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Abstract: This paper describes a research aimed at problematizing cases of indication to the use of Base Ten Blocks (BTB) and Green Chips (GC) for the teaching and learning of the Arabic numeral system and of arithmetic operations. The cases analyzed are present in the five collections of Elementary School mathematics textbooks selected by the National Program of Textbook (PNLD), which were the most purchased by the Ministry of Education (MEC) in 2016, to supply the system of public education in Brazil during the triennium 2016-2018. The identified indications were classified in three types, and two of them were particularly problematized for presenting some type of limitation or confusion. Only one of the five collections studied did not present problems regarding manipulatives.

*Keywords***:** Base 10 number system, Multi-digit addition, Place value, Measurement-based approach, Spatial reasoning, Representation, Physical manipulatives.

Introduction

From all the numeral systems developed by humankind, the Hindu-Arabic system probably became hegemonic because it offered some facilities almost non-existent at the time. According to Boyer (1996), the Indians, responsible for the creation of this technology, articulated three principles that, although they existed before, had not yet been articulated in one single numeral system. These principles are: the decimal base, the place value notation, and the encrypted form–or a different symbol–for each of the first ten numbers, named Anka (litteraly meaning "mark") for digits from one to nine, and Sunya (literally, "void") for the symbol zero, as stated by Datta and Singh (1935).

The great advantage of the system lies, nevertheless, in its capability of allowing one symbol to represent different quantities. That is owed to the place value notation, the most important concept of the numeral system, according to Sun and Bussi (2018). Moreover, beyond its practicality, the system also spares the waste of mental labor (Sharma, 1993).

The Arabic numeral system is a knowledge taught in schools all around the world (Yong, 1996), and it is certainly necessary that it is fully comprehended by children (Sharma, 1993; Nurnberger-Haag,

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2018). However, this knowledge presents a "deceptive simplicity" (Dutta, 2002, p. 08), and the learning process does not always happen in a peaceful manner, for due to its complex nature (Thomas, 2004; Kamii, 1986) it is not easily apprehended by the students. Given the aforementioned difficulties, researchers such as Froebel, Montessori, Dienes, Cuisenaire, among others, either developed or adapted a series of objects and manipulatives that could work as mechanisms to aid children's comprehension of abstract mathematical concepts. In continuation with those developments, nowadays researchers and teachers have created, recreated and adapted a plethora of manipulatives to aid the learning of mathematics, such as place value pocket charts, bundles of popsicle sticks or soda straws, various models of counters, etc.

We have been able to establish from our researches that, because of their presence in the daily practices of mathematics teachers, manipulatives are constantly remembered in mathematics textbooks during the first school years of the most diverse countries in the world, including Brazil. Some researchers corroborate this affirmation and reiterate that in general textbooks support the use of this type of material (Nührenborger & Steinbring, 2008; Nacarato, 2005). Apart from a few studies on the limitation of abacus with colored beads (Bartolini Bussi, 2011; Bartolini Bussi et al, 2018A) or Base Ten Blocks (Ladel and Kortenkamp, 2016, 2016b), specialized literature has not presented discussions on possible mistakes and errors regarding some specific manipulatives, much less when it comes to mistakes and errors propagated by textbooks and teacher training materials.

Having noticed this gap, in 2012 we initiated a research aimed at problematizing indications to the use of manipulatives, for the purpose of teaching the Hindu-Arabic system, and their presence on textbooks. This text presents a part of this broader study, as delimited below.

Accordingly, this paper reports a research aimed at problematizing the cases of indication to the use of Base Ten Blocks[2] (BTB) and Green Chips (GC)[3] for the teaching and learning of the Arabic numeral

[2] Also known as multibase arithmetic blocks (MAB) or Dienes blocks. In Brazil they are known as golden material. Throughout the text we adopt the terminology Base Ten Block or BTB.

[3] Didactic material presented exclusively in Ápis collection – Kindergarten Mathematics. As the author does not ascribe a name to the manipulative in question, we conventionally call it Green Chips.

system and of arithmetic operations. The cases analyzed are present in the five collections of Elementary School mathematics textbooks selected by the National Program of Textbook (PNLD), which were the most purchased by the Ministry of Education (MEC) in 2016, to supply the system of public education in Brazil during the triennium of 2016-2018. The choice of the triennium took place because that was the last complete term for a PNLD edition, therefore the period was already a source of consolidated data.

Hence, after some explanation about the two main themes developed in this paper, “mathematics textbooks” and “manipulatives in the teaching and learning of mathematics”, we describe the methodology that guided the research, and finally present and discuss the collected data.

Mathematics Textbooks

The textbook is a book designed to “provide an authoritative pedagogical version of an area of knowledge” (Stray, 1994, p. 02). In general, this type of material is very common in daily school life. In fact, Valverde et al. (2002) affirm that, although there are evidences pointing that school experiences vary considerable among different countries, some of the elements that compose the school environment are virtually universal, textbooks being one of them. As reported by the authors, only teachers and students would be more omnipresent than textbooks at schools.

The books are not simply present at schools, they are considered by some researchers as the main didactical resource (Pepin, Gueudet & Trouche, 2013; Fan et al., 2004; Reys et al., 2003, Van den Han & Heinze, 2018; Lepik, Grevholm & Viholainen, 2015; Mullis et al., 2012), the main tool (Hadar & Ruby, 2019) or at least one of the most important tools used by teachers (Rezat, 2010) and they significantly impact their instructional decisions (Valverde et al., 2002). At the same time, textbooks are important for students as well. This type of material is, for some teachers, the only resource students will have access to in their classes (Lepik, Grevholm & Viholainen, 2015), being perhaps the main learning source, both inside and outside of school (Fan et al., 2004), or still a substantial factor in the learning of students (Van den Han e Heinze, 2018). This way, the importance of textbooks for teachers and students around the world is, to a greater or lesser degree, recognized by several studies.

If all over the world textbooks have taken the lead in mathematics teaching and learning processes, it appears to be a reasonable question to enquire the way teachers relate to these teaching materials.

Reflecting upon the use of textbooks by teachers, Nicol and Crespo (2006) developed a research in an undergraduate teaching training program, there they investigated how preservice teachers use and interpret textbooks. Even though the study was conducted with preservice teachers, the results may allow for an overall understanding of mathematics teachers' relations with teaching materials. The authors pointed that teachers took several different approaches to the use of textbooks. They classified this use in three categories: adherence, elaboration, and creation. In the category adherence, textbooks are considered "authorities" that will indicate what to teach and how to do it. Usually only textbooks are selected to support the work, and the activities proposed in them undergo little or no adaptations. The few adaptations turn out to be very superficial. In this category the teachers do not see themselves as resources. In the category elaboration, although textbooks are considered the main teaching resources, they are not the only ones. They are viewed as guides when it comes to what to teach and how to teach it. From conceptual and contextual elaborations, lessons, tasks, and textbook exercises are elaborated and expanded. In this category teachers consider themselves as resources. At last, in the category creation, textbooks are considered one of the many resources available, and are used in a critical and innovative way. Both potentialities and limitations are identified in these curricular materials. In this category the teacher optimizes the teaching structures creating adequate problems, as well as contextualizing the activities.

Based on the classification proposed by Nicol and Crespo (2006), our concerns turn to the teachers who, in their use of textbooks, usually conform to the description of the category "adherence". The dependency on textbooks relates to preservice teachers' fear of not knowing sufficiently the subject matter they need to teach (Bush, 1987; Schoen, LaVenia, Ozsoy, 2019). So the textbook would be the solution, since it would alleviate any deficiencies (Bush, 1987). In this case it is possible to identify a type of authority that teachers attribute to the authors of textbooks (Ewing, 2004; Ball & Fieman-Nemser, 1988; 1998; Jamieson-Proctor & Byrne, 2008), as well as to the textbooks themselves.

That added to the fact that textbooks are seen as mirrors of the curriculum, exerting a strong influence in the choice of the contents that are going to be taught and learned (Lepik, Grevholm & Viholainen, 2015), thus strongly impacting classroom practices (Valverde et al., 2002), we come to the conclusion that discussions on the quality of textbooks are of extreme urgency.

Reinforcing this preoccupation, another perspective points that textbooks are of significative influence in students' learning experiences (Polikoff, 2015, Chingos & Whitehurst, 2012; Tarr et al, 2006; Hadar & Ruby, 2019; Van den Han & Heinze, 2018; Valverde et al., 2002). To Chingos & Whitehurst (2012) the selection of teaching materials, textbooks among them, will greatly affect students' learning. Some teachers believe that students will learn mathematics if the textbook is followed faithfully (Schoen, LaVenia, Ozsoy, 2019), others, that the different opportunities presented in these didactic materials affect students' performance regarding the cognitive domain (Hadar, 2017; Hadar & Ruby, 2019) and that the textbooks methods will necessarily affect the instructions (Reys, Reys & Chavez, 2004).

Gracin (2008) understands that textbook tasks may influence the way students think. From this we deduct that if some textbook tasks present some type of problem or dubious and diffuse interpretation, it could reflect on the learning process of children. Hence, although it is not our goal to address the difficulties children may face when dealing with misleading or problematic elements found in textbooks, we are interested in problematizing tasks related to the presentation or indication to the use of certain manipulatives found in Elementary School mathematics textbooks.

Manipulatives in the Teaching and Learning of Mathematics

When it comes to the educational field, manipulatives can be understood as "any physical, pictorial or virtual objects used as resources in the teaching of a certain knowledge " (Silveira, Powel & Grando, 2020, p. 01, in press). According to these authors, "manipulatives" is a broader expression, thus encompassing the other classifications for these types of resourses, wheter they are physical or virtual.

Based on Bartolini Bussi and Boni's (2003) work, which did not share exactly the same objectives, Silveira, Powel and Grando (2020, in press), after some adaptation, introduced a classification according to four categories, in which they differenciate the most varied types of manipulatives that are often used as resources for the teaching of mathematics. They are:

> *didactically constructed materials*, that include all types of material artificially created by educators to simulate relations that stimulate the construction of mathematical ideas. These materials may be physical, as it is the case of Dienes Blocks, Cuisenaire rods, and circular geoplanes, or pictorial such as Dienes Blocks' representations that appear in some textbooks; *cultural instruments inherited from tradition*, which have accompanied and aided the theoretical developments of mathematics, such as the abacus, the soroban, the ruler and the compass; *objects taken from daily life*, that to a certain extent attest fragments of mathematical knowledge, such as string, coins, toys, sticks, seeds or stones and the *virtual manipulatives*, that include dynamic visual representations of mathematical objects, daily life objects or physical manipulatives, produced in the field of information and communications technology, that can be manipulated from mouse devices, joysticks, touchpads or touch screens. (Silveira, Powel & Grando, 2020, p. 01, in press)

Regardless of the category considered, the use of these objects has been the subject of discussion for numerous researchers (Baroody, 1989; Ball, 1992; Thompson & Lambdin, 1994; Clements & Mcmillen, 1996; Uttal, Scudder & DeLoache, 1997; Kilpatrick, Swafford & Findell, 2001; Moyer, 2001; Kamii, Lewis & Kirkland, 2001; Moyer & Jones, 2004; McNeil & Jarvin, 2007; Brown, McNeil e Glenberg, 2009; Kaminski & Sloutsky, 2013).

For some researchers, manipulatives are widely accepted in mathematics classes, especially in the early years (Kilpatrick, Swafford e Findell, 2001), and they work as a sort of catalyst for deepening mathematical understanding (Marshall & Swan, 2008). Researchers affirm that manipulatives may provide links that help students to relate their knowledge and informal experience to mathematical abstractions (Kilpatrick, Swafford & Findell, 2001), or that they help students to understand abstract mathematical concepts, because they assist in the connection between concepts and more informal and concrete ideas (Uribe-Flórez e Wilkins, 2010). There are still those who understand that manipulatives aid students that have difficulties in comprehending abstract symbols (Moyer e Jones, 2004) and, in this same sense, help to make invisible mathematical concepts visible (Golafshani, 2013), or help to make mathematics more real for students (Baroody, 1989). According to Furner, Yahya and Duffy (2005) the use of manipulatives aids children in the building of connections between concrete objects and abstract concepts.

Furthermore, researchers alert to possible problems concerning the use of manipulatives. For Kilpatrick, Swafford and Findell (2001) the use of such objects should not be seen as an end in itself. When utilized during a sufficient amount of time, they work as a means to allow for children's construction of meanings and connections. To the same end, Ball (1992) presents her concerns regarding the idea of manipulatives' magical ability to enlighten and to make students understand, causing the mathematical knowledge to arise automatically. The mathematical knowledge does not reside or exist in the wood, plastic or cardboard of manipulative materials (Ball, 1992; Kamii, Lewis & Kirkland, 2001), as some teachers, who already know the concepts they want to teach, think. To touch and manipulate these materials is not enough for children to draw correct conclusions (Ball, 1992). According to this researcher, the ways to work with manipulatives, the purposes of use, and the dialogues and interactions in the activities are as important as the object being utilized. From this vantage point, Uttal, Scudder and DeLoache (1997) also criticize the understanding that manipulatives magically aid teachers. For them, the use of manipulatives can even be positive, but such objects do not possess the key "to unblock the mysteries of mathematics" (p. 50). Other researchers also adhere to the notion that manipulatives do not carry in their physicality the meaning of mathematical ideas (Clements & Mcmillen, 1996). This way, Moyer (2001) states that manipulatives are not inherently bearers of meaning or insight. It is students' reflections on their actions that can aid them in the construction of meanings.

Some researchers claim that manipulatives will only be useful if they are not used as toys. Based on several studies, they understand that teachers should use manipulatives less frequently if they are too rich in details or too familiar to children's context outside school (as toys, for instance). They affirm that students may decide to play with the manipulatives at the expense of enhancing their knowledge of mathematics (McNeil & Jarvin, 2007; McNeil et al, 2009; Brown, McNeil & Glenberg, 2009). Aligned with this perspective, Kaminski and Sloutsky (2013) state that those who design educational materials should limit the inclusion of "extraneous perceptual information" in their creations, avoiding that children have their attention diverted from the concept that is going to be taught. Manipulatives "stripped off perceptual characters or irrelevant attributes" help children to concentrate their attention in the relation with the mathematical concept presented (Laski, et al, 2015). In accordance, Uttal, Scudder & DeLoache (1997) also consider these are the best types of manipulatives.

Apart from these concerns, researchers understand that it may be counterproductive to use manipulatives without being certain that students understand the relation these objects establish with the mathematical concepts studied. If this harmony is not established, children may feel overloaded when faced with the necessity of separately learning two systems–the manipulatives system and the system of mathematical rules and conventions–at the same time (Uttal, Scudder & DeLoache, 1997). In the same direction, Brown, McNeil & Glenberg (2009) state that, when using manipulatives, it is important that students understand they are not entering a new system, isolated from mathematics. On the contrary, it should be clear to them that they are using concrete objects to aid their understanding of the symbolic system they are studying – mathematics.

Baroody (1989) makes clear his concern over the notion that the use of manipulatives is enough or necessary to promote a meaningful learning of mathematics. For him, this warning should be raised in the packaging of manipulatives. Willingham (2017) also understands that the efficacy of learning enabled by a certain manipulative depends on the way the teacher encourages its usage. Aligned with these ideas, Uttal (2003) affirms that the role of the teacher is key to establish whether the manipulatives will aid, hinder, or make no difference in children's understanding of mathematics. According to him, it is the teacher's role to conduct the orientation in an effective way. Even if all the steps are properly executed, when manipulatives are used in a routine basis, children's learning may not be potentialized (Clements & Mcmillen, 1996).

Studies on the use of manipulatives seem to converge to one perspective: manipulatives can be useful or useless (Kamii, Lewis, & Kirkland, 2001), depending on a series of elements, for instance: how these materials are designed, how they are used, and how the instructions to their use are given by teachers. Some materials may have already been designed with limitations that hinder their utilization. This is the case, for example, of spike abaci (Bartolini Bussi, 2011; Silveira, 2014; Bartolini Bussi et al, 2018). Relative value stops existing at the expense of the absolute value embedded in the colors of the beads, that starts to work with values of different powers of ten at each color. Other materials can be misused depending on the guidelines provided, either by teachers or by textbook authors. These cases specifically involve the use of Base Ten Blocks or similar manipulatives in correlation to place value charts (Silveira, 2014, 2016, 2018; Ladel & Kortenkamp, 2016, 2016b). This last case will be the subject of further discussion in this study.

Methodology

The outcomes presented here are part of a larger research that aimed at problematizing the indications to the use of manipulatives, with the objective of teaching the Arabic system, in textbooks and diverse types of books, notebooks or handouts oriented to the training of teachers of Elementary School. Even though we have encountered indications to the use of several types of manipulatives–such as Base Ten Blocks, Green Chips, bundles of sticks and straws, money and abaci–, in order to provide greater detail, this paper will address Base Ten Blocks and Green Chips only, given the proximity between these manipulatives, as shall be explained. The investigation materials analysed were delimited as well, only textbooks from the five collections most purchased by MEC, from the last term of PNLD, PNLD 2016, were analysed. The textbooks chosen were utilized in schools during the years 2016, 2017 and 2018.

To determine MEC's most purchased textbook collections during the period, we solicited access to information through the governmental webpage, and received the answer a few days later. A spreadsheet containing the information requested was sent. From these data we elaborated the following table:

Table 1
Total of copies of each collection acquired by MEC in the triennium 2016-2018[4]

Name of the collection	Number of copies acquired by MEC in the triennium 2016-2018
Ápis – Matemática	5.454.251
Projeto Coopera Matemática	2.885.793
Porta Aberta (Edição Renovada) Matemática	2.452.296
Projeto Buriti Matemática	2.448.325

[4] With a total of 3.576.208 textbooks, "Novo Girassol: Saberes e Fazeres do Campo" was the second most bought collection in the period, according to the worksheet sent by MEC. This collection has been excluded from our sample because it was selected by PNLD Campo 2016, a PNLD edition specific for rural schools.

Novo Bem-Me-Quer Matemática	1.622.131

Each copy was consulted page by page, and the goal was to observe the form of indication to the use of manipulatives, not to determine the number of occurrences of certain indications. In the analysis, we took as a base the notion that manipulatives with added values, either by convention, as in Green Chips, or by evidence, as in Base Ten Blocks, already present in the sum of the values of the pieces the final result of the count and, this way, they are not positional. Thereby, we took as problematic those cases when the manipulatives were inserted in labeled tables or charts, such as place value charts. Below, we introduce the two manipulatives examined and, in sequence, we present and discuss the data collected.

Base Ten Blocks and Green Chips

Base Ten Blocks are known in Brazil as "Golden Material", closely resembling the name Maria Montessori attributed to the manipulative of her own design, the "Golden Bead Material". The name was motivated by the color of the golden beads' raw material, that looked like orange-colored glass (similar to amber), as seen in **Figure 1**.

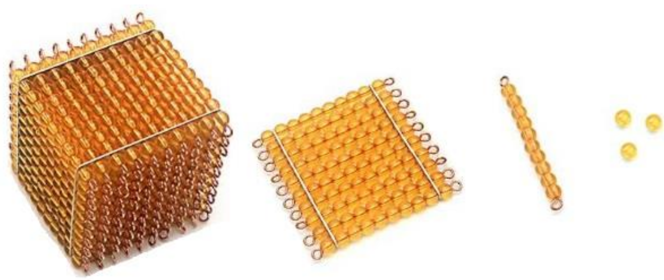


Figure 1: Golden Beads designed by Maria Montessori.

The first modification of the Golden Beads Material could have been implemented by Helena Lubienska de Lenval, who worked with Montessori. Lubienska could have designed the material using wood in order to reduce production costs (Alves, 2019). Catherine Stern could also have produced a similar manipulative during the 1950s (Griffiths, Black & Gifford, 2017). It is important to stress that these manipulatives are means to the same end, the two were configured to aid student's understanding of base ten, one of the most basic principles of our numeral system. However, when it comes to manipulation, they show some differences. For instance, the Montessorian material can be disassembled and reassembled, it is possible to remove all of the 1,000 beads that compose the cube, although it would not be an easy task, as we can observe in **Figure 1**, but in the case of wooden blocks, such as Base Ten Blocks, the same cannot be done. Why? Because the units that compose longs, flats and cubes are carved in a piece of wood, that is, they are glued together.

The blocks, as they became known today, were especially designed by Dienes (1960). Based on the mathematical variability principle, the designer's objective was to facilitate the understanding of what he named the "bare bones of place-value concept" (Dienes, 1960, p. 53). In other words, the author designed the Multibase Arithmetic Blocks (M.A.B) to facilitate the understanding that the main principle of a positional system, the place value, cannot be lost even when we vary the digits, the powers and the base. Historically, the author presents the material in five boxes, each one containing the blocks for powers b^0,〚 b〛^1,〚 b〛^2 and〚 b〛^3, organized according to the bases 3, 4, 5, 6 and 10 (Dienes, 1960). As previously exposed, it is clear that Dienes had a different goal. With the overcoming of the Modern Mathematics Movement, supported by Dienes, multibase blocks, as boxes of blocks for the five indicated bases, disappeared from schools and dealers' catalogs (Schmittau & Vagliardo, 2006) and became "outdated" (Nataraj & Thomas, 2007).

In any case, one of the five boxes of multibase blocks by Dienes was never as popular as today. According to Chan et al. (2017), Base Ten Blocks, such as designed by Dienes, have long been adopted all over the world to aid children's understanding of the base ten system. Some researchers (Mcneil & Jarvin, 2007) consider them a resourceful material for the teaching and learning of numbers and operations. Since they are not rich in interesting perceptual details, it is much easier for students to focus on the mathematical concept at stake. In other words, they are considered free of distractors (Bartolini Bussi, 2011). Namely, Base Ten Blocks are composed by blocks that represent the quantities 1, 10, 100 e 1000. The little cubes represent units; longs represent tens; flats, hundreds; and the cube, thousands.

Very present in Brazilian schools, the use of this manipulative is broadly recommended in Elementary School textbooks

Figure 2: Base Ten Blocks (according Dienes design).

In a research involving a model of a developmental sequence of conceptual structures for two-digit numbers, Fuson, Smith, and Cicero (1997) discuss and support what they call children's conceptual tools that, according to our understanding, would be a system of recording inspired by longs and units that comprise Base Ten Blocks. In their notebooks, children drew vertical lines – called ten-sticks – and dots, to represent longs and units, respectively. According to these authors, children can solve more difficult problems using such representations or drawings. Fuson (2009) named these representations Math drawings. For her, math drawings were simplified drawings capable of capturing certain important aspects of mathematics. Rivera (2014, 2015), brings images of these representations, that she calls "Fuson math drawings". Although these representations do not appear in the textbooks analyzed in this research, they are present in textbooks used in the USA, in collections such as "Into Math" and "Go Math!". In several activities, children are invited to draw "quick pictures", that would be drawings close to Fuson's math drawnings. They are also called "Pictorial Base Ten", the nomenclature adopted in this paper.

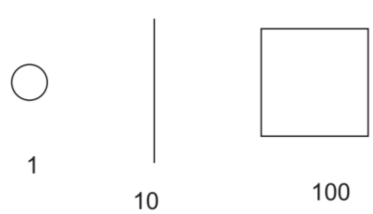


Figure 3: Fuson math drawings, Quick pictures or Pictorial Base Ten.

While pictorial base ten appears as a pictorial representation for Base Ten Blocks, without many details, the Green Chips, the other manipulative analyzed in this study, seems to take the opposite approach. They seem to have been designed to give substance to pictorial base ten. In fact, Green Chips, proposed by the author of collection "Ápis - Alfabetização Matemática" and observed only in this collection, are presented in the teacher's handbook of the second year as a type of register for Base Ten Blocks. According to the author, drawing Base Ten Blocks may be a difficult task for students. Green Chips, however, are not only a pictorial representation. They are available on the support material of collection "Ápis - Alfabetização Matemática", to be cut out and manipulated. They are composed by paper chips in the shapes presented in **Figure 4**. Circular chips correspond to units, rectangular chips represent the tens and the hundreds are represented by squares. There are no chips to represent the thousands in the material.

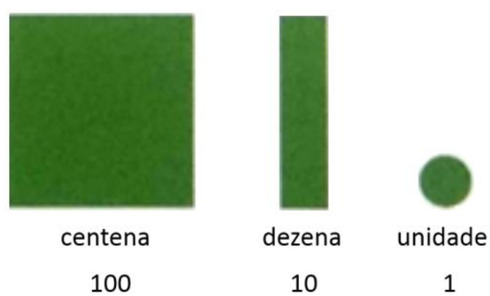


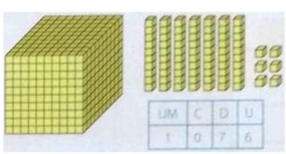
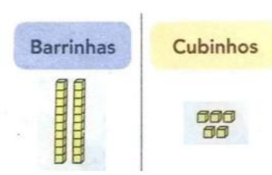
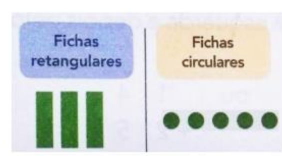
Figure 4: Green Chips
Source: third grade textbook from collection "Ápis – Alfabetização matemática". p. 58.

Although they have the same goal–to aid the understanding of base ten–these two manipulatives have a different nature. In Base Ten Blocks the values of each blocks are not conventional, because they are evident from the division carved in the wood. In other words, a long, that seems to have ten little cubes glued together, will probably have ten times the volume of the little cube, and a flat will have approximately 10 times the volume of a long. In Green Chips the values exist by convention. It is easy to realize that the values 1, 10 and 100 for the three models of paper chips are random, since neither the area of the square chip is ten times the area of the rectangular chip nor the area of the rectangular chip is ten times the area of the circular one.

Presentation and Discussion of Data

Conforming to the way they are introduced in the textbooks surveyed and in the teachers‘ handbooks, the indications to the use of Base Ten Blocks and Material Verde as support materials for the understanding of the Arabic System can be categorized in three types. In Type 1, Base Ten Blocks and Green Chips are not framed by tables or labeled charts. In the activities these materials are organized in decreasing order of blocks and chips. In type 2 indications, the blocks or chips are acomodated inside place value charts. In these cases, the columns are labeled with the names of the orders: Hundreds, Tens and Ones or their abbreviations H, T and O. In type 3 indications, the blocks or chips of the materials are also acomodated inside charts or labeled tables. But in these cases, the labels refer to the name of the material. When Base Ten Blocks are represented, the columns are titled as Flats, Longs and cubes, and when Green Chips are indicated, the columns are titled as square chips, rectangular chips, and circular chips. In Table 2 we present examples of indications to the use of Base Ten Blocks and Green Chips according to the three types of instructions described.

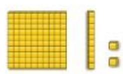
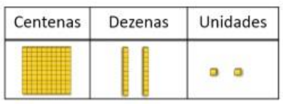
Table 2
Types of indication to the use of Base Ten Blocks and Green Chips present in the textbooks surveyed.[5].

Type 1	Type 2	Type 3
		
 C D U 1 4 2		

[5] The words Centena - C, Dezena - D and Unidade - U are the Portuguese words for Hundreds - H, Tens - T and Units - U. The words Barrinhas and Cubinhos are the words for Longs and Units in Brazilian Portuguese. The expressions Fichas retangulares e Fichas circulares are the expressions for Rectangular chips and Circular chips in Brazilian Portuguese.

Table 3 informs the manipulatives indicated and the type of indication present in each of the collections surveyed.

Table 3
Types of use of manipulatives by the collection of textbooks researched.

	Ápis Matemática	Projeto Coopera Matemática	Porta Aberta Matemática	Projeto Buriti Matemática	Novo Bem-Me-Quer Matemática
	X	X	X	X	X
					X
	X		X	X	X
	X				
	X				
	X				

Type 1 indications to the use of Base Ten Blocks are found in all the collections. This is the only type of orientation brought by collection "Projeto Coopera Matemática". In the case of type 1 indications to the use of Green Chips, they are also present in collection "Ápis - Alfabetização Matemática". This use of manipulatives is in accordance with the reason why they were designed, that is, to aid the understanding of the structural framework of a base, such as the set of Dienes Blocks. In the case of Base Ten Blocks, in which the value of each block is self-evident, or can be confirmed through the combination of other blocks, or Green Chips, in which the chips' values were previously accorded, the goal is to aid children's understanding that ten blocks of the same shape or ten identical chips, can be exchanged for a compact block or another chip, as long as their value correspond to the value of those former ten blocks or chips. This idea will be employed later, when the blocks and chips are used to teach the operating principles of addition and subtraction algorithms.

To understand a little more about the use of Base Ten Blocks and Green Chips, let us go back to our numeral system. Thinking about the three main principles of the Arabic numeral system, presented by Boyer (1996), although in social life the result of their combination is already a part of children's lives from early age, at school these three principles are not presented at the same time, and the reasons for that are obvious. The school wishes to teach how this system works or to better put it, how does this quantity record factory works, and in order to make sense of the "decimal base" concept, children need to understand the additive and multiplicative principles, at least.

So, first children are taught the number symbols that make up the non-positional part of the system, namely the digits from 0 to 9. At this moment the aim is that children can connect a random symbol, once the digits bear no logical relation to the quantity they signify, to collections of objects or, in the case of zero, to the absence of quantity. From this point, when it is desired that children advance in the counting of larger collections, the operation of the decimal base must be taught. It is at this moment that Base Ten Blocks, or perhaps Green Chips, seem to offer a good contribution.

It is expected that the manipulation of these objects will help children to understand the exchange process. Teachers need to bear in mind that Base Ten Blocks are not positional materials. For Varelas and Becker (1997), the value of a block has no relation to its spatial distribution. The authors are saying that a handful of blocks are worth the same value regardless of their place or the way they are arranged. So, Pimm (2018) presents a simple argument that will help our understanding. For him, in order to establish whether a system that uses the same symbols to represent quantities is positional or not, it is necessary that we take a group of these symbols (we understand that two different symbols are enough for comprehension) and mix them. If the configured value changes depending on the symbols' arrangement, then the system is positional. If the value does not change, it is not a positional system. To exemplify, let us take some symbols of the Egyptian numeral system to register a certain quantity. Is the value altered if we change their order?

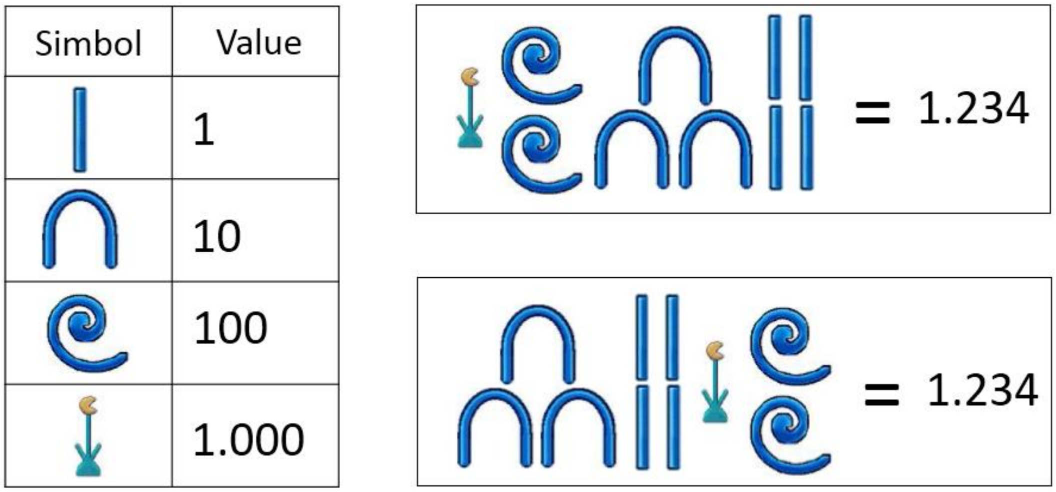


Figure 5: Arrangement of Egyptian numbers.

We realize it is not. The value recorded is the same in both cases. The record of Egyptian numbers is organized in a way so that the symbols representing higher powers of ten are placed on the left of those representing lower powers of ten. However, because the value of each symbol has been previously established by convention, the way the symbols are arranged in the writing of quantities is not related to their position, it aims to facilitate reading and interpretation. Indeed, observing the two arrangements previously presented, although both represent the same quantity, it is easier to identify the value of the first than to identify the value of the second. Therefore, we understand that the Egyptian numeral system is not positional.

For Pimm (2018), the process of codification that sustain numeral systems such as the Egyptian is identical to the codification process embedded in Base Ten Blocks. Our only disagreement is the fact that in the Egyptian numeral system the values ascribed to each symbol seem random. Although the same thing happens to Green Chips, it is not the case with Base Ten Blocks, whose value is explicit in the object’s materiality. Since the values of the symbols, in their arbitrary or motivated attributions, are invariable, the system does not need to establish a positional organization to guarantee the register of quantities. **Figure 6** applies the same idea of **Figure 5** to Base Ten Blocks and Green Chips:

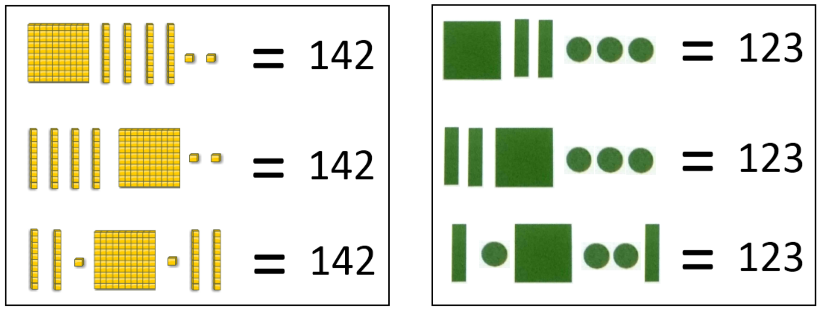


Figure 6: Organization of quantity record with Base Ten Blocks and Green Chips.

When the child begins to understand the value of blocks and chips, and how the exchanges occur, another journey begins: it is time to introduce the concept of relative value. It is necessary to show how the place-value system allows for the register of quantities by "recycling" the ten digits, that can be infinitely rearranged to register any quantities. Facing a set of blocks, the child is expected to identify the quantity they represent and to register it using Arabic numbers. Even though it is written in a simplified way, we understand this is not an easy process, nevertheless the cognitive processes involved in the learning of numbers and numeral systems are not our subject of study.

Hence, we conclude it is not a problem when textbooks indicate the use of Base Ten Blocks and Green Chips without inserting them in tables or labeled charts, even if they are always arranged in respect to their magnitude order: blocks and chips that represent a higher power of ten are always to the left of those corresponding to a lower power of ten.

A thorough problematization is called for by type 2 indications to the use of Base Ten Blocks or Green Chips–the said indications insert or superpose blocks and chips inside place value charts. Thus, we raise the debate on the place value principle, one of the components of the Arabic numeral system.

The Place Value Chart (PVC) is a calculating machine and it highlights that all positional systems are comprised by additive and multiplicative principles. Non-positional symbols with conventional values, the digits from 0 to 9 in the case of the Arabic system, are inserted in columns to count the number of different powers of ten that will result in the record of the desired quantity. Thus, these symbols or digits possess, each of them, a conventional value, known as absolute value. 7, for example, always corresponds to 7 units, as well as five 5 always corresponds to 5 units. Each digit, however, takes on a relative value, which is related to the type of 'thing' being counted. When inserted in a position, a column, a digit starts to correspond to a certain "packet size" from that same column, that is after all a power of ten. To put it differently, considering whole numbers only, a digit 7 inserted in the third column from left to right corresponds to 7 "packets" of 100 units, that is 7×10^2. The result of the calculation generates the relative value of 7 in this position: 700. Here we have an invariable rule: for a positional system to work, non-positional symbols with conventional values (including a symbol that represents the absence of quantity) shall be inserted in positions. In a base n system, each of these positions carries a base n potency and, starting from zero, the exponents increase one unit at each new column to the left.

To teach the Arabic system we need to stress that the same digit can take different relative values in a same number. For this understanding to take place, the register of quantities larger than nine units will always appear inserted in a place-value chart. We ask children to multiply each digit by the power of ten relative to its position, and after that we instruct them to add those values, thus obtaining the initial number. Hence, the place-value chart is a fundamental structure for children to understand how it is possible that the same digit represents different quantities. **Figure 7** exemplifies the matter.

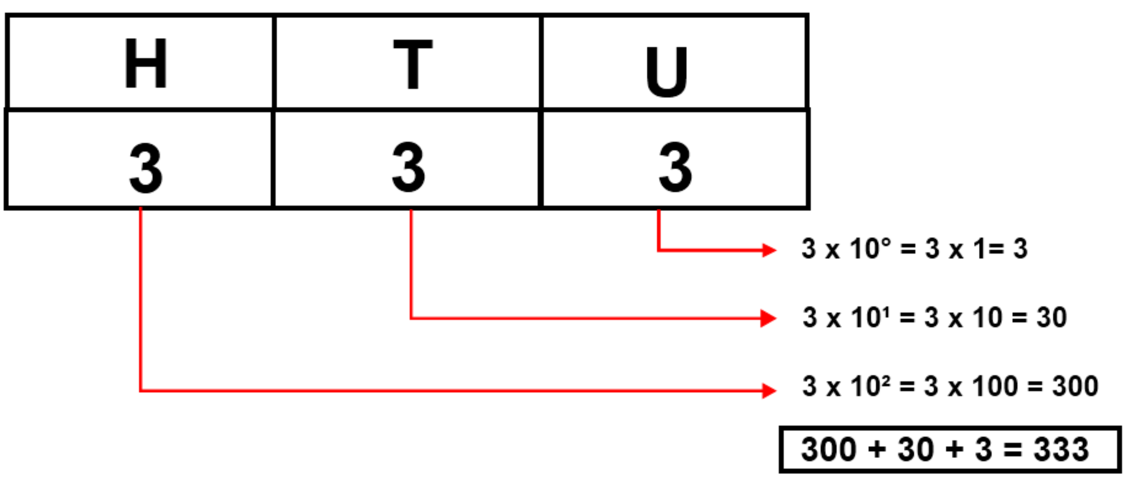


Figure 7: Place value chart and the additive and multiplicative structures in a number.

Thus, what is the meaning of inserting blocks or chips, objects with conventional or evident values, in a place value chart? It means the creation of an evident conflict. If a block or chip that is worth 100 units (flat or square chip) is inserted in the order of the hundreds in a place value chart, what is supposed to happen? One may either consider that its absolute value is now 1 unit – which is absolutely incoherent, since students are constantly reminded that flats and square chips are worth 100 units – or its relative value becomes 10.000 units, for 100 (the added value of the flat or the square chip) multiplied by 100 (multiplicative factor of the order of hundreds) equals 10.000.

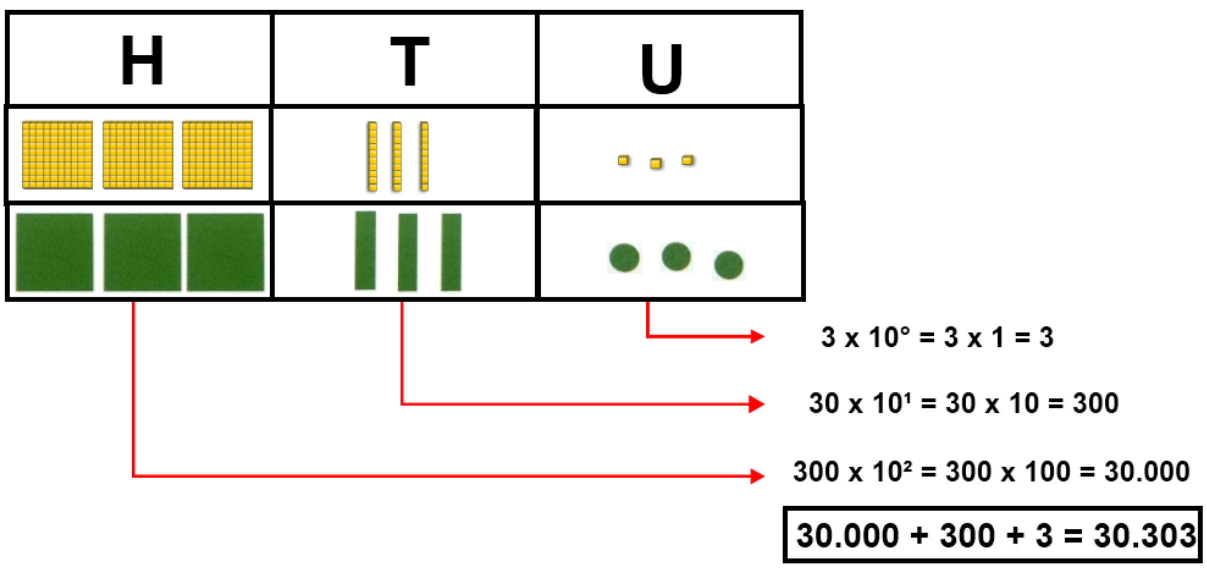


Figure 8: Place value chart and the additive and multiplicative structures in a number represented by manipulatives.

Type 2 representations, with different manipulatives and pictorial materials, are present in several publications, and not only in textbooks, but also books and handbooks (instructional or not) for teacher training (Toledo & Toledo, 1997; Brasil, 2007; Brasil , 2008; National Center on Intensive Intervention, 2015) and academic articles published in academic proceeding or high impact journals. Examples can be seen in the works of Fuson (1986), Murata and Stewart (2017), Fraivillig (2017). In some cases, we read or listen to the argument that even faced with activities that explore Type 2, many students are used to solving the problem without further difficulties. Baroody (1989), however, warns that it is perfectly possible that students learn to use the manipulatives in a mechanical way, so as to obtain the desired answer, even if they do not understand the concepts being taught.

In contestation, Silveira (2014, 2016, 2018, 2019), Ladel and Kortenkamp (2016, 2016b) affirm that these forms of representation are deceitful, because at the moment a child understands place value, and that is exactly our goal, such representations become false. The authors affirm yet that this is a recurring error when added value materials are utilized.

In regard to Base Ten Blocks, Ladel and Kortenkamp (2016) suggest that they should not be used in place value charts. Conversely, Silveira (2014, 2016, 2018, 2019) understands that all manipulatives or pictorial materials that possess added value can certainly be used within place value charts. The primary condition however is that only pieces that represent units should be used. The image below portrays this possibility.

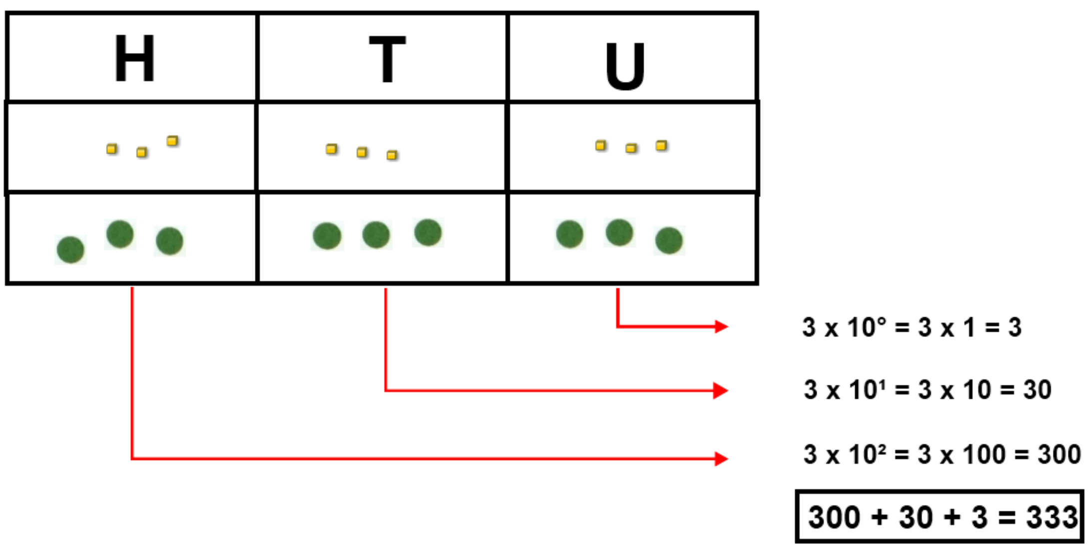


Figure 9: Place value chart and manipulatives representing a number.

Type 3 indications demand some attention as well. In these indications the labels H, T and O or Hundreds, Tens and Ones are replaced either by the blocks' names, for Base Ten Blocks, or by the chips' names, for Green Chips. The intention of the authors is probably to aid children in the organization of blocks and chips and to facilitate the counting. Notwithstanding, this usage may lead to confusion.

Figure 10 indicates a way to organize the work with Base Ten Blocks. Apparently in this case the aim of the chart is to help the child to count and consequently to make additive operations. Beside each line in the chart there is a numeric value that ought to correspond to the value of the blocks in that same line. It appears to be an interesting form of organization but, although it is not a place value chart, it brings objects that have been through a process of codification in previous classes, which ensures them a determined value. At this point the child is expected to perceive that these objects are more than pieces of wood, they represent quantities. So, what would be the meaning of these three flats that appear in the first line of the first column? Are they 3 longs, 3 tens or 30 units?

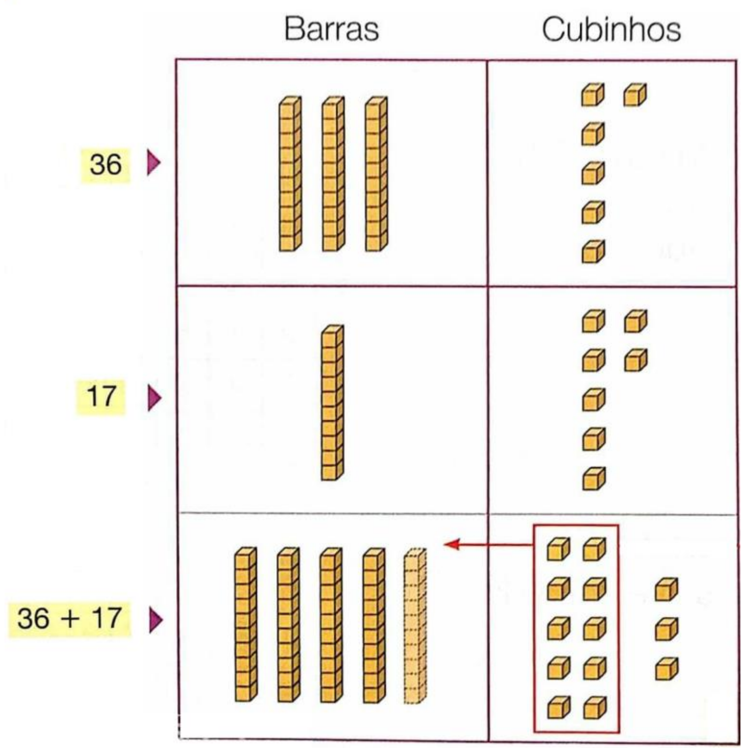


Figure 10: Labeled chart used to exemplify a sum with two parcels usis Base Ten Blocks.
Source: Adapted from collection Projeto Buriti Matemática – 3rd grade.

Although it seems logical for those who know the manipulatives and know a lot about numbers, is it also simple for a child in the learning stage? Corroborating this information, Ball (1992) affirms that because we have already developed this mathematical understanding, we can “see” mathematical ideas represented properly in some objects. Meanwhile children, on the other hand, may see other things. To the same end, Thompson and Lambdin (1994) alert to the possibility that students may have multiple interpretations about certain manipulatives. According to these authors, teachers need to be aware when presuming that students can see in manipulatives what they are supposed to see. The discussion on the use of manipulatives brought forward by this paper draws attention to some complexities: at a given moment, children are expected to look at a bar, for example, and to see 1 ten or 10 units. Later, in a different activity, they are expected to see only longs.

Figure 11 presents a comparison between two possibilities for the organization of a certain amount of Green Chips in a labeled chart. In one of the cases we use the chips, in the other we use digits from the Arabic system.

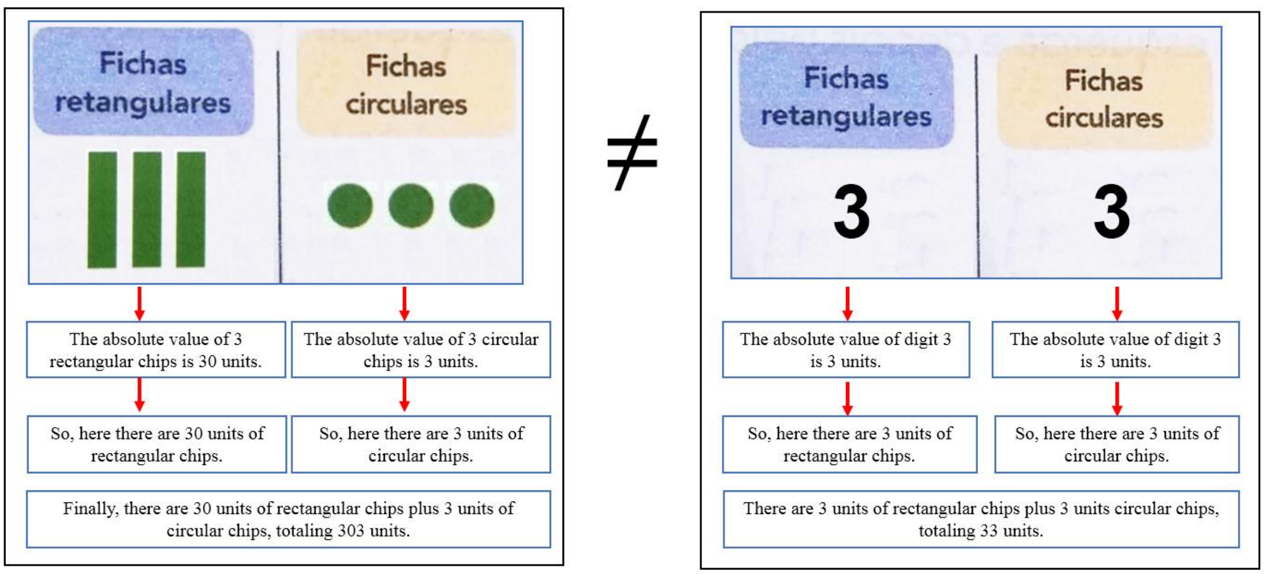


Figure 11: Comparison between Type 3 indications to use with Green Chips and with Arabic system digits.

Gluck (1991) presented directions to the use of an organization chart, designed for blocks of different orders of magnitude – she also gave instructions on how to build it. Although the author did not report more complex problems, she prefers not to add labels to each columns of the chart, because they seem to distract students. Added to that, labeled charts–in which blocks, or chips are placed–set an environment that may hinder and mislead children's learning.

Final Remarks

Based on a literature review, this paper initially offers some considerations on the use of textbooks by mathematics teachers. In sequence, we attempt to demonstrate how books present themselves as fundamental tools, and how teachers value and respect their contributions. The next step was to evince that textbooks have given central importance to the use of manipulatives, such as Base Ten Blocks, in their propositions, probably influencing teachers' practices. In a second moment, and still supported by literature, we introduce a discussion about manipulatives, showing their importance and their possible limitations of use in the teaching and learning of mathematics. The research reported here is based on and justified by these considerations.

While the textbooks studied here have been through the scrutiny of PNLD consultants, Reys, Reys and Chávez (2004) affirm that it is uncommon for textbooks consumers to demand evidences of the efficacy of these didactic materials from the publishers. According to the authors, textbooks' editors hardly ever offer scientific proof regarding the efficacy of the textbook's use in classrooms. These authors, as well as the findings presented in this study, offer indicatives that reinforce the need of a thorough investigation of the textbooks used in schools.

Furthermore, it is also necessary to stress the importance of attention when using manipulatives. Although they may be important allies in mathematics classes, if poorly designed or misused they become prejudicial to children's learning, because represent poorly, or fail to represent, the intended relationship or mathematical notion. When textbooks bring equivocal or confuse indications to the use of manipulatives, we observe a worsening of the situation. Books wield a form of authority on teachers and may endorse uses and representations that we believe to be equivocal or confuse. They can generate in the teacher a certain ease in developing the activities, which sometimes keeps them from questioning the use they make of manipulatives, even when faced with students' learning difficulties.

Therefore, we understand that mathematics textbooks are didactic materials that need to undergo strict scrutiny, by their authors and editors, by the institutions and programs that evaluate them, and also by researchers interested in the theme.

Finally, we understand that the outcomes of this research contribute to the field of mathematics education in many research fronts. One of them concerns the study of textbooks, didactic materials which are extremely present in classrooms all over the world. Added to that, this paper offers an encompassing discussion on the use of manipulatives in classrooms, highlighting limitations regarding the indications to the use of these materials. These two research fronts are of significant importance for the field. We leave new gaps still and, as a result, new possibilities of research remain open. It is necessary to know if and in what way the problematic cases reported in this paper affect the learning of children. The call is launched.

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