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# Leaning against the bubble. Can theoretical models match the empirical evidence?<sup>1</sup>

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**Abstract:** By estimating a Markov-switching model, we provide new evidence on the nonlinear effects of monetary policy shocks on asset prices and on their bubble component. We show that regime-dependence is mainly driven by the states affecting the interest rate equation. We also show that, following a positive interest rate shock, an OLG model of asset price bubbles with credit frictions and sticky prices may predict an increase in the real rate, a recession/deflation and an increase in the bubble value. This result, which is new to the theoretical literature, matches both the previously existing and our empirical evidence.

JEL Classification: E13, E32, E44, E52, G12

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# 1 Introduction

The aim of this paper is twofold: on the one side, we provide new evidence on the effects of monetary policy shocks on asset prices and on their bubble component; on the other side, we show that the consideration of credit frictions and sticky prices may allow theoretical models of asset price bubbles to replicate the empirical finding that positive monetary policy shocks may increase stock prices. The importance of this issue is demonstrated by the inability of the recent literature on bubbles to: (1) agree on whether and how monetary policy should respond to perceived deviations of asset prices from fundamentals; (2) replicate the limited existing evidence on the effect of interest rate changes on asset price bubbles (Galí and Gambetti, 2015).

At the beginning of this Century, there existed a widespread consensus on the view that central banks should abstain from intervening in the possible presence of bubbles in the stock (or other assets) markets by changing the monetary interest rates. This was mostly due to the possible unintended consequences of this policy and to the difficulty to detect actual bubble episodes (Bernanke & Gertler, 1999 and 2001). Price and financial stability were perceived as complementary objectives and monetary policy should hence remain focused on inflation control (Taylor 2008) and should only intervene eventually, with policies aimed at "cleaning up the mess" left by the bubble burst. This view changed after the financial crisis of 2008, making room for a new conventional view on the correct monetary policy stance, described as "leaning against the wind" (LAW). According to this view, central banks should actively act to curb the emergence of bubbles by raising the monetary interest rates when the observed increase in asset prices seems to be driven by a bubble component.

The recent theoretical literature fuelled a heated debate on the effectiveness of the LAW policy. This debate was stimulated by Galí's (2014) early critique to the new conventional view. Provided that a rational bubble grows at the rate of interest, an increase in this rate reduces the fundamental price (the present discounted value of future dividends), but increases the bubbly component of the asset price. If the bubble size is large, the second effect may dominate over the

first one and an interest rate hike may end up increasing the asset price. As a consequence, a strong interest rate response to bubbles may raise both asset price volatility and the bubble component, and it would hence be optimal, in some instances, to lower interest rates in the face of a growing bubble. The reaction to this critique has been wide, but the flavor of the debate can be here provided through selected examples.

Miao et al. (2018) confirmed the preference for the LAW, as in their model bubbles respond to shocks on impact and an increase in interest rates is able to dampen their dynamics. Dong et al. (2017) obtained mixed results depending on the kind of shock hitting the economy (total factor productivity, sentiment or nominal shock) and on the monetary policy adopted (standard rule vs. strict inflation targeting). In some situations it is optimal to attach a negative reaction coefficient on asset price movement even if the welfare improvement is usually small (with respect to that reached under standard policy rules), thus confirming the early finding of Bernanke and Gertler (1999). In the conclusion of their work, Dong et al. (2017) state that some of their results confirm "the early finding of Bernanke and Gertler (1999, 2001) and provide a theoretical foundation for the conventional wisdom on the role of monetary policy in managing asset bubbles". However, in the face of an expansionary monetary policy shock, when the central bank adopts standard rules (i.e., not responding to asset price movements) they obtain an increase in asset price bubble, a response that is at odds with that presented by Galì (2014) and with the empirical evidence provided by Galì and Gambetti (2015). Ikeda (2018) shows instead that bubbles produce both an inflationary and a deflationary effect: the increase in firms' borrowing capacity allowed by the bubble stimulates production and increases marginal cost, but the fall in borrowing costs reduces marginal cost. In the presence of a shock to the bubble size, the bubble does not sensibly react to monetary tightening and the monetary policy that strongly responds to nominal output is close to optimal.

Some recent papers have claimed that Galì's (2014) results rely on the assumption that there exists no outside liquidity (like non-contingent government bonds) in positive supply. Both Allen et al. (2018), which is a real bubble model, and Hirano et al. (2018), which is a New Keynesian bubble model with infinitely lived

agents, show that the LAW reduce bubbles even in the steady state, thus providing a formal rationale for the new conventional view. Allen et al. (2018) demonstrate that, in the presence of outside liquidity, the lean against the wind policy dampens bubbles, but contracts the economy and leaves fewer resources to buy assets and/or to finance productive activities. In this environment, the new conventional policy is hence to be preferred, not to contrast the distortions that arise when bubbles are present, but to mitigate the harm that is caused when bubbles burst.

Modelling issues aside,<sup>2</sup> the theoretical impasse is accentuated by: (i) the limited attempt to confront the theoretical results with the empirical findings; (ii) the inability of the models that have carried out such an exercise (e.g., Dong et al., 2017) to replicate the important, but limited, empirical findings which are available on the effects of monetary policy shocks on stock prices. In our opinion, the empirical results on the relation between monetary policy and asset price bubbles should not be disregarded, as they can help us to discriminate among theoretical models on the basis of their ability to replicate the empirical findings and to disentangle the effects of interest rate changes in the presence of bubble components in asset prices.

We contribute to this literature from both the empirical and the theoretical perspective. Empirically, we extend Galì and Gambetti's (2015) investigation to the consideration of a discrete representation of the sources of non-linearity observed in the data. We estimate a Markov-switching structural vector autoregressive (MS-SVAR) model using updated information (1960Q1-2019Q4) for the same set of variables, i.e. real output and dividends, output and commodity price indexes, the federal funds rate and the real stock price index. The monetary policy shock is identified by imposing the same recursive scheme on the contemporaneous model structure (Christiano et al., 2005). Compared to Galì and Gambetti's (2015) analysis, based on a time-varying coefficients (TVC) SVAR, our empirical

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<sup>2</sup>These issues relate, for example, to: (i) monetary policy ineffectiveness in long-run models (Martin and Ventura, 2016; Ikeda and Phan, 2016); (ii) the difficulty of including productive capital in "perpetual youth" frameworks (Galì, 2017); (iii) the preference to be given to the backward-looking sunspot solution around a stable bubbly steady state vs. the forward-looking minimal state variable solution around an unstable bubbly steady state; (iv) the differences produced by the adoption of an infinite-horizon framework vs. an overlapping generations (OLG) model (Miao et al., 2018).

strategy allows us to investigate the factors that are responsible for their main finding, i.e., that after 1990 monetary policy shocks lead to persistent increases in the bubble component over a protracted time period.

By selecting a parsimonious regime-switching structure from a set of increasingly complex candidate models, we find evidence that the preferred structure points to few sources of nonlinearity. One source of time-variation affects all model variables through the stochastic model component (variance of structural disturbances) and two independent sources of instability pertain to the interest rate and the stock price equations. The main results can be summarized as follows. First, the recurrent states have a clear interpretability in terms of known historical events of crisis, monetary policy changes and financial deepening. On this basis, we label the states in the stochastic component as *variability states* and those in the systematic component as *monetary policy states* and *financial states*. Second, from state-dependent impulse responses and forecast error variance decompositions, we confirm Galì and Gambetti's (2015) result about the responses of the fundamental and the bubble components to an exogenous monetary tightening, specifying however that it inflates the bubble mainly under a "normal times" monetary policy regime. This outcome holds irrespectively of the prevailing financial regime, suggesting a different interpretation of their finding. Our result does not contrast with the view that the conditional stock price dynamics depend on the relative size of the bubble component, increasing with the real interest rate. However, it provides an additional insight on the transmission mechanism, by pointing to the conduct of monetary policy and to its effectiveness for the management of the business cycle: within the "normal times" monetary policy regime, which is shown to be a recurrent state that does not hold only in the "great moderation period" (Sims and Zha, 2006), and with the onset of the financial deepening regime, monetary policy shocks lead to more persistent increase in both nominal and real interest rates, thus more sharply affecting output and real dividend dynamics. Increased policy effectiveness under "normal times", more than the financial state being in place, turns out to be the main determinant of the result originally achieved by Galì and Gambetti (2015) and detailed by our analysis. Robustness checks show that our results continue to hold irrespectively of the sample period being considered, of

the consideration of states in the stock price equation, and of slight modifications of the identification strategy.

On the theoretical side, we adopt the model developed in Ciccarone et al. (2019), which is a simple OLG scheme similar to those employed in several recent contributions ( e.g., Galí, 2014; Martin and Ventura, 2016; Ikeda and Phan, 2016). The model includes heterogeneous households, some of which are borrowers and others are lenders in the credit market, as well as physical capital accumulation. Financial frictions are present and are modeled along the lines of Martin et al. (2012) and Martin and Ventura (2015, 2016); these imperfections in the credit market play an important role, as the agents looking for funds to be used for productive investments face a significant constraint: the amount of credit that can be obtained by borrowers is provided by lenders on the basis of the amount of collateral investors can pledge. Nominal frictions in the formation of final goods' prices are also included and provide a role for monetary policy in fixing the nominal interest rate. Furthermore, bequests from old borrowers to young borrowers make it possible to adopt a realistic numerical version of the model. The main theoretical result is that this model, in line with the empirical evidence, following a positive shock to the nominal interest rate predicts: (a) an increase in the real rate; (b) recession/deflation; (c) a raise in the bubble value. This conclusion does not provide straightforward support to the new conventional LAW monetary policy.

The paper is structured as follows. In the next section, we present our empirical strategy and findings. We first describe the specification of the sources of nonlinearity in the MS-SVAR and we then discuss the emerging state probabilities and results from stochastic simulations. Robustness checks are briefly summarized. In section **3**, we briefly describe the theoretical model. In section **4**, we present the dynamics produced in this model by a shock to the nominal interest rate and compare them with the VAR based impulse responses. Section **5** concludes.

## **2 Empirical evidence**

Our empirical analysis is based on a nonlinear SVAR including the same set of variables considered in Gali and Gambetti (2015) and with the monetary policy shock

being identified by imposing the same recursive structure. Gali and Gambetti’s (2015) analysis is based on a TVC-SVAR estimated on US quarterly data for the period 1960Q1–2011Q4. Its general result is that there are protracted episodes in which stock prices, after a short-term drop, increase persistently in response to an exogenous tightening of monetary policy. The monetary policy shock leads to a persistent drop in the fundamental component of real stock prices and to an increase in the bubble component. The positive response of the bubble clearly increases after 1990, a result which is interpreted as evidence of a dependence of the stock price on the relative size of the bubble component.

Differently from Gali and Gambetti’s (2015) analysis, in which the time-evolving model structure is based on a TVC-SVAR consistent with smooth structural change, the nonlinearity we consider in our structural model follows Markov-switching processes (Sims and Zha, 2006; Sims, Waggoner and Zha, 2008). Our choice for such a modelling strategy is based on two main considerations: i) the observed uneven dynamics of financial variables generally display abrupt, instead of smooth, time variation, consistent with discrete structural changes in market beliefs; ii) a regime-switching model may provide a clearer picture of the events underlying the time-variation in both the stochastic and the systematic model components, enhancing the interpretability of results. Based on the estimated benchmark MS-SVAR, whose switching structure is selected among a set of model competitors displaying an increasingly complex nonlinear structure, the empirical analysis employs smoothed probabilities, state-dependent moments, impulse response functions (IRFs) and forecast error variance decompositions to characterize the uneven transmission dynamics over time triggered by an exogenous tightening of monetary policy.

## 2.1 The MS-SVAR

The MS-SVAR is estimated with the Bayesian method considering quarterly US data spanning the period 1960:Q1-2019:Q4. Six variables are included in the MS-SVAR: log-real GDP  $y_t$ , log-real dividends  $d_t$ , the log deflator for GDP  $p_t^y$ , the log-deflator for non-energy commodities  $p_t^c$ , the federal funds rate  $r_t$  and the log-



real SP500 index  $q_t$ . For the policy rate, we get rid of the zero-lower bound issue by considering the Wu and Xia (2016, 2020) shadow interest rate for the 2009-2016 time interval.<sup>3</sup>

Since the volatility of the model variables may depend on changes in both the stochastic component and in the model’s structural coefficients, evidence can be uninformative for the identification of the actual source of non-linearity in time-varying SVARs (Benati and Surico, 2009; Fernandez-Villaverde, Guerron\_Quintana and Rubio-Ramirez, 2010; 2015). In order to minimize the risk of diluting the emerging changes in the systematic component (model coefficients) into changes in the stochastic component (heteroskedasticity), we limit the nonlinear structure to a compact size, by allowing for no more than three states in the Markov-chains affecting the stochastic and systematic model components. Model selection of the regime-switching structure is based on measures of fit across competing models, where the set of candidates is specified by combining a selection of different switching structures. Table 1 reports the log Marginal Data Density (MDD) of the tested models, compared according to the MDD measure provided by the Muller sampling method (Liu, Waggoner and Zha, 2011).

(INSERT TABLE 1 HERE)

The regime-switching model with three variance states in the first Markov chain and two coefficient states in the interest rate equation (3V-2C) outperforms the constant coefficient model and all alternative nonlinear model structures<sup>4</sup>. The model with three states in shocks’ variability and two states in both the monetary policy and the stock price equations (3V-2C-2C), with a second-high MDD, is however only marginally dominated by the preferred model. Even though this result signals that the regime-switching dynamics in the stock price equation does not improve the model fit on that implying a more parsimonious specification of

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<sup>3</sup>A detailed description of variables’ definitions and data sources is presented in the Appendix 1.

<sup>4</sup>This result is not fully consistent with that obtained by Sims and Zha (2006), suggesting that no relevant monetary policy regimes emerge when allowing for a rich characterization of the time-varying volatility and considering a monetary aggregate in the policy reaction function. We do not experiment on this issue in order to keep our analysis aligned to its main goal.

the coefficient's nonlinear structure, given the importance of the nonlinearity in the stock price equation for the scopes of our analysis, we keep the model with the more complex regime-switching dynamics as the benchmark structure<sup>5</sup>. We thus consider three channels of instability in the SVAR: i) the first one affects the stochastic component (thus driving the time-varying covariance matrix and capturing shocks' heteroskedasticity) through the independent three-state Markov chain  $s_t^V$ . We name the states in this chain *volatility states*. ii) the second chain affects the monetary policy rate equation, through the two-state latent variable  $s_t^{MP}$ , which defines the *monetary policy states*; iii) the third chain affects the stock price equation, through the two-state latent variable  $s_t^{SP}$ , which defines the *financial states*. By merging the two coefficient's chains in the composite process  $s_t^C = \{s_t^{MP}, s_t^{SP}\}$ , the MS-SVAR model is as follows:

$$\mathbf{y}'_t \mathbf{A}_0(s_t^C) = \mathbf{c}(s_t^C) + \sum_{i=1}^{\rho} \mathbf{y}'_{t-i} \mathbf{A}_i(s_t^C) + \boldsymbol{\epsilon}'_t \boldsymbol{\Sigma}^{-1}(s_t^V) \quad (1)$$

where  $\mathbf{y}'_t = \left[ y_t \quad d_t \quad p_t^y \quad p_t^c \quad r_t \quad q_t \right]$ ,  $\mathbf{c}(s_t^C)$  is the vector of constants,  $\mathbf{A}_0(s_t^C)$  is the invertible contemporaneous correlations matrix,  $\mathbf{A}_i(s_t^C)$  denotes the dynamic cross-correlation matrices for each lag term  $\rho$ , and  $\boldsymbol{\Sigma}$  is a diagonal matrix. Following the standard practice for quarterly observations, we fix  $\rho = 5$  and adopt Litterman's (1986) random walk prior, consistently with the stochastic properties of the variables. The calibration of prior's hyperparameters follows Sims, Waggoner and Zha's (2008), who provide a benchmark for quarterly data MS-SVARs<sup>6</sup>. A multivariate normal distribution for the orthogonal structural shocks  $\boldsymbol{\epsilon}_t$  is assumed:

$$P(\boldsymbol{\epsilon}_t | \mathbf{Y}^{t-1}, \mathbf{S}_t, \boldsymbol{\theta}, q) = \mathcal{N}(\boldsymbol{\epsilon}_t | \mathbf{0}_n, I_n) \quad (2)$$

where the structural shocks' standard deviations are given by the diagonal elements

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<sup>5</sup>Results for the model displaying the highest MDD are reported in the Appendix' additional results section,

<sup>6</sup>Specifically, we adopt the following hyperparameter' structure:  $\mu_1 = 1$ ;  $\mu_1 = 1$ ;  $\mu_1 = 0.1$ ;  $\mu_1 = 1.2$ ;  $\mu_1 = 1$ ;  $\mu_1 = 1$ . The results are however robust to reasonable changes in these values. The estimation results are generated with one million Gibbs sampling replications. The first 100,000 draws are discarded as burn-in, and then one in every ten draws is retained.

of the matrix  $\Sigma^{-1}(s_t^V)$ ,  $\theta$  denotes the vector of the model’s structural parameters,  $\mathbf{S}_t$  and  $\mathbf{Y}^{t-1}$  collect past information on the latent processes and data, respectively. The transition probabilities from state  $i$  to state  $j$ ,  $q_{i,j}$  are collected in the composite transition matrix  $Q = (q_{i,j})_{(i,j) \in H \times H} \in \mathfrak{R}^{h^2}$ , where  $H = \{1 \dots h\}$  is the set of possible regimes for  $s_t$ , and  $Q = Q^C \otimes Q^V$ <sup>7</sup>.

The identification of monetary policy shocks is achieved by imposing the same exclusion restrictions on the contemporaneous structure adopted by Gali and Gambetti (2015), which in turn inherit the identifying restrictions suggested by Christiano et al. (2005). Given a triangular structure for  $\mathbf{A}'_0$ , the ordering of the variables in  $\mathbf{y}_t$  imposes a definite structural meaning to an otherwise contemporaneously recursive system with no economic content: only stock prices respond contemporaneously to the federal funds rate shock.

## 2.2 Regimes

Figure 1 reports the smoothed probabilities evaluated at the posterior mode for the Markov chains emerging under the benchmark model in the extended sample 1960:1-2019:4<sup>8</sup>. The top two panels display states’ probabilities for the first and the third states, capturing *intermediate* and *high* shocks’ sizes. The gray areas denote periods in which these states are active. The *low variability* state can be obtained considering the periods in which the intermediate and high variability states are jointly inactive. To enhance regimes’ interpretation, the red line displays the evolution of the real stock price over time. The third panel displays the probabilities for the states in the first coefficients’ Markov chain, i.e., those driven by latent factors shifting the interest rate equation. There is clear evidence of recurrent switches in the federal funds rate, affecting mainly the first half of the sample. Gray areas in this case identify the first state of the Markov chain on the interest rate equation, which characterizes periods of relatively lower and less volatile interest rates, depicted by the red line. The bottom panel shows the state

<sup>7</sup>The prior for the transition probabilities is a Dirichlet with parameters implying a symmetric prior average duration of regimes of six quarters. The results are robust to significant variations in the prior regime’s duration.

<sup>8</sup>Syntetic results for the reduced sample ending in 2011Q4 are reported in the part of the Appendix dedicated to the robustness checks.

probabilities for the first state in the second coefficient's chain, affecting the stock price equation, whose evolution is depicted by the red line. In this case, gray areas tend to identify periods in which downturns in real stock prices are observed.

(INSERT FIG 1 HERE)

Variance and coefficient regime-switches have a story to be told in terms of known events. The *high variance state* (third state in the variance's Markov chain -  $3v$ ) is associated with periods of increased variability in stock prices, i.e., those characterizing the time interval between the two oil crises, the early Eighties' crisis period and the 2008-10 financial turmoil. The *intermediate variance* state (first state in the variance's chain -  $1v$ ) characterizes the second half of the Eighties, including the stock market crash event in 1987. The remaining periods, thus the financial deepening era taking place in the first part of the Nineties and enduring until the 2008 financial crisis, are characterized by a *low variance* state (second state in variance's chain -  $2v$ ).

The states affecting the interest rate equation also have a clear interpretation. Given the time alignment of probability switches with the observed major changes in the federal funds rate, we interpret these changes as reflecting structural breaks in the monetary policy reaction rule. The gray areas in this case identify a *normal times monetary policy* state (first state in the first coefficients' chain -  $1c$ ), whereas the complementary state, which is in place in periods during which the federal funds rate experienced substantial changes, identifies a *crisis times monetary policy* state (second state in the first coefficients' chain -  $2c$ ). Regime-specific contemporaneous correlations, conditional moments and standard deviations of shocks show that the interest rate is on average much higher during the *crisis times* state.<sup>9</sup> It is worth noting that *normal* and *crisis times monetary policy* states do not characterize a definite and prolonged time interval as implicit in the literature on the so-called "great moderation". They instead emerge as recurrent states that are present over the entire time window, even though with a much higher probability before the Nineties. Under this perspective, our result confirms Sims and Zha's (2006) finding

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<sup>9</sup>See the Appendix for details

of an alternate characterization of monetary policy. The main difference is that in our case this emerges in the model’s systematic component, indicating recurrent shifts in the conduct of monetary policy, more than in shocks size. Monetary policy switches towards a *crisis times* regime are apparent in the late Sixties/early Seventies, in the periods following the first oil crisis, twice during the first half of the Eighties (Volker era) and after the last decade’s financial turmoil, characterized by the adoption of unconventional monetary policies to escape the zero-lower-bound episode. The negative shadow interest rate during these periods clearly captures these policy changes.

The states characterizing the stock price equation display frequent probability switches centered around stock price turning points. In this case, gray areas identify a *crisis times financial state* (first state in the second coefficients’ chain -  $1c$ ), characterized by relatively lower values for the conditional means of stock prices and higher conditional variability of structural shocks.<sup>10</sup>

### 2.3 Impulse responses and variance decompositions

We evaluate the regime-specific conditional dynamics generated by an unexpected increase in the federal funds rate considering regime-dependent impulse response functions (IRFs) and forecast error variance decompositions (FEVDs). In order to enhance the reading of results, we condition IRFs and FEVDs to the second variability state  $2v$ , labeled as *low variability state*. Note that variance states do not affect the model-generated transmission dynamics, which depend only on the model’s coefficients. The relative contribution of the monetary policy shock to each variable’s variance is also unaffected by such a re-scaling.

The crucial question here is whether the central result achieved by Galì and Gambetti (2015) - i.e., that an increase in the nominal interest rate may inflate the bubbly component in real asset prices - is confirmed when considering a different nonlinear model and an extended sample. Figure 2 shows the regime-specific IRFs of the model’s variables to a monetary tightening. We also compute and report

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<sup>10</sup>Information about transition probabilities across regimes, conditional moments and contemporaneous correlations are reported in the Appendix for each regime emerging from the combination of the different variance and coefficients’ states.

the response of the implied real interest rate. Figure 3 depicts the responses of the fundamental and bubble components in real stock prices, obtained by means of the same forward decomposition relation. The IRFs clearly signal that Galì and Gambetti's (2015) result is generally confirmed, and that its size is regime-specific.

(INSERT FIG 2 AND 3 HERE)

The increase in the bubble component following the policy tightening emerges irrespective of the regime being in place. However, the bubble response is stronger under a *normal times* monetary policy state and is only marginally affected by the active financial state. Note that this result does not contrast with the view that the conditional stock price dynamics depend on the relative size of the bubble component, increasing with the real interest rate. However, it provides an additional insight in the transmission mechanism of the monetary policy shock, pointing to a different interpretation of Galì and Gambetti's (2015) finding. Our results suggest that the key factor for the rational bubble's dynamics is the endogenous component of monetary policy (synthesizing its conduct) and its effectiveness for the management of the business cycle: in the *normal times* monetary policy regime, policy shocks lead to increased responses in the model's variables. The policy regime prevailing after 1990, i.e., the *normal times* policy regime, is characterized by smoother and much more persistent policy shocks with respect to what emerges under the *crisis times* policy regime, leading to greater and more persistent negative responses of output and real dividends. The stock price negatively reacts to the monetary tightening, but its contraction is dampened with respect to that of real dividends and to the (persistent) increase in the real interest rate. As a consequence, the fundamental component decreases more than the stock price and the bubble component goes up. The fact that this result emerges irrespectively of the prevailing state for the stock price equation clarifies that the time-varying conditional dynamics obtained by Galì and Gambetti (2015), pointing to heightened responses of the bubble after the Nineties, is more the result of shifts in the conduct and effectiveness of monetary policy than of the prevalence of a given financial regime.

Table 2 reports the regime-specific FEVDs, indicating the fraction of variance explained by the monetary policy shocks at the one-year and five-year time horizons. The FEVDs confirm that the relevance of the policy shock for output variability is higher under *normal times* monetary policy regimes ( $1c - 1c$  and  $1c - 2c$ ) than under *crisis times* policy regimes ( $2c - 1c$  and  $2c - 2c$ ). The increase in the fraction of variance explained by the shock is close to 30% when the *low* stock price regime is in place ( $. - 1c$ ) and to 44% when the *high* stock price regime is active ( $. - 2c$ ). The second coefficient Markov chain is instead responsible in determining changes in the fraction of variance of dividends, commodity prices and real stock prices. However, whereas for dividends and commodity prices the fraction of variance explained by the policy shock is higher when the *normal times* stock price regime is in place ( $1c - 2c$  and  $2c - 2c$ ), for real stock prices the opposite evidence clearly emerges. In this case, the fraction of variance explained by the policy shock is much higher under the *crisis times* stock price regime ( $1c - 1c$  and  $2c - 1c$ ). At the five-year horizon, the percentage of variance due to policy shocks is 12% in a *normal times* policy - *crisis times* stock price regime ( $1c - 1c$ ) and only 2% in a *normal times* policy - *normal times* stock price regime ( $1c - 2c$ ). A comparable change is observed by considering the shift from a *crisis times* monetary - *crisis times* stock price regime ( $2c - 1c$ ) to a *crisis times* monetary - *normal times* stock price regime.

(INSERT TABLE 2 HERE)

In order to detect whether our result owes more to the regime-specific response of monetary policy to stock prices than to a more generally "passive" conduct of monetary policy in normal times (suggested by the smoother and more persistent response of both the nominal and the real interest rate to the policy shock), we re-estimate the model under the over-identifying assumption that the interest rate does not respond to the real stock price, either contemporaneously (this is implicit in the recursive identification scheme, given variable's ordering), or dynamically; This check can be implemented by using the methodology described in Sims *et al.* (2008). Figures 4 shows that the responses of the fundamental and bubble components to the policy shock are qualitatively unaffected by the alternative

identification scheme.

(INSERT FIG 4 HERE)

## 2.4 Robustness

The robustness of results is evaluated in two major directions. First, by considering the MDD-maximizing model, we verify whether our results depend on the specific regime-switching structure being selected for the benchmark model. The model with the highest MDD is characterized by a three-state Markov-chain for structural disturbances and by a two-state Markov chain for the interest rate equation. Second, we re-estimate the MDD-maximizing model considering a reduced sample of quarterly observations for the period 1960Q1-2011Q4, i.e., the same dataset employed by Galì and Gambetti (2015). Smoothed probabilities and regime-specific IRFs, reported in the robustness section of the Online Technical Appendix, show that our result continue to hold irrespectively of the different regime-switching structure and of the sample period being considered.

## 3 The theoretical model

The exercise carried out in the next section is based on a slightly modified version of the model described in Ciccarone et al. (2019), which we here synthetically describe.<sup>11</sup> The economy is populated by overlapping generations (OLG) of agents living for two periods; within each period, young and old agents coexist in equal and unchanging proportion. This OLG framework is characterized by two main elements: (i) frictional financial markets coupled with the presence of physical capital accumulation; (ii) sticky prices. Etherogeneity among households allows for the existence of borrowers and lenders in the credit market. Borrowers can make productive investment in physical capital and can also trade, among themselves, an additional asset which is modeled as “pure” bubble, analogous to a pyramid scheme. Due to asymmetric information between creditors and debtors, and to the

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<sup>11</sup>See the Appendix for details



absence of state-contingent securities, the amount of credit that can be obtained by borrowers varies with the amount of collaterals that can be pledged, which depends also on the (expected) value of the bubbly asset. Due to price stickiness, monetary policy can affect the real macroeconomic variables.

The economy produces one intermediate good and a continuum of differentiated final goods. The intermediate good is produced by one representative firm using physical capital and labor, both traded in competitive markets. The intermediate output is sold in a perfectly competitive market to a continuum of monopolistically competitive final producers, each producing an imperfectly substitutable final good. Final goods can be consumed or transformed into new physical capital. The nominal prices of the final goods are sticky.

Within generations, agents may be savers or lenders. The agents of the “savers” type work when young, consume the final goods when young and when old, and save part of their labor income when young to purchase credit contracts paying nominal interest. The firms producing the final output are owned by the old savers, who pass them on as a bequest at the end of their lives, when young savers enter their old age. Profits and interest payments on credit contracts finance the consumption of old savers.

When young, the other group of agents (borrowers/entrepreneurs) exploits investment opportunities, represented by both productive (“fundamental”) and non-productive (“bubbly”) assets, and finance this expenditure by borrowing in the financial sector and by using the resources left to them as a bequest by the borrowers of the previous generation.

Productive investment adds to the capital stock, which the young borrowers buy from the old borrowers at the end of the period. A representative intermediate firm rents physical capital from borrowers and hands to them, when they become old, the remuneration of capital. Bubbly assets are valued on the expectation of their re-sale value. Each generation starts (issues) new bubbles with random initial value, which are traded in the market for bubbles, alongside the old bubbles started by previous generations and sold to the young one.

In the credit market, identical and perfectly competitive banks accept the deposits demanded by savers and use them to supply the loans demanded by bor-

rowers at the nominal loan rate. At the end of each period, loans and deposits (plus interest) are paid back, banks' balance sheets clear and banks shut down, to open again at the beginning of the next period. Savers can hence hold two types of financial assets: money, supplied by the Central Bank, and bank deposits. The Central Bank sets the rate of interest on deposits<sup>12</sup> by following a dynamic rule to be described in the next section.

In the market for bubbles, old borrowers supply the outstanding bubbles issued in the previous period and can start (issue) new bubbles. Both types of bubbles are demanded by the young borrowers. Young savers, who supply labor inelastically, enter period  $t$  without previously accumulated cash holdings and deposits, receive money wage income and deposit at banks. At the end of each period, deposits are repaid, together with interest earnings. A cash-in-advance constraint requires agents to allocate money balances and money wage income for consumption, net of the deposits they make at banks. The old savers receive the aggregate profits obtained from retailer firms and from banks, and are not interested in carrying financial assets to the future (bequest motive).

Credit market imperfections affect the behavior of banks, which may not always obtain the full repayment of the loans provided to the borrowers. To obtain loans, borrowers must then provide credit intermediaries with collaterals. They can pledge only a fraction of their future resources, but can create and exchange a bubbly asset that can also be used as collateral. The overall guarantee provided by borrowers eliminates the need for banks to add a risk premium, on top of the riskless rate, in the rate of interest on loans.

### 3.1 The market for bubbles

The equilibrium between demand (by the young borrowers) and supply (by the old borrowers) of bubble in every time period is  $B_{t+1} = B_t + B_{t+1}^N$ , where  $B_{t+1} \geq 0$  is

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<sup>12</sup>This is equivalent to allowing households to buy government assets and to hold bank deposits, which represent a form of private liquidity (Aksoy et al., 2013). As riskless deposits and riskless non-contingent government bonds are perfect substitutes in the savers' portfolios, assuming no arbitrage conditions, the deposit rate always equals the government bond rate (Freixas and Rochet, 1997). The central bank is then also setting the nominal interest rate on public (outside) liquidity (in zero net supply), reflecting in reduced form open market operations.

the physical amount of the bubble supplied at  $t + 1$  and  $B_{t+1}^N$  represents the newly issued bubbles. The bubble equilibrium equation can be expressed as:

$$Q_{t+1} = R_{t+1}^B Q_t + Q_{t+1}^N \quad (3)$$

where  $P_{t+1}^B$  is the real price of the bubble,  $R_{t+1}^B = P_{t+1}^B/P_t^B$  is the real factor of return on the bubble and  $Q_t = P_t^B B_t$  is the real value of the bubble.

### 3.2 Savers/lenders

The preferences of the representative saver are specified by the following utility function:<sup>13</sup>

$$\mathcal{U}_t^s = \frac{(C_{1,t}^s)^{1-\gamma_s} - 1}{1-\gamma_s} + \beta \frac{(\mathbb{E}_t C_{2,t+1}^s)^{1-\gamma_s} - 1}{1-\gamma_s} \quad (4)$$

where  $\gamma_s \in [0; 1)$  is the inverse of the intertemporal elasticity of substitution,  $\beta \in (0; 1)$  is the subjective discount factor and  $\mathbb{E}_t$  represents the (conditional) expectation operator.  $C_{1,2}^s$  is an index of the saver's aggregate consumption of the final goods in the two periods:

$$C_{1,t}^s = \left( \int_0^1 C_{1,t}^s(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}; \quad C_{2,t+1}^s = \left( \int_0^1 C_{2,t+1}^s(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5)$$

where  $\epsilon$  is the elasticity of substitution between the  $j$ -types of final goods.

The budget constraints are:

$$C_{1,t}^s = \frac{W_t}{P_t} - L_t^{S \text{ real}}; \quad C_{2,t+1}^s = L_t^{S \text{ real}} \frac{P_t}{P_{t+1}} (1 + i_t) + \Pi_{t+1}^R \quad (6)$$

where  $L_t^{S \text{ real}}$  is the amount of savings,  $\frac{W_t}{P_t}$  is the real wage and  $\Pi_{t+1}^R$  are real profits.

The optimization problem of savers is then:

$$\max_{C_{1,t}^s, C_{2,t+1}^s, L_t^{S \text{ real}}} \mathcal{U}_t^s \quad \text{s.t.: (6)}$$

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<sup>13</sup>This corresponds to an Epstein-Zin utility function when agents are "risk neutral". See Martin and Ventura (2015).

from the first order conditions of this problem the supply of funds (expressed in real terms) can be obtained:

$$L_t^{S\text{real}} = \frac{\beta^{\frac{1}{\gamma_s}}}{\beta^{\frac{1}{\gamma_s}} + (\mathbb{E}_t R_{t+1})^{1-\frac{1}{\gamma_s}}} \left( \frac{W_t}{P_t} - \frac{\mathbb{E}_t (\Pi_{t+1}^R)}{\beta^{\frac{1}{\gamma_s}} (\mathbb{E}_t R_{t+1})^{\frac{1}{\gamma_s}}} \right) \quad (7)$$

### 3.3 Borrowers/investors

We assume that, differently from savers, borrowers want to leave some resources  $S_{t+1} \geq 0$  to the next generation:<sup>14</sup>

$$\mathcal{U}_t^b = \frac{(C_{1,t}^b)^{1-\gamma_b} - 1}{1-\gamma_b} + \beta \frac{(\mathbb{E}_t C_{2,t+1}^b)^{1-\gamma_b} - 1}{1-\gamma_b} + \eta \frac{(\mathbb{E}_t S_{t+1})^{1-\gamma_b} - 1}{1-\gamma_b} \quad (8)$$

where the parameter  $\gamma_b \in (0; 1)$  can be different from  $\gamma_s$  and  $\eta > 0$ . The young borrower can also use part of his/her resources to buy investment goods whose aggregate index is:  $I_t = \left( \int_0^1 I_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$ :

The borrowers' budget constraints can be written as:

$$C_{1,t}^b = \frac{L_t^D}{P_t} + S_t - Q_t - I_t - (1 - \delta_K) K_t; \quad (9)$$

$$C_{2,t+1}^b = r_{t+1}^k K_{t+1} + (1 - \delta_K) K_{t+1} - S_{t+1} + R_{t+1}^B Q_t + Q_{t+1}^N - \frac{1 + i_t^L}{P_{t+1}} L_t^D \quad (10)$$

where  $L_t^D$  is the agents' demand for funds;  $\frac{1+i_t^L}{P_{t+1}} L_t^D$  is the amount to be repaid when old;  $Q_t$  is the amount of the bubble purchased when young and  $R_{t+1}^B Q_t + Q_{t+1}^N$  represents the accruals from selling the bubble when old, i.e., the bubble purchased when young augmented with its factor of return, plus the value of the newly created (and sold) bubble. The rate  $r_{t+1}^k$  is the rental price of physical capital, so that  $r_{t+1}^k K_{t+1}$  is the physical capital income obtained by the old agent. The amount  $(1 - \delta_K) K_{t+1}$  represents the value of the remaining capital stock (net of

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<sup>14</sup>The inclusion of the bequest term is needed in order to calibrate the steady state of the model at business cycle frequency.

depreciation, at the constant rate  $\delta_K \in (0; 1)$ ) that old agents sell to young agents.

Finally, the following capital accumulation constraint holds:

$$K_{t+1} = I_t + (1 - \delta_K) K_t \quad (11)$$

*Credit market imperfections.* Credit market imperfections affect the behavior of banks, which may not always obtain the full repayment of the loans (capital plus interest) provided to the borrowers,  $L_t^D (1 + i_t^L)$ , due, e.g., to a risk of bankruptcy leading to default, or forms of misbehavior by the borrowers. As a consequence, borrowers cannot obtain loans without providing credit intermediaries with collaterals given by the sum of a fraction  $\phi \in (0; 1)$  of their future resources and of the re-sell value of their bubbly asset  $B_t$ .

The banks' problem can then be written as:

$$\max_{L_t^D} \Pi_t^{bank} = (1 + i_t^L) L_t^D - (1 + i_t) D_t; \quad \text{s.t. } D_t = L_t^D$$

and the optimality condition implies:  $i_t^L = i_t$ .

Being  $D_t = L_t^D = L_t^S$ , it follows that the borrowing constraint - which we here assume to hold with equality - can be written as:

$$\frac{(1 + i_t)}{P_{t+1}} L_t^D = \phi (r_{t+1}^k + 1 - \delta_K) K_{t+1} + R_{t+1}^B Q_t + Q_{t+1}^N \quad (12)$$

The optimization problem of the borrowers is:

$$\max_{I_t, L_t^{D \text{ real}}, Q_t, Q_{t+1}^N, S_{t+1}} \mathcal{U}^b \quad \text{s.t. } (9), (10), (12) \quad (13)$$

First of all, if the collateral constraint always holds, the demand for credit funds will be represented by equation (12), rewritten with the appropriate expectation operators:

$$L_t^{D \text{ real}} = \frac{1}{\mathbb{E}_t R_{t+1}} [\phi \mathbb{E}_t (r_{t+1}^k K_{t+1}) + \phi (1 - \delta_K) K_{t+1} + \mathbb{E}_t Q_{t+1}] \quad (14)$$

From the first order conditions for a maximum, we derive the equilibrium con-

dition in asset (and credit) markets:

$$\mathbb{E}_t R_{t+1}^B = \mathbb{E}_t R_{t+1} \quad (15)$$

From the first order conditions of (13) with respect to  $I_t$ , we get:

$$K_{t+1} = \frac{\beta^{\frac{1}{\gamma_b}} (L_t^D \text{real} - Q_t + S_t) + [(1 - \phi) (\mathbb{E}_t R_t^K)]^{-\frac{1}{\gamma_b}} S_{t+1}}{\beta^{\frac{1}{\gamma_b}} + [(1 - \phi) (\mathbb{E}_t R_t^K)]^{1 - \frac{1}{\gamma_b}}} \quad (16)$$

where the return factor is:  $R_t^K = r_t^k + 1 - \delta_K$ . The first order conditions of (13) with respect to capital determine also the following relation:

$$\mathbb{E}_t R_{t+1}^K > \mathbb{E}_t R_{t+1} = \mathbb{E}_t R_{t+1}^B$$

which must be satisfied in equilibrium<sup>15</sup>.

Finally, the first order condition of (13) with respect to  $S_{t+1}$  (recall that  $S_t$  is predetermined for the young agent) leads to:

$$S_t = (1 - \phi) \left[ 1 + \left( \frac{\beta}{\eta} \right)^{\frac{1}{\gamma_b}} \right]^{-1} R_t^K K_t$$

that is, the amount of the bequest is proportional to the net resources  $R_t^K K_t$  deriving from capital ownership.

### 3.4 Intermediate and final firms

*Intermediate firm.* The firm's production technology is of the Cobb-Douglas type:

$$X_t = F(K_t; N_t) = AK_t^\alpha (g^t N_t)^{1-\alpha}; \quad \alpha \in (0; 1) \quad (17)$$

where  $A > 0$  is a scale factor. We assume that the economy experiences a growth process driven by exogenous (Harrod-neutral) technical progress embodied in the

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<sup>15</sup>This condition guarantees that borrowers are willing to borrow funds in the credit market, as the return on the real capital they can buy with them is greater than the factor of return on credit that will be paid to savers.

growth rate  $g > 1$ . The intermediate firm profit, expressed in real terms, is:

$$\Pi_t^X = \frac{P_t^X}{P_t} A K_t^\alpha (g^t N_t)^{1-\alpha} - \frac{W_t}{P_t} N_t - r_t^k K_t$$

where  $P_t^X/P_t$  is the real price of the intermediate good. The demand functions for inputs, stemming from profit maximization (together with  $N_t = 1$ ), are equal to:

$$\begin{aligned} \frac{W_t}{P_t} &= g^t (1 - \alpha) A \left( \frac{K_t}{g^t} \right)^\alpha \frac{P_t^X}{P_t} \\ r_t^k &= \alpha A \left( \frac{K_t}{g^t} \right)^{\alpha-1} \frac{P_t^X}{P_t} \end{aligned}$$

*Final goods producers.* The production function of the  $j$ -th producer is linear in the unique input  $X_t(j)$ :

$$Y_t(j) = X_t(j) \quad (18)$$

and the monopolist faces a demand for the  $j$ -th good equal to:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} (C_t + I_t) \quad \text{where } C_t \equiv C_{1,t}^b + C_{2,t}^b + C_{1,t}^s + C_{2,t}^s \quad (19)$$

The final producer's real profit is:  $\Pi^R(j)_t = \frac{P_t(j)}{P_t} Y_t(j) - \frac{P_t^X}{P_t} X_t(j)$ , where  $\frac{P_t^X}{P_t} X_t(j)$  is the real cost of production. Hence the firm's marginal cost  $mc_t$  is  $mc_t = \frac{P_t^X}{P_t}$ .

The monopolist sets the price  $P_t^o(j)$  so as to solve the problem:

$$\max_{P_t^o(j)} \Pi^R(j)_t \quad \text{s.t. (18),(19)}$$

and the real price of the individual good  $j$  writes:

$$\frac{P_t^o(j)}{P_t} = \left( \frac{\epsilon}{\epsilon - 1} \right) mc_t. \quad (20)$$

As we assume nominal rigidities, in every  $t$  some of the prices  $P_t^o(j)$  can be equal to a value set some period in the past. We may hence assume that, in general, the average price index  $P_t$  is a function not only of the current  $mc_t$ , but also of the

level of prices and marginal costa expected in the past:<sup>16</sup>

$$P_t = P(mc_t; \mathbb{E}_{t-1-s} P_t; \mathbb{E}_{t-1-s} mc_t)_{s=0,1,2,\dots,\infty} \quad (21)$$

Under a flexible price regime in which  $P_t^o(j) = P_t$ , equation (20) univocally sets the value of the marginal cost  $mc_t = \mu = 1 - \frac{1}{\epsilon} \in (0; 1)$ ,  $\forall t$ , where  $\frac{1}{\mu}$  is the mark-up over production costs.

### 3.5 Stationary state

In the stationary states of our economy, the endogenous variables (indicated without the subscript  $t$ ) are constant through time and prices are fully flexible, so that  $P(j) = P$ ,  $mc = \mu$  and  $y(j) = y = \int_0^1 y(j) dj = \bar{y} = x$ . We also assume zero trend inflation:  $\pi = 0$ . Under stationarity, the equilibrium system can be reduced to the following set of equations:

$$\begin{aligned} gk &= \frac{(l^D - q) \left( \eta^{\frac{1}{\gamma_b}} + \beta^{\frac{1}{\gamma_b}} \right) + \eta^{\frac{1}{\gamma_b}} (1 - \phi) (\mu \alpha A k^\alpha + 1 - \delta_K) k}{\left( \eta^{\frac{1}{\gamma_b}} + \beta^{\frac{1}{\gamma_b}} \right) + [(1 - \phi) (\mu \alpha A k^\alpha + 1 - \delta_K)]^{1 - \frac{1}{\gamma_b}}} \quad (22) \\ l^S &= \frac{\beta^{1/\gamma_s}}{\beta^{1/\gamma_s} + R^{1 - \frac{1}{\gamma_s}}} \left[ \mu (1 - \alpha) - \frac{g(1 - \mu)}{\epsilon (\beta R)^{1/\gamma_s}} \right] A k^\alpha; \\ l^D &= \frac{g}{R} (\phi \mu \alpha A k^\alpha + \phi (1 - \delta_K) k + q); \\ l^D &= l^S; \quad q = \frac{R}{g} q + q^N; \end{aligned}$$

In general, we can define two classes of stationary solutions: a *non-bubbly* and a *bubbly stationary equilibrium*, i.e., a vector  $(l^D, l^S, k, R, q)$  that solves equations (22) under, respectively, the assumption:  $q^N = 0$ , and  $q^N > 0$ . Our interest focuses on the bubbly stationary equilibrium, where the bubble market  $q = \left( \frac{g}{g-R} \right) q^N$  poses a constraint on any possible bubbly stationary state: given  $q^N \geq 0$ , the interest rate on the credit (and the bubble) market must be small enough:  $g > R = R^B$ . This condition is necessary for the young agents to be able to buy the bubble

<sup>16</sup>We use the “sticky information” model; see the Appendix for details.



(using their resources and the lent funds).

In order to simplify the analysis and to highlight the underlying relationships between the main endogenous variables, we choose a specific form for the new bubble creation process, linking it to the economy's size through the parameter  $\omega > 0$ :  $q^N = \omega y = \omega A k^\alpha$ . Under this assumption, from the equilibrium  $l^D = l^S$  we obtain:

$$\phi(1 - \delta_K) = \left\{ \frac{\beta^{1/\gamma_s}}{\beta^{1/\gamma_s}/R + R^{-\frac{1}{\gamma_s}}} \left[ \frac{\mu(1 - \alpha)}{g} - \frac{1}{\epsilon(\beta R)^{1/\gamma_s}} \right] - \phi\mu\alpha - \frac{\omega g}{g - R} \right\} A k^{\alpha-1} \quad (23)$$

Furthermore, as we also assume that  $\gamma_b = 1$ , the borrowers' accumulation equation is:

$$\begin{aligned} r^k &= \alpha\mu(Ak^{\alpha-1}) = \mu\alpha \cdot f_B(R) \\ f_B(R) &= \frac{g \left(1 + \frac{1}{\beta+\eta}\right)}{\frac{g}{R}(\phi\mu\alpha + \omega) + \left(\frac{(1-\phi)\eta}{\eta+\beta}\right)\mu\alpha} - \frac{(1 - \delta_K) \left[ \left(\frac{(1-\phi)\eta}{\eta+\beta}\right) + \phi\frac{g}{R} \right]}{\frac{g}{R}(\phi\mu\alpha + \omega) + \left(\frac{(1-\phi)\eta}{\eta+\beta}\right)\mu\alpha} \end{aligned} \quad (24)$$

In order to understand the effects of  $\omega$  on the stationary value of capital  $k$ , and hence on  $y$ , we look at equation (24) written in the form  $Ak^{\alpha-1} = f_B(R)$ , solve it with respect to  $k$  and compute the derivative  $dk(R)/d\omega$ . In this economy, the value of  $\omega$  rules the size of the bubble in the (locally unique) stationary state. It affects capital ( $k$ ) and identifies an expansionary (crowding-in) stationary regime and a contractionary (crowding-out) stationary regime through the working of three channels.

- *Credit demand channel*: an increase in the bubbly asset (a higher  $\omega$ ) slackens the collateral constraint and allows borrowers to demand more funds and to invest more (positive effect on  $k$ ).
- *Price channel*: an increase in  $\omega$  increases the cost of borrowed funds, which leads borrowers to demand less funds and to invest less (negative effect on  $k$ ).
- *Asset allocation channel*: an increase in  $\omega$  increases the quantity of the bub-

bly asset to be purchased, which crowds-out expenditures in productive investment (negative effect on  $k$ ).

The final effect of an exogenous increase of  $\omega$  on  $k$  (i.e., whether the economy is in a crowding-in or in a crowding-out regime) is determined by the relative size of these three channels. Furthermore, if it is  $\omega = 0$ , the economy sets itself into a “no bubble” stationary state in which the level of capital  $k_{NB}$  is generally lower than that of a “bubbly” economy ( $\omega > 0$ ).<sup>17</sup> The model can then be expressed as a linear approximation around the steady state<sup>18</sup> and used for numerical investigations. We are here particularly interested in the response of key endogenous variables to changes in the nominal interest rate set by the monetary authority. To this issue we now turn our attention.

## 4 Effects of monetary policy shocks on asset price bubbles

In the introduction, we stressed that a model’s ability to replicate the existing empirical findings on the effects of monetary policy on asset bubbles should be conceived as a relevant factor to consider when comparing different theoretical frameworks. In line with this belief, in this section we provide a comparison of the dynamics displayed by our model economy with both our original empirical evidence discussed in section (2) and that provided by Galì and Gambetti (2015). To this aim, we simulate the effects of a shock to the nominal interest rate on the real macroeconomic variables and on the bubble size. We then compare them, at least from a qualitative point of view, with the empirical evidence, to shed some lights on the transmission mechanism of monetary policy in a context where bubbles can affect the economic environment in a complex way, as exemplified by the three channels discussed in the previous section.

In order to calculate the predicted dynamic responses of the model economy to an exogenous shock to the nominal interest rate (i.e., the authority’s instrument  $i_t$ ),

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<sup>17</sup>This is true for the range of parameter’s values used in the numerical application of the model.

<sup>18</sup>See appendix for the complet set of linearised equations.

we assume that no stochastic shock hits the bubble size and replace the monetary policy rule employed in Ciccarone et al. (2019) with the following one:<sup>19</sup>

$$\begin{aligned} \widehat{(1+i_t)} &= \rho_i \widehat{(1+i_{t-1})} + (1-\rho_i) (\delta_\pi \pi_t + \delta_y \widehat{y}_t^{gap} + \delta_q \widehat{q}_t) + e_t^i \quad \text{where: (25)} \\ e_t^i &= \rho_{ei} e_{t-1}^i + \varepsilon_t^i, \quad \text{and} \quad \varepsilon_t^i \sim i.i.d. (0; \sigma_{\varepsilon_i}^2) \end{aligned}$$

where  $\widehat{(1+i_t)}$  is the deviation of the monetary policy factor from its steady state value,  $\delta_\pi > 1$  and  $\delta_y \geq 0$  are the usual policy reaction parameters to inflation and output gap deviations, and  $\delta_q \geq 0$  denotes the policy reaction to the bubble component ( $\delta_q > 0$  indicates a LAW policy). This (empirical) rule makes the interest rate react not only to changes in the inflation rate  $\pi_t$  and in the output gap  $\widehat{y}_t^{gap}$ , but there is also a smoothing parameter  $\rho_i$  which captures a well known feature of real data, that is the persistence of the policy rate path.

We compute the impulse-response functions to the monetary policy shock<sup>20</sup> under the crowding in regime:  $\omega = 0.00535$  (but we obtain analogous results under the crowding out regime) and under different parameterizations of the Taylor rule: a benchmark, textbook-like, rule with  $\delta_q = \rho_i = 0$ ; a LAW rule with  $\rho_i = 0$  and  $\delta_q = 0.5$ ; an empirical rule with  $\rho_i > 0$  and  $\delta_q = 0$ .<sup>21</sup>

(INSERT FIG 5 HERE)

In line with the empirical evidence presented in section (2), under all the specification of the Taylor rule we have considered, following a positive shock to the nominal interest rate, the model predicts a recession/deflation, together with an increase in the real rate  $R_t$ . More importantly, the direction of the reaction of the bubble  $q_t$  is the same as that emphasized in Section (2) and by Galí and Gambetti (2015).

The economic mechanism underlying these results is as follows: The shock on  $e_t^i$  has a direct impact, due to the nominal rigidity, on the real interest rate  $R_t$ . In

<sup>19</sup>The percentage deviations from the stationary state are indicated with a hat, e.g., for the generic (trendless) variable  $z_t$ :  $\widehat{z}_t = \frac{z_t - \bar{z}}{\bar{z}}$ .

<sup>20</sup>The parameters of the interest rate shock are set as:  $\rho_i = 0.2$  and  $\sigma_{\varepsilon_i}^2 = 0.01$ . (The other parameters are the same as those employed in Ciccarone et al. (2019))

<sup>21</sup>A sensitivity analysis (available from the authors) for different values of  $\delta_q$ ,  $\rho_i$ , and for  $\delta_\pi$ ,  $\delta_y$ , confirms the general results to be discussed below.

addition to the standard demand channel acting through the Euler equation and favoring future consumption, the increase in the real interest rate generates two additional effects, which are related to the channels mentioned in section **3**: on the one side, the demand for credit is reduced, due to the working of the price channel (the increased cost of borrowed funds); on the other side, the value of the bubble rises and the allocation channel re-directs more resources towards the purchase of the bubble. These recessive channels more than compensate the effect of the increase in the bubble size  $q_t$  on credit demand, due to the slackening of the collateral constraint which allows borrowers to demand more funds (the credit demand channel); the prevalence of recessionary drivers is related to the fact that the shock is straightforwardly directed to the interest rate and does not directly affect the bubble size. The outcome is hence a recession/deflation coupled with a raise in the bubble value. The policy reaction to the fall in income  $y_t$  and inflation  $\pi_t$  tends to mitigate the recessionary effects of  $e_t^i$  (see equation (25)) by reducing the amplitude of the downturn of these two variables, as confirmed by an analogous experiment carried out with greater  $\delta_\pi$  and  $\delta_y$ .<sup>22</sup>

As for the role played by the different parameterizations of the Taylor rule, we can note that the recession/deflation and the increase in the real rate  $R_t$  (and hence of the bubble component) are magnified when  $\rho_i > 0$  and/or  $\delta_q > 0$ , as compared to the benchmark case. The reason is that, following the exogenous increase in the policy rate, the policy reaction is dampened by the presence of the smoothing parameter  $\rho_i$  (only a fraction  $1 - \rho_i$  of the target variable deviations contribute to set the endogenous reaction of the policy rate) and/or by the presence of a LAW policy reaction ( $\delta_q > 0$ ). Under LAW, the signal sent from inflation and output gap deviations, which calls for a containment of  $R_t$ 's dynamics, is attenuated by the increase in the bubble component.<sup>23</sup> In both cases, the response of the Central Bank results in an effort which is too feeble in stabilizing the real interest rate. Consequently, the deviations of  $R$  from its steady state value are greater and

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<sup>22</sup>This experiment is available from the authors.

<sup>23</sup>As in more standard cases, the general equilibrium effects, together with the high level of  $\delta_\pi$  (as required by the Taylor principle), contribute to generate the reduction of  $i_t$  at impact in response to the shock  $e_t^i$ : when  $\delta_q > 0$ , the decreasing dynamics of the nominal interest rate is driven by a sharper fall of inflation coupled with a stronger increase in the real rate.

this explains both the greater severity of the recession/deflation and the more pronounced increase in the bubbly component. By contrast, the stronger is the policy reaction to the fall in the output gap and in the inflation rate, the milder is the recessionary effect of  $e_t^i$ ; this implies a smaller deviation of the real interest rate from its stationary value, coupled with a smaller increase in the bubble's value.

The IRF analysis of the theoretical model suggests a simple interpretation of the policy regimes singled out by the MS-VAR analysis of Section (2): the normal times monetary policy regime which is recurrent especially after the 1990's, suggests that a Central Bank which is too timid in its reaction to exogenous shocks hitting the interest rate could be at the origin of both recessions and growth in the bubble value. Notably, such a regime seems to prevail during the great moderation, although it is not limited to this period. This can be interpreted as a signal that the low variability of the policy rates during these periods may be due to the absence of relevant exogenous shocks rather than to a higher capacity of the Central Bank to isolate the system from exogenous disturbances. The Central Bank's stance seems to move away from the *normal times monetary policy* regime when stronger real shocks start hitting the system. Following these episodes, the Central Bank tends to intervene more aggressively and this dampens both the recessionary and the "bubble effect" of an exogenous tightening of the policy rate.

## 5 Conclusions

The aim of this paper was to put to a test the intuition that credit frictions and sticky prices may allow theoretical models to replicate the scarce existing empirical evidence on the effects of monetary policy on asset prices. To this aim, we have first contributed to enrich that evidence, estimating a regime-switching SVAR model over an updated data sample including the same US variables employed in Gali and Gambetti's (2015) analysis. The analysis confirmed their result: an unexpected monetary tightening determines an increase in the stock price's bubble component. This result qualitatively holds irrespectively of the regime being in place, with the size of the bubble's response being time-dependent. Our analysis specifies however

that the main element responsible for the emerging nonlinearity in the effects is a regime which is characterized by a recurrent state affecting the monetary policy equation, that we denote as a "normal times" monetary policy regime. The analysis has shown that this outcome holds irrespectively of the prevailing financial state, suggesting that the conduct of monetary policy and its effectiveness are mainly responsible for the regime-dependence. Empirical results are shown to be robust to changes in the time interval being considered, in the nonlinear model structure and in the identification strategy.

We have then evaluated the ability of a model economy with frictional financial markets, sticky prices and a form of etherogeneity among household - which splits them between borrowers and lenders in the credit market - to replicate the relevant empirical findings. The existence of financial frictions and the absence of state-contingent securities require borrowers to provide financial intermediaries with collaterals in order to obtain the credit needed to carry out productive investments and to buy bubbly assets. Price stickiness allows monetary policy to affect the real macroeconomic variables by using the nominal interest rate as the policy instrument.

We have studied the dynamic responses of the model which obtain in this environment following an exogenous increase in the nominal interest rate, assuming that the monetary authority does not react to the bubble. The results we have obtained can be summarized as follows: following a shock to the nominal interest rate, the model predicts an increase in the real interest rate, a recession/deflation and an increase in the bubble value. The economic explanation of these results is that the exogenous shock increases the real interest rate; this reduces the demand for credit via the price channel; as the value of the bubble rises, the allocation channel re-directs more resources towards the purchase of this asset; these two recessive channels more than compensate the expansionary effect on demand generated via the credit demand channel and the outcome is a recession/deflation coupled with a raise in the bubble value.

The dynamic responses generated by the model well conform to those singled out by Galì and Gambetti (2015) for the United States economy: the response of a bubble to a recessionary policy shock is positive and growing, especially after 1990,

and an exogenous tightening of monetary policy leads to a persistent increase in both nominal and real rates, and to a decline in GDP and in the GDP deflator. It should be stressed, in particular, that the direction of the reaction of the bubble value in the theoretical model closely replicates that of Galì and Gambetti (2015).

The dynamic responses of the model are also in line with the original empirical findings we have provided in section (2) and suggest that in “normal times” the timid reaction of the Central Bank to recessionary shocks hitting the interest rates favours recessions and inflates the bubble value. The low variability of the policy rates during the Great Moderation should accordingly be due to good luck rather than to the Monetary Authorities’ ability to effectively react to exogenous disturbances. When stronger real shocks started to hit the economy, the *crisis times monetary policy* adopted by Central Banks dampened both the recessionary and the “bubble effect” of an exogenous tightening of the policy rate.

Our conclusion is that frictional financial markets and sticky prices are crucial characteristics of the economy that theoretical models should not disregard in order to improve our understanding of the effects of monetary policy on asset price bubbles and to increase the models’ ability to replicate the existing empirical findings.

## References

- [1] Allen, F., Barlevy, G. and Gale, D., et al., 2018. A Theoretical Model of Leaning Against the Wind. Federal Reserve Bank of Chicago, Working paper No. 2017-16.
- [2] Aksoy, Y., Basso, H. and Coto-Martinez, J., 2013. Lending Relationships and Monetary Policy, *Economic Inquiry* 51, 368-393.
- [3] Benati, L and Surico, P., 2009. VAR Analysis and the Great Moderation, *American Economic Review*, 99, 1636-1652
- [4] Bernanke, B.S. and Gertler, M., 1999. Monetary Policy and Asset Price Volatility, in New Challenges for Monetary Policy, *Federal Reserve Bank of Kansas City*, 77-128.
- [5] Bernanke, B.S. and Gertler, M., 2001. Should Central Banks Respond to Movements in Asset Prices? *American Economic Review*, 91, 253-257.
- [6] Chib, S., 1995. Marginal Likelihood from the Gibbs Output, *Journal of the American Statistical Association*, 90, 1313-1321.
- [7] Christiano, L.J., Eichenbaum, M. and Evans, C.L., 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113, 1-45.
- [8] Ciccarone, G., Giuli, F., Marchetti, E., 2019. Should central banks lean against the bubble? The monetary policy conundrum under credit frictions and capital accumulation, *Journal of Macroeconomics*, 59, 195–216.
- [9] Dong, F., Miao, J., and Wang, P., 2017. Asset Bubbles and Monetary Policy, working paper, Boston University.
- [10] Freixas, X. and Rochet, J.C., 1997. *Microeconomics of Banking*, MIT Press, Cambridge, Massachusetts.



- [11] Fernández-Villaverde, J. Guerron-Quintana, P. and Rubio-Ramirez, J.F., 2010. Fortune or Virtue: Time-Variant Volatilities Versus Parameter Drifting in U.S. Data,. *FRB of Philadelphia Working Paper No. 10-14*, Available at SSRN: <https://ssrn.com/abstract=1600862> or <http://dx.doi.org/10.2139/ssrn.1600862>
- [12] Fernández-Villaverde, J. Guerron-Quintana, P. and Rubio-Ramirez, J.F., 2015. Estimating dynamic equilibrium models with stochastic volatility, *Journal of Econometrics*, 185, 216-229.
- [13] Galì, J., 2014. Monetary Policy and Rational Asset Price Bubbles, *American Economic Review* 104, 721–752.
- [14] Galì, J., 2017. Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations, mimeo.
- [15] Galì, J., Gambetti, L., 2015. The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence, *American Economic Journal: Macroeconomics* 7, 233–257.
- [16] Hirano, T., Ikeda, D. and Phan, T., 2018, Risky Bubbles, Public Debt and Monetary Policy, working paper.
- [17] Ikeda, D., 2018. Monetary Policy, Inflation and Rational Asset Price Bubbles, working paper.
- [18] Ikeda D., Phan T., 2016. Toxic Asset Bubbles, *Economic Theory*, 61, 241–271.
- [19] Liu, Z., Waggoner, D.F. and Zha, T., 2011. Sources of macroeconomic fluctuations: A regime-switching DSGE approach, *Quantitative Economics*, 2, 251-301.
- [20] Mankiw, G., Reis, R. (2002) Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve, *Quarterly Journal of Economics*, 117, 1295-1328.

- [21] Martin, A., Ventura, J., 2016. Managing Credit Bubbles, *Journal of the European Economic Association* 14, 753-789.
- [22] Miao, J., Shen, Z. and Wang, P., 2018. Monetary Policy and Rational Asset Price Bubbles Redux, mimeo.
- [23] Sims, C.A. and Zha, T., 2006. Were There Regime Switches in U.S. Monetary Policy? *American Economic Review*, 96, 54-81.
- [24] Sims, C.A., Waggoner, D.F. and Zha, T., 2008. Methods for inference in large multiple-equation Markov-switching models. *Journal of Econometrics*, 146, 255-274.
- [25] Taylor, J.B., 2008. The Costs and Benefits of Deviating from the Systematic Component of Monetary Policy, Keynote address at the Federal Reserve Bank of San Francisco, Conference on Monetary Policy and Asset Markets, February 22, available at: <https://web.stanford.edu/~johntayl/Onlinepaperscombinedbyyear/2008/>
- [26] Wu, J.C. and Xia, F.D., 2016. Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit, and Banking*, 48, 253-291.
- [27] Wu, J.C. and Xia, F.D., 2020. Shadow fed funds rate. Downloadable at <https://sites.google.com/view/jingcynthiawu/shadow-rates>

## 6 Appendix

### 6.1 Data

We use quarterly US data spanning the period 1960:Q1-2019:Q4. The six variables included in the MS-SVAR are the same employed by Galí and Gambetti (2015): log-real GDP  $y_t$ , log-real dividends  $d_t$ , the log deflator for GDP  $p_t^y$ , the log-deflator for non-energy commodities  $p_t^c$ , the federal funds rate  $r_t$  and the log-real SP500 index  $q_t$ . For the policy rate, we get rid of the zero-lower bound issue by considering

the Wu and Xia (2016, 2020) shadow interest rate for the 2009-2016 time interval. Table A1.1 below summarizes the variable's data sources and their transformations used in the estimates. The federal funds rate, real output and its deflator are all taken from the Federal Reserve Economic database (FRED), dividends and stock prices are taken from the updated Shiller' stock market database, and non energy commodity prices are taken from the World Bank's (WB) The Pink Sheet historical database on commodity prices.

Table A1.1 Data sources and their transformations

Variable	Definition	Source	Transf.
$y_t$	Real GDP	FRED	$\log(Y_t)$
$d_t$	Real SP Comp. dividends	Shiller' Stock Market data	$\log(\frac{D_t}{P_t^y})$
$p_t^y$	GDP deflator	FRED	$\log(P_t^y)$
$p_t^c$	Non energy comm. pr.	WB - The Pink Sheet	$\log(P_t^c)$
$r_t$	Federal funds rate (shadow rate)	FRED (Wu and Xia, 2020)	—
$q_t$	Real SP Comp. stock price	Shiller' Stock Market data	$\log(\frac{Q_t}{P_t^y})$

## 6.2 Model

As for price dynamics, we adopt the scheme developed by Mankiw and Reis (2002), which is based on the imperfect (and "sticky") diffusion of information among the monopolistic producers through time. The linearized version of the individual "target price"  $P_t^o(j)$  set by the firm is:

$$\hat{P}_t^o(j) = \hat{P}_t + \hat{m}c_t(j)$$

where, for each variable  $z_t$ :  $\hat{z}_t = \frac{z_t - \bar{z}}{\bar{z}}$ . According to the sticky information hypothesis, a firm using an information set dated period  $s$  sets today a price equal to:  $\hat{d}_t(s) = E_{t-s} \hat{P}_t^o(j)$ . In each period only a fraction  $(1 - \rho_p)$  of firms obtain a new information set, whereas the remaining  $\rho_p \in (0; 1)$  continue to fix prices on the basis of the old one. Hence, by denoting with  $\hat{d}_t(s) = E_{t-s} \hat{P}_t^o(j)$  the price set

today, the average price level will be equal to:

$$\hat{P}_t = (1 - \rho_p) \sum_{s=0}^{\infty} \rho_p^s \hat{d}_t(s) = (1 - \rho_p) \sum_{s=0}^{\infty} \rho_p^s E_{t-s} [\hat{P}_t + \hat{m}c_t]$$

This equation can be reduced to the "sticky information" Phillips Curve (where:  $\hat{P}_t - \hat{P}_{t-1} \simeq \pi_t$ ):

$$\pi_t = \left( \frac{1 - \rho_p}{\rho_p} \right) \hat{m}c_t + (1 - \rho_p) \sum_{s=0}^{\infty} \rho_p^s [E_{t-1-s} (\pi_t + \Delta \hat{m}c_t)] \quad (26)$$

The other linearized equations are the following ones. Those of the credit market are:

$$\beta^{1/\gamma_s} w \hat{w}_t + \frac{g\pi^R}{R^{1/\gamma_s}} \mathbb{E}_t \left( \frac{1}{\gamma_s} \hat{R}_{t+1} - \hat{\pi}_{t+1}^R \right) = l^S \left( \beta^{1/\gamma_s} + R^{1-\frac{1}{\gamma_s}} \right) \hat{l}_t^S \quad (27)$$

$$\begin{aligned} & + l^S \left( 1 - \frac{1}{\gamma_s} \right) R^{1-\frac{1}{\gamma_s}} \mathbb{E}_t \hat{R}_{t+1} \\ \frac{l^D R}{g} \left( \hat{l}_t^D + \mathbb{E}_t \hat{R}_{t+1} \right) & = \phi \mu \alpha A k^\alpha \left( \hat{m}c_{t+1} + \alpha \hat{k}_{t+1} \right) \quad (28) \\ & + \phi (1 - \delta_K) k^* \hat{k}_{t+1} + q \mathbb{E}_t \hat{q}_{t+1} \\ \hat{l}_t^D & = \hat{l}_t^S \end{aligned}$$

The equilibrium accumulation equations and the dynamics of  $\hat{s}$  are:

$$\begin{aligned} g s^q s_{t+1}^q & = g (1 - \phi) r^k \left\{ k \left( 1 - \frac{1}{\gamma_b} \right) + \frac{s^q}{\gamma_b [(1 - \phi) R^K]} \right\} \left[ (\alpha - 1) \hat{k}_{t+1} + \hat{m}c_t \right] \\ & + [(1 - \phi) R^K \beta]^{\frac{1}{\gamma_b}} g k \hat{k}_{t+1} + g (1 - \phi) R^K k \hat{k}_{t+1} \\ & - [(1 - \phi) R^K \beta]^{\frac{1}{\gamma_b}} \left( \hat{l}_t^D - q \hat{q}_t + s^q s_t^q \right); \end{aligned} \quad (29)$$

$$\begin{aligned} \hat{s}_t^q & = \frac{1}{\mu \alpha A k^{\alpha-1} + 1 - \delta_K} \left[ \mu \alpha A k^{\alpha-1} \left( \hat{m}c_t + \alpha \hat{k}_t \right) + (1 - \delta_K) \hat{k}_t \right] \\ & = \frac{r_k}{R^K} \left( \hat{m}c_t + \alpha \hat{k}_t \right) + \left( \frac{1 - \delta_K}{R^K} \right) \hat{k}_t. \end{aligned}$$

The other equations of the real side of the economy are:

$$\begin{aligned}
\widehat{gk}_{t+1} &= (g - 1 + \delta_k) \widehat{inv}_t + (1 - \delta_k) \widehat{k}_t; \\
\widehat{y}_t &= \alpha \widehat{k}_t; \quad \widehat{w}_t = \alpha \widehat{k}_t + \widehat{mc}_t \\
\pi^R \widehat{\pi}_t^R &= y \widehat{y}_t - Ak^\alpha \mu \left( \widehat{mc}_t + \alpha \widehat{k}_t \right); \\
\widehat{R}_{t+1} &= \left( 1 + \widehat{i}_t \right) - \pi_{t+1};
\end{aligned}$$

The bubble dynamics is:

$$\widehat{q}_{t+1} = \frac{R}{g} \left( \mathbb{E}_t \widehat{R}_{t+1} + \widehat{q}_t \right) \tag{30}$$

We then add to the linearized model the dynamics of savers' consumption and the aggregate resource constraint:

$$\begin{aligned}
c_2^s \widehat{c}_{2,t}^s &= \frac{Rl^S}{g} \left( \widehat{l}_{t-1}^S + \widehat{R}_t \right) + \pi^R \widehat{\pi}_t^R; \\
y \widehat{y}_t &= c_2^s \widehat{c}_{2,t}^s + \widehat{c}_{1,t}^s c_1^s + c_1^s \widehat{c}_{1,t}^s + c_2^s \widehat{c}_{2,t}^s + inv \cdot \widehat{inv}_t
\end{aligned}$$

as well as the dynamics of borrowers's consumption:

$$\begin{aligned}
c_1^b \widehat{c}_{1,t}^b &= l^D \widehat{l}_t^D + s^{q*} \widehat{s}_t^q - inv \cdot \widehat{inv}_t - (1 - \delta_K) k^* \widehat{k}_t - q \widehat{q}_t; \tag{31} \\
c_2^b \widehat{c}_{2,t+1}^b &= \mu \alpha A k^\alpha \left( \widehat{mc}_{t+1} + \alpha \widehat{k}_{t+1} \right) + q \widehat{q}_{t+1} \\
&\quad - \frac{Rl^D}{g} \left( \widehat{l}_t^D + \widehat{R}_{t+1} \right) + (1 - \delta_K) k^* \widehat{k}_{t+1} - s^{q*} \widehat{s}_{t+1}^q
\end{aligned}$$

Table 1 - Measures of fit for model selection. Constant coefficients (Chib, 1995):

The table reports the log marginal densities for each model:

Model	2V	3V	2V-2C	3V-2C
Muller MDD	3105.22	3213.75	3160.81	3221.24

Model	2V-2C-2C	3V-2C-2C	3V-3C-2C	3V-3C-3C
Muller MDD	3182.83	3220.37	3219.17	3206.45

2V: two variance states only.      3V: three variance states only.

2V-2C: two variance states and two coeff. states (interest rate equation).

3V-2C: three variance states and two coeff. states (interest rate equation).

2V-2C-2C: two var. states, two coeff. states (interest rate/stock price).

3V-2C-2C: three var. states, two coeff. states (interest rate/stock price).

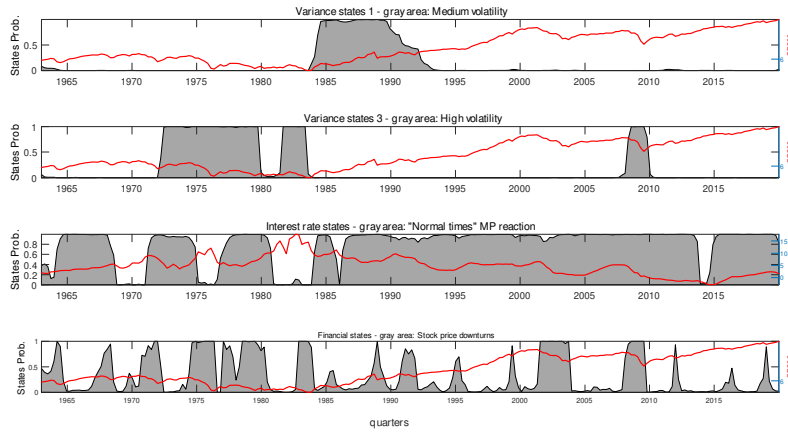
3V-3C-2C: three var. states, three/two coeff. states, (int. rate/stock price).

3V-3C-3C: three var. states, three coeff. states (interest rate/stock price).

Table 2 - FEVDs - monetary policy shock

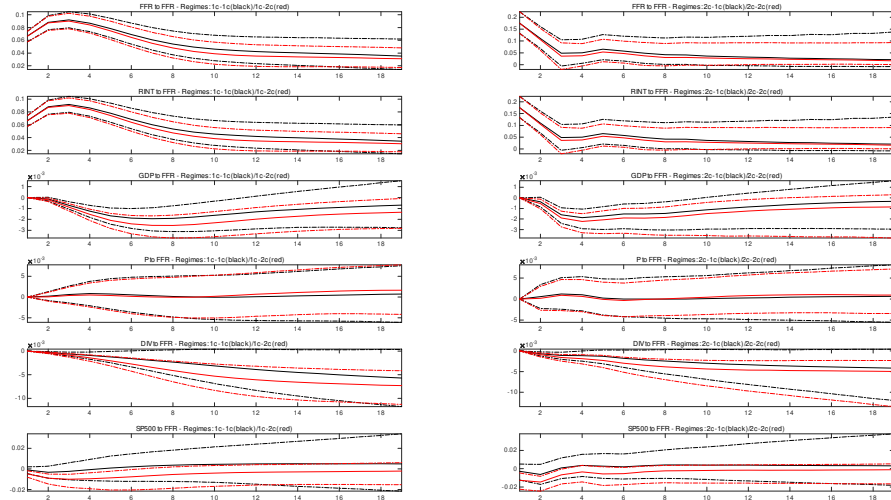
	$1c - 1c$	$1c - 2c$	$2c - 1c$	$2c - 2c$	$1c - 1c$	$1c - 2c$	$2c - 1c$	$2c - 2c$
	Real output				Dividends			
1y	0.196	0.177	0.183	0.163	0.034	0.041	0.028	0.035
5y	0.371	0.328	0.285	0.227	0.022	0.052	0.026	0.067
	Output price deflator				Commodity price deflator			
1y	0.039	0.041	0.069	0.077	0.008	0.006	0.199	0.211
5y	0.166	0.210	0.154	0.195	0.021	0.035	0.174	0.236
	Federal funds rate				Stock price			
1y	0.688	0.724	0.445	0.488	0.034	0.010	0.076	0.025
5y	0.299	0.355	0.206	0.248	0.121	0.020	0.154	0.026

Figure 1 - States' smoothed probabilities



The figure shows the smoothed probabilities evaluated at the posterior mode. Grey areas in the top two panels denote intermediate and high shocks' sizes. Grey areas in the third panel denote the normal times monetary policy state. Grey areas in the fourth panel depict financial stress regimes.

Figure 2 - Regime-specific IRFs to a positive monetary policy shock

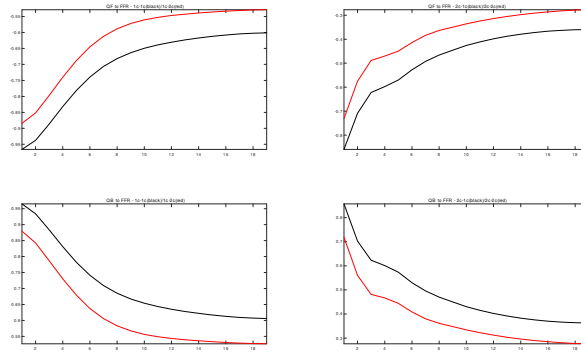


Responses are conditional to the regime in place on the VAR coefficients.

The two columns report the impulse responses to a monetary policy shock in the different regimes.

Figure 3 - Regime-specific IRFs to a positive monetary policy shock.

Fundamental and bubble components

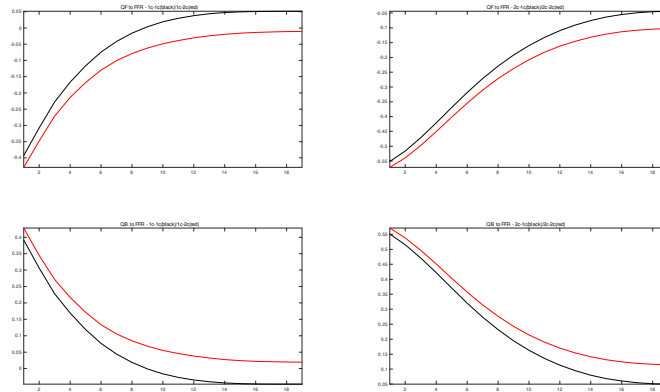


Responses are conditional to the regime in place on the VAR coefficients.

The two columns report the impulse responses to a monetary policy shock in the different regimes.



Figure 4 - Regime-specific IRFs to a positive monetary policy shock  
 Fundamental and bubble components - No SP500 in policy rule



Responses are conditional to the regime in place on the VAR coefficients.

The two columns report the impulse responses to a monetary policy shock in the different regimes.

Figure 5 - IRFs of  $\{\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{q}_t, \widehat{1+i}_t\}$  to an exogenous shock  $e_t^i$   
 (The VAR variables' names are indicated in brackets)

