Data driven learning model predictive control of offshore wind farm

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Abstract

This paper presents a data-driven control approach for maximizing the total power generation of the offshore wind farm by using a recently developed learning model predictive control (LMPC) algorithm. The control is designed by coordinating yaw angle control actions of wind turbines to mitigate the wake interactions among the turbines for increasing the total farm power production, which is termed as wake redirection. This paper mainly focuses on designing the architecture and methodology of the LMPC for wind farm, including a unified wind turbine wake interaction model, the LMPC for minimizing an iteration cost function, the recursive feasibility, stability and convergence analysis. Extensive comparative studies are conducted to verify the performance of the LMPC in comparison with the existing model predictive control (MPC) method under the same wind speed conditions. The results show that the wind farm yields up to 15% more power production by using the LMPC than the conventional MPC.

Keywords: Wind farm; Learning model predictive control; Wake interaction; Wake redirection; FLORIS wind farm model.

I. INTRODUCTION

Wind energy is increasingly becoming one of the most popular renewable energy sources around the world and will fulfill one third of the world's electricity need by 2050 [1]. The global investment share of offshore wind in the wind energy has risen steadily from 10% in 2013 to 25% in 2016 [2]. An offshore wind farm is a cluster of wind turbines that are collected together in each other's proximity to reduce maintenance costs. However, wind turbines in an offshore wind farm experience up to 40% power losses due to the aerodynamic wake interaction [3] which is generated from the upwind turbines and therefore can lower the wind power production of downwind turbines inside the wake region. The wake interaction is generally characterized by increased turbulence and reduced wind flow velocity, and is highly dependent on the number of wind turbines and atmospheric conditions including wind speed, wind direction, and turbine control settings such as blade pitch angles, generator torques, and yaw angles [4].

The challenges in mitigating complex wake effects in offshore wind farms and therefore increasing wind farm power production have motivated the development of a wide variety of approaches in the wind energy research area. One approach is to carefully optimize the wind farm layout by allowing enough space between turbine locations in the design phase. However, this optimization may be limited due to the stochastic nature of continuously varying wind conditions. Besides the design optimization, another possible approach, is to down regulate individual turbines from their peak power points in order to maximize the wind farm power production as a whole. However, this turbine downregulation method maybe highly suboptimal due to the poor understanding and

mitigation of turbine-wake interactions in an offshore wind farm. Therefore, the efforts should be made towards joint setting of control variables or coordinating control settings of individual wind turbines for reducing power losses due to wake interactions and hence maximizing the total wind farm power generation. Actually, the axial induction factors and the yaw-offset angles of individual wind turbines can be employed as the control variables to mitigate the wake effects. The axial induction factor can be generally regulated by adjusting the generator torque and the blade pitch angle in a wind turbine. The yaw-offset angle, defined as the misalignment angle between the wind rotor and the wind direction, can be intentionally and intelligently yawed out of the wind direction to deflect the wake trajectory away from downwind turbines, thereby increasing the wind power productions of downstream wind turbines. The method of adjusting the yaw-offset angles can be generally termed as the *wake redirection control*. Although the control of both the induction factors and the yaw-offset angles can be conducted simultaneously, the wake redirection control is easy to implement and has much greater potential in maximizing the total wind farm power production.

Recently, the active regulation of induction factor and yaw angle for mitigating the wake interference and improving the total wind farm power production have been a hot topic and different control perspectives have been reported in the literature. For example, the gradient ascent algorithm [5], the game-theoretic search algorithm [6], the simultaneous perturbation stochastic approximation method [7], and the maximum power point tracking methods [8] have been proposed for maximizing the wind farm power production, and some of them have been evaluated by using large eddy simulation model such as the Parallelized Large-eddy simulation Model (PALM) [9]. In [10], an optimization approach was employed to determine the optimal control actions for maximizing the wind farm power based on the sequential quadratic programing. However, the optimization was designed based on analytical wind farm power functions with simplified wake models, which cannot precisely reflect the wind farm conditions. In [11], the optimal turbine yaw-offset angles were derived by using a parametric wind farm power function that was constructed from a high-fidelity Computational Fluid Dynamics (CFD) simulation. However, a large amount of wind turbine parameters and environmental variables need to be specified in the CFD model.

In [12], the feasibility of using the Bayesian Ascent (BA) algorithm for determining the coordinated optimal control actions for a wind farm was evaluated with different number of wind turbines and time-varying wind conditions. However, this parametric study was based on analytical wind farm power function, and the used BA algorithm may incur an excessive number of iterations when scaling up to optimize a large number of wind turbines. In [13], the Bayesian optimization (BO) was used to find the optimal inputs for a wind farm in the context of exploration and exploitation based sequential decision-making. However, the BO needs the prior description of the target function using Gaussian process and requires a large amount of data points to reach the optimal operational conditions. In addition, the BO may not perform well with noisy measurement data and cannot be used in real-time control applications due to constraints in the (conventional) sampling strategies in BO. In [14], the game theory and cooperative control were used to optimize wind farm energy production based on the framework of enmeshed decision making where individual

turbines represent the decision makers. In [15], a constrained time efficient closed-loop wind farm control approach was introduced and evaluated in a parameter-varying realistic wind farm flow model. In [16], two decentralized discrete adaptive filtering algorithms were proposed to optimize the total wind farm power output with only limited information sharing among neighbor turbines and without using the wind farm power generation model. The fast convergence to the optimal total power generation and high efficiency of the algorithms were demonstrated by simulation. In [17], the influence of optimal control parameters on wind farm optimization was quantified and the simulation results indicated that up to 21% gain in wind farm power production could be achieved by using optimization. In [18], an optimization under uncertainty (OUU) method was formulated to find the optimal wake redirection strategy in the presence of yaw angle uncertainty. A utility-scale two-turbine test case was used to demonstrate the method, and the results indicated that the OUU solution improved the overall annual average energy by 0.2% and produced fewer extreme yaw situations than the deterministic solution when considering the realistic uncertainty.

In [19], a constrained model predictive control (MPC) was proposed for maximizing the total wind farm power production by using a two-dimensional dynamic wind farm model to predict wake interactions. The wind turbine axial induction factors were used as control inputs and an adjoint approach was utilized to compute the optimal induction factors. The control effectiveness of the approach was demonstrated based on a 2×3 wind farm. In [20], a control algorithm was proposed to maximize the wind farm power reserve by distributing the power contribution of each wind turbine. The evaluation of the algorithm was conducted by simulations of 12 wind turbines. In [21], a closed-loop receding horizon controller was proposed based on a time-varying one-dimensional wake model to provide secondary frequency regulation in a wind farm. The controller was designed by using wind speed measurements of each wind turbine as feedback and was then tested in large eddy simulations of an 84-turbine wind farm. In [22], a distributed MPC scheme was proposed and solved by distributed optimization at every time step. Two MPC versions respectively with hard constraint and soft constraint of the farm-wide power output were implemented and compared. An alternating direction method of multipliers and a dual decomposition scheme were compared in the distributed optimization. Four exemplary scenarios was used to test the performance of the distributed MPC controller and the distributed optimization methods. The results implied that the use of the distributed MPC in wind farms is viable. In [23], a distributed MPC was presented for the optimal active power control of a wind farm that was equipped with fast and short-term energy storage system. A gradient method via dual decomposition was used to implement the distributed MPC. Case studies were performed to evaluate the effectiveness of the distributed MPC whose efficiency is independent from the wind farm size.

This paper aims to develop a new wake redirection control strategy based on the recent learning model predictive control (LMPC) approach [25], [26] to improve the performance of the traditional MPC in maximizing the total offshore wind farm power production. The LMPC is a data driven approach that can iteratively improve the wind farm power production and minimize the control action burden by learning the input trajectories and state cost from previous iterations. The LMPC is designed and

implemented iteratively by solving a finite-time constrained optimization problem that can minimize a finite-time predicted cost while handling state and inputs constraints. The LMPC scheme is reference-free and is able to improve its performance (in comparison with MPC) by learning from previous iterations. A convex safe set and a terminal cost function, learnt from previous iterations, are used in the LMPC to allow for the consideration of the long term planning and can guarantee the stability, recursive feasibility and performance improvement of the control object. Moreover, the converged steady-state trajectory is locally optimal for an approximation of the infinite horizon control problem.

The solution of the LMPC in the wind farm control is a sequence of optimized joint yaw angles that can be used to minimize the cost function while satisfying the recursive stability and convergence performance. By using extensive co-calculations based on the data-driven FLORIS (FLOw Redirection and Induction in Steady-state) tool developed by NREL (National Renewable Energy Laboratory) and the MATLAB software, greatly improved performance of the LMPC method in maximizing the offshore wind farm power capture has been demonstrated based on a comparison with the existing MPC method. The former results in up to 15% more power production than the latter.

The rest of the paper is organized as follows: in section II, the wind farm model and the control problem are formulated. In section III, the LMPC design and related stability proof are presented. In section IV, the validations and discussions of the developed LMPC wind farm control strategy are presented, based on the FLORIS model. Section V includes the conclusion of this paper.

II. Wind Farm Model and Problem Formulation

In order to derive and implement the LMPC for a wind farm, it is necessary to formulate a wind farm model based on the wind turbine wake model and wake interaction model among the turbines. Also, it is essential to study the effects of yaw offset angles on the wind farm power production to formulate the predictive control problem.

A. Wind Turbine Wake Model

The commonly used and computationally efficient wind turbine wake model is the Jensen model (or the Park wake model) [24] which assumes that the wind velocity in a wake is uniform and expands proportionally to the axial downstream distance from the turbine rotor. As shown in Fig. 1, for a turbine i within a wind farm, the wake behind this turbine can be divided into three separate regions, i. e. near wake (q = 1), far wake (q = 2), and mixing regions (q = 3), while each region has a diameter $D_{w,i,q}$, (q=1, 2, 3) [11]. The near-wake region is influenced by the local chord Reynolds number and the turbine tip speed ratio. In the far wake region, the wind velocity deficits with the distance from the turbine rotor. In the mixing region, due to the turbulence-induced mixing, the wind velocity will gradually recover to the free-stream velocity.

The wind velocity behind a turbine i is modeled as

$$V_{w,i}(x,y) = V_\infty \left[1 - 2a_i c_i(x,y)\right] \tag{1}$$

where V_∞ is the free-stream inflow wind velocity, a_i is the axial induction factor of the turbine i that can be defined as the relative amount of velocity drop at the turbine rotor with respect to the inflow velocity, $c_i(x,y)$ is a piecewise wake decay coefficient that can be defined as follows

$$c_i(x,y) = \begin{cases} c_{i,1}(x) \text{ when } \left|y - y_{w,i}(x)\right| \le D_{w,i,1}(x)/2 \\ c_{i,2}(x) \text{ when } D_{w,i,1}(x)/2 < \left|y - y_{w,i}(x)\right| \le D_{w,i,2}(x)/2 \\ c_{i,3}(x) \text{ when } D_{w,i,2}(x)/2 < \left|y - y_{w,i}(x)\right| \le D_{w,i,3}(x)/2 \\ 0 \quad \text{when } \left|y - y_{w,i}(x)\right| > D_{w,i,3}(x)/2 \end{cases} \tag{2}$$

where $c_{i,q}(x), q = 1,2,3$ is the local wake decay coefficient for each region, $y_{w,i}(x)$ is the position of the wake centerline when $x > X_i$.

The local wake decay coefficient $c_{i,q}(x), q = 1,2,3$ is

$$c_{i,q}(x) = \left[\frac{D_i}{D_i + 2k_e m_{v,q}(u_i)(x - X_i)}\right]^2, q = 1,2,3 \tag{3}$$

where D_i is the rotor diameter of the turbine i, k_e and $m_{v,q}(u_i)$ are the wake expansion coefficients, $u_i(t)$ is the control input for the turbine, which is the yaw angle of the turbine in the wake redirection control.

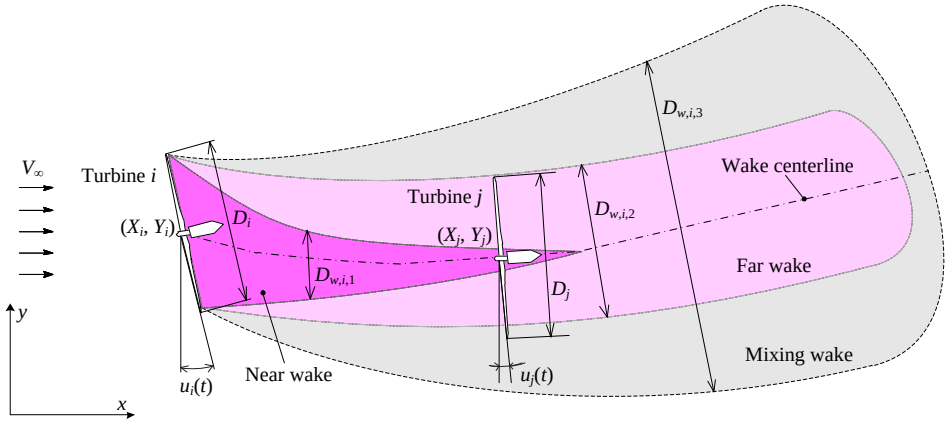

Fig. 1 Schematic of the wake regions behind the turbine i

The coefficient $m_{v,q}(u_i)$ is directly related to the yaw angle offset via the following relationship

$$m_{v,q}(u_i) = \frac{M_{v,q}}{\cos(a_v + b_v u_i(t))}, q = 1,2,3 \tag{4}$$

where $M_{v,q}$, a_v, and b_v are constant.

The diameters of the wake regions $D_{w,i,q}$ in (2) is

$$D_{w,i,q}(x) = \max\left(D_i + 2k_e m_{e,q}(x - X_i), 0\right) \tag{5}$$

where the scaling parameter $m_{e,q}, q=1,2,3$ denotes the wake expansion rate of the three regions.

The position of the wake centerline is determined by combining the yaw-induced and rotation-induced wake lateral offsets with respect to the hub coordinate of the turbine *i* as follows

$$\begin{cases} y_{w,i}(x) = Y_i + \delta y_{w,rot,i}(x) + \delta y_{w,yaw,i}(x,u_i,a_i) \\ \delta y_{w,rot,i}(x) = a_d + b_d(x - X_i) \\ \delta y_{w,yaw,i}(x,u_i,a_i) = \int_0^{x-X_i} \tan(\xi_i)\mathrm{d}x \\ \xi_i = \dfrac{2a_i(1-a_i)\cos^2(u_i)\sin(u_i)}{\left(1+2k_d\dfrac{x-X_i}{D_i}\right)^2} \end{cases} \tag{6}$$

where $\delta y_{w,rot,i}(x)$ and $\delta y_{w,yaw,i}(x,u_i,a_i)$ are respectively the rotation and yaw induced offsets of the wake centerline position. The coefficients a_d, b_d and k_d are constant in determining the centerline position.

B. The Wake Interaction Model

Considering a turbine *j* in the wake regions of the turbine *i*, the overlapping areas between the turbine rotor and the three wake regions is shown in Fig. 2. Then, the effective inflow wind velocity of the turbine *j* is readily calculated by weighting the wake regions by the overlapping areas with the turbine rotor. Therefore, the effective inflow speed V_j of the downstream turbine *j* takes the following form [11]

$$V_j = V_\infty\left(1-2\sqrt{\sum_{i\in\aleph:X_i<X_j}\left(a_i\sum_{q=1}^{3}c_{i,q}(X_j)\min\left(\frac{A_{i,j,q}^{ol}}{A_j},1\right)\right)}\right) \tag{7}$$

where (X_j, Y_j) denotes the coordinate of the turbine *j*, $A_{i,j,q}^{ol}, q=1,2,3$ is the overlapping areas of the three wake regions with the turbine rotor, A_j is the turbine rotor area.

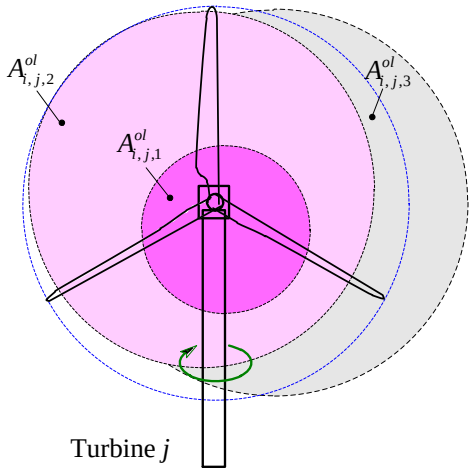

Fig. 2 Schematic of the overlapping areas of the turbine *j*

As illustrated in (7), the inflow wind velocity of the turbine *j* is not only related to its own yaw angle, but also is influenced by the yaw angle of the upstream turbine *i*, which clearly indicates that the inflow wind power of the downstream wind turbine *j* is affected by the yaw angle control inputs of itself and the upstream turbines. Therefore, the changes of the operational conditions of upstream turbines influence the inflow wind velocity of a downstream wind turbine through wake interactions, which therefore influence the power extraction of downstream turbine. The wind velocity of upstream turbines that are located in the front of a wind farm and that are not influenced by other turbines can be determined as $V_j = V_\infty$.

C. The Control Problem Formulation

For a large-scale wind farm with *N* wind turbines denoted by the set $\aleph = \{1, 2, ..., N\}$, each wind turbine *j* is characterized by its rotor diameter D_j, induction factor a_j, inflow wind velocity $V_j(t)$ and yaw angle offset $u_j(t)$.

Based on the actuator disc theory, the power extracted from a single turbine *j* is equivalent to the amount of power extracted by the rotor disc as follows [12]

$$P_j(t) = \frac{\pi \rho D_j^2}{8}\left[V_j(t)\cos\left(u_j(t)\right)\right]^3 C_{pj}\left(a_j, u_j(t)\right) \tag{8}$$

where $P_j(t)$ is the power extracted from the turbine *j*, ρ is the air density, and [16]

$$C_{pj}(a_j, u_j) = 4a_j(1-a_j)^2\cos(u_j(t))^2 \tag{9}$$

is the power coefficient of the turbine.

The total wind power extraction $x(t)$ of the wind farm is an aggregation of the powers produced by all the wind turbines

$$x(t) = \sum_{j=1}^{N} P_j(t) = \sum_{j=1}^{N} \frac{\pi \rho D_j^2}{8}\left[V_j(t)\cos\left(u_j(t)\right)\right]^3 C_{pj}\left(a_j, u_j(t)\right) \tag{10}$$

In the case of wake redirection control, by assuming the constant induction factor a_j and the constant variation rate of the yaw angle offset $\dot{u}_j(t)$, the time derivative of the wind farm power is determined based on (8)~(10) as follows

$$\begin{aligned}\dot{x}(t) = &\sum_{j=1}^{N} \frac{3\pi\rho D_j^2 a_j(1-a_j)^2\left[V_j(t)\cos\left(u_j(t)\right)\right]^2\cos(u_j(t))^2}{2}\left[\begin{array}{l}\dot{V}_j(t)\cos\left(u_j(t)\right) - \\ V_j(t)\dot{u}_j(t)\sin\left(u_j(t)\right)\end{array}\right] \\ &-\sum_{j=1}^{N}\pi\rho D_j^2 a_j(1-a_j)^2 u_j(t)\left[V_j(t)\cos\left(u_j(t)\right)\right]^3\sin(u_j(t))^2\dot{u}_j(t)\end{aligned} \tag{11}$$

Therefore, by using a constant time interval Δt for the time differential operation in (11), the time derivative of the wind farm power in (11) can be reasonably represented as

$$\dot{x}(t) = \frac{x(t+\Delta t) - x(t)}{\Delta t} \Rightarrow x(t+\Delta t) = x(t) + \dot{x}(t)\Delta t \Rightarrow x(t+\Delta t) = f_p\left(x(t), u_j(t)\right), \forall j \in \aleph \tag{12}$$

By observing (9)-(12) and the wake model in (1)-(7), the wind farm power can be generally characterized by the function

$f_p\left(x(t),u_j(t)\right)$ in (12) and is only directly related to the joint yaw angle offset of the turbines within the wind farm in the wake redirection control case. Therefore, by properly regulating the joint yaw angles of the turbines, the state of the wind farm power extraction can be varied accordingly, which forms the control problem of yaw angel redirection control in maximizing the wind farm power generation.

In practice, the nonlinear function $f_p\left(x(t),u_j(t)\right)$ in (12) can be well parameterized and accurately represented by using the FLORIS wind farm model with each wind turbine represented by the offshore 5 MW NREL wind turbine model. As a consequence, with the power function being explicitly defined above, the LMPC scheme can be employed to solve the wind farm power maximization problem.

III. The LMPC for Wind Farm

In this section, the LMPC approach [25], [26] is explored for developing the wake redirection control to maximize the total wind farm power production. The LMPC is designed and implemented iteratively by solving a finite-time constrained optimization problem. The solution is a sequence of optimized joint yaw angles that can be used to minimize an iteration cost function of the wind farm power generation. Then, the LMPC is proved to be recursively feasible and stable at each successive iteration, and the iteration cost is guaranteed to be non-increasing at all time instants and iterations.

A. The LMPC Design

The wind farm power maximization problem formulated in the above section is equivalent to the minimization of its reciprocal $\frac{1}{x(t+\Delta t)}$ or $\frac{1}{x(t)}$.

In order to transform the continuous time domain wind farm state equation in (12) into the discrete time domain to facilitate the LMPC design, the following variables are defined

$$\begin{cases} x_{t+1}=\dfrac{1}{x(t+\Delta t)}; \\ x_t=\dfrac{1}{x(t)}; \\ u_t=\left[u_1(t),..,u_j(t),..,u_N(t)\right] \\ \Delta t=1. \end{cases} \tag{13}$$

where x_{t+1} and x_t denote the state variables at time instants t and t+1, respectively, u_t denotes the joint set of the yaw angles at time instant t.

Based on (13), (12) can be transformed into the discrete time domain as follows

$$x_{t+1}=\frac{1}{x(t+\Delta t)}=\frac{1}{x(t)+\dot{x}(t)\Delta t}\Rightarrow x_{t+1}=\frac{\frac{1}{x(t)}}{1+\frac{\dot{x}(t)}{x(t)}\Delta t}\Rightarrow x_{t+1}=\frac{x_t}{1+\dot{x}(t)x_t} \tag{14}$$

Therefore, based on (10)-(14), the state equation of the wind farm is represented as

$$x_{t+1}=f(x_t,u_t) \tag{15}$$

where $f(\cdot)$ represents the nonlinear relationship between the control input u_t and state variable x_{t+1}, and can be characterized by the function $f_p\left(x(t),u_j(t)\right)$ in (12) or represented by using the FLORIS wind farm model.

In general, the nonlinear function $f(\cdot)$ is continuous at all time instants, and the state and inputs are subject to necessary constraints.

At the *j*th iteration of the LMPC task, the state trajectory and the associated control input sequence are

$$\begin{cases}\boldsymbol{x}^j=\left[x_0^j,.,x_t^j,..,x_{T^j}^j\right];\\ \boldsymbol{u}^j=\left[u_0^j,..,u_t^j,..,u_{T^j-1}^j\right].\end{cases} \tag{16}$$

where x_t^j and u_t^j denote the state trajectory and control input at the time instant *t* of the *j*th iteration, T^j is the time instant at which the control task is completed.

At each *j*th iteration, the state trajectory is assumed to start from the same initial state. Then, by using the stored data in (16), a convex safe set CS^j at the *j*th iteration can be constructed [25]. Thus,

$$CS^j=\mathrm{Conv}\left(\bigcup_{i=0}^{j}\bigcup_{t=0}^{T^i}x_k^i\right) \tag{17}$$

Actually, the convex safe set CS^j can be defined as the convex hull of the union of the stored data as follows

$$CS^j=\left\{x\in\mathbf{R}:\exists\lambda_k^i\geq 0,\sum_{i=0}^{j}\sum_{t=0}^{T^i}\lambda_k^i=1,\sum_{i=0}^{j}\sum_{t=0}^{T^i}\lambda_k^i x_k^i=x\right\} \tag{18}$$

where λ_k^i denotes the Lagrange multiplier associated with each recorded data in the convex safe set.

Also, based on the stored data in (16), a continuous cost-to-go function can be defined as

$$Q_k^j=h(x_k^j,u_k^j)+Q_{k+1}^j \tag{19}$$

where Q_k^j is the cost-to-go function defined at the *k* time instant of the *j*th iteration. At the time instant T^j when the control task is completed, the cost to go function can be defined as

$$Q_{T^j}^j=h(x_{T^j}^j,0),\forall j>0 \tag{20}$$

For the wind farm control task, the cost function in (19) and (20) can be defined as

$$h(x_k^j, u_k^j) = (x_k^j)^{\mathrm{T}} Q x_k^j + (u_k^j)^{\mathrm{T}} \boldsymbol{R} u_k^j \tag{21}$$

where Q=10000, $\boldsymbol{R}$=$\boldsymbol{I}_N$ ($\boldsymbol{I}_N$ is a $N\times N$ dimension identity matrix) are the constant coefficients, respectively.

Then, the Q-function to be optimized can be defined based on (19)-(21) as follows

$$\begin{aligned} &Q^{j,*}(x) = \min_{\lambda_k^j \in [0,1]} \sum_{i=0}^{j} \sum_{k=0}^{T^i} \lambda_k^i Q_k^i \\ &\text{s.t.} \begin{cases} \sum_{i=0}^{j} \sum_{k=0}^{T^i} \lambda_k^i = 1 \\ \sum_{i=0}^{j} \sum_{k=0}^{T^i} \lambda_k^i x_k^i = x \end{cases} \end{aligned} \tag{22}$$

The above Q-function defined in (22) can be used as the terminal cost in an infinite horizon optimal control problem. Therefore, the control problem defined in the section II-C can be equivalently transformed into the infinite horizon optimal control problem at each jth iteration as follows [26]

$$J_{t\to t+n}^j(x_t^j) = \min_{U_t^j,\, \lambda^j} \left[\sum_{k=t}^{t+n-1} h(x_{k/t}^j, u_{k/t}^j) + \sum_{i=0}^{j-1} \sum_{k=0}^{T^i} \boldsymbol{\lambda}_k^i Q_k^i \right] \tag{23}$$

where n is the prediction time horizon.

The cost function defined in (23) includes a running cost $\sum_{k=t}^{t+n-1} h(x_{k/t}^j, u_{k/t}^j)$ and the cost-to-go function $\sum_{i=0}^{j-1} \sum_{k=0}^{T^i} \boldsymbol{\lambda}_k^i Q_k^i$. The control problem is also subject to the constraints of initial condition, dynamics constraint, state and input constraints that can be represented as

$$\begin{cases} x_{t/t}^j = x_t^j; \\ x_{k+1/t}^j = f(x_{k/t}^j, u_{k/t}^j); \\ \sum_{i=0}^{j-1} \sum_{k=0}^{T^i} \lambda_k^i = 1, \lambda_k^i \ge 0; \\ \sum_{i=0}^{j-1} \sum_{k=0}^{T^i} \lambda_k^i x_k^i = x_{t+n/t}; \\ u_{\min} \le u_{k/t}^j \le u_{\max}, \forall k = t, ..., t+n-1. \end{cases} \tag{24}$$

where $u_{\min}$ and $u_{\max}$ represent the upper and lower bounds on the yaw control inputs. The constraints defined in (24) also enforce the latest predicted state $x_{t+n/t}$ into the convex hull of the recorded states.

By solving (23) and (24), the optimal control sequence of the yaw angle offset can be obtained at the time instant t of the jth iteration as follows

$$U_t^{j,*} = \left[u_{t/t}^{j,*}, ..., u_{t+n-1/t}^{j,*} \right] \tag{25}$$

Consequently, the optimal control input applied to the wind farm at the time instant t of the jth iteration is obtained as

$$u_t = u_{t/t}^{j,*} \tag{26}$$

Then, the infinite horizon optimal control problem proceeds and is repeated at the time instant $t + 1$ based on the optimal control signal in (26) and the new state variable $x_{t+1/t+1} = x_{t+1}^{j}$ until the maximum iteration number is exceeded.

The above infinite horizon optimal control problem defined in (23) and (24) is generally difficult to solve by using the traditional semidefinite programming and linear matrix inequalities especially when considering that the FLORIS wind farm model is implicitly involved in the problem. The constraints defined by the FLORIS wind farm model cannot be explicitly described by linear equations and many nonlinear relationships actually exist in the optimization problem. Therefore, the commercial or non-commercial semidefinite programming (SDP) solvers cannot be used. Then, the genetic algorithm (GA) is used to find the global optimal solution of the control input $u_{t/t}^{j,*}$ at time instant t of the jth iteration. As a derivative free and powerful optimization tool inspired by evolutionary biology, the GA is designed based on the natural selection mechanism that includes the operators of selection, crossover, mutation, and inversion in a population candidate [27]. After a sufficiently high number of generations and population evolutions, the GA is capable of the global optimal solution with a satisfactory fitness level. In order to eliminate the heavy computational burden in the GA, the GA parameters including the generation number, the population size and the tolerance constraint can be carefully chosen to save time.

B. The Recursive Feasibility and Stability

The recursive feasibility and stability of the LMPC controlled wind farm system can be proved based on the designed convex safe set and the cost-to-go function. Also, there exists an asymptotically stable equilibrium point for the closed loop system at every iteration $j \geq 1$ [26].

Considering the system (15) controlled by the LMPC in (26) and state update in (15), one obtains

$$x_{t+1/t}^{*,j} = x_{t+1}^{j} \tag{27}$$

Given the related optimal input sequence and the optimal trajectory obtained by using (23) and (24), the cost function is obtained as

$$J_{0\to n}^{j}(x_{t+1/t}^{*}) = J_{0\to n}^{j}(x_{t+1}^{j}) \tag{28}$$

Therefore, by defining $Q^{j-1}(x_{t+n/t}^{j}) = \sum_{i=0}^{j-1}\sum_{k=0}^{T^i} \boldsymbol{\lambda}_k^i Q_k^i$ and based on (27) and (28), the optimal cost can be used as a Lyapunov function as follows

$$\begin{aligned} J_{0\to n}^{j}(x_t^j) &= \min_{U_t^j,\, x_t^j}\left[\sum_{k=0}^{n-1} h(x_{k/t}^{j}, u_{k/t}^{j}) + Q^{j-1}(x_{n/t}^{j})\right] = h(x_{t/t}^{j,*}, u_{t/t}^{j,*}) + \sum_{k=1}^{n-1} h(x_{t+k/t}^{j,*}, u_{t+k/t}^{j,*}) + Q^{j-1}(x_{t+n/t}^{j,*}) \\ &= h(x_{t/t}^{j,*}, u_{t/t}^{j,*}) + \sum_{k=1}^{n-1} h(x_{t+k/t}^{j,*}, u_{t+k/t}^{j,*}) + h(x_{t^*}^{i^*}, u_{t^*}^{i^*}) + Q^{j-1}(x_{t^*+1}^{i^*}) \geq h(x_{t/t}^{j,*}, u_{t/t}^{j,*}) + J_{0\to n}^{j}(x_{t+1/t}^{j,*}) \end{aligned} \tag{29}$$

where ($i*$, $t*$) in (29) is defined as

$$(i^*, t^*) = \arg\min J^i_{0\to\infty}(x^i_t), \forall x \in CS^i \tag{30}$$

Therefore,

$$J^j_{0\to n}(x^{j,*}_{t+1/t}) - J^j_{0\to n}(x^j_t) \le -h(x^{j,*}_{t/t}, u^{j,*}_{t/t}) < 0 \tag{31}$$

Due to the positive definitiveness of $h(\cdot)$ in (21) and the continuity of $J^j_{0\to n}(\cdot)$, it is obvious that the cost function in (31) is non-increasing along the closed loop trajectory and will reach an asymptotically stable point when $t\to\infty$.

C. The Convergence Analysis

The convergence property of the LMPC controlled wind farm can be analyzed based on the proof of the recursive feasibility and stability in section III-B. The cost defined in (23) will be non-increasing along the closed loop LMPC controlled wind farm system and will converges to a steady state trajectory x^∞ when time $t\to\infty$ and the iteration $j\to\infty$. Then, the steady state yaw angle control input will be $u^\infty = \lim_{j\to\infty} u^j$ which is the global optimal solution for the infinite horizon optimal control problem in (23) and (24).

The interested readers can refer to [28] for more details about the convergence analysis of the LMPC.

IV. Validations and Discussions

In order to investigate and ascertain the efficiency and effectiveness of the LMPC designed for wake redirection control, computational experiments are conducted by using the NREL FLORIS tool and the MATLAB software. The experiments have been conducted under eleven different scenarios with different freestream wind speed inputs. A traditional MPC has been designed for comparison.

A. Computational Experiments

The computational experiments are carried out based on the co-calculations of MATLAB and the FLORIS tool when the inflow freestream wind speeds vary from 6 m/s to 11 m/s.

The FLORIS wind farm model is written in Python language and is designed to provide a controls-oriented and computationally inexpensive tool that models the turbine interactions and the steady-state wake characteristics in a wind farm. The FLORIS model is designed based on the Jensen model and the wake deflection by Jiménez et al., and hence is sufficiently fast to perform wake redirection control while retaining enough accuracy [29], [30]. The NREL 5 MW offshore wind turbine model with the rated wind speed of 12 m/s has been adopted in the FLORIS tool and the turbine yaw angles are bounded from 0° to 15°.

As shown in Fig. 3, the simulated wind farm consists of four wind turbines aligned in two rows with 800 m spacing in the downwind direction and 630 m spacing in the crosswind direction. The freestream wind speed comes from the front-left with the

direction of 270°, and turbines 1 and 3 are in the upstream direction while the turbine 2 and 4 are in the downstream wind direction. The four turbines can interact fully through the wakes of the upstream turbines and therefore represent the realistic scenario for wake redirection control. The incoming freestream wind speed varies from 6 m/s to 11 m/s and has the turbulence intensity of 5% at the hub height, which represents the below-rated wind speed condition for the wind farm power optimization.

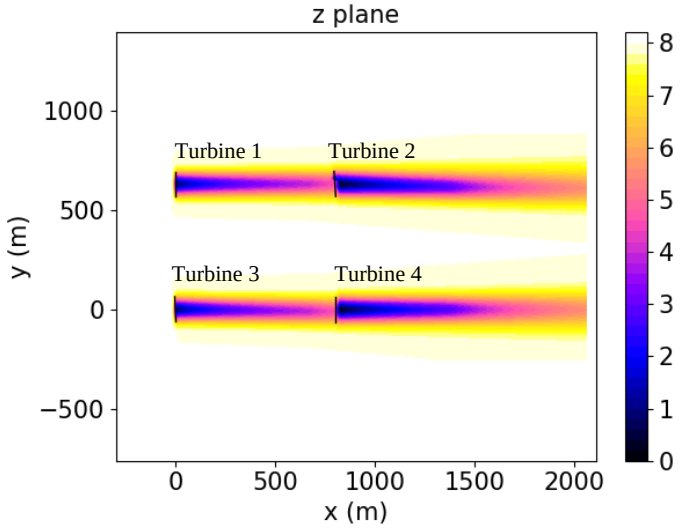

Fig. 3 The FLORIS wind turbine layout and wind speed distribution

The LMPC algorithm has been designed and implemented in the MATLAB which calls the FLORIS tool by using the function "*py.importlib.import_module*" while the FLORIS tool defines an objective function with the turbine yaw angle vector as input and the total wind farm power as output.

The genetic optimization tool (gatool) in the MATLAB has been used to solve the optimal control problem defined in (23) and (24) for the LMPC implementation. The parameters of the genetic optimization have been carefully chosen in the "*gaoptimset*" such that the LMPC problem can be solved efficiently with enough accuracy. For example, the number of generations has been set as 30, the number of population size has been set as 50, and the operation parameters for crossover and mutation can be set as 0.6 and 0.2, respectively. The gatool calls the FLORIS tool for objective function evaluations at each time step with turbine yaw settings as input and the calculated total wind farm power from the FLORIS tool as output. Other key parameters for the LMPC have been set as: the controller horizon is 3 while the iteration number is 5. In order to further verify the effectiveness of the LMPC, a traditional MPC has been designed for comparison.

B. Results and Analysis

As shown in Fig. 4, the total wind farm power increases with the increasing incoming wind speed from 6 m/s to 11 m/s with 0.5 m/s interval . it is obvious that the designed LMPC produces more power than the traditional MPC in the whole wind speed range. At 6 m/s, LMPC increases the power capture by around 7.8% compared with the conventional MPC via the optimal turbine yaw angle settings. This increase becomes more significant in the high-speed range of the wind and it reaches around 15% at 11 m/s.

The increase may be attributed to the fact that the LMPC can be used to find the optimal partial-wake operational point (through iterative learning) where the wake-induced power losses are reduced due to less wake interactions among wind turbines. The results also indicate that the LMPC method may be more suitable for the case of higher inflow wind speed.

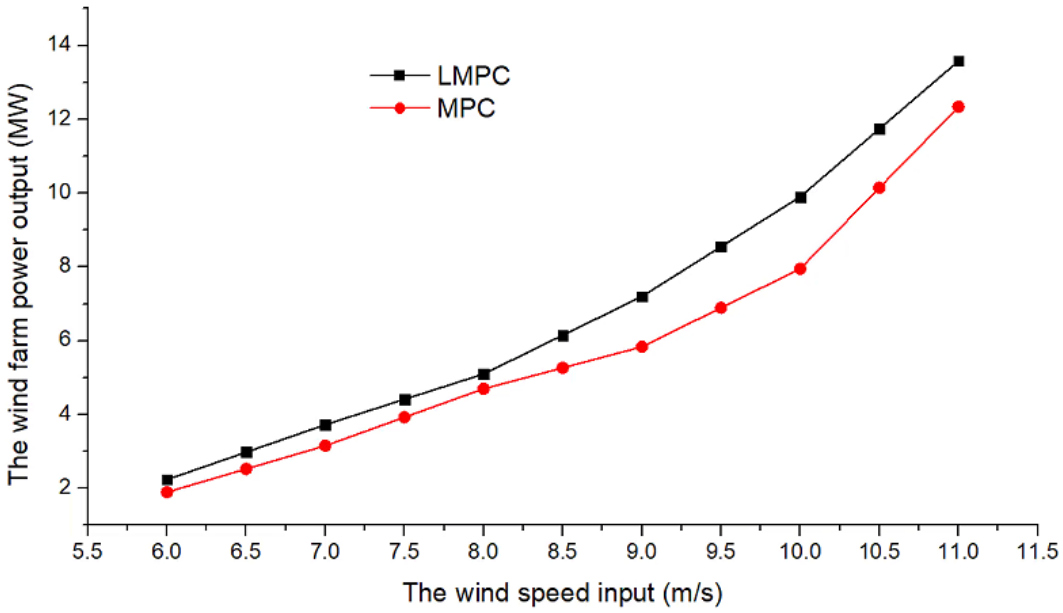

Fig. 4 The total wind farm power generations by using the two control algorithms

Fig. 5 demonstrates the evolution of the iteration cost of the LMPC under the incoming freestream wind speed of 8 m/s, which is the typical result of all the wind speed scenarios. As shown in this figure, the iteration cost decreases almost monotonically with the iterations and reaches the steady value of 858.2 in two iterations. This result demonstrates the recursive stability and convergence of the LMPC, as mentioned in the sections III-B and C.

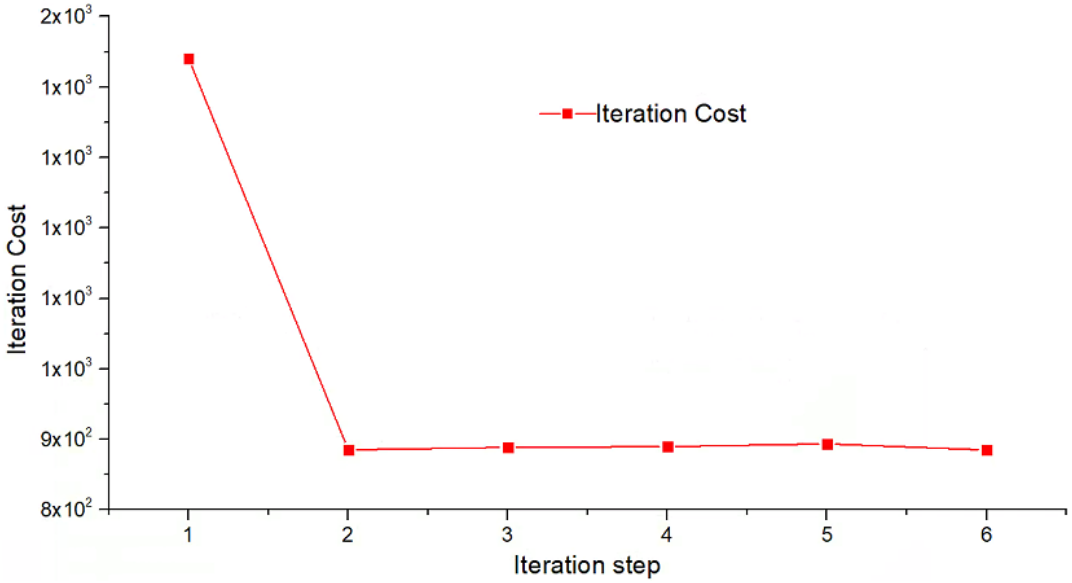

Fig. 5 The evolution of the iteration cost of the LMPC under the incoming freestream wind speed of 8 m/s

Fig. 6 illustrates the LMPC of wind farm energy extraction with the evolution of the total wind farm power production over the

prediction horizon at wind speed of 8 m/s. As shown in this figure, the total wind farm power evolves from 6.5 MW to 5.2 MW with the decreasing iteration cost in Fig. 5. Although the 6.5 MW wind farm power is greater than the converged value of 5.2 MW, its iteration cost involving the yaw angle actions is higher than the case at 5.2 MW as shown in Fig 6. It is clear that the LMPC can be used to achieve a good trade-off between the control action cost and the optimal wind farm generation via executing several trial expenses as shown in Fig. 6.

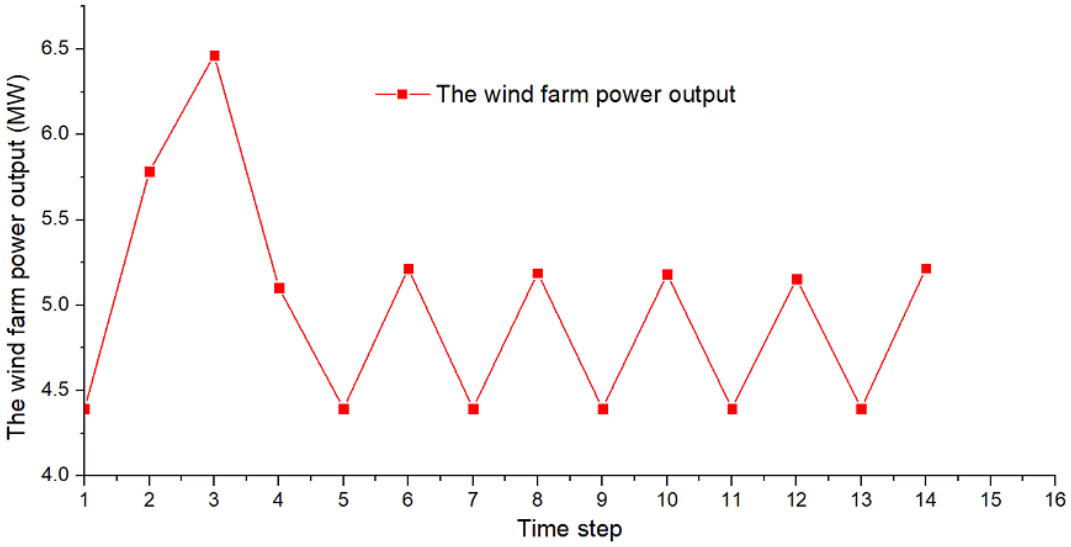

Fig. 6 The total wind farm power generation using the LMPC under the wind speed input of 8 m/s

Fig. 7 depicts the calculated time series of the optimal yaw angle settings in the four-turbine example at freestream wind speed V_∞=8 m/s. As shown in the figure, the trajectories of the yaw angles of the wind turbines 1, 2, 3 and 4 respectively evolve to the optimal and steady values of 15°, 0.2238°, 14.83° and 3.4697° after the time step 5, which further demonstrates the stability and convergence properties of the LMPC in section III. The yaw angles for front wind turbine 1 and 3 are set around the maximum value of 15° so that their blades can be yawed out of the incoming wind and more wind energy can be captured by wind turbine 2 and 4 that have very small yaw angles. In this case, the total wind farm power output can be improved. The fluctuations of the turbine yaw angles after the time step 5 may be caused by the self-similar Gaussian model in the new version FLORIS model, which may have an impact on the total wind farm power output. Despite this, the converged values of the yaw angle settings are required to sufficiently minimize the iteration cost function defined in (23) for the wind farm in a desirable manner in particularly for wind farm system with fast dynamics.

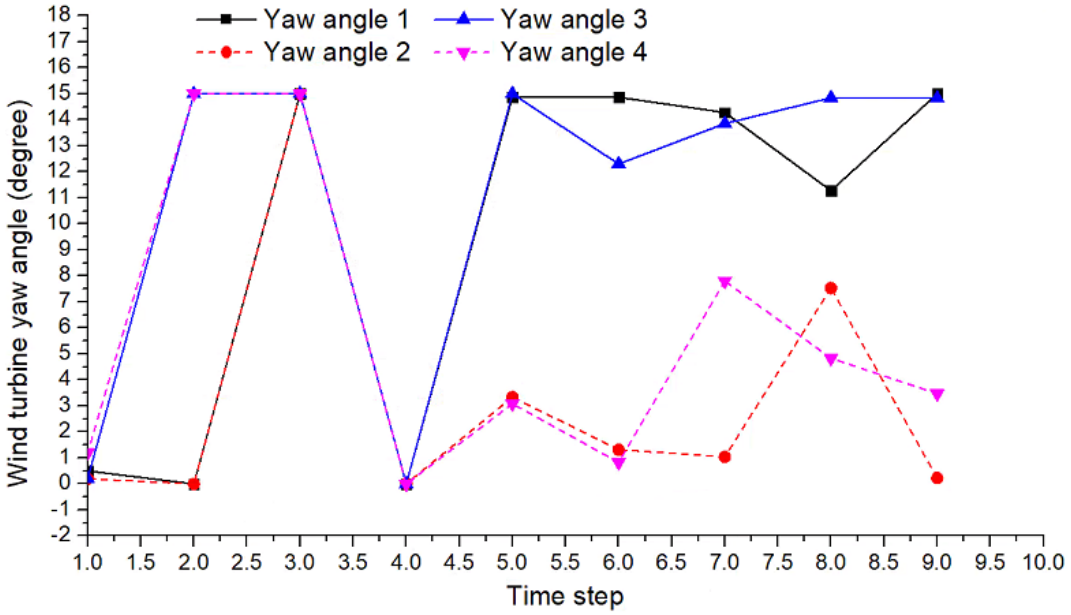

Fig. 7 The wind turbine yaw angles in the LMPC under the wind speed input of 8 m/s

V. Conclusions

The data-driven LMPC has been proposed for increasing the total wind farm power production of offshore wind farm over a prediction horizon by iteratively executing the yaw angle control actions when considering the aerodynamic wake interactions among wind turbines. The LMPC can ensure the desired closed-loop control performance and improve execution efficiency simultaneously by learning from previous iterations. The detailed dynamic wake model and wake interaction model in a wind farm have been presented. The design details have been presented by recursively constructing the convex safe set and a terminal cost function for guaranteeing the control performance at each iteration. The convergence and global stability of the LMPC have also been proved. The effectiveness of the LMPC approach has been validated based on the co-calculations of the NREL FLORIS tool and the MATLAB. The results demonstrated that the LMPC converged fast to the optimal control set points of the yaw angles. In comparison to the conventional MPC, the wind farm yielded up to 15% more power production by using the LMPC.

We mention that the FLORIS model is a control-oriented steady-state wake model, which only provides information for the wake at a given wind condition. Therefore, the results in the work can be further improved by using dynamic CFD simulation tools like PALM or SOWFA. In the future work, we will investigate how to implement the control by using these dynamic CFD simulation tools for a large-scale wind farm.

VI. Acknowledgement

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