W.H edited the manuscript. A.W.H. contributed to the Quantum Computing  
499 section. P.S.E., J.W., S.B., M.G., M.J.M., M.B.G. contributed content to the genetics and  
500 genomics sections, and A.A., G.S., J.T.B., and J.M. to the imaging and behavioral phenotyping  
501 subsections.

502

## 503 **References**

- 504 1. Mohseni, M. *et al.* Commercialize quantum technologies in five years. *Nature* **543**, 171–174 (2017).
- 505 2. Arute, F. *et al.* Quantum supremacy using a programmable superconducting processor. *Nature* **574**,  
506 505–511 (2019).
- 507 3. U.S. House. 115th Congress. H.R.6227, U.S. National Quantum Initiative Act. *Washington:*  
508 *Government Printing Office* (2018).
- 509 4. Monroe, C., Raymer, M. G. & Taylor, J. The U.S. National Quantum Initiative: From Act to action.  
510 *Science* **364**, 440–442 (2019).
- 511 5. Thew, R., Jennewein, T. & Sasaki, M. Focus on quantum science and technology initiatives around  
512 the world. *Quantum Sci. Technol.* **5**, 010201 (2019).
- 513 6. Amin, M. H., Andriyash, E., Rolfe, J., Kulchytskyy, B. & Melko, R. Quantum Boltzmann Machine.  
514 *Physical Review X* **8**, 021050 (2018).
- 515 7. Khoshaman, A., Vinci, W., Denis, B., Andriyash, E. & Amin, M. H. Quantum variational autoencoder.

- 516 *Quantum Science and Technology* **4**, 1–12 (2019).
- 517 8. Giovannetti, V., Lloyd, S. & Maccone, L. Quantum random access memory. *Phys. Rev. Lett.* **100**,  
518 160501 (2008).
- 519 9. Monras, A., Beige, A. & Wiesner, K. Hidden Quantum Markov Models and non-adaptive read-out of  
520 many-body states. *arxiv.org/abs/1002.2337* (2010).
- 521 10. Nielsen, M. A. & Chuang, I. L. *Quantum Computation and Quantum Information*. (Cambridge  
522 University Press, 2010).
- 523 11. Alexeev, Y. *et al.* Quantum Computer Systems for Scientific Discovery.  
524 <http://arxiv.org/abs/1912.07577> (2019).
- 525 12. Van Meter, R., Itoh, K. M. & Ladd, T. D. Architecture-dependent execution time of Shor’s algorithm.  
526 <https://arxiv.org/abs/quant-ph/0507023v2> 183–188 (2008) doi:10.1142/9789812814623\_0029.
- 527 13. Durr, C. & Hoyer, P. A Quantum Algorithm for Finding the Minimum. *arxiv.org/abs/quant-*  
528 *ph/9607014* (1996).
- 529 14. Brassard, G., Høyer, P., Mosca, M. & Tapp, A. Quantum amplitude amplification and estimation. in  
530 *Contemporary Mathematics: Quantum Computation and Information* (eds. Lomonaco, S. J. &  
531 Brandt, H. E.) 53–74 (American Mathematical Society, 2002). doi:10.1090/conm/305/05215.
- 532 15. Farhi, E., Goldstone, J., Gutmann, S. & Sipser, M. Limit on the speed of quantum computation in  
533 determining parity. *Phys. Rev. Lett.* **81**, 5442–5444 (1998).
- 534 16. Li, Z., Li, J., Dattani, N. S., Umrigar, C. J. & Chan, G. K. L. The electronic complexity of the ground-  
535 state of the FeMo cofactor of nitrogenase as relevant to quantum simulations. *J. Chem. Phys.* **150**,  
536 024302 (2019).
- 537 17. Grover, L. K. A fast quantum mechanical algorithm for database search. [http://arxiv.org/abs/quant-](http://arxiv.org/abs/quant-ph/9605043)  
538 *ph/9605043* (1996).
- 539 18. Ambainis, A. & Kokainis, M. Quantum algorithm for tree size estimation, with applications to

- 540 backtracking and 2-player games. *Proceedings of the 49th Annual ACM SIGACT Symposium on*  
541 *Theory of Computing - STOC 2017* 989–1002 (2017) doi:10.1145/3055399.3055444.
- 542 19. Farhi, E., Goldstone, J., Gutmann, S. & Sipser, M. Quantum Computation by Adiabatic Evolution.  
543 <http://arxiv.org/abs/quant-ph/0001106> (2000).
- 544 20. Kadowaki, T. & Nishimori, H. Quantum annealing in the transverse Ising model. *Phys. Rev. E* **58**,  
545 5355–5363 (1998).
- 546 21. Farhi, E., Goldstone, J. & Gutmann, S. A Quantum Approximate Optimization Algorithm.  
547 <http://arxiv.org/abs/1411.4028> 1–16 (2014).
- 548 22. Harrow, A. W. Small quantum computers and large classical data sets.  
549 <http://arxiv.org/abs/2004.00026> (2020).
- 550 23. Arunachalam, S. & de Wolf, R. A Survey of Quantum Learning Theory.  
551 <https://arxiv.org/abs/1701.06806v3> (2017).
- 552 24. Outeiral, C. *et al.* The prospects of quantum computing in computational molecular biology. *WIREs*  
553 *Comput Mol Sci* **4**, 406 (2020).
- 554 25. Perdomo-Ortiz, A., Dickson, N., Drew-Brook, M., Rose, G. & Aspuru-Guzik, A. Finding low-energy  
555 conformations of lattice protein models by quantum annealing. *Sci. Rep.* **2**, 1–7 (2012).
- 556 26. Li, R. Y., Di Felice, R., Rohs, R. & Lidar, D. A. Quantum annealing versus classical machine learning  
557 applied to a simplified computational biology problem. *npj Quantum Information* **4**, 14 (2018).
- 558 27. Kandala, A. *et al.* Hardware-efficient variational quantum eigensolver for small molecules and  
559 quantum magnets. *Nature* **549**, 242–246 (2017).
- 560 28. Dorner, R., Goold, J. & Vedral, V. Towards quantum simulations of biological information flow.  
561 *Interface Focus* **2**, 522–528 (2012).
- 562 29. Foss-Feig, J. H. *et al.* Searching for Cross-Diagnostic Convergence: Neural Mechanisms Governing  
563 Excitation and Inhibition Balance in Schizophrenia and Autism Spectrum Disorders. *Biol. Psychiatry*

- 564           **81**, 848–861 (2017).
- 565   30. Li, H. & Homer, N. A survey of sequence alignment algorithms for next-generation sequencing.  
566       *Brief. Bioinform.* **11**, 473–483 (2010).
- 567   31. Srinivasan, S., Downey, C. & Boots, B. Learning and Inference in Hilbert Space with Quantum  
568       Graphical Models. *32nd Conference on Neural Information Processing Systems (NIPS 2018)*,  
569       *Montréal, Canada.* (2018).
- 570   32. Howie, B. N., Donnelly, P. & Marchini, J. A flexible and accurate genotype imputation method for  
571       the next generation of genome-wide association studies. *PLoS Genet.* **5**, e1000529 (2009).
- 572   33. McConnell, M. J. *et al.* Intersection of diverse neuronal genomes and neuropsychiatric disease: The  
573       Brain Somatic Mosaicism Network. *Science* **356**, eaal1641 (2017).
- 574   34. Kathuria, K., Ratan, A., McConnell, M. & Bekiranov, S. Implementation of a Hamming distance–like  
575       genomic quantum classifier using inner products on ibmqx2 and ibmq\_16\_melbourne. *Quantum*  
576       *Machine Intelligence* **2**, 1–26 (2020).
- 577   35. Griffiths, R. C. & Marjoram, P. An Ancestral Recombination Graph. in *Progress in population*  
578       *genetics and human evolution* 257–270 (Springer, 1997).
- 579   36. Li, H. & Durbin, R. Inference of human population history from individual whole-genome  
580       sequences. *Nature* **475**, 493–496 (2011).
- 581   37. Li, H. & Durbin, R. Fast and accurate long-read alignment with Burrows-Wheeler transform.  
582       *Bioinformatics* **26**, 589–595 (2010).
- 583   38. Dobin, A. *et al.* STAR: Ultrafast universal RNA-seq aligner. *Bioinformatics* **29**, 15–21 (2013).
- 584   39. Ramesh, H. & Vinay, V. String Matching in  $O(Vn+Vm)$  Quantum Time. [http://arxiv.org/abs/quant-](http://arxiv.org/abs/quant-ph/9605043)  
585       *ph/9605043* 1–7 (2000).
- 586   40. Montanaro, A. Quantum Pattern Matching Fast on Average. *Algorithmica* **77**, 16–39 (2017).
- 587   41. Yang, J., Lee, S. H., Goddard, M. E. & Visscher, P. M. GCTA: A tool for genome-wide complex trait



- 588 analysis. *Am. J. Hum. Genet.* **88**, 76–82 (2011).
- 589 42. Harrow, A. W., Hassidim, A. & Lloyd, S. Quantum algorithm for linear systems of equations. *Phys.*  
590 *Rev. Lett.* **103**, 1–4 (2009).
- 591 43. Wiebe, N., Braun, D. & Lloyd, S. Quantum algorithm for data fitting. *Phys. Rev. Lett.* **109**, 1–5 (2012).
- 592 44. Gamazon, E. R. *et al.* A gene-based association method for mapping traits using reference  
593 transcriptome data. *Nat. Genet.* **47**, 1091–1098 (2015).
- 594 45. Wang, D. *et al.* Comprehensive functional genomic resource and integrative model for the human  
595 brain. *Science* **362**, eaat8464 (2018).
- 596 46. Robinson, R. W. Counting Unlabeled Acyclic Graphs. in *Combinatorial Mathematics V. Lecture Notes*  
597 *in Mathematics, Vol. 622* (ed. Little, C. H. C.) (Springer, 1977). doi:10.1007/BFb0069178.
- 598 47. Zhang, B. & Horvath, S. A General Framework for Weighted Gene Co-Expression Network Analysis.  
599 *Stat. Appl. Genet. Mol. Biol.* **4**, 17 (2005).
- 600 48. O’Malley, D., Vesselinov, V. V., Alexandrov, B. S. & Alexandrov, L. B. Nonnegative/Binary matrix  
601 factorization with a D-Wave quantum annealer. *PLoS One* **13**, e0206653 (2018).
- 602 49. Fischl, B. FreeSurfer. *Neuroimage* **62**, 774–781 (2012).
- 603 50. Reinen, J. M. *et al.* The human cortex possesses a reconfigurable dynamic network architecture that  
604 is disrupted in psychosis. *Nat. Commun.* **9**, 1–15 (2018).
- 605 51. Demirtaş, M. *et al.* Hierarchical Heterogeneity across Human Cortex Shapes Large-Scale Neural  
606 Dynamics. *Neuron* **101**, 1181–1194.e13 (2019).
- 607 52. Deco, G. *et al.* How Local Excitation-Inhibition Ratio Impacts the Whole Brain Dynamics. *Journal of*  
608 *Neuroscience* **34**, 7886–7898 (2014).
- 609 53. Deco, G., Senden, M. & Jirsa, V. How anatomy shapes dynamics: a semi-analytical study of the brain  
610 at rest by a simple spin model. *Front. Comput. Neurosci.* **6**, 1–7 (2012).
- 611 54. Childs, A. M. & Liu, J.-P. Quantum spectral methods for differential equations.

- 612 <https://arxiv.org/abs/1901.00961v1> 1–29 (2019).
- 613 55. Berry, D. W., Childs, A. M., Ostrander, A. & Wang, G. Quantum Algorithm for Linear Differential  
614 Equations with Exponentially Improved Dependence on Precision. *Commun. Math. Phys.* **356**,  
615 1057–1081 (2017).
- 616 56. Leyton, S. K. & Osborne, T. J. A quantum algorithm to solve nonlinear differential equations.  
617 <https://arxiv.org/abs/0812.4423v1> 1–11 (2008).
- 618 57. Dezfouli, A., Morris, R., Ramos, F., Dayan, P. & Balleine, B. W. Integrated accounts of behavioral and  
619 neuroimaging data using flexible recurrent neural network models. *32nd Conference on Neural  
620 Information Processing Systems (NeurIPS 2018), Montréal, Canada.* 1–10 (2018)  
621 doi:10.1101/328849.
- 622 58. Farhi, E. & Neven, H. Classification with Quantum Neural Networks on Near Term Processors.  
623 <http://arxiv.org/abs/1802.06002> 1–21 (2018).
- 624 59. Havlíček, V. *et al.* Supervised learning with quantum-enhanced feature spaces. *Nature* **567**, 209–  
625 212 (2019).
- 626 60. National Academies of Sciences Engineering, A. M. *Quantum Computing: Progress and Prospects.*  
627 <http://dx.doi.org/10.17226/25196> (2019) doi:10.17226/25196.
- 628 61. Schuld, M. & Killoran, N. Quantum Machine Learning in Feature Hilbert Spaces. *Phys. Rev. Lett.* **122**,  
629 40504 (2019).
- 630 62. Schuld, M., Fingerhuth, M. & Petruccione, F. Implementing a distance-based classifier with a  
631 quantum interference circuit. *Europhys. Lett.* **119**, 60002 (2017).
- 632 63. Schuld, M., Bocharov, A., Svore, K. & Wiebe, N. Circuit-centric quantum classifiers.  
633 *arXiv:1804.00633v1* (2018).
- 634 64. Lambert, N. *et al.* Quantum biology. *Nat. Phys.* **9**, 10–18 (2013).
- 635 65. Tegmark, M. Importance of quantum decoherence in brain processes. *Phys. Rev. E* **61**, 4194–4206

636 (2000).

637 66. Marr, D. C. & Poggio, T. From Understanding Computation to Understanding Neural Circuitry.

638 *Neurosci. Res. Program Bull.* **15**, 470–488 (1977).

## 639 **Figure Legends**

640 **Figure 1. Concepts in Quantum Computing. A.** Conceptual illustration of bit vs. qubit.  
641 The state of a qubit can be represented by a point on the unit sphere with the North and  
642 South poles corresponding to the states 0 and 1 of a classical bit. **B.** The state space of  
643 3 qubits is a  $2^3$ -dimensional complex vector. **C.** Classical (Number Field Sieve (NFS)  
644 algorithm) and quantum (Beckman-Chari-Devabhaktuni-Preskill (BCDP) implementation  
645 of Shor's algorithm) runtimes for factoring integers. Shor's algorithm for quantum  
646 computers yields an exponential speedup over the best-known classical algorithm  
647 (Panel C from ref <sup>12</sup>).  
648

649 **Figure 2. Complexity of linking levels of analyses from genomics to human**  
650 **behavior.** The challenge consists, in part, of the need to interrogate the  
651 enormous search space for determining the mapping across levels, which  
652 constitutes a many-to-many probabilistic problem. Computational innovation will  
653 be a key effort to help close these gaps. Figure adapted with permission from  
654 ref. <sup>29</sup>.  
655

## 656 **Boxes**

### 657 **Box 1: Glossary of Terms**

#### 658 **Biological:**

- 659 ○ *Single nucleotide Polymorphisms (SNPs).* Germline (inherited) mutations in a genome  
660 where the identity of a single nucleotide is changed relative to a reference genome, and  
661 whose prevalence in a population is dependent on the pattern of their inheritance.
- 662 ○ *Genetic recombination.* Exchange of segments between separate genomes or  
663 chromosomes, or different regions of the same chromosome, by the creation of single- (eg.  
664 viruses) or double-stranded (eg. humans) breaks and subsequent ligation of the crossed  
665 segments.
- 666 ○ *Genome-Wide Association Study (GWAS).* Identification of mutations in a population with  
667 statistically significant associations to the occurrence of a studied phenotype.
- 668 ○ *Quantitative Trait Loci (QTL).* Mutations in a genome or population with statistically  
669 significant association to the occurrence of a studied endophenotype, i.e. a phenotype at  
670 the sub-organism level, for example, cell- or tissue-level gene expression.

671 **Machine Learning:**

- 672 ○ *Hidden Markov Models (HMMs)*. Stochastic latent state method to model a linear  
673 sequence of observations as a probabilistic sequence of underlying state transitions and  
674 state-to-observation emissions.
- 675 ○ *Boltzmann Machines*. Generative classical neural network model, based on an energy  
676 function containing local (unary) and pairwise terms over an underlying undirected graph.  
677 Recently, the model has been extended to replace the classical energy with a quantum  
678 Hamiltonian to form a Quantum Boltzmann Machine (QBM)<sup>6</sup>.
- 679 ○ *Variational Auto-Encoders (VAEs)*. Generative neural network model, incorporating a latent  
680 space which is mapped to observed variables by a learned feedforward classical neural  
681 network. Latent space can be a classical (Gaussian) or quantum (QBM)<sup>7</sup> distribution.

682 **Quantum Computing:**

- 683 ○ *Quantum Superposition*. A fundamental principle of quantum mechanics whereby the  
684 overall state of a system (e.g., electron in an atom, qubit, etc.) is in a linear combination of  
685 orthogonal basis states (e.g., lowest energy state, next excited state, etc.). For example, if  
686  $|0\rangle$  denotes the lowest energy state of a qubit and  $|1\rangle$  an excited state of a qubit, the state  
687 of the qubit,  $|\psi\rangle$ , can be in a superposition of basis states:  $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$ .
- 688 ○ *Quantum Random Access Memory (qRAM)*. In analogy with random access memory (RAM)  
689 which uses  $n$  bits to address  $2^n$  distinct memory cells, qRAM uses  $n$  qubits to address any  
690 quantum superposition of  $2^n$  memory cells<sup>8</sup>.
- 691 ○ *Quantum Annealing (QA)*. A technique for minimizing a function  $f$  using a low-temperature  
692 quantum system whose energy corresponds to  $f$ , along with an auxiliary field which is  
693 slowly turned off. The auxiliary field attempts to create superpositions between nearby  
694 qubit strings, similar to equally weighting possible solutions, and facilitates “quantum  
695 tunneling” (i.e. transition of a quantum state between nearby low-energy strings even  
696 through regions of higher energy) to arrive at a minimum of  $f$  relatively efficiently once  
697 turned off.
- 698 ○ *Hidden Quantum Markov Model (HQMMs)*. The quantum analogue of HMMs, where the  
699 sequence of quantum operations is such that information of the state transition and  
700 emission probabilities of the qubits can be retained even after partial measurement of the  
701 system (i.e. measurements do not collapse the entire system)<sup>9</sup>.
- 702

703 **Box 2: Computational Opportunities for the Future**

704 Existing quantum algorithms, for example, function minimization, are often written in terms of  
705 abstract and highly general functions. If biological applications can help motivate specific,  
706 mathematically well-posed tasks, then it may be the case that targeted quantum algorithm  
707 development can lead to improvement. While this promise is discussed at length in the  
708 following section in the context of the study of the human brain, here we briefly introduce  
709 some of the key areas of ongoing research in quantum computing, related to and providing the  
710 context for applications in biology.

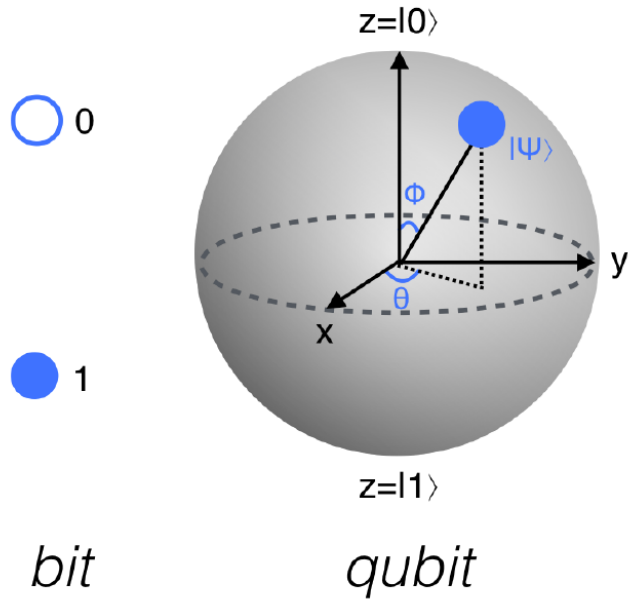
711 *Optimization in biomolecular problems*. There has also been considerable interest in extending  
712 QC to biomolecular and biological problems<sup>24</sup>. In several cases, small examples of biological

713 problems have been mapped to combinatorial optimization problems. A QA approach was  
714 employed in the exploration of the coarse-grained folding landscape of a six-amino acid  
715 peptide, within a 2D lattice framework<sup>25</sup>. QA was also evaluated against a set of classical  
716 methods on an optimization problem involving the search for the consensus DNA sequence  
717 motif of transcription factor binding<sup>26</sup>. In this instance, the authors trained a classifier  
718 (sequence is binding or non-binding) and a ranking algorithm (ranking sequences by binding  
719 affinity), finding a slight improvement of QA over classical approaches in the classification  
720 problem, and similar performance for the ranking task.

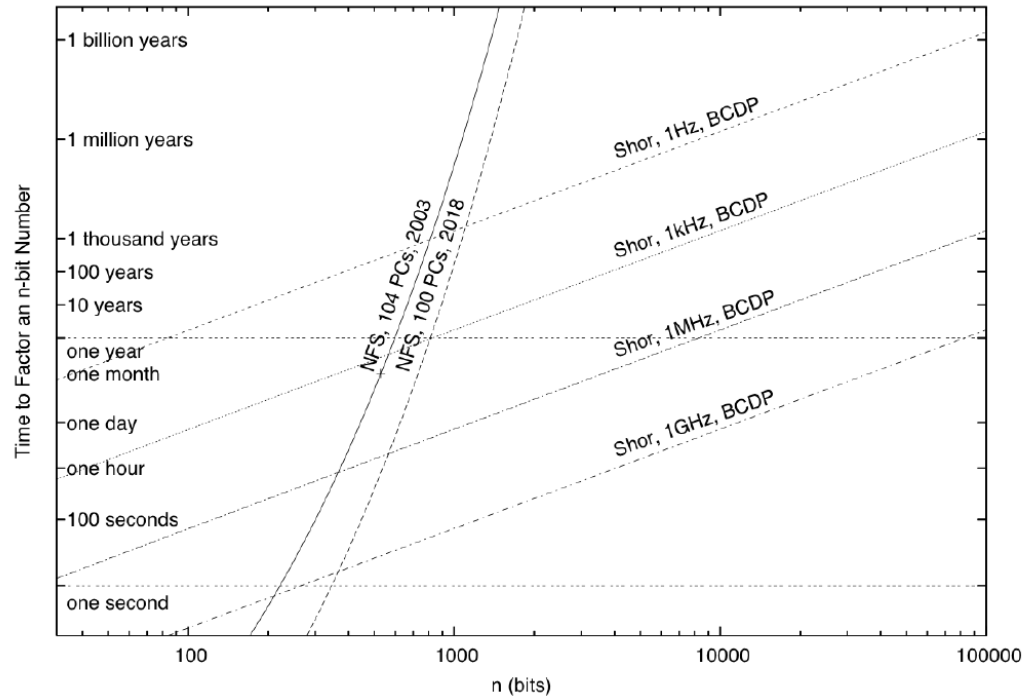
721 *Simulation of classical and quantum systems.* There have been successful demonstrations of the  
722 application of quantum computation to problems in chemistry. A Variational Quantum  
723 Eigensolver (VQE) approach was used<sup>27</sup> to estimate the ground state energies of small  
724 molecules as a function of their component atomic separations. Briefly, short quantum circuits  
725 define a variational ansatz of trial solutions for the ground state and the circuit parameters are  
726 varied to minimize the energy using algorithms such as gradient descent. While the complexity  
727 of simulating quantum dynamics on quantum computers is well understood and is usually  
728 tractable, the success of VQE will depend on the quality of the ansatz and is an active area of  
729 ongoing research.

730 Quantum simulation of chemical reactions is known in principle to be possible on a quantum  
731 computer and as the practical details are fleshed out, this is expected to be an important  
732 application of quantum computers for applications both inside and outside of biology. One  
733 particular strength is in modeling dynamics, and there is evidence that energy transport and  
734 electron transport in biological molecules involves quantum effects that could potentially be  
735 more accurately modeled by a quantum simulation<sup>28</sup>.

736  
737 **Competing interests statement:** The authors declare that they have no competing financial  
738 interest.

**A****B**

$$\begin{pmatrix} \alpha_{000} \\ \alpha_{001} \\ \alpha_{010} \\ \alpha_{011} \\ \alpha_{100} \\ \alpha_{101} \\ \alpha_{110} \\ \alpha_{111} \end{pmatrix}$$

**C**

# Computational Complexity Across Level of Analysis: Many-to-Many Mapping Problem

