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Direct product of Neutrosophic INK-Algebras

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Abstract: In this paper, first we define the notion direct product of neutrosophic sets in INK-algebras, neutrosophic set, neutrosophic INK-ideals, neutrosophic closed INK-ideals and direct product of neutrosophic INK-ideals in INK-algebras. We prove some theorems which show that there is some relation between these notions. Finally, we define the INK-subalgebra of INK-algebra and then we give related theorem about the relationship between their Images and direct product of neutrosophic INK-ideals.

Keywords: INK-algebra; neutrosophic set; direct product of neutrosophic INK-subalgebra; direct product of neutrosophic INK-ideal.

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1. Introduction

In 1986, Atanassov Introduced the Intuitionistic fuzzy set and later intuitionistic fuzzy set was applied in BCI/BCK-algebra, Introduced by Imai and Iseki in the 1980s. Following this, various researchers published articles using the intuitionistic fuzzy set concept. In 2005, Smarandache invented the new notion of the neutrosophic set in 1998 and it is a common code from the intuitionistic fuzzy set [1-8] and [15-20]. This has been followed by a lot of researchers publishing various articles over the last few years. In [9], [10], [11], [13], [14] and [12] Kaviyarasu et. al published an article using the fuzzy concept set in INK-algebra and later in solve they neutrosophic set in INK-algebra. In this paper we have introduced a new code using two different neutrosophic sets called direct product of neutrosophic sets in INK-algebra. We are also examining the relationship between neutrosophic INK- subalgebra and neutrosophic INK-ideal and its conditions.

2. Preliminaries

Before we begin our study, we will give the definition and useful properties of INK-algebras.

Definition 2.1: An algebra $(X, *, 0)$ is called a INK-algebra if it satisfies the following conditions for any ą, ъ $\in X$.

i) $((ą * ъ) * (ą * c)) * (z * ъ) = 0$
ii) $((ą * c) * (ъ * c)) * (ą * ъ) = 0$
iii) $ą * 0 = ą$
iv) $ą * ъ = 0$ and $ъ * ą = 0$ $imply$ $ą = ъ$.

where "*" is a binary operation and the "0" is a constant of X.

Definition 2.2: A non-empty subset S of a INK-algebra (X, *, 0) is said to be a INK-subalgebra of X, if $ą * ъ \in S$, whenever $ą, ъ \in X$.

Definition 2.3: Let (X, *, 0) be a INK-algebra. A nonempty subset $ꞁ$ of X is called an ideal of X if it satisfies

i) $0 \in ꞁ$

ii) $ą * ъ \in ꞁ$ and $ъ \in ꞁ$ imply $ą \in ꞁ$ for all $ą, ъ \in X$.

Definition 2.4: Let I be a non-empty subset of a INK- algebra X. Then $ꞁ$ is called a INK-ideal of X, if

i) $0 \in ꞁ$

ii) $((c * ą) * (c * ъ)) \in ꞁ$ and $ъ \in ꞁ$ imply $ą \in ꞁ$ for all $ą, ъ, c \in X$.

Definition 2.5: A neutrosophic set ∧ in a nonempty set X is a structure of the form
$∧ = \{(X, ⋌^{T, ꞁ, ꜰ}(ą)) \mid ą \in X\}$, where $⋌^{T}: X \to [0,1]$ is a truth membership function $⋌^{ꞁ}: X \to [0,1]$ is a indeterminate membership function and $⋌^{ꜰ}: X \to [0,1]$ is a false membership function.

Definition 2.6: A neutrosophic set ∧ in X is called a neutrosophic INK-subalgebra of X if it satisfies the following conditions, for all $ą, ъ \in X$.

i) $⋌^{T}(ą * ъ) \geq min\{⋌^{T}(ą), ⋌^{T}(ъ)\}$

ii) $⋌^{ꞁ}(ą * ъ) \leq max\{⋌^{ꞁ}(ą), ⋌^{ꞁ}(ъ)\}$

iii) $⋌^{ꜰ}(ą * ъ) \geq min\{⋌^{ꜰ}(ą), ⋌^{ꜰ}(ъ)\}$

Definition 2.7: A neutrosophic set ∧ in X is called a neutrosophic ideal of X if it satisfies the following conditions, for all $ą, ъ \in X$.

i) $⋌^{T}(0) \geq ⋌^{T}(ą)$, $⋌^{ꞁ}(0) \leq ⋌^{ꞁ}(ą)$ and $⋌^{ꜰ}(0) \geq ⋌^{ꜰ}(ą)$

ii) $⋌^{T}(ą) \geq min\{⋌^{T}(ą * ъ), ⋌^{T}(ъ)\}$

iii) $⋌^{ꞁ}(ą) \leq max\{⋌^{ꞁ}(ą * ъ), ⋌^{ꞁ}(ъ)\}$

iv) $⋌^{ꜰ}(ą) \geq min\{⋌^{ꜰ}(ą * ъ), ⋌^{ꜰ}(ъ)\}$.

Definition 2.8: A neutrosophic set ∧ in X is called a neutrosophic INK-ideal of X if it satisfies the following conditions, for all $ą, ъ, z \in X$.

i) $⋌^{T}(0) \geq ⋌^{T}(ą)$, $⋌^{ꞁ}(0) \leq ⋌^{ꞁ}(ą)$ and $⋌^{ꜰ}(0) \geq ⋌^{ꜰ}(ą)$

ii) $⋌^{T}(ą) \geq min\{⋌^{T}((c * ą) * (c * ъ)), ⋌^{T}(ъ)\}$

iii) $⋌^{ꞁ}(ą) \leq max\{⋌^{ꞁ}((c * ą) * (c * ъ)), ⋌^{ꞁ}(ъ)\}$

iv) $⋌^{ꜰ}(ą) \geq min\{⋌^{ꜰ}((c * ą) * (c * ъ)), ⋌^{ꜰ}(ъ)\}$.

3. Direct product of Neutrosophic INK-subalgebra and INK-ideal

Definition 3.1: Let ⋎ and ⋏ are two neutrosophic sets in INK-algebras X_1 and X_2. The direct product of neutrosophic sets ⋎ and ⋏ is defined by $⋎ \times ⋏ = \langle ⋌^{T, ꞁ, ꜰ}{}_{(⋎ \times ⋏)} \rangle$ and defined by

i) $⋌^{T}{}_{(⋎ \times ⋏)}(ą, ъ) = min\{⋌^{T}{}_{⋎}(ą), ⋌^{T}{}_{⋏}(ъ)\}$

ii) $⋌^{ꞁ}{}_{(⋎ \times ⋏)}(ą, ъ) = max\{⋌^{ꞁ}{}_{⋎}(ą), ⋌^{ꞁ}{}_{⋏}(ъ)\}$

iii) $⋌^{ꜰ}{}_{(⋎ \times ⋏)}(ą, ъ) = min\{⋌^{ꜰ}{}_{⋎}(ą), ⋌^{ꜰ}{}_{⋏}(ъ)\}$

For all $ą, ъ \in X$.

Definition 3.2: A neutrosophic sets $⋎ \times ⋏ = \langle ⋌^{T, ꞁ, ꜰ}{}_{(⋎ \times ⋏)} \rangle$ of X_1 and X_2 is called direct product of neutrosophic INK-subalgebra of $X_1 \times X_2$, if

i) $⋌^{T}{}_{(⋎ \times ⋏)}((ą_1, ъ_1) * (ą_2, ъ_2)) \geq min\{⋌^{T}{}_{(⋎ \times ⋏)}(ą_1, ъ_1), ⋌^{T}{}_{(⋎ \times ⋏)}(ą_2, ъ_2)\}$

ii) $⋌^{ꞁ}{}_{(⋎ \times ⋏)}((ą_1, ъ_1) * (ą_2, ъ_2)) \leq max\{⋌^{ꞁ}{}_{(⋎ \times ⋏)}(ą_1, ъ_1), ⋌^{ꞁ}{}_{(⋎ \times ⋏)}(ą_2, ъ_2)\}$

iii) $\measuredangle^{F}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) \geq min\ \{\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1),\ \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$
for all $a, ъ \in X$.

Definition 3.3: A neutrosophic sets $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ of X_1 and X_2 is called direct product of neutrosophic INK-ideal of $X_1 \times X_2$, if

i) $\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(0,0) \geq \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a, ъ)$
ii) $\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(0,0) \leq \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a, ъ)$
iii) $\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(0,0) \geq \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a, ъ)$
iv) $\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1) \geq min\{\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(((a_3, ъ_3) * (a_1, ъ_1)) * ((a_3, ъ_3) * (a_2, ъ_2))),\ \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$
v) $\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1) \leq max\{\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(((a_3, ъ_3) * (a_1, ъ_1)) * ((a_3, ъ_3) * (a_2, ъ_2))),\ \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$
vi) $\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1) \geq min\ \{\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(((a_3, ъ_3) * (a_1, ъ_1)) * ((a_3, ъ_3) * (a_2, ъ_2))),\ \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$
for all $a,\ ъ \in X$.

Definition 3.4: A neutrosophic sets $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ of X_1 and X_2 is called direct product of neutrosophic closed INK-ideal of $X_1 \times X_2$, if it satisfies (Def 3.3 iv, v, vi) the following condition

i) $\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(0,0) * (a, ъ) \geq \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a, ъ)$
ii) $\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(0,0) * (a, ъ) \leq \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a, ъ)$
iii) $\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(0,0) * (a, ъ) \geq \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a, ъ)$, for all x, y $\in$ X.

Theorem 3.5: Let $\curlyvee$ and $\curlywedge$ be two neutrosophic INK-subalgebras of X_1 and X_2. Then $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ *i*s a neutrosophic INK-subalgebra of $X_1 \times X_2$.

Proof. For any $(a_1, ъ_1), (a_2, ъ_2) \in X_1 \times X_2$. Then

$$\begin{aligned}
\measuredangle^{T}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &= \measuredangle^{T}_{(\curlyvee \times \curlywedge)}((a_1 * a_2),\ (ъ_1 * ъ_2)) \\
&= \min\{\measuredangle^{T}_{\curlyvee}((a_1 * a_2),\ \measuredangle^{T}_{\curlywedge}(ъ_1 * ъ_2))\} \\
&\geq min\{min\{\measuredangle^{T}_{\curlyvee}(a_1),\ \measuredangle^{T}_{\curlyvee}(a_2)\},\ \min\{\measuredangle^{T}_{\curlywedge}(ъ_1),\ \measuredangle^{T}_{\curlywedge}(ъ_2)\}\} \\
&= min\{min\{\measuredangle^{T}_{\curlyvee}(a_1),\ \measuredangle^{T}_{\curlywedge}(ъ_1)\},\ \min\{\measuredangle^{T}_{\curlyvee}(a_2),\ \measuredangle^{T}_{\curlywedge}(ъ_2)\}\} \\
\measuredangle^{T}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &\geq min\ \{\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1),\ \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\},
\end{aligned}$$

$$\begin{aligned}
\measuredangle^{I}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &= \measuredangle^{I}_{(\curlyvee \times \curlywedge)}((a_1 * a_2),\ (ъ_1 * ъ_2)) \\
&= max\{\measuredangle^{I}_{\curlyvee}((a_1 * a_2),\ \measuredangle^{I}_{\curlywedge}(ъ_1 * ъ_2))\} \\
&\leq max\{maa\{\measuredangle^{I}_{\curlyvee}(a_1),\ \measuredangle^{I}_{\curlyvee}(a_2)\},\ max\{\measuredangle^{I}_{\curlywedge}(ъ_1),\ \measuredangle^{I}_{\curlywedge}(ъ_2)\}\} \\
&= max\{maa\{\measuredangle^{I}_{\curlyvee}(a_1),\ \measuredangle^{I}_{\curlywedge}(ъ_1)\},\ max\{\measuredangle^{I}_{\curlyvee}(a_2),\ \measuredangle^{I}_{\curlywedge}(ъ_2)\}\} \\
\measuredangle^{I}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &\leq max\ \{\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1),\ \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}
\end{aligned}$$

And

$$\begin{aligned}
\measuredangle^{F}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &= \measuredangle^{F}_{(\curlyvee \times \curlywedge)}((a_1 * a_2),\ (ъ_1 * ъ_2)) \\
&= min\{\measuredangle^{F}_{\curlyvee}((a_1 * a_2),\ \measuredangle^{F}_{\curlywedge}(ъ_1 * ъ_2))\} \\
&\geq min\{min\{\measuredangle^{F}_{\curlyvee}(a_1),\ \measuredangle^{F}_{\curlyvee}(a_2)\},\ min\{\measuredangle^{F}_{\curlywedge}(ъ_1),\ \measuredangle^{F}_{\curlywedge}(ъ_2)\}\} \\
&= min\{min\{\measuredangle^{F}_{\curlyvee}(a_1),\ \measuredangle^{F}_{\curlywedge}(ъ_1)\},\ min\{\measuredangle^{F}_{\curlyvee}(a_2),\ \measuredangle^{F}_{\curlywedge}(ъ_2)\}\} \\
\measuredangle^{F}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) * (a_2, ъ_2)) &\geq min\ \{\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_1, ъ_1),\ \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}.
\end{aligned}$$

Hence, $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-subalgebra of $X_1 \times X_2$.

Theorem 3.6: Let $\curlyvee$ and $\curlywedge$ be two neutrosophic INK-ideals of X_1 and X_2. Then $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$

is a neutrosophic INK-ideal of $X_1 \times X_2$.

Proof. For any (a_1, a_2, a_3) and $(ъ_1, ъ_2, ъ_3) \in X_1 \times X_2$.

Then $\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(0,\ 0) = min\ \{\measuredangle^{T}_{\curlyvee}(0),\ \measuredangle^{T}_{\curlywedge}(0)\}$

$\geq min\ \{\measuredangle^{T}_{\curlyvee}(a),\ \measuredangle^{T}_{\curlywedge}(ъ)\}$

$= \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a,\ ъ)$,

$\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(0,\ 0) = max\ \{\measuredangle^{I}_{\curlyvee}(0),\ \measuredangle^{I}_{\curlywedge}(0)\}$

$\leq max\ \{\measuredangle^{I}_{\curlyvee}(a),\ \measuredangle^{I}_{\curlywedge}(ъ)\}$

$= \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a,\ ъ)$,

And

$\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(0,\ 0) = min\ \{\measuredangle^{F}_{\curlyvee}(0),\ \measuredangle^{F}_{\curlywedge}(0)\}$

$\geq min\ \{\measuredangle^{F}_{\curlyvee}(a),\ \measuredangle^{F}_{\curlywedge}(ъ)\}$

$= \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a,\ ъ)$.

Now (a_1, a_2, a_3) and $(ъ_1, ъ_2, ъ_3) \in X_1 \times X_2$.

$\measuredangle^{T}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) = min\{\measuredangle^{T}_{\curlyvee}(a_1),\ \measuredangle^{T}_{\curlywedge}(ъ_1)\}$

$\geq min\{min\{\measuredangle^{T}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{T}_{\curlyvee}(a_2)\},\ min\{\measuredangle^{T}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2)),\ \measuredangle^{T}_{\curlywedge}(ъ_2)\}\}$

$= min\{min\{\measuredangle^{T}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{T}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2))\},\ min\ \{\measuredangle^{T}_{\curlyvee}(a_2),\ \measuredangle^{T}_{\curlywedge}(ъ_2)\}\}$

$= min\ \{\measuredangle^{T}_{(\curlyvee \times \curlywedge)}\ ((a_3, ъ_3) * ((a_1, ъ_1)),\ ((a_3, ъ_3) * ((a_2, ъ_2)),\ \measuredangle^{T}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$,

$\measuredangle^{I}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) = max\{\measuredangle^{I}_{\curlyvee}(a_1),\ \measuredangle^{I}_{\curlywedge}(ъ_1)\}$

$\leq max\{max\{\measuredangle^{I}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{I}_{\curlyvee}(a_2)\},\ max\ \{\measuredangle^{I}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2)),\ \measuredangle^{I}_{\curlywedge}(ъ_2)\}\}$

$= max\{max\{\measuredangle^{I}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{I}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2))\},\ max\{\measuredangle^{I}_{\curlyvee}(a_2),\ \measuredangle^{I}_{\curlywedge}(ъ_2)\}\}$

$= max\ \{\measuredangle^{I}_{(\curlyvee \times \curlywedge)}\ ((a_3, ъ_3) * ((a_1, ъ_1)),\ ((a_3, ъ_3) * ((a_2, ъ_2)),\ \measuredangle^{I}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$,

And

$\measuredangle^{F}_{(\curlyvee \times \curlywedge)}((a_1, ъ_1) = min\{\measuredangle^{F}_{\curlyvee}(a_1),\ \measuredangle^{F}_{\curlywedge}(ъ_1)\}$

$\geq min\{min\{\measuredangle^{F}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{F}_{\curlyvee}(a_2)\},\ min\{\measuredangle^{F}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2)),\ \measuredangle^{F}_{\curlywedge}(ъ_2)\}\}$

$= min\{min\{\measuredangle^{F}_{\curlyvee}((a_3 * a_1) * ((a_3 * a_2)),\ \measuredangle^{F}_{\curlywedge}((ъ_3 * ъ_1) * ((ъ_3 * ъ_2))\},\ min\{\measuredangle^{F}_{\curlyvee}(a_2),\ \measuredangle^{F}_{\curlywedge}(ъ_2)\}\}$

$= min\ \{\measuredangle^{F}_{(\curlyvee \times \curlywedge)}\ ((a_3, ъ_3) * ((a_1, ъ_1)),\ ((a_3, ъ_3) * ((a_2, ъ_2)),\ \measuredangle^{F}_{(\curlyvee \times \curlywedge)}(a_2, ъ_2)\}$.

Hence, $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Theorem 3.7: Let $\curlyvee$ and $\curlywedge$ be two neutrosophic closed INK-ideals of X_1 and X_2. Then $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ *is* a neutrosophic closed INK-ideal of $X_1 \times X_2$.

Proof. By using the theorem 3.6. $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Now for any $a, ъ \in X_1 \times X_2$, then

$\measuredangle^{T}_{(\curlyvee \times \curlywedge)}(0, 0) * (a, ъ) = \measuredangle^{T}_{(\curlyvee \times \curlywedge)}((0 * a), (0 * ъ))$

$= min\ \{\measuredangle^{T}_{\curlyvee}((0 * a),\ \measuredangle^{T}_{\curlywedge}(0 * ъ))\}$

$\geq min\ \{\measuredangle^{T}_{\curlyvee}(a),\ \measuredangle^{T}_{\curlywedge}(ъ))\}$,

$\measuredangle^{I}_{(\curlyvee \times \curlywedge)}(0, 0) * (a, ъ) = \measuredangle^{I}_{(\curlyvee \times \curlywedge)}((0 * a), (0 * ъ))$

$= max\ \{\measuredangle^{I}_{\curlyvee}((0 * a),\ \measuredangle^{I}_{\curlywedge}(0 * ъ))\}$

$\leq max\ \{\measuredangle^{I}_{\curlyvee}(a),\ \measuredangle^{I}_{\curlywedge}(ъ))\}$

And

$\measuredangle^{F}_{(\curlyvee \times \curlywedge)}(0, 0) * (a, ъ) = \measuredangle^{F}_{(\curlyvee \times \curlywedge)}((0 * a), (0 * ъ))$

$= min\ \{\measuredangle^{F}_{\curlyvee}((0 * a),\ \measuredangle^{F}_{\curlywedge}(0 * ъ))\}$

$\geq min\ \{\measuredangle^{F}_{\curlyvee}(a),\ \measuredangle^{F}_{\curlywedge}(ъ))\}$, Hence, $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic closed INK-ideal of $X_1 \times X_2$.

Theorem 3.8: Let $\curlyvee$ and $\curlywedge$ be two neutrosophic INK-ideals of X_1 and X_2. Then

$\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)},\ \overline{\measuredangle}^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Proof. Since by theorem 3.7, $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$. Then,

$$\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(0,0) \geq \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$1 - \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(0,0) \geq 1 - \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$\overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}(0,0) \leq \overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}),$$
$$\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(0,0) \leq \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$1 - \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(0,0) \leq 1 - \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$\overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}(0,0) \geq \overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$

And

$$\measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(0,0) \geq \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$1 - \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(0,0) \geq 1 - \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ})$$
$$\overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}(0,0) \leq \overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}).$$

Now $(a_1, a_2, a_3), (\text{ъ}_1, \text{ъ}_2, \text{ъ}_3) \in X_1 \times X_2$.

$$\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \geq \min\{\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$1 - \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \geq 1 - min\ \{\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \leq max\{1 - \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ 1 - \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \leq max\{\overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \overline{\measuredangle}^{T}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$

$$\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \leq max\{\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$1 - \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \leq 1 - max\{\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \geq min\{1 - \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ 1 - \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \geq min\{\overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \overline{\measuredangle}^{I}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$

$$\measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \geq \min\{\measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$1 - \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a_1, \text{ъ}_1) \geq 1 - min\ \{\measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \leq max\{1 - \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ 1 - \measuredangle^{F}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}$$
$$\overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}(a, \text{ъ}) \leq max\{\overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}((a_3, \text{ъ}_3) * ((a_1, \text{ъ}_1)) * ((a_3, \text{ъ}_3) * ((a_2, \text{ъ}_2)),\ \overline{\measuredangle}^{F}{}_{(\curlyvee \times \curlywedge)}(a_2, \text{ъ}_2)\}.$$

Hence, $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)},\ \overline{\measuredangle}^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Theorem 3.9: Let $\curlyvee \times \curlywedge = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$. Then

$(\curlyvee \times \curlywedge)^m = \langle \measuredangle^{T,\ I,\ F}{}_{(\curlyvee \times \curlywedge)^m} \rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Proof. For any $(a, \text{ъ}) \in X_1 \times X_2$.

Then $\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(0,\ 0) \geq \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a,\ \text{ъ})$

$$\{\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(0,\ 0)\}^m \geq \{\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a,\ \text{ъ})\}^m$$
$$\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(0,\ 0)^m \geq \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)}(a,\ \text{ъ})^m$$
$$\measuredangle^{T}{}_{(\curlyvee \times \curlywedge)^m}(0,\ 0) \geq \measuredangle^{T}{}_{(\curlyvee \times \curlywedge)^m}(a,\ \text{ъ})$$

$$\{\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(0,\ 0)\}^m \leq \{\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a,\ \text{ъ})\}^m$$
$$\measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(0,\ 0)^m \leq \measuredangle^{I}{}_{(\curlyvee \times \curlywedge)}(a,\ \text{ъ})^m$$

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)^m}(0,\ 0) \le \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)^m}(a,\ b)$

And

$\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,\ 0)\}^m \ge \{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a,\ b)\}^m$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,\ 0)^m \ge \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a,\ b)^m$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)^m}(0,\ 0) \ge \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)^m}(a,\ b)$.

Now $(a_1, a_2, a_3), (b_1, b_2, b_3) \in X_1 \times X_2$.

$\{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}((a_1, b_1)\}^m \ge min\ \{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}((a_3, b_3) * ((a_1, b_1)), ((a_3, b_3) * ((a_2, b_2)),\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)\}^m$

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a_1, b_1)^m \ge min\ \{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2)))^m,\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)^m\}$

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)^m}(a_1, b_1) \ge min\ \{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)^m}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2))),\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)^m}(a_2, b_2)\}$,

$\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}((a_1, b_1)\}^m \le\ max\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}((a_3, b_3) * ((a_1, b_1)), ((a_3, b_3) * ((a_2, b_2)),\ \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)\}^m$

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a_1, b_1)^m \le\ max\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2)))^m,\ \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)^m\}$

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)^m}(a_1, b_1) \le\ max\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)^m}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2))),\ \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)^m}(a_2, b_2)\}$

And

$\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}((a_1, b_1)\}^m \ge min\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}((a_3, b_3) * ((a_1, b_1)), ((a_3, b_3) * ((a_2, b_2)),\ \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)\}^m$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a_1, b_1)^m \ge min\ \{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2)))^m,\ \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)^m\}$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)^m}(a_1, b_1) \ge min\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)^m}(((a_3, b_3) * (a_1, b_1)) * ((a_3, b_3) *\ (a_2, b_2))),\ \curlywedge^{m}{}_{(\curlyvee\times\curlywedge)^m}(a_2, b_2)\}$.

Hence, $(\curlyvee\times\curlywedge)^m\ =\ \langle\curlywedge^{T,\ I,\ F}{}_{(\curlyvee\times\curlywedge)^m}\rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Theorem 3.10: Let $\curlyvee\times\curlywedge = \langle\curlywedge^{T,\ I,\ F}{}_{(\curlyvee\times\curlywedge)}\rangle$ and $\Upsilon\ \times\ \Gamma\ =\ \langle\curlywedge^{T,\ I,\ F}{}_{(\Upsilon\times\Gamma)}\rangle$ is a neutrosophic INK-ideal of X_1 *and* X_2. Then $(\curlyvee\times\curlywedge) \cap (\Upsilon\ \times\ \Gamma)\ =\ \langle\curlywedge^{T,\ I,\ F}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}\rangle$ is a neutrosophic INK-ideal of $X_1 \times X_2$.

Proof. For any $(a, b) \in X_1 \times X_2$.

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \ge \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a,\ b)$ *and* $\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \ge \curlywedge^{T}{}_{(\Upsilon\times\Gamma)}(a,\ b)$

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,\ 0),\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \ge \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a,\ b),\ \curlywedge^{T}{}_{(\Upsilon\times\Gamma)}(a,\ b)$

$min\{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,0),\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(0,0)\}\ \ge min\{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a,b), \curlywedge^{T}{}_{(\Upsilon\times\Gamma)}(a,b)\}$

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(0,0) \ge \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(a,b)$,

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \le \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a,\ b)$ *and* $\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \le \curlywedge^{I}{}_{(\Upsilon\times\Gamma)}(a,\ b)$

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,\ 0),\ \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \le \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a,\ b),\ \curlywedge^{I}{}_{(\Upsilon\times\Gamma)}(a,\ b)$

$maa\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,0),\ \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(0,0)\}\ \le maa\{\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)}(a,b),\ \curlywedge^{I}{}_{(\Upsilon\times\Gamma)}(a,b)\}$

$\curlywedge^{I}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(0,0) \le \curlywedge^{I}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(a,b)$,

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,0) \ge \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a,b)$ *and* $\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,0) \ge \curlywedge^{F}{}_{(\Upsilon\times\Gamma)}(a,b)$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,\ 0),\ \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,\ 0) \ge \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a,\ b),\ \curlywedge^{F}{}_{(\Upsilon\times\Gamma)}(a,\ b)$

$min\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,0),\ \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(0,0)\}\ \ge min\{\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)}(a,b),\ \curlywedge^{F}{}_{(\Upsilon\times\Gamma)}(a,b)\}$

$\curlywedge^{F}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(0,0) \ge \curlywedge^{F}{}_{(\curlyvee\times\curlywedge)\cap(\Upsilon\times\Gamma)}(a,b)$.

Now $(a_1, a_2, a_3), (b_1, b_2, b_3) \in X_1 \times X_2$.

$\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a_1, b_1) = min\ \{\curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}((a_3, b_3) * ((a_1, b_1)),\ ((a_3, b_3) * ((a_2, b_2)),\ \curlywedge^{T}{}_{(\curlyvee\times\curlywedge)}(a_2, b_2)\}$,

$\curlywedge^{T}{}_{(\Upsilon\times\Gamma)}(a_1, b_1) = min\ \{\curlywedge^{T}{}_{(\Upsilon\times\Gamma)}((a_3, b_3) * ((a_1, b_1)),\ ((a_3, b_3) * ((a_2, b_2)),\ \curlywedge^{T}{}_{(\Upsilon\times\Gamma)}(a_2, b_2)\}$.

$$\{\text{⋌}^{T}_{(\Upsilon\times\lambda)}((a_1,\text{ъ}_1),\ \text{⋌}^{T}_{(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)\}$$
$$\geq \begin{Bmatrix} \min\{\text{⋌}^{T}_{(\Upsilon\times\lambda)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{T}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2) \\ \min\{\text{⋌}^{T}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{T}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$

$$\geq \min \begin{Bmatrix} \min\{\text{⋌}^{T}_{(\Upsilon\times\lambda)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\ \text{⋌}^{T}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3) \\ *(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\min\{\ \text{⋌}^{T}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2),\ \text{⋌}^{T}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$

$$\text{⋌}^{T}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)$$
$$\geq \{\text{⋌}^{T}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\text{⋌}^{T}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}\ (a_2,\text{ъ}_2)\},$$

$$\text{⋌}^{I}_{(\Upsilon\times\lambda)}(a_1,\text{ъ}_1)=\max\ \{\text{⋌}^{I}_{(\Upsilon\times\lambda)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),\ ((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{I}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2)\},$$
$$\text{⋌}^{I}_{(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)=\max\ \{\text{⋌}^{I}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),\ ((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{I}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2)\}.$$
$$\{\text{⋌}^{I}_{(\Upsilon\times\lambda)}((a_1,\text{ъ}_1),\ \text{⋌}^{I}_{(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)\}$$
$$\leq \begin{Bmatrix} \max\{\text{⋌}^{I}_{(\Upsilon\times\lambda)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{I}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2) \\ \max\{\text{⋌}^{I}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{I}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$

$$\leq \max \begin{Bmatrix} \max\{\text{⋌}^{I}_{(\Upsilon\times\lambda)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\ \text{⋌}^{I}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3) \\ *(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\max\{\ \text{⋌}^{I}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2),\ \text{⋌}^{I}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$
$$\text{⋌}^{I}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)$$
$$\leq \{\text{⋌}^{I}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\text{⋌}^{I}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}\ (a_2,\text{ъ}_2)\}$$

And

$$\text{⋌}^{F}_{(\Upsilon\times\lambda)}(a_1,\text{ъ}_1)=\min\ \{\text{⋌}^{F}_{(\Upsilon\times\lambda)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),\ ((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2)\},$$
$$\text{⋌}^{F}_{(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)=\min\ \{\text{⋌}^{F}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),\ ((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2)\}.$$
$$\{\text{⋌}^{F}_{(\Upsilon\times\lambda)}((a_1,\text{ъ}_1),\ \text{⋌}^{F}_{(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)\}$$
$$\geq \begin{Bmatrix} \min\{\text{⋌}^{F}_{(\Upsilon\times\lambda)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2) \\ \min\{\text{⋌}^{F}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3)*((a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*((a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$

$$\geq \min \begin{Bmatrix} \min\{\text{⋌}^{F}_{(\Upsilon\times\lambda)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\Gamma)}\ ((a_3,\text{ъ}_3) \\ *(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\min\{\ \text{⋌}^{F}_{(\Upsilon\times\lambda)}(a_2,\text{ъ}_2),\ \text{⋌}^{F}_{(\Upsilon\times\Gamma)}(a_2,\text{ъ}_2) \end{Bmatrix}$$

$$\text{⋌}^{F}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}(a_1,\text{ъ}_1)$$
$$\geq \{\text{⋌}^{F}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}((a_3,\text{ъ}_3)*(a_1,\text{ъ}_1)),((a_3,\text{ъ}_3)*(a_2,\text{ъ}_2)),\ \text{⋌}^{F}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}\ (a_2,\text{ъ}_2)\}.$$

Hence, $(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)=\langle\text{⋌}^{T,\ I,\ F}_{(\Upsilon\times\lambda)\cap(\Upsilon\times\Gamma)}\rangle$ is a neutrosophic INK-ideal of $X_1\times X_2$.

4. Conclusion

In this paper we applied the notion of direct product of neutrosophic set to INK-ideal of INK-algebra. We hhave Introduced the direct product the concept of neutrosophic INK-algebra and a direct product of closed neutrosophic INK-ideal, and have Investigated several properties. We have provided conditions for a direct product of neutrosophic set to be a direct product of neutrosophic INK-ideal in INK-algebra.

References

1. Smarandache, F. Neutrosophic set-a generalization of the Intuitionistic fuzzy set. *International Journal of Pure Applied Mathematics,* **2005**, 24 (3), 287-297.

2. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **1986**, 20(1), 87-96.
3. Jun, Y.B. Neutrosophic subalgebras of several types in BCK/BCI-algebras. *Annals of Fuzzy Mathematics. Information*, **2017**, 14(1), 75–86.
4. Songsaeng, M.; Iampan, A. Neutrosophic Set Theory Applied to UP-Algebra. *European Journal of Pure and Applied Mathematics*. **2019**, 12(4), 1382-1409.
5. Jun, Y. B.; Smarandache, F.; Bordbar. H. Neutrosophic N-structures applied to BCK/BCI-algebras, *Information.* **2017**, 8(4),128.
6. Jun, Y. B.; Smarandache, F.; Song, S-Z.; Khan, M. Neutrosophic positive Implicative N-ideals in BCK-algebras. *Axioms,* **2018**, **7**(1), 3.
7. Zadeh, L. A. Fuzzy sets, *Information control*. **1965**, **8**, 338-353.
8. Ozturk, M. A.; Jun, Y. B. Neutrosophic Ideals in BCK/BCI-algebras based on neutrosophic points, *Journal of The International Mathematical Virtual institute*, **2018**, 8, 1–17.
9. Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Fuzzy p-ideal in INK-algebra, *Journal of XI'an University of Architecture & Technology,* **2018**, 12 (3), 4746-4752.
10. Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Intuitionistic fuzzy translation on INK-Algebra, *Advances in Mathematics: Scientific Journal*, **2020**, 9 (1), 295–303.
11. Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Direct product of intuitionistic fuzzy INK-ideals of INK-algebras, *Material Today: Proceedings,* **2019**, 16 (2), 449-455. doI.org/10.1016/j.matpr.2019.05.114.
12. Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Neutrosophic set in INK-Algebra, *Advances in Mathematics: Scientific Journal*, **2020**, 9 (7), 4345-4352.
13. Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Fuzzy subalgebras and fuzzy INK-ideals in INK-algebras, *International Journal of Pure and Applied Mathematics,* **2017**, 113(6), 47-55.
14. Kaviyarasu, M.; Indhira, K. Review on BCI/BCK-algebras and development, *International Journal of Pure and Applied Mathematics*, **2017,** 117(11), 377-383.
15. Smarandache, F. Neutro Algebra is a Generalization of Partial Algebra, *International Journal of Neutrosophic Science*, **2020**, 2(1), 08-17.
16. Smarandache, F. Introduction to Neutro Algebraic Structures and Anti Algebraic Structures (revisited), *Neutrosophic Sets and Systems*, **2020**, 31, 1-16.
17. Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. *Applied Sciences*, **2000**, 10(4), 1202.
18. Abdel-Basst, M., Mohamed, R., & Elhoseny, M. (2020). <? covid19?> A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans. *Health Informatics Journal*, 1460458220952918.
19. Abdel-Basset, M., Gamal, A., Chakrabortty, R. K., & Ryan, M. A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations: A case study. *Journal of Cleaner Production*, *280*, 124462.
20. Abdel-Basst, M., Mohamed, R., & Elhoseny, M. A novel framework to evaluate innovation value proposition for smart product-service systems. *Environmental Technology & Innovation*, 2020, 101036.
21. Remya. P. B and Francina Shalini. A, Neutrosophic Vague Binary BCK/BCI-algebra, *Neutrosophic Sets and Systems*, **2020**, 35, pp. 45-67. DOI: 10.5281/zenodo.3951639.
22. Muralikrishna. P and S. Manokaran. S, MBJ -Neutrosophic-Ideal of-Algebra, *Neutrosophic Sets and Systems*, 2020, **35**, pp. 99-118. DOI: 10.5281/zenodo.3951647.

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