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Abstract**:** The field of the history of mathematics education, inscribed in the field of research on mathematics education in Brazil, has been developed as a result of the work of many groups of researchers, dedicated to varied themes under the light of various theoretical and methodological approaches. While it is impossible to describe in detail such studies in a single article, the objective of this text is to offer an overview of research results on a specific topic – mathematics in secondary education and training of teachers to teach at this level, over the years.
The current situation of the country regarding education in general is related to the past and different Brazilian political periods: Colony, Empire and Republic. We will succinctly examine the beginnings of Brazilian basic school education before the Republic, seeking to highlight characteristics of secondary school studies, especially regarding the teaching of mathematics. Then, we will focus on the same themes during the period after the establishment of the Republican regime. Finally, we will discuss the historical trajectory of the training mathematics teachers who work on secondary education in the country.

*Keywords***:** History of mathematics education in Brazil. History of secondary education in Brazil. History of the formation and practice of mathematics teachers in Brazil.

Introduction

In recent years, research on the history of mathematics education conducted in Brazil has undergone a unique qualitative improvement. Numerous groups, working with the most diverse themes and theoretical foundations, have not only been promoting research, but also supporting and promoting the creation of regional, national, and international journals and organizing events, as well as stimulating the creation of new research groups. While it is impossible to detail in a single article the numerous themes and references mobilized in such research work; our intention is to give the reader an overview of how Brazilian investigations, in the specific field of the history of mathematics education, within the field of mathematics education, have developed studies that, taken together, enable the understanding of the history of Brazilian education in general, and particularly the teaching of mathematics over the years. We chose the specific theme of secondary education and presented a narrative of what has been investigated

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by several researchers within various theoretical and methodological perspectives regarding this schooling period. We believe that far from exhausting the theme or detailing all the different approaches used to investigate it, this overall picture of secondary education will give the reader an overview that will enable a more clear characterization of educational movements and policies implemented in Brazil over the years. It can also contribute to the understanding these factors aiming at more specific and historically founded interventions regarding current initiatives, which are set in the very disturbing present political scenario, with the rise of the far-right to power with the 2018 elections.

The current organization of the Brazilian educational system was established by *Lei de Diretrizes e Bases da Educação Nacional* (Guidelines and Bases of National Education), promulgated on December 20, 1996, almost five hundred years after the country was discovered and colonization by the Portuguese began. In addition to early-childhood education for children up to five years old, the structure of basic education in Brazil, according to legislation in force, is comprised of elementary school (nine years) and high school (three years).

The expressions "primary education" and "secondary education" to name levels of education are no longer used in the country. However, they became remarkable markers for referencing two distinct stages of school education. The first involves the most elementary education, which originally comprised four grades and was, to some extent, the level completed by most of the population who had access to education. Currently, this level corresponds to the first part of elementary school, from first to fifth year. Much smaller numbers of Brazilians were attending the later stage, usually known as secondary education, which was a preparatory phase for higher education. In today's educational system, what was previously denominated secondary education nowadays encompasses the last four years of elementary school and the three years of high school.

With a territory of continental dimensions and the most varied physical landscapes (climate, geography, vegetation), and demographic, economic, political, social and cultural conditions, Brazil has experienced three major periods from a political standpoint: as a Colony of Portugal (1500-1822), as an Empire (1822-1889) and a Republic (since 1889).

Social inequality, still pronounced today, marks the trajectory of Brazil since the colonial era. It is imperative to keep such inequalities in mind, which have a strong impact on the educational levels reached by the population, in any study of the history of education in Brazil, and particularly an examination of the path of secondary mathematics education in the country. One of the main contributing factors to social inequality, including educational factors, lies in the existence and endurance of slavery – initially of indigenous, but mainly of African peoples, from around 1530 to 1888. For more than 350 years, those peoples were excluded from the few existing schooling institutions in pre-Republican Brazil. Combined with the different characteristics of the economic development models of the country, which led to the increase of poverty, concentration of income, disparities in access to services, uncontrolled urbanization and violence, the legacy of slavery played an unquestionable role in the current levels of schooling of the Brazilian population.

When the Republic was established in 1889, approximately 85% of Brazilian adults were illiterate (Romanelli, 2005). According to data of *Instituto Brasileiro de Geografia e Estatística*[2], in 2018, a 6.8% illiteracy rate[3] persisted among the population aged 15 years or older, whereas the rate reached 18.6% among those over 60 years of age. Basic and compulsory school education, which includes the entire secondary education, had been completed by less than half (47.4%) of the population over 25, which amounted to about 130 million people. The rates also show that Caucasians significantly outperform Blacks, and that are differences between men (45%) and women (49,5%). It is also essential to highlight the large regional discrepancies regarding the levels of education among the Brazilian population. In summary, after 130 years, since Brazil became a Republic, the completion of secondary studies continues to be achieved precariously in the country, with many disparities in terms of color or race.

This situation stems from the past and the different political periods of Brazil. Due to the space limitations of the present article, in order to focus on the history of mathematics in secondary education and teacher

[2] Data derived from *Pesquisa Nacional por Amostra de Domicílios Contínua* (Continuous National Household Sample Survey) 2016-2018 and can be obtained at the website of *Instituto Brasileiro de Geografia e Estatística* (www.ibge.gov.br).

[3] This percentage corresponded to a total of 11,3 million people.

education, it will be necessary to present aspects of the educational trajectory of the country through time in a general and simplified way. We will briefly examine the beginnings of Brazilian schooling before the Republic, seeking to highlight characteristics of secondary school studies and especially mathematics. Then, we will focus on the same themes after the establishment of the Republican regime. Finally, we will discuss the historical path of teacher-training courses for teaching mathematics at secondary level in the country.

From Colony to Republic (1500-1889): Mathematics in the embryonic stage of secondary education in Brazil

Since the beginning of the colonization - sometimes romantically called "the discovery", schooling in Brazil had been an almost exclusive prerogative of priests of the Society of Jesus, who arrived in the country in 1549 and, led by Father Manuel da Nóbrega, created the first elementary school in the city of Salvador, currently the capital of the state of Bahia. The Jesuit education network expanded with the foundation of elementary schools in other locations, as well as high schools, gradually established in various places[4].

It has been reported that after the first rudiments of arithmetic were taught by elementary schools, continuing education in Jesuit institutions was centered on the classical humanities, with little room for mathematical knowledge and great emphasis on Latin. Starting in 1648, concurrently to the education provided by the Jesuits, the Portuguese Crown hired specialists in military courses to teach and train personnel for working in fortifications aimed to defend Brazilian lands. The discovery of gold led to growing concerns about defending Portuguese interests, and to the development of military education, for which mathematical knowledge was indispensable (Valente, 1999).

[4] These schools were created in Porto Seguro and Ilhéus, also in Bahia, and in the now states of Espírito Santo and São Paulo (São Vicente and São Paulo de Piratininga). The schools were initially founded in three Northeastern states – Bahia, in 1556, Pernambuco, in the city of Olinda, in 1558, and Maranhão (1622), as well as in the Southeaster Rio de Janeiro, in 1567, and São Paulo, in 1631. Later, other such schools were also created in other regions (Shigunov & Maciel, 2008).

In 1759, Marquis of Pombal, then Prime Minister of Portugal, expelled the Jesuits from all colonies and Brazil was left with a few schools, run by other religious orders, in addition to military educational institutions. To fill the gap created by the departure of the Jesuits, in 1772, the Portuguese government created the so-called "royal classes", where grammar, Latin, Greek, philosophy and rhetoric were taught in isolation, and later the mathematical disciplines of arithmetic, algebra and geometry. What is known about this phase is that mathematics classes were few, and attendance was scanty (Miorim, 1998).

While Brazil remained a colony of Portugal, and even after, there were seminaries and schools maintained by religious orders for secondary education, private schools and tutors, as well as the so-called Lyceums in a few places. In 1808, the relocation of the king of Portugal and the entire court to the country, due to the invasion of Napoleon, significantly modified the educational and cultural characteristics of the Brazilian scenario, resulting in the creation of several institutions, including those responsible for training civil and military engineers.

In 1822, Brazil became independent of Portugal by an act of Prince D. Pedro, who became the first ruler of the first constitutional monarchy then instituted. Although the first constitution of the country, promulgated in 1824, and in force throughout the imperial period, guaranteed gratuitous primary education to all Brazilians, only on October 15, 1827, did Congress vote the first bill regarding national public instruction in Brazil. That law stated that there would be first-letter schools in every city, town, and populated area[5].

[5] It is important to highlight a relevant aspect related to this initial level of schooling. In 1834, the government promoted a decentralization of teaching, which transferred the responsibility of elementary "first letters" education to the administrative divisions represented by the provinces. As a result, it was not possible to establish a school system capable of serving the population of the entire country. The marks of exclusion in Brazil are old, as the colonizing metropolis was against the Protestant Reformation and, thus, considered natives as barbarians and black slaves as property of their masters. For this significant portion of the population, education was virtually expendable. These circumstances were associated with the natural difficulties of providing educational institutions in an immense, unpopulated country, with enormous distances (Cury, 2003).

At the beginning of the 19th century, several secondary education institutions trained students for admission exams to the few establishments which provided military training or higher education. For law schools, in particular, the exams dealt with several subjects, among which geometry. Valente (2004) emphasizes that this requirement changed the role of mathematical knowledge in education: previously regarded as merely technical-instrumental, necessary for trade and military training, such knowledge became one of the components of overall culture.

In 1837, Bernardo Pereira de Vasconcelos, Minister of the Empire, inspired by the organization of French schools, created the first public secondary school in Rio de Janeiro, where the court resided, which was called *Imperial Colégio de Pedro II*, designed to function as both a boarding school and a day school. Studies at this institution lasted between 6 and 8 years and included, above all, classic-humanistic disciplines. However, modern languages, natural and physical sciences, history, and mathematics, comprising Arithmetic, Algebra, Geometry and, later, Trigonometry were also taught. Students who passed all subjects in the course were exempted from exams to be admitted to higher education courses. Despite the predominance of literary and humanistic disciplines, mathematics remained present in all grades of the course at *Imperial Colégio de Pedro II*, in the various reforms that modified its educational plan over the years.

Imperial Colégio Pedro II became the model institution for secondary education in Brazil and, until 1873, students from other provinces had to go to Rio de Janeiro to take their exams, which were centralized there (Veiga, 2007). Subsequently, legislation authorized the application of such exams in the provinces.

Although secondary education was offered in several institutions during the empire, the establishment of graded secondary education was slow in the country and all instruction was carried out with a view to preparatory exams. After the creation of *Pedro II*, there were attempts to require a certificate of completion of secondary studies, but throughout the 19th century, passing the various exams was sufficient to gain entry to higher education. Many students who passed these exams did not finish secondary school and, as a result, graded secondary education would not develop (Valente, 2004).

Secondary studies were carried out almost exclusively by the male economic elite of the country, who were preparing to occupy political-administrative positions, or pursue higher education. Women from upper classes were generally educated for household activities and social roles in either lay or religious women's schools, or at home, with the help of foreign tutors. They learned the first letters, French, music, piano, and homemaking skills[6]. Women from lower classes could attend elementary school instruction, teacher-training courses in the so-called *Escolas Normais*, and vocational schools.

Secondary education in Brazil during the First Republican period (1889-1930)

The period between 1889 and 1930 is usually referred to as the First Republic. Brazil was the last western country to abolish slavery, and dethrone the monarchy associated with it a year later. According to Schwarz (2012), the Republic emerged under circumstances that promised equality and citizenship and whose model of civilization was represented by France. As previously mentioned, when the Republic started, the situation of education was intolerable in a country where most of the population was illiterate. Benjamin Constant (1836-1891) was the first head of the governmental agency responsible for education: The Ministry of Education, Post and Telegraph. In 1890, he launched an educational reform that became known with his name. That reform solely encompassed public education at primary and secondary levels in the Federal District, then situated in Rio de Janeiro.

At secondary level, legislation aimed to break away from the humanist and literary tradition by adopting a curriculum that favored scientific and mathematical disciplines. Benjamin Constant and the group of Brazilian military men who led the movement for the installation of the Republic adhered to the ideas of French philosopher Auguste Comte (1798-1857). In line with Comte's positivist ideology, mathematics was considered the most important of sciences; the discipline acquired great relevance in Benjamin Constant's proposal for educational reform, particularly in the seven years that then comprised secondary education (Miorim, 1998).

[6] In the 1880, some women started studying at *Pedro II*. In 1887, Rita Lobato was the first woman in the country to graduate as a medical doctor, in Rio de Janeiro; the only woman in her class (Bandeira, 2013).

In the beginning of the Brazilian Republic, French ideas and practices strongly resonated, in education, as well as in other areas, and the teaching of mathematics was not different. The influence of textbooks by French authors was enormous and lasted from the beginning of secondary education in Brazil onto the first three decades of the republican period (Valente, 1999).

At the beginning of the 20th century, preparatory exams began to be organized according to the higher education courses sought by candidates. A list of contents to be used in the tests required for admission to the various existing courses included arithmetic, algebra, geometry, and trigonometry.

> For instance, for admission to medical and surgical sciences, the candidate should take exams in arithmetic, algebra up to 1st degree equations, geometry, and rectilinear trigonometry. For the course in Law and social sciences, arithmetic and geometry; for the polytechnic school, only arithmetic; for pharmacy, arithmetic, algebra, up to 1st degree equations, geometry though only plane; for dentistry, arithmetic up to and including proportions; fine arts, arithmetic, algebra, geometry and rectilinear trigonometry (Valente, 2004, p. 108).

After the institution of the Republic, *Colégio Imperial D. Pedro II*, which was renamed as the *Ginásio Nacional* (National Gymnasium), constituted the golden standard for secondary education in general, and particularly mathematics. From 1901, institutions whose didactic-pedagogical practices were the same as of National Gymnasium were authorized to grant their graduating students the title of bachelor. In all such schools, mathematics studies were marked by preparatory exams. After 1916, schools outside Rio de Janeiro were able to set up examining boards for preparatory courses locally, and organized their courses for that purpose each year. The operation of schools in which the completion of secondary education granted automatic entry to higher education, such as the former *Colégio Pedro II*, was also guided by preparatory exams. Thus, regular students of such schools took, for example, the arithmetic exam to continue their studies, whereas the same exam could be taken by an external student who wished, through the so-called *graded* exams, to overcome the content related to arithmetic of the preparatory exams (Valente , 2004).

Since the reform implemented by Benjamin Constant, several educational reforms were carried out in Brazil until 1930, year which inaugurated significant political change in the country. However, according to Miorim (1998), there were no relevant changes in secondary education, which “continued to

be seen as intended for the preparation of liberal professions, such as law, medicine and engineering" (p. 89). Until the late 1920s, preparatory exams played a paramount role in the organization of mathematics content in secondary school.

The beginning of the Republic in Brazil was marked by turbulent events, with several social riots in the countryside and cities. Although slavery was abolished, there were no social policies to support freed slaves and, simultaneously, influential discriminatory racial theories spread until the 1930s. Schwarcz (2012) points out the pressure of an unsustainable burden on blacks and mestizos. Theories proposed deterministic models of social interpretation that hierarchized races and condemned miscegenation.

> According to such models, the explanation for the lack of professional and social achievement of black people and freed slaves could be found in science, or rather, in race, and not in the living conditions or the immediate past. In fact, the joint and massive spread of these theories pushed the post-Abolition debate away from issues of citizenship and equality, in the name of biological reasons and arguments (Schwarcz, 2012, p. 61).

Educational exclusion, largely originated in the practice of slavery and in the permanently prejudiced and segregating character of Brazilian society, persisted, despite the hope that had arisen from the proclamation of the Republic. In 1920, the rate of illiterate Brazilians over 15 years of age was 65% (Braga & Mazzeu, 2017). A privileged minority who would later go to university attended secondary education. Teachers at this level and, among them, mathematics teachers, did not have specific training for the job, as there were no institutions destined for such preparation. Engineers and graduates from military courses taught mathematics at Brazilian secondary schools (Valente, 2008).

Since the proclamation of the Republic, education has been considered a central political problem, seen as essential for overcoming the country's underdevelopment. Until 1988, political citizenship required reading and writing proficiency, as the previous Brazilian constitutions vetoed the vote of illiterate citizens. The propagation of schooling was strongly defended based on the idea that the path to the development of the country resided in the access to education of large portions of the population. In the 1920s, amid profound political, economic, and social change, reforms in the education

system were carried out in several Brazilian states, as well as the Federal District, encompassing primary education and teacher-training for that level. The changes effected by state and Federal District laws were linked to the pedagogical movement known, among other denominations, as *New School* or *Progressive School*[7].

Such movement sought to implement, in primary school, ideas being developed in Europe and the United States since the 19th century, through the work of several educators from different countries[8]. Miorim (1998) highlights two fundamental ideas shared by different new school views: the "activity principle" and the "principle of introduction of real-life situations into school", which brought changes to teaching in the early years of schooling, and specific reflections on how to approach mathematics.

According to Miorim (1998), this movement of pedagogical renewal did not immediately affect secondary education, whose practice continued to be guided "by bookish teaching, unrelated to students' live, based on memorization and passive assimilation of content" (p. 90).

In 1908, the fourth International Congress of Mathematics took place in Rome, in which an international commission[9], chaired by the German mathematician Felix Klein (1849-1925), was created to deal with teaching issues. One of the goals of such commission was to carry out a study on mathematics in secondary education in several countries, including Brazil. Constitution of such a commission points to the existence of a first international movement for the modernization of instruction. The main proposals of this movement were: promote the unification of mathematical content covered in school into a single discipline, emphasize the practical applications of mathematics and introduce the teaching of differential and integral calculus at secondary level.

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[7] According to Veiga (2007), the terms "modern school", "progressive school" and "labor school" were also used.

[8] Although New School was nurtured by a wide spectrum of theories, some principles have been established as identifying features. According to Vidal (2003, p. 501), such principles were "the centrality of the child in the learning relationships, respect for hygienic norm in disciplining students' bodies, and their gestures, scientificity of school regarding social knowledge and behavior and the laudation of observation, intuition, and the construction of students' knowledge".

[9] This was CIEM (Comission Internationale de l'Enseignement Mathématique) or IMUK (Internationalen Mathematischen Unterrichts Kommission), which since 1954 became known as ICMI – International Comission on Mathematical Instruction.

Secondary mathematics education in Brazil after 1930

The Vargas Era (1930-1945)

Due to dissatisfaction with the results of the 1930 presidential election, a movement led by the states of Rio Grande do Sul, Minas Gerais and Paraíba, with crucial military participation, took over the government of the country. With this takeover, dubbed the Revolution of 1930, Getúlio Vargas (1882-1954), a politician from Rio Grande do Sul, rose to power. From 1930 to 1934, Vargas was the head of a Provisional Government. In 1934, he was elected president of the republic by the National Constituent Assembly. In 1937, a *coup d'état* established the so-called *Estado Novo* regime, through which Vargas, supported by the military, became a dictator who extinguished political parties and closed congress. In 1945, opposition to that government succeeded in obtaining the resignation of the chief of state, and free elections for the legislature and presidency were held. After 15 years, Brazil was again constituted under rule of law.

The period between 1930-1945 became known as the Vargas era, "because it was the time for the construction and consolidation of the figure of its maximum leader, Getúlio Vargas" (Gomes, 2013, p. 26). It was a time of tremendous political, social, and economic change. During the Provisional Government, the Ministry of Education and Health was created, presented as one of the initiatives that substantiated the projects of the new government, along with the Ministry of Labor, Industry and Commerce, dubbed the "ministry of the revolution". This was clearly a new interventionist orientation of the state in economic and social matters (Gomes, 2013a, p. 279). A series of decrees to regulate education became known by the name of the first occupant of the new ministry: tradition led this national legislation to be called the Francisco Campos Reform of 1931.

An essential characteristic of the proposal for secondary education of the Francisco Campos reform consisted in recognizing the training character of this level of education, in contrast to its previous nature of preparation for higher education, accessible only to the elite. This characterization of secondary

education as a formation stage is explicit in the explanatory statement of Minister Francisco Campos (1891-1968) to the head of government, Getúlio Vargas, in April 1931:

> The purpose of secondary education is, in fact, much broader than that usually attributed to it. As a rule, secondary education has been considered among us as a simple tool for preparing candidates for higher education, thus ignoring its eminently educational function, which consists, precisely, in the development of the faculties of appreciation, judgment and criteria, essential to all branches of human activity, and particularly in the training of intelligence, by putting problems in their exact terms and seeking their most appropriate solutions[10].

For mathematics teaching, the reform brought major changes, with the introduction of modernizing ideas, inspired by the proposals of Felix Klein, approved in 1928, for the program of Colégio Pedro II[11] by Euclides Roxo (1890-1950) one of the institution's teachers. According to that proposal the former disciplines arithmetic, algebra, geometry, and trigonometry, which were taught by different teachers and used different textbooks, were unified into a new discipline called mathematics, thoroughly detailed in the Francisco Campos reform. The aim of teaching mathematics was presented at the very beginning.

> The teaching of mathematics aims to develop students' spiritual culture through the knowledge of mathematical processes, at the same time, enabling concise and rigorous reasoning through clear exposition of thought in precise language.
> Furthermore, in order to meet the immediate interest of its usefulness and the educational value of its methods, it will seek, not only to awaken in students the ability to resolve and act promptly and attentively, but also favor the development of the ability to understand and analyze quantitative and spatial relations, necessary for application in various domains of practical life, as well as the exact and deep interpretation of the objective world[12].

It further stressed the need to always keep in mind the degree of mental development of students and their interests while teaching, and insisted that their activity be constant, so that students were

[10] *Novíssimo Programa do Ensino Secundário* (New Secondary Education Program) (as per art.10, decree n. 19.890 on April 18, 1931). Rio de Janeiro, 1931.
[11] The institution which had its name changed, after the promulgation of the Republic, resumed the use of the former name in 1911.
[12] *Novíssimo Programa do Ensino Secundário* (New Secondary Education Program) (as per art.10, decree n. 19.890 on April 18, 1931). Rio de Janeiro, 1931.

"discoverers and not passive recipients of knowledge[13]". Therefore, it recommended renouncing "the practice of memorization without reasoning, abusive presentation of definitions and rules and systematic study of ready-made demonstrations". In addition, it highlighted that teaching should result from intuition; particularly geometry, whose study of formal demonstrations should be preceded by experimentation and construction activities. The concept of function was valued as a "core teaching idea", first presented intuitively and gradually developed throughout the grades.

There were also specific guidelines and a list of contents for arithmetic, algebra, and geometry, for the five grades of the elementary course, following the four-year primary course. Mathematics was included in these five years, and in the last grade, teaching of basic differential and integral calculus, limit, derivative and integral, was prescribed, according to Felix Klein's recommendations. Students who wished to go to university should then take a complementary course, which lasted two years, directed towards the type of course to which they sought admission. For the careers in medicine, pharmacy and dentistry, mathematics would be taught in one year; for engineering, chemistry, and architecture there was a two-year course.

Revolutionary compared to the prevailing teaching tradition, the proposal for mathematics of the Francisco Campos reform was attacked in many ways (Miorim, 1998). Teachers at the time had difficulties adapting, which was exacerbated, at first, by the lack of textbooks in line with the new guidelines. Some defended the teaching of classical humanities, especially Latin, and strongly criticized what they considered to be an excess of content in the reform program, as well as the merger of mathematical disciplines into a single course. Mathematics teachers who favored previous teaching system, in which mathematics was conceived mainly as a mental discipline, believed that the new proposal, which began impacting some textbooks of a more intuitive character, demeaned teaching[14].

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[13] The quotation marks in this paragraph are excerpts transcribed from Francisco Campos reform proposal.

[14] The chief representative of this group was a teacher of *Colégio Pedro II*, Joaquim Inácio de Almeida Lisboa, who attacked Euclides Roxo, the person in charge of the mathematics reform, through many articles published in newspapers at the time.

From 1942 to 1946, Brazilian education underwent new reforms, through a series of decrees that created important industrial and commercial education institutions, respectively *Serviço Nacional de Aprendizagem Industrial* (National Service for Industrial Education) and *Serviço Nacional de Aprendizagem Comercial* (National Service for Commercial Education)[15], as well as standardized industrial, commercial, primary, secondary, teacher-training and agricultural schools. That set of decrees became known as the Gustavo Capanema reform, named after the Minister of Education in office between 1934 and 1945. Secondary education, regulated in 1942 through *Lei Orgânica do Ensino Secundário* (Organic Law of Secondary Education), was organized in two cycles: the four-year long junior high school, and the three-year long high school, with two areas of knowledge: classic and scientific. In addition to teacher-training, whose objective was to train teachers for primary school, a secondary technical-vocational branch was created, subdivided into industrial, commercial, and agricultural. This set of reforms displayed a centralizing and dualistic character in the sense that it separated secondary education, destined for the elites, from professional education, for the lower classes, as only secondary school graduates had the right to apply to higher education (Saviani, 2007).

The Organic Law of Secondary Education was accompanied by a ministerial ordinance, dated July 17, 1942, which established the programs for the disciplines taught in junior high school, a part of secondary education. Unlike what had happened with the Francisco Campos reform, the Gustavo Capanema reform did not detail such programs. The ordinance simply presented lists of contents, without any methodological indications for addressing the various issues. The mathematics programs for the first two grades were divided into two areas: intuitive geometry and practical arithmetic, while those of the last two grades included content related to algebra and deductive geometry separately. After the Francisco Campos reform, several textbook collections were published containing five volumes whose objective was to comply with the provisions of that proposal for the elementary course. After the Capanema reform,

[15] Senac was created after Getúlio Vargas was ousted, on January 10, 1946. According to Romanelli (2005), however sanctioned by the new government, the institution seems to have been designed by the previous administration.

authors and publishers reorganized such collections into four volumes and relaunched them on the market to meet the new structure of secondary education (Valente, 2005).

It was also in the Vargas Era that the specific training of teachers for secondary school through higher education began in Brazil. In 1934, the first courses were established at the Faculty of Philosophy, Sciences and Arts of the University of São Paulo, including those for certifying mathematics teachers. Noteworthy, in the same period, was the creation of the University of the Federal District, in Rio de Janeiro, in 1935, which also included the training of mathematics teachers for secondary education.

From the end of the Vargas Era to the New Math Movement

Starting in 1950s, school subjects, including mathematics, began to change. The transformation of the economic, social, and cultural conditions in Brazil and the possibility of access to school began to require changes in the functioning and purposes of the institution, which had an impact on the teaching of the various disciplines. The audience was modified with the inclusion in education of lower classes students, who had long been claiming the right to schooling. It was a democratization of the school, which also started to receive children of the working-class, and the number of students in primary and secondary education increased enormously. There was, therefore, a significant change in school and pedagogical conditions, as well as cultural needs and requirements (Soares, 2011).

As a matter of fact, and due to factors other than those just mentioned, the teaching of mathematics in Brazil would change a lot at the end of the 1950s, when the first national teaching congresses were held in the country. At that time, many mathematicians and mathematics teachers became involved in the New Math Movement, considered the second international movement to modernize mathematics education, with repercussions in many countries.

Some events in Europe and the United States are often remembered as milestones of the modernization movement. The first was the launch of the Earth's first artificial satellite, Sputnik, by the Soviet Union, in 1957. The United States government, which, since the early 1950s had been mobilizing towards curriculum reforms in the areas of science and mathematics, in order to overcome the gap

between that curriculum and the scientific-technological progress of the time, intensified its efforts and funding to develop pedagogic resources, including books, and disseminate the new proposals internally and abroad (Búrigo, 1989; Fiorentini, 1995; Miorim, 1998). At the same time, in Europe, especially France, mathematicians and educators promoted events and propagated an innovative idea for teaching mathematics.

In 1959, twenty countries sent experts to a conference promoted by the European Organization for Economic Cooperation in Royaumont, France, to discuss proposals for changes in the teaching of mathematics at secondary level. The aim was to renew teaching by introducing more recently developed aspects of mathematics into the curriculum, that is, from the 18th century onwards. The foundations of the modernist movement were laid at that conference. In addition to the introduction of more recently produced knowledge, it was advocated that the precision of mathematical language be emphasized, and a new approach to traditional content was proposed, which included the language of sets, relations, and algebraic structures. It was also recommended that content be graded according to the modern logical construction of mathematics and the properties of operations be highlighted instead of placing emphasis on computational skills.

For Rios, Búrigo and Oliveira Filho (2011), the existence of teaching modernization projects in several European countries, in the early 1960s, suggests that after the Royaumont seminar the renewal movement gained momentum. The authors mention another event as important for the spread of the modernizing movement in Latin America, the First Inter-American Mathematical Conference, an initiative of ICMI, sponsored by the Organization of American States, and by the North American National Science, Ford and Rockefeller Foundations. They also emphasize the relevance of the participation of an agency of UNESCO created in Brazil[16] in 1946, widely responsible for the contact of secondary teachers with North American initiatives to modernize the teaching of mathematics.

[16] The *Instituto Brasileiro de Educação, Ciência e Cultura* (Brazilian Institute of Education, Science, and Culture).

In several states, groups of teachers were trained to tutor other teachers regarding the new proposed guidelines. Osvaldo Sangiorgi, the leader of a particularly important group (Mathematics Teaching Study Group), founded in São Paulo, in 1961, had conducted an internship at the University of Kansas, in the United States. Other prominent groups were the Study Group for Mathematics Education, in Porto Alegre; the Group for Study and Research in Mathematics Education, in Rio de Janeiro; the Center for Study and Dissemination of Mathematics Education, in Paraná; and the group of Bahia, organized by Professor Omar Catunda.

After the 3rd Brazilian Congress on Mathematics Education, held in Rio de Janeiro, in 1959, with 500 teachers from 18 states in attendance, most subsequent events addressed mainly the ideas and practices regarding the so-called "modernization of Mathematics" in Brazil. [17].

One of the main objectives of the New Math Movement was to integrate the fields of arithmetic, algebra, and geometry in teaching by including set language, algebraic structures and relations, and functions. The need to give more importance to the logical and structural aspects of mathematics was also emphasized, as opposed to the pragmatic characteristics that predominated in teaching at that time, reflected in the presentation of rules without justification and in the mechanization of procedures. The objective was that the knowledge conveyed at school reflected the fact that mathematics had become more precise and logically based since the 19th century (Miguel, Fiorentini & Miorim, 1992). Whereas, for geometry, the proposal was to replace the classic approach inspired by Euclid's *Elements* by the focus of geometric transformations, with the study of the concepts of vector, vector space and linear transformation.

It is essential to define the context in which the ideas for modernizing the teaching of mathematics have increasingly penetrated the Brazilian secondary education scenario. As Búrigo, Rios

[17] In 1962, during the 4th Brazilian Congress of Mathematics Teaching, in Belém do Pará, the Grupo de Estudos do Ensino da Matemática (Mathematics Teaching Study Group) presented some experiences conducted with New Math guidelines, as well as a program for mathematics in secondary education, based in modernizing ideas. In 1966, the 5th Brazilian Congress of Mathematics Teaching, held in São José dos Campos, state of São Paulo, focused specifically on the implementation of New Math in Brazil, and included advocates of the modernizing reform from other countries, such as Marshall Stone, from the United States, and Georges Papy, from Belgium, among others (Miorim, 1998).

and Oliveira Filho (2011) point out, such ideas are associated with a European and American movement to increase the appreciation for scientific education at a time of accelerated technological innovation and the need to train scientists and technicians. For Brazil, investing in science education was part of a developmental project or ideology, favored by the military dictatorship which had been established in the country in 1964. Búrigo et al. (2011) cite studies on the effects of the presence of the military regime on the diffusion and institutionalization of the modernizing movement for teaching mathematics and reflect on that theme:

> The secrecy and informality which pervaded many of the decisions made at the time are important traits of the authoritarianism in the period which deserve special attention from researchers, for two reasons. The first consequence of such informality is that the understanding of the processes cannot be obtained solely from official documents, or even written records. It would be necessary, therefore, to resort to the memory of individuals who participated or witnessed such decision-making. The second effect that must be noted is the combination of visible hierarchies and veiled threats. In a context where any contestation could be punished, it is quite possible that guidelines and determinations coming "from above" would be confusing and interpreted by schoolteachers as impositions (Búrigo et al., 2011, p. 52).

The researchers claim there was no evidence, at the time of their study, that the Modern Mathematics Movement, in its early years, correlated to a planned policy by federal or state agencies. However, they warn that the ambiguity of such bodies regarding consent and support without much cost, and the manifestation of a defined position regarding the movement could have been used as an advantage. Governments could benefit from the dissemination of the idea of stimulating scientific education, “without assuming the burden of defending a controversial curricular proposal, not fully understood, or whose complete implementation would be more costly” (Búrigo, Rios & Oliveira Filho, p. 50).

Many collections of textbooks, published since 1963, played an important role in the dissemination of such modernist ideas. According to Búrigo et al. (2011), the role played by books can be explained by their direct and almost immediate impact on classrooms and reach in regions far from large urban centers in Brazil, thus providing local teachers with opportunities for self-improvement. In addition,

the production of textbooks increased a lot in the 1960s, which marked a moment of modernization in the Brazilian publishing sector (Miorim, 2005).

Most mathematics books based on the structural organization of numerical sets, usually started by approaching sets, in which the presence of symbolic language was strongly evident. Only later were numerical sets discussed, in the following order: natural, whole, rational, and real, with emphasis on the inclusion relationship of each numerical set onto the next. The approach to numerical sets emphasized the structural properties of the operations defined in them, concentrating on addition and multiplication, associativity, commutativity, neutral and inverse elements, and distributiveness of multiplication in relation to addition.

Miorim (2005) underlines the difficulties of textbook authors in devising an approach in line with the modernist ideology, especially regarding geometry, as the new works did not differ much from what had been done before. Each author or group of authors worked differently on geometric content. However, there was a common aspect in the presentation of such content in many books: the use of the language of sets. School geometry was translated by the authors into their work according to their own pedagogical experiences and readings of modernist proposals. In the Brazilian scenario, there were different proposals, some closer, other more distant, from what the modernization movement advocated for teaching geometry. Matos and Leme da Silva (2011) illustrate this issue by focusing on two different works for secondary school - the collection signed by Osvaldo Sangiorgi, published in the 1960s, and a set of texts for an experimental course developed in the state of Bahia, edited in 1971 as a collection of books called *Ensino Atualizado de Matemática – Curso Ginasial (Modern Mathematics Teaching – High School)*[18]. According to the analysis of Matos and Leme da Silva, Sangiorgi made few changes to the approach of geometry, using the language of sets and the proposition of an experimental approach before the deductive approach. Without mentioning vector spaces and with superficial references to geometric transformations, his collection distanced itself from the ideas of the modernization movement. Within the

[18] The production of that work was headed by two university professors: Omar Catunda and Martha Dantas.

context of that movement in Brazil, the author offered a solution which was closer to the debates within the country than to those in international forums. On the other hand, the work of the authors from Bahia represented a bold innovation, for adhering explicitly to the treatment of geometry, proposing the study of related geometry and geometric transformations before Euclidean geometry.

Nonetheless, it can be said that the modes of appropriation of the ideas of the movement contributed to the mischaracterization of the traditional axiomatic-deductive approach to geometry in favor of an eclectic approach in which demonstrations have become less demanded than before. Several authors attribute a decrease in the presence of geometry in pedagogical practices carried out in schools to the dissemination of the ideas of the New Math Movement, both because of the important role acquired by algebra and by the difficulties teachers faced in implementing modernist proposals for geometry.

In the history of the organization of Brazilian education, many changes were brought about by *Lei de Diretrizes e Bases para o Ensino de 1º e 2º graus* (Guidelines and Bases for Elementary and Secondary Education), in 1971, which reorganized schooling into two levels. The first level, comprised of eight years, merged the former primary school and junior high. Students were no longer required to take the Admission Exam, which enabled them to continue their studies after the first four years of schooling. The second level (*high school*) was proposed as a vocational course, seeking to divert part of the demand for higher education, which did not offer enough spots for all high school graduates. For Pavanello (1993), it was not possible to carry out this vocational training in public schools, which lacked human and material resources, while private schools, interpreting the legislation according to their interests, maintained preparatory courses for higher education. In the 1970s and 1980s, partly due to the expansion of the school network, without measures for providing quality teacher-training on a large scale, in a context in which algebra assumed a predominant role, geometry virtually disappeared.

In the late 1970s, criticism of the New Math Movement arose in many countries. People of significant credibility among mathematicians, such as Morris Kline, in the United States, and René Thom, in France, took a stand against the proposals of the movement because, according to them, the emphasis was placed on mathematics for the sake of mathematics, on its formalism and structural aspects, as well

as the excessive concern with language and symbols. In Brazil, the criticism of modern mathematics and the discussion of its failure in teaching in the late 1970s and early 1980s occurred at a time of renewal of educational ideals, brought about by the end of the military dictatorship.

After the Modern Mathematics Movement

In relation to the content required for mathematics at the level previously referred to as primary school, which included four years of secondary school (junior high), alternatives to the modernist ideology arose, such as that represented by the curriculum proposal of the state of São Paulo, in 1986. Centered on three major themes - numbers, measurement and geometry, its characteristics opposed those prevailing during the predominance of concepts associated with modern mathematics. Among them were the concern with a historical approach to the themes, emphasis on understanding concepts, taking into account the development of the students, emphasis on the importance of geometry and elimination of the emphasis given to sets, symbolic language as well as rigor and precision in mathematical language.

At the end of the 20th century and in the first two decades of the 21st, other events impacted secondary education and the training of mathematics teachers for this level in Brazil. This is a reference to the implementation of postgraduate programs in mathematics at universities, starting in 1971, and the creation of specific postgraduate courses in mathematics education, at specialist level, master's, and doctorate, in several Brazilian states, starting in 1987. Many local, state and national mathematics education meetings were also important and, in 1988, the foundation of the Brazilian Society of Mathematical Education, a civil society of scientific and cultural character, whose main purpose is to bring together professionals in the area of mathematics education and similar fields. The members of the Brazilian Society of Mathematical Education are researchers, teachers and students who work in education at basic and higher levels in Brazil.

As previously stated, in 1996, the current *Lei de Diretrizes e Bases da Educação Nacional* (Guidelines and Bases of National Education) was enacted, which contains the main parameters related to education in the country, including structure.

The changes that occurred regarding the recommendations for teaching mathematics combined to the crisis of the New Math Movement, the emergence and development of the area of mathematics education, with the conduction of a significant amount of research that contemplated many trends and the most diverse contexts where mathematics is taught, have had repercussions on the most recent curricular proposals. Among those, the most important is the *Parâmetros Curriculares Nacionais para o Ensino Fundamental* (National Curricular Parameters for Elementary Education), published in 1997-1998. Shortly after, similar proposals for high school, youth and adult education and indigenous education emerged. All these proposals have incorporated the results of academic research in mathematics education in Brazil and abroad, since the late 1970s. They share some common elements, such as the need to incorporate information and communication technologies, games and concrete materials, the history of mathematics, and, above all, the aim that mathematics in elementary school has real meaning for students, going beyond simple preparation for careers that they may eventually choose. Recently, the Ministry of Education has produced new curricular documents, called the *Base Nacional Comum Curricular* (*Common National Core Curriculum*), which involve all basic education disciplines in Brazil, thus, including mathematics in secondary education[19]. The format of the National Curricular Parameters differs from the Common National Core Curriculum because the former offers more general guidelines, while in the latter, the content to be dealt with in each schoolyear is specified in detail. The Common National Core Curriculum has been widely criticized by educators, especially researchers in mathematics education[20]. The deadline stablished for its final implementation in schools is the year 2020.

[19] The document regarding Elementary School was published in 2017.

[20] For Passos and Nacarato (2018), the Common National Core Curriculum represents a rupture in the previous path of advancement in the proposals for teaching mathematics. The authors point to the incompleteness and contradictions between the introductory discourse and the specific skills for teaching mathematics. They also consider that the proposals of the document regarding some topics are presented

Teacher-training for secondary school mathematics

Training and practice of mathematics teachers in Brazil are associated with the country's political periods. As previously mentioned, institutional preparation of teachers for secondary school began only with the creation of a course at the University of São Paulo, in 1934. That university, as well as the University of Distrito Federal, established in 1935, in Rio de Janeiro, were the first two institutions created after the Francisco Campos Reform, in 1931. At the time, specific legislation on higher education highlighted the urgency of offering professional training for teachers, particularly those teaching in secondary education.

Extinguished in1939 for political reasons, the University of the Federal District, which trained teachers on all secondary education[21] disciplines, was closed and incorporated into the University of Brazil. Then, *Faculdade Nacional de Filosofia* (The National College of Philosophy) was organized from the already existing Faculdade de Filosofia, Ciências e Letras do Rio de Janeiro (College of Philosophy, Sciences and Arts of Rio de Janeiro). That college begun training mathematics teachers for secondary school. The model was the same for all the courses which trained teachers at *Faculdade de Filosofia, Ciências e Letras da USP* (College of Philosophy, Sciences and Arts of the University of São Paulo). Known as "3 + 1", the model for such courses concentrated the courses on mathematics and physics in the first three years, relegating pedagogical subjects to the last year (general didactics, special didactics, educational psychology, school administration, biological foundations of education and sociological foundations of education) which was necessary to complement the training of the future teacher. The

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in a very simplified way, and do not consider the results of research on mathematics education in Brazil. Pinto (2017), made another crucial observation when commenting the silence of the document regarding high school on already consolidated theoretical and methodological approaches in the field of mathematics education. Examples are ethnomathematics, mathematical modelling, and the history of mathematics, which constitute valuable contributions to pedagogical practice. Thus, the proposal of the *Base Nacional Comum Curricular* represents a retraction to a previous perspective, in which the curriculum was established as a fixed model that viewed teaching as mere acceptance of guidelines provided by specialists.

[21] The teacher-training model of the University of the Federal District differed greatly from that adopted by the University of São Paulo. Details about the specific case of mathematics are presented in Gomes (2010).

model adopted at the University of São Paulo, became hegemonic in Brazil, since both the National College of Philosophy and the College of Philosophy, Sciences and Arts had a great influence on the initial organization of teacher-training courses created elsewhere in the country. The dissociation between specific content and pedagogical training of teachers, which was consolidated in that model, enshrined a separation still ingrained in educational institutions today (Gatti; Barreto, André & Almeida, 2019).

Since the 1930s, the development of institutional actions for training mathematics teachers in Brazil has expanded considerably. The expanding need to offer education to a growing population, and the urbanization of the country have continuously led to the creation of courses and programs to prepare teachers in many different places. The immenseness of the territory and the physical, human, economic, political, social and cultural diversity of the country, the many changes made to educational legislation at municipal, state and national levels, and the great geographical inequalities[22] are reflected in the availability of teacher-training opportunities. Thus, while in parts of southeastern and southern states the creation and implementation of courses took place between the 1930s and 1960s, and those courses have long been consolidated, in the northeast, the middle west and the north, the development has been more recent and, consequently, even today, the demands for academically qualified teachers are higher in some places. Such characteristics of the Brazilian historical-geographical scenario suffice to indicate that historically investigating the formation and practice of mathematics teachers presents a promising challenge in the research of the history of mathematical education. The project *Mapeamento da Formação e Atuação de Professores que ensinam/ensinaram Matemática no Brasil* (Mapping the formation and practice of teachers who teach / taught mathematics in Brazil), carried out by the group *História Oral e Educação Matemática* (Oral History and Mathematics Education) since 2002[23] aims to

[22] Brazil is a federation comprised of 26 states and a Federal District, where the country's capital, Brasília, is located. The states and the Federal District are grouped into five regions (north, northeast, mid-west, southeast, and south) based on physical, economic, political, cultural similarities.

[23] In the project, approximately 50 master's and doctoral studies were completed by 2018. To a large extent, these investigations focus on specific institutions and programs for initial or in-service qualification of mathematics teachers, aiming to collect stories about the creation and development of these institutions and programs.

study the formation and practice of mathematics teachers in Brazil without approaching it as an all-encompassing and single theme. The work developed by that group has been marked by oral history methodology (Garnica & Vianna, 2019), in which interviews are conducted and recorded, narratives are transcribed and textualized[24].

The investigations use scale variation, placing the focus on truly diverse scenarios, from the human, political, economic, socio-cultural, and educational viewpoints. This enables the perception of connections that are not evident when the training and practice of mathematics teachers in Brazil are approached from a general and universalizing perspective, which views the situation throughout the country as a mere mirror of those teaching preparation models first instituted.

The project has shown that the training and practice of mathematics teachers in Brazil are significantly conditioned to the place where the research is conducted. By highlighting and valuing peculiarities and specificities, the oral history methodology has made it possible to discern many differences. However, the accounts and their interpretation, combined with the analyses of diversified written documentation and the comparison of the results of various investigations, also shed light onto common issues. In fact, there are aspects that stand out in the interviewees' accounts in many surveys. In diversified scenarios, three characteristics have been revealed that go hand in hand, and which we will comment below; they are the deficiency, the urgency, and the discontinuity of mathematics teacher-training actions in Brazil.

The demand for teachers in Brazil, between the 1930s and 1960s, increased at a higher rate compared to the growth of university mathematics teacher-training courses for secondary education. Data from the Ministry of Education reported that in 1957 only 16% of teachers working at that level of education had earned degrees (Búrigo, Rios & Oliveira Filho, 2011). At a time of fast expansion in education, a government program created in the second government of Getúlio Vargas, in 1953,

[24] Transcriptions are transformed into textualizations that are edited versions in which speech is reorganized and modified in order to smooth out the marks of oral language and obtain a more fluent text, without changing the gist of what was said by the interviewees.

Campanha de Aperfeiçoamento e Difusão do Ensino Secundário (Campaign for Improvement and Dissemination of Secondary Education) promoted emergency courses to train teachers in many states, during the 1960s and 1970s. Held during school break periods ou Summer and Winter breaks, such courses aimed to prepare those who were already teaching for a sufficiency exam that would enable them to teach various disciplines, including mathematics.

The Campaign for Improvement and Dissemination of Secondary Education continued to provide such courses for the duration of the military regime in Brazil, which lasted from 1964 to the mid-1980s. In that period, there was a great expansion of private institutions, which started to give out teaching degrees. Pavanello (1993) stresses that, due to the expansion of the public school network and the educational policies at that period, beginning in 1968, faster associate mathematics degrees programs were created to meet urgent demands. However, according to the author, such courses were not adequate regarding the preparation for teaching geometry, and this was as one of the factors that contributed to the long-lasting shortcomings of the discipline in Brazilian secondary schools.

Although the Campaign for the Improvement and Dissemination of Secondary Education ceased to exist in the early 1970s, similar emergency teacher-training courses continued to be provided in order to license large numbers of teachers to teach mathematics who did not have the opportunity to take specific courses as they lived and worked in regions far from the large urban centers where university education had been consolidated for over 50 years.

Many such campaigns were carried out because, in addition to the growth of the population and the number of schools, several legislative acts of the federal government changed the requirements for working as a teacher and certificates became mandatory. However, even in the case of institutions that created courses to train mathematics teachers through a conventional model (Almeida, 2015; Cury, 2014; Martins-Salandim, 2012; Paiva, 2016), research by the Oral History and Mathematical Education Group has shown that the foundation and the subsequent development of such courses sought to meet an urgent need, due to the great expansion in schooling which occurred in Brazil, especially since the 1960s. Hence,

interviewees' accounts decisively highlight the signs of deficiency and urgency in the training of mathematics teachers.

In the emergency measures, observed in studies such as Cury (2011), Fernandes (2011) and Gonzales (2017), a third component - discontinuity - became clear. In general, such courses, developed through agreements between government agencies and universities, and conducted during the teachers' vacation period, aimed to definitively confer a title that legally enabled the professional to teach. Often, according to the above-mentioned researches, conducted in different locations and states, the courses have been developed in an improvised manner, without adequate didactic material and sufficient physical structure. Teacher-training programs are disconnected because they are interrupted by changes in government (Gonzales, 2017). Therefore, transience and impermanency mark this third typical component of mathematics teacher education efforts in Brazil, accompanying the two previously mentioned components – deficiency and urgency.

Final considerations

Space limitations of the present article permit only an overview on the research about the history of secondary education and mathematics teacher-training for this level in Brazil. This history, characterized essentially by the diversity of the country, has been constituted as a fertile ground for research still being conducted. In addition to the traditional written documentary sources, a fundamental contribution has been made by sources produced from oral accounts. Oral history has been shedding light onto the way teachers receive and put into practice directions emanating from pedagogical movements or educational reforms. In general, historiographic research with oral history methodology has emphasized that educational agents "assign meaning to the various influences to which they are exposed and own them, parameterizing their actions based on such attributed meanings" (Souza & Garnica, 2013, p. 371).

From the end of the 20th century to the beginning of the current century, Brazil underwent radical changes in its political, social, economic, and cultural dimensions. Education has always been linked to

the demands and characteristics of the societies that support it, and the teaching of mathematics is part of that education. In every historical moment, mathematics, as any other discipline, is woven by external factors - social, political, cultural, and economic conditions that constitute schooling and teaching - and by internal factors related to the nature of knowledge in a specific area. As in other fields, in mathematics the constitution of such internal factors has increased, not only regarding to content, as knowledge about the nature of teaching and learning processes and the training of professionals in the field of mathematics education has had a strong impact on curricular and didactic-pedagogical proposals and resources.

In Brazil today, one of the greatest demands for education and for improving mathematics teaching is training teachers to cater to a large and diverse population. In the 2000s, teacher-training courses, both at undergraduate and graduate levels, were considerably expanded. Distance initial training programs for mathematics teachers at the *Universidade Aberta do Brasil (Brazilian Open University)*, funded by the federal government, have been an important example for about ten years. It is unfortunate to point out that, while this paper was being written, and during the eighteen months that have elapsed since the beginning of the current extreme right-wing government, of exacerbated neoliberal orientation, budget-cuts and political persecution of educational institutions have led to serious setbacks on secondary education and teacher training. Now, at present, in the short run, it is not possible to foresee perspectives for overcoming these problems that characterize Brazilian education, especially school mathematics.

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