Physics of Excited Nucleons (NSTAR 2013)
International Journal of Modern Physics: Conference Series
Vol. 26 (2014) 1460097 (5 pages)
© The Authors
DOI: 10.1142/S2010194514600970

World Scientific
www.worldscientific.com

Int. J. Mod. Phys. Conf. Ser. 2014.26. Downloaded from www.worldscientific.com
by UNIVERSITY OF GENT TOEGEPASTE WISKUNDE & INFORMATICA (ATTN: PROF DR G. CROMBEZ) on 01/27/14. For personal use only.

INCOMPLETENESS OF COMPLETE KAON PHOTOPRODUCTION*

JAN RYCKEBUSCH and TOM VRANCX

Department of Physics and Astronomy, Ghent University,
Proeftuinstraat 86, B-9000 Gent, Belgium

Published 14 January 2014

A possible roadmap for reaching a status of complete information in $\gamma p \to K^+\Lambda$ is outlined.

Keywords: Kaon photoproduction; complete measurements.

PACS numbers: 11.80.Cr, 13.60.Le, 24.10.i, 25.20.Lj

1. Introduction

Thanks to recent technological advances in producing high-quality polarized monochromatic photon beams, and in developing polarized nucleon targets, it becomes possible to measure a sufficiently large amount of single- and double-polarization observables in pion and kaon photoproduction from the nucleon. As a result, a status of complete quantum mechanical information of meson photoproduction comes within reach. Measurements are complete whenever they enable one to determine unambiguously all amplitudes of the underlying reaction process at some specific kinematics.

We consider the $\gamma p \to K^+\Lambda$ reaction as a prototypical example of pseudoscalar-meson photoproduction from the proton. The transversity amplitudes $b_i (i = 1, 2, 3, 4)$ express the transition matrix elements in terms of the p and Λ spinors (with quantization axis perpendicular to the reaction plane) and of linear photon polarizations (J_x, J_y). We propose[1] to use normalized transversity amplitudes (NTA) $a_i \equiv \frac{b_i}{\sqrt{\sum_i |b_i|^2}}$ to perform an amplitude analysis of the single- and double-polarization observables. The NTA provide complete information after determining the differential cross section. The corresponding polarization observables can be expressed in terms of linear and nonlinear equations of bilinear products of the

*This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 3.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

Int. J. Mod. Phys. Conf. Ser. 2014.26. Downloaded from www.worldscientific.com
by UNIVERSITY OF GENT TOEGEPASTE WISKUNDE & INFORMATICA (ATTN: PROF DR G. CROMBEZ) on 01/27/14. For personal use only.

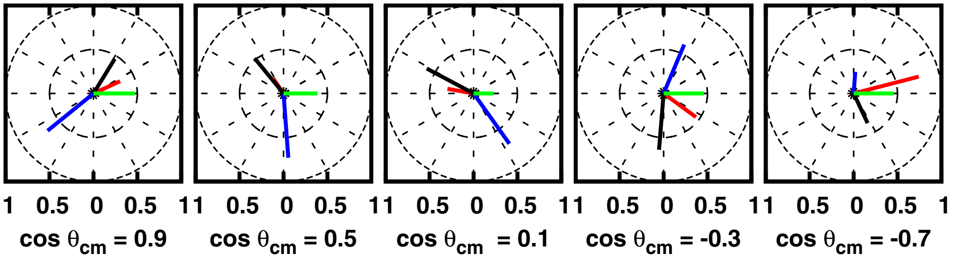

Fig. 1. The RPR-2011 predictions for the NTA (the definitions are given in Ref.[1]) for the $\gamma p \to K^+\Lambda$ reaction at $W = 1900$ MeV. The NTA are displayed as vectors: $a_1 = r_1 e^{i\alpha_1}$ (red), $a_2 = r_2 e^{i\alpha_2}$ (blue), $a_3 = r_3 e^{i\alpha_3}$ (black), $a_4 = r_4$ (green). We adopt the convention that α_4=0.

a_i. For a given kinematical setting determined by the meson angle θ_{cm} and the invariant mass W, the $a_i = r_i e^{i\alpha_i}$ are fully determined by six real numbers conveniently expressed as three real moduli r_i and three real relative phases $\alpha_i - \alpha_4$. All observables are invariant under a transformation of the type $\alpha_i \to \alpha_i + \beta$, with β an arbitrarily chosen overall phase. In Fig. 1 we show predictions for the NTA at $W = 1900$ MeV and various θ_{cm}. The adopted model for $\gamma p \to K^+\Lambda$ is the Regge-plus-Resonance (RPR) approach in its most recent version RPR-2011.[2,3] The model has a Reggeized t-channel background and the s-channel resonances $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $D_{13}(1900)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$. The RPR approach provides a low-parameter framework with predictive power for K^+ and K^0 photoproduction on the proton and the neutron.[4]

2. Extracting the Moduli and Phases From $\gamma p \to K^+\Lambda$ Data

An obvious advantage of using the transversity amplitudes is that linear equations connect the moduli r_i of the NTA to the single-polarization observables $\{\Sigma, T, P\}$

$$\begin{cases} r_1 = \frac{1}{2}\sqrt{1+\Sigma+T+P}, \\ r_2 = \frac{1}{2}\sqrt{1+\Sigma-T-P}, \\ r_3 = \frac{1}{2}\sqrt{1-\Sigma-T+P}, \\ r_4 = \frac{1}{2}\sqrt{1-\Sigma+T-P}\,. \end{cases} \tag{1}$$

Accordingly, a measurement of (Σ, T, P) at given $(W, \cos\theta_{\mathrm{cm}})$ allows one to infer the moduli $r_i(W, \cos\theta_{\mathrm{cm}})$ of the NTA. The GRAAL collaboration provides $p(\gamma, K^+)\Lambda$ data for $\{\Sigma, T, P\}$ at 66 $(W, \cos\theta_{\mathrm{cm}})$ combinations in the ranges $1.65 \lesssim W \lesssim 1.91$ GeV ($\Delta W \approx 50$ MeV) and $-0.81 \lesssim \cos\theta_{\mathrm{cm}} \lesssim 0.86$ ($\Delta\cos\theta_{\mathrm{cm}} \approx 0.3$). Figure 2 shows the extracted r_i at three θ_{cm} intervals along with the RPR-2011 predictions. For a few kinematic points the r_i could not be retrieved from the data. This occurs whenever one or more arguments of the square roots in Eq. (1) become negative

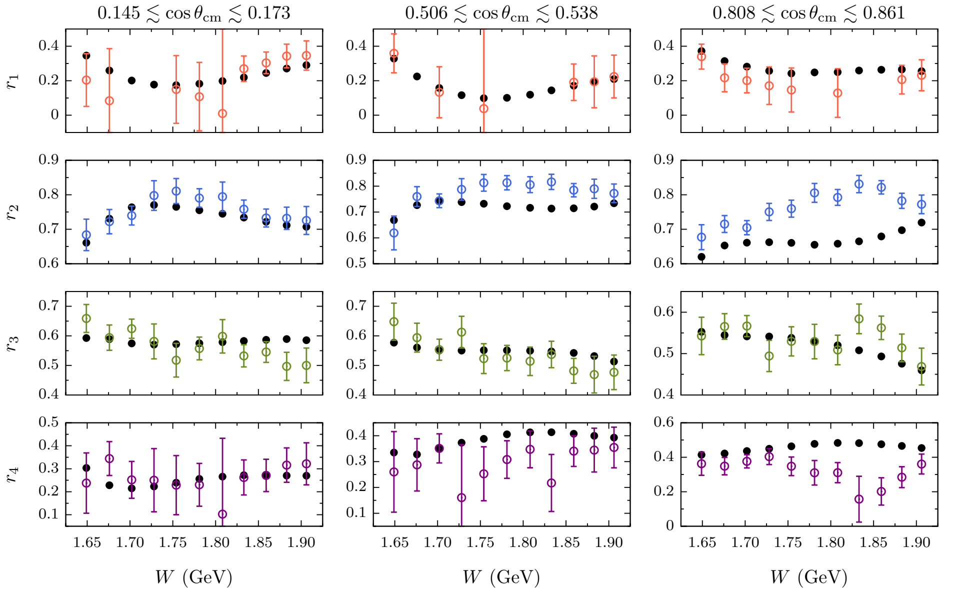

Fig. 2. The energy dependence of the moduli r_i of the normalized transversity amplitudes for the $\gamma p \to K^+\Lambda$ reaction. The data are extracted from the GRAAL results for the single-polarization observables reported in Ref.[5] The dots are the bin-centered RPR-2011 predictions.

due to finite experimental error bars. The RPR-2011 model offers a fair description of the W dependence of the extracted r_i except for the most forward angles at $W \approx 1.85$ GeV. Furthermore, the data confirm the predicted dominance of the r_2.

Inferring the NTA phases α_i from data requires measured double asymmetries. Complete sets of the first kind, which involve seven observables (e.g. $\{\Sigma, P, T, C_x, O_x, E, F\}$), lead to the following set of nonlinear equations for the phases[6,1]

$$\begin{cases} r_1 r_4 \sin\delta_1 + r_2 r_3 \sin\Delta_{23} &= -\frac{C_x}{2}, \\ r_1 r_4 \cos\delta_1 + r_2 r_3 \cos\Delta_{23} &= +\frac{O_x}{2}, \\ r_1 r_3 \cos\Delta_{13} - r_2 r_4 \cos\delta_2 &= +\frac{E}{2}, \\ r_1 r_3 \sin\Delta_{13} + r_2 r_4 \sin\delta_2 &= -\frac{F}{2}, \\ \delta_1 + \Delta_{23} - \delta_2 - \Delta_{13} &= \quad 0 \end{cases} \tag{2}$$

where $\delta_i \equiv \alpha_i - \alpha_4$ and $\Delta_{ij} = \delta_i - \delta_j$. Solutions to the above set of nonlinear equations gives the phases $(\delta_1, \delta_2, \Delta_{13}, \Delta_{23})$ for given moduli (r_1, r_2, r_3, r_4). We stress that single-polarization observables are part of any complete set as they provide the information about the moduli. Double polarization observables are required to get access to the phases. In all practical situations one has $\delta_1 + \Delta_{23} - \delta_2 - \Delta_{13} \approx 0$.

Int. J. Mod. Phys. Conf. Ser. 2014.26. Downloaded from www.worldscientific.com by UNIVERSITY OF GENT TOEGEPASTE WISKUNDE & INFORMATICA (ATTN: PROF DR G. CROMBEZ) on 01/27/14. For personal use only.

Int. J. Mod. Phys. Conf. Ser. 2014.26. Downloaded from www.worldscientific.com
by UNIVERSITY OF GENT TOEGEPASTE WISKUNDE & INFORMATICA (ATTN: PROF DR G. CROMBEZ) on 01/27/14. For personal use only.

Finite error bars introduce a bias for the choices made with regard to the reference phase (here, α_4) for the above equations. A consistent set of estimators $\widetilde{\delta}_i^{\alpha_j}$ for the independent phases (insensitive to choices made with regard to the reference phase) has been proposed in Ref.[1]

To date, the published double polarization observables for $\gamma p \to K^+\Lambda$ do not allow one to extract the phases of the NTA. We have conducted studies with pseudo-data generated by the RPR-2011 model for $\gamma p \to K^+\Lambda$. We have considered ensembles of 200 pseudo-data sets each containing samples of 50 events for the asymmetries $\{\Sigma, P, T, C_x, O_x, E, F\}$. The pseudo-data are drawn from Gaussians with the RPR-2011 prediction as mean and a given σ_{exp} as standard deviation. The retrieved (r_i, δ_i) do not necessarily comply with the input amplitudes. There are various sources of error: (i) imaginary solutions for the moduli ; (ii) imaginary solutions for the phases; (iii) incorrect solutions which stem from the fact that $\delta_1 + \Delta_{23} - \delta_2 - \Delta_{13} = 0$ cannot be exactly obeyed for data with finite errors. We find that the amount of incorrect and imaginary solutions is much larger for the phases than for the moduli. The frequency of finding imaginary solutions can be dramatically reduced by improving on the experimental resolution σ_{exp}.

3. Conclusions

We have sketched a possible roadmap for reaching a status of complete information in pseudoscalar-meson photoproduction. We suggest that the use of transversity amplitudes is tailored to the situation that experimental information about (Σ, P, T) is more abundant (and most often more precise) than for the double polarization observables. Linear equations connect $\{\Sigma, P, T\}$ to the moduli r_i of the NTA. An analysis of $\{\Sigma, T, P\}$ data for $\gamma p \to K^+\Lambda$ from GRAAL allowed us to extract the r_i in the majority of considered $(W, \cos\theta_{\text{cm}})$ combinations. Extracting the NTA independent phases is far more challenging as they are connected to the double asymmetries by means of nonlinear equations. It has been suggested[7] that over-complete sets which involve more than seven polarization observables may provide a solution to tackle the problem of extracting the relative phases of the amplitudes from the data.

Acknowledgments

This work is supported by the Research Council of Ghent University and the Flemish Research Foundation (FWO Vlaanderen).

References

1. T. Vrancx, J. Ryckebusch, T. Van Cuyck, P. Vancraeyveld, *Phys. Rev. C* **87**, 055205 (2013).
2. L. De Cruz, J. Ryckebusch, T. Vrancx, P. Vancraeyveld, *Phys. Rev. C* **86**, 015212 (2012).

3. L. De Cruz, T. Vrancx, P. Vancraeyveld, J. Ryckebusch, *Phys. Rev. Lett.* **108**, 182002 (2012).
4. P. Vancraeyveld, L. De Cruz, J. Ryckebusch, T. Vrancx, *Nucl. Phys.* **A897**, 42 (2013).
5. A. Lleres *et al.* (GRAAL Collaboration), *Eur. Phys. J.* **A31**, 79 (2007) and **A 39**, 149 (2009).
6. Wen-Tai Chiang and F. Tabakin, *Phys. Rev. C* **55**, 2054 (1997).
7. R. L. Workman, M. W. Paris, W. J. Briscoe, L. Tiator, S. Schumann, M. Ostrick, and S. S. Kamalov, *Eur. Phys. J.* **A47**, 143 (2011).

Int. J. Mod. Phys. Conf. Ser. 2014.26. Downloaded from www.worldscientific.com
by UNIVERSITY OF GENT TOEGEPASTE WISKUNDE & INFORMATICA (ATTN: PROF DR G. CROMBEZ) on 01/27/14. For personal use only.