Programme for International Student Assessment

A teacher's guide to PISA mathematical literacy

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Acknowledgements

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Introduction

The Programme for International Student Assessment (PISA) is an international assessment of the skills and knowledge of 15-year olds. A project of member countries of the Organisation for Economic Co-operation and Development (OECD), it has taken place at three year intervals since 2000. Detailed reports of Australian students' performance, their attitudes and beliefs towards mathematics in PISA can be found in the full reports written to inform the wider educational community. In December 2013 the results of the most recent PISA assessment, PISA 2012, will be released.

The full national reports can be found, along with much more information about PISA, at www.acer.edu.au/ozpisa.

After each three-year cycle, a number of items from the assessment are released by the OECD so that educators are able to see how the assessment is constructed.   By combining these released items with a description of Australian students' performance on the items, and providing an overall picture of achievement in the subject area, this report (and the companion reports on reading literacy and scientific literacy) aims to enable teachers to gain a deeper understanding of PISA, and to use the results of the assessment to inform their teaching.

More and more, policy makers are using the results of studies such as PISA to make decisions about education – for example the Australian Government's *National Plan for School Improvement* establishes a new target to place Australia in the top five countries in the world in reading, numeracy and science by 2025 (see www.betterschools.gov.au). It is important that practitioners and others understand the assessments which underpin the goals, and think about how they are able to make a difference to the outcomes of Australian children.

The aim of this report is to provide this understanding, and encourage discussion about assessment achievement, and benchmarking within the wider educational community.

PISA … what is it?

PISA is a key part of Australia's National Assessment Program (NAP). Alongside NAPLAN, which is a census of students at Years 3, 5, 7 and 9, nationally representative samples of students participate in three national assessments in science literacy, civics and citizenship, and ICT literacy. Together with these, nationally representative samples of Australian students also participate in two international studies as part of the NAP (Figure 1.1). These studies enable Australia to benchmark our students in reading, mathematical and scientific literacy against similar samples of students in more than 60 other countries.

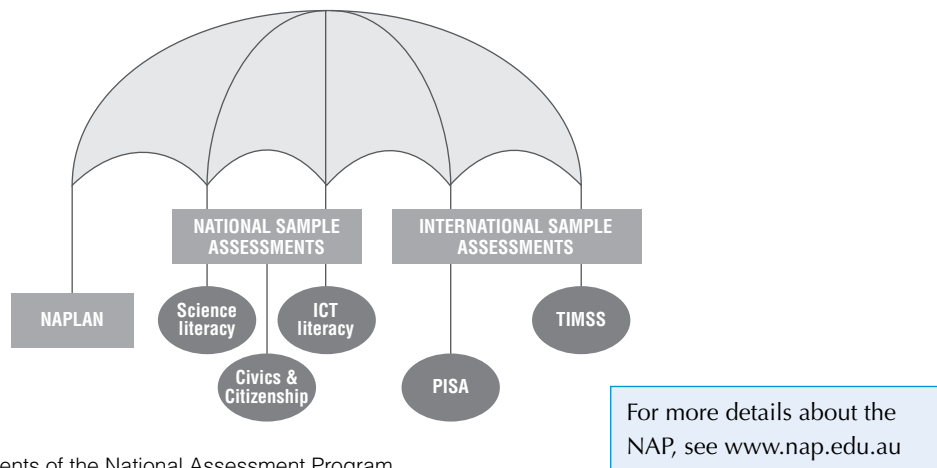PISA is a key part of the MCEECDYA National Assessment Program

Figure 1.1 Components of the National Assessment Program

For more details about the NAP, see www.nap.edu.au

PISA was designed to assist governments in monitoring the outcomes of education systems in terms of student achievement on a regular basis and within an internationally accepted common framework, in other words, to allow them to compare how students in their countries were performing on a set of common tasks compared to students in other countries. In this way, PISA helps governments to not only understand, but also to enhance, the effectiveness of their educational systems and to learn from other countries' practices.

Students aged 15 were chosen as the target group in PISA as compulsory schooling ends at this age in many countries.

PISA seeks to measure how well young adults, at age 15 and therefore near the end of compulsory schooling in most participating education systems, have acquired and are able to use knowledge and skills in particular areas to meet real-life challenges.

In addition to assessing facts and knowledge, PISA assesses students' ability to use their knowledge to solve real-world problems. Thus, the term 'literacy' is used, since it implies not only knowledge of a domain, but also the ability to apply that knowledge.

As part of PISA, students complete an assessment including items testing reading literacy, mathematical literacy and scientific literacy. In each cycle of PISA, one of the cognitive areas is the main focus of the assessment, with most of the items focussing on this area and fewer items on the other two areas (although still enough items to provide links between years) (see Figure 1.2 – shading indicates the major domain of the cycle).   Students also complete an extensive background questionnaire, and school principals complete a survey describing the context of education at their school, including the level of resources in the school, qualifications of staff and teacher morale.

PISA 2000	PISA 2003	PISA 2006	PISA 2009	PISA 2012
Reading Literacy	Reading Literacy	Reading Literacy	Reading Literacy	Reading Literacy
Mathematical Literacy	Mathematical Literacy	Mathematical Literacy	Mathematical Literacy	Mathematical Literacy
Scientific Literacy	Scientific Literacy	Scientific Literacy	Scientific Literacy	Scientific Literacy

Figure 1.2 Cycles of PISA and the major and minor domains of assessment for each cycle

The reporting of the findings from PISA focuses on issues such as:

◗ How well are young adults prepared to meet the challenges of the future?

◗ Can they analyse, reason and communicate their ideas effectively?

◗ What skills do they possess that will facilitate their capacity to adapt to rapid societal change?

◗ Are some ways of organising schools or school learning more effective than others?

◗ What influence does the quality of school resources have on student outcomes?

◗ What educational structures and practices maximise the opportunities of students from disadvantaged backgrounds?

◗ How equitable is the provision of education within a country or across countries?

What do PISA students and schools do?

Cognitive Assessment

> Students completed a pen-and-paper assessment and a context questionnaire.

In PISA 2009, the majority of the assessment was devoted to reading literacy, with mathematical literacy and scientific literacy assessed to a lesser extent. Participating students each completed a two-hour paper-and-pen assessment.

A sub-sample of students who participated in the paper-and-pen assessment also completed an assessment of digital reading literacy, which used the information technology infrastructure (computer laboratories) at schools.

The data for this report are based on PISA 2003, as mathematical literacy was the major focus in that cycle. The format of the testing parallels the testing for 2009, with the majority of the assessment devoted to mathematical literacy.

Context questionnaire

The data collected in the 35-minute Student Questionnaire provide an opportunity to investigate factors that may influence performance and consequently give context to the achievement scores. Responses to a set of 'core' questions about the student and their family background, (including

> Students completed a background survey and principals a school survey. The survey results provide rich context for the achievement data.

age, year level and socioeconomic status) are collected during each assessment. In PISA 2003 students were also asked about their engagement with mathematics, learning strategies and aspects of instruction.

Information at the school-level was collected through a 30-minute online School Questionnaire, answered by the principal (or the principal's designate). The questionnaire sought descriptive information about the school and information about instructional practices.

Participants in PISA 2009

Although PISA was originally created by OECD governments, it has become a major assessment tool in many regions and countries around the world. Since the first PISA assessment in 2000, the number of countries or economic regions who have participated from one PISA cycle to the next has increased. Sixty-five countries participated in PISA 2009, comprising 34 OECD countries and 31 partner countries/economies (Figure 1.3).
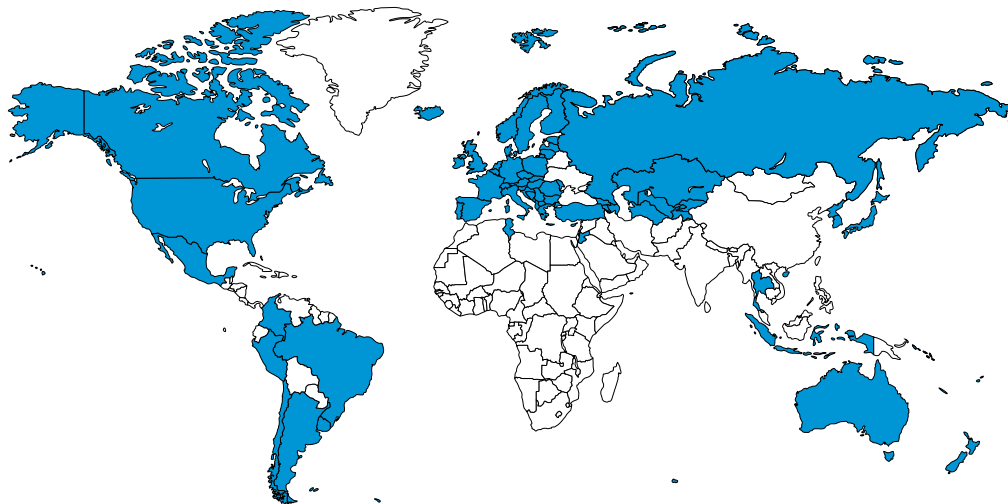

Figure 1.3 Countries participating in PISA 2009

OECD countries: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States of America.

Partner countries/economies: Albania, Argentina, Azerbaijan, Brazil, Bulgaria, Chinese Taipei, Colombia, Croatia, Dubai (UAE), Hong Kong-China, Indonesia, Jordan, Kazakhstan, Kyrgyzstan, Latvia, Liechtenstein, Lithuania, Macao-China, Montenegro, Panama, Peru, Qatar, Romania, Russian Federation, Serbia, Shanghai-China, Singapore, Thailand, Trinidad and Tobago, Tunisia, Uruguay.

Schools and students

The target population for PISA is students who are 15 years old and enrolled at an educational institution, either full- or part-time, at the time of testing. In most countries, 150 schools and 35 students in each school were randomly selected to participate in PISA. In some countries, including Australia, a larger sample of schools and students participated. In Australia's case, a larger sample provides the ability to report reliable results for each state and territory and for Indigenous students. The larger PISA sample is also used as the next cohort for the Longitudinal Survey of Australian Youth (LSAY). The Australian sample for PISA 2009 consisted of 353 schools and 14,251 students. The sample for PISA 2003 was 321 schools and 12,551 students.

> It's important that a range of schools is selected and that a range of students is selected from within schools. This way we are able to get an accurate picture of the whole Australian student population.

This report

This report is one of a series of three reports that focus on Australian students' performance on the PISA items that have been released in each of the assessment domains: reading literacy, mathematical literacy and scientific literacy. Further information about PISA in Australia is available from the national PISA website - www.acer.edu.au/ozpisa while further details about Australia's participation and performance in PISA 2009 is available in Challenges for Australian Education: Results from PISA 2009.

This report focuses on mathematical literacy. The mathematics items presented here were released for public viewing after the PISA 2003 assessment, and have not been used in subsequent assessments. No other mathematics items have been released. The performance results for the items that are presented in this report are thus based on the PISA 2003 cohort.

Chapter 2 of this report provides a brief overview of the PISA Mathematics Framework, so that educators gain an understanding of the context in which the questions for the assessment are written, and an overview of Australia's results in the PISA 2003 international assessment. Chapter 3 provides all of the released items in mathematics for PISA, along with marking guides, examples of responses and the performance of Australian students and that of students in comparison countries on these items. The focus of Chapter 4 is the context behind achievement: attitudes, engagement and learning strategies.

Mathematical literacy in PISA

How is mathematical literacy defined in PISA?

The PISA mathematical literacy domain is concerned with the capacities of students to analyse, reason and communicate ideas effectively as they pose, formulate, solve and interpret mathematical problems in a variety of situations. The PISA assessment framework defines mathematical literacy as:

> … an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

In this conception, mathematical literacy is about meeting life needs. Mathematical literacy is expressed through using and engaging with mathematics, making informed judgements, and understanding the usefulness of mathematics in relation to the demands of life.

How is mathematical literacy measured in PISA?

The PISA framework for mathematical literacy is organised into three broad components: the situations and contexts in which problems are located and that are used as sources of stimulus material; the mathematical content to which different problems and questions relate, and which are organised by certain overarching ideas; and the mathematical competencies that must be activated to connect the real world (in which problems are generated) with mathematics, and then used to solve the problems. The three components are shown in Figure 2.1.
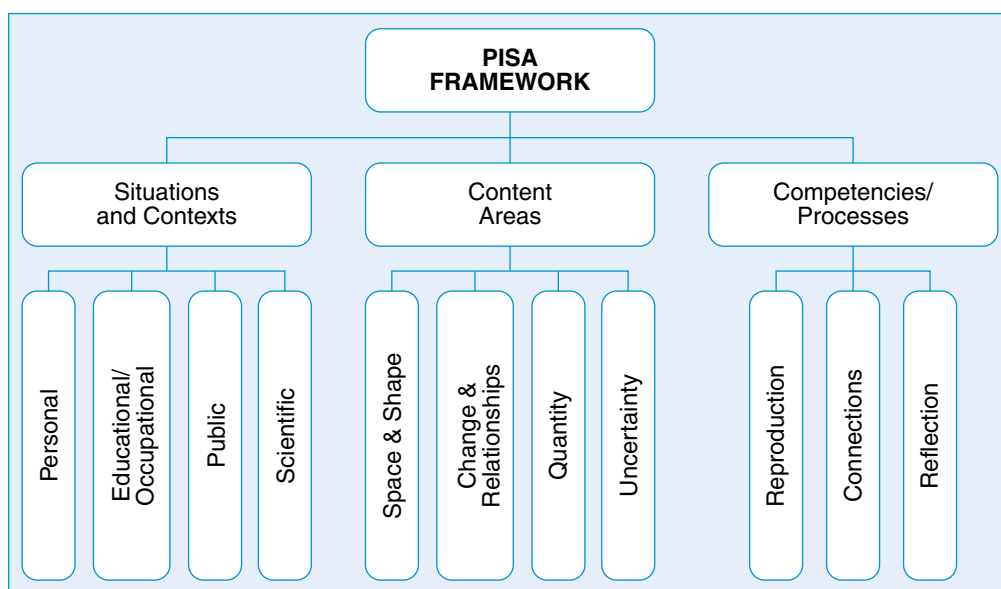
Figure 2.1 The components of the mathematical literacy framework

Situations and context

An important aspect of mathematical literacy is engagement with mathematics: using and doing mathematics in a variety of situations. Students were shown written materials that described various situations that students could conceivably confront, and which required them to apply their mathematical knowledge, understanding or skill to analyse and deal with the situation. Four situations are defined in the PISA mathematical literacy framework: personal, educational/ occupational, public and scientific. The situations differ in terms of how directly each problem affects students' lives; that is, the proximity of the connection between the student and the problem context.

For example, personal situations are closest to the student and are characterised by the direct perceptions involved. The situations also differ in the extent to which the mathematical aspects are explicit. Some tasks in the assessment refer only to mathematical objects, symbols or structures, and make no reference to matters outside the mathematical world. This reflects the strong emphasis in the PISA mathematical literacy assessment on exploring the extent to which students can both identify mathematical features of a problem presented in a non-mathematical context, and activate their mathematical knowledge to explore and solve such a problem.

Mathematical content

The PISA framework defines mathematical content in terms of four broad knowledge domains, referred to as 'overarching ideas', which reflect historically established branches of mathematical thinking and underpin mathematical curricula in education systems throughout the world. Together, these broad content areas cover the range of mathematics that 15-year-old students need as a foundation for life and for further extending their horizon in mathematics. The four overarching ideas are as follows:

◗ *Space and shape* - which draws on the curriculum of geometry. Looking for similarities and differences, recognising shapes in different representations and different dimensions, understanding the properties of objects and their relative positions, and the relationship between visual representations (both two- and three-dimensional) and real objects.

◗ *Change and relationships* - which relates most closely to the curriculum area of algebra. Recognising relationships between variables and thinking about relationships in a variety of forms including symbolic, algebraic, graphical, tabular and geometric.

◗ *Quantity* - understanding of relative size, recognition of numerical patterns, and the use of numbers to represent quantities and quantifiable attributes of real world objects (counting and measuring).

◗ *Uncertainty* – solving problems related to data and chance, which generally correspond to statistics and probability in school curricula.

In PISA 2003, results were reported for each of these four overarching ideas, as well as for mathematical literacy overall. Separate reporting by subscale is not possible for mathematical literacy in 2006 and 2009, and so results from 2003 will be used in this report.

Mathematical processes

While the overarching ideas define the main areas of mathematics that are assessed in PISA, they do not make explicit the mathematical processes that students apply as they attempt to solve problems. The PISA mathematics framework uses the term *mathematisation* to define the cycle of activity for investigating and solving real-world problems. Beginning with a problem situated in reality, students must organise it according to mathematical concepts. They progressively trim away the reality in order to transform the problem into one that is amenable to direct mathematical solution. Students can then apply specific mathematical knowledge and skills to solve the mathematical problem before using some form of translation of the mathematical results into a solution that works for the original problem context; for example, this may involve the formulation of an explanation or justification of proof.

Various competencies are called into play as the mathematisation process is employed. Each of these competencies can be processed at different levels of mastery. The PISA mathematical literacy framework discusses and groups the competencies into three clusters: the *reproduction cluster* (which involves the reproduction of practised knowledge); the *connections* cluster (which builds on the reproduction cluster by applying problem solving to situations that are not routine but still familiar); and the *reflection* cluster (which involves reflecting about the process needed or used to solve a problem). Key features of the competency clusters are shown in Figure 2.2.

Reproduction Cluster	Connections Cluster	Reflection Cluster
Reproducing representations, definitions and facts Interpreting simple, familiar representations Performing routine computations and procedures Solving routing problems	Integrating and connection across content, situations and representations Non-routine problem solving, translation Interpretation of problem situations and mathematical statements Using multiple well-defined methods Engaging in simple mathematical reasoning	Complex problem solving and posing Reflecting on, and gaining insight into, mathematics Constructing original mathematical approaches Communicating complex arguments and complex reasoning Using multiple complex methods Making generalisations

Figure 2.2 PISA competency clusters[1]

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1   Source: Sheil, G., Perkins, R., Close, S. & Oldham, E. (2007). *PISA mathematics: A teacher's guide*. Dublin: Government Publications (Table 2.1, p.6).

The structure of the assessment

Item response formats

Detailed information about the construction of assessment booklets and the marking of PISA items can be found in the national report, available from www.acer.edu.au/ozpisa.

The item response formats in the PISA assessment are similar across literacy domains. Students are presented with a series of units, consisting of one or more items related to a piece of text or a diagram accompanied by a text. Some items require students to select the correct answer, using a basic or complex multiple-choice item format. Other items involve students having to construct a response. There are three different types of constructed response items – short response items (students are required to provide a response that is numeric or another fixed form); open constructed response items (students write an explanation of their results that illustrates aspects of the methods and thought processes they used to answer the question); and closed response items (students give evidence of the calculations they employed to complete the answer).

Distribution of items

A total of 85 mathematical literacy items were used in PISA 2003, with almost half the items included in the 2006 and 2009 PISA assessments. The common items assessed in each cycle provide a link that enables the monitoring of 15-year-old mathematical literacy performance across and within countries over time. Of the 85 items, 17 were multiple-choice items, 11 were complex multiple-choice items, 13 were closed-constructed response items, 21 were open-constructed response items; and 23 were short response items.

Scaling the mathematical literacy tasks

The scale of mathematical literacy was constructed using Item Response Theory, with mathematical literacy items ranked by difficulty and linked to student proficiency. Using such methods means that the relative ability of students taking a particular test can be estimated by considering the proportion of test items they answer correctly, while the relative difficulty of items in a test can be estimated by considering the proportion of students getting each item correct. On this scale, it is possible to estimate the location of individual students, and to describe the degree of mathematical literacy that they possess.

The relationship between items and students on the mathematical literacy scale (shown in Figure 2.3) is probabilistic. The estimate of student proficiency reflects the kinds of tasks they would be expected to successfully complete. A student whose ability places them at a certain point on the PISA mathematical literacy scale would most likely be able to successfully complete tasks at or below that location, and increasingly more likely to complete tasks located at progressively lower points on the scale, but would be less likely to be able to complete tasks above that point, and increasingly less likely to complete tasks located at progressively higher points on the scale.
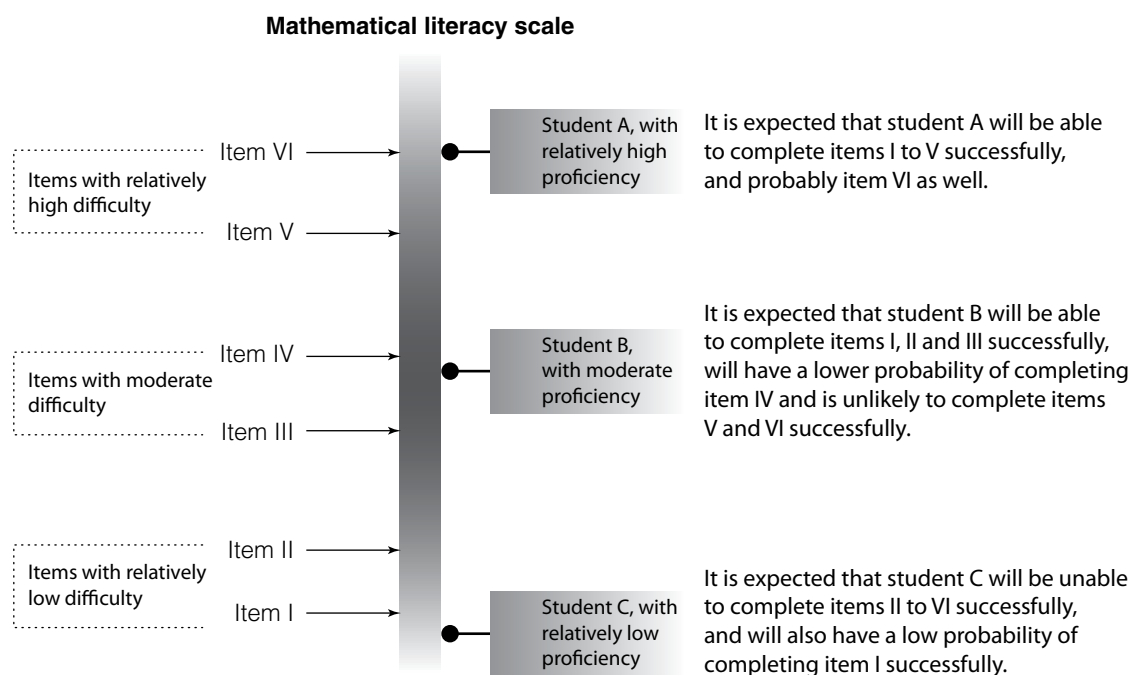
Mathematical literacy scale

Figure 2.3 The relationship between items and students on the mathematical literacy scale

Reporting mathematical literacy performance: mean scores and proficiency levels

The results for all countries for PISA 2000 – 2009 are available through the international and national reports (www.acer.edu.au/ozpisa). The following section of this report will provide a brief overview of Australia's results compared to those of some other countries, and will give the reader an idea of how Australian students perform on this assessment compared to:

◗ other native English speaking countries (Canada, New Zealand, United States);

◗ Finland (highest scoring country previously);

◗ high-achieving Asian neighbours (Hong Kong – China, Korea, Shanghai – China, Singapore); and

◗ the OECD average.

Mean scores and distribution of scores

Student performance in PISA is reported in terms of statistics such as mean scores and measures of distributions of achievement, which allow for comparisons against other countries and subgroups. Mean scores provide a summary of student performance and allow comparisons of the relative standing between different student subgroups. In PISA 2003, the mean score across participating OECD countries was set at 500 score points with a standard deviation of 100, and in PISA 2009 the OECD average was 496 score points.[2] This mean score is the benchmark against which future mathematical performance in PISA is compared.

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2   The OECD average reflects the mean score for all OECD countries. The OECD average can change from each PISA assessment because the number of participating countries differs (for eg. in 2003, there were 30 OECD countries and in 2009 this had increased to 34 OECD countries) and also because the overall performance for a country can change.

Figure 2.4 shows the scores of the countries listed above relative to the OECD average, for PISA 2009 mathematical literacy. All countries that are annotated with an asterisk (*) scored at a level significantly higher than the OECD average, and the countries whose bars are shaded in dark blue are those whose scores were significantly higher than those of Australia.
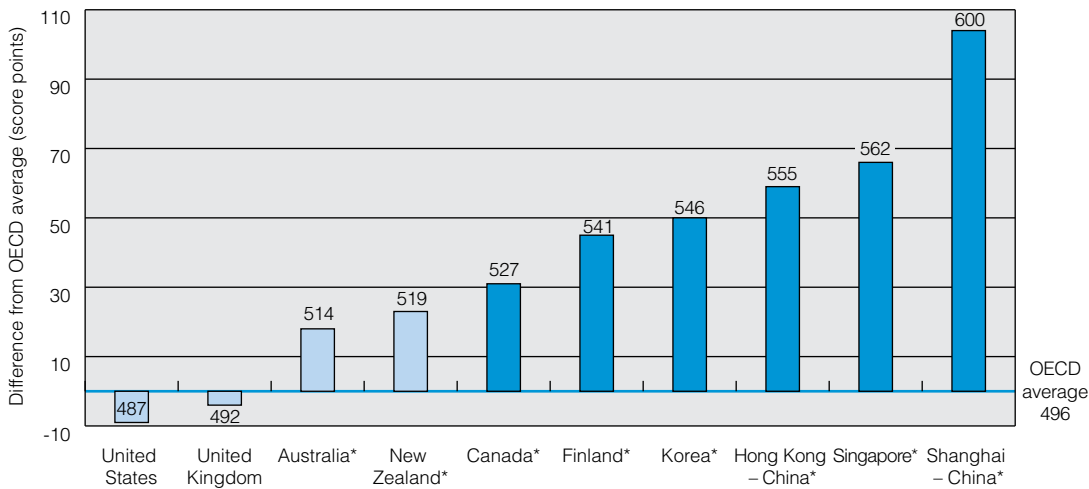

Figure 2.4 PISA 2009 Mathematical literacy achievement comparison with OECD average score

There were statistically significant gender differences in mathematical literacy performance in many participating countries, with males outscoring females by 12 score points, on average, across the OECD. The difference in the average performance of females and males in Australia was a significant 10 score points, similar to that seen in Canada (12 score points) but substantially lower than that in the United Kingdom (21 score points) or the United States (20 score points).

Interpreting such data can be challenging. We know what the mean and standard deviation are, but what does this mean in practical terms? Fortunately we are able to get a rough measure of how many score points comprise a year of schooling, given that 15-year-old students are often in adjacent grades.

> For Australia, in mathematical literacy, one year of schooling was found to be the equivalent of 41 score points.

Looking at the difference between the scores of students in Shanghai – China and those in Australia, the score difference of 86 score points translates to just over two years of schooling.

Programme for International Student Assessment

Proficiency levels

While mean scores provide a comparison of student performance on a numerical level, proficiency levels provide a description of the knowledge and skills that students are typically capable of displaying. This produces a picture of the distribution of student performance within a country (or other groups of students) across the various proficiency levels. In PISA 2003, six levels of proficiency for mathematical literacy were defined, which have remained unchanged for subsequent cycles.[3] The continuum of increasing mathematical literacy (with Level 6 as the highest and Level 1 as the lowest proficiency level) is shown in Figure 2.5, along with the summary descriptions of the kinds of mathematical competencies associated with the different levels of proficiency. A difference of 62 score points represents one proficiency level on the PISA mathematical literacy scale.

	Students at this level can ...
Level 6	conceptualise, generalise, and utilise information; are capable of advanced mathematical thinking and reasoning; have a mastery of symbolic and formal mathematical operations and relationships; formulate and precisely communicate their findings, interpretations and arguments.
Level 5	develop and work with models for complex situations; select, compare, and evaluate appropriate problem solving strategies for dealing with complex problems; work strategically using broad, well-developed thinking and reasoning skills; reflect on their actions and formulate and communicate their interpretations and reasoning.
Level 4	work effectively with explicit models for complex concrete situations; select and integrate different representations, including symbolic ones; utilise well-developed skills and reason flexibly; construct and communicate explanations and arguments.
Level 3	execute clearly described procedures, including those that require sequential decisions; select and apply simple problem solving strategies; interpret and use representations; develop short communications reporting these.
Level 2	interpret and recognise situations in contexts that require no more than direct inference; extract relevant information from a single source and make use of a single representational mode; employ basic procedures; make literal interpretations of the results.
Level 1	answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined; identify information and carry out routine procedures according to direct instructions in explicit situations; perform actions that are obvious and follow immediately from the given stimuli.
Below Level 1	not demonstrate even the most basic types of mathematical literacy that PISA measures. These students are likely to be seriously disadvantaged in their lives beyond school.

Figure 2.5 Summary descriptions of the six proficiency levels on the overall mathematical literacy scale

Students who scored below 358 score points are placed below Level 1. This is not to say that these students were incapable of performing any mathematical operation, but they were unable to utilise mathematical skills in a given situation as required by the easiest PISA tasks. Their pattern of answers was such that they would be expected to be able to solve fewer than half of the tasks in a test made up solely of questions drawn from Level 1. These students are likely to have serious difficulties in using mathematics to benefit from further education and learning opportunities in life.

Internationally, Level 2 has been defined as a 'baseline' proficiency level, as it represents a standard level of mathematical literacy proficiency where students begin to demonstrate the kind of skills that enable them to actively use mathematics as stipulated by the PISA definition.

The percentage of students at each of the six proficiency levels and the proportion not achieving the lowest proficiency level is shown in Figure 2.6. Australia is doing reasonably well in mathematical literacy, with 16 per cent not achieving the lowest levels described by MCEECDYA as being an acceptable standard.

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3   Information about how the proficiency levels are created is available in Chapter 2 of *Challenges for Australian Education: Results from PISA 2009*.
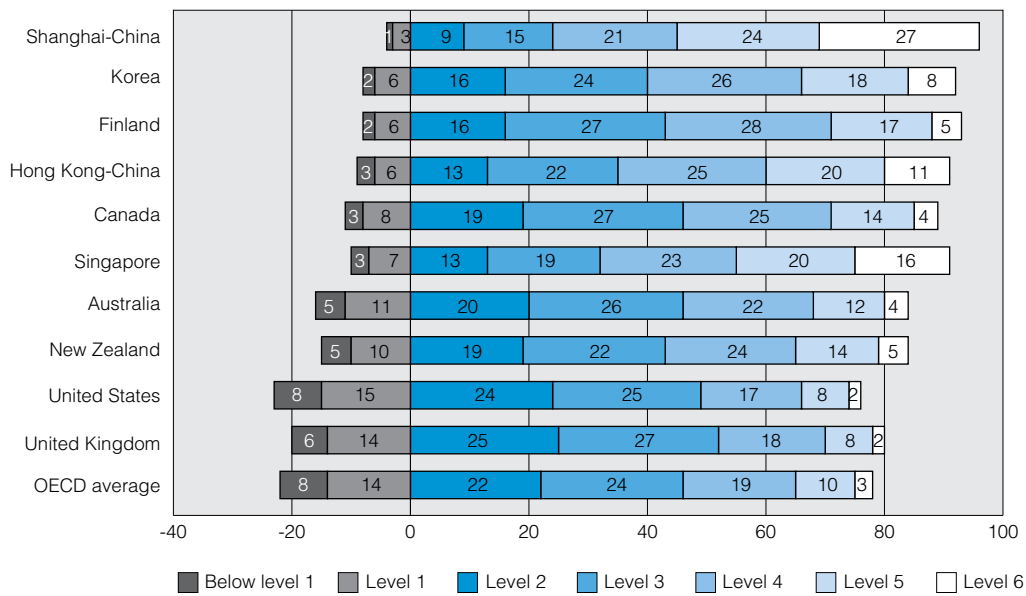
Figure 2.6 Proportions of students at mathematical proficiency levels for Australia and comparison countries

However it is also evident from the figure that Australia has a substantially higher proportion of students in the lower mathematical literacy levels than some other countries, and a lower proportion of students in the higher levels of achievement. Both need to be addressed if Australia's achievement is to improve.

Gender differences

The proportions of females and males at each of the mathematical literacy proficiency levels in Australia and across the OECD countries are shown in Figure 2.7.
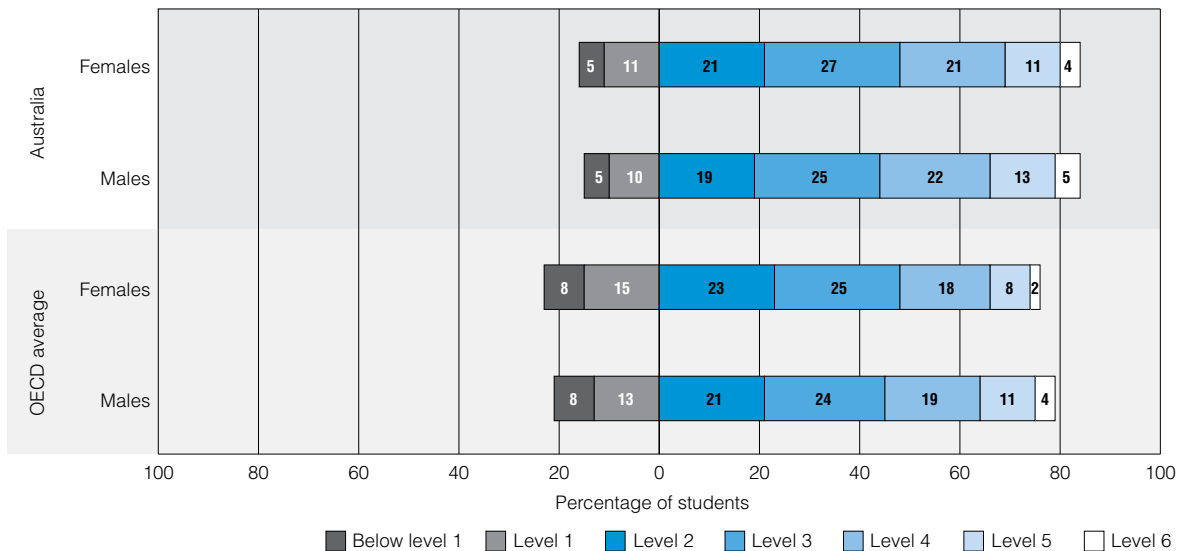

Figure 2.7 Proportions of students at mathematical proficiency levels by gender, Australia and OECD average

◗ A slightly larger proportion of male than female students in Australia achieved at the higher proficiency levels, and about the same proportion of each at the lower proficiency levels.

◗ In Australia, 15 per cent of females and 18 per cent of males reached Level 5 or 6, compared to 10 per cent of females and 15 per cent of males across OECD countries.

Performance on the mathematical literacy subscales

Earlier in this chapter, we described the components of mathematical literacy (subscales) – *Space and shape*, *Change and relationships*, *Quantity*, and *Uncertainty*. The difference between Australian male and female students' scores and the OECD average on each of these is shown in Figure 2.8.
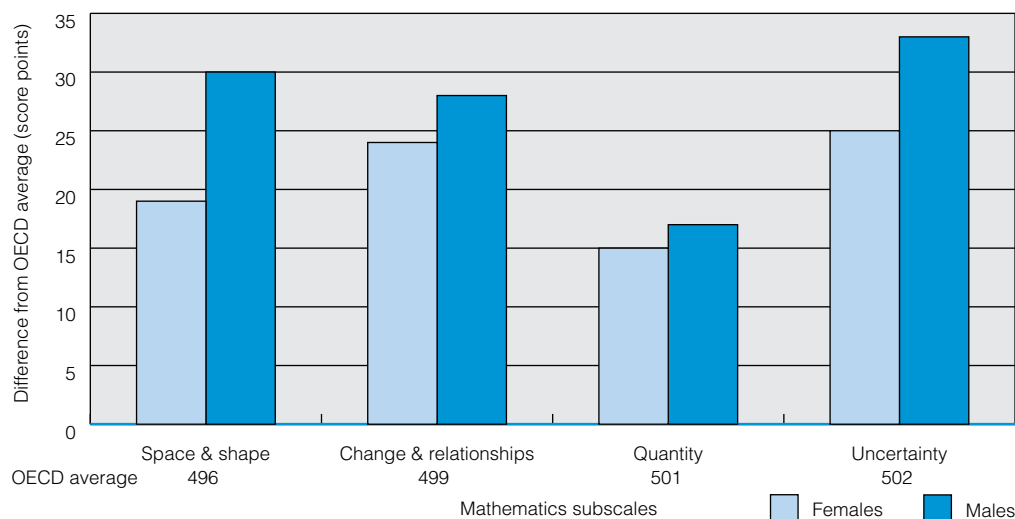

Figure 2.8 Performance on mathematics subscales for Australian students relative to OECD average

◗ Australian students scored significantly better than the OECD average on each of the subscales.

◗ Australian students performed relatively better overall on *uncertainty* tasks, relatively less well on *quantity* tasks.

◗ The largest gender differences are apparent in *space and shape* and *uncertainty*, in which males scored substantially as well as significantly higher than females.

Performance over time

One of the main aims of PISA is to examine student performance over time so that policy makers can monitor learning outcomes in both a national and international context. PISA 2003 defined the mathematical literacy framework in detail, and so comparisons can be made to this point in time.

A handful of countries saw an improvement in their mathematical literacy scores from 2003 to 2009, but Australia's average score declined by a significant 10 points. There was no statistical difference between the proportion of Australian students achieving Level 2, the 'baseline' proficiency level, in 2003 and 2009; however, the number of Australian students achieving Level 5 or above, in the top performing category, had dropped from 20 to 16 per cent.

Results for other groups of students within Australia

Indigenous students:

◗ achieved a mean score of 441 points, compared to a mean score of 517 points for non-Indigenous students. The difference in scores is the equivalent of almost two years of schooling.

◗ were underrepresented at the higher end of the mathematical literacy proficiency scale. Just four per cent achieved at or above Level 5, compared to 17 per cent of non-indigenous Australian students and 13 per cent of students on average across the OECD achieved this level.

◗ were over-represented at the lower end of the mathematical literacy proficiency scale. Forty per cent failed to reach Level 2, compared to 22 per cent of students across the OECD and 15 per cent of non-Indigenous students in Australia.

Students with a language background other than English:

◗ performed at a similar level to students who spoke English as their main language, with mean scores of 517 points and 516 points respectively.

◗ were more likely than students with an English-speaking background to achieve at the higher proficiency levels 5 or 6 (21% and 16% respectively).

◗ were more likely than students with an English speaking background to not reach Level 2, (20% and 14% respectively).

Students from the lowest quartile of socioeconomic background:

◗ achieved a mean score of 471 points compared to students in the highest quartile who achieved a mean score of 561 points.

> This difference was equivalent to almost three full years of schooling.

◗ were overrepresented at lower levels of achievement and underrepresented at higher levels. Just six per cent of students in the lowest quartile compared with 29 per cent of students in the highest quartile achieved at or above Level 5, while five per cent of students in the highest quartile of socioeconomic background, compared to more than one quarter (28%) of students in the lowest quartile, failed to reach Level 2.

Students in metropolitan areas:

◗ performed at a significantly higher level than students in schools from provincial areas, who in turn performed at a significantly higher level than students attending schools in remote areas.

> The score differences equate to about 2/3 of a school year between students in metropolitan and provincial schools and a further year higher than those in remote schools.

◗ were more likely to achieve at the higher proficiency levels - 18 per cent from metropolitan schools, 12 per cent from provincial schools and eight per cent of students from remote schools, achieved at or above Level 5.

◗ were less likely to achieve at the lower proficiency levels - 15 per cent of those in metropolitan schools, 19 per cent in provincial schools, and 33 per cent of students in remote schools failed to reach Level 2.

Points to ponder

◗ Do you think there are substantial differences in the performance of different groups of students in your school, as described in this chapter?

◗ What are some reasons you can think of that would help explain gender differences in mathematical literacy?

◗ One of the things that Australia needs to do to improve our overall mathematical literacy is to address the issue of the underachievement of disadvantaged students. What are some ways that schools can help students who are from lower levels of socioeconomic background?

◗ Australian students perform relatively well on items to do with statistics and chance, and relatively less well on items to do with measurement. Do you see similar patterns in your class or school? As both are significant areas of the curriculum, can you think of why this might be so, and suggest ways of improving students' understanding of measurement?

Chapter 3

Sample mathematical literacy items and responses

A selection of sample questions is provided in this section to show the types of items that have been included in the assessment as well as to illustrate the various aspects of the PISA mathematical literacy framework (the overarching ideas, competencies and situations) and the wide range of complexity involved in such tasks. Table 3.1 presents a map of the sample mathematical literacy items included in this section of the report. The most difficult items are located at the top of the figure, at the higher proficiency levels, and the least difficult, at the lower levels, at the bottom. Each of the items is placed in the relevant proficiency level according to the difficulty of the item (the number in brackets), and the content area they are assessing.

Proficiency level	Content Area			
	Space and Shape	Change and Relationships	Quantity	Uncertainty
6	CARPENTER Question 1 (687)	WALKING Question 2 (723) (full credit)		ROBBERIES Question 1 (694)
669.3 score points				
5		WALKING Question 2 (666) (partial credit 2) WALKING Question 1 (611)		
607.0 score points				
4		WALKING Question 2 (605) (partial credit 1)	EXCHANGE RATE Question 3 (586)	
544.7 score points				
3	NUMBER CUBES Question 1 (503)			
482.4 score points				
2			EXCHANGE RATE Question 2 (439)	
420.1 score points				
1			EXCHANGE RATE Question 1 (406)	
357.8 score points				

Figure 3.1 Sample items and cut-off score points for the mathematical literacy proficiency scale

In each of the graphs in this chapter, the bars represent the difference between the proportion of students in the country that answered correctly and the OECD average proportion of students that answered correctly. Countries are asterisked (*) if this proportion is <u>significantly</u> different to the OECD average, and bars are shaded dark blue if the proportion is <u>significantly</u> different to the proportion of Australian students. Comparisons will be made to all of the countries used in the previous chapter other than Shanghai-China and Singapore, who did not participate in PISA in 2003.

Exchange Rate

The unit 'Exchange Rate' consisted of three items involving number operations (multiplication and division) set in the overarching Quantity area and in a public context. The concept of foreign exchange rates, and the possibility of both increasing and decreasing movements, formed the basis of this constructed response unit. Exposure to the operation and use of exchange rates may not be common to all students but the concept can be seen as belonging to skills and knowledge required in the global economy.

Exchange Rate Question 1

The first item in 'Exchange Rate' required students to interpret a simple, explicit mathematical relationship (the exchange rate for 1 Singapore dollar/1 South African rand), and then apply a small reasoning step to apply the relationship directly to 3000 Singapore dollars, using the calculation (3000 x 4.2). This item is set in a relatively familiar context and the direct application of well-known mathematical knowledge places this item at proficiency level 1. The following answer is an example of a student response that was awarded full credit.

Figure 3.2 shows the proportion of students in Australia and each of the comparison countries who answered this item correctly.
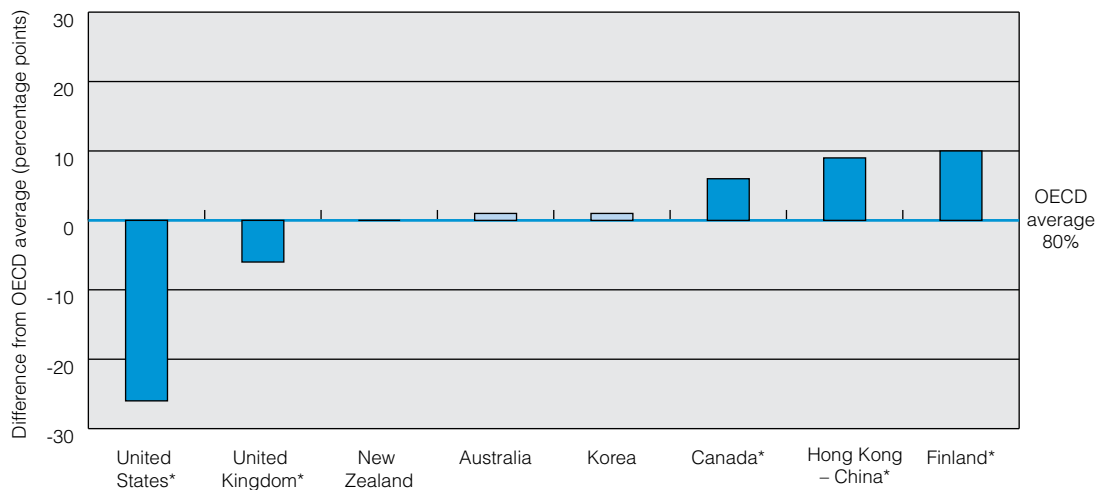
Figure 3.2 Proportion of students providing correct response to Exchange Rate question 1: Australia and comparison countries

◗ Just over 80 per cent of Australian students in 2003 were able to answer question 1 from the Exchange Rate unit correctly. This proportion was approximately equal to the average across OECD countries.

◗ A significantly lower proportion of Australian students answered this question correctly compared to students from Canada, Finland and Hong Kong-China.

◗ A significantly higher proportion of Australian students answered this question correctly than the United Kingdom and the United States.

Exchange Rate Question 2

The second item in 'Exchange Rate' was also a short constructed response item, which required a limited form of mathematisation (understanding a simple text) as well as deciding that division was the correct procedure.

Students were required to interpret a simple, explicit mathematical relationship and only a small reasoning step was required to apply the relationship directly to 3900 South African rand using a calculation (3900/4.0). This question belonged to the reproduction competency cluster and proficiency level 2. An example of a correct student response is provided below.
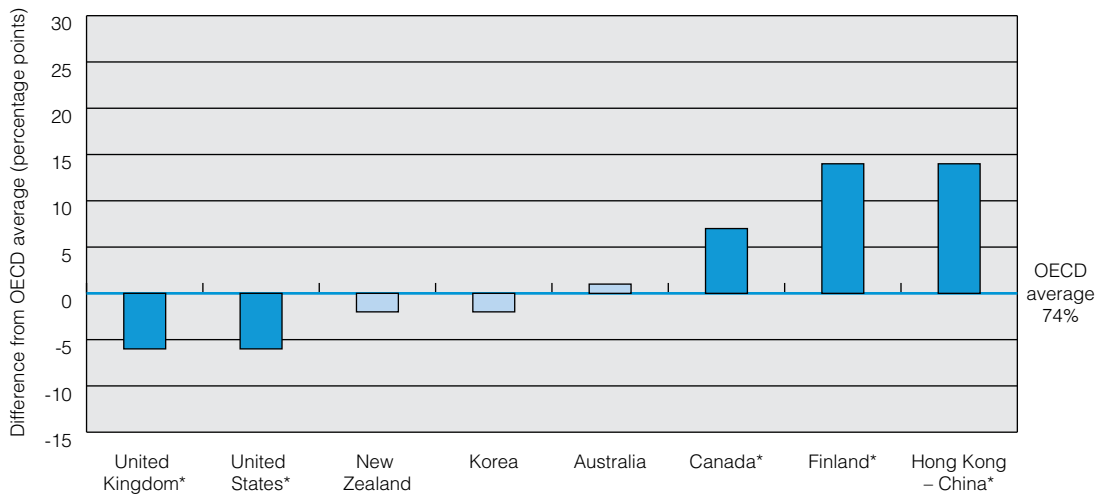
Figure 3.3 Proportion of students providing correct response to Exchange Rate question 2: Australia and comparison countries

◗ Three-quarters of Australian students responded to question 2 from the Exchange Rate unit correctly, which was a similar proportion to that found across OECD countries, on average.

◗ Australia had a significantly lower proportion of students responding correctly than Canada, Finland and Hong Kong-China.

◗ More Australian students than students from the United Kingdom and the United States answered this question correctly.

Exchange Rate Question 3

The mathematics required to solve the problem in this open constructed response item was more demanding as students needed to reflect on the concept of exchange rate movements and the subsequent consequences. The required procedural knowledge was more complex, and involved students applying flexible reasoning and reflection.

The student example below achieved full credit. Students had to interpret the specified change in the exchange rate and apply basic computational skills or quantitative comparison skills to solve the problem. Students also needed to provide an explanation of their conclusion. This item belongs to the reflection cluster and represents proficiency level 4.

> Exchange Rate Question 3 is an example of a quantity item. Australian students did not perform as well on these tasks compared to other content tasks.

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

Yes it was Mei-Ling's favour because she got more than what she would have if it was at 4.2 ZAR. per SGD.

example
3900 ÷ 4.2 = 928.5 SGD
3900 ÷ 4 = 975 SGD

Students found this item more difficult than the previous two questions in this unit, with about half of the Australian students in 2003 successfully answering this question.
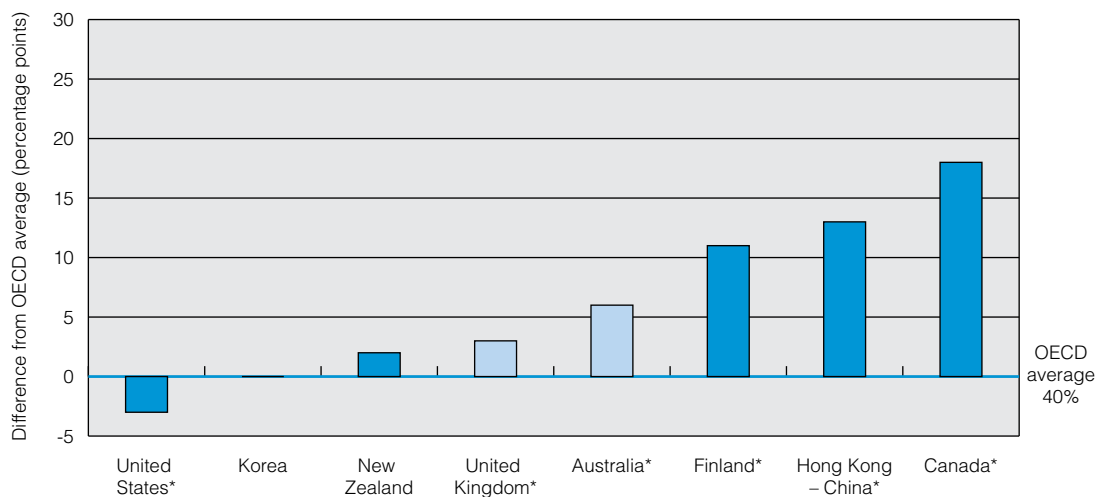

Figure 3.4 Proportion of students providing correct response to Exchange Rate question 3: Australia and comparison countries

◗ The proportion of Australian students who answered question 3 correctly was significantly higher than the average proportion of students across OECD countries.

◗ While the proportion of Australian students responding correctly is significantly lower than the proportion of students in Canada, Finland and Hong Kong-China, the proportion is significantly higher than in New Zealand, the United States and Korea.

Number Cubes

During their education, students would have encountered many games and activities, whether formal or informal, that use number cubes or dice. Somewhat challenging was the problem posed below, which required spatial insight or mental visualisation technique, as students needed to imagine how the four planes of number cubes, if reconstructed into a three-dimensional number cube, obey the numerical construction rule given in the information (i.e. two opposite sides have a total of seven dots).

This problem required the encoding and spatial interpretation of two-dimensional objects, interpretation of the connected three-dimensional object, and checking certain basic computational relations. Thus this item fits within the connections competency cluster, an essential part of mathematical literacy, because students live in three-dimensional space and are often confronted with two-dimensional representations.

Full credit was given to students who correctly identified the four expected results, as shown in the example below. This complex multiple-choice item is situated in a personal context, is placed in the overarching area of Space and Shape and illustrates proficiency level 3.
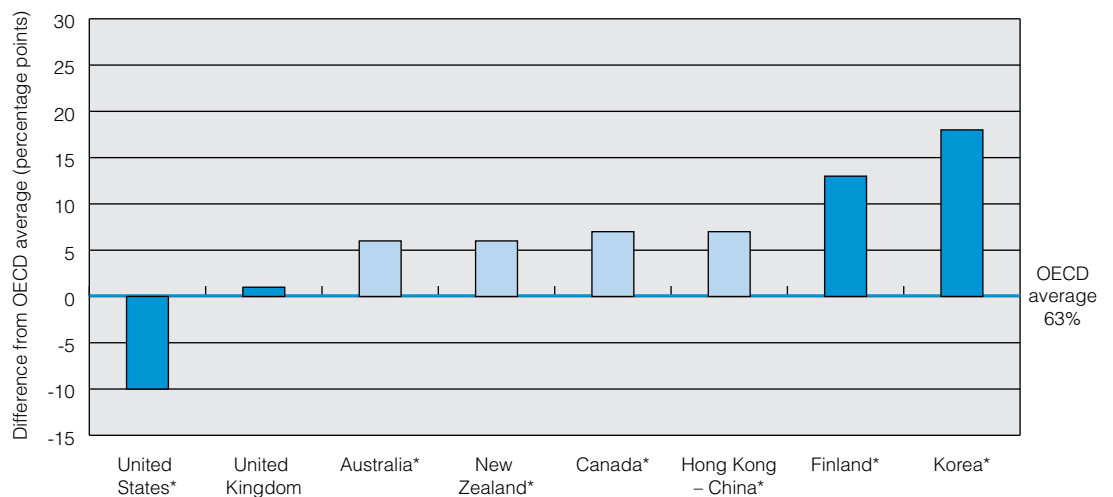
Figure 3.5 Proportion of students providing correct response to Number Cubes question 1: Australia and comparison countries

◗ Almost 70 per cent of Australian students answered question 2 from the Number Cubes correctly which was significantly more than the average proportion of students for OECD countries.

◗ The proportion of Australian students who answered this question correction was significantly higher than students from the United Kingdom and the United States, but was lower than those from Finland and Korea.

Walking

Reflecting on embedded mathematics from daily life is part of acquiring mathematical literacy and the unit 'Walking' is an example of this phenomenon. Students would be familiar with seeing their footprints in sand or soil but prob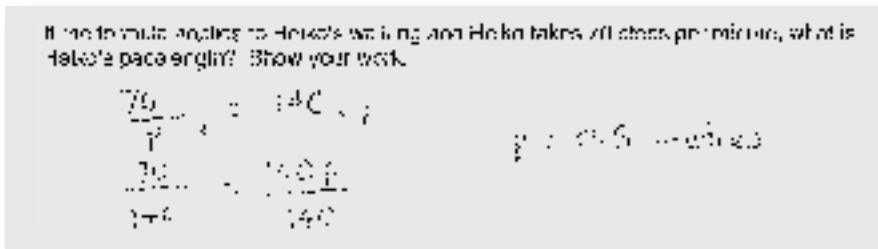ably would not have given much thought to the relationship between the 'number of steps taken per minute' and 'pace length'.

The two questions in this unit were open constructed response items, in the Change and Relationships area and situated in a personal context.

Walking Question 1

The first item required problem solving by asking students to make use of a formal algebraic expression – substituting a simple formula and carrying out a routine calculation: if 70/p = 140 what is the value of p? Students needed to recognise that as the pace length increases, so the number of steps per minute will decrease, and in order to gain credit for this item students needed to carry out the actual calculation.

This item belongs to the reproduction competency cluster and illustrates Level 5 proficiency. The following example gained full credit for showing the correct substitution of numbers in the formula, along with the correct answer, and the scoring guide follows.

Walking scoring – Question 1

Full Credit
0.5 m or 50 cm, $\frac{1}{2}$ (unit not required).

▸ 70 / p = 140

▸ 70 = 140 p

▸ p = 0.5.

▸ 70/140.

Partial Credit
Correct substitution of numbers in the formula, but incorrect answer, or no answer.

▸ $\frac{70}{p}$ = 140 [substitute numbers in the formula only].

▸ $\frac{70}{p}$ = 140

▸ 70 = 140 p

▸ p = 2 [correct substitution, but working out is incorrect].

OR Correctly manipulated the formula into $P=n/140$, but no further correct working.
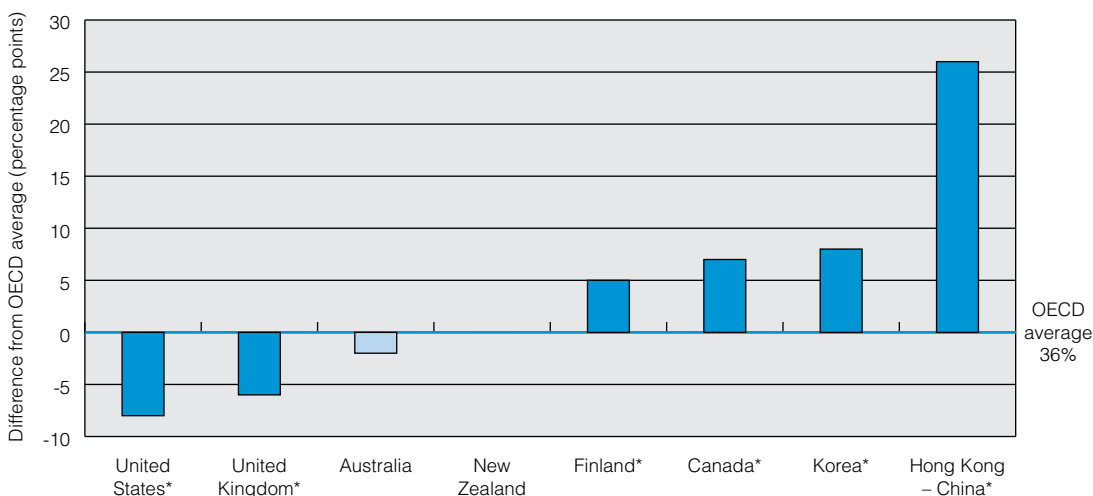
No Credit

▸ Other responses

▸ 70 cm.

Figure 3.6 Proportion of students providing correct response to Walking question 1: Australia and comparison countries

◗ Around one-third of Australian students received full credit for their answer to this question.

◗ The proportion of Australian students who answered this question correctly was significantly higher than the proportions of students from the United Kingdom or the United States who did so.

Walking Question 2

The second item in 'Walking' also involved the relationship between 'the number of steps per minute' and 'pace length', but with the added requirement of using a non-routine calculation. Students needed to calculate the number of steps per minute when the pace length is given (0.8m), which requires proper substitution: $n/0.80 = 140$ and the observation that this equals $n = 140 \times 0.80 = 112$ (steps per minute).

More than routine operations were required here, with substitution in an algebraic expression being used followed by manipulating the resulting formula, in order to carry out the required calculation. The next step required going beyond the observation that the number of steps is 112, as the question also asked for the speed per minute – the subject walks $112 \times 0.80 = 89.6$ metres, so his speed is 89.6 metres/minute. The final step is to transform this speed in metres/minute into kilometres/hour – a more common unit of speed.

Full credit for this item illustrates the high level of skills and knowledge required at proficiency level 6. Only one-fifth of Australian students received full credit for their response. Students providing the above explanations were given full credit as they showed they were able to complete the conversions and provide a correct answer in both the requested units. This problem is rather complex and belongs to the connections competency cluster. Not only is use of a formal algebraic expression required, but also completing a sequence of different but connected calculations that need proper understanding of transforming formulae and units of measure. The following sample response was awarded full credit.

Bernard knows his pace length is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

Students who scored at a high level of partial credit for this item demonstrated high Level 5 ability with a difficulty of 666 score points, only 3 score points below Level 6. Although students were able to go further than finding the number of steps per minute, and made some progress towards the conversions, their final responses were not entirely correct or remained incomplete.

A lower level of partial credit was given when students showed they had understood the formula and correctly substituted the appropriate values, finding the number of steps per minute. These responses were placed at the top part of Level 4– just below the boundary of Level 5.

Walking scoring – Question 2

Full Credit
Correct answers (unit not required) for both metres/minute and km/hour: n = 140 x .80 = 112. Per minute he walks 112 x .80 metres = 89.6 metres. His speed is 89.6 metres per minute. So his speed is 5.38 or 5.4 km/hr.

Full credit given as long as both correct answers are given (89.6 and 5.4), whether working out is shown or not. Note that errors due to rounding are acceptable. For example, 90 metres per minute and 5.3 km/hr (89 X 60) are acceptable.

◗ 89.6, 5.4.

◗ 90, 5.376 km/h.

◗ 89.8, 5376 m/hour [note that if the second answer is given without units, it should be coded as partial credit].

Partial Credit (2-point)
As for full credit but fails to multiply by 0.80 to convert from steps per minute to metres per minute. For example, his speed is 112 metres per minute and 6.72 km/hr.

◗ 112, 6.72 km/h.

OR The speed in metres per minute correct (89.6 metres per minute) but conversion to kilometres per hour incorrect or missing.

◗ 89.6 metres/minute, 8960 km/hr.

◗ 89.6, 5376.

◗ 89.6, 53.76.

◗ 89.6, 0.087 km/h.

◗ 89.6, 1.49 km/h.

OR Correct method (explicitly shown) with minor calculation error(s) not covered by Code 21 and Code 22. No answers correct.

◗ n=140 x .8 = 1120; 1120 x 0.8 = 896. He walks 896 m/min, 53.76km/h.

◗ n=140 x .8 = 116; 116 x 0.8 =92.8. 92.8 m/min -> 5.57km/h.

Partial Credit (1-point)
Only 5.4 km/hr is given, but not 89.6 metres/minute (intermediate calculations not shown).

◗ 5.4.

◗ 5.376 km/h.

◗ 5376 m/h.

OR *n* = 140 x .80 = 112. No further working out is shown or incorrect working out from this point.

◗ 112.

◗ n=112, 0.112 km/h.

◗ n=112, 1120 km/h.

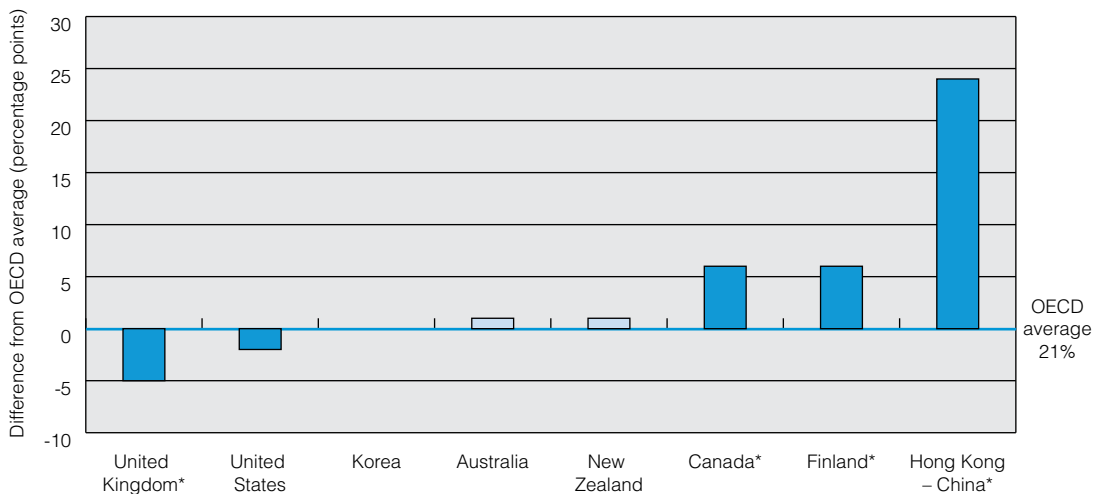◗ 112 m/min, 504 km/h.

No Credit

◗ Other responses.

Figure 3.7 Proportion of students providing correct response to Walking question 2: Australia and comparison countries

- Just over one fifth of Australian students received credit for their response to this item.
- The proportion of Australian students that received credit was significantly higher than the proportions of students from the United Kingdom and the United States.
- Greater proportions of students in Canada, Finland and Hong Kong–China received credit on this item compared to students in Australia.

Robberies

The unit 'Robberies', situated in the public context, provided a graphical representation showing the number of robberies within a two-year period, along with a statement made by a reporter. This type of item is frequently presented in the media where graphics have been used to support a predetermined message.

The item involved data interpretation, placing it in the overarching area of Uncertainty and in the connections competency cluster, as students needed to rely on reasoning and interpretation competencies together with communication skills. Students were asked, using an open constructed response, to consider the reporter's statement and with the use of the graph explain whether the statement fitted the data.

An example of a full credit response is shown below. To obtain full credit, students had to indicate that the statement was not reasonable and explain their judgment in appropriate detail. Answers had to focus on an increase given by the exact number of robberies in absolute terms and also in relative terms.
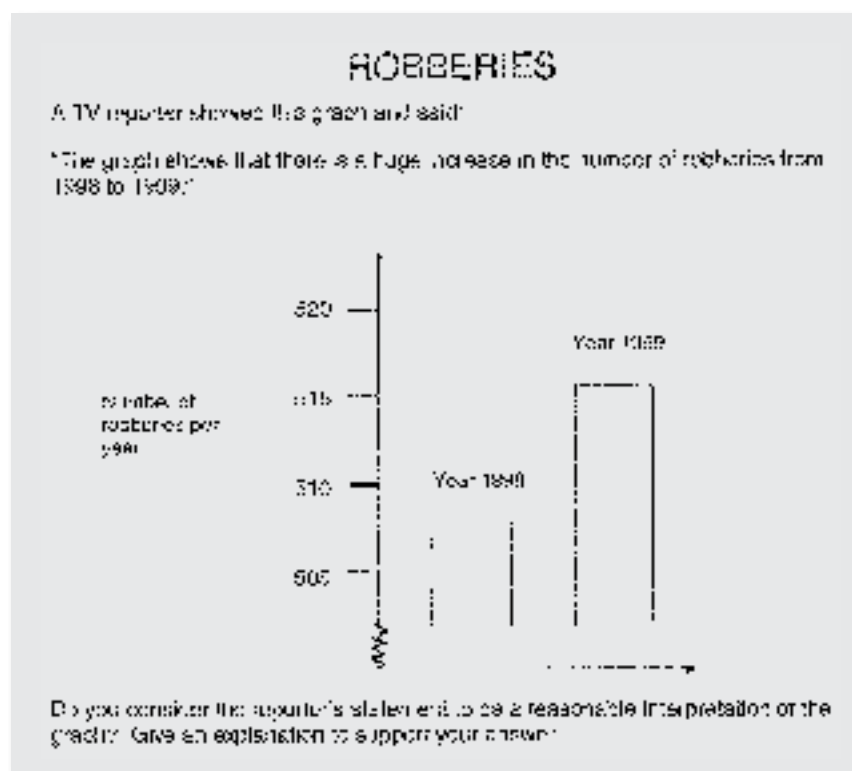
Robberies Question 1 is an example of an uncertainty item. This kind of task was a relative strength for Australian students compared to other content areas tasks.

This item illustrated a proficiency at Level 6. The question required students to be able to communicate an argument based on interpretation of data, using some proportional reasoning in a statistical context.

Robberies scoring – Question 1

Note: The use of NO in these codes includes all statements indicating that the interpretation of the graph is NOT reasonable. YES includes all statements indicating that the interpretation is reasonable. Please assess whether the student's response indicates that the interpretation of the graph is reasonable or not reasonable, and do not simply take the words "YES" or "NO" as criteria for codes

Full Credit
No, not reasonable. Focuses on the fact that only a small part of the graph is shown.

◗ Not reasonable. The entire graph should be displayed.

◗ I don't think it is a reasonable interpretation of the graph because if they were to show the whole graph you would see that there is only a slight increase in robberies.

◗ No, because he has used the top bit of the graph and if you looked at the whole graph from 0 – 520, it wouldn't have risen so much.

◗ No, because the graph makes it look like there's been a big increase but you look at the numbers and there's not much of an increase.

◗ No, because the graph makes it look like there's been a big increase but you look at the numbers and there's not much of an increase.

OR No, not reasonable. Contains correct arguments in terms of ratio or percentage increase.

◗ No, not reasonable. 10 is not a huge increase compared to a total of 500.

◗ No, not reasonable. According to the percentage, the increase is only about 2%.

◗ No. 8 more robberies is 1.5% increase. Not much in my opinion!

◗ No, only 8 or 9 more for this year. Compared to 507, it is not a large number.

OR Trend data is required before a judgement can be made.

◗ We can't tell whether the increase is huge or not. If in 1997, the number of robberies is the same as in 1998, then we could say there is a huge increase in 1999.

◗ There is no way of knowing what "huge" is because you need at least two changes to think one huge and one small.

Partial Credit
No, not reasonable, but explanation lacks detail. Focuses ONLY on an increase given by the exact number of robberies, but does not compare with the total.

◗ Not reasonable. It increased by about 10 robberies. The word "huge" does not explain the reality of the increased number of robberies. The increase was only about 10 and I wouldn't call that "huge".

◗ From 508 to 515 is not a large increase.

◗ No, because 8 or 9 is not a large amount.

◗ Sort of. From 507 to 515 is an increase, but not huge.

[Note that as the scale on the graph is not that clear, accept between 5 and 15 for the increase of the exact number of robberies.]

OR No, not reasonable, with correct method but with minor computational errors.

◗ Correct method and conclusion but the percentage calculated is 0.03%.

No Credit
No, with no, insufficient or incorrect explanation.

◗ No, I don't agree.

◗ The reporter should not have used the word "huge".

◗ No, it's not reasonable. Reporters always like to exaggerate.

OR Yes, focuses on the appearance of the graph and mentions that the number of robberies doubled.

◗ Yes, the graph doubles its height.

◗ Yes, the number of robberies has almost doubled.

OR Yes, with no explanation, or explanations other than those above.
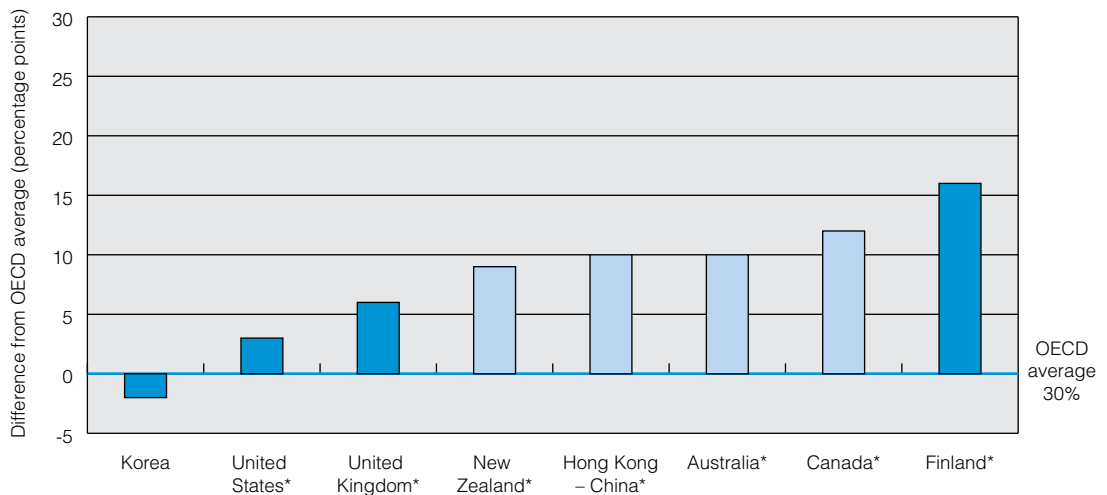
OR Other responses.

Figure 3.8 Proportion of students providing correct response to Robberies question 1: Australia and comparison countries

◗ Forty per cent of Australian students answered this question correctly, receiving either full or partial credit for their responses.

◗ This proportion was higher than that recorded by the United Kingdom, the United States, Korea and across OECD countries on average, but significantly lower than that for Finland.

Carpenter

'Carpenter', also a complex multiple-choice item, fits into the educational context and belongs to the Space and Shape area. Students were presented with four possible designs for garden beds and were asked to determine if each design could be made with 32 metres of timber.
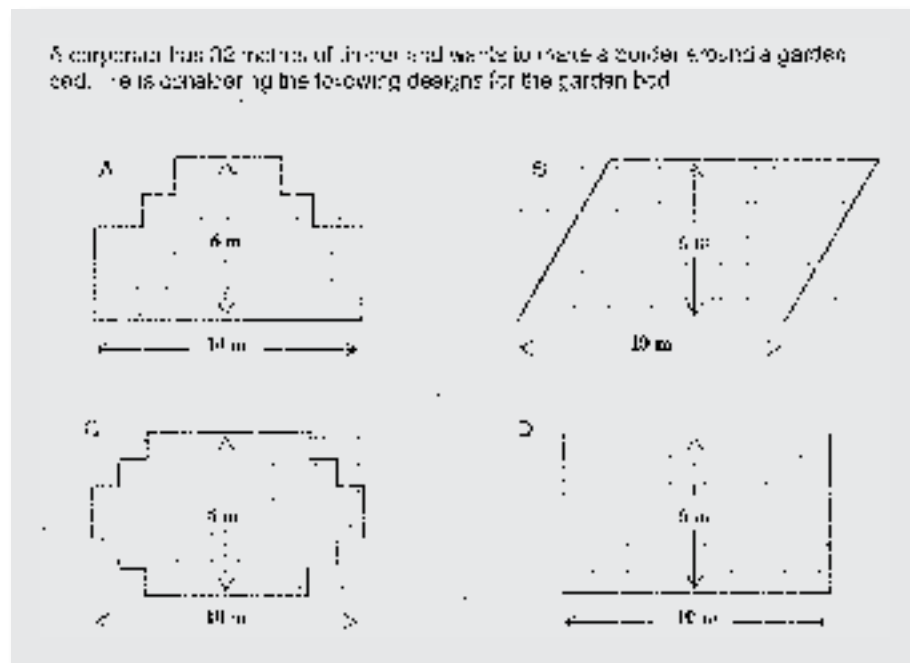
Students needed to rely on their geometric knowledge, not only recognising the three rectangular shapes but also the parallelogram and that it requires more than 32 metres of timber. This use of geometric insight and argumentation skills and technical geometric knowledge makes this one of the more difficult items at Level 6.

To obtain full credit, as shown below, students had to correctly identify which of the garden beds could be constructed. Partial credit was given when students correctly identified three of the four answers. A quarter of Australian students were awarded full credit for their response to this question.

Figure 3.9 Proportion of students providing correct response to Carpenter question 1: Australia and comparison countries

◗ The proportion of Australian students who correctly answered this question was significantly higher than across the OECD countries on average.

◗ More students from Hong Kong-China and Korea than from Australia answered this question correctly, while fewer students in the United Kingdom and the United States gave the correct answer.

Other findings

◗ Differences in the proportion of male and female students who answered individual items correctly were found in only two of the released items presented here: Question 2 in the Number Cubes unit (placed at Proficiency Level 3), and Question 1 in the Carpenter unit, a difficult item (placed at Proficiency Level 6).

Attitudes, engagement and strategies

This chapter looks at several aspects of student learning that are of interest to teachers. Ensuring students have positive attitudes towards learning, and that they have the motivation and capacity to continue learning throughout their lives, are important outcomes of learning in their own right. In addition, helping students to overcome mathematics anxiety, and assisting students in developing strong and positive learning strategies will help them become effective learners of mathematics. This chapter looks at these cognitive, affective and attitudinal aspects of learning mathematics.

This chapter will examine, for Australian students:

◗ *Students' engagement with mathematics*. This is related both to students' own interest and enjoyment and to external incentives.

◗ *Students' beliefs about themselves*. This includes students' views about their own competence and learning characteristics in mathematics, as well as attitudinal aspects, which have both been shown to have a considerable impact on the way they set goals, the strategies they use and their performance.

◗ *Students' anxiety in mathematics*, which is common among students in many countries and is known to affect performance.

◗ *Students' learning strategies*. This considers what strategies students use during learning. Also of interest is how these strategies relate to motivational factors and students' self-related beliefs as well as to students' performance in mathematics.

Engagement with mathematics

Interest and enjoyment in mathematics

Students were asked to think about their views on mathematics and indicate their agreement on the following statements:

◗ I enjoy reading about mathematics

◗ I look forward to my mathematics lessons

◗ I do mathematics because I enjoy it

◗ I am interested in the things I learn in mathematics.

Students' responses to these items were combined into an *interest and enjoyment in mathematics* index. Australia's mean on this index was about the same as that for the OECD overall. In Australia, as well as in many other countries, there was a significant gender difference in enjoyment in mathematics, with males being significantly more interested and enjoying mathematics to a greater extent than females. There was a positive correlation in Australia with achievement on the mathematical literacy part of the assessment. This was around 0.2, and the

difference between the students who scored in the highest quartile on this index and those whose scores were in the lowest quartile was around 50 score points, which is the equivalent of a year of schooling.

Figure 4.1 shows the responses of all Australian students to the items on the *interest and enjoyment of mathematics* scale. The item that stands out in this analysis is "I look forward to maths lessons", to which a total of 83 per cent of students responded positively.
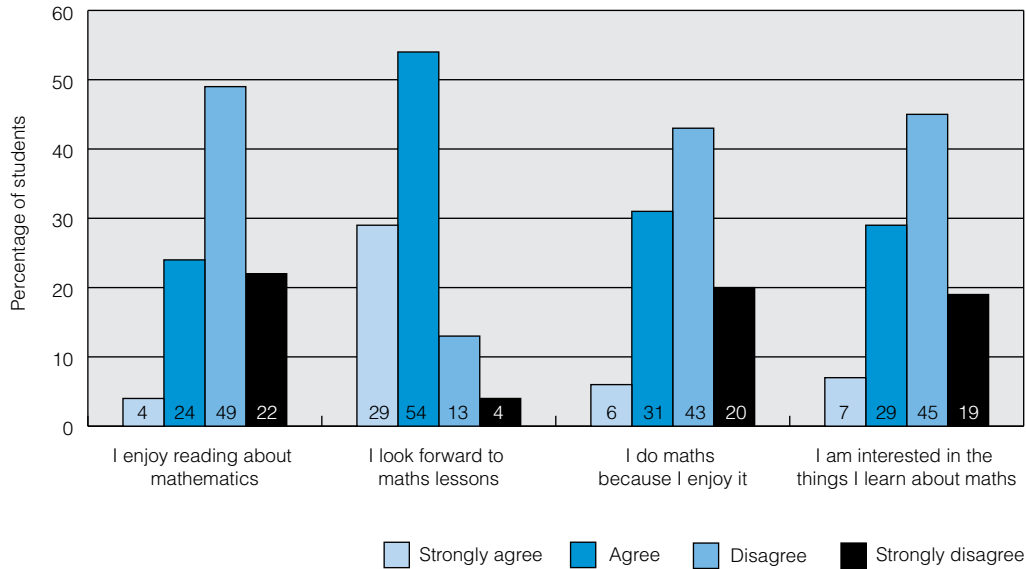
Kudos to maths teachers!!

Figure 4.1 Responses of Australian students to interest and enjoyment of mathematics items

However there are gender differences in achievement, and there are corresponding gender differences in attitudes towards mathematics. Australian male and female students' responses to one of the items in this scale: "I do maths because I enjoy it", is presented in Figure 4.2. Across Australia just 37 per cent of all students agreed or strongly agreed that they enjoyed doing mathematics.

The proportion of females is significantly higher than the proportion of males who strongly disagree with this statement, while the proportion of males who agree with the statement is significantly higher than the proportion of females. The relationship between enjoyment and doing well is also clear here (although the direction of the relationship is not): the difference between the average scores of students who strongly agreed that they enjoyed mathematics and those who strongly disagreed was about the equivalent of a year of schooling, for both male and female students.
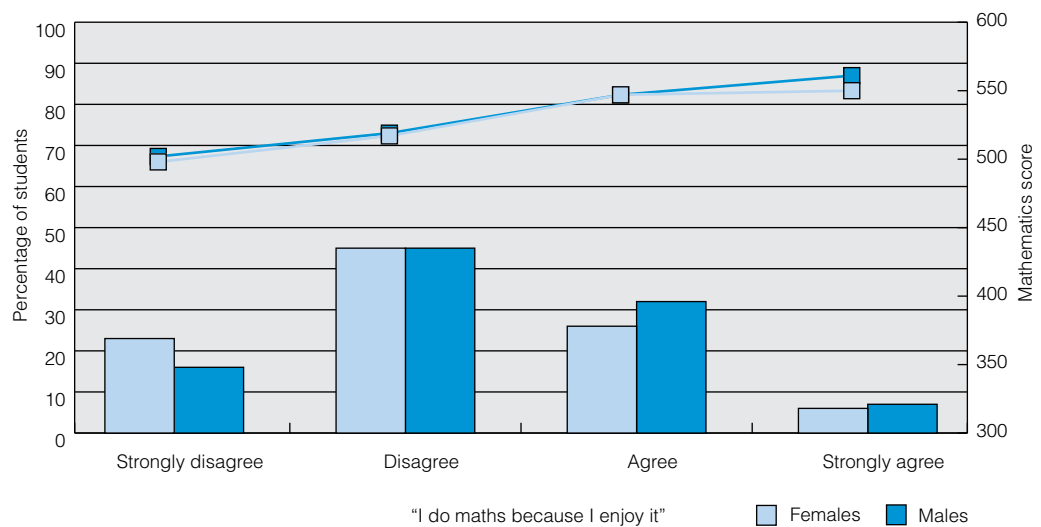
Figure 4.2 Enjoyment of reading and achievement in mathematical literacy for male and female Australian students

Instrumental motivation

While it appears that enjoyment and interest in mathematics is a bit of a mixed bag with students, the overwhelming majority of Australian students could see a long-term benefit to studying the subject. Instrumental or extrinsic motivation was measured in PISA 2003 by asking students their level of agreement with the following:

◗ Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.

◗ Learning mathematics is important because it will help me with the subjects that I want to study further on in school.

◗ Mathematics is an important subject for me because I need it for what I want to study later on.

◗ I will learn many things in mathematics that will help me get a job.

Overall, the Australian mean on the instrumental motivation index was significantly higher than the OECD mean, indicating that Australian students perceived it as more important than was the average across the OECD. Of interest is that countries such as Japan, Korea and Hong Kong-China, particularly the females in those countries, all reported significantly lower levels of instrumental motivation than on average across the OECD. In Australia, the mean for males on the instrumental motivation index was significantly higher than for females. This is perhaps not surprising given that males have a greater tendency to go on to further study in disciplines that demand an understanding of mathematics.[4]

Figure 4.3 shows the proportion of students agreeing (combining strongly agree and agree) and disagreeing (combining strongly disagree with disagree) with each of the instrumental motivation statements.

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4   Watt, H.M.G. (2010). Gender and occupational choice. In J.C. Chrisler, D.R. McCreary (Eds.), *Handbook of Gender Research in Psychology* (pp. 379–400), New York: Springer.
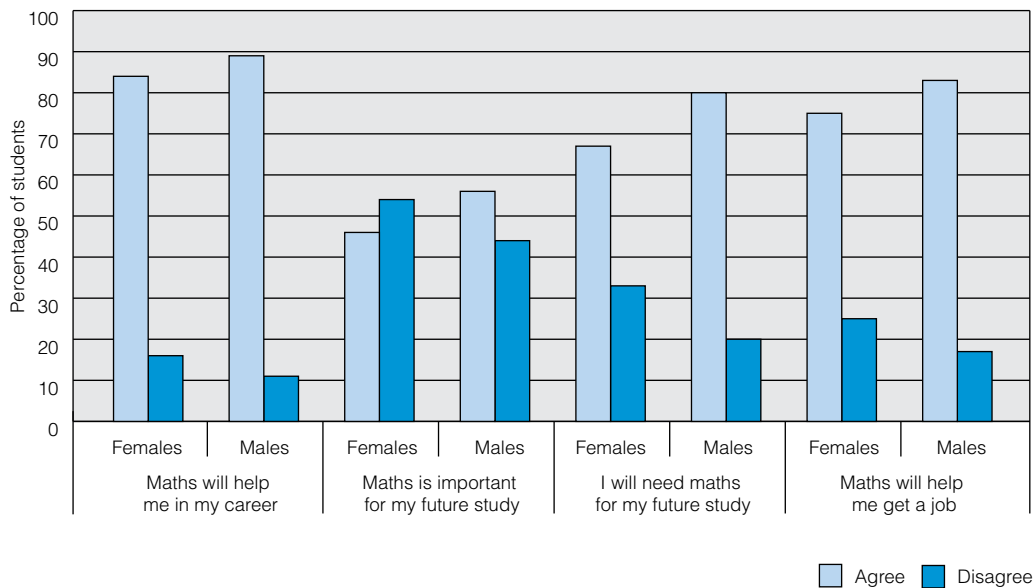
Figure 4.3 Level of agreement with instrumental motivation items

◗ While not reporting the same level of agreement as males, female students in Australia also indicated a belief that mathematics is relevant to their futures. Two-thirds of 15-year-old females think that they will need mathematics in their future study, and three quarters of females (75%) believe that mathematics will be relevant to their getting a job.

> How does this relate to the students in your school? Do they have a realistic idea of what maths will be needed in the career they are aiming for, or the effects of dropping maths prior to Year 12?

Students' beliefs about themselves

Autonomous learning requires both a critical, realistic assessment of the difficulty of a task, and the ability to invest enough energy in a task to accomplish it. As they progress through school, students form views about their own competence and learning abilities. These views have been shown to have considerable impact on the way a student sets goals, uses strategies and evaluates his or her own performance.

PISA collected information on mathematics self-efficacy, mathematics self-concept and mathematics anxiety. Mathematics self-efficacy relates to a student's beliefs about their capability to successfully learn mathematics. Self-efficacy may play an important role in learning because it provides the foundation for motivation and influences the level of effort and persistence a student applies to performing a task and attaining a particular outcome. Mathematics self-concept relates to a student's perception of their own mathematical competence, and belief in one's own abilities is highly relevant to successful learning (Marsh, 1993[5]). Mathematical anxiety is a third factor assessed in PISA. Students can perceive mathematics in general or specific mathematical tasks as being potentially intimidating. Subsequently, students may feel helpless and uneasy.

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5   Marsh, H.W. (1993). The multidimensional structure of academic self-concept: Invariance over gender and age. *American Education Research Journal*, *30*(4), 841–860.

Mathematics self-efficacy

Students were asked to what extent they believe in their own ability to manage learning situations effectively and to overcome difficulties by indicating their confidence in completing a range of mathematical tasks:

◗ Using a bus or train timetable to work out how long it would take to get from one place to another.

◗ Calculating how much cheaper a TV would be after a 30% discount.

◗ Calculating how many square metres of tiles you need to cover a floor.

◗ Understanding graphs presented in newspapers.

◗ Solving an equation like 3x+5=17.

◗ Finding the actual distance between two places on a map with a 1:10,000 scale.

◗ Solving an equation like 2(x+3)=(x+3)(x-3).

◗ Calculating the petrol consumption rate of a car.

Overall, Australia's mean was significantly higher than the OECD average, but the score for males was significantly higher than the score for females. This latter finding was the case for all countries, but there are substantial differences in the extent of the gender difference. In Australia males scored significantly higher than the OECD average, females at around the OECD average. Self-efficacy had the strongest relationship with achievement, with students in the highest quarter on the index in Australia scoring 132 score points higher than students in the lowest quarter of the index. This is the equivalent of more than three years of schooling.

Figure 4.4 shows the percentage of male and female Australian students who were confident or very confident in completing each of the tasks in the self-efficacy items, along with the PISA mathematical literacy scores of these students.
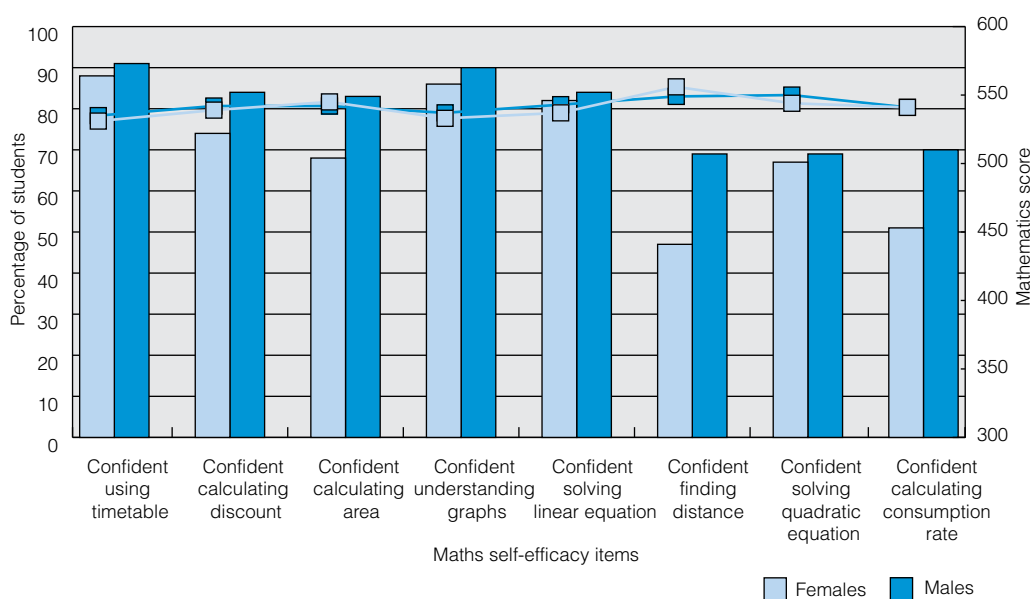

Figure 4.4 Confidence in mathematics and achievement for Australian students, by gender

◗ Notable is that for males and females with equally high levels of confidence, the achievement scores in mathematics were not significantly different.

◗ On some items: using a timetable, solving both linear and quadratic equations, there was little difference in the proportion of males and females who were confident of being able to complete these tasks. Approximately 20 per cent fewer females reported being confident finding a distance using a scale and calculating the petrol consumption rate of a car than males.

Can you think of a reason that females have less confidence in their ability to solve these types of problem?

Mathematics self-concept

PISA collected information on student beliefs about their own mathematical competence. There is research about the learning process that has shown that students need to believe in their own capacities before making the necessary investment in learning strategies that can lead to improved performance.[6]

Students were asked about how they felt when studying mathematics by indicating their level of agreement with the following statements:

◗ I am just not good at mathematics.

◗ I get good marks in mathematics.

◗ I learn mathematics quickly.

◗ I have always believed that mathematics is one of my best subjects.

◗ In my mathematics class, I understand even the most difficult work.

Australia's mean on the self-efficacy index was significantly higher than the OECD average – meaning Australian students are generally more confident of their abilities in mathematics than is the average across the OECD. In all countries, males had a significantly higher score on this index than females. Self-efficacy is strongly associated with achievement in mathematics, with 100 score point difference between the scores of those students in the lowest quarter of the index and those in the highest quarter. This is the equivalent of about 2 ½ years of schooling.

The percentage of male and female students who agreed or strongly agreed with each of these items (other than the first, for which the proportion who disagreed or strongly disagreed was used), along with their mathematical literacy scores, is shown in Figure 4.5.
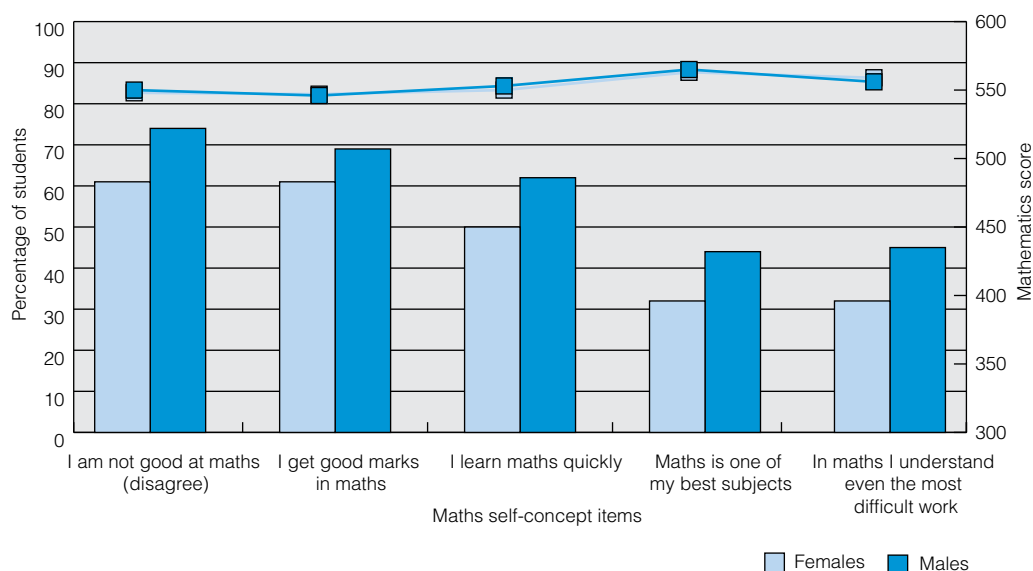

Figure 4.5 Proportion of student agreement with mathematics self-concept items

6   Zimmerman, B.J. (1999). Commentary: Toward a cyclically interactive view of self-regulated learning. *International Journal of Educational Research*, *31*(6), 545–551.

◗ Students are less positive about their mathematical competence in general than in their ability to solve particular problems. However more than half had a strong self-concept in mathematics – fewer agreed that mathematics is one of their best subjects or that they understand the most difficult work.

◗ Male students were more likely to agree to each of the items, showing higher levels of confidence in mathematics overall.

◗ There were no gender differences in achievement when the level of self-concept was the same.

Do you talk explicitly to students about their self-confidence?

Can you think of ways to get female students to have more accurate self-confidence?

Mathematics anxiety

Students were asked about feelings of helplessness and the emotional stress they have when dealing with mathematics. PISA collected information about students' mathematics anxiety by asking them to think about mathematics and answer to what extent they agreed with the following statements:

◗ I often worry that it will be difficult for me in mathematics classes.

◗ I get tense when I have to do mathematics homework.

◗ I get nervous doing mathematics problems.

◗ I feel helpless when doing a mathematics problem.

◗ I worry that I will get poor marks in mathematics.

The items were used to construct an index representing *mathematics anxiety*. Australian students reported slightly lower levels of mathematics anxiety than on average across the OECD, however the level of anxiety for females was significantly higher than the OECD average and the level for males significantly lower than the OECD average. Females reported significantly higher levels of mathematics anxiety in all countries other than Poland and Serbia.

A strong negative association was found between mathematics anxiety and mathematics performance. Students reporting a high level of mathematics anxiety performed at a lower level than students reporting a low level of mathematics anxiety. There was an 86 score point difference between students in the lowest quarter on the mathematics anxiety index and those in the highest quarter on the index. This difference equates to around two years of schooling.

Figure 4.6 shows the proportion of male and female students who agreed or strongly agreed with each of the items on the mathematics anxiety scale. The black line at the top of the figure shows the scores for students who did *not* agree with the item – on three out of the five items this was fortunately less than 50 per cent of students.
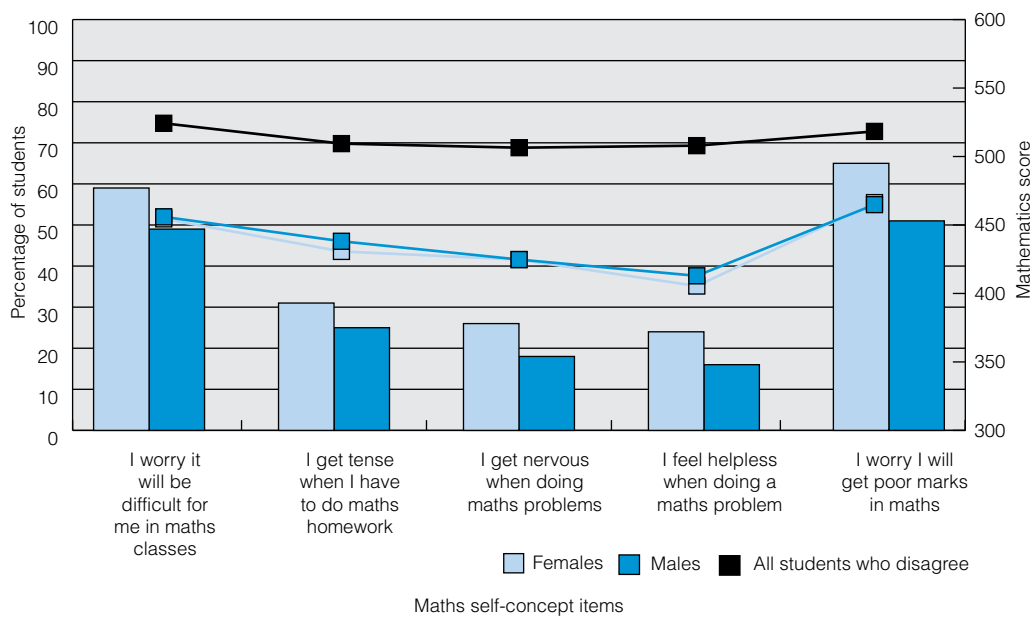
Figure 4.6 Proportion of student agreement with mathematics anxiety items

◗ Female students agreed to a greater extent on each of the mathematics anxiety items.

◗ The levels of helplessness are relatively low. Scores for both males and females are lowest for students who agreed with this item.

◗ Major issues for female students were worrying about mathematics classes being difficult and that they would get poor marks in mathematics. Combined with the previous finding that two-thirds of female students agree or strongly agree that they need mathematics for their future lives, this sets up a tension for those students who have a moderate or strong level of mathematics anxiety, which may potentially have ramifications on students' decisions to pursue higher level mathematics or careers involving mathematics.

How do you think your students would respond to these items?

Do you think that the females in your classes have substantially higher levels of maths anxiety than the males? What factors might contribute to girls worrying more about their performance in mathematics than boys in Australian classrooms?

What strategies do you know that could help students who have problems with maths anxiety?

Students' learning strategies

Learning is more than acquiring knowledge, it involves being able to process information efficiently, relate it to existing knowledge and apply it to different situations. Students need to take an active role in managing and regulating their own learning. PISA focuses on three kinds of learning strategies – memorisation, elaboration and control strategies. Students provided information about their learning strategies by indicating their agreement to a range of statements.

Memorisation strategies

Memorisation strategies include rote learning facts or rehearsal of examples. If the learner's goal is simply retrieval of information, then this strategy is adequate, however it rarely leads to deep understanding. To achieve understanding, new information must be integrated into a learner's prior understanding.

Students were asked to think about the different ways of studying mathematics and to what extent they agreed with the following statements:

◗ I go over some problems in mathematics so often that I feel as if I could solve them in my sleep.

◗ When I study for mathematics, I learn as much as I can off by heart.

◗ In order to remember the method for solving a mathematics problem, I go through examples again and again.

◗ To learn mathematics, I try to remember every step in a procedure.

These items were combined to form an index for *memorisation strategies*. One of the highest performing countries in mathematical literacy, Korea, had one of the lowest means on this index, and all countries that scored at a higher level than Australia in mathematical literacy had scores on this index significantly lower than the OECD average. However all English-speaking countries had scores that were significantly <u>higher</u> than the OECD average. On this index, there were no gender differences amongst Australian students.

Australian students' responses to the items comprising the memorisation strategy index are shown in Figure 4.7 (agree and strongly agree combined into agree, and strongly disagree and disagree into disagree).
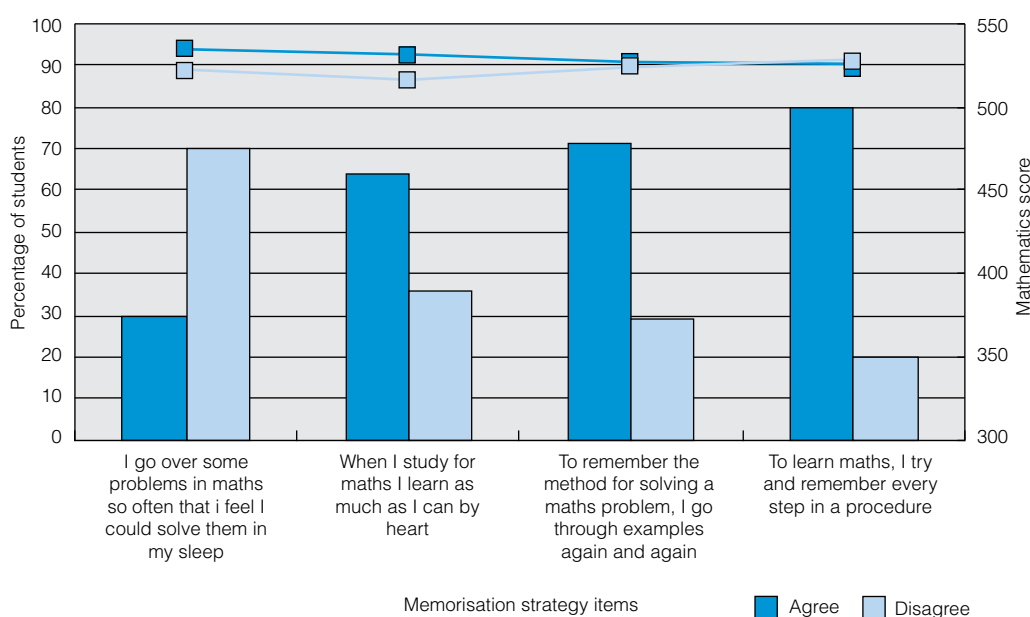

Figure 4.7 Proportion of student agreement with memorisation strategy items

◗ Many students showed a strong use of a variety of memorisation strategies. Eighty per cent of Australian students said that they learn mathematics by trying to remember every step in a procedure, for example.

◗ Australian students showed a much greater reliance on rote learning, with 64 per cent agreeing that they studied for mathematics by learning as much as possible by heart, compared to 45 per cent on average across the OECD.

◗ Students who agreed to the first two items scored, on average, higher in mathematical literacy than those who disagreed. On the last two items, there was no difference in scores between those who agreed and those who disagreed.

Elaboration strategies

Elaboration strategies involve a student integrating new information with their existing knowledge base or prior learning, by exploring how the material relates to things learned in other contexts, or how the information could be applied in other contexts. In doing so, they acquire an understanding of new information, rather than the more superficial *memorisation* strategies.

The *elaboration strategies* index is based on students' responses to:

◗ When I am solving mathematics problems, I often think of new ways to get the answer.

◗ I think how the mathematics I have learnt can be used in everyday life.

◗ I try to understand new concepts in mathematics by relating them to things I already know.

◗ When I am solving a mathematics problem, I often think about how the solution might be applied to other interesting questions.

◗ When learning mathematics, I try to relate the work to things I have learnt in other subjects.

Australia's mean on this index was not significantly different to the average over the OECD. Figure 4.8 shows the proportion of students agreeing (combining strongly agree and agree) and disagreeing (combining strongly disagree with disagree) with the items on this index.
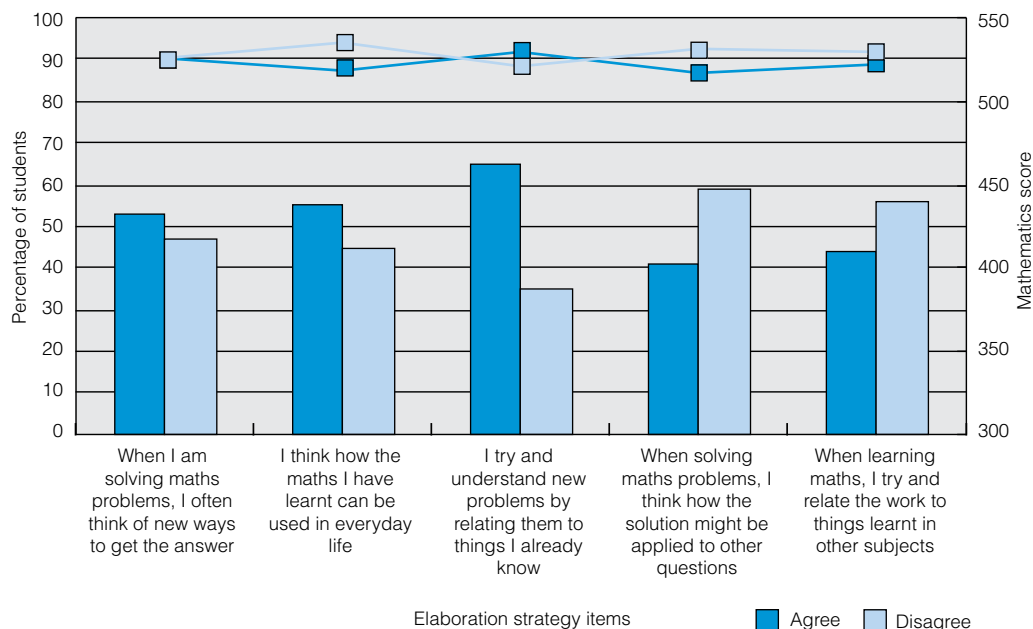

Figure 4.8 Proportion of student agreement with elaboration strategy items

◗ The patterns of response are similar to those over the OECD on average.

◗ Australian males were more likely than Australian females to agree to the items in this index, resulting in a significantly higher average score on the elaboration strategy index.

◗ In general, Australian students do not try to relate work they are doing to other subjects or to other questions.

Control strategies

Students who use control strategies are able to manage their own learning: they check what they have learned, assess what they still need to learn and adapt information they have learned to new situations. The *control strategies* index was constructed using the student responses to the following statements:

◗ When I study for a mathematics test, I try to work out what are the most important parts to learn.

◗ When I study mathematics, I make myself check to see if I remember the work I have already done.

◗ When I study mathematics, I try to figure out which concepts I still have not understood properly.

◗ When I cannot understand something in mathematics, I always search for more information to clarify the problem.

◗ When I study mathematics, I start by working out exactly what I need to learn.

Australian females were slightly more likely to use control strategies than Australian males, and more likely than on average across the OECD. Figure 4.9 shows the proportion of Australian students agreeing (combining strongly agree and agree) and disagreeing (combining strongly disagree with disagree) with the items on the control strategies index.
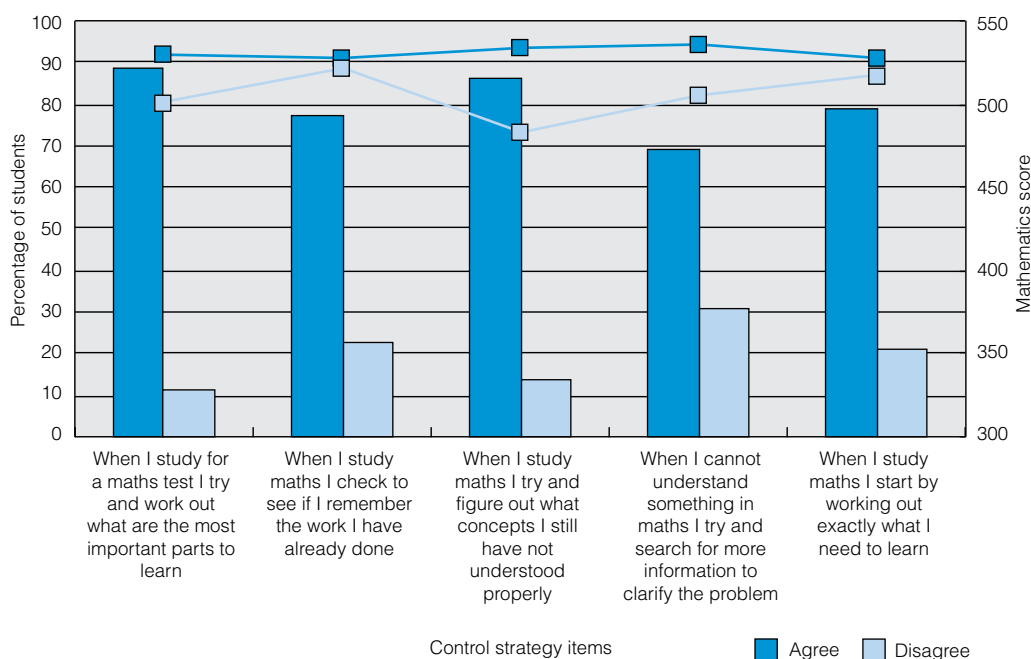

Figure 4.9 Proportion of student agreement with control strategy items

◗ Australian students overwhelmingly agreed with each of the items comprising the control strategies index.

◗ Australian females were more likely to use control strategies than Australian males, and this resulted in an average score on control strategies for females that was higher than the OECD average, and significantly higher than that for males.

◗ The relationship between control strategies and performance varied between countries, with some questions about how students interpret the questions. In Australia there was a moderate relationship between achievement and the control strategies index, and students in the highest quarter of the index scored 23 points higher than students in the lowest quarter of the index.

Final Words

While Australian students continue to perform at a high level comparative to the rest of the world in mathematical literacy, there was a significant decline in scores between PISA 2003 and PISA 2009. PISA 2012 provides a return to mathematical literacy as the main focus of the assessment and so will provide further information about Australia's current position in mathematics performance.   Australia's participation in international studies allows these comparisons to be made, and the national data allow patterns to be seen that are often not obvious at a local level.

Of particular concern is the decline in performance of our high achieving students. In PISA 2003, 20 per cent of students achieved proficiency levels 5 or 6 – in PISA 2009 this had declined to 16 per cent. Related directly to this is the proportion of low achievers (students achieving below proficiency level 2). In PISA 2003, 14 per cent of Australia's students were achieving at the level deemed by the OECD to put them at risk of not having acquired the skills necessary for being a productive and active 21st century citizen. In PISA 2009 this had increased slightly to 16 per cent of students. Are we teaching too much to the middle? Are we not extending those capable students enough, and are we addressing the needs of low achieving students?

Broadly, the proportions of students at the lower levels of achievement are strongly linked to socioeconomic opportunities. Forty per cent of our Indigenous students (compared to 15 per cent of non-Indigenous students) and 28 per cent of students from the lowest quartile of socioeconomic background (compared to five per cent from the highest quartile) are not achieving the basic level of mathematical literacy (i.e. not achieving proficiency level 2). Are there particular strategies that can be used to scaffold the performance of these groups of students? What resources might these students be lacking that could help their learning and engagement?

Participation in mathematics in upper secondary school has been declining for a number of years, and the proportion of students entering courses in science, technology, engineering and mathematics (STEM) at the tertiary level is concerning.[7] Some of the reasons for students not pursuing mathematics past compulsory level may lie in the findings of the last chapter of this report.

The issue of gender differences in mathematics achievement is an important one. For several decades now, since campaigns in the late 1970s to increase participation of young women in mathematics and science related fields, there have been no gender differences in Australia, but they seem to have crept back. There is much information in this report that can help teachers and schools to consider the way that mathematics is taught in schools and the messages that young women receive.

The surveys of students provide some valuable information that may assist in improving outcomes for all students.

◗ The data from PISA shows that students who are interested in and enjoy mathematics are more likely to be doing better at it than those who are not. Recognising the reciprocal relationship
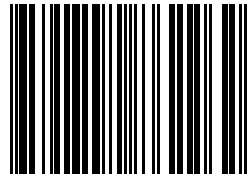
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7   Marginson, S, Tytler, R, Freeman, B and Roberts, K (2013). *STEM: Country comparisons* . Report for the Australian Council of Learned Academies, www.acola.org.au.

here, how do we engage students more with mathematics so that they <u>want</u> to do it past the compulsory years?

◗ The data also show that enjoyment is not a necessary precursor to high achievement in mathematics – understanding the role mathematics plays in a students' future also plays a key role, and one that teachers and schools are able to assist with by explicitly relating students' learning to the real world.

◗ Gender differences need to be addressed. A smaller proportion of female students than male students achieve at the higher proficiency levels and a larger proportion achieve at the lower proficiency levels.

◗ Teachers can support students' mathematics learning by providing direct and explicit instructions about strategies for understanding mathematics and tackling problems.