Neutrosophic Sets and Systems

Volume 38 Article 13

11-23-2020

A Novel Approach: Neutro-Spot Topology and Its Supra Topology With Separation Axioms and Computing the Impact on COVID-19

Chinnadurai V

Sindhu M. P

Neutrosophic Sets and Systems,Vol.38,2020
University of New Mexico

A Novel Approach: Neutro-Spot Topology and Its Supra Topology With Separation Axioms and Computing the Impact on COVID-19

Chinnadurai V[1], Sindhu M P[2]

[1] Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu, India; kv.chinnaduraimaths@gmail.com
[2] Department of Mathematics, Karpagam College of Engineering, Coimbatore, Tamilnadu, India; sindhu.mp@kce.ac.in

Correspondence: kv.chinnaduraimaths@gmail.com

Abstract: Neutrosophic soft set is a parametric set of uncertainty, whereas the neutrosophic soft point is an exceptional type of it which used highly to explore the separation axioms. In this study, the impression of neutrosophic soft topological space is stretched to a new topology which contains neutrosophic soft points as its elements and named as neutro-spot topological space. One more topology is defined on the complement of neutrosophic soft points which satisfies the condition of supra topological space and named as neutro-supra spot topological space. Also, defined the notion of interior and closure, and are approached in a different way, along with the concept of subspace topology of such topological spaces. Some related properties have been proved and disproved with counterexamples. Moreover, the approach to separation axioms in such spaces has been presented with descriptive examples. The current epidemic situation discussed as a real life application in decision making problem to detect the major impact of COVID-19 and recover them quickly. The affected people investigated by the doctors according to their symptoms and other medical issues. The process of solving specified in the algorithm and the estimation formula stated for calculation. The appropriate treatment is provided for affected people as per the estimated value.

Keywords: Neutro-spot topological space; neutro-spot absolute interior; neutro-supra spot topological space; neutro-supra spot absolute closure; neutro-spot subspace topological space; neutro-supra spot subspace topological space; neutro-spot $T_{i=0,1,2.}$-spaces and neutro-supra spot $T_{i=0,1,2.}$-spaces; decision making problem on COVID-19.

1. Introduction

The values of three independent membership degrees such as truth, falsity, and indeterminacy, consigned to each element of a set which characterized to neutrosophic set (NS) as originated by (1998) Smarandache [16, 17], which is a simplification of a fuzzy set (FS) defined by (1965) Zadeh [36], and intuitionistic fuzzy set (IFS) created by (1986) Atanassov [34]. It turns out to be a valuable mathematical utensil to examine formless, faulty, unclear data. In recent years many researchers have further expanded and developed the theory and application of NSs [1, 10, 12-15]. Also, (2017) Smarandache [18] originated a new trend set called plithogenic set (PS) and others developed [4, 8, 11].

A soft set (SS) is a study of parameterization of vagueness introduced by (1999) Molodtsov [32]. Later (2013) Maji [30] pooled two-hybrid sets NS and SS, and urbanized its construction as neutrosophic soft sets (NSSs). This type of set is extended by different researchers [2, 7, 9] and its application in decision making (DM) problems [3]. Topology plays a vital role among all these types of sets such as fuzzy topology (FT), soft topology (ST), fuzzy soft topology (FST), neutrosophic topology (NT) [31], etc. Likewise, (2017) Bera & Mahapatra [26, 6] created the conception of neutrosophic soft topological spaces (NSTSs) and others [19, 20, 24]. Neutrosophic soft point (NSP) is a special type of NSS, which gives rise to the concept of separation axioms on NSTS [23, 25], and their application to DM was considered in [21, 22, 28]. Mashhour et al. [35] (1983) diminished the conditions of general topology, termed as supra topological spaces (SpTSs). The real-life application of SpTS is applied and defined on various sets such as FS [33], SS [29, 27], NS [5], and so on.

The major contribution of this work is to initiate a topology on NSPs, whose open sets defines the concept of the interior on it. The complement of NSP is defined and named as neutrosophic soft whole set. Such type of sets obeys the condition of supra topology, and so generated new types of supra topology, whose open sets defines the concept of closure on it. Some essential definitions and remarkable properties are studied with appropriate examples. On the other hand, the impact of separation axioms is examined in both the topologies with suitable examples. The recent wide-ranging problem extended as a major application to provide appropriate treatment for COVID-19 patients. Those people are under investigation as stated by its symptoms and other medical issues. Also, provided the solving procedure and formulae for calculation. The main aim of this DM problem to recover them speedily via proper treatment.

This study is prearranged as follows. Some important definitions related to the study are presented in part 2. Part 3 introduces the definition of neutro-spot topology, its interior, its subspace topology with fundamental properties, and related examples. Part 4 introduces the definition of neutro-supra spot topology, its closure, its subspace topology with fundamental properties, and related examples. Part 5 extends this study to separation axioms on both the topologies with explanatory examples. Part 6 solves the DM problem to detect the impact of COVID-19. The algorithm and formulae are presented to find the final result and provided proper treatment for them. At last, concluded with few ideas for upcoming work in part 7.

2. Preliminaries

In this part, some essential definitions connected to this work are pointed.

Definition 2.1 [26] Let *V* be an initial universe set, *E* be a set of parameters, and *P*, *Q* is any two NSSs over (*V*, *E*). Then a NSS *P* over *V* is a set defined by a set-valued function $f\left({}^{(E)}P\right)$ representing a mapping $f\left({}^{(E)}P\right): E \to NS(V)$ where $f\left({}^{(E)}P\right)$ is called the approximate function of the NSS *P* and *NS*(*V*) is a family of NS over *V*.

$$P = f\left({}^{(E)}P\right) = \left\{\left(e, < v, T_{f\left({}^{(e)}P\right)}(v), I_{f\left({}^{(e)}P\right)}(v), F_{f\left({}^{(e)}P\right)}(v)) >: v \in V\right): e \in E\right\}.$$

Definition 2.2 [24] A NSS *P* over (*V*, *E*) is said to be null NSS if $T_{f\left({}^{(e)}P\right)}(v) = 0$, $I_{f\left({}^{(e)}P\right)}(v) = 0$, $F_{f\left({}^{(e)}P\right)}(v) = 1$, $\forall e \in E$, $\forall v \in V$. It is denoted by ϕ_t.

A NSS *P* over (*V*, *E*) is said to be absolute NSS if $T_{f\left({}^{(e)}P\right)}(v) = 1$, $I_{f\left({}^{(e)}P\right)}(v) = 1$, $F_{f\left({}^{(e)}P\right)}(v) = 0$, $\forall e \in E$, $\forall v \in V$. It is denoted by 1_t.

Clearly, $(\phi_t)^c = 1_t$ and $(1_t)^c = \phi_t$.

Definition 2.3 [25] Let *V* be a universe and *E* be a set of parameters. Let *K* be a NSS over (*V*, *E*). Let *e* be an element of *E* and let

$$K(e') = f\left({}^{(E)}K\right)(e') = \begin{cases} (\alpha,\beta,\gamma), & if\ e' = e \\ (0,0,1), & if\ e' \neq e \end{cases} \quad \text{for all } e' \in E .$$

Then $K(e')$ is called NSP over (*V*, *E*) and $K = \bigcup_{e' \in E} K(e')$. That is, a NSS is the union of its NSPs.

3. Neutro-Spot Topology

This part defines the neutro-spot topology, neutro-spot absolute interior, and its subspace topology with some properties and examples.

Definition 3.1 Let *V* be a universe and *E* be a set of parameters. Let *NSP*(*V*, *E*) be the family of all NSPs over *V*. Then $\tau_t \subset NSP(V, E)$ is said to be neutro-spot topology (NSPT) over (*V*, *E*) if it satisfies the following conditions

(i) $\phi_t, 1_t \in \tau_t$.

(ii)the finite intersection of NSPs in τ_t belongs to τ_t.

The trio (*V*, τ_t, *E*) is said to be neutro-spot topological space (NSPTS) over (*V*, *E*). Elements in τ_t are called neutro-spot open sets (NSPOSs).

Example 3.2 A survey is taken based on the characteristics of houses owned by the people living in a slum area. Let $V = \{p_1, p_2, p_3\}$ be the set of sample people living in different areas in the slum and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters of houses, where e_1 = neat, e_2 = beautiful, e_3 = compact, and e_4 = large. Let $\tau_t = \{\phi_t, 1_t, K_1, K_2, K_3\}$. According to the survey, the results have proceeded in the form of NSPs K_1, K_2, K_3 over *V* as follows:

$$K_1 = \begin{cases} f\left({}^{(e_1)}K_1\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_2)}K_1\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_3)}K_1\right) = \{< p_1,(0,0,1) >, < p_2,(.3,.4,.2), < p_3,(0,0,1) >\} \\ f\left({}^{(e_4)}K_1\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \end{cases};$$

$$K_2 = \begin{cases} f\left({}^{(e_1)}K_2\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_2)}K_2\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_3)}K_2\right) = \{< p_1,(.5,.2,.6) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_4)}K_2\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \end{cases} \text{ and}$$

$$K_3 = \begin{cases} f\left({}^{(e_1)}K_3\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_2)}K_3\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \\ f\left({}^{(e_3)}K_3\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(.1,.2,.3) >\} \\ f\left({}^{(e_4)}K_3\right) = \{< p_1,(0,0,1) >, < p_2,(0,0,1), < p_3,(0,0,1) >\} \end{cases}.$$

Here $\phi_t \cap K_1 = \phi_t$, $\phi_t \cap K_2 = \phi_t$, $\phi_t \cap K_3 = \phi_t$, $1_t \cap K_1 = K_1$, $1_t \cap K_2 = K_2$, $1_t \cap K_3 = K_3$, $K_1 \cap K_2 = \phi_t$, $K_1 \cap K_3 = \phi_t$, $K_2 \cap K_3 = \phi_t$.

Then K_1, K_2 and K_3 are NSPOSs.

Thus (*V*, τ_t, *E*) is a NSPTS over (*V*, *E*).

Proposition 3.3 Let (*V*, τ_{t1}, *E*) and (*V*, τ_{t2}, *E*) be two NSPTSs over (*V*, *E*). Then (*V*, $\tau_{t1} \cap \tau_{t2}$, *E*) is also a NSPTS over (*V*, *E*).

Proof. Let (*V*, τ_{t1}, *E*) and (*V*, τ_{t2}, *E*) be two NSPTSs over (*V*, *E*).

(i)Obviously, $\phi_t, 1_t \in \tau_{t1} \cap \tau_{t2}$.

(ii)Let $K_1, K_2 \in \tau_{t1} \cap \tau_{t2}$.

Then $K_1, K_2 \in \tau_{t1}$ and $K_1, K_2 \in \tau_{t2}$.

$\Rightarrow$ $K_1 \cap K_2 \in \tau_{t1}$ and $K_1 \cap K_2 \in \tau_{t2}$.

$\Rightarrow$ $K_1 \cap K_2 \in \tau_{t1} \cap \tau_{t2}$.

Thus (V, $\tau_{t1} \cap \tau_{t2}$, E) is a NSPTS over (V, E).

Remark 3.4 Let (V, τ_{t1}, E) and (V, τ_{t2}, E) be two NSPTSs over (V, E). Then (V, $\tau_{t1} \cup \tau_{t2}$, E) is not NSPTS over (V, E).

Example 3.5 Let $V = \{g_1, g_2, g_3\}$, $E = \{e_1, e_2, e_3, e_4\}$. Let $\tau_{t1} = \{\phi_t, 1_t, K\}$ and $\tau_{t2} = \{\phi_t, 1_t, L\}$ where the NSPs K and L over V are defined as

$$K = \begin{cases} f\left({}^{(e_1)}K\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \\ f\left({}^{(e_2)}K\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \\ f\left({}^{(e_3)}K\right) = \{< g_1, (0,0,1) >, < g_2, (.7,.6,.2), < g_3, (0,0,1) >\} \\ f\left({}^{(e_4)}K\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \end{cases} \quad \text{and}$$

$$L = \begin{cases} f\left({}^{(e_1)}L\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \\ f\left({}^{(e_2)}L\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \\ f\left({}^{(e_3)}L\right) = \{< g_1, (0,0,1) >, < g_2, (.3,.9,.5), < g_3, (0,0,1) >\} \\ f\left({}^{(e_4)}L\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1), < g_3, (0,0,1) >\} \end{cases} .$$

Then $\tau_{t1} \cup \tau_{t2} = \{\phi_t, 1_t, K, L\}$ is not an NSPTS over (V, E), since $K \cap L \notin \tau_{t1} \cup \tau_{t2}$.

Thus (V, $\tau_{t1} \cup \tau_{t2}$, E) is not NSPTS over (V, E).

Proposition 3.6 Let K and L be any two NSPs on NSPTS (V, τ_t, E) over (V, E). Then

(i) $(K \cup L)^c = K^c \cap L^c$.

(ii) $(K \cap L)^c = K^c \cup L^c$.

Proof. Straight forward.

Proposition 3.7 Let (V, τ_t, E) be a NSPTS over (V, E) and

$$\tau_t = \{K_i : K_i \in NSP(V,E)\} = \left\{\left\langle e, f\left({}^{(E)}K\right) : K_i \in NSP(V,E)\right\rangle\right\}$$

where $f\left({}^{(E)}K\right) = \left\{\left\langle v, T_{f\left({}^{(e)}K\right)}(v), I_{f\left({}^{(e)}K\right)}(v), F_{f\left({}^{(e)}K\right)}(v)\right\rangle : v \in V, e \in E\right\}$.

Define

$$\tau_{t1} = \left\{\left[T_{f\left({}^{(e)}K\right)}(V)\right]_{e \in E}\right\},$$

$$\tau_{t2} = \left\{\left[I_{f\left({}^{(e)}K\right)}(V)\right]_{e \in E}\right\} \text{ and}$$

$$\tau_{t3} = \left\{\left[F_{f\left({}^{(e)}K\right)}(V)\right]^c_{e \in E}\right\}.$$

Then τ_{t1}, τ_{t2} and τ_{t3} are FSTs on (V, E).

Proof. Let (V, τ_t, E) be a NSPTS over (V, E).

(i) Since $\phi_t, 1_t \in \tau_t$,

$\Rightarrow 0,1 \in \tau_{t1}, 0,1 \in \tau_{t2}, 1,0 \in \tau_{t3}$.

(ii)Let $K_1, K_2 \in \tau_t$. Then $K_1 \cap K_2 \in \tau_t$.

That is,

$K_1 \cap K_2 = \left\{ \left\langle \min \left[T_{f\left({}^{(e)}K_1\right)}(V), T_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E}, \min \left[I_{f\left({}^{(e)}K_1\right)}(V), I_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E}, \max \left[F_{f\left({}^{(e)}K_1\right)}(V), F_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\rangle \right\} \in \tau_t$.

Thus

$$\left\{ \min \left[T_{f\left({}^{(e)}K_1\right)}(V), T_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\} \in \tau_{t1},$$
$$\left\{ \min \left[I_{f\left({}^{(e)}K_1\right)}(V), I_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\} \in \tau_{t2} \text{ and}$$
$$\left\{ \min \left[F_{f\left({}^{(e)}K_1\right)}(V), F_{f\left({}^{(e)}K_2\right)}(V) \right]^c_{e\in E} \right\} \in \tau_{t3}.$$

(iii)Let $K_i \in \tau_t$, where $i \in \mathrm{I}$. Then $\bigcup_{i\in \mathrm{I}} K_i \in \tau_t$.

That is,

$$\bigcup_{i\in \mathrm{I}} K_i = \left\{ \left\langle \max \left[T_{f\left({}^{(e)}K_i\right)}(V) \right]_{e\in E}, \max \left[I_{f\left({}^{(e)}K_i\right)}(V) \right]_{e\in E}, \min \left[F_{f\left({}^{(e)}K_i\right)}(V) \right]_{e\in E} \right\rangle \right\}_{i\in \mathrm{I}} \in \tau_t.$$

Thus

$$\left\{ \max \left[T_{f\left({}^{(e)}K_i\right)}(V) \right]_{e\in E} \right\}_{i\in \mathrm{I}} \in \tau_{t1},$$
$$\left\{ \max \left[I_{f\left({}^{(e)}K_i\right)}(V) \right]_{e\in E} \right\}_{i\in \mathrm{I}} \in \tau_{t2} \text{ and}$$
$$\left\{ \max \left[F_{f\left({}^{(e)}K_i\right)}(V) \right]^c_{e\in E} \right\}_{i\in \mathrm{I}} \in \tau_{t3}.$$

Hence τ_{t1}, τ_{t2} and τ_{t3} are FSTs on (*V, E*).

Remark 3.8 The following example illustrates that the converse of Proposition 3.7 is not true.

Example 3.9 Let $V = \{g_1, g_2, g_3\}$, $E = \{e_1, e_2, e_3, e_4\}$. Let $\tau_t = \{\phi_t, 1_t, K_1, K_2\}$ where the NSPs K_1 and K_2 over *V* are defined as

$$K_1 = \begin{cases} f\left({}^{(e_1)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left({}^{(e_2)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left({}^{(e_3)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(.7,.6,.2), < g_3,(0,0,1) >\} \\ f\left({}^{(e_4)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \end{cases} \text{ and}$$

$$K_2 = \begin{cases} f\left({}^{(e_1)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left({}^{(e_2)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left({}^{(e_3)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(.3,.9,.5), < g_3,(0,0,1) >\} \\ f\left({}^{(e_4)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \end{cases}.$$

Then

$$\tau_{t1} = \left\{ \left[T_{f\left({}^{(e)}\phi_t\right)}(V), T_{f\left({}^{(e)}1_t\right)}(V), T_{f\left({}^{(e)}K_1\right)}(V), T_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\},$$
$$\tau_{t2} = \left\{ \left[I_{f\left({}^{(e)}\phi_t\right)}(V), I_{f\left({}^{(e)}1_t\right)}(V), I_{f\left({}^{(e)}K_1\right)}(V), I_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\} \text{ and}$$
$$\tau_{t3} = \left\{ \left[F_{f\left({}^{(e)}\phi_t\right)}(V), F_{f\left({}^{(e)}1_t\right)}(V), F_{f\left({}^{(e)}K_1\right)}(V), F_{f\left({}^{(e)}K_2\right)}(V) \right]_{e\in E} \right\}$$

are FSTs on (*V, E*), where

$$\begin{aligned} \tau_{t1} = \{ & \langle e_1,(0,0,0),(1,1,1),(0,0,0),(0,0,0)\rangle, \langle e_2,(0,0,0),(1,1,1),(0,0,0),(0,0,0)\rangle, \\ & \langle e_3,(0,0,0),(1,1,1),(0,.7,0),(0,.3,0)\rangle, \langle e_4,(0,0,0),(1,1,1),(0,0,0),(0,0,0)\rangle \} \end{aligned} \text{ and so on.}$$

Thus $\tau_t = \{\phi_t, 1_t, K_1, K_2\}$ is not NSPT over (*V, E*), since $K_1 \cap K_2 \notin \tau_t$.

Proposition 3.10 Let (V, τ_t, E) be a NSPTS over (*V, E*) and

$$\tau_t = \{K_i : K_i \in NSP(V,E)\} = \left\{ \left\langle e, f\left({}^{(E)}K\right) : K_i \in NSP(V,E) \right\rangle \right\}$$

where $f\left({}^{(E)}K\right)=\left\{\left\langle v, T_{f\left({}^{(e)}K\right)}(v), I_{f\left({}^{(e)}K\right)}(v), F_{f\left({}^{(e)}K\right)}(v)\right\rangle : v\in V, e\in E\right\}$.

Define

$\tau_{t1}=\left\{T_{f\left({}^{(e)}K\right)}(V)\right\}_{e\in E}$,

$\tau_{t2}=\left\{I_{f\left({}^{(e)}K\right)}(V)\right\}_{e\in E}$ and

$\tau_{t3}=\left\{F_{f\left({}^{(e)}K\right)}(V)\right\}_{e\in E}$ as FSTs on (*V*, *E*).

Then $\tau_{t1}\cup\tau_{t3}$ and $\tau_{t2}\cup\tau_{t3}$ are not FSTs on (*V*, *E*).

Proposition 3.10 is illustrated by the following example.

Example 3.11 Let $V=\{g_1,g_2,g_3\}$, $E=\{e_1,e_2,e_3,e_4\}$. Let $\tau_t=\{\phi_t,1_t,K_1,K_2,K_3\}$ where the NSPs K_1,K_2,K_3 over *V* are defined as

$$K_1=\begin{cases} f\left({}^{(e_1)}K_1\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_2)}K_1\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(.7,.6,.2)>\}\\ f\left({}^{(e_3)}K_1\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_4)}K_1\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\end{cases};$$

$$K_2=\begin{cases} f\left({}^{(e_1)}K_2\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_2)}K_2\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(.8,.7,.5)>\}\\ f\left({}^{(e_3)}K_2\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_4)}K_2\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\end{cases} \text{ and}$$

$$K_3=\begin{cases} f\left({}^{(e_1)}K_3\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_2)}K_3\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(.7,.6,.5)>\}\\ f\left({}^{(e_3)}K_3\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\\ f\left({}^{(e_4)}K_3\right)=\{<g_1,(0,0,1)>,<g_2,(0,0,1),<g_3,(0,0,1)>\}\end{cases}.$$

Here $\phi_t\cap K_1=\phi_t$, $\phi_t\cap K_2=\phi_t$, $\phi_t\cap K_3=\phi_t$, $1_t\cap K_1=K_1$, $1_t\cap K_2=K_2$, $1_t\cap K_3=K_3$, $K_1\cap K_2=K_3$, $K_1\cap K_3=K_3$, $K_2\cap K_3=K_3$.

Then K_1,K_2 and K_3 are NSPOSs.

Thus (*V*, τ_t, *E*) is a NSPTS over (*V*, *E*).

Then

$$\tau_{t1}=\{\langle e_1,(0,0,0),(1,1,1),(0,0,0),(0,0,0),(0,0,0)\rangle,\langle e_2,(0,0,0),(1,1,1),(0,0,.7),(0,0,.8),(0,0,.7)\rangle,\\ \langle e_3,(0,0,0),(1,1,1),(0,0,0),(0,0,0),(0,0,0)\rangle,\langle e_4,(0,0,0),(1,1,1),(0,0,0),(0,0,0),(0,0,0)\rangle\},$$

and so on, are FSTs on (*V*, *E*).

But, $\tau_{t1}\cup\tau_{t3}$ and $\tau_{t2}\cup\tau_{t3}$ are not FSTs on (*V*, *E*).

Proposition 3.12 Let (*V*, τ_t, *E*) be a NSPTS over (*V*, *E*). Then

${}^{e}\tau_{t1}=\left\{T_{f\left({}^{(e)}K\right)}(V):K\in\tau_t\right\}$,

${}^{e}\tau_{t2}=\left\{I_{f\left({}^{(e)}K\right)}(V):K\in\tau_t\right\}$ and

${}^{e}\tau_{t3}=\left\{F_{f\left({}^{(e)}K\right)}(V):K\in\tau_t\right\}$

for each $e\in E$, define FTs on (*V*, *E*).

Proof. Follows from Proposition 3.7.

Remark 3.13 The following example illustrates that the converse of Proposition 3.12 is not true.

Example 3.14 Consider Example 3.9.
Then

$$^{e_1}\tau_{t1} = \left\{T_{f\left({}^{(e_1)}\phi_t\right)}(V), T_{f\left({}^{(e_1)}1_t\right)}(V), T_{f\left({}^{(e_1)}K_1\right)}(V), T_{f\left({}^{(e_1)}K_2\right)}(V)\right\},$$

$$^{e_1}\tau_{t2} = \left\{I_{f\left({}^{(e_1)}\phi_t\right)}(V), I_{f\left({}^{(e_1)}1_t\right)}(V), I_{f\left({}^{(e_1)}K_1\right)}(V), I_{f\left({}^{(e_1)}K_2\right)}(V)\right\} \text{ and}$$

$$^{e_1}\tau_{t3} = \left\{F_{f\left({}^{(e_1)}\phi_t\right)}(V), F_{f\left({}^{(e_1)}1_t\right)}(V), F_{f\left({}^{(e_1)}K_1\right)}(V), F_{f\left({}^{(e_1)}K_2\right)}(V)\right\}$$

are FTs on *V*, where

$$^{e_1}\tau_{t1} = \{(0,0,0),(1,1,1),(0,0,0),(0,0,0)\} \text{ and so on.}$$

Consequently, $\left\{^{e_2}\tau_{t1}, {}^{e_2}\tau_{t2}, {}^{e_2}\tau_{t3}\right\}$ and $\left\{^{e_3}\tau_{t1}, {}^{e_3}\tau_{t2}, {}^{e_3}\tau_{t3}\right\}$ are fuzzy tritopologies on *V*.
Thus $\tau_t = \{\phi_t, 1_t, K_1, K_2\}$ is not NSPT over (*V*, *E*), since $K_1 \cap K_2 \notin \tau_t$.

Definition 3.15 Let (V, τ_t, E) be a NSPTS over (*V*, *E*) and $L \in NSP(V,E)$ be any NSP. Then the neutro-spot absolute interior of *L* is denoted by $\widetilde{L}_\circ$ and defined as
(i) $\widetilde{L}_\circ = \{K : K \in NSPOS \,\&\, K \subseteq L\}$ i.e., the union of all neutro-spot open subsets of *L*.
(ii)

$$\widetilde{L}_\circ = \left\{\left(\widetilde{T}_{\circ f\left({}^{(e)}L\right)}, \widetilde{I}_{\circ f\left({}^{(e)}L\right)}, \widetilde{F}_{\circ f\left({}^{(e)}L\right)}\right)_{e\in E}\right\} = \left\{\left(\max_i T_{f\left({}^{(e)}K_i\right)}, \max_i I_{f\left({}^{(e)}K_i\right)}, \min_i F_{f\left({}^{(e)}K_i\right)}\right)_{e\in E} : K_i \in \tau_t \,\&\, f\left({}^{e}K_i\right) \subseteq f\left({}^{e}L\right)\right\}.$$

Example 3.16 Let $V = \{g_1, g_2\}$, $E = \{e_1, e_2, e_3\}$. Let $\tau_t = \{\phi_t, 1_t, K_1, K_2, K_3\}$ where the NSPs K_1, K_2, K_3 over *V* are defined as

$$K_1 = \begin{cases} f\left({}^{(e_1)}K_1\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_2)}K_1\right) = \{< g_1,(.9,.4,.2)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_3)}K_1\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \end{cases};$$

$$K_2 = \begin{cases} f\left({}^{(e_1)}K_2\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_2)}K_2\right) = \{< g_1,(0,0,.1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_3)}K_2\right) = \{< g_1,(0,0,1)>, < g_2,(.6,.1,.8)\} \end{cases} \text{ and}$$

$$K_3 = \begin{cases} f\left({}^{(e_1)}K_3\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_2)}K_3\right) = \{< g_1,(.4,.3,.5)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_3)}K_3\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \end{cases}.$$

Here $\phi_t \cap K_1 = \phi_t$, $\phi_t \cap K_2 = \phi_t$, $\phi_t \cap K_3 = \phi_t$, $1_t \cap K_1 = K_1$, $1_t \cap K_2 = K_2$, $1_t \cap K_3 = K_3$, $K_1 \cap K_2 = \phi_t$, $K_1 \cap K_3 = K_3$, $K_2 \cap K_3 = \phi_t$.
Then K_1, K_2 and K_3 are NSPOSs.
Thus (V, τ_t, E) is a NSPTS over (*V*, *E*).
Let $L \in NSP(V,E)$ be any NSP defined as

$$L = \begin{cases} f\left({}^{(e_1)}L\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_2)}L\right) = \{< g_1,(.9,.5,.1)>, < g_2,(0,0,1)\} \\ f\left({}^{(e_3)}L\right) = \{< g_1,(0,0,1)>, < g_2,(0,0,1)\} \end{cases}.$$

Then $\phi_t, K_1, K_3 \subseteq L$.

Thus $\widetilde{L}_\circ = \phi_t \bigcup K_1 \bigcup K_3 = K_1$.

Also,

$f\left(^{(e_1)}K_1\right), f\left(^{(e_1)}K_2\right), f\left(^{(e_1)}K_3\right) \subseteq f\left(^{(e_1)}L\right)$,

$f\left(^{(e_2)}K_1\right), f\left(^{(e_2)}K_2\right), f\left(^{(e_2)}K_3\right) \subseteq f\left(^{(e_2)}L\right)$ and

$f\left(^{(e_3)}K_1\right), f\left(^{(e_3)}K_3\right) \subseteq f\left(^{(e_3)}L\right)$.

Then

$\left(\widetilde{T}_{\circ f\left(^{(e_1)}L\right)}, \widetilde{I}_{\circ f\left(^{(e_1)}L\right)}, \widetilde{F}_{\circ f\left(^{(e_1)}L\right)}\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1) >\}$,

$\left(\widetilde{T}_{\circ f\left(^{(e_2)}L\right)}, \widetilde{I}_{\circ f\left(^{(e_2)}L\right)}, \widetilde{F}_{\circ f\left(^{(e_2)}L\right)}\right) = \{< g_1, (.9,.4,.2) >, < g_2, (0,0,1) >\}$ and

$\left(\widetilde{T}_{\circ f\left(^{(e_3)}L\right)}, \widetilde{I}_{\circ f\left(^{(e_3)}L\right)}, \widetilde{F}_{\circ f\left(^{(e_3)}L\right)}\right) = \{< g_1, (0,0,1) >, < g_2, (0,0,1) >\}$.

Thus $\left\{\left(\widetilde{T}_{\circ f\left(^{(e)}L\right)}, \widetilde{I}_{\circ f\left(^{(e)}L\right)}, \widetilde{F}_{\circ f\left(^{(e)}L\right)}\right)_{e \in E}\right\} = K_1$.

Theorem 3.17 Let (V, τ_t, E) be a NSPTS over (V, E) and $L, S \in NSP(V, E)$ be any two NSPs. Then,

(i) $\widetilde{L}_\circ \subseteq L$ and $\widetilde{L}_\circ$ is the largest NSPOS.

(ii) $L \subseteq S \Rightarrow \widetilde{L}_\circ \subseteq \widetilde{S}_\circ$.

(iii) $\widetilde{L}_\circ$ is a NSPOS i.e., $\widetilde{L}_\circ \in \tau_t$.

(iv) L is a NSPOS iff $\widetilde{L}_\circ = L$.

(v) $\left(\widetilde{L}_\circ\right)_\circ = \widetilde{L}_\circ$.

(vi) $\left(\widetilde{\phi}_{t\circ}\right)_\circ = \widetilde{\phi}_{t\circ}$ and $\left(\widetilde{1}_{t\circ}\right)_\circ = \widetilde{1}_{t\circ}$.

(vii) $\left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ = \widetilde{L}_\circ \bigcap \widetilde{S}_\circ$.

(ix) $\widetilde{L}_\circ \bigcup \widetilde{S}_\circ \subseteq \left(\widetilde{L} \bigcup \widetilde{S}\right)_\circ$.

Proof. Let (V, τ_t, E) be a NSPTS over (V, E) and $L, S \in NSP(V, E)$ be any two NSPs.

(i) Follows from Definition 3.15.

(ii) Let $L \subseteq S$. Then $\widetilde{L}_\circ \subseteq L \subseteq S \Rightarrow \widetilde{L}_\circ \subseteq S$ and $\widetilde{S}_\circ \subseteq S$.

Since $\widetilde{S}_\circ$ is the largest NSPOS contained in S, hence $\widetilde{L}_\circ \subseteq \widetilde{S}_\circ$.

(iii) Follows from Definition 3.15.

(iv) Let L be a NSPOS. Then $\widetilde{L}_\circ$ is the largest NSPOS which contained in L is equal to L.

Hence $\widetilde{L}_\circ = L$.

Conversely, assume that $\widetilde{L}_\circ = L$.

By (iii), $\widetilde{L}_\circ \in \tau_t$.

Then L is a NSPOS.

(v) Let $\widetilde{L}_\circ = K$. Then $K \in \tau_t$ iff $\widetilde{K}_\circ = K$.

Thus $\left(\widetilde{L}_\circ\right)_\circ = \widetilde{L}_\circ$.

(vi) Since $\phi_t, 1_t \in \tau_t$ and by (iv), hence $\left(\widetilde{\phi}_{t\circ}\right)_\circ = \widetilde{\phi}_{t\circ}$ and $\left(\widetilde{1}_{t\circ}\right)_\circ = \widetilde{1}_{t\circ}$.

(vii) $L \bigcap S \subseteq L$ and $L \bigcap S \subseteq S$.

$\Rightarrow \left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ \subseteq \widetilde{L}_\circ$ and $\left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ \subseteq \widetilde{S}_\circ$.

$\Rightarrow \left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ \subseteq \widetilde{L}_\circ \bigcap \widetilde{S}_\circ$.

Also, $\widetilde{L}_\circ \subseteq L$ and $\widetilde{P}_\circ \subseteq P$.

Then $\widetilde{L}_\circ \bigcap \widetilde{S}_\circ \subseteq L \bigcap S$.

Since $\left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ \subseteq L \bigcap S$ and it is the largest NSPOS, then $\widetilde{L}_\circ \bigcap \widetilde{S}_\circ \subseteq \left(\widetilde{L} \bigcap \widetilde{S}\right)_\circ$.

Hence $\left(\widetilde{L} \cap \widetilde{S}\right)_{\circ} = \widetilde{L}_{\circ} \cap \widetilde{S}_{\circ}$.

(vii) $L \subseteq L \cup S$ and $S \subseteq L \cup S$.

$\Rightarrow \widetilde{L}_{\circ} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{\circ}$ and $\widetilde{S}_{\circ} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{\circ}$.

$\Rightarrow \widetilde{L}_{\circ} \cup \widetilde{S}_{\circ} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{\circ}$.

Definition 3.18 Let (V, τ_t , E) be a NSPTS over (V, E) where τ_t is a NSPT over (V, E) and $L \in NSP(V,E)$ be any NSP. Then the subspace topology on NSPTS is denoted by τ_t^L and defined as $\tau_t^L = \{L \cap K_i : K_i \in \tau_t\}$ and $\phi_t, 1_t \in \tau_t^L$. Thus (V, τ_t^L, E) is a neutro-spot subspace topological space (NSPSTS) of (V, τ_t , E), where τ_t^L is also a NSPT over (V, E).

Example 3.19 Let $V = \{g_1, g_2, g_3\}$, $E = \{e_1, e_2\}$. Let $\tau_t = \{\phi_t, 1_t, K_1, K_2, K_3\}$ where the NSPs K_1, K_2, K_3 over V are defined as

$$K_1 = \begin{cases} f\left(^{(e_1)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}K_1\right) = \{< g_1,(0,0,1) >, < g_2,(.1,.4,.5), < g_3,(0,0,1) >\} \end{cases} ;$$

$$K_2 = \begin{cases} f\left(^{(e_1)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}K_2\right) = \{< g_1,(0,0,1) >, < g_2,(.4,.7,.8), < g_3,(0,0,1) >\} \end{cases} \text{ and}$$

$$K_3 = \begin{cases} f\left(^{(e_1)}K_3\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}K_3\right) = \{< g_1,(0,0,1) >, < g_2,(.1,.4,.8), < g_3,(0,0,1) >\} \end{cases} .$$

Here $\phi_t \cap K_1 = \phi_t$, $\phi_t \cap K_2 = \phi_t$, $\phi_t \cap K_3 = \phi_t$, $1_t \cap K_1 = K_1$, $1_t \cap K_2 = K_2$, $1_t \cap K_3 = K_3$, $K_1 \cap K_2 = K_3$, $K_1 \cap K_3 = K_3$, $K_2 \cap K_3 = K_3$.

Then K_1, K_2 and K_3 are NSPOSs.

Thus (V, τ_t , E) is a NSPTS over (V, E).

Define $L_1 \in NSP(V,E)$ as

$$L_1 = \begin{cases} f\left(^{(e_1)}L_1\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}L_1\right) = \{< g_1,(0,0,1) >, < g_2,(.7,.3,.6), < g_3,(0,0,1) >\} \end{cases} .$$

Thus we denote $L_1 \cap \phi_t = \phi_t$, $L_1 \cap 1_t = L_1$, $L_1 \cap K_1 = L_2$, $L_1 \cap K_2 = L_3$, $L_1 \cap K_3 = L_4$.

Then the NSPs L_2, L_3, L_4 over V are defined as

$$L_2 = \begin{cases} f\left(^{(e_1)}L_2\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}L_2\right) = \{< g_1,(0,0,1) >, < g_2,(.1,.3,.6), < g_3,(0,0,1) >\} \end{cases} ;$$

$$L_3 = \begin{cases} f\left(^{(e_1)}L_3\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}L_3\right) = \{< g_1,(0,0,1) >, < g_2,(.4,.3,.8), < g_3,(0,0,1) >\} \end{cases} \text{ and}$$

$$L_4 = \begin{cases} f\left(^{(e_1)}L_4\right) = \{< g_1,(0,0,1) >, < g_2,(0,0,1), < g_3,(0,0,1) >\} \\ f\left(^{(e_2)}L_4\right) = \{< g_1,(0,0,1) >, < g_2,(.1,.3,.8), < g_3,(0,0,1) >\} \end{cases} .$$

Then $\tau_t^L = \{\phi_t, 1_t, L_1, L_2, L_3, L_4\}$ is a NSPST over (V, E).

Thus (V, τ_t^L, E) is a NSPSTS of (V, τ_t , E).

4. Neutro-Supra Spot Topology

This part defines the neutro-supra spot topology, neutro-supra spot absolute closure, and its subspace topology with some properties and examples.

Definition 4.1 Let *V* be a universe and *E* be a set of parameters. Then the complement of NSP is said to be neutrosophic soft whole set (NSWS) over (*V*, *E*).

Definition 4.2 Let *V* be a universe and *E* be a set of parameters. Let *NSWS*(*V*, *E*) be the family of all neutrosophic soft whole sets (NSWSs) over *V*. Then $(\tau_t)' \subset NSWS(V,E)$ is said to be neutro-supra spot topology (NSSPT) on *V* if it satisfies the following conditions

(i) $\phi_t, 1_t \in (\tau_t)'$.

(ii)the arbitrary union of NSWSs in $(\tau_t)'$ belongs to $(\tau_t)'$.

Then $(\tau_t)'$ is said to be a neutro-supra spot topology (NSSPT) over (*V*, *E*), and the trio (*V*, $(\tau_t)'$, *E*) is said to be a neutro-supra spot topological spaces (NSSPTSs) over (*V*, *E*). Elements in $(\tau_t)'$ are called neutro-supra spot open sets (NSSPOSs).

Example 4.3 Consider Example 3.2. Here $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c, (K_3)^c\}$.
Then NSWSs $(K_1)^c, (K_2)^c, (K_3)^c$ over *V* are defined as

$$(K_1)^c = \begin{cases} f\left((e_1)(K_1)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_2)(K_1)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_3)(K_1)^c\right) = \{< p_1,(1,1,0)>, < p_2,(.2,.6,.3), < p_3,(1,1,0)>\} \\ f\left((e_4)(K_1)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \end{cases};$$

$$(K_2)^c = \begin{cases} f\left((e_1)(K_2)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_2)(K_2)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_3)(K_2)^c\right) = \{< p_1,(.6,.8,.5)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_4)(K_2)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \end{cases} \text{ and}$$

$$(K_3)^c = \begin{cases} f\left((e_1)(K_3)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_2)(K_3)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \\ f\left((e_3)(K_3)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(.3,.8,.1)>\} \\ f\left((e_4)(K_3)^c\right) = \{< p_1,(1,1,0)>, < p_2,(1,1,0), < p_3,(1,1,0)>\} \end{cases}.$$

Here $\phi_t \cup (K_1)^c = (K_1)^c$, $\phi_t \cup (K_2)^c = (K_2)^c$, $\phi_t \cup (K_3)^c = (K_3)^c$, $1_t \cup (K_1)^c = 1_t$, $1_t \cup (K_2)^c = 1_t$, $1_t \cup (K_3)^c = 1_t$, $(K_1)^c \cup (K_2)^c = 1_t$, $(K_1)^c \cup (K_3)^c = 1_t$, $(K_2)^c \cup (K_3)^c = 1_t$.
Then $(K_1)^c, (K_2)^c$ and $(K_3)^c$ are NSSPOSs.
Thus (*V*, $(\tau_t)'$, *E*) is a NSSPTS over (*V*, *E*).

Proposition 4.4 Let (*V*, $(\tau_{t1})'$, *E*) and (*V*, $(\tau_{t2})'$, *E*) be two NSSPTSs over (*V*, *E*). Then (*V*, $(\tau_{t1})' \cap (\tau_{t2})'$, *E*) is also a NSSPTS over (*V*, *E*).
Proof. Let (*V*, $(\tau_{t1})'$, *E*) and (*V*, $(\tau_{t2})'$, *E*) be two NSSPTSs over (*V*, *E*).
(i)Obviously, $\phi_t, 1_t \in (\tau_{t1})' \cap (\tau_{t2})'$.

(ii)Let $\{K_i \in NSWS(V,E) : i \in \mathrm{I}\} \in (\tau_{t1})' \cap (\tau_{t2})'$.

Then $\{K_i\} \in (\tau_{t1})'$ and $\{K_i\} \in (\tau_{t2})'$.

$\Rightarrow \bigcup_{i\in \mathrm{I}} K_i \in (\tau_{t1})'$ and $\bigcup_{i\in \mathrm{I}} K_i \in (\tau_{t2})'$.

$\Rightarrow \bigcup_{i\in \mathrm{I}} K_i \in (\tau_{t1})' \cap (\tau_{t2})'$.

Thus (V, $(\tau_{t1})' \cap (\tau_{t2})'$, E) is a NSSPTS over (V, E).

Remark 4.5 Let (V, $(\tau_{t1})'$, E) and (V, $(\tau_{t2})'$, E) be two NSSPTSs over (V, E). Then (V, $(\tau_{t1})' \cup (\tau_{t2})'$, E) is not NSSPTS over (V, E).

Example 4.6 Consider Example 3.5. Here $(\tau_{t1})' = \{\phi_t, 1_t, K^c\}$ and $\tau_{t2} = \{\phi_t, 1_t, L^c\}$ where the NSWSs K^c and L^c over V are defined as

$$K^c = \begin{cases} f\left({}^{(e_1)}K^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_2)}K^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_3)}K^c\right) = \{< g_1,(1,1,0) >, < g_2,(.2,.4,.7), < g_3,(1,1,0) >\} \\ f\left({}^{(e_4)}K^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \end{cases} \quad \text{and}$$

$$L^c = \begin{cases} f\left({}^{(e_1)}L^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_2)}L^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_3)}L^c\right) = \{< g_1,(0,0,1) >, < g_2,(.5,.1,.3), < g_3,(0,0,1) >\} \\ f\left({}^{(e_4)}L^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \end{cases}.$$

Then $(\tau_{t1})' \cup (\tau_{t2})' = \{\phi_t, 1_t, K^c, L^c\}$ is not an NSSPTS over (V, E), since $K^c \cup L^c \notin (\tau_{t1})' \cup (\tau_{t2})'$.

Thus (V, $(\tau_{t1})' \cup (\tau_{t2})'$, E) is not NSSPTS over (V, E).

Proposition 4.7 Let K and L be any two NSWSs on NSSPTS (V, $(\tau_t)'$, E) over (V, E). Then

(i) $(K \cup L)^c = K^c \cap L^c$.

(ii) $(K \cap L)^c = K^c \cup L^c$.

Proof. Straight forward.

Proposition 4.8 Let (V, $(\tau_t)'$, E) be a NSSPTS over (V, E) and

$$(\tau_t)' = \{K_i : K_i \in NSWS(V,E)\} = \left\{\left\langle e, f\left({}^{(E)}K\right) : K_i \in NSWS(V,E)\right\rangle\right\}$$

where $f\left({}^{(E)}K\right) = \left\{\left\langle v, T_{f\left({}^{(e)}K\right)}(v), I_{f\left({}^{(e)}K\right)}(v), F_{f\left({}^{(e)}K\right)}(v)\right\rangle : v \in V, e \in E\right\}$.

Define

$(\tau_{t1})' = \left\{T_{f\left({}^{(e)}K\right)}(V)\Big|_{e\in E}\right\}$,

$(\tau_{t2})' = \left\{I_{f\left({}^{(e)}K\right)}(V)\Big|_{e\in E}\right\}$ and

$(\tau_{t3})' = \left\{F_{f\left({}^{(e)}K\right)}(V)\Big|^{c}_{e\in E}\right\}$.

Then $(\tau_{t1})', (\tau_{t2})'$ and $(\tau_{t3})'$ are FSTs on (V, E).

Proof. Let (V, $(\tau_t)'$, E) be a NSSPTS over (V, E).

(i) Since $\phi_t, 1_t \in (\tau_t)'$,

$\Rightarrow 0,1 \in (\tau_{t1})', 0,1 \in (\tau_{t2})', 1,0 \in (\tau_{t3})'$.

(ii)Let $\{K_i \in NSWS(V,E) : i \in \mathrm{I}\} \in (\tau_t)'$.

Then $\bigcup_{i\in I} K_i \in (\tau_t)'$.

That is,

$$\bigcup_{i\in I} K_i = \left\{ \left\langle \sup\left[T_{f\left({}^{(e)}K_i\right)}(V)\right]_{e\in E}, \sup\left[I_{f\left({}^{(e)}K_i\right)}(V)\right]_{e\in E}, \inf\left[F_{f\left({}^{(e)}K_i\right)}(V)\right]_{e\in E} \right\rangle \right\}_{i\in I} \in (\tau_t)'.$$

Thus

$$\left\{\sup\left[T_{f\left({}^{(e)}K_i\right)}(V)\right]_{e\in E}\right\}_{i\in I} \in (\tau_{t1})' ,$$

$$\left\{\sup\left[I_{f\left({}^{(e)}K_i\right)}(V)\right]_{e\in E}\right\}_{i\in I} \in (\tau_{t2})' \text{ and}$$

$$\left\{\sup\left[T_{f\left({}^{(e)}K_i\right)}(V)\right]^{c}_{e\in E}\right\}_{i\in I} \in (\tau_{t3})'.$$

(iii) Let $K_1, K_2 \in (\tau_t)'$.

Then $K_1 \cap K_2 \in (\tau_t)'$.

That is,

$$K_1 \cap K_2 = \left\{ \left\langle \min\left[T_{f\left({}^{(e)}K_1\right)}(V), T_{f\left({}^{(e)}K_2\right)}(V)\right]_{e\in E}, \min\left[I_{f\left({}^{(e)}K_1\right)}(V), I_{f\left({}^{(e)}K_2\right)}(V)\right]_{e\in E}, \max\left[F_{f\left({}^{(e)}K_1\right)}(V), F_{f\left({}^{(e)}K_2\right)}(V)\right]_{e\in E} \right\rangle \right\} \in (\tau_t)'.$$

Thus

$$\left\{\min\left[T_{f\left({}^{(e)}K_1\right)}(V), T_{f\left({}^{(e)}K_2\right)}(V)\right]_{e\in E}\right\} \in (\tau_{t1})',$$

$$\left\{\min\left[I_{f\left({}^{(e)}K_1\right)}(V), I_{f\left({}^{(e)}K_2\right)}(V)\right]_{e\in E}\right\} \in (\tau_{t2})' \text{ and}$$

$$\left\{\min\left[F_{f\left({}^{(e)}K_1\right)}(V), F_{f\left({}^{(e)}K_2\right)}(V)\right]^{c}_{e\in E}\right\} \in (\tau_{t3})' .$$

Hence $(\tau_{t1})'$, $(\tau_{t2})'$ and $(\tau_{t3})'$ are FSTs on (*V*, *E*).

Remark 4.9 The following example illustrates that the converse of Proposition 4.8 is not true.

Example 4.10 Consider Example 3.9. Here $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c\}$ where the NSWSs $(K_1)^c$ and $(K_2)^c$ over *V* are defined as

$$(K_1)^c = \begin{cases} f\left({}^{(e_1)}(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_2)}(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_3)}(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(.2,.4,.7), < g_3,(1,1,0) >\} \\ f\left({}^{(e_4)}(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \end{cases} \quad \text{and}$$

$$(K_2)^c = \begin{cases} f\left({}^{(e_1)}(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_2)}(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left({}^{(e_3)}(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(.5,.1,.3), < g_3,(1,1,0) >\} \\ f\left({}^{(e_4)}(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \end{cases} .$$

Then

$$(\tau_{t1})' = \left\{\left[T_{f\left({}^{(e)}\phi_t\right)}(V), T_{f\left({}^{(e)}1_t\right)}(V), T_{f\left({}^{(e)}(K_1)^c\right)}(V), T_{f\left({}^{(e)}(K_2)^c\right)}(V)\right]_{e\in E}\right\} ,$$

$$(\tau_{t2})' = \left\{\left[I_{f\left({}^{(e)}\phi_t\right)}(V), I_{f\left({}^{(e)}1_t\right)}(V), I_{f\left({}^{(e)}(K_1)^c\right)}(V), I_{f\left({}^{(e)}(K_2)^c\right)}(V)\right]_{e\in E}\right\} \text{ and}$$

$(\tau_{t3})' = \left\{ F_{f\left({}^{(e)}\phi_t\right)}(V), F_{f\left({}^{(e)}1_t\right)}(V), F_{f\left({}^{(e)}(K_1)^c\right)}(V), F_{f\left({}^{(e)}(K_2)^c\right)}(V) \Big|_{e\in E} \right\}$ are FSTs on (V, E),

where

$$\begin{aligned}(\tau_{t1})' = \big\{ &\langle e_1, (0,0,0), (1,1,1), (1,1,1), (1,1,1)\rangle, \langle e_2, (0,0,0), (1,1,1), (1,1,1), (1,1,1)\rangle,\\ &\langle e_3, (0,0,0), (1,1,1), (1,.2,1), (1,.5,1)\rangle, \langle e_4, (0,0,0), (1,1,1), (1,1,1), (1,1,1)\rangle \big\}\end{aligned}$$ and so on.

Thus $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c\}$ is not NSSPT over (V, E), since $(K_1)' \cap (K_2)' \notin (\tau_t)'$.

Proposition 4.11 Let $(V,\ (\tau_t)',\ E)$ be a NSSPTS over (V, E) and

$$(\tau_t)' = \{K_i : K_i \in NSWS(V,E)\} = \left\{\left\langle e, f\left({}^{(E)}K\right) : K_i \in NSWS(V,E)\right\rangle\right\}$$

Where

$$f\left({}^{(E)}K\right) = \left\{\left\langle v, T_{f\left({}^{(e)}K\right)}(v), I_{f\left({}^{(e)}K\right)}(v), F_{f\left({}^{(e)}K\right)}(v)\right\rangle : v\in V, e\in E\right\}.$$

Define

$(\tau_{t1})' = \left\{ T_{f\left({}^{(e)}K\right)}(V) \Big|_{e\in E} \right\}$,

$(\tau_{t2})' = \left\{ I_{f\left({}^{(e)}K\right)}(V) \Big|_{e\in E} \right\}$ and

$(\tau_{t3})' = \left\{ F_{f\left({}^{(e)}K\right)}(V) \Big|^{c}_{e\in E} \right\}$ are FSTs on (V, E).

Then $(\tau_{t1})' \cup (\tau_{t3})'$ and $(\tau_{t2})' \cup (\tau_{t3})'$ are not FSTs on (V, E).

Proposition 4.11 is illustrated by the following example.

Example 4.12 Consider Example 3.11. Here $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c, (K_3)^c\}$ where the NSWSs $(K_1)^c, (K_2)^c, (K_3)^c$ over V are defined as

$$(K_1)^c = \begin{cases} f\left({}^{(e_1)}(K_1)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_2)}(K_1)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (.2,.4,.7) >\} \\ f\left({}^{(e_3)}(K_1)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_4)}(K_1)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \end{cases};$$

$$(K_2)^c = \begin{cases} f\left({}^{(e_1)}(K_2)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_2)}(K_2)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (.5,.3,.8) >\} \\ f\left({}^{(e_3)}(K_2)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_4)}(K_2)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \end{cases} \text{ and}$$

$$(K_3)^c = \begin{cases} f\left({}^{(e_1)}(K_3)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_2)}(K_3)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (.5,.4,.7) >\} \\ f\left({}^{(e_3)}(K_3)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \\ f\left({}^{(e_4)}(K_3)^c\right) = \{< g_1, (1,1,0) >, < g_2, (1,1,0), < g_3, (1,1,0) >\} \end{cases}.$$

Here $\phi_t \cup (K_1)^c = (K_1)^c$, $\phi_t \cup (K_2)^c = (K_2)^c$, $\phi_t \cup (K_3)^c = (K_3)^c$, $1_t \cup (K_1)^c = 1_t$, $1_t \cup (K_2)^c = 1_t$, $1_t \cup (K_3)^c = 1_t$, $(K_1)^c \cup (K_2)^c = (K_3)^c$, $(K_1)^c \cup (K_3)^c = (K_3)^c$, $(K_2)^c \cup (K_3)^c = (K_3)^c$.

Then $(K_1)^c, (K_2)^c$ and $(K_3)^c$ are NSSPOSs.

Thus $(V,\ (\tau_t)',\ E)$ is a NSSPTS over (V, E).

Then

$$(\tau_{t1})' = \{\langle e_1,(0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\rangle,\langle e_2,(0,0,0),(1,1,1),(1,1,.2),(1,1,.5),(1,1,.5)\rangle, \langle e_3,(0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\rangle,\langle e_4,(0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\rangle\},$$

and so on, are FSTs on (*V*, *E*).
But, $(\tau_{t1})' \cup (\tau_{t3})'$ and $(\tau_{t2})' \cup (\tau_{t3})'$ are not FSTs on (*V*, *E*).

Proposition 4.13 Let (*V*, $(\tau_t)'$, *E*) be a NSSPTS over (*V*, *E*). Then

$$({}^e\tau_{t1})' = \{T_{f({}^{(e)}K)}(V) : K \in (\tau_t)'\},$$
$$({}^e\tau_{t2})' = \{I_{f({}^{(e)}K)}(V) : K \in (\tau_t)'\} \text{ and}$$
$$({}^e\tau_{t3})' = \{F_{f({}^{(e)}K)}(V) : K \in (\tau_t)'\}$$

for each $e \in E$, define FTs on (*V*, *E*).
Proof. Follows from Proposition 4.8.

Remark 4.14 The following example illustrates that the converse of Proposition 4.13 is not true.

Example 4.15 Consider Example 4.10.
Then

$$({}^{e_1}\tau_{t1})^c = \{T_{f({}^{(e_1)}\phi_t)}(V), T_{f({}^{(e_1)}1_t)}(V), T_{f({}^{(e_1)}(K_1)^c)}(V), T_{f({}^{(e_1)}(K_2)^c)}(V)\} ,$$

$$({}^{e_1}\tau_{t2})^c = \{I_{f({}^{(e_1)}\phi_t)}(V), I_{f({}^{(e_1)}1_t)}(V), I_{f({}^{(e_1)}(K_1)^c)}(V), I_{f({}^{(e_1)}(K_2)^c)}(V)\} \text{ and}$$

$$({}^{e_1}\tau_{t3})^c = \{F_{f({}^{(e_1)}\phi_t)}(V), F_{f({}^{(e_1)}1_t)}(V), F_{f({}^{(e_1)}(K_1)^c)}(V), F_{f({}^{(e_1)}(K_2)^c)}(V)\} \text{ are FTs on } V,$$

where

$$({}^{e_1}\tau_{t1})' = \{(0,0,0),(1,1,1),(1,1,1),(1,1,1)\} \text{ and so on.}$$

Consequently, $\{({}^{e_2}\tau_{t1})^c, ({}^{e_2}\tau_{t2})^c, ({}^{e_2}\tau_{t3})^c\}$ and $\{({}^{e_3}\tau_{t1})^c, ({}^{e_3}\tau_{t2})^c, ({}^{e_3}\tau_{t3})^c\}$ are fuzzy tritopologies on *V*.
Thus $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c\}$ is not NSSPT over (*V*, *E*), since $(K_1)' \cap (K_2)' \notin (\tau_t)'$.

Definition 4.16 Let (*V*, $(\tau_t)'$, *E*) be a NSSPTS over (*V*, *E*) and $L \in NSWS(V,E)$ be any NSWS. Then the neutro- supra spot absolute closure of *L* is denoted by $\widetilde{L}_-$ and defined as
(i) $\widetilde{L}_- = \{K : K \in NSSPOS \,\&\, K \subseteq L\}$ i.e., the intersection of all neutro-supra spot open subsets of *L*.
(ii)

$$\widetilde{L}_- = \{\langle \widetilde{T}_{-f({}^{(e)}L)}, \widetilde{I}_{-f({}^{(e)}L)}, \widetilde{F}_{-f({}^{(e)}L)}\rangle_{e\in E}\} = \{\langle \min_i T_{f({}^{(e)}K_i)}, \min_i I_{f({}^{(e)}K_i)}, \max_i F_{f({}^{(e)}K_i)}\rangle_{e\in E} : K_i \in (\tau_t)' \,\&\, f({}^eK_i) \subseteq f({}^eL)\}.$$

Example 4.17 Consider Example 3.16. Here $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c, (K_3)^c\}$ where the NSWSs $(K_1)^c, (K_2)^c, (K_3)^c$ over *V* are defined as

$$(K_1)^c = \begin{cases} f({}^{(e_1)}(K_1)^c) = \{<g_1,(1,1,0)>, <g_2,(1,1,0)\} \\ f({}^{(e_2)}(K_1)^c) = \{<g_1,(.2,.6,.9)>, <g_2,(1,1,0)\} \\ f({}^{(e_3)}(K_1)^c) = \{<g_1,(1,1,0)>, <g_2,(1,1,0)\} \end{cases};$$

$$(K_2)^c = \begin{cases} f\left(^{(e_1)}(K_2)^c\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \\ f\left(^{(e_2)}(K_2)^c\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \\ f\left(^{(e_3)}(K_2)^c\right) = \{< g_1,(1,1,0)>,< g_2,(.8,.9,.6)\} \end{cases} \text{ and}$$

$$(K_3)^c = \begin{cases} f\left(^{(e_1)}(K_3)^c\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \\ f\left(^{(e_2)}(K_3)^c\right) = \{< g_1,(.5,.7,.4)>,< g_2,(1,1,0)\} \\ f\left(^{(e_3)}(K_3)^c\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \end{cases} .$$

Here $\phi_t \cup (K_1)^c = (K_1)^c$, $\phi_t \cup (K_2)^c = (K_2)^c$, $\phi_t \cup (K_3)^c = (K_3)^c$, $1_t \cup (K_1)^c = 1_t$, $1_t \cup (K_2)^c = 1_t$, $1_t \cup (K_3)^c = 1_t$, $(K_1)^c \cup (K_2)^c = 1_t$, $(K_1)^c \cup (K_3)^c = (K_3)^c$, $(K_2)^c \cup (K_3)^c = 1_t$.

Then $(K_1)^c, (K_2)^c$ and $(K_3)^c$ are NSSPOSs.

Thus (*V*, $(\tau_t)'$, *E*) is a NSSPTS over (*V*, *E*).

Let $L \in NSWS(V,E)$ be any NSWS defined as

$$L = \begin{cases} f\left(^{(e_1)}L\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \\ f\left(^{(e_2)}L\right) = \{< g_1,(.1,.5,.9)>,< g_2,(1,1,0)\} \\ f\left(^{(e_3)}L\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)\} \end{cases} .$$

Then $1_t, (K_1)^c, (K_3)^c \supseteq L$.

Thus $\widetilde{L}_- = 1_t \cup (K_1)^c \cup (K_3)^c = (K_1)^c$.

Also,

$$f\left(^{(e_1)}(K_1)^c\right), f\left(^{(e_1)}(K_2)^c\right), f\left(^{(e_1)}(K_3)^c\right) \supseteq f\left(^{(e_1)}L\right),$$

$$f\left(^{(e_2)}(K_1)^c\right), f\left(^{(e_2)}(K_2)^c\right), f\left(^{(e_2)}(K_3)^c\right) \supseteq f\left(^{(e_2)}L\right) \text{ and}$$

$$f\left(^{(e_3)}(K_1)^c\right), f\left(^{(e_3)}(K_3)^c\right) \supseteq f\left(^{(e_3)}L\right).$$

Then

$$\left(\widetilde{T}_{-f\left(^{(e_1)}L\right)}, \widetilde{I}_{-f\left(^{(e_1)}L\right)}, \widetilde{F}_{-f\left(^{(e_1)}L\right)}\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)>\},$$

$$\left(\widetilde{T}_{-f\left(^{(e_2)}L\right)}, \widetilde{I}_{-f\left(^{(e_2)}L\right)}, \widetilde{F}_{-f\left(^{(e_2)}L\right)}\right) = \{< g_1,(.2,.6,.9)>,< g_2,(1,1,0)>\} \text{ and}$$

$$\left(\widetilde{T}_{-f\left(^{(e_3)}L\right)}, \widetilde{I}_{-f\left(^{(e_3)}L\right)}, \widetilde{F}_{-f\left(^{(e_3)}L\right)}\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0)>\} .$$

Thus $\left\{\left(\widetilde{T}_{-f\left(^{(e)}L\right)}, \widetilde{I}_{-f\left(^{(e)}L\right)}, \widetilde{F}_{-f\left(^{(e)}L\right)}\right)_{e \in E}\right\} = (K_1)^c$.

Theorem 4.18 Let (*V*, $(\tau_t)'$, *E*) be a NSSPTS over (*V*, *E*) and $L, S \in NSWS(V,E)$ be any two NSWSs. Then,

(i) $L \subseteq \widetilde{L}_-$ and $\widetilde{L}_-$ is the smallest NSSPOS.

(ii) $L \subseteq S \Rightarrow \widetilde{L}_- \subseteq \widetilde{S}_-$.

(iii) $\widetilde{L}_-$ is a NSSPOS i.e., $\widetilde{L}_- \in (\tau_t)^c$.

(iv) L is a NSSPOS iff $\widetilde{L}_- = L$.

(v) $\left(\widetilde{L}_-\right)_- = \widetilde{L}_-$.

(vi) $\left(\widetilde{\phi}_{t-}\right)_{-} = \widetilde{\phi}_{t-}$ and $\left(\widetilde{1}_{t-}\right)_{-} = \widetilde{1}_{t-}$.

(vii) $\left(\widetilde{L} \cup \widetilde{S}\right)_{-} = \widetilde{L}_{-} \cup \widetilde{S}_{-}$.

(ix) $\left(\widetilde{L} \cap \widetilde{S}\right)_{-} \subseteq \widetilde{L}_{-} \cap \widetilde{S}_{-}$.

Proof. Let (V, $(\tau_t)'$, E) be a NSSPTS over (V, E) and $L, S \in NSWS(V,E)$ be any two NSWSs.

(i) Follows from Definition 4.16.

(ii) Let $L \subseteq S$. Then $L \subseteq \widetilde{L}_{-}$ and $P \subseteq \widetilde{S}_{-} \Rightarrow L \subseteq S \subseteq \widetilde{S}_{-} \Rightarrow L \subseteq \widetilde{S}_{-}$.

Since $\widetilde{L}_{-}$ is the smallest NSSPOS containing L, hence $\widetilde{L}_{-} \subseteq \widetilde{S}_{-}$.

(iii) Follows from Definition 4.16.

(iv) Let L be a NSSPOS. Then $\widetilde{L}_{-}$ is the smallest NSSPOS which containing L is equal to L.

Hence $\widetilde{L}_{-} = L$.

Conversely, assume that $\widetilde{L}_{-} = L$.

By (iii), $\widetilde{L}_{-} \in (\tau_t)^c$.

Then L is a NSSPOS.

(v) Let $\widetilde{L}_{-} = K$. Then $K \in (\tau_t)^c$ iff $\widetilde{K}_{-} = K$.

Thus $\left(\widetilde{L}_{-}\right)_{-} = \widetilde{L}_{-}$.

(vi) Since $\phi_t, 1_t \in (\tau_t)^c$ and by (iv), hence $\left(\widetilde{\phi}_{t-}\right)_{-} = \widetilde{\phi}_{t-}$ and $\left(\widetilde{1}_{t-}\right)_{-} = \widetilde{1}_{t-}$.

(vii) $L \subseteq L \cup S$ and $S \subseteq L \cup S$.

$\Rightarrow \widetilde{L}_{-} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{-}$ and $\widetilde{S}_{-} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{-}$.

$\Rightarrow \widetilde{L}_{-} \cup \widetilde{S}_{-} \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{-}$.

Also, $L \subseteq \widetilde{L}_{-}$ and $S \subseteq \widetilde{S}_{-}$.

Then $L \cup S \subseteq \widetilde{L}_{-} \cup \widetilde{S}_{-}$.

Since $L \cup S \subseteq \left(\widetilde{L} \cup \widetilde{S}\right)_{-}$ and it is the smallest NSSPOS, then $\left(\widetilde{L} \cup \widetilde{S}\right)_{-} \subseteq \widetilde{L}_{-} \cup \widetilde{S}_{-}$.

Hence $\left(\widetilde{L} \cup \widetilde{S}\right)_{-} = \widetilde{L}_{-} \cup \widetilde{S}_{-}$.

(vii) $L \cap S \subseteq L$ and $L \cap S \subseteq S$.

$\Rightarrow \left(\widetilde{L} \cap \widetilde{S}\right)_{-} \subseteq \widetilde{L}_{-}$ and $\left(\widetilde{L} \cap \widetilde{S}\right)_{-} \subseteq \widetilde{S}_{-}$.

$\Rightarrow \left(\widetilde{L} \cap \widetilde{S}\right)_{-} \subseteq \widetilde{L}_{-} \cap \widetilde{S}_{-}$.

Definition 4.19 Let (V, $(\tau_t)'$, E) be a NSSPTS over (V, E) where $(\tau_t)'$ is a NSSPT over (V, E) and $L \in NSWS(V,E)$ be any NSWS. Then the subspace topology on NSSPTS is denoted by $(\tau_t^L)'$ and defined as $(\tau_t^L)' = \{L \cap K_i : K_i \in (\tau_t)'\}$ and $\phi_t, 1_t \in (\tau_t^L)'$. Thus ($V$, $(\tau_t^L)'$, E) is a neutro-supra spot subspace topological space (NSSPSTS) of (V, $(\tau_t)'$, E), where $(\tau_t^L)'$ is also a NSSPT over (V, E).

Example 4.20 Consider Example 3.19. Here $(\tau_t)' = \{\phi_t, 1_t, (K_1)^c, (K_2)^c, (K_3)^c\}$ where the NSWSs $(K_1)^c, (K_2)^c, (K_3)^c$ over V are defined as

$$(K_1)^c = \begin{cases} f\left((e_1)(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left((e_2)(K_1)^c\right) = \{< g_1,(1,1,0) >, < g_2,(.5,.6,.1), < g_3,(1,1,0) >\} \end{cases} ;$$

$$(K_2)^c = \begin{cases} f\left((e_1)(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(1,1,0), < g_3,(1,1,0) >\} \\ f\left((e_2)(K_2)^c\right) = \{< g_1,(1,1,0) >, < g_2,(.8,.3,.4), < g_3,(1,1,0) >\} \end{cases} \text{ and}$$

$$(K_3)^c = \begin{cases} f\left({}^{(e_1)}(K_3)^c\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0),< g_3,(1,1,0)>\} \\ f\left({}^{(e_2)}(K_3)^c\right) = \{< g_1,(1,1,0)>,< g_2,(.8,.6,.1),< g_3,(1,1,0)>\} \end{cases}.$$

Here $\phi_t \cup (K_1)^c = (K_1)^c$, $\phi_t \cup (K_2)^c = (K_2)^c$, $\phi_t \cup (K_3)^c = (K_3)^c$, $1_t \cup (K_1)^c = 1_t$, $1_t \cup (K_2)^c = 1_t$, $1_t \cup (K_3)^c = 1_t$, $(K_1)^c \cup (K_2)^c = (K_3)^c$, $(K_1)^c \cup (K_3)^c = (K_3)^c$, $(K_2)^c \cup (K_3)^c = (K_3)^c$.

Then $(K_1)^c, (K_2)^c$ and $(K_3)^c$ are NSSPOSs.

Thus (V, $(\tau_t)'$, E) is a NSSPTS over (V, E).

Define $L_1 \in NSWS(V,E)$ as

$$L_1 = \begin{cases} f\left({}^{(e_1)}L_1\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0),< g_3,(1,1,0)>\} \\ f\left({}^{(e_2)}L_1\right) = \{< g_1,(1,1,0)>,< g_2,(.6,.7,.7),< g_3,(1,1,0)>\} \end{cases}.$$

Thus we denote $L_1 \cap \phi_t = \phi_t$, $L_1 \cap 1_t = L_1$, $L_1 \cap (K_1)^c = L_2$, $L_1 \cap (K_2)^c = L_3$, $L_1 \cap (K_3)^c = L_4$.

Then the NSWSs L_2, L_3, L_4 over V is defined as

$$L_2 = \begin{cases} f\left({}^{(e_1)}L_2\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0),< g_3,(1,1,0)>\} \\ f\left({}^{(e_2)}L_2\right) = \{< g_1,(1,1,0)>,< g_2,(.5,.6,.7),< g_3,(1,1,0)>\} \end{cases};$$

$$L_3 = \begin{cases} f\left({}^{(e_1)}L_3\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0),< g_3,(1,1,0)>\} \\ f\left({}^{(e_2)}L_3\right) = \{< g_1,(1,1,0)>,< g_2,(.6,.3,.7),< g_3,(1,1,0)>\} \end{cases} \text{ and}$$

$$L_4 = \begin{cases} f\left({}^{(e_1)}L_4\right) = \{< g_1,(1,1,0)>,< g_2,(1,1,0),< g_3,(1,1,0)>\} \\ f\left({}^{(e_2)}L_4\right) = \{< g_1,(1,1,0)>,< g_2,(.6,.6,.7),< g_3,(1,1,0)>\} \end{cases}.$$

Then $(\tau_t^L)' = \{\phi_t, 1_t, L_1, L_2, L_3, L_4\}$ is a NSSPST over (V, E).

Thus (V, $(\tau_t^L)'$, E) is a NSSPSTS of (V, $(\tau_t)'$, E).

5. Separation Axioms

This part is split into two parts as separation axioms on NSPTS and NSSPTS are defined with examples.

5.1. Separation Axioms on NSPTS

Definition 5.1.1 Let (V, τ_t , E) be a NSPTS over (V, E) where τ_t is a NSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSPOSs R and S such that

$p \in R$ and $p \cap S = \phi_t$ or

$q \in S$ and $q \cap R = \phi_t$,

Then (V, τ_t , E) is said to be a neutro-spot T_0 -space.

Definition 5.1.2 Let (V, τ_t , E) be a NSPTS over (V, E) where τ_t is a NSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSPOSs R and S such that

$p \in R$ and $p \cap S = \phi_t$ and

$q \in S$ and $q \cap R = \phi_t$,

Then (V, τ_t, E) is said to be a neutro-spot T_1-space.

Definition 5.1.3 Let (V, τ_t, E) be a NSPTS over (V, E) where τ_t is a NSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSPOSs R and S such that

$p \in R$, $q \in S$ and $R \cap S = \phi_t$,

Then (V, τ_t, E) is said to be a neutro-spot T_2-space.

Theorem 5.1.4 Let (V, τ_t, E) be a NSPTS over (V, E). Every neutro-spot T_2-space is also a neutro-spot T_1-space and every neutro-spot T_1-space is also a neutro-spot T_0-space.

Proof. Follows from Definitions 5.1.1, 5.1.2, and 5.1.3.

Example 5.1.5 Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$. Let $\tau_t = \{\phi_t, 1_t, R_1, R_2, R_3, R_4\}$ where the NSPs R_1, R_2, R_3, R_4 over V are defined as

$$R_1 = \begin{cases} f\left({}^{(e_1)}R_1\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}R_1\right) = \{< v_1, (.7,.4,.2) >, < v_2, (0,0,1)\} \end{cases} ;$$

$$R_2 = \begin{cases} f\left({}^{(e_1)}R_2\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}R_2\right) = \{< v_1, (.3,.5,..6) >, < v_2, (0,0,1)\} \end{cases} ;$$

$$R_3 = \begin{cases} f\left({}^{(e_1)}R_3\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}R_3\right) = \{< v_1, (.3,.4,.6) >, < v_2, (0,0,1)\} \end{cases} \text{ and}$$

$$R_4 = \begin{cases} f\left({}^{(e_1)}R_4\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}R_4\right) = \{< v_1, (0,0,1) >, < v_2, (.8,.4,.5)\} \end{cases}.$$

Here $\phi_t \cap R_1 = \phi_t$, $\phi_t \cap R_2 = \phi_t$, $\phi_t \cap R_3 = \phi_t$, $\phi_t \cap R_4 = \phi_t$, $1_t \cap R_1 = R_1$, $1_t \cap R_2 = R_2$, $1_t \cap R_3 = R_3$, $1_t \cap R_4 = R_4$, $R_1 \cap R_2 = R_3$, $R_1 \cap R_3 = R_3$, $R_1 \cap R_4 = \phi_t$, $R_2 \cap R_3 = R_3$, $R_2 \cap R_4 = \phi_t$, $R_3 \cap R_4 = \phi_t$.

Then R_1, R_2, R_3 and R_4 are NSPOSs.

Thus (V, τ_t, E) is a NSPTS over (V, E).

Let p and q be any distinct NSPs which are defined as

$$p = \begin{cases} f\left({}^{(e_1)}p\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}p\right) = \{< v_1, (.1,.4,.7) >, < v_2, (0,0,1)\} \end{cases} \text{ and}$$

$$q = \begin{cases} f\left({}^{(e_1)}q\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}q\right) = \{< v_1, (0,0,1) >, < v_2, (.2,.1,.5)\} \end{cases}.$$

Hence (V, τ_t, E) is a neutro-spot T_2-space, also a neutro-spot T_1-space and a neutro-spot T_0-space.

5.2. Separation Axioms on NSSPTS

Definition 5.2.1 Let $(V, (\tau_t)', E)$ be a NSSPTS over (V, E) where $(\tau_t)'$ is a NSSPT over (V, E). Let p and q be any distinct NSPs. If there exists NSSPOSs R and S such that

$p \in R$ and $p \cap \cong \phi_t$ or

$q \in S$ and $q \cap R \cong \phi_t$,
Then $(V,\ (\tau_t)',\ E)$ is said to be a neutro-supra spot T_0-space.

Definition 5.2.2 Let $(V,\ (\tau_t)',\ E)$ be a NSSPTS over (V, E) where $(\tau_t)'$ is a NSSPT over (V, E). Let *p* and *q* be any distinct NSPs. If there exists NSSPOSs *R* and *S* such that

$p \in R$ and $p \cap S \cong \phi_t$ and

$q \in S$ and $q \cap R \cong \phi_t$,
Then $(V,\ (\tau_t)',\ E)$ is said to be a neutro-supra spot T_1-space.

Definition 5.2.3 Let $(V,\ (\tau_t)',\ E)$ be a NSSPTS over (V, E) where $(\tau_t)'$ is a NSSPT over (V, E). Let *p* and *q* be any distinct NSPs. If there exists NSSPOSs *R* and *S* such that

$p \in R$, $q \in S$ and $R \cap S \cong \phi_t$,
Then $(V,\ (\tau_t)',\ E)$ is said to be a neutro-supra spot T_2-space.

Theorem 5.2.4 Let $(V,\ (\tau_t)',\ E)$ be a NSSPTS over (V, E). Every neutro-supra spot T_2-space is also a neutro-supra spot T_1-space and every neutro-supra spot T_1-space is also a neutro-supra spot T_0-space.
Proof. Follows from Definitions 5.2.1, 5.2.2, and 5.2.3.

Example 5.2.5 Consider Example 5.1.5. Here $(\tau_t)' = \{\phi_t, 1_t, (R_1)^c (R_2)^c (R_3)^c (R_4)^c\}$ where the NSWSs $(R_1)^c, (R_2)^c, (R_3)^c, (R_4)^c$ over *V* is defined as

$$(R_1)^c = \begin{cases} f\left({}^{(e_1)}(R_1)^c\right) = \{< v_1, (1,1,0) >, < v_2, (1,1,0)\} \\ f\left({}^{(e_2)}(R_1)^c\right) = \{< v_1, (.2,.6,.7) >, < v_2, (1,1,0)\} \end{cases};$$

$$(R_2)^c = \begin{cases} f\left({}^{(e_1)}(R_2)^c\right) = \{< v_1, (1,1,0) >, < v_2, (1,1,0)\} \\ f\left({}^{(e_2)}(R_2)^c\right) = \{< v_1, (.6,.5,.3) >, < v_2, (1,1,0)\} \end{cases};$$

$$(R_3)^c = \begin{cases} f\left({}^{(e_1)}(R_3)^c\right) = \{< v_1, (1,1,0) >, < v_2, (1,1,0)\} \\ f\left({}^{(e_2)}(R_3)^c\right) = \{< v_1, (.6,.6,.3) >, < v_2, (1,1,0)\} \end{cases} \text{ and}$$

$$(R_4)^c = \begin{cases} f\left({}^{(e_1)}(R_4)^c\right) = \{< v_1, (1,1,0) >, < v_2, (1,1,0)\} \\ f\left({}^{(e_2)}(R_4)^c\right) = \{< v_1, (1,1,0) >, < v_2, (.5,.6,.8)\} \end{cases}.$$

Here $\phi_t \cup (R_1)^c = (R_1)^c$, $\phi_t \cup (R_2)^c = (R_2)^c$, $\phi_t \cup (R_3)^c = (R_3)^c$, $\phi_t \cup (R_4)^c = (R_4)^c$, $1_t \cup (R_1)^c = 1_t$, $1_t \cup (R_2)^c = 1_t$, $1_t \cup (R_3)^c = 1_t$, $1_t \cup (R_4)^c = 1_t$, $(R_1)^c \cup (R_2)^c = (R_3)^c$, $(R_1)^c \cup (R_3)^c = (R_3)^c$, $(R_1)^c \cup (R_4)^c = 1_t$, $(R_2)^c \cup (R_3)^c = (R_3)^c$, $(R_2)^c \cup (R_4)^c = 1_t$, $(R_3)^c \cup (R_4)^c = 1_t$.
Then $(R_1)^c, (R_2)^c, (R_3)^c$ and $(R_4)^c$ are NSSPOSs.
Thus $(V,\ (\tau_t)',\ E)$ is a NSSPTS over (V, E).
Let *p* and *q* be any distinct NSPs which are defined as

$$p = \begin{cases} f\left({}^{(e_1)}p\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1)\} \\ f\left({}^{(e_2)}p\right) = \{< v_1, (.001,.002,.987) >, < v_2, (0,0,1)\} \end{cases} \text{ and}$$

$$q=\begin{cases} f\left({}^{(e_1)}q\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1)\} \\ f\left({}^{(e_2)}q\right)=\{<v_1,(0,0,1)>,<v_2,(.002,.001,.998)\} \end{cases}.$$

Hence (V, $(\tau_t)'$, E) is a neutro-supra spot T_2-space, also a neutro-supra spot T_1-space and a neutro-supra spot T_0-space.

6. DM Problem to Detect the Impact on COVID-19

In this part, the DM problem explains the COVID-19 situation and detected its impact on corona virus patients to undergoing exact treatment for them according to their medical report. The process of evaluation is pointed out in the algorithm and formula defined for computing the result.

Definition 6.1 Let (V, τ_{t1}, $E1$) and (V, τ_{t2}, $E2$) be two NSPTSs over (V, $E1$) and (V, $E2$), respectively where V is the set of risk factors and $E1$, $E2$ are two different parametric sets of medical issues. Let M be a corona virus patient, were ${}^{E1}M \in \tau_{t1}$ and ${}^{E2}M \in \tau_{t2}$ are two NSPs.

Then for each $v \in V$, the Risk State Value (RSV) of $M(v)$ is given as:

$$RSV\left[M(v)^{(E1,E2)}\right]=\left|\left(\frac{A-C}{2}\right)\times\left(1-\frac{B}{2}\right)\right|, \tag{6.1.1}$$

where

$$A=\sum_{i\in I}T_{f\left({}^{(s_i)}M\right)}(v)+\sum_{j\in I}T_{f\left({}^{(r_j)}M\right)}(v),$$

$$B=\sum_{i\in I}I_{f\left({}^{(s_i)}M\right)}(v)+\sum_{j\in I}I_{f\left({}^{(r_j)}M\right)}(v),$$

$$C=\sum_{i\in I}F_{f\left({}^{(s_i)}M\right)}(v)+\sum_{j\in I}F_{f\left({}^{(r_j)}M\right)}(v)$$

and for all $s_i \in E1$, $r_j \in E2$.

Then $\forall v \in V$, the Total Risk State Value (TRSV) of M is given as:

$$TRSV(M)=\sum_n\left(RSV\left[M(v_n)^{(E1,E2)}\right]\right), \tag{6.1.2}$$

Algorithm

Step 1: List the set of risk factors $v \in V$.
Step 2: List two different parametric sets, say $E1$ and $E2$, where $E1$ represents the symptoms of COVID-19 and $E2$ represents the pre-medical issues.
Step 3: Pick out the people affected by COVID-19, say M.
Step 4: Go through the medical status of each patients.
Step 5: Test their corona virus symptoms ($E1$) and categories its risk factors (V).
Step 6: Collect those data in the form of NSPs.
Step 7: Define a NSPT τ_{t1} and so (V, τ_{t1}, $E1$) is a NSPTS over (V, $E1$).
Step 8: Check the pre-medical issues of each patient ($E2$) and categories its risk factors (V).
Step 9: Collect those data in the form of NSPs, which satisfies the condition of NSPT τ_{t2}.
Step 10: Define a NSPTS (V, τ_{t2}, $E2$) over (V, $E2$).
Step 11: Use the formula 6.1.1 to calculate the RSV of $M(v)$, for each $v \in V$, and tabulate it.
Step 12: Use the formula 6.1.2 to calculate the TRSV of M, for all $v \in V$, and tabulate it.

Step 13: (i) If TRSV(M) is greater than 2.5, then M undergoes ventilation treatment.

(ii) If TRSV(M) lies between 1.5 and 2.5, then M needs to be hospitalized.

(iii) If TRSV(M) is less than 1.5, then M should be self-isolated at home.

Step 14: Step 13 is also concluded according to the risk factor V.

Step 15: If two or more patients have the same TRSV, then each patient required the same treatment.

Problem 6.2 The survey on COVID-19 patients tested in a particular area. Its low-level risk scenarios are talking to someone face to face, walking, jogging, cycling, etc., The medium level risk scenarios are grocery shopping. The high-level risk scenarios are restaurants, public bathrooms, indoor spaces, and common areas. The very high-level risk scenarios are schools, colleges, parties, weddings, cinemas, and workplaces. The people who are affected by COVID-19 are tested by the doctors according to their symptoms and are under investigation. Also, considered other medical issues. They are categorized with some risk factors. Our problem is to recover them quickly by giving appropriate treatment for those affected people.

1. Let $V=\{v_1,v_2,v_3,v_4\}$ be the set of risk factors, where v_1 – low risk, v_2 – medium risk, v_3 – high risk and v_4 – very high risk.

2. Let $E1=\{s_1,s_2,s_3,s_4\}$ and $E2=\{r_1,r_2,r_3,r_4\}$ be two different parametric sets. The set $E1$ represents the symptoms of COVID-19 such as s_1 – fever, s_2 – dry cough, s_3 – chest pain, and s_4 – shortness of breath. The set $E2$ represents pre-medical issues such as r_1 – diabetes, r_2 – blood pressure, r_3 – cardiac diseases, and r_4 – respiratory diseases.

3. Let M_1, M_2, M_3 and M_4 be the people affected by COVID-19.

4. Go through the medical status of each patient.

5. First test their corona virus symptoms ($E1$) and categories its risk factors (V).

6. Those data are collected in the form of NSPs, are as follows:

$$^{E1}M_1=\begin{cases} f\left(^{(s_1)}M_1\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_2)}M_1\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_3)}M_1\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_4)}M_1\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),,<v_3,(0,0,1)><v_4,(.6,.5,.2)>\} \end{cases};$$

$$^{E1}M_2=\begin{cases} f\left(^{(s_1)}M_2\right)=\{<v_1,(0,0,1)>,<v_2,(.5,.2,.6),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_2)}M_2\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_3)}M_2\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_4)}M_2\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),,<v_3,(0,0,1)><v_4,(0,0,1)>\} \end{cases};$$

$$^{E1}M_3=\begin{cases} f\left(^{(s_1)}M_3\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_2)}M_3\right)=\{<v_1,(.3,.2,.8)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_3)}M_3\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_4)}M_3\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),,<v_3,(0,0,1)><v_4,(0,0,1)>\} \end{cases} \text{ and}$$

$$^{E1}M_4=\begin{cases} f\left(^{(s_1)}M_4\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_2)}M_4\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(0,0,1)>\} \\ f\left(^{(s_3)}M_4\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),<v_3,(0,0,1)>,<v_4,(.8,.6,.3)>\} \\ f\left(^{(s_4)}M_4\right)=\{<v_1,(0,0,1)>,<v_2,(0,0,1),,<v_3,(0,0,1)><v_4,(0,0,1)>\} \end{cases}.$$

7. Then $\tau_{t1}=\left\{^{E1}M_1,\ ^{E1}M_2,\ ^{E1}M_3,\ ^{E1}M_4\right\}$ is a NSPT.

Thus (V, τ_{t1}, $E1$) is a NSPTS over (V, $E1$).

8. Then, check the pre-medical issues of each patients ($E2$) and categories its risk factors (V).
9. Those data are collected in the form of NSPs, are as follows:

$$^{E2}M_1 = \begin{cases} f\left({}^{(r_1)}M_1\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_2)}M_1\right) = \{< v_1, (.6,.7,.4) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_3)}M_1\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_4)}M_1\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1),, < v_3, (0,0,1) >< v_4, (0,0,1) >\} \end{cases};$$

$$^{E2}M_2 = \begin{cases} f\left({}^{(r_1)}M_2\right) = \{< v_1, (0,0,1) >, < v_2, (.4,.3,.4), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_2)}M_2\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_3)}M_2\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_4)}M_2\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1),, < v_3, (0,0,1) >< v_4, (0,0,1) >\} \end{cases};$$

$$^{E2}M_3 = \begin{cases} f\left({}^{(r_1)}M_3\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_2)}M_3\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_3)}M_3\right) = \{< v_1, (.4,.4,.2) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_4)}M_3\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1),, < v_3, (0,0,1) >< v_4, (0,0,1) >\} \end{cases} \text{ and}$$

$$^{E2}M_4 = \begin{cases} f\left({}^{(r_1)}M_4\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_2)}M_4\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_3)}M_4\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1), < v_3, (0,0,1) >, < v_4, (0,0,1) >\} \\ f\left({}^{(r_4)}M_4\right) = \{< v_1, (0,0,1) >, < v_2, (0,0,1),, < v_3, (.7,.5,.3) >< v_4, (0,0,1) >\} \end{cases}.$$

10. Then $\tau_{t2} = \{{}^{E2}M_1, {}^{E2}M_2, {}^{E2}M_3, {}^{E2}M_4\}$ is a NSPT.
Thus (V, τ_{t2}, $E2$) is a NSPTS over (V, $E2$).
11. By using the formula 6.1.1, the RSV of $M(v)$ are calculated, for each $v \in V$.
These values are tabulated in the following table.

Table 6.2.1. RSV Table

	v_1	v_2	v_3	v_4
M_1	1.755	0	0	1.43
M_2	0	1.75	0	0
M_3	1.625	0	0	0
M_4	0	0	1.59	1.855

12. By using the formula 6.1.2, the TRSV of M are calculated, for all $v \in V$.
These values are tabulated in the following table.

Table 6.2.2. TRSV Table

	v_1	v_2	v_3	v_4	**TRSV**
M_1	1.755	0	0	1.43	**3.185**
M_2	0	1.75	0	0	1.75
M_3	1.625	0	0	0	1.625
M_4	0	0	1.59	1.855	**3.445**

13. Here TRSV(M_1) = 3.185 and TRSV(M_4) = 3.445, which are greater than 2.5.

Thus they both are at a very high-risk stage.
So, they each should be treated in ventilation for quick recovery.
Next, TRSV(M_2) = 1.75 and TRSV(M_3) = 1.625, which are greater than 1.5.
Here M_2 is under medium risk stage, and so need to be hospitalized.
Even though the TRSV of M_3 lies between 1.5 and 2.5, M_3 is under the low-risk stage.
So, M_3 should be self-isolated at home itself.

7. Conclusions

In this paper, NSPTS and NSSPTS are introduced and defined a subspace topology on them. Along with its absolute interior of NSPT and absolute closure of NSSPT are also defined. Few properties are examined with illustrative examples. This study extended to introduce a concept of separation axioms of NSPTS and NSSPTS are defined as neutro-spot $T_{i=0,1,2.}$ -spaces and neutro-supra spot $T_{i=0,1,2.}$ -spaces respectively with related examples. Additionally, the DM problem explains the COVID-19 situation and detected its impact on corona virus patients to undergoing exact treatment for them according to their medical report. The process of evaluation is pointed out in the algorithm and formula defined for computing the result. The appropriate treatment is provided for affected people as per the estimated value. Some more practical applications of such types of topologies can be explored for future work. Many more sets like open sets, closed sets, rough sets, crisp sets, etc., can be developed on NSPTS and NSSPTS. Later these concepts will step ahead on multi-criteria DM problems by upcoming researchers.

References

1. Abishek, G.; Saurabh, S.; Rakesh, K. B. On Parametric Diverdence Measure of Neutrosophic Sets with its Application in Decision-Making Models. Neutrosophic Sets and Systems **2019**, 29, 101-120.
2. Anjan, M. Vague–Valued Possibility Neutrosophic Vague Soft Expert Set Theory and Its Applications. Neutrosophic Sets and Systems **2019**, 29, 142-157.
3. Abhishek, G.; Rakesh, K. B. Technique for Reducing Dimensionality of Data in Decision-Making Utilizing Neutrosophic Soft Matrices. Neutrosophic Sets and Systems **2019**, 29, 129-141.
4. Shazia, R.; Madiha, Q.; Muhammad, S.; Smarandache, F.; Bakhtawar, A. K. Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique. Neutrosophic Sets and Systems **2019**, 28, 34-50.
5. Jayaparthasarathy, G.; Little Flower, V. F.; Arockia Dasan, M. Neutrosophic Supra Topological Applications in Data Mining Process. Neutrosophic Sets and Systems **2019**, 27, 80-97.
6. Bera, T.; Mahapatra, N. K. On Neutrosophic Soft Topological Space. Neutrosophic Sets and Systems **2018**, 19, 3-15.
7. Cruz, R. A.; Irudhayam, F. N. Neutrosophic Soft Cubic Set in Topological Spaces. Neutrosophic Sets and Systems **2018**, 23, 23-44.
8. Smarandache, F. Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. Neutrosophic Sets and Systems **2018**, 22, 168-170.
9. Bera, T.; Mahapatra, N. K. On Neutrosophic Soft Prime Ideal. Neutrosophic Sets and Systems **2018**, 20, 54-75.
10. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Smarandache, F. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems **2019**, 43(2), 38.
11. Abdel-Basset, M.; Mohamed, R.; Zaied, A. E. N. H.; Smarandache, F. A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. Symmetry **2019**, 11(7), 903.

12. Abdel-Basset, M.; Saleh, M.; Gamal, A.; Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing **2019**, 77, 438-452.
13. Abdel-Basset, M.; Nabeeh, N. A.; El-Ghareeb, H. A.; Aboelfetouh, A. Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems **2019**, 1-21.
14. Abdel-Baset, M.; Chang, V.; Gamal, A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry **2019**, 108, 210-220.
15. Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision Making. Symmetry **2018**, 10, 116.
16. Smarandache, F. Neutrosophy, Neutrosophic Probability, Set and Logic. American Research Press, Rehoboth, USA **1998,** 105.
17. Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics **2005**, 24, 287-297.
18. Smarandache, F. Plithogeny, Plithogenic Set, Logic, Probability, and Statistics; Infinity Study **2017**, 141.
19. Chinnadurai, V.; Sindhu, M. P. A Novel Approach for Pairwise Separation axioms on Bi-Soft Topology Using Neutrosophic Sets and An Output Validation in Real Life Application. Neutrosophic Sets and Systems **2020**, 35, 435-463.
20. Chinnadurai, V.; Sindhu, M. P. Generalization of Level Sets in Neutrosophic Soft Sets and Points: A New Approach. Tathapi **2020**, 19 (50), 54-81.
21. Chinnadurai, V.; Smarandache, F.; Bobin, A. Multi-Aspect Decision-Making Process in Equity Investment Using Neutrosophic Soft Matrices. Neutrosophic Sets and Systems **2020**, 31, 224-241.
22. Chinnadurai, V.; Swaminathan, A.; Bobin, A.; Kathar Kani, B. Generalised Interval Valued Intuitionistic Fuzzy Soft Matrices and Their Application to Multicriteria Decision Making. Recent Trends in Pure and Applied Mathematics **2019**, AIP Conf. Proc. 2177, 020019-1 −020019-11.
23. Mehmood, A.; Nadeem, F.; Nordo, G.; Zamir, M.; Park, C.; Kalsoom, H.; Jabeen, S.; Khan, M.I.; Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces, Neutrosophic Sets and Systems **2020**, 32, 38-51.
24. Taha, Y.O.; Cigdem, G.A.; Sadi, B. A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces. Communications in Mathematics and Applications **2019**, 10(3), 481-493.
25. Taha, Y.O.; Cigdem, G.A.; Sadi. B. Separation axioms on neutrosophic soft topological spaces. Turkish Journal of Mathematics **2019**, 43, 498-510.
26. Bera, T.; Mahapatra, N.K. Introduction to neutrosophic soft topological space. OPSEARCH **2017**, 54, 841-867.
27. Abd El-latif, A. M. Soft supra strongly generalized closed sets. Journal of Intelligent & Fuzzy Systems **2016**, 31(3), 1311-1317.
28. Faruk, K. Neutrosophic Soft Sets with Applications in Decision Making. International Journal of Information Science and Intelligent Systems **2015**, 4(2), 1-20.
29. El-Sheikh S. A.; Abd El-latif, A.M. Decompositions of some types of supra soft sets and soft continuity. International Journal of Mathematics Trends and Technology **2014**, 9(1), 37–56.
30. Maji, P.K. Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics **2013**, 5, 157-168.
31. Salma, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. IOSR Journal of Applied Mathematics **2012**; 3, 31-35.
32. Molodtsov, D. Soft set theory-first results. Computers and Mathematics with Applications **1999**, 37, 19-31.
33. Abd El-monsef, M. E.; Ramadan, A. E. On fuzzy supra topological spaces. Indian Journal of Pure and Applied Mathematics **1987**, 18(4), 322–329.
34. Atanassov, K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems **1986**, 20, 87-96.
35. Mashhour, A. S.; Allam, A. A.; Mohmoud, F. S.; Khedr, F. H. On supra topological spaces. Indian Journal of Pure and Applied Mathematics **1983**, 14(4), 502–510.
36. Zadeh, L.A. Fuzzy sets. Information and Control **1965**, 8, 338-353.

Received: June 9, 2020. Accepted: Nov 23, 2020