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A Two Stage Interval-valued Neutrosophic Soft Set Traffic Signal Control Model for Four Way Isolated signalized Intersections

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Abstract. One of the major problems of both developed and developing countries is traffic congestion in urban road transportation systems.Some of the adverse consequences of traffic congestion are loss of productive time ,delay in transportation,increase in transportation cost,excess fuel consumption,safety of people,increase in air pollution level and disruption of day-to-day activities.Researches have shown that among others,traditional traffic control system is one of the main reasons for traffic congestion at traffic junctions.Most countries through out the world use pre-timed/ fixed cycle time traffic control systems.But these traffic control systems do not give an optimal signal time setting as they do not take into account the time dependent heavy traffic conditions at the junctions.They merely use a predetermined sequence or order for both signal phase change and time setting.Some times this also leads to more congestion at the junctions.As an improvement of fixed time traffic control method , fuzzy logic traffic control model was developed which takes into account the current traffic conditions at the junctions and works based on fuzzy logic principle under imprecise and uncertain conditions.But as a real life situation,in addition to uncertainty and impreciseness there is also indeterminacy in traffic signal control constraints which fuzzy logic can not handle.The aim of this research is to develop a new traffic signal control model that can solve the limitations of fixed time signal control and fuzzy logic signal control using a flexible approach based on interval-valued neutrosophic soft set and its decision making technique,specially developed for this purpose.We have developed an algorithm for controlling both phase change and green time extension / termination as warranted by the traffic conditions prevailing at any time.This algorithm takes into account the existing traffic conditions,its uncertainty and indeterminacy.The decision making technique developed allows both phase change and green time setting to be managed dynamically ,depending on the current traffic intensity and queuing of vehicles at different lanes ,as opposed to an order or a pre-determined sequence followed in existing traffic control models.

Keywords: Signal control; soft set; neutrosophic set; interval-valued neutrosophic set ;interval-valued neutrosophic soft set

1. **INTRODUCTION**

Most of the cities of developing countries suffer from traffic congestion in urban road transportation systems.Consequently this leads to loss of productive time,delay in transportation,increase in operating cost of transportation and excess fuel consumption. Increase in number of vehicles, in sufficient roads, quality of roads and traditional traffic signal control systems are the major causes for traffic congestion in big cities[1].

Some of the adverse consequences of traffic congestion include increase in the level of pollution ,safety of the people,disruption of day to day life /daily activities ,health burden for the individuals as well as government and private organizations.In early days as well as at present ,traffic is controlled by hand signs by traffic police or by signals and markings called the traditional traffic control systems.Researches have established that unless otherwise implemented properly the traditional traffic control system can contribute more to the congestion at intersections[2].

As a real world phenomenon traffic congestion involves uncertainty and impreciseness and this paved the way for the use of fuzzy logic controller in traffic control systems.Fuzzy logic is one of the most appropriate tool to handle imprecise characteristics accurately and scientifically.Fuzzy controller model makes use of expert's experience and knowledge in traffic control field to develop the linguistic protocol that produces input/output for the control system.Imprecise,inexact and linguistic traffic terms such as 'heavy traffic','moderate traffic', 'light traffic' and 'low traffic' can be manipulated using fuzzy logic controller to estimate signal timings and sequences.Currently most signalized intersections in almost all developing countries use fixed time traffic controllers or pre- timed traffic lights.The traffic lights change phase at a constant cycle time in fixed traffic light controller,without taking into account the peak period or highly varying traffic intensity with respect to time.Pre-timed traffic light also causes traffic congestion as it is incapable of detecting traffic intensity at the junction and allow the vehicles waiting in the lanes to cross the junction as per the urgency necessitated by the traffic conditions prevailing at that time.

The present day traffic signal controller models suffer from indeterminacy due to various factors like unawareness of the problem,inaccurate and imperfect data and poor forecasting in addition to uncertainty in the constraints.To overcome this we need to develop a new mathematical tool that can effectively address the problem of traffic congestion.The drawbacks of the present day traffic control models and their failure to address the problem of urban traffic congestion have motivated us to carryout this research.The main aim of this research is to develop a new traffic signal model that can overcome the limitations of present day traffic control models including fuzzy logic signal control.

Smarandache[41] introduced the concept of neutrosophic sets in 1998. Neutrosophic set(NS) is an extension of classical,fuzzy and intutisionistic fuzzy sets.A neutrosophic set is characterized by its components truth membership degree,indeterminacy membership degree and falsity membership degree ,which are considered to be independent of each other.These three membership degrees are more suitable to represent indeterminate and inconsistent information.Based on the pioneering work of Smarandache ,Wang et al.[3] introduced the idea of interval-valued neutrosophic set (IVNS), an extension of neutrosophic set which is found to be more suitable for many real life applications like image processing,decision making,supply chain management,water resource management and medical diagnosis.Interval-valued neutrosophic set is more realistic,flexible and practical than neutrosophic set and is described by three intervals namely a membership interval,an indeterminacy interval and a non-membership interval.Thus IVNS provides a more reasonable and structured mathematical framework to cope with indeterminate and inconsistent information.

In a developed society, people are more concerned about their health. Thus, improvement of medical field application has been one of the greatest active study areas. Medical statistics show that heart disease is the main reason for morbidity and death in the world.The physician's job is difficult because of having too many factors to analyze in the diagnosis of heart disease. Besides, data and information gained by the physician for diagnosis are often partial and immersed. Recently, health care applications with the Internet of Things (IoT) have offered different dimensions and other online services. These applications have provided a new platform for millions of people to receive benefits from the regular health tips to live a healthy life. With this idea in their mind Abdel-Basset et al. [26, 27, 28, 29] proposed a neutrosophic multi criteria decision making (NMCDM) technique and applied this technique to medical diagnosis.

Many researchers have combined neutrosophic set with other mathematical structures to produce different hybrid structures and used them in suitable applications.These hybrid structures include neutrosophic soft set ,intuitionistic soft set ,generalized neutrosophic soft set and interval valued neutrosophic soft set. In this research we design a new model to facilitate smooth flow of traffic at isolated four way signalized intersections using a flexible approach based on interval-valued neutrosophic soft sets and its decision making technique developed for this purpose.We developed an algorithm that can dynamically control both of the main activities of traffic control namely ,phase change and green time setting as necessitated by the traffic conditions prevailing at the time of consideration.Based on the decision making technique specifically developed for this purpose ,this algorithm makes use of the existing traffic conditions and its uncertainty as well as indeterminacy to effectively control the phase change and green time setting to facilitate smooth flow of traffic at four way intersections.

The traffic signal control model ,when implemented properly could eliminate the drawbacks of the present day traffic signal control models and make a complete solution to the problem of urban traffic congestion at four way signalized intersections.

To the best of our knowledge no research work seems to have been carried out so far on the performance of fixed time or pre-timed traffic signal controllers as well as on the use of fuzzy logic controller at four way traffic intersections in interval valued neutrosophic soft set environment.

2. REVIEW OF LITERATURE

Molodtsov [4],introduced the concept of soft sets for dealing with fuzzy,uncertain and ambiguously defined objects and outlined some problems that could be tackled using soft sets.The major advantage of soft set is its ability to use parametrization which was lacking with other tools used in the study of uncertainty.

Smarandache [5] defined the notion of neutrosophic set as a generalization of intutionistic fuzzy set.He explained the difference between neurosophic set and intutionistic fuzzy set with illustrative examples. Wang et al.[6]defined the single valued neutrosophic set and proposed a framework for this set. They also defined the set theoretic operations and established some properties of this set.

Aggarwal et al.[7] discussed how neutrosophic logic can be utilized for modeling and control and proposed a block diagram for neutrosophic inference systems.

Maji [8] gave an application of neutrosophic set to a decision making problem on object recognition in imprecise environment.Maji [9] defined the notion of neutrosophic soft set and studied some properties of this set.Multi-criteria decision making using weighted correlation coefficient and weighted cosine similarity measure under single valued neutrosophic environment was studied by Ye [10].

Broumi and Smarandache [11] developed a new multi attribute decision making method based on neutrosophic trapezoidal lingustic weighted arithmetic and geometric averaging operators.They have demonstrated the new method with numerical example.Broumi et al.[12]have presented the definition of neutrosophic parametrized soft set and its operations.They have defined some aggregation operators to develop a soft decision making technique which is more efficient.Sahin [13]generalized the concept of neutrosophic soft set and gave an application of generalized neutrosophic soft set in decision making.Deli [14]further generalized the concepts of soft set ,fuzzy soft set ,interval valued fuzzy soft set,intutionistic fuzzy soft set,interval valued intutionistic fuzzy soft set and neutrosophic soft set by introducing interval valued neutrosophic soft set.He defined some operations on this set and developed a decision making method based on level soft sets and provided an illustrative example for the proposed method.Deli and Cogman [15]defined some operations on interval valued neutrosophic soft sets and developed a decision making method based on interval valued neutrosophic soft set and illustrated the method by example.Zhang et al.[16] defined some aggregation operators on interval valued neutrosophic set and developed a method for multi-criteria decision making using these operators.They have also illustrated the working of the method by example.Ye [17] explained the difficulty in applying the single valued and interval valued neutrosophic sets and introduced the concept of simplified neutrosophic set .He defined three vector similarity measures in vector space and applied them in a multi-criteria decision making problem with simplified neutrosophic information.Illustrative example is also provided.Ye [18] shown that simplified neutrosophic set is a sub class of neutrosophic set and defined some operational laws and aggregation operators such as weighted arithmetic and weighted geometric average operators.Based on these operators and cosine similarity measure,he developed a multicriteria decision making method and using numerical example illustrated the ranking order of alternatives to select the ideal and best alternatives.Ye [19] defined the hamming and euclidean distances between interval valued neutrosophic sets and the similarity measures between them.He used the similarity measures to rank the alternatives and to determine the best one and demonstrated the method with example.Smarandache [20] introduced the concept of generalized interval neutrosophic soft set and discussed some operations on this set.He also presented an application of generalized interval neutrosophic soft set in decision making problem.Deli and Broumi [21]have redefined the notion of neutrosophic soft set and its operations.They defined neutrosophic soft matrix and their operators and explained the use of this matrix in storing a neutrosophic set in computer memory.They have also developed a decision making method based on neutrosophic soft matrix. Biswas et al.[22] have introduced the concept of single valued trapezoidal neutrosophic number .They defined the value and ambiguity indices of truth,indeterminacy and falsity membership functions of this number.A new ranking method of these numbers based on the two indices is developed and this ranking technique is applied to a multi-criteria decision making problem with a numerical illustration.Tian et al.[23] developed a method for multi-criteria decision making by combining simplified neutrosophic sets and normalized Bonferoni mean operators.They first introduced a comparison method for simplified neutrosophic linguistic numbers by employing linguistic scale function. Then they defined a normalized weighted Bonferoni mean operator and studied its properties .Based on the mean operator they developed a multi-criteria decision making method and illustrated with an example.Ye [24]introduced new exponential operational laws of interval valued neutrosophic set .He proposed an interval neutrosophic weighted exponential aggregation operator and its dual and developed a decision making method based on these operators.Yin-Xiang Ma et al.[25] developed a medical treatment selection method based on prioritized harmonic mean operators in an interval neutrosophic linguistic environment.They defined two aggregation operators based on harmonic mean and used them to develop an interval neutrosophic linguistic multi-criteria group decision making method and applied it to a practical treatment selection problem.

Abdel-Basset [26] proposed a novel framework based on computer supported diagnosis and IoT to detect and monitor heart failure infected patients, where the data are obtained from various other sources. The proposed healthcare system aims at obtaining better precision of diagnosis with ambiguous information and suggested neutrosophic multi criteria decision making (NMCDM) technique to aid patient and physician to know if patient is suffering from heart failure. The proposed model is validated by numerical examples on real case studies. The experimental results indicate that the proposed system provides a viable solution that can work at wide range and a new platform to millions of people getting benefit over decreasing of mortality and cost of clinical treatment related to heart failure.

Abdel-Basset et al.[27] suggested a novel framework based on computer propped diagnosis and IoT to detect and observe type-2 diabetes patients. The recommended healthcare system aims to obtain a better accuracy of diagnosis with mysterious data. The overall experimental results indicated the validity and robustness of the proposed algorithms.

Abdel-Basset et al.[28] used a plithogenic multi criteria decision making (MCDM) strategy and VIKOR (Vlsekriterijumska Optimizacija I Kompromisno Resenje) technique to arrive at a methodological procedure to assess the infirmary serving under the framework of plithogenic theory ,which is more general than fuzzy,intuitionistic fuzzy and neutrosophic theory.In plithogenic theory ,the ambiguity ,incomplete information ,qualitative information,approximate evaluation ,imprecision and uncertainty are addressed with semantic expressions determined by plithogenic numbers and computing of contradiction degrees of attribute values.The theory is applied to 3 private and 2 general hospitals in Zagazig and found that the serving efficiency of private medical centers is superior than that of the general medical centers.

In some real world decision making environment ,similarity plays a vital role and unexpected outcomes from the decision making point of view.Based on this Abdel-Basset et al.[29] proposed the cosine similarity measures and weighted cosine similarity measures for bipolar and interval valued bipolar neutrosophic set and presented a two multi attribute decision making techniques based on the proposed measures .The feasibility of the proposed measures are verified using numerical examples and used for diagnosing bipolar disorder diseases. Thamaraiselvi and Santhil,[30] introduced a mathematical representation of a transportation problem in neutrosophic environment and proposed a new method for solving transportation problems with indeterminate and inconsistent information.Liang et al.[31] proposed a single valued trapezoidal neutrosophic preference relation as a strategy for tackling multicriteria decision making problems based on two aggregation operators, namely single valued trapezoidal weighted arithmetic and geometric average operators.Deli and Subas [32]have presented a methodology for solving multi-attribute decision making problems using single valued neutrosophic numbers.They have defined cut sets of these numbers and applied these cuts to single valued trapezoidal and triangular neutrosophic numbers and studied some properties.They developed a ranking method using the concepts of values and ambiguities and developed a multi-attribute decision making method using single valued trapezoidal neutrosophic numbers.Limin Su[33] developed a project procurement method selection under interval neutrosophic environment.He defined a similarity measure to handle this selection and applied it to develop a decision making model.A case study is presented to show the applicability of the proposed approach .The results are compared with three of the existing methods to show the superiority of the proposed method.Broumi et al.[34] proposed a new score function for interval valued neutrosophic numbers and used it to solve the shortest path problem.They proposed novel algorithms to find the neutrosophic shortest path by considering interval valued neutrosophic number,trapezoidal and triangular interval valued neutrosophic numbers for finding the length of the path in a network with illustrative examples.The effectiveness of the proposed algorithms are explained by a comparative analysis with the existing methods.Hongwa Qin and Xiagin Ma [35] proposed a new and complete system evaluation method based on interval valued fuzzy soft set.This method has four components viz data collection and processing,combination of data sets ,parameter reduction and decision making.They have

applied the method to three real life evaluation systems.Poyen Fb et al.[36] proposed a frame work for an intelligent traffic control system. They proposed a mechanism in which the time period of green light and red light are assigned on the basis of the density of the traffic present at that time.The density is calculated using proximity infrared sensors and the glowing time of green light is assigned with the help of micro controller.Ye[37] presented the concepts of neutrosophic linear equations ,neutrosophic matrix and operations on such matrices.He proposed some methods to solve the system of neutrosophic linear equations.He has applied the method to solve the system of linear equations arising in indeterminate traffic flow.Broumi[38] has explained the importance of traffic management to ensure safe and peaceful travel for people.He defined some weighted aggregation operations on type 2 fuzzy sets and interval neutrosophic sets and constructed an improved score function for interval neutrosophic numbers.He proposed a method for traffic flow control based on this score function.Javed Alam and Pandey [40] proposed a two stage traffic light system for real-time traffic monitoring to dynamically manage both the phase and green time of traffic lights for an isolated signalized intersection with the objective of minimizing the average vehicle delay in different traffic flow rates.Software has been developed in MATLAB to simulate the situation of an isolated signalized intersection based on fuzzy logic. Simulation results verify the performance of the proposed two stage traffic light system using fuzzy logic.
In this research we develop a new traffic signal control model.This model makes use of interval-valued neutrosophic soft set and its decision making technique developed specifically for this purpose.The model consists of two stages and can dynamically manage both phase change and green signal time extension for real time traffic monitoring.

3. PRELIMINARY CONCEPTS

In this section we present the necessary preliminary ideas and some basic results needed for the present research work.We start from the definition of a soft set.

3.1 **Soft Set[4]**

Let X be the universal set under consideration and E be a set of parameters which represents the attributes,properties or characteristics of objects in X.Let $P(X)$ denote the power set of X and $A \subseteq E$. A pair (F, A) is called a soft set over X where F is a mapping given by $F : A \to P(X)$.That is a soft set over X is a parametrized family of subsets of X.
For each $e \in A$,$F(e)$ may be considered as the set of e-approximate elements of X,called the value set of e.It is clear that soft set is not a set in the classical sense.

3.2 **Complement of a soft set** [4]

The complement of a soft set (F, A) denoted by $(F, A)^c$ is defined as $(F, A)^c = (F^c, \sim A)$,where $F^c :\sim A \to P(X)$ is the mapping given by $F^c(\alpha) = X - F(\sim \alpha), \forall \alpha \in \sim A$.Obviously $(F^c)^c = F$ and $((F, A)^c)^c = (F, A)$.

3.3 **Null soft set** [4]

A soft set (F, A) over$X$ is said to be a null soft set denoted by $\tilde{\Phi}$,if $\forall e \in A, F(e) = \tilde{\Phi}$ (null set).

3.4 **Absolute soft set** [4]

A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$, if $\forall e \in \tilde{A}$, $F(e) = X$.

3.5 **Soft subset and soft super set** [4]

Let (F, A) and (G, B) be two soft sets over a common universe X.We say that (F, A) is a soft subset of (G, B) if

(i) $A \subseteq B$ and

$(ii)\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

In such a case we write $(F, A)\tilde{\subseteq}(G, B)$.

(F, A) is called a soft super set of (G, B) if (G, B) is a soft subset of (F, A).In this case we write $(F, A)\tilde{\supseteq}(G, B)$.

3.6 **Soft Equality** [4]

Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

3.7 **Single valued neutrosophic set**[6]

Let X be the universal set.A neutrosophic set A in X is characterized by a truth membership function μ_A, an indeterminacy membership function υ_A and a falsity membership function ω_A,where $\mu_A, \upsilon_A, \omega_A$: $X \to [0, 1]$ are functions and $\forall x \in X$,$x \equiv x(\mu_x, \upsilon_x, \omega_x) \in A$ is a single valued neutrosophic element of A.

A single valued neutrosophic set A(SVNS in short) over a finite universe $X = \{x_1, x_2, ..., x_n\}$ can be represented as $A = \sum_{i=1}^{n} < \mu_A(x_i), \upsilon_A(x_i), \omega_A(x_i) > /x_i$.

The three membership functions form the fundamental concepts of neutrosophic set and they are independently and explicitly quantified subject to the following conditions.

$$0 \leq \mu_A(x), \upsilon_A(x), \omega_A(x) \leq 1 \text{ and}$$
$$0 \leq \mu_A(x) + \upsilon_A(x) + \omega_A(x) \leq 3 \ \forall x \in X.$$

3.8 **Complement of a single valued neutrosophic set** [6]

The complement of a SVNS A denoted by A^c is defined by $\mu_{A^c}(x) = \omega_A(x)$,$\upsilon_{A^c}(x) = 1 - \upsilon_A(x)$ and $\omega_{A^c}(x) = \mu_A(x)$ for all $x \in X$

3.9 **Containment and equality of SVNS defined on the same universe** X[6]**.**

(i) A is contained in B denoted as $A\tilde{\subseteq}B$ if and only if $\mu_A(x) \leq \mu_B(x)$; $\upsilon_A(x) \leq \upsilon_B(x)$ and $\omega_A(x) \geq \omega_B(x)\forall x \in X$.

(ii) A and B are equal ,that is., $A = B$ if and only if $A\tilde{\subseteq}B$ and $B\tilde{\subseteq}A$.

3.10 **Union and Intersection of SVNS** [6]

Let A and B be two SVNS defined on a common universe X .Then the union of A and B ,written as $A \cup B = C$ is defined by

$$\mu_C(x) = max(\mu_A(x), \mu_B(x))$$
$$\upsilon_C(x) = max(\upsilon_A(x), \upsilon_B(x))\text{and}$$

$$\omega_C(x) = min(\omega_A(x), \omega_B(x))\forall x \in X.$$

The intersection of A and B ,denoted by $A \cap B = C$ is defined by

$$\mu_C(x) = min(\mu_A(x), \mu_B(x)),$$
$$\upsilon_C(x) = min(\upsilon_A(x), \upsilon_B(x)) \text{ and}$$
$$\omega_C(x) = max(\omega_A(x), \omega_B(x))\forall x \in X.$$

3.11 **Interval Valued Neutrosophic Set** [39]

Definition: For an arbitrary sub interval set A of $[0,1]$ we define $\underline{A}$ = inf of A and $\overline{A}$ = sup of A. Let X be the universal set.An interval valued neutrosophic set A in X is characterized by a truth membership function μ_A , an indeterminacy membership function υ_A and a falsity membership function ω_A for each element $x \in X$ where

$$\mu_A(x) = [\underline{\mu}_A(x), \overline{\mu}_A(x)],\ \upsilon_A(x) = [\underline{\upsilon}_A(x), \overline{\upsilon}_A(x)],\ \omega_A(x) = [\underline{\omega}_A(x), \overline{\omega}_A(x)] \text{ are closed sub-intervals of } [0,1].$$

Thus $A = < \mu_A(x), \upsilon_A(x), \omega_A(x) > /x; x \in X$.

3.12 **Empty and universal IVNS** [39]

Let A be an IVNS over the universal set X.

(i) A is called an empty IVNS if $A = \overline{\Phi}_N = < [0,0], [1,1], [1,1] > /x; \forall x \in X$ (ii) A is called a universal IVNS if $A = \overline{E} = < [1,1], [0,0], [0,0] > /x; \forall x \in X$

3.13 **Containment** [39]

Let A and B be two IVNS over a common universe X.Then A is contained in B if and only if

$$\underline{\mu}_A(x) \le \underline{\mu}_B(x); \overline{\mu}_A(x) \le \overline{\mu}_B(x); \underline{\upsilon}_A(x) \le \underline{\upsilon}_B(x); \overline{\upsilon}_A(x) \le \overline{\upsilon}_B(x); \underline{\omega}_A(x) \ge \underline{\omega}_B(x); \overline{\omega}_A(x) \ge \overline{\omega}_B(x) \quad \forall x \in X.$$

In this case we write $A \subseteq B$

3.14 **Union and Intersection of IVNS** [39]

let A and B be two IVNS defined over a common universe X.The union of A and B denoted by$A \tilde{\cup} B$ is defined as

$$A\tilde{\cup}B = \{< [max(\underline{\mu}_A(x), \underline{\mu}_B(x)), max(\overline{\mu}_A(x), \overline{\mu}_B(x))],$$
$$[max(\underline{\upsilon}_A(x), \underline{\upsilon}_B(x)), max(\overline{\upsilon}_A(x), \overline{\upsilon}_B(x))],$$
$$[min(\underline{\omega}_A(x), \underline{\omega}_B(x), min(\overline{\omega}_A(x), \overline{\omega}_B(x))] > /x; \forall x \in X\}$$

Similarly the intersection of A and B denoted by $A\tilde{\cap}B$ is defined by

$$A\tilde{\cap}B = \{< [min(\underline{\mu}_A(x), \underline{\mu}_B(x)), min(\overline{\mu}_A(x), \overline{\mu}_B(x))],$$
$$[min(\underline{\upsilon}_A(x), \underline{\upsilon}_B(x)), min(\overline{\upsilon}_A(x), \overline{\upsilon}_B(x))],$$
$$[max(\underline{\omega}_A(x), \underline{\omega}_B(x), max(\overline{\omega}_A(x), \overline{\omega}_B(x))] > /x; \forall x \in X\}$$

3.15 **Complement of IVNS** [39]

The complement A^c of an IVNS A is defined as $A^c = \{< [\underline{\omega}_A(x), \overline{\omega}_A(x)], [1 - \overline{\upsilon}_A(x), 1 - \underline{\upsilon}_A(x)], [\underline{\mu}_A(x), \overline{\mu}_A(x)] > /x; \forall x \in X\}$

It is clear that $A\tilde{\cup}B$,$A\tilde{\cap}B$ and A^c are all IVNS over X.

3.16 **Neutrosophic Soft Set** [20]

Let X be the universal set and E be the set of parameters ,$A \subseteq E$.Let $P(X)$ denote the set of all neutrosophic subsets of X. A pair (F, A) is called a neutrosophic soft set over X ,where F is a mapping given by $F : A \to P(X)$.For each $e \in A$,$F(e)$ is a neutrosophic subset of X.

3.17 **Union and Intersection of neutrosophic Soft Set** [20]

Let (F, A) and (G, B) be two neutrosophic soft sets over (X, E).The union of (F, A) and $(G, B)$is defined as the neutrosophic soft set (H, C) where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \text{F(e)} & \text{if} e \in A - B \\ \text{G(e)} & \text{if } e \in B - A \\ \text{F(e)}\cup\text{G(e)} & \text{if } e \in A \cup B \end{cases} \tag{1}$$

We write $(H, C) = (F, A)\tilde{\cup}(G, B)$.

The intersection of (F, A) and (G, B) denoted by (H, C) where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \text{F(e)} & \text{if} e \in A - B \\ \text{G(e)} & \text{if } e \in B - A \\ \text{F(e)}\cap\text{G(e)} & \text{if } e \in A \cap B \end{cases} \tag{2}$$

We write $(H, C) = (F, A)\tilde{\cap}(G, B)$.

Interval valued neutrosophic soft set is a hybrid structure combining interval-valued neutrosophic set and soft set.The first quantifies the indeterminacy and the second provides the parametrization tool.Interval valued neutrosophic soft set (IVNSS) is a better tool to express uncertainty than the other variants of fuzzy or soft sets.The mathematical definition of IVNSS is given below.

3.18 **Interval Valued neutrosophic Soft Set** [15]

Let X be the universal set and E be a set of parameters ,$A \subseteq E$. Let IVNS (X) denote the set of all interval-valued neutrosophic sets over X. A pair (G, A) is called an interval -valued neutrosophic soft set (IVNSS) over X where G is a mapping given by $G : A \to IVNS(X)$.For $e \in A, G(e)$ is an interval-valued neutrosophic set in X, called e-approximate value set.Thus for all $e \in A, G(e) =< x, \mu^e(x), \upsilon^e(x), \omega^e(x) >\in IVNS(X)$.

3.19 **Null and absolute IVNSS** [15]

An IVNSS(G, A) of X is called a null or empty IVNSS denoted by $\tilde{\phi_N}$ if and only if $\forall e \in A$

$$\mu_e^G(x) = [0, 0], \upsilon_e^G(x) = [1, 1], \omega_e^G(x) = [1, 1] \forall x \in X$$

(G, A) is said to be an absolute IVNSS denoted by $\tilde{A_N}$ if and only if

$$\mu_e^G(x) = [1, 1], \upsilon_e^G(x) = [0, 0], \omega_e^G(x) = [0, 0] \forall x \in X$$

For examples and illustration of related operations one can refer to the respective references.

4. THE PROPOSED TWO STAGE IVNSS TRAFFIC SIGNAL CONTROL MODEL

4.1 Introduction

It has been reported in the literature that fuzzy logic controller performs better when compared with pre-timed or fixed time controllers.However,in both the latter mentioned signal controllers the phase changes occur in a sequential order without any consideration on the current traffic conditions at the intersection.In this research it is proposed to implement a two stage traffic light control system for real time traffic monitoring to dynamically manage both phase change as well as extension of green signal time for an isolated signalized intersection with the objective of minimizing the average vehicle delay under varying traffic flow rates.Thus in the proposed model the phase change may not be in a sequential order among the roads at the intersection.IVNSS model has the ability to imitate the human intelligence for controlling traffic flow.It allows implementation of real life rules similar to the way human mind would work.This model is based on concepts graded to handle uncertainties and impreciseness to facilitate relatively smooth traffic flow.The graded concepts are more useful since real time situations in traffic control are very often difficult to describe precisely and are not deterministic.

IVNSS model allows linguistic expressions and inexact data to be manipulated in designing signal timings and phase change intervals.We consider an isolated traffic signal intersection with four approaches north,south,east and west ,whose green time extension or termination at the traffic junction are tested during rush hour against the two parameters 'average number of vehicles entering the junction along the lanes with current green signal '(arrival rate)and 'the saturation flow rate' in the previous cycle.These input parameters are used to construct two neutrosophic sets on input variables ,viz 'the quantity of traffic on the arrival side' and the quantity of traffic on the departure side' to estimate the output variable 'the extension time for the green signal' based on weight criteria.Thus based on the current traffic conditions the IVNSS model can estimate the output of the neutrosophic controller either to extend or terminate the ongoing green light signal.If there is no extension of the ongoing green time ,there will be an immediate phase change of the traffic light allowing the flow of traffic from an alternate lane.

4.2 Design Criteria and Constraints

The design of IVNSS traffic signal controller depends on the experience and knowledge of experts in traffic control to formulate the linguistic protocol and to generate the input variables for the traffic signal control system.Vehicle detectors are installed on 'upstream line' and 'stop line'.The number of approaching vehicles for each approach during a given time interval can be estimated using the detectors.

The following assumptions are made for designing the IVNSS traffic control system:

[1] The junction is an isolated four way intersection with traffic flowing from north,west, south and east directions.

[2]When traffic moves from north and south,that from west and east stops,and vice-versa.

[3]Right and left turns are permitted

[4]The IVNSS controller observes the density of north and south traffic as one side and west and east traffic as another side.

[5]The minimum and maximum green time in a cycle is specified.

[6]If the north and south side is green ,this would be the arrival side while the west and east would be considered as the queuing side,and vice-versa.Then the output variable would be the extension time needed for the green light on the arrival side based on some weight criteria.

[7]In the proposed two stage traffic signal system,it is assumed that phase composition is pre-determined and the phase sequence as well as signal timings are changeable.

4.3 **Structure of the Two Stage Traffic Signal Controller Model.**

The general structure of an isolated four way intersection is illustrated in Figure 1.Each lane is equipped with two electromagnetic sensors.The first sensor is located behind each traffic light and the second is located behind the first at a distance S.The first sensor counts the number of cars passing the traffic light and the second counts the number of cars coming to the intersection.The number of cars waiting at the traffic light of each road is determined by the difference of the readings between the two sensors of that road.Each traffic light is a proximity sensor and can only sense the presence of a car in front of it waiting at the junction.The isolated intersection considered in Figure 1 is characterized by four phases as shown in Figure 2 with eight lanes.Each phase has two lanes.As discussed earlier the objective of phase design is to separate the conflicting movements at the intersection into various phases in such a way that there is no conflict in the movement during any phase.For example ,when phase 1 is enabled,only the vehicles of lane $EL_1$of the road direction east(E)and lane WL_1 of the road direction west (W)can go either straight or turn left ,while all other lanes will have red light to stop movement of vehicles.

The flow rate and saturation flow rate of each phase is obtained as the maximum of the flow rates and saturation flow rates intended as the number of vehicles during the green/red light in their respective lanes as shown.

$$\text{Phase1} = \text{maximum (EL1, WL1)}$$
$$\text{Phase2} = \text{maximum (EL2, WL2)}$$
$$\text{Phase3} = \text{maximum (NL1, SL1)}$$
$$\text{Phase4} = \text{maximum (NL2, SL2)}$$

Table 1 summarizes the notation adopted in the two stage traffic control system.

4.4 **Data Collection and Processing.**

The necessary crisp data for the input parameters are obtained from vehicular detectors installed at traffic intersections.In order to derive useful information from the detector data ,it is important to be selective and to use data that can give meaningful information about actual traffic flow.Factors which influence the data include the setting of signal group(red or green)and locating detection area.The

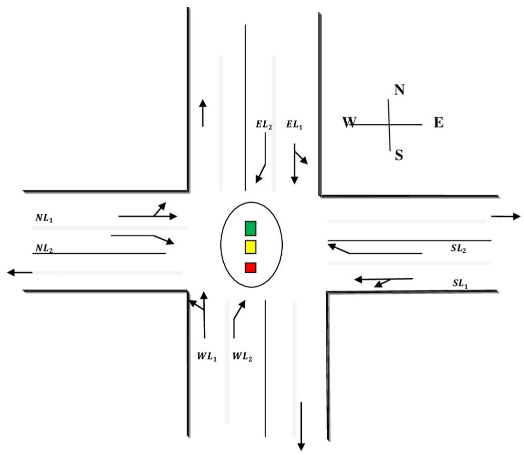

FIGURE 1. General Structure of the Intersection

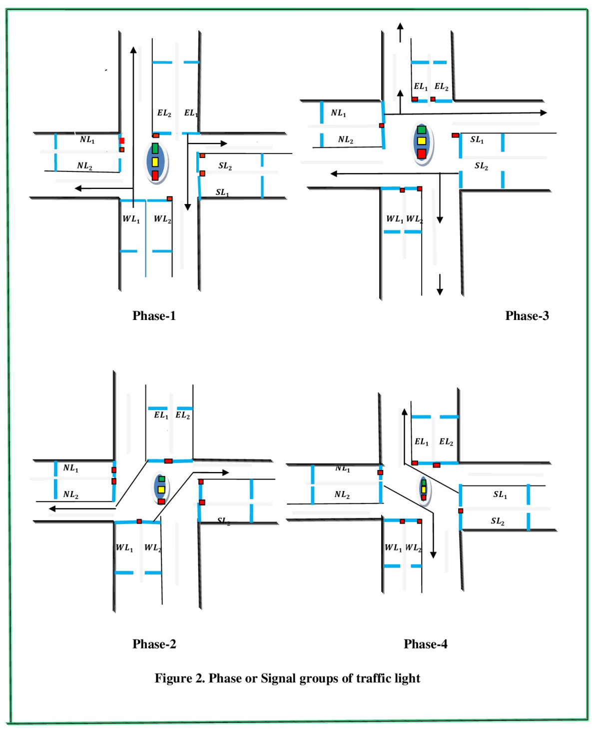

Figure 2. Phase or Signal groups of traffic light

FIGURE 2. Phases/signal groups of Traffic light.

proposed IVNSS model uses the following two neutrosophic variables as input.

4.4.1 **Saturation flow rate (vehicle/hour/green)**

Saturation Flow Rate is the number of vehicles that would pass through the intersection when they approach signal which stays with green for an entire hour. Obviously, certain aspects of the traffic and the roadway will affect the saturation flow rate of the approach. If the approach has very narrow lanes, traffic will naturally provide longer gaps between vehicles, which will reduce our saturation flow rate. If there is large number of turning movements, or large number of trucks and busses, your saturation

Symbol	Lane movement	Allowed
EL_1	East Lane-1	Straight and left
EL_2	East Lane-2	right
WL_1	West Lane-1	Straight and left
WL_2	West Lane-2	right
NL_1	North Lane-1	Straight and left
NL_2	North Lane-2	right
SL_1	South Lane-1	Straight and left
SL_2	South Lane-2	right

TABLE 1. Notation of different phases/signal groups

flow rate will be reduced [64].

On another hand, the saturation flow rate (s) for a lane group is the maximum number of vehicles from that lane group that can pass through the intersection during one hour of continuous green under the prevailing traffic and roadway conditions.

4.4.2.**Average arrival rate (veh/min/lane)**

The detection area is located S meters down stream of the stop line of each approach,to record arrival rate. During the green period, the detector records the average arrival rate every minute and the average arrival rate for that period is used as an input.The output variable is the extension time of green signal time for the phase based on weight criteria.

4.5 **Description of Two Stage IVNSS Traffic Signal Control Model(IVNSSTSC)**.

In the proposed IVNSS traffic signal control model there are two different modules, namely the phase selection decision module (PSDM) and the extension time calculation module(ETCM) for the selection of phase and the extension of green signal duration.

4.5.1 **Phase Selection Decision Module(PSDM)**

This stage performs the selection of the most appropriate phase /signal group for the next cycle with highest traffic urgency as the next phase to switch on using traffic data obtained from the previous cycle based on the input parameters.The proposed IVNSS model uses crisp traffic information from traffic sensors located at traffic junction for estimating the input parameters; saturation flow rate and average arrival rate collected during the previous cycle in order to decide on the number of seconds of green signal time required by each set of signal groups (phases) during the next cycle.The crisp data obtained has to be transformed into neutrosophic data using neutrosophication /normalization techniques.

The output parameter or decision criteria of the proposed IVNSSTSC is weight of the signal group .Weight is an indicator of the degree of need/urgency of the phase or signal group (SG) that requires green signal.For example,if the weight of SG_1 is 18 and that of SG_2 is 12 ,then SG_1 needs green time more urgently than SG_2.

4.5.2 **Extension Time Calculation Module (ETCM)**

This stage calculates the green light time i.e., extension time of the signal group or phase which has highest urgency based on the weight value obtained in PSDM stage.

The weight values of SG_1, SG_2, SG_3 and SG_4 are used to calculate the total green time in a cycle.The weight values of each signal group and the total green time in a cycle are used to estimate the duration of green time that a signal group requires in the next cycle.Both total green time and green time of each signal group/phase are calculated as follows.

$$\text{Total Green Time } (TGT) = (\sum C_n - MinW) \times (\frac{MaxGT - MinGT}{MaxW - MinW}) + MinGT$$

The green time of each signal group is obtained by

$$\text{Green Time of } SG_n = \frac{C_n \times TGT}{\sum C_n}$$

where $\sum C_n$is the total weight of signal groups ,$MinW$ and $MaxW$ are the minimum and maximum values of weights respectively.$MinGT$ and $MaxGT$ are the minimum and maximum values of green time in a cycle and n is the group index $(n = 1, 2, 3, 4)$.

The proposed IVNSSTSC model consists of the following components namely,(i)data collection(ii)neutrosophication or normalization (iii)combination of data set (iv)parameter reduction and (v)decision making.

A flow chart of the proposed IVNSSTSC model is presented below.

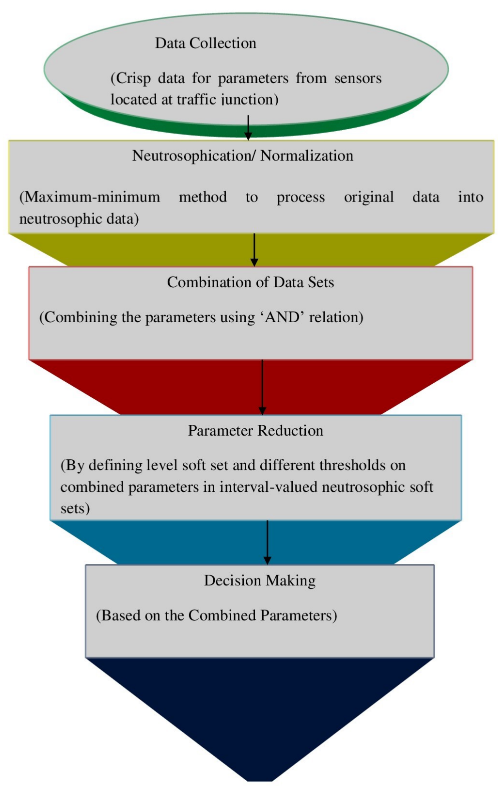

Figure 3. Structure of the Proposed Two Stage traffic Signal Control Model Using IVNSS

4.6 **Neutrosophication /Normalization**

In order to asses and evaluate a new system ,we need to have original data .In the present case to analyze the working of the proposed IVNSSTSC system it is necessary to collect data from traffic sensors installed at traffic junction or traffic management directorate or using any other video graphic measures.But the data collected by this method will not be in the form of IVNSS.Hence it is essential to transform the original data into corresponding IVNSS.

Interval-valued data are used to describe truth,indeterminacy and falsity membership degrees by identifying their lower and upper limits in the unit interval$[0,1]$.Consequently in this research the maximum-minimum neutrosophication/normalization method discussed by Hongwa Qin et al.[35] for IVFSS is extended to IVNSS.In the maximum-minimum normalization method there are more than one evaluators who give the scores of the objects aiming at one parameter.

The algorithm for the method is given below.

Maximum-Minimum Normalization Algorithm [35]

(i) Input original data : the set of objects $U = \{x_1, x_2, ..., x_n\}$; the set of parameters $E = \{e_1, e_2, ..., e_m\}$. **(ii)** For any $x_i \in U$ and for every $e_k \in E$, sort the data in ascending order.

(iii) Find the maximum and minimum values of data for every $x_i \in U$ and $e_k \in E$.

(iv) Transform the maximum and minimum evaluation scores into sub-intervals of $[0,1]$ and normalize them as upper and lower membership degrees of the corresponding IVNSS.

(v) Get the IVNSS for the evaluation system.

When the original data are numeric ,the maximum and minimum evaluation scores are taken as the highest and lowest limits of such an evaluation respectively.Maximum and minimum evaluation scores can be transformed into sub-intervals of $[0,1]$,which are considered as the normalized upper and lower membership degrees of membership in the corresponding IVNSS.

4.7 **Defining Interval-Valued Neutrosophic Set for the Data**

In this study we divide the input parameters into a finite number of states (levels) and construct the IVNSS based on this division.The parameter average arrival rate (e_1) is divided into five states/interval-valued neutrosophic set:very high, high, medium, low and very low.The saturation flow rate (e_2) is divided into four states/interval-valued neutrosophic set:very high, high, medium and low.The input data (traffic conditions) are first transformed into interval-valued neutrosophic data using the maximum-minimum normalization method for (e_1) and (e_2).This is carried out using the questionnaire from domain experts;their options could be a degree of " good performance" a degree of "indeterminacy" and a degree of "poor performance" with respect to each parameter.The interval valued neutrosophic soft set (F, A) describes the "average number of vehicles entering the junction " evaluated with respect to the levels of the parameter e_1 ,where $A = \{a_1, a_2, a_3, a_4, a_5\}$ in which a_1-stands for very high,a_2-stands for high,a_3-stands for medium, a_4- stands for low and a_5-stands for very low.

Similarly the interval-valued neutrosophic soft set (G, B) describes the 'the saturation flow rate' evaluated with respect to the levels of the parameter e_2 ,where $B = \{q_1, q_2, q_3, q_4\}$ in which q_1-represents very high, q_2 -represents high, q_3-represents medium and q_1-represents low.

Thus $X = \{SG_1, SG_2, SG_3, SG_4\}$ are the signal groups under consideration ,$E = \{$ average arrival rate (e_1),saturation flow rate $(e_2)\}$ is the parameter set ,A is an IVNS describing the states of e_1 ,along each signal group and B is an IVNS describing the states of e_2 along each signal group.

4.8. **Combination of Data Set**

In this section we define some operations on IVNSS and establish some properties of IVNSS.

4.8.1 **Cartesian Product of Two IVNSS**[12]

Let X be the universal set ,E be a set of parameters and $A, B \subseteq E$ .Let (F, A) and (G, B) be two IVNSS defined on X.Then the cartesian product of $(F, A)$and (G, B) denoted by $(H, A \times B)$ is defined as $H : A \times B \to IVNSS(X)$,where $H(a, b) = F(a) \cap G(b)$.

An interval valued neutrosophic relation from (F, A) to (G, B) is an interval-valued neutrosophic soft subset of $(H, A \times B)$.

4.8.2. **AND and OR operations**[12]

Let (F, A) and (G, B) be two IVNSS over the common universe X.Then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is an IVNSS defined as

$(F, A) \wedge (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ $\forall(\alpha, \beta) \in A \times B$.

That is,

$$\begin{gathered} H(\alpha,\beta)(X) =< [min(\underline{\mu}^{(x)}_{F(\alpha)}(x), \underline{\mu}^{(x)}_{G(\alpha)}(x)), min(\overline{\mu}^{(x)}_{F(\alpha)}(x), \overline{\mu}^{(x)}_{G(\alpha)}(x))], \\ [min(\underline{\upsilon}^{(x)}_{F(\alpha)}(x), \underline{\upsilon}^{(x)}_{G(\alpha)}(x)), min(\overline{\upsilon}^{(x)}_{F(\alpha)}(x), \overline{\upsilon}^{(x)}_{G(\alpha)}(x))], \\ [max(\underline{\omega}^{(x)}_{F(\alpha)}(x), \underline{\omega}^{(x)}_{G(\alpha)}(x)), max(\overline{\omega}^{(x)}_{F(\alpha)}(x), \overline{\omega}^{(x)}_{G(\alpha)}(x))] > /\forall(\alpha,\beta) \in A \times B \text{ and } x \in X. \end{gathered}$$

Similarly ,(F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is an IVNSS defined as $(F, A) \vee (G, B) = (J, A \times B)$ where $J(\alpha, \beta) = F(\alpha) \cup G(\beta) \forall(\alpha, \beta) \in A \times B$.

That is,

$$\begin{gathered} J(\alpha,\beta)(X) =< [max(\underline{\mu}^{(x)}_{F(\alpha)}(x), \underline{\mu}^{(x)}_{G(\alpha)}(x)), max(\overline{\mu}^{(x)}_{F(\alpha)}(x), \overline{\mu}^{(x)}_{G(\alpha)}(x))], \\ [max(\underline{\upsilon}^{(x)}_{F(\alpha)}(x), \underline{\upsilon}^{(x)}_{G(\alpha)}(x)), max(\overline{\upsilon}^{(x)}_{F(\alpha)}(x), \overline{\upsilon}^{(x)}_{G(\alpha)}(x))], \\ [min(\underline{\omega}^{(x)}_{F(\alpha)}(x), \underline{\omega}^{(x)}_{G(\alpha)}(x)), min(\overline{\omega}^{(x)}_{F(\alpha)}(x), \overline{\omega}^{(x)}_{G(\alpha)}(x))] > /\forall(\alpha,\beta) \in A \times B \text{ and } x \in X. \end{gathered}$$

4.9. **Parameter Reduction and Decision making**[15]

In this section we develop a parameter reduction algorithm of IVNSS for the smooth flow of traffic at isolated four way intersections. The algorithm produces a soft set from an IVNSS.For this we first define the concept of level sets for IVNSS. The notion of level set presents a flexible approach to IVNSS based decision making for facilitating the flow of traffic in an efficient way.The algorithm is designed to solve a decision making problem based on IVNSS using level soft sets.Level soft sets play a vital role in connecting IVNSS and crisp soft sets.Using level soft sets we do not deal with IVNSS directly ,but work with crisp soft sets derived from them after choosing certain thresholds or decision strategies such as top/bottom level decision rules or mid level decision rules.As per the algorithm ,the choice value of the object in a level soft set is in fact the number of fair attributes which belong to that object on the premise that the degree of truth membership of x with respect to the parameter e is not less than the truth membership levels, the degree of indeterminacy membership of x with respect to the parameter e is not more than the indeterminacy levels and the degree of falsity membership of x with respect to the parameter e is not more than the falsity membership levels.

4.9.1 **Relation From IVNSS**[15]

Let $(G, A) \in IVNSS(X)$.Then a relation form of (G, A) is defined by

$R_{(G,A)} = \{(r_{(G,A)}(e, x)/(e, x); r_{(G,A)}(e, x) \in IVNS(X), e \in A, x \in U\}$ where $r_{(G,A)} : E \times X \to IVNS(X) and r_{(G,A)}(e, x) = G_e(x) for all x \in X, e \in A$.

That is $r_{(G,A)}(e, x) = G_e(x)$ is characterized by truth membership function μ,indeterminacy membership function υ, and falsity membership function ω. For each point $e \in E$, and $x \in X$ and μ, υ, ω are interval valued subsets of $[0, 1]$.

4.9.2. **Example**[15]

Let $X = \{x_1, x_2\}$ be the set of houses under consideration and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters where $e_1 =$ cheap ,$e_2 =$ beautiful,$e_3 =$ in the green surroundings, $e_4 =$ costly and $e_5 =$ large respectively. Based on experts evaluation, an interval valued neutrosophic soft set (G, A) is defined and presented in tabular form as follows.

U	x_1	x_2
e_1	$[0.5, 0.8], [0.5, 0.9], [0.2, 0.5]$	$[0.4, 0.8], [0.2, 0.5], [0.5, 0.6]$
e_2	$[0.5, 0.8], [0.2, 0.8], [0.3, 0.7]$	$[0.1, 0.9], [0.6, 0.7], [0.2, 0.3]$
e_3	$[0.2, 0.7], [0.1, 0.5], [0.5, 0.8]$	$[0.5, 0.7], [0.1, 0.4], [0.6, 0.7]$
e_4	$[0.4, 0.5], [0.4, 0.9], [0.4, 0.9]$	$[0.3, 0.4], [0.6, 0.7], [0.1, 0.5]$
e_5	$[0.1, 0.7], [0.5, 0.6], [0.1, 0.5]$	$[0.6, 0.7], [0.2, 0.4], [0.3, 0.7]$

Table 2. The tabular representation of the IVNSS (G,A)

For this example $r_{(G,A)}(e, x) = G_e(x)$is given below

$$G_(e_1)(x_1) =< [0.5, 0.8], [0.5, 0.9], [0.2, 0.5] >$$
$$G_(e_1)(x_2) =< [0.4, 0.8], [0.2, 0.5], [0.5, 0.6] >$$
$$G_(e_2)(x_1) =< [0.5, 0.8], [0.2, 0.8], [0.3, 0.7] >$$
$$G_(e_2)(x_2) =< [0.1, 0.9], [0.6, 0.7], [0.2, 0.3] >$$
$$G_(e_3)(x_1) =< [0.2, 0.7], [0.1, 0.5], [0.5, 0.8] >$$
$$G_(e_3)(x_2) =< [0.5, 0.7], [0.1, 0.4], [0.6, 0.7] >$$
$$G_(e_4)(x_1) =< [0.4, 0.5], [0.4, 0.9], [0.4, 0.9] >$$
$$G_(e_4)(x_2) =< [0.3, 0.4], [0.6, 0.7], [0.1, 0.5] >$$
$$G_(e_5)(x_1) =< [0.1, 0.7], [0.5, 0.6], [0.1, 0.5] >$$

$$G_(e_5)(x_2) =< [0.6, 0.7], [0.2, 0.4], [0.3, 0.7] >$$

The notion of level soft set and different thresholds on the parameters of interval valued intuitoinistic fuzzy soft sets were introduced by Zhang et al.[16].Irfan Deli [14] has extended these concepts to interval valued neutrosophic soft sets.In this work we extend the notion of level soft sets to IVNSS and different thresholds to group decision making by combining the input parameter sets.

4.9.3. **Level Soft set of IVNSS** [15]

Let $(G, A) \in IVNSS(X)$ .For interval valued subsets $\alpha, \beta, \gamma \subseteq [0, 1]$,the $\langle \alpha, \beta, \gamma \rangle-$level soft subset of (G, A) denoted by $((G, A); \langle (\alpha, \beta, \gamma) \rangle)$, is defined as

$((G, A); < (\alpha, \beta, \gamma) >) = \{(e_i, (x_j \in X; \mu(x_{ij}) = 1)); e_i \in E\}$ where,

$$\mu(x_{ij}) = \begin{cases} 1 & \text{if } (\alpha, \beta, \gamma) \tilde{\lesseqgtr} G_{e_i}(x_j) \\ 0 & \text{otherwise for all} x_j \in X \end{cases} \tag{3}$$

If $(\alpha, \beta, \gamma) \leqslant G_{e_i}(x_j)$,it means that the degree of truth membership of x with respect to the parameter e is not less than α ,the degree of indeterminacy of x with respect to e is not more than β ,and the degree of falsity of x with respect to e is not more than γ.

In practical application of IVNSS α, β, γ are the thresholds pre-established by the decision maker reflecting his requirements on truth,indeterminacy and falsity membership levels respectively.

4.9.4 **Example**

For the Example 4.9.2,the $([0.3, 0.4], [0.5, 0.7], [0.6, 0.8])$ level soft subset of (G, A) is

$((G, A); \langle [0.3, 0.4], [0.5, 0.7], [0.6, 0.8] \rangle) = \{(e_1, u_2), (e_3, u_2), (e_5, u_1)\}$

4.9.5 **Average Threshold of an IVNSS**[15]

Let $(G, A) \in$ IVNSS(X) .For the given (G, A) ,we define the average threshold denoted by$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}$ as

$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)} : A \to IVNS(X)$ by

$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_i) = \sum_{x \in X} G_{e_i}(x) / \mid x \mid$ for all $e_i \in A$.

By average level decision rule we mean using the avg-threshold of the IVNSS in decision making procedure.

4.9.6 **Example**[15]

For the Example 4.9.2,the avg-threshold neutrosophic set is

$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_1) = \sum_{i=1}^{2} e_{e_1}(x_i) / \mid x \mid = \langle [0.45, 0.8], [0.35, 0.7], [0.35, 0.55] \rangle$$
$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_2) = \sum_{i=1}^{2} e_{e_2}(x_i) / \mid x \mid = \langle [0.3, 0.85], [0.4, 0.75], [0.25, 0.5] \rangle$$
$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_3) = \sum_{i=1}^{2} e_{e_3}(x_i) / \mid x \mid = \langle [0.35, 0.7], [0.1, 0.45], [0.55, 0.75] \rangle$$
$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_4) = \sum_{i=1}^{2} e_{e_4}(x_i) / \mid x \mid = \langle [0.35, 0.45], [0.5, 0.8], [0.25, 0.7] \rangle$$
$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)}(e_5) = \sum_{i=1}^{2} e_{e_5}(x_i) / \mid x \mid = \langle [0.35, 0.7], [0.35, 0.5], [0.2, 0.6] \rangle$$

Thus

$$\langle \alpha, \beta, \gamma \rangle^{Avg}_{(G,A)} = \{\langle [0.45, 0.8], [0.35, 0.7], [0.35, 0.55] \rangle / e_1,$$

$$\langle[0.3, 0.85], [0.4, 0.75], [0.25, 0.5]\rangle/e_2,$$
$$\langle[0.35, 0.7], [0.1, 0.45], [0.55, 0.75]\rangle/e_3,$$
$$\langle[0.35, 0.45], [0.5, 0.8], [0.25, 0.7]\rangle/e_4,$$
$$\langle[0.35, 0.7], [0.35, 0.5], [0.2, 0.6]\rangle/e_5\}$$

For this example $((G, A); \langle\alpha, \beta, \gamma\rangle^{Avg}_{(G,A)})$ =empty.

4.9.7 **Max-min-min threshold of an IVNSS**[15]

Let $(G, A) \in$ IVNSS (X).For the given (G, A) we define an interval-valued neutrosophic set $\langle\alpha, \beta, \gamma\rangle^{Mmm}_{(G,A)} : A \rightarrow$IVNSS (x) by

$$\langle(\alpha, \beta, \gamma)\rangle^{Mmm}_{(G,A)} = \{\langle[max_{x\in X}\{\underline{\mu}^{(x)}_{G(e_i)}\}, max_{x\in X}\{\overline{\mu}^{(x)}_{G(e_i)}\}],$$
$$[min_{x\in X}\{\underline{\upsilon}^{(x)}_{G(e_i)}\}, min_{x\in X}\{\overline{\upsilon}^{(x)}_{G(e_i)}\}],$$
$$[min_{x\in X}\{\underline{\omega}^{(x)}_{G(e_i)}\}, min_{x\in X}\{\overline{\omega}^{(x)}_{G(e_i)}\}]\rangle/e_i; e_i \in E\}$$

$\langle\alpha, \beta, \gamma\rangle^{Mmm}_{(G,A)}$ is called the maximum -minimum threshold of the IVNSS (G, A).By Mmm-level decision rule we mean using the maximum-minimum-minimum threshold level soft set in IVNSS based decision making.

For this example$((G, A); \langle\alpha, \beta, \gamma\rangle^{Avg}_{(G,A)})$ =empty.

4.9.8 **Min-min-min threshold of an IVNSS**[15]

Let $(G, A) \in$ IVNSS (X).For the given (G, A) the min-min-min threshold denoted by $\langle\alpha, \beta, \gamma\rangle^{mmm}_{(G,A)}$: $A \rightarrow$IVNSS(x) is an interval valued neutrosophic set defined as

$$\langle(\alpha, \beta, \gamma)\rangle^{mmm}_{(G,A)} = \{\langle[min_{x\in X}\{\underline{\mu}^{(x)}_{G(e_i)}\}, minx \in X\{\overline{\mu}^{(x)}_{G(e_i)}\}],$$
$$[min_{x\in X}\{\underline{\upsilon}^{(x)}_{G(e_i)}\}, min_{x\in X}\{\overline{\upsilon}^{(x)}_{G(e_i)}\}],$$
$$[min_{x\in X}\{\underline{\omega}^{(x)}_{G(e_i)}\}, min_{x\in X}\{\overline{\omega}^{(x)}_{G(e_i)}\}]\rangle/e_i; e_i \in E\}$$

By mmm-level decision rule we mean using the min-min-min threshold level soft set in IVNSS based decision making.

4.10 **The Proposed Two Stage IVNSS Algorithm for Traffic Signal control**

In this research the following algorithm is developed for controlling the flow of traffic using the proposed IVNSS decision making technique.

Steps of the Algorithm:

(i) Input the data set $X = \{SG_1, SG_2, SG_3, SG_4\}$ and the control parameters $E = \{e_1, e_2\}$, where SG_1, SG_2, SG_3 andSG_4 are the four signal groups or phases under consideration , e_1 = Average arrival rate on lanes with current green light in veh/min/lane, e_2 =maximum queue length on lane with red light,which may receive green signal in the next phase,in veh/lane.

(ii) Data Collection: Collect all the necessary crisp data for the input parameters from traffic sensors.

(iii) Neutrosophication/Normalization:Transform the crisp data into related IVNSS data.

(iv) Obtain the IVNSS (F, A) and (G, B) as explained in section 4.7.

(v) Combination of data sets: Use (F, A) and (G, B) to find (F, A) AND (G, B). As there are five levels in A and four levels in B ,we have $5\times4 = 20$ values of the form $e_{ij} = a_i \wedge q_j; i = 1, 2, 3, 4, 5 and j = 1, 2, 3, 4$.

Let the resultant IVNSS of $(F,A)AND(G,B) = (K,R)$.

(vi) Parameter reduction:Input a threshold interval valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{avg}_{(K,R)} or \langle\alpha,\beta,\gamma\rangle^{Mmm}_{(K,R)} or \langle\alpha,\beta,\gamma\rangle^{mmm}_{(K,R)}$.

Using avg-level decision rule (or Mmm-level decision rule or mmm-level decision rule) for making decision based on the resultant IVNSS (K,R).

(vii) Compute the avg-level soft set$((K,R);\langle(\alpha,\beta,\gamma)\rangle^{avg}_{(K,R)})(or\langle\alpha,\beta,\gamma\rangle^{mmm}_{(K,R)}$ or $\langle\alpha,\beta,\gamma\rangle^{mmm}_{(K,R)})$

(viii) Obtain the tabular form of the level soft set (K,R) calculated in step **(vii)**.

(ix) Compute the choice values (weight) c_i of SG_i for every i in X.

(x) Select k such that $c_k = max_{i\in X} C_i$. The optimal decision is to select signal group SG_k for the next phase.

(xi) Based on the choice value (weight) determine the extension time for each phase or signal group and extension time for total cycle length as explained in Section 4.5.

Note: If C_k attains maximum value for more than one signal group SG_k ,then any one such group can be chosen arbitrarily .Alternatively,when more than one optimal k exists ,we may go back to step **(vi)** and change the threshold (or decision rule) so that a unique optimal choice remains.

5. Verification of the Model

5.1 Traffic Control and Traffic Signal timing in Addis Ababa City.

Addis Ababa city road authority (AACRA)has installed traffic light signal controllers at many of the road junctions in the city to control and regulate traffic flow.At each of these junctions fixed time cycle traffic management system is employed to control traffic signals.These traffic control systems do not consider the congestion in lanes or need to extend or terminate green signal times due to congestion or no traffic.At times these signals are turned off and the phase change is controlled manually by traffic police officers who use predetermined sequential order to control traffic flow.Over 500 traffic police officers are deployed in the city every day.It is true that efficient and experienced traffic police officers can adjust the signal timings according to traffic intensity ,especially during peak hours.On the other hand ,the office that is responsible for traffic control in the city sets the length of each phase group in a cycle or the length of green light signal in a cycle to control traffic at the junction .Usually the maximum time given for green light in a signal group is 120 seconds with a maximum total time of 8 minutes for one full cycle. The minimum time given for green light in the city is 12 seconds.The maximum time is given for intersections that suffer from heavy traffic during the peak hours.

5.2 Site Selection and Description of the Verification Area

Addis Ababa, the capital city of Ethiopia has an estimated population of 5.5 millions and is located in the horn of Africa.The city is also the seat of African Union (AU),African Economic Commission (AEC) and more than 120 embassies of different countries.It is reported that about 80 percent of the vehicles in Ethiopia are found in Addis Ababa with an yearly growth rate of 5 percent.The total road length of the city is 1329.5 kilometers ,out of which 29.7 percent is paved and 70.3 percent unpaved.According to

AACRA ,the city currently has 67 km of road with in 100 sq.km,which is minimal when compared to Nairobi ,Kenya where 155 km of road is found with in 100 sq.km.The city experiences traffic congestion at different intersections throughout the day ,the average traffic congestion intensity in Addis Ababa city expressed in vehicle minute or person minute is very high and the result shows that on the average about 38 vehicle days and 352 person days are wasted at each intersection leg or congestion spot per day.As per the information obtained from Addis Ababa city traffic management directorate office ,about 30 traffic intersections are identified out of which 14 traffic junctions are awarded installation of technologically advanced signal lights with traffic detectors.Unless otherwise implemented with proper care this could be a major cause for congestion.It has been identified that signal phase improvement is one of the most useful and cost effective method to reduce congestion.

For verifying the proposed IVNSS traffic signal control model and to compare the performance of existing fixed cycle time traffic signal controller with the proposed model ,the main traffic network in the inner city of Addis Ababa ,namely St. Stifanos traffic junction is selected.This junction consists of four roads :Bambis (West),Betemengist (south),Meskel square (East) and Dembel(north).The geometry of the intersection is shown in figure below.Existing delays and traffic volumes are measured on typical working days.The data collected in the field are used to estimate the optimal signal plans.The optimized delay and signal timings are compared with the existing values to evaluate the performance of the proposed IVNSS traffic control model.

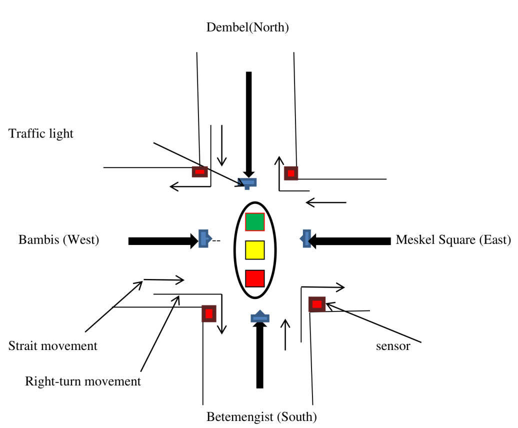

Fig.4. Geometry of the Intersection at St.Stifanos Junction

FIGURE 4. Geometry of the Intersection at St. Stifanos Junction

5.3 **Verification of the Model using the Sample Data from the Selected Traffic Junction**

The steps of the algorithm presented in Section 4.10 are applied to the sample data obtained from St. Stifanos junction .The decision making procedure of the Algorithm is verified with the data collected as follows:

Step **(i)** $X = \{SG_1, SG_2, SG_3, SG_4\}$ where SG_1, SG_2, SG_3 and SG_4 are the four signal groups explained in Section 4.3.The parameter set is $E = \{e_1, e_2\}$ as defined in Section 4.4

Step(ii) A sample data obtained from vehicular detector at St. Stifanos junction is tabulated below.

X	e_1	e_2
SG_1	61	52
SG_2	54	79
SG_3	64	60
SG_4	59	62

Table 3. Sample Data obtained From Vehicular Detector at St. Stifanos Junction

Step(iii) The evaluation scores obtained from experts with respect to all roads and all levels of the parameter e_1 are given below.

For the signal group SG_1 with respect to a_1 obtained from five experts

(SG_1, a_1).

Truth membership degree (μ)	5.8	7.9	8.3	8.7	5.1
Indeterminacy membership degree (υ)	3.3	1.4	1.7	2.1	4.4
falsity membership degree (ω)	4.1	2.4	2.6	2.1	3.7

Similarly the data collected from experts for other combinations are tabulated below.

(SG_1, a_2)

μ	9.1	6.3	7.1	6.9	8.1
υ	1.3	3.4	3.7	4.1	2.8
ω	1.9	3.9	3.6	3.1	2.3

(SG_1, a_3)

μ	4.1	3.4	2.8	3.1	4.3
υ	4.3	5.4	2.9	4.1	4.0
ω	4.1	5.9	4.6	5.1	3.9

(SG_1, a_4)

μ	2.7	2.1	1.9	2.0	3.0
υ	2.3	1.4	1.5	2.0	1.0
ω	6.1	6.9	6.5	7.1	5.5

(SG_1, a_5)

μ	2.8	1.9	2.3	1.7	3.1
υ	2.3	1.1	1.2	1.6	2.1
ω	7.3	8.9	9.5	9.1	7.7

(SG_2, a_1)

μ	2.8	4.9	1.3	4.5	2.1
υ	5.3	4.5	3.7	6.1	7.4
ω	7.8	8.9	8.6	9.1	7.7

(SG_2, a_2)

μ	3.5	3.9	2.3	4.7	4.1
υ	5.3	6.4	5.5	3.8	4.7
ω	4.5	5.8	6.6	7.1	4.5

(SG_2, a_3)

μ	4.8	8.9	5.3	8.5	5.1
υ	7.3	8.4	5.7	6.1	5.4
ω	3.1	2.5	2.1	1.4	3.7

(SG_2, a_4)

μ	8.8	7.5	6.5	8.5	5.5
υ	6.3	4.5	5.6	7.1	5.4
ω	1.3	2.3	5.1	8.1	1.4

(SG_2, a_5)

μ	4.5	3.9	8.5	3.7	4.6
υ	4.3	5.4	2.7	5.5	4.6
ω	5.1	6.5	1.6	4.7	3.9

(SG_3, a_1)

μ	6.8	8.7	5.3	6.2	4.1
υ	8.3	3.5	4.7	7.1	6.4
ω	6.1	1.9	6.6	2.8	4.7

(SG_3, a_2)

μ	7.8	6.6	6.3	8.2	5.8
υ	5.7	6.5	3.7	5.1	8.4
ω	2.1	3.9	4.6	2.0	4.2

(SG_3, a_3)

μ	5.8	4.7	9.3	7.2	4.1
υ	3.3	4.5	5.7	7.1	8.5
ω	5.1	2.9	1.6	3.8	4.7

(SG_3, a_4)

μ	5.5	2.7	5.4	3.2	6.2
υ	5.2	2.5	4.5	3.1	6.4
ω	6.1	8.7	6.4	7.3	3.7

(SG_3, a_5)

μ	3.6	2.7	5.3	2.2	4.1
υ	4.3	2.5	4.6	5.1	7.4
ω	7.3	8.8	5.4	8.0	4.7

(SG_4, a_1)	μ	4.8	6.2	5.3	6.2	4.3
	υ	4.6	7.5	1.7	8.2	5.7
	ω	6.3	2.9	7.6	2.8	4.5

(SG_4, a_2)	μ	6.4	9.7	3.3	8.4	5.4
	υ	8.1	3.2	4.3	7.6	6.2
	ω	3.1	1.6	8.6	2.1	4.5

(SG_4, a_3)	μ	8.5	7.6	6.3	7.4	5.4
	υ	2.1	7.2	8.2	5.6	2.2
	ω	1.1	3.5	4.7	5.6	1.5

(SG_4, a_4)	μ	6.1	5.7	8.3	2.7	5.8
	υ	8.1	7.5	4.3	7.3	6.4
	ω	5.2	7.6	1.6	8.5	4.7

(SG_4, a_5)	μ	5.5	4.9	8.3	3.4	5.3
	υ	2.5	1.9	7.4	3.3	5.2
	ω	6.3	8.5	4.7	8.4	5.5

Step(iv) Using the maximum-minimum neutrosophication/normalization technique the maximum-minimum evaluation scores for all the above combinations are calculated and tabulated below.

	scores	Max.	Min.
(SG_1, a_1)	μ	8.7	5.1
	υ	4.4	1.4
	ω	4.1	2.1

	scores	Max.	Min.
(SG_1, a_2)	μ	9.1	6.3
	υ	4.1	1.3
	ω	3.9	1.9

	scores	Max.	Min.
(SG_1, a_3)	μ	4.3	2.8
	υ	5.4	2.7
	ω	5.9	3.9

	scores	Max.	Min.
(SG_1, a_4)	μ	3.0	1.9
	υ	2.3	1.0
	ω	7.1	5.5

	scores	Max.	Min.
(SG_1, a_5)	μ	3.1	1.7
	υ	2.3	1.1
	ω	9.5	7.3

	scores	Max.	Min.
(SG_2, a_1)	μ	4.9	1.3
	υ	7.4	3.7
	ω	9.1	7.5

	scores	Max.	Min.
(SG_2, a_2)	μ	4.7	2.3
	υ	6.4	3.8
	ω	7.1	4.5

	scores	Max.	Min.
(SG_2, a_3)	μ	8.9	4.8
	υ	8.4	5.4
	ω	3.7	1.4

	scores	Max.	Min.
(SG_2, a_4)	μ	8.8	5.5
	υ	7.1	4.5
	ω	8.1	1.3

	scores	Max.	Min.
(SG_2, a_5)	μ	8.5	3.1
	υ	5.5	2.7
	ω	6.5	1.6

	scores	Max.	Min.
(SG_3, a_1)	μ	8.7	4.1
	υ	8.3	3.5
	ω	6.6	1.9

	scores	Max.	Min.
(SG_3, a_2)	μ	8.2	5.8
	υ	8.4	3.7
	ω	4.6	2.0

	scores	Max.	Min.
(SG_3, a_3)	μ	9.3	4.1
	υ	6.4	2.5
	ω	8.7	3.7

	scores	Max.	Min.
(SG_3, a_4)	μ	6.2	2.7
	υ	6.4	2.5
	ω	8.7	3.7

	scores	Max.	Min.
(SG_3, a_5)	μ	5.3	2.2
	υ	7.4	2.5
	ω	8.8	4.7

	scores	Max.	Min.
(SG_4, a_1)	μ	6.2	4.3
	υ	8.2	1.7
	ω	7.6	2.8

	scores	Max.	Min.
(SG_4, a_2)	μ	9.7	3.3
	υ	8.1	3.2
	ω	8.6	1.6

	scores	Max.	Min.
(SG_4, a_3)	μ	8.5	5.4
	υ	8.2	2.1
	ω	5.6	1.1

(SG_4, a_4)

scores	Max.	Min.
μ	8.3	2.7
υ	8.1	4.3
ω	8.5	1.6

(SG_4, a_5)

scores	Max.	Min.
μ	8.3	3.4
υ	7.4	1.9
ω	8.5	4.7

Similarly the data collected from experts for the parameter e_2 are presented below after maximum-minimum transformation.

(SG_1, q_1)

scores	Max.	Min.
μ	8.3	3.5
υ	8.1	2.6
ω	6.8	2.2

(SG_1, q_2)

scores	Max.	Min.
μ	8.6	4.4
υ	8.7	2.7
ω	8.6	1.5

(SG_1, q_3)

scores	Max.	Min.
μ	9.3	5.7
υ	8.2	3.7
ω	7.9	1.7

(SG_1, q_4)

scores	Max.	Min.
μ	8.7	3.6
υ	8.2	2.8
ω	9.1	1.4

(SG_2, q_1)

scores	Max.	Min.
μ	9.2	5.4
υ	7.4	2.8
ω	5.5	1.9

(SG_1, q_2)

scores	Max.	Min.
μ	9.1	5.7
υ	8.2	3.2
ω	7.1	2.6

(SG_2, q_3)

scores	Max.	Min.
μ	7.3	3.4
υ	7.5	2.8
ω	7.5	4.2

(SG_2, q_4)

scores	Max.	Min.
μ	5.7	1.5
υ	7.2	3.1
ω	8.1	4.5

(SG_3, q_1)

scores	Max.	Min.
μ	8.6	4.4
υ	8.2	2.2
ω	7.6	3.5

(SG_3, q_2)

scores	Max.	Min.
μ	8.3	5.4
υ	7.7	4.2
ω	7.6	1.6

(SG_3, q_3)

scores	Max.	Min.
μ	7.6	3.6
υ	7.6	2.6
ω	7.7	2.5

(SG_3, q_4)

scores	Max.	Min.
μ	7.6	2.6
υ	6.6	2.5
ω	8.4	4.2

(SG_4, q_1)

scores	Max.	Min.
μ	7.4	2.5
υ	7.8	3.1
ω	7.3	4.5

(SG_4, q_2)

scores	Max.	Min.
μ	8.6	3.7
υ	7.4	3.4
ω	6.4	2.5

(SG_4, q_3)

scores	Max.	Min.
μ	8.3	3.4
υ	7.4	3.2
ω	5.6	2.5

(SG_4, q_4)

scores	Max.	Min.
μ	8.7	2.6
υ	7.4	2.4
ω	7.5	1.7

Step (v) These maximum-minimum evaluation scores are converted into sub-intervals of $[0, 1]$and the tabular representation of (F, A) and (G, B) are given below.

X	a_1	a_2	a_3	a_4	a_5
SG_1	$[0.51, 0.87], [0.14, 0.44], [0.21, 0.41]$	$[0.63, 0.91], [0.13, 0.41], [0.19, 0.39]$	$[0.28, 0.43], [0.27, 0.54], [0.39, 0.59]$	$[0.19, 0.3], [0.1, 0.23], [0.55, 0.71]$	$[0.17, 0.31], [0.11, 0.23], [0.73, 0.95]$
SG_2	$[0.13, 0.49], [0.37, 0.74], [0.75, 0.91]$	$[0.23, 0.47], [0.38, 0.64][0.45, 0.71]$	$[0.48, 0.89], [0.54, 0.84], [0.14, 0.37]$	$[0.55, 0.88], [0.45, 0.71], [0.13, 0.81]$	$[0.37, 0.85], [0.27, 0.55], [0.16, 0.65]$
SG_3	$[0.41, 0.87], [0.35, 0.83], [0.19, 0.66]$	$[0.58, 0.82], [0.37, 0.84], [0.2, 0.46]$	$[0.41, 0.93], [0.33, 0.85], [0.16, 0.51]$	$[0.27, 0.62], [0.25, 0.64], [0.37, 0.87]$	$[0.22, 0.53], [0.25, 0.74], [0.47, 0.88]$
SG_4	$[0.43, 0.62], [0.17, 0.82], [0.28, 0.76]$	$[0.33, 0.97], [0.32, 0.81], [0.16, 0.86]$	$[0.54, 0.85], [0.21, 0.82], [0.11, 0.56]$	$[0.27, 0.83], [0.43, 0.81], [0.16, 0.85]$	$[0.34, 0.83], [0.19, 0.74], [0.47, 0.85]$

TABLE 4. Tabular Representation of (F,A)

Step(vi) Calculate (F, A)AND $(G, B) = (K, R)$ and present the tabular form of (K, R).

X	q_1	q_2	q_3	q_4
SG_1	$[0.35, 0.83], [0.26, 0.81], [0.22, 0.68]$	$[0.44, 0.86], [0.27, 0.87], [0.15, 0.86]$	$[0.57, 0.93], [0.37, 0.82], [0.17, 0.79]$	$[0.36, 0.87], [0.28, 0.82], [0.14, 0.91]$
SG_2	$[0.54, 0.92], [0.28, 0.78], [0.19, 0.55]$	$[0.57, 0.91], [0.32, 0.82], [0.26, 0.71]$	$[0.34, 0.73], [0.28, 0.75], [0.42, 0.75]$	$[0.15, 0.57], [0.31, 0.72], [0.45, 0.81]$
SG_3	$[0.44, 0.86], [0.22, 0.82], [0.35, 0.76]$	$[0.54, 0.83], [0.42, 0.77], [0.16, 0.76]$	$[0.36, 0.79], [0.26, 0.76], [0.25, 0.77]$	$[0.26, 0.76], [0.25, 0.66], [0.42, 0.84]$
SG_4	$[0.25, 0.74], [0.31, 0.78], [0.45, 0.73]$	$[0.37, 0.86], [0.34, 0.74], [0.25, 0.64]$	$[0.34, 0.83], [0.32, 0.74], [0.25, 0.56]$	$[0.26, 0.87], [0.24, 0.74], [0.17, 0.75]$

TABLE 5. Tabular Representation of (G,B)

X	e_{11}	e_{12}	e_{13}	e_{14}
SG_1	$[0.35, 0.83], [0.26, 0.80], [0.22, 0.68]$	$[0.44, 0.86], [0.27, 0.87], [0.21, 0.86]$	$[0.51, 0.87], [0.37, 0.82], [0.21, 0.79]$	$[0.36, 0.87], [0.28, 0.82], [0.21, 0.91]$
SG_2	$[0.13, 0.49], [0.37, 0.75], [0.75, 0.91]$	$[0.13, 0.49], [0.37, 0.82], [0.75, 0.91]$	$[0.13, 0.49], [0.37, 0.75], [0.75, 0.91]$	$[0.13, 0.49], [0.37, 0.74], [0.75, 0.91]$
SG_3	$[0.41, 0.86], [0.35, 0.83], [0.35, 0.76]$	$[0.41, 0.83], [0.42, 0.83], [0.19, 0.76]$	$[0.36, 0.79], [0.35, 0.83], [0.25, 0.77]$	$[0.26, 0.76], [0.35, 0.83], [0.42, 0.84]$
SG_4	$[0.25, 0.62], [0.31, 0.82], [0.45, 0.76]$	$[0.37, 0.62], [0.34, 0.82], [0.28, 0.76]$	$[0.34, 0.62], [0.32, 0.82], [0.28, 0.76]$	$[0.26, 0.62], [0.24, 0.82], [0.28, 0.76]$
X	e_{21}	e_{22}	e_{23}	e_{24}
SG_1	$[0.35, 0.83], [0.26, 0.80], [0.22, 0.68]$	$[0.44, 0.86], [0.27, 0.87], [0.19, 0.86]$	$[0.57, 0.91], [0.37, 0.82], [0.19, 0.79]$	$[0.36, 0.87], [0.28, 0.82], [0.19, 0.91]$
SG_2	$[0.23, 0.47], [0.38, 0.78], [0.45, 0.71]$	$[0.23, 0.47], [0.38, 0.82], [0.45, 0.71]$	$[0.23, 0.47], [0.38, 0.75], [0.45, 0.75]$	$[0.13, 0.49], [0.37, 0.74], [0.75, 0.91]$
SG_3	$[0.44, 0.82], [0.37, 0.84], [0.35, 0.76]$	$[0.54, 0.82], [0.42, 0.84], [0.20, 0.76]$	$[0.36, 0.79], [0.37, 0.84], [0.25, 0.77]$	$[0.26, 0.76], [0.35, 0.83], [0.42, 0.84]$
SG_4	$[0.25, 0.74], [0.32, 0.81], [0.45, 0.86]$	$[0.33, 0.86], [0.34, 0.81], [0.25, 0.86]$	$[0.33, 0.83], [0.32, 0.81], [0.25, 0.86]$	$[0.26, 0.62], [0.24, 0.82], [0.28, 0.76]$
X	e_{31}	e_{32}	e_{33}	e_{34}
SG_1	$[0.28, 0.43], [0.27, 0.81], [0.39, 0.68]$	$[0.28, 0.43], [0.27, 0.87], [0.39, 0.86]$	$[0.28, 0.43], [0.37, 0.82], [039, 0.79]$	$[0.28, 0.43], [0.28, 0.82], [0.39, 0.91]$
SG_2	$[0.48, 0.89], [0.54, 0.84], [0.19, 0.55]$	$[0.48, 0.89], [0.54, 0.84], [0.26, 0.71]$	$[0.34, 0.73], [0.54, 0.84], [0.42, 0.75]$	$[0.15, 0.57], [0.54, 0.84], [0.45, 0.81]$
SG_3	$[0.41, 0.86], [0.33, 0.85], [0.35, 0.76]$	$[0.41, 0.83], [0.42, 0.85], [0.16, 0.76]$	$[0.36, 0.79], [0.33, 0.85], [0.25, 0.77]$	$[0.26, 0.76], [0.33, 0.85], [0.42, 0.84]$
SG_4	$[0.25, 0.74], [0.31, 0.82], [0.45, 0.73]$	$[0.37, 0.85], [0.34, 0.82], [0.25, 0.64]$	$[0.34, 0.83], [0.32, 0.82], [0.25, 0.56]$	$[0.26, 0.85], [0.24, 0.82], [0.17, 0.75]$
X	e_{41}	e_{42}	e_{43}	e_{44}
SG_1	$[0.19, 0.3], [0.26, 0.81], [0.55, 0.71]$	$[0.19, 0.30], [0.27, 0.87], [0.55, 0.86]$	$[0.19, 0.30], [0.37, 0.82], [0.55, 0.79]$	$[0.19, 0.30], [0.28, 0.82], [0.55, 0.91]$
SG_2	$[0.54, 0.88], [0.45, 0.74], [0.19, 0.81]$	$[0.55, 0.88], [0.45, 0.82], [0.26, 0.81]$	$[0.34, 0.73], [0.45, 0.75], [0.42, 0.81]$	$[0.15, 0.57], [0.45, 0.72], [0.45, 0.81]$
SG_3	$[0.27, 0.62], [0.25, 0.82], [0.37, 0.87]$	$[0.27, 0.62], [0.42, 0.77], [0.37, 0.87]$	$[0.27, 0.62], [0.26, 0.76], [0.37, 0.87]$	$[0.26, 0.62], [0.25, 0.66], [0.42, 0.87]$
SG_4	$[0.25, 0.74], [0.43, 0.81], [0.45, 0.85]$	$[0.27, 0.83], [0.43, 0.81], [0.25, 0.85]$	$[0.27, 0.83], [0.43, 0.81], [0.25, 0.85]$	$[0.26, 0.83], [0.43, 0.81], [0.17, 0.85]$
X	e_{51}	e_{52}	e_{53}	e_{54}
SG_1	$[0.17, 0.31], [0.26, 0.81], [0.73, 0.95]$	$[0.17, 0.31], [0.27, 0.87], [0.73, 0.95]$	$[0.17, 0.31], [0.37, 0.82], [0.73, 0.95]$	$[0.17, 0.31], [0.28, 0.82], [0.73, 0.95]$
SG_2	$[0.37, 0.85], [0.28, 0.74], [0.19, 0.65]$	$[0.37, 0.85], [0.32, 0.82], [0.26, 0.71]$	$[0.34, 0.73], [0.28, 0.75], [0.42, 0.75]$	$[0.15, 0.57], [0.31, 0.72], [0.45, 0.81]$
SG_3	$[0.22, 0.53], [0.31, 0.78], [0.47, 0.88]$	$[0.22, 0.53], [0.42, 0.77], [0.47, 0.88]$	$[0.22, 0.53], [0.26, 0.76], [0.47, 0.88]$	$[0.22, 0.53], [0.25, 0.74], [0.47, 0.88]$
SG_4	$[0.25, 0.74], [0.31, 0.78], [0.47, 0.95]$	$[0.34, 0.83], [0.34, 0.74], [0.47, 0.95]$	$[0.34, 0.83], [0.32, 0.74], [0.47, 0.95]$	$[0.26, 0.83], [0.24, 0.74], [0.47, 0.85]$

TABLE 6. Tabular Representation of $(K, R) = (F, A) \wedge (G, B)$

Step (vii) Evaluate the threshold interval-valued neutrosophic set $\langle \alpha, \beta, \gamma \rangle^{avg}_{(K,R)}$ and tabulate it.

$$\langle \alpha, \beta, \gamma \rangle^{avg}_{(K,R)} = \{\langle [0.285, 0.7], [0.3225, 0.81] \rangle, [0.4425, 0.7775] \rangle / e_{11},$$
$$< [0.3375, 0.7] \rangle, [0.35, 0.835], [0.3575, 0.8225] > / e_{12},$$
$$< [0.335, 0.6925], [0.3525, 0.805], [0.3725, 0.8325] > / e_{13},$$
$$< [0.2525, 0.685], [0.31, 0.8025], [0.415, 0.855] > / e_{14},$$
$$< [0.3175, 0.715], [0.3325, 0.81], [0.3675, 0.7525] > / e_{21},$$
$$< [0.385, 0.7525], [0.3525, 0.835], [0.2725, 0.7975] > / e_{22},$$
$$< [0.3225, 0.75], [0.36, 0.805], [0.285, 0.7925] > / e_{23},$$
$$< [0.2525, 0.695], [0.31, 0.8025], [0.41, 0.88] > / e_{24},$$
$$< [0.405, 0.73], [0.3625, 0.83], [0.345, 0.68] > / e_{31},$$
$$< [0.385, 0.75], [0.3925, 0.845], [0.265, 0.7425] > / e_{32},$$
$$< [0.33, 0.695], [0.39, 0.8325], [0.3275, 0.7175] > / e_{33},$$
$$< [0.2375, 0.6525], [0.3475, 0.8325], [0.3575, 0.8275] > / e_{34},$$

$< [0.3125, 0.635], [0.3475, 0.795], [0.39, 0.81] > /e_{41},$

$< [0.32, 0.6575], [0.3925, 0.8175], [0.3575, 0.8475] > /e_{42},$

$< [0.2675, 0.62], [0.3775, 0.785], [0.345, 0.83] > /e_{43},$

$< [0.215, 0.58], [0.2525, 0.7525], [0.3975, 0.86] > /e_{44},$

$< [0.2525, 0.6075], [0.29, 0.7775], [0.465, 0.8575] > /e_{51},$

$< [0.275, 0.63], [0.3375, 0.8], [0.4825, 0.8725] > /e_{52},$

$< [0.2675, 0.6], [0.3075, 0.7675], [0.5225, 0.8825] > /e_{53},$

$< [0.2, 0.56], [0.27, 0.755], [0.53, 0.8725] > /e_{54}\}.$

Step(viii) Compute avg-level soft set$((K,R);\langle\alpha,\beta,\gamma\rangle^{avg}_{(K,R)})$

$((K,R);\langle\alpha,\beta,\gamma\rangle^{avg}_{(K,R)}) =$

$\{(e_{11}, \{SG_1\}), (e_{13}, \{SG_3\}), (e_{33}, \{SG_4\}), (e_{34}, \{SG_4\}), (e_{11}, \{SG_1\}), (e_{53}, \{SG_2\}),$

$(e_{54}, \{SG_3, SG_4\}\}$

The tabular form of the level soft set $((K,R);\langle\alpha,\beta,\gamma\rangle^{avg}_{(K,R)})$

U	SG_1	SG_2	SG_3	SG_4
e_{11}	1	0	0	0
e_{12}	0	0	0	0
e_{13}	0	0	1	0
e_{14}	0	0	0	0
e_{21}	0	0	0	0
e_{22}	0	0	0	0
e_{23}	0	0	0	0
e_{24}	0	0	0	0
e_{31}	0	0	0	0
e_{32}	0	0	0	0
e_{33}	0	0	0	1
e_{34}	0	0	0	1
e_{41}	0	0	0	0
e_{42}	0	0	0	0
e_{43}	0	0	0	0
e_{44}	0	0	0	0
e_{51}	1	0	0	0
e_{52}	0	0	0	0
e_{53}	0	1	0	0
e_{54}	0	0	1	1

Table 7. The Tabular Representation of The Level Soft Set (K, R)

Step(ix) Compute the choice values (weights) c_i for SG_i for $i = 1, 2, 3, 4$.

$c_1 = \sum_{SG_1} e_{ij} = 2, c_2 = \sum_{SG_2} e_{ij} = 1, c_3 = \sum_{SG_3} e_{ij} = 2, c_4 = \sum_{SG_4} e_{ij} = 3$

Step (x)$c_4 = max\{c_1, c_2, c_3, c_4\}$

Hence the optimal decision is to select SG_4 for extension of green signal time.

Step(xi) Based on the weights obtained in Step (ix) determine the extension time for each phase or signal group .

Total Green Time $(TGT) = (\sum C_n - MinW) * (\frac{MaxST - MinST}{MaxW - MinW}) + MinST$

$= (8-1) \times (\frac{120-12}{3-1}) + 12 = 390 seconds.$

where

- $\sum C_n$ is the total weight of signal groups $= 2 + 1 + 2 + 3 = 8$
- $MinW = 1$ and $MaxW = 3$ are the minimum and maximum values of weights respectively.
- $MinST = 12$ seconds and $MaxST = 120$ seconds,are minimum and maximum value of green time in a cycle.
- n is the group index $(n = 1, 2, 3, 4)$.

The green time of each signal group is

- Green Time of $SG_1 = \dfrac{C_1 * TGT}{\sum C_n} = \dfrac{2 \times 390}{8} = 97.5$ seconds
- Green Time of $SG_2 = \dfrac{C_2 * TGT}{\sum C_n} = \dfrac{1 \times 390}{8} = 48.75$ seconds
- Green Time of $SG_3 = \dfrac{C_3 * TGT}{\sum C_n} = \dfrac{2 \times 390}{8} = 97.5$ seconds
- Green Time of $SG_4 = \dfrac{C_4 * TGT}{\sum C_n} = \dfrac{3 \times 390}{8} = 146.25$ seconds.

6. A Comparative Analysis

In this section we make a comparative study of the proposed two stage traffic control model with the existing traffic control systems such as traditional traffic traffic control system ,fixed cycle or pre-timed traffic control models.Researches have established that traditional traffic control systems contribute to traffic congestion as they are one of the main reasons for congestion if not implemented properly.

The present day urban traffic control system uses fixed cycle time or pre-timed signal control.In this case the phase change occurs sequentially and the green times are fixed. The system neither takes into account the varying traffic intensity with respect to time nor the peak hour heavy traffic in to consideration.The present day traffic control systems also suffer from indeterminacy due to various factors like unawareness of the problem,inaccurate and imperfect data,poor forecasting and uncertainty in the constraints.Due to this delay or waiting time at traffic intersections increase and at time mounts even up to 5 or 6 full cycle times depending on the intensity of traffic.Some developed countries use fuzzy logic controllers to regulate traffic at intersections.Though fuzzy logic is capable of handling uncertainty and impreciseness in data ,it can not cope up with indeterminacy.This drawback can be overcome by the proposed two stage traffic control model as it takes into account indeterminacy and dynamically manages the traffic flow by extending/terminating green light timings and effecting phase changes as necessitated by the traffic intensity at that time,not necessarily sequentially as followed in present day traffic control techniques.

7. CONCLUSION

In this research we have developed an algorithm for traffic signal control based on interval-valued neutrosophic soft set data .This algorithm can control both activities namely phase change and green time duration dynamically taking into consideration of the current traffic intensity and queue of vehicles estimated lingustically using experts opinions and converting this data into interval valued neutrosophic soft sets. Based on the decision making technique developed ,the algorithm makes use of the existing traffic conditions together with its uncertainty and indeterminacy to control and facilitate smooth flow of traffic at four way signalized isolated intersections. The algorithm is verified with the sample data collected at St.Stifanos four way isolated junction in Addis Ababa city .As a future research direction we are developing a simulation technique to generate data and to test and validate the proposed traffic signal control model.

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