Making connections to the big ideas in mathematics:

Promoting proportional reasoning

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Proportional Reasoning

- an **elusive** but **pervasive** concept
- required for successful operation in rational number topics
  - e.g., fractions, decimals, percentages
- …and other topics
  - e.g., scale drawing, probability, trigonometry
- …and other subjects
  - e.g., drawing timelines in history; understanding speed in science).

Overview

- Defining proportional reasoning
- Background literature on research into proportional reasoning in mathematics
- A recent Australian research project on proportional reasoning
- Classroom examples for promoting proportional reasoning

Prop

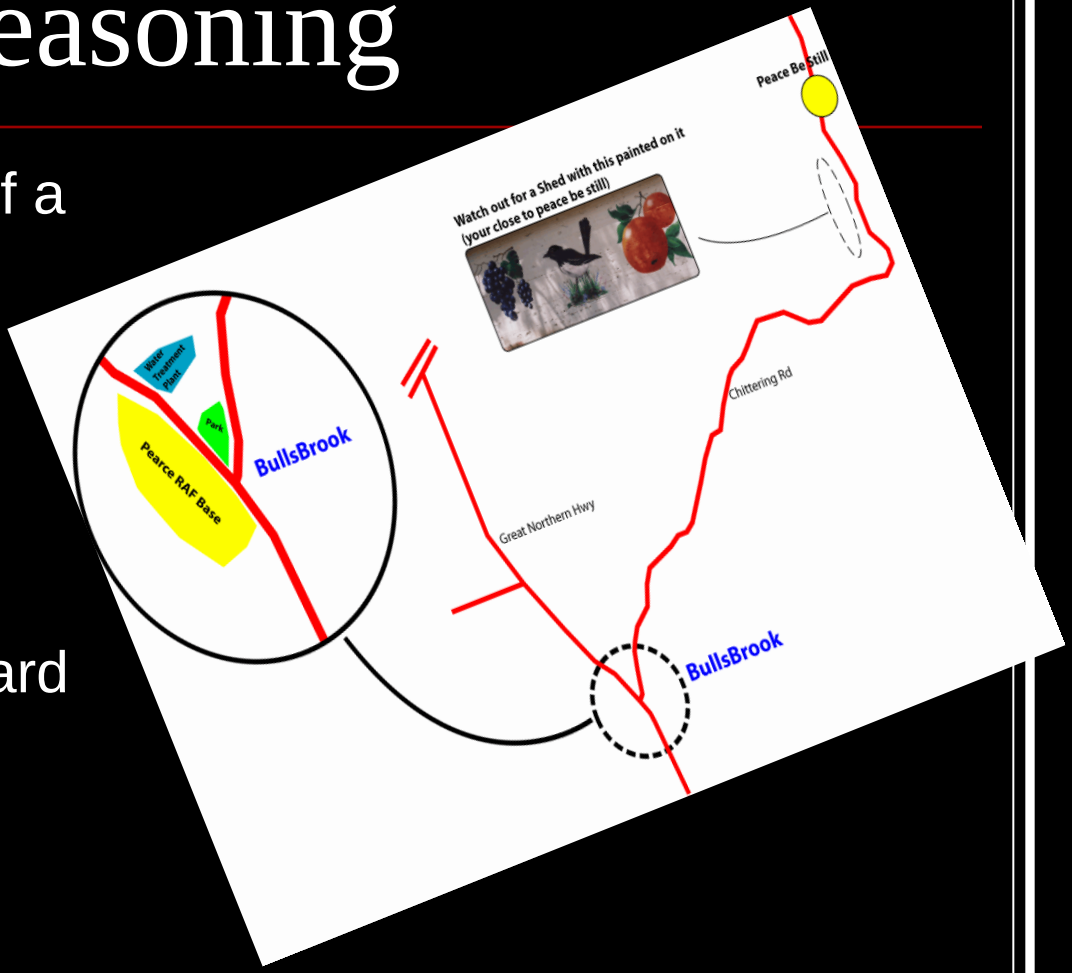
"Which is the better buy?"

$4.00
1 kg
20 Loads

$6.79
1.5 kg
30 Loads

Proportional Reasoning

- Drawing of a plan view of a house
- a “mud map” of the path from home to school
- or a plan of the school yard



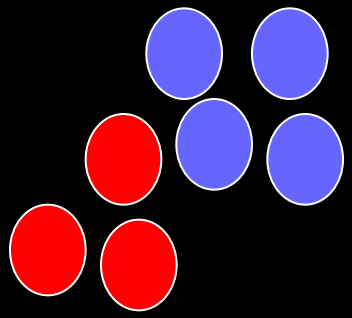
Proportional Reasoning

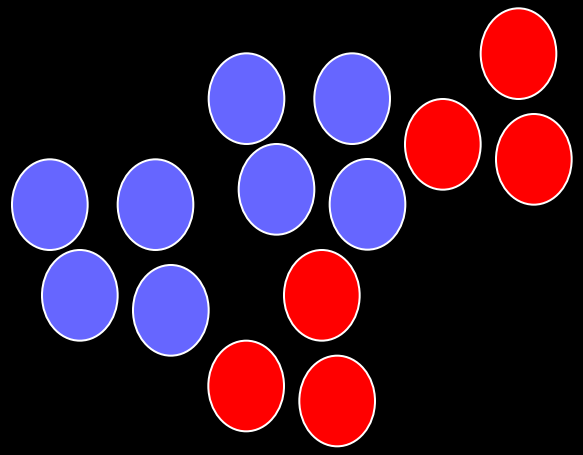
- Sharing four pizzas between three people

- or two chocolate bars between five people

Proportional Reasoning

- Determining whether there is more chance of selecting red from a collection of 3 red and 4 blue compared to a collection of 6 red and 8 blue.





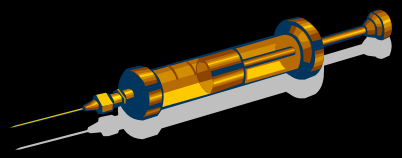
Proportional reasoning

- Knowing why whole potatoes take longer to cook than French fries

- Or why a baby locked in a car on a hot day suffers more than an adult in the same circumstances.

Real consequences

- administration of incorrect doses in medicine



- failure to accurately convert between units of metric and imperial units of measure

- inaccurate mix of chemicals in pesticides

Proportional Reasoning

- The ability to understand situations of comparison
- Multiplicative Thinking
  - Ability to see situations in a multiplicative sense rather than an additive sense
  - Flexibility in thinking about numbers and situations involving number

Additive vs multiplicative thinking

- *Additive Thinking* -Describing a change from 2 to 10 as an addition of 8

- *Multiplicative Thinking* – Describing a change from 2 to 10 as multiplying by 5

Incorrect additive thinking

- A
  ev
  require 5 sheets of coloured paper to complete their art work. The child is asked how many sheets of coloured paper will be needed if there were 4 children.

Seven sheets of coloured paper

Proportional Reasoning and the Mathematics Curriculum

- “Proportional reasoning is the capstone of children’s elementary school arithmetic and the cornerstone of all that is to follow”
  - (Lesh, Post and Behr, 1988, p. 93-94)

Proportional reasoning key points

- one of the most commonly applied mathematics concepts in the real world
- when misapplied, can have disasterous consequences
- estimated that 50% of the adult population cannot reason proportionally
- proportional reasoning tasks are hard
- proportional reasoning takes a long time to develop

Why is proportional reasoning hard?

- “...requires firm grasp of various rational number concepts such as order and equivalence, the relationship between the unit and its parts, the meaning and interpretation of ratio, and issues dealing with division, especially as this relates to dividing smaller numbers by larger ones.”
  - (Post, Behr & Lesh, 1988, p. 80)

Characteristics of a proportional reasoner

- “A proportional reasoner has the mental flexibility to approach problems from multiple perspectives and at the same time has understandings that are stable enough not to be radically affected by large or “awkward” numbers, or the context within which a problem is posed”.

  - (Post, Behr & Lesh, 1988, p. 80)

Intertwined with many mathematical concepts

- English & Halford (1995): "*Fractions are the building blocks of proportion*" (p. 254).
- Behr et al. (1992): "*the concept of fraction order and equivalence and proportionality are one component of this very significant and global mathematical concept*" (p. 316)
- Streefland: (1985): "*Learning to view something 'in proportion', or 'in proportion with...' precedes the acquisition of the proper concept of ratio*" (p. 83).

Building blocks: multiplication and division

- Developing students' understanding of ratio and proportion is difficult because the concepts of multiplication, division, fractions and decimals are the building blocks of proportional reasoning, and students' knowledge of such topics is generally poor
  - (Lo & Watanabe, 1997).

The development of proportional reasoning

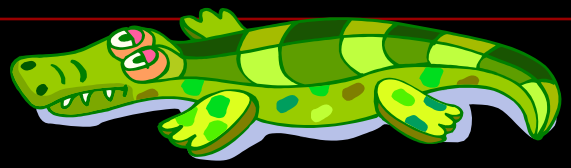
- a gradual process
- underpinned by increasingly more sophisticated multiplicative thinking
- characterised by the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms
  - (Lamon, 2005).

Relative and absolute thinking

- Consider the stories of the two crocodiles…

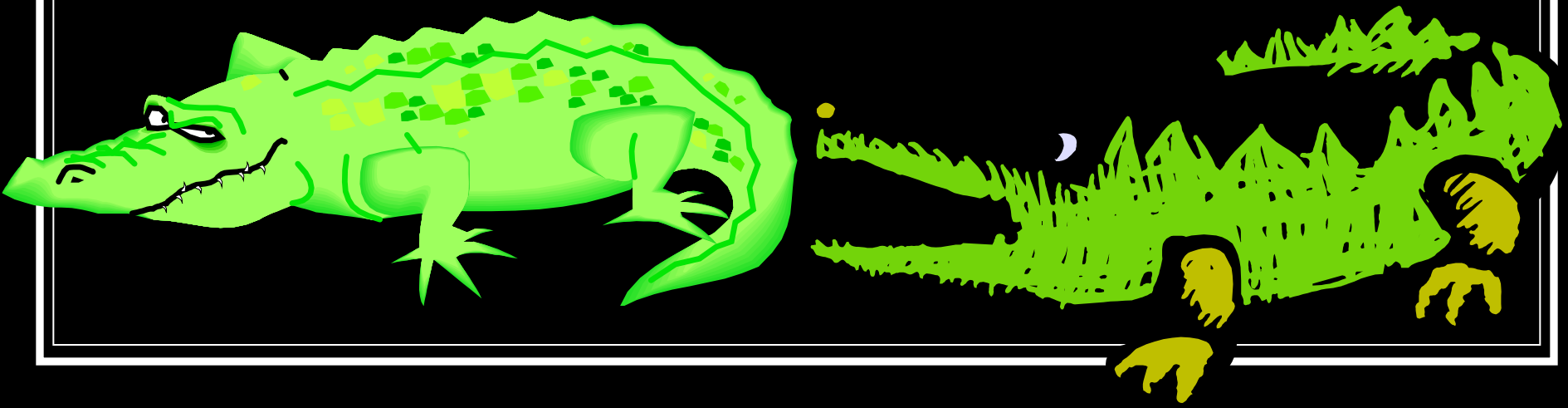
Prickles and Tiny

- *At the zoo, there are two long-term resident crocodiles that have affectionately been named Prickles and Tiny. When they arrived at the zoo, Prickles was 4 metres long and Tiny was 5 metres long. Five years later, both crocodiles are now fully grown. Prickles is 7 metres long and Tiny is 8 metres long.*



- *Prickles (4 metres) Tiny (5 metres)*

- *Prickles (7 metres) Tiny (8 metres)*



Have both grown the same amount?

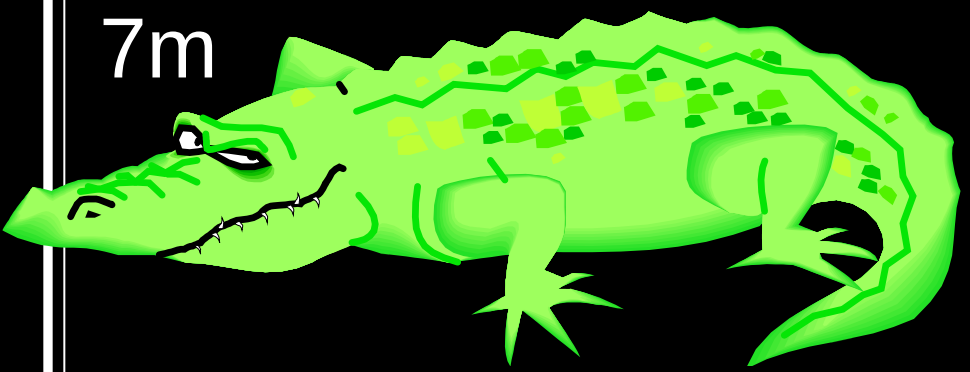
- Absolutely - yes
- Relatively - no

Prickles

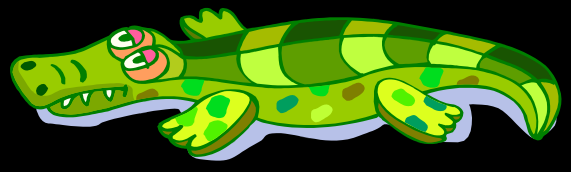
4m

Prickles has grown 3/4 of his beginning length

7m



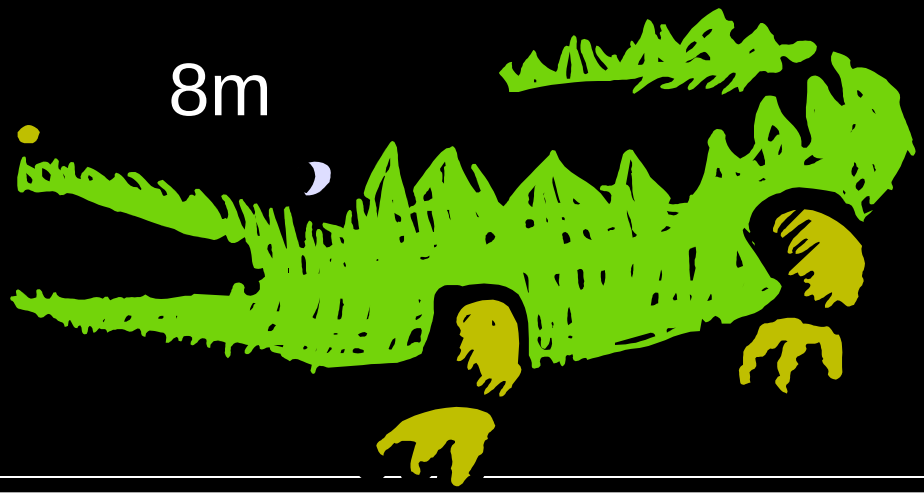
Tiny



5m

Tiny has grown 3/5 of his original length

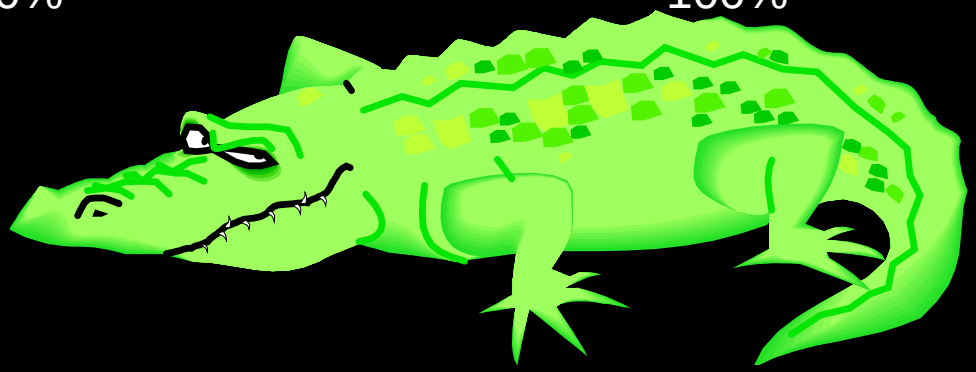
8m



Who has grown the most?

- 3/4 is greater than 3/5
- Prickles has grown more than Tiny
- …relatively

Another representation



0m

4m

7m

8m

double in size

0%

100%

200%

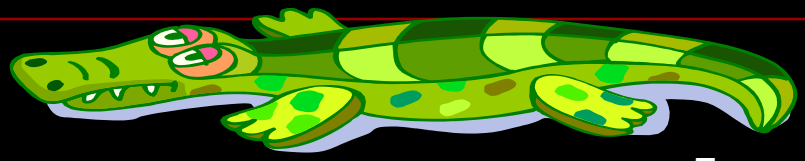
current size

3/4 increase

75% increase

175% of original size

Another representation of Tiny



0m

5m

8m

10m

double in size

0%

100%

200%

current size

3/5 increase

60% increase

160% of original size

Early proportional reasoning strategies

- Children's intuitive strategies for solving proportion problems typically are additive (Hart, 1981).

Moving from additive thinking

- The teacher's role is to build on students' intuitive additive strategies and guide them towards building multiplicative structures.

- Strong multiplicative structures develop as early as the second grade for some children, but are also seen to take time to develop to a level of conceptual stability, often beyond fifth grade
  - (Clark & Kamii, 1996).

Why is proportion hard?

- Streefland (1985): “Ratio is introduced too late to be connected with mathematically related ideas such as equivalence of fractions, scale, percentage” (p. 78).
- English & Halford (1995): proportional reasoning is taught in isolation and thus remains unrelated to other topics.
- Behr et al. (1992): “…the elementary school curriculum is deficient by failing to include the basic concepts and principles relating to multiplicative structures necessary for later learning in intermediate grades” (p. 300).

- Research has shown that students' (and teachers') understanding of proportion is generally poor (e.g., Behr et al., 1992; Fisher, 1988; Hart, 1981).

Non-Calculator Allowed

Year 7

19% correct

Determines missing value in table and represents it as a percentage

24 Ann recorded the colour of 50 cars in this table.

Write your answer in the box.

Car colour	Number of cars
White	25
Blue	4
Yellow	5
Red	?
TOTAL	50

What percentage of the cars is red? ____ %

Year 9 – 34% correct

Key points and implications for instruction

Proportion concept is intertwined with many mathematical concepts

Developing understanding of proportional relationships takes a long time

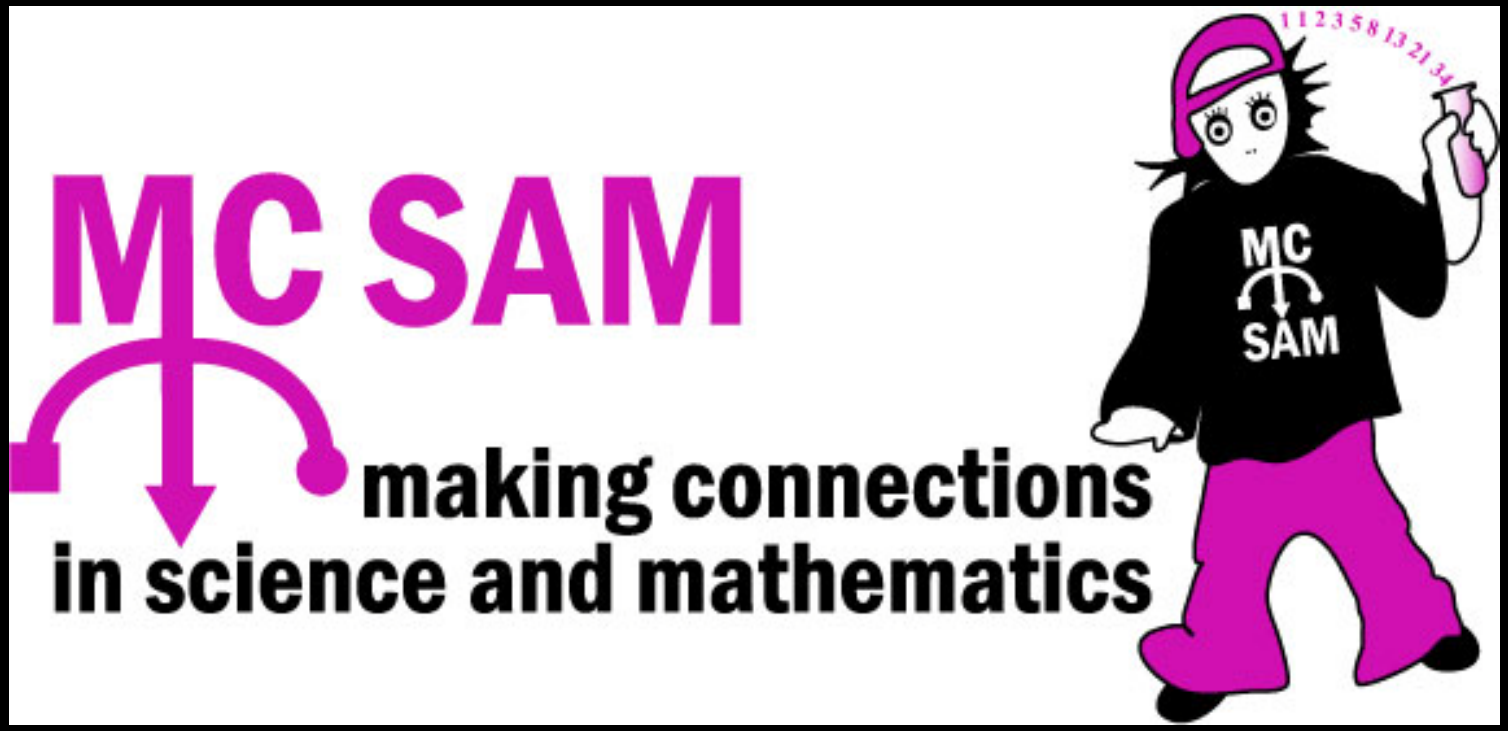
NEED TO:

Make explicit the proportional nature of various rational number topics

Revisit topics continually

Building proportional reasoning must be through multiple perspectives (Post et al., 1988).

UQ Research Project (2007-2010)



THE UNIVERSITY OF QUEENSLAND

MC SAM
making connections in science and mathematics

Research Team

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UQ

Dr Tony Wright
UQ

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ACU

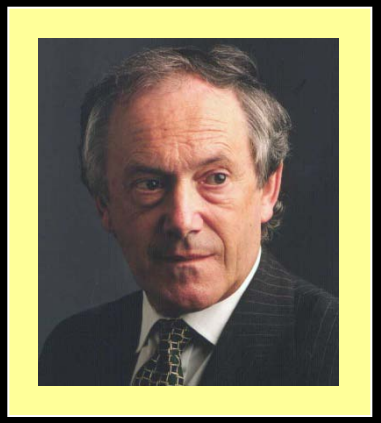
Mr Geoff Hilton
UQ

The Key Project Question

What are the major connections between mathematics and science?

Connections in maths & science

"……determining where mathematics ends and science begins is as difficult and pointless as mapping the edge of the morning mist."



Peter Atkins Galileo's Finger: The Ten Great Ideas of Science, 2003, Oxford Press.

- Scale drawing, fractio[illegible]s, algebra, density, mol[illegible]rity, [illegible]ecimals, percent[illegible]es, [illegible] ra[illegible]o, rate, speed, ga[illegible] [illegible]ons, curr[illegible] [illegible]ng, trig ratios [illegible] [illegible]ents, [illegible] [illegible]es, chem[illegible] [illegible]ion, solar syst[illegible], [illegible] co[illegible]versions, scales of dista[illegible]ce, timelines, floating, sinking, gravity, acceleration, velocity, pulleys and levers, …

Proportional Reasoning

Assessing Proportional Reasoning

- Keeping It In Proportion (KIIP)
  - Pen and paper test
  - “Explain your thinking”
  - ~800 MYS students (Grades 4-9)

Scoring System

- 2 digit code

00 no answer given; no strategy

Coding scheme

- GIVEN ANSWER
1. Correct
2. Incorrect
0. No response

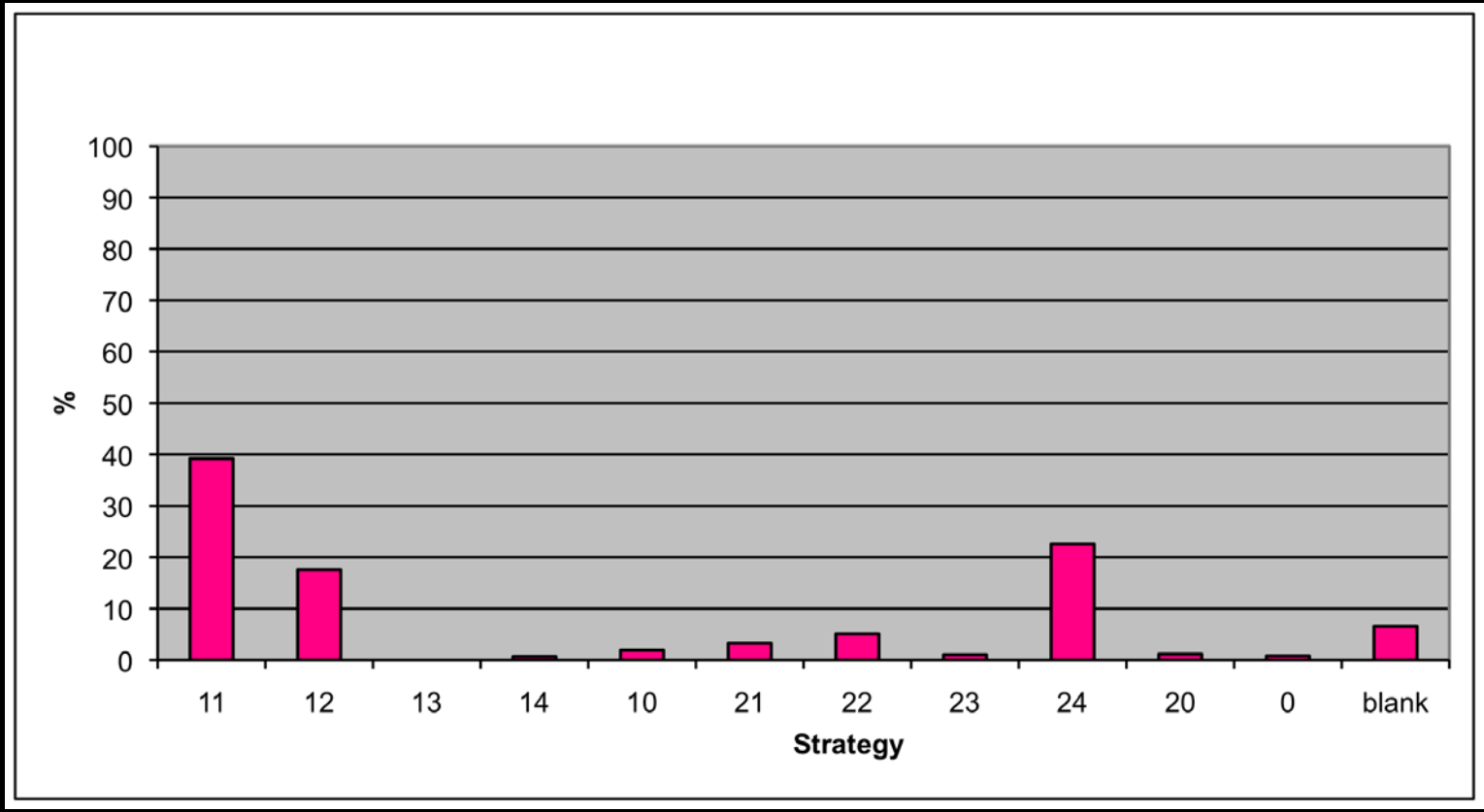
- STRATEGY
1. Multiplicative
2. Repeated addition, (skip counting. use of table)
3. Additive
4. Inappropriate
0. No response

Butterflies

To feed 2 butterflies the zoo needs 5 drops of nectar per day. How many drops would they need each day for 12 butterflies?

Show all your working and explain your answer in as much detail as possible.

Butterflies



Sticky Mess

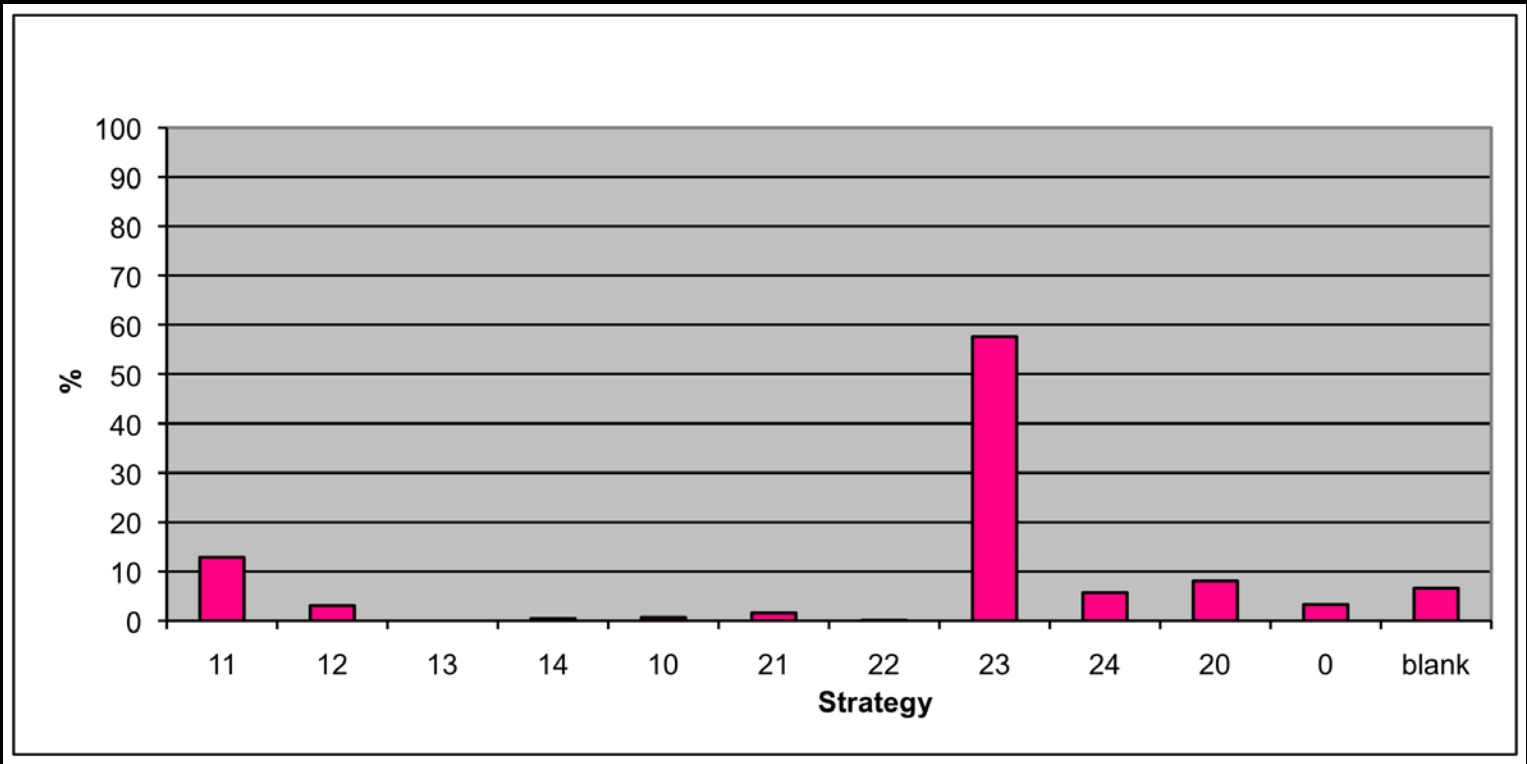
A recipe for a sticky mess needs 4 cups of sugar and 10 cups of flour. You decide to make a larger amount of the recipe, and have 6 cups of sugar. How many cups of flour will you need for the recipe to work?

So, the situation is this:

4 cups sugar and 10 cups flour for the original recipe

6 cups sugar and ___ cups flour for the larger quantity

Sticky Mess


100
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22
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24
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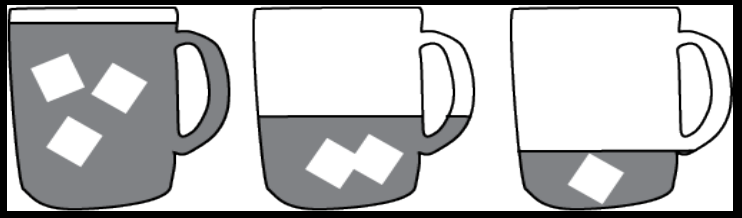
0
blank
Strategy

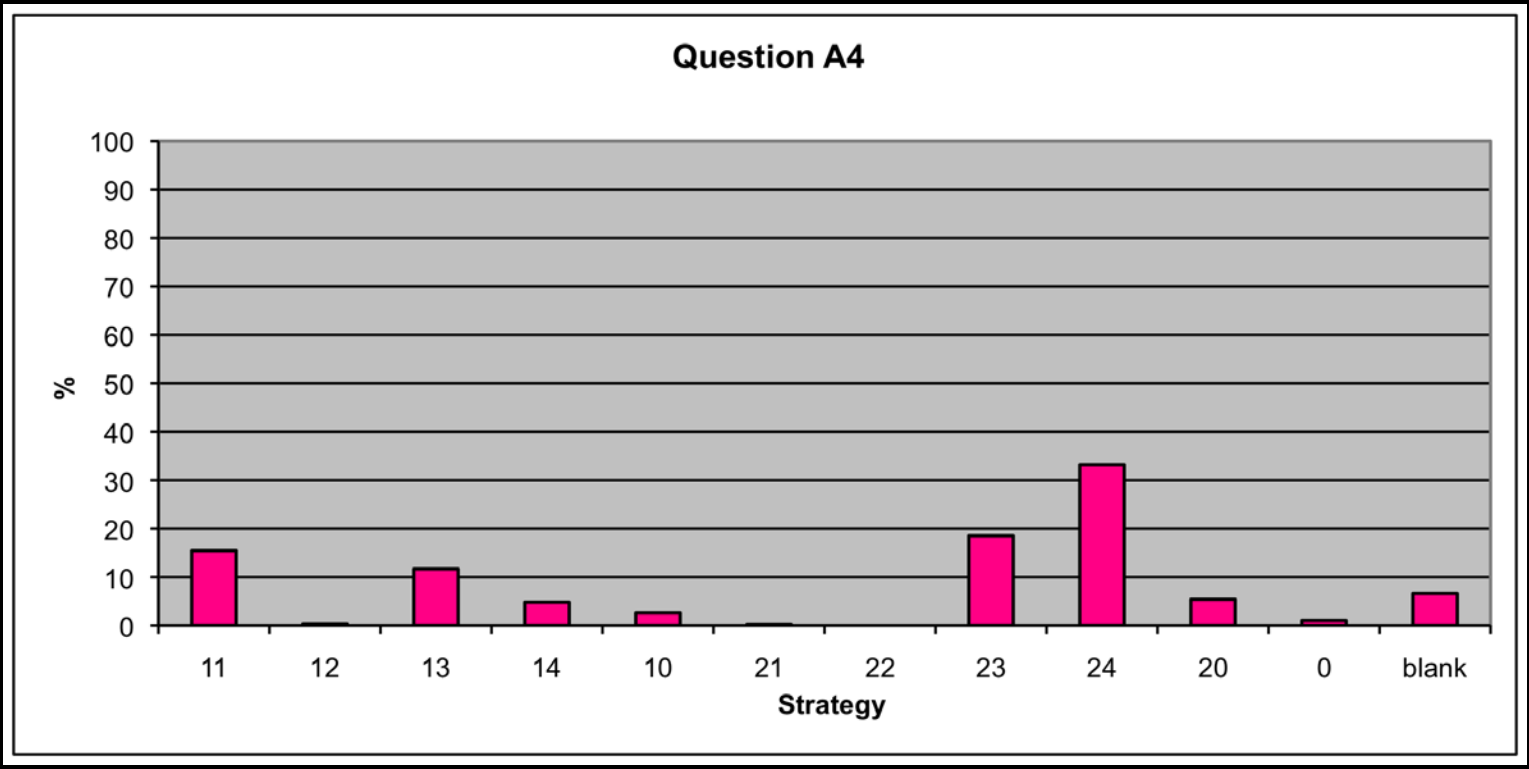
Three Cups

Three cups have different amounts of water. The first is full, the second one half full, and the third is one third full. The first cup has three spoons of sugar stirred in, the second has two spoons, and the third has one spoon of sugar. Which is the sweetest?

- ❑ cup A
- ❑ cup B
- ❑ cup C
- ❑ they’re all the same
- ❑ there is not enough information to be sure
- Explain the reason for your choice:



Three cups


Question A4
100
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24
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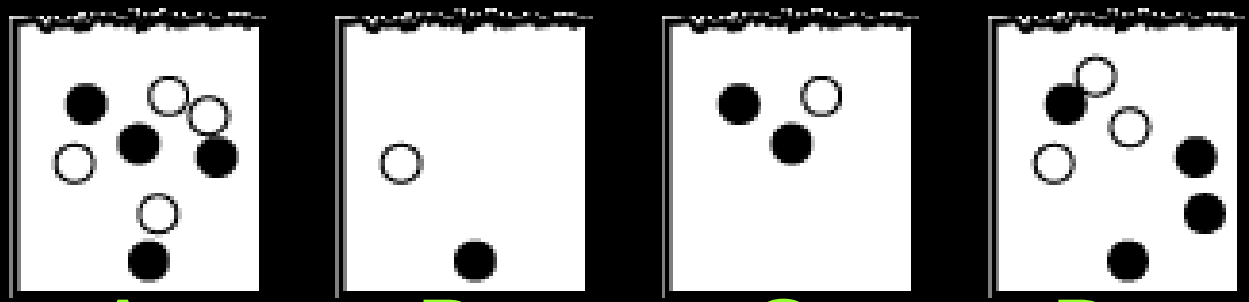
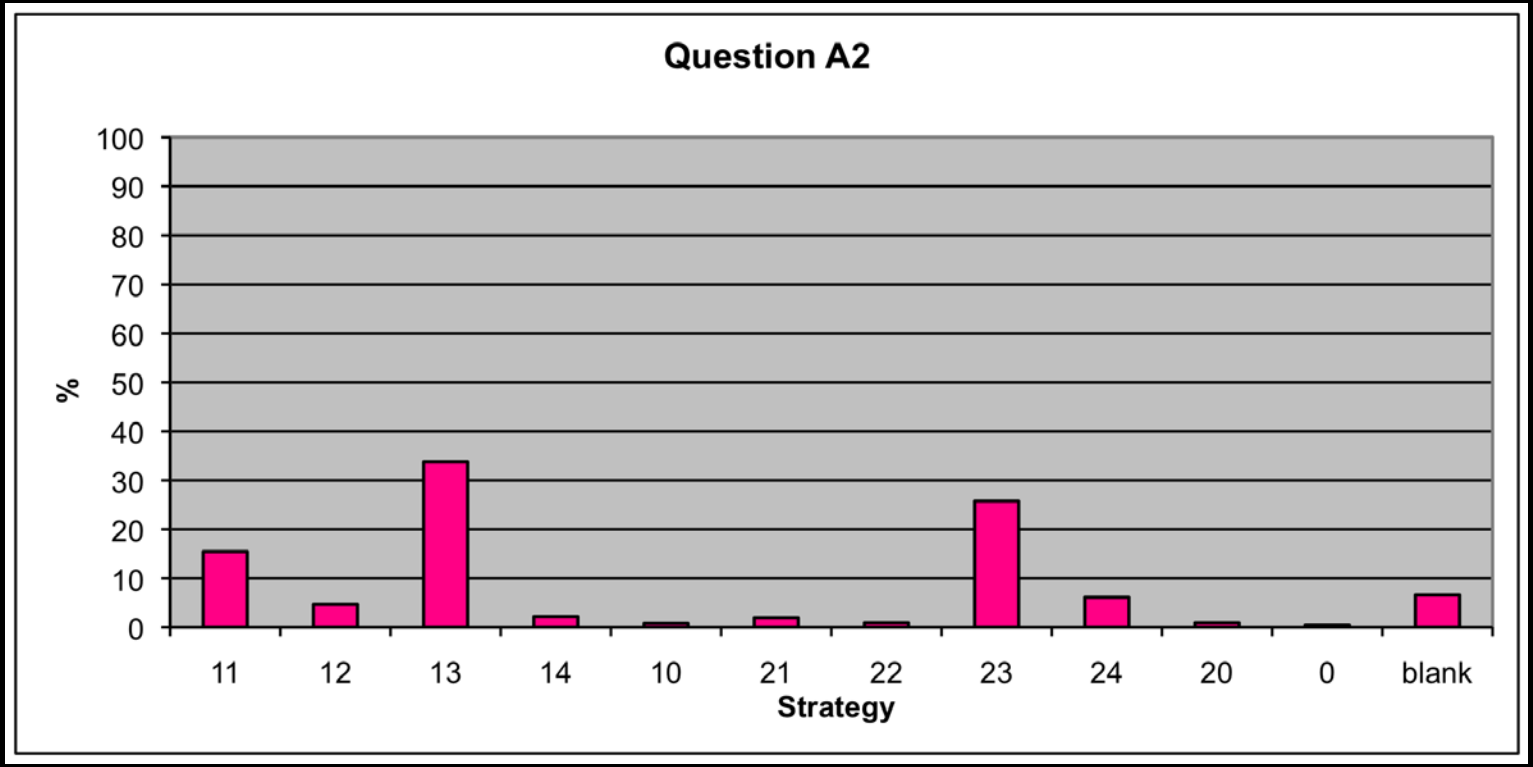
0
blank
Strategy

Chance (En)counters

- You are presented with four paper bags containing black and white counters. The number of counters in each bag is provided in the diagrams below. You are blindfolded, the bags are shaken, and you are asked to select a counter from each bag. From which bag do you have the best chance of selecting a black counter?



Chance (en)counters

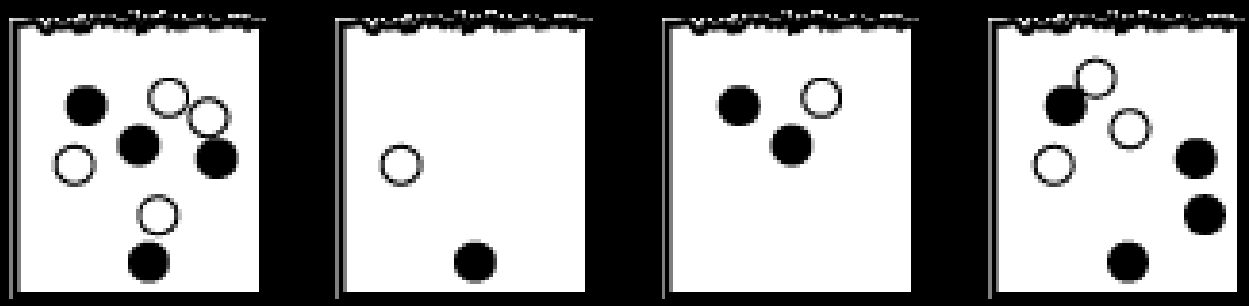


Creative Thinking

- Some students' responses were very creative
- Such responses remind us of the active way that students' minds work when they are doing mathematics assessments

Chance (En)counters

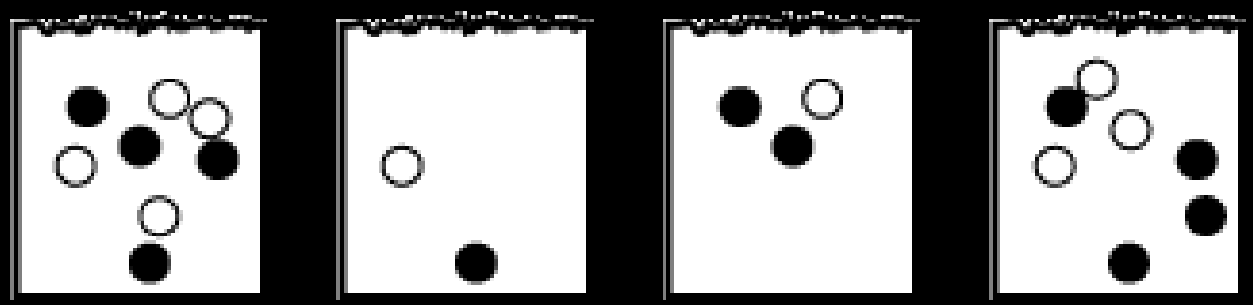
- I'll pick off the top, so more chance with C. One of the black counters is at the top of the bag and the other is down lower.



A B C D

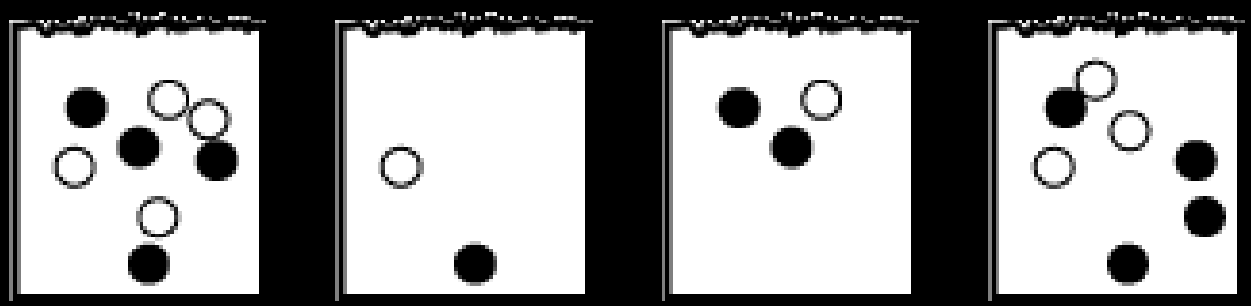
More Chance (En)counters

- B because there are only 2 and one is white and one is black and so I think most people dig to the bottom and the bottom is where the black is…



More Chance (En)counters

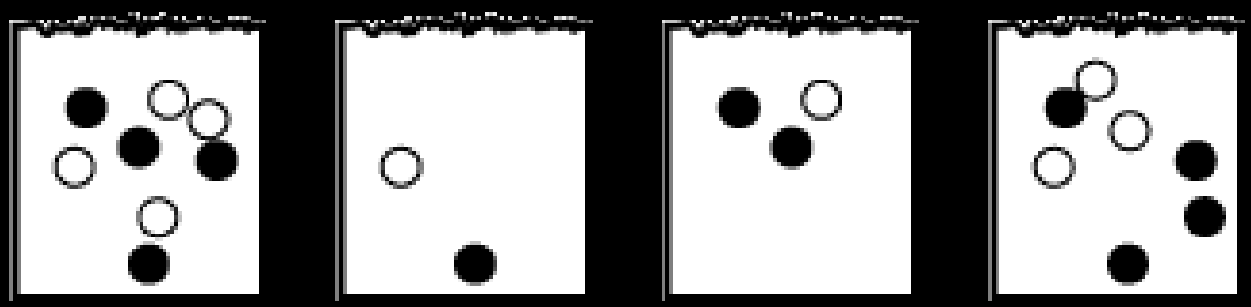
- A because most of the black counters are on top of the bag.



A B C D

More Chance (En)counters

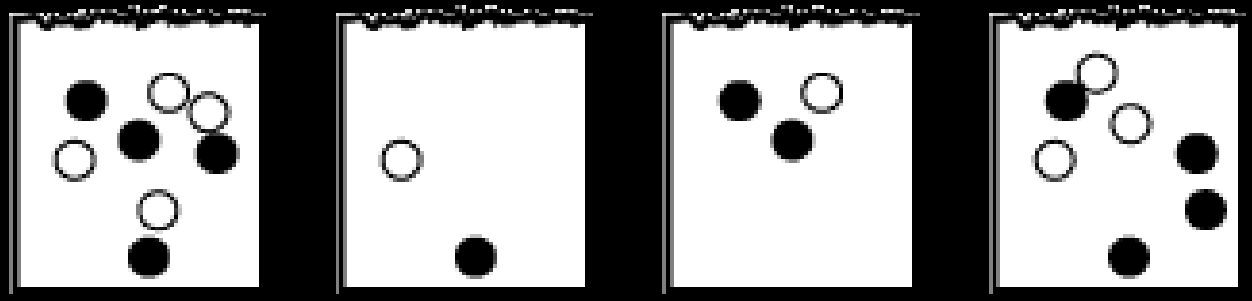
- C because there is less weight than the rest.



A B C D

More Chance (En)counters

- A because if you shake it, it makes more noise than the other ones so yeah, I chose Bag A.



A B C D

Washing Days

A
$4.00
1 kg
20 loads

B
$6.50
1.5 kg
30 loads

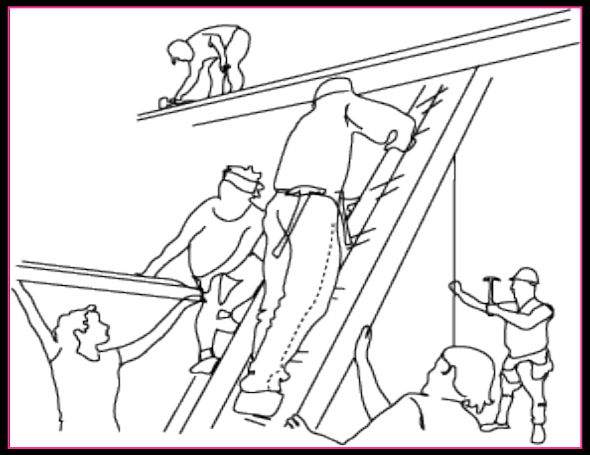
Which is better value, A or B?

Explain your thinking.

It doesn't really matter as long as it gets your clothes clean

Garage Building

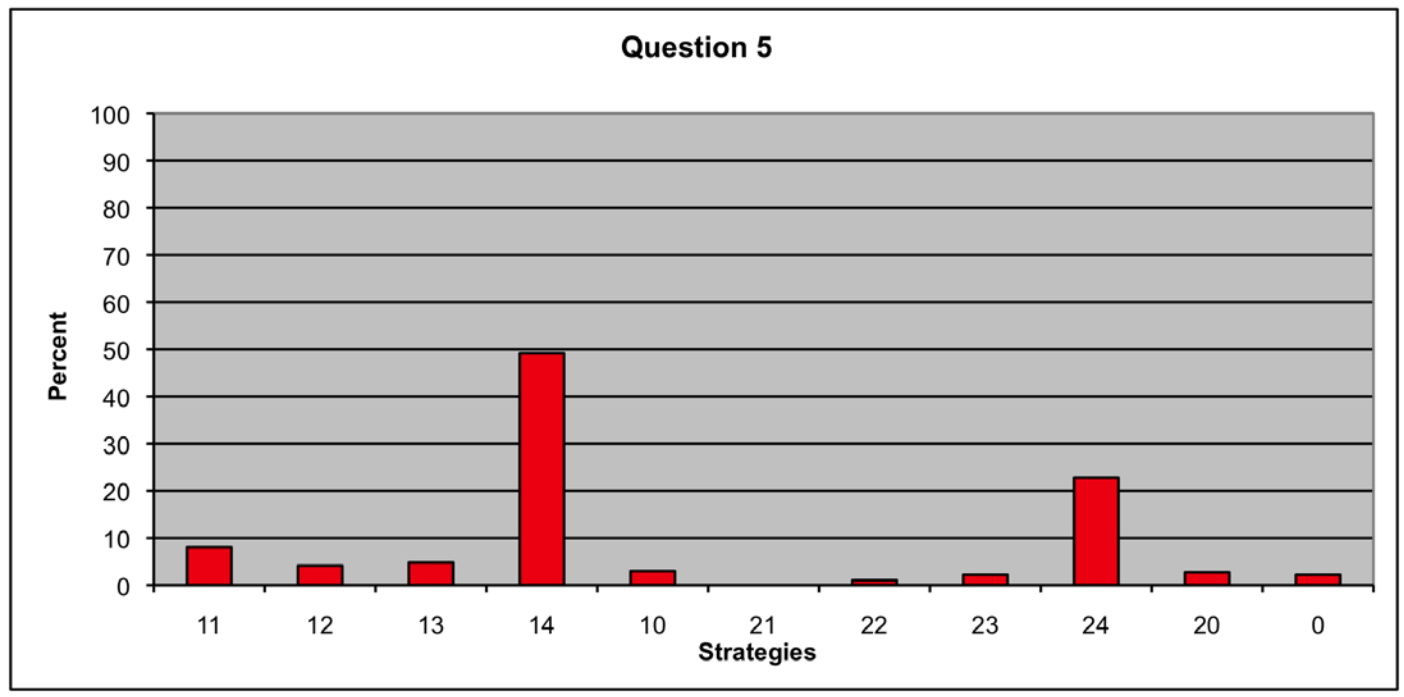
- Six people can build a garage in 3 days. Assuming that all of the people work at the same rate, how many people would it take to build the garage in 1 day?
- ❑ 18
- ❑ 2
- ❑ 12
- ❑ 9
- Explain the reason for your choice



Inverse proportion

- Inverse proportion is rarely addressed in early MYS mathematics curriculum
- We were surprised by the students' results to this task…

Garage building

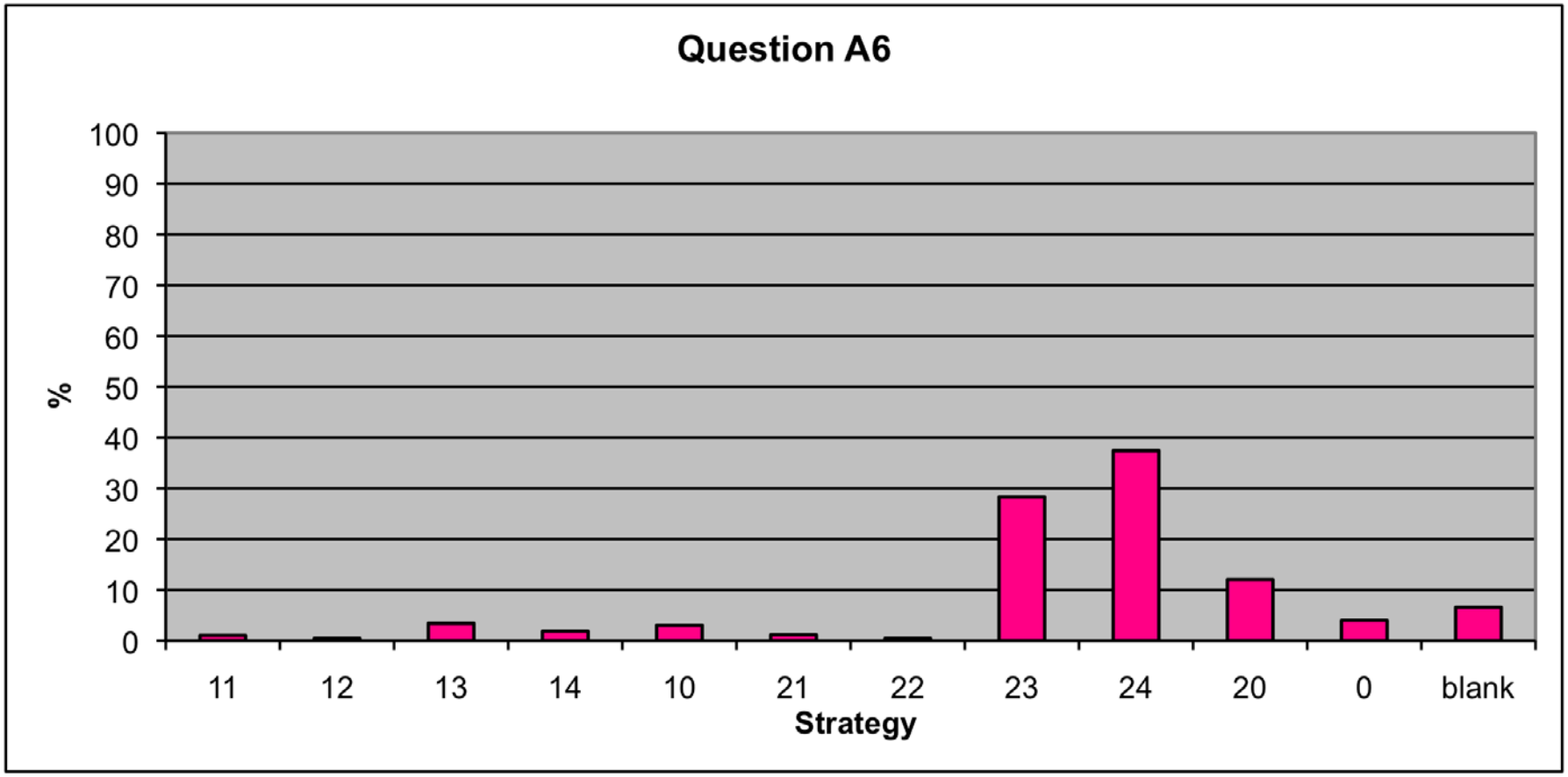


Right Answer, Wrong Thinking

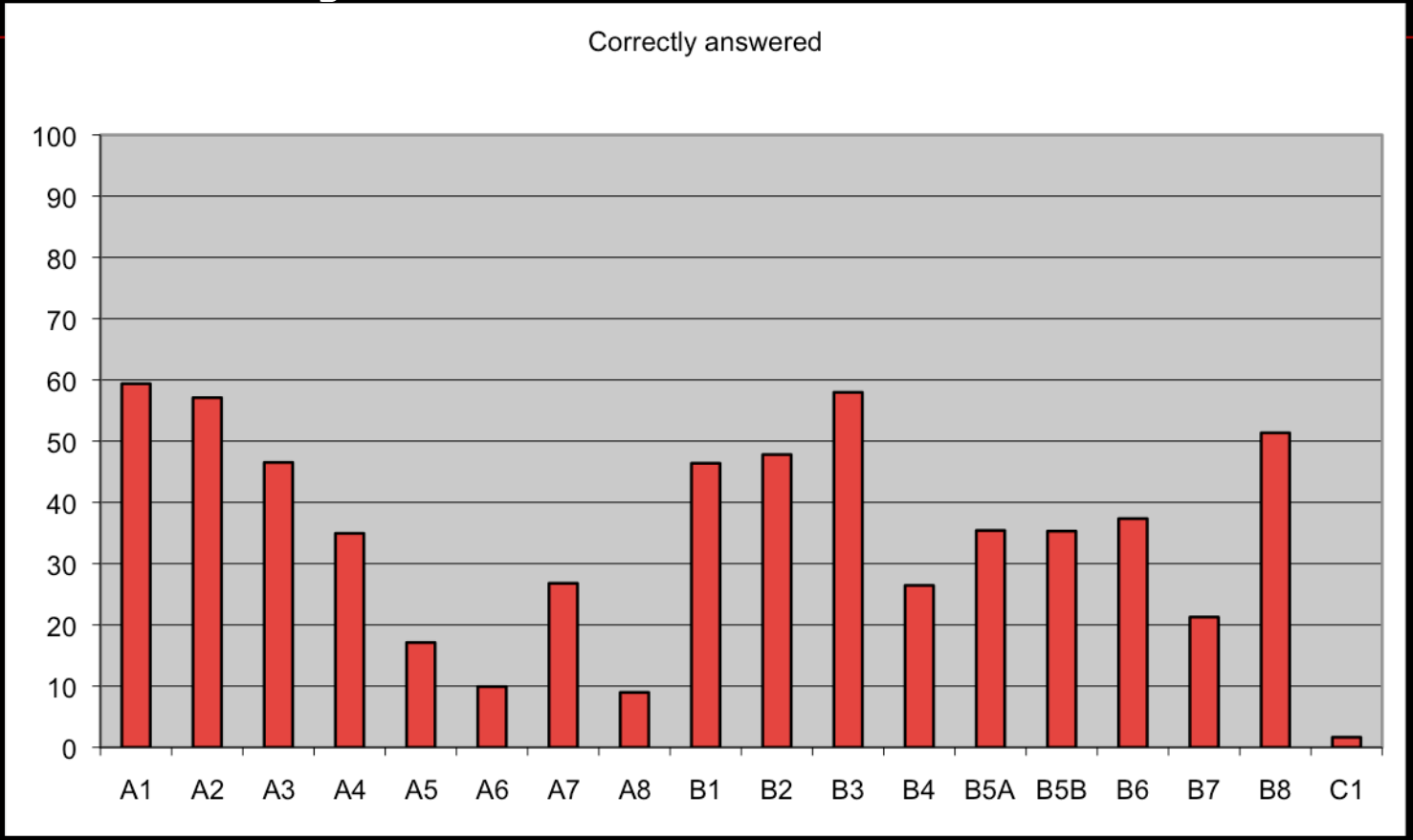
- 3 days 6 people
- 2 days 12 people (should be 9 people)
- 1 day 18 people

Revised Version

Six people can paint a fence in 3 days. Assume that all of the people work at the same rate. How many people would it take to paint the fence in 2 days?

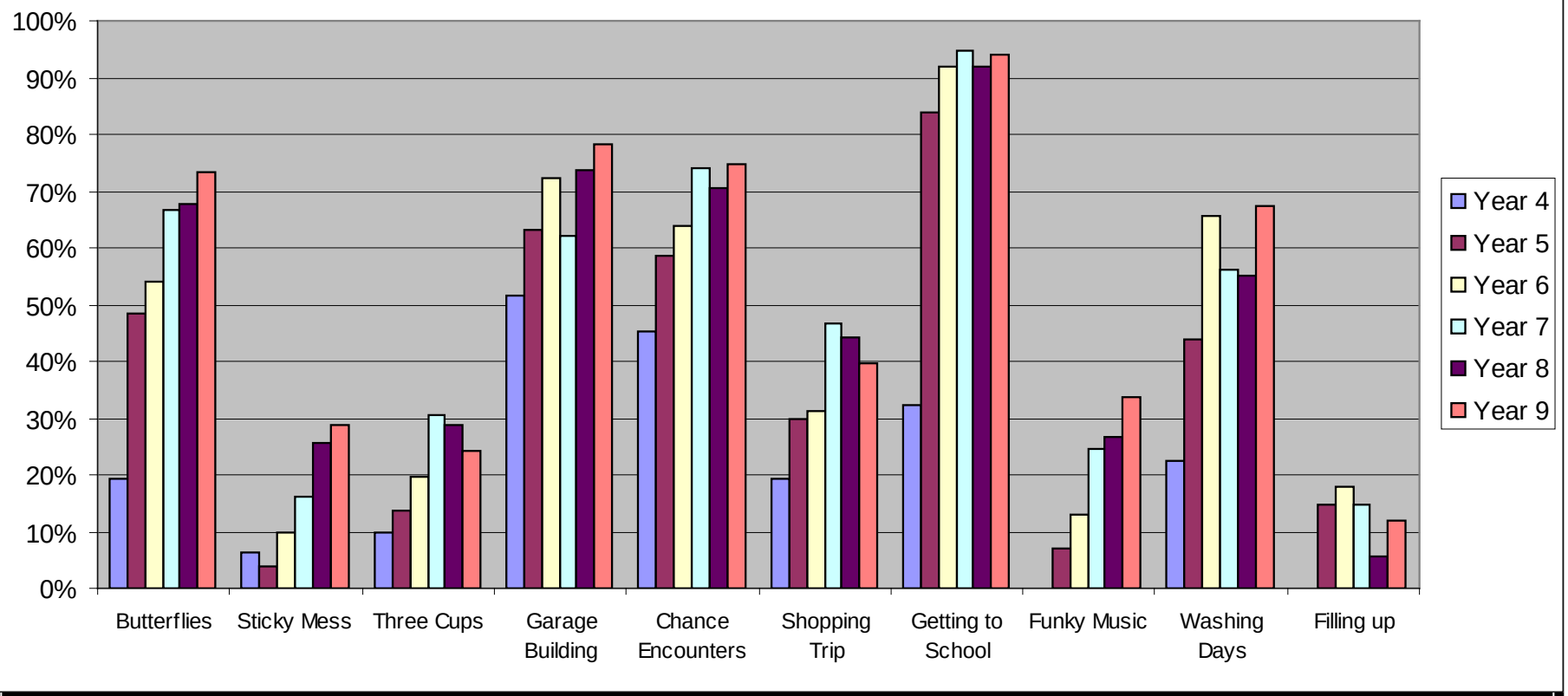


Correctly answered



C1 result is skewed because not all students were offered test.

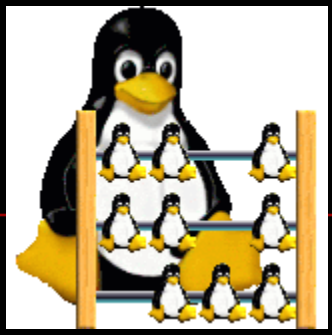
Right Answers all years



Promoting proportional reasoning

- Classroom activities that readily link to both the mathematics and/or science curriculum

Penguins

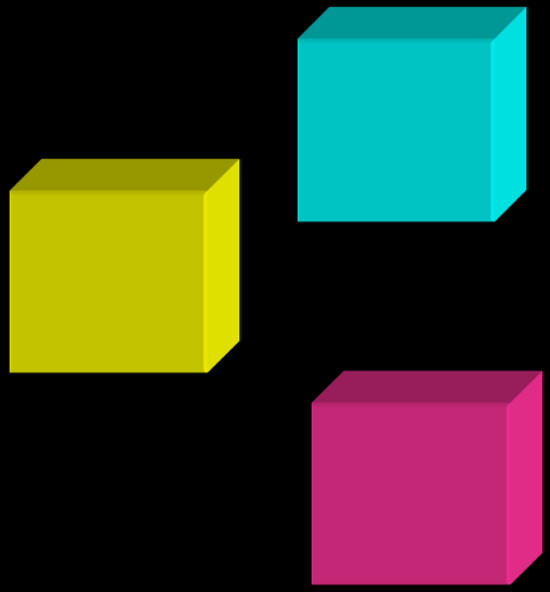


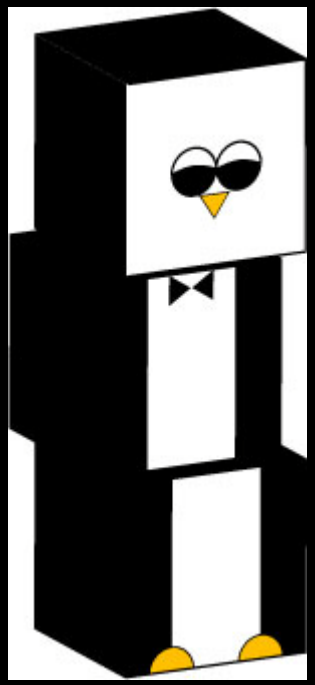
- **Why do penguins huddle?**

March of the penguins

Construct some penguins

- Use three cubes to create a penguin





Penguins

No of Penguins	Surface Area	Volume	Ratio SA : V (SA÷V)
1	14	3	4.6:1
9	54	27	2:1
25	90	75	1.2:1

Link to real situations

- Why do whole potatoes take longer to cook than potato chips?

- Why are McDonald's chips so slim?

Charge over baby in car

BORONIA police have again warned parents not to leave their children alone in cars for any length of time.

The warning comes after police charged a woman with conduct endangering serious injury after passersby called police to the Studfield shopping centre car park, Rowville, in early January.

Police found a two-month-old baby who had reportedly been alone in the car for up to 45 minutes.

Senior Sergeant Steve Rosewarne urged people to phone 000 if they noticed a distressed child alone in a car.

■ A FERNTREE Gully man has been charged with assault, and traffic and cultivating offences after a domestic disturbance at Ferntree Gully, on Friday, November 7. Seven police officers attended the house before the man was arrested at 11pm.

Police Beat

■ POLICE have charged a man from The Basin with possession of fireworks after complaints from nearby residents about the constant noise from fireworks being set off.

■ A ROUTINE call over shoplifting offences led police to an "Aladdin's cave" of allegedly stolen goods. Boronia police's Constable Brendon de Schwartz said the goods valued between $35,000 and $45,000 had been found at a Ringwood home on Tuesday, February 11.

Police have charged a 65-year-old man with a number of theft and proceeds of crime offences dating back as far as two years ago, and breach of a suspended sentence. Police are still looking for the owners of the goods, believed to have been stolen from a number of shops.

■ BORONIA police have had a number of calls regarding fireworks being set off in the area of the Dandenong Ranges National Park. Sen-Sgt Rosewarne said they risked starting a bushfire, and asked people who knew of illegal fireworks being used to report this to police.

More police briefs on Page 27.

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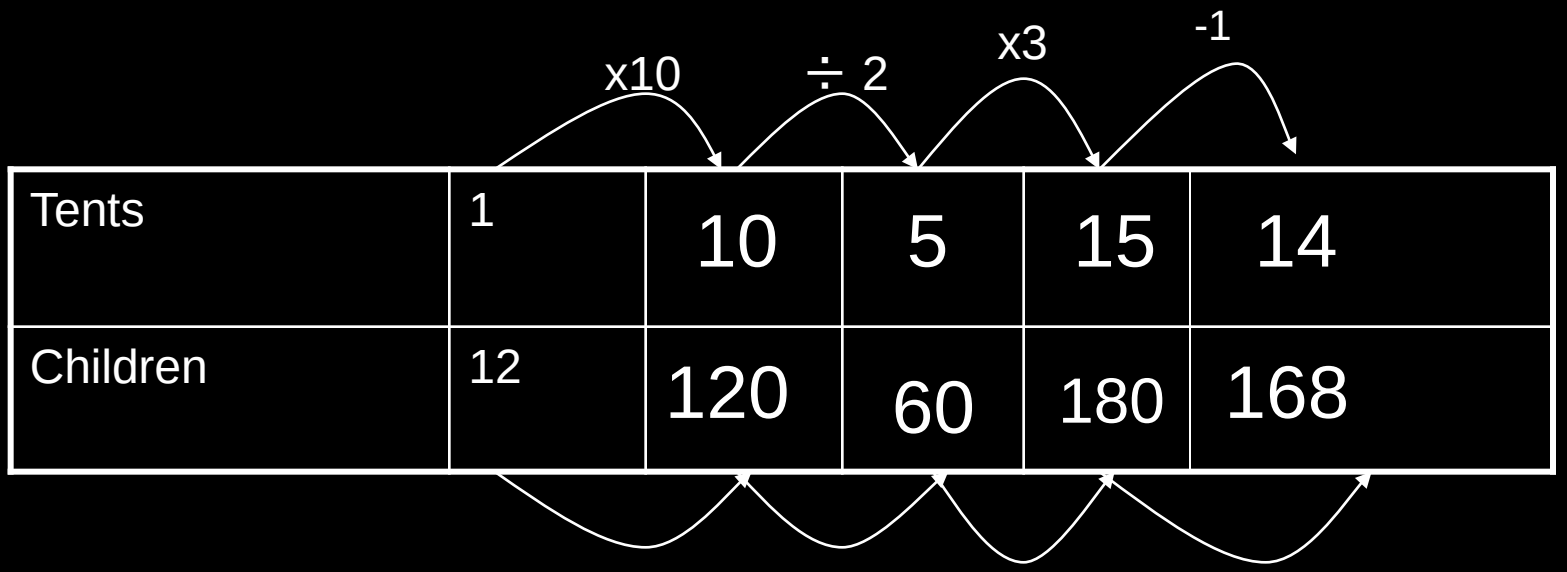
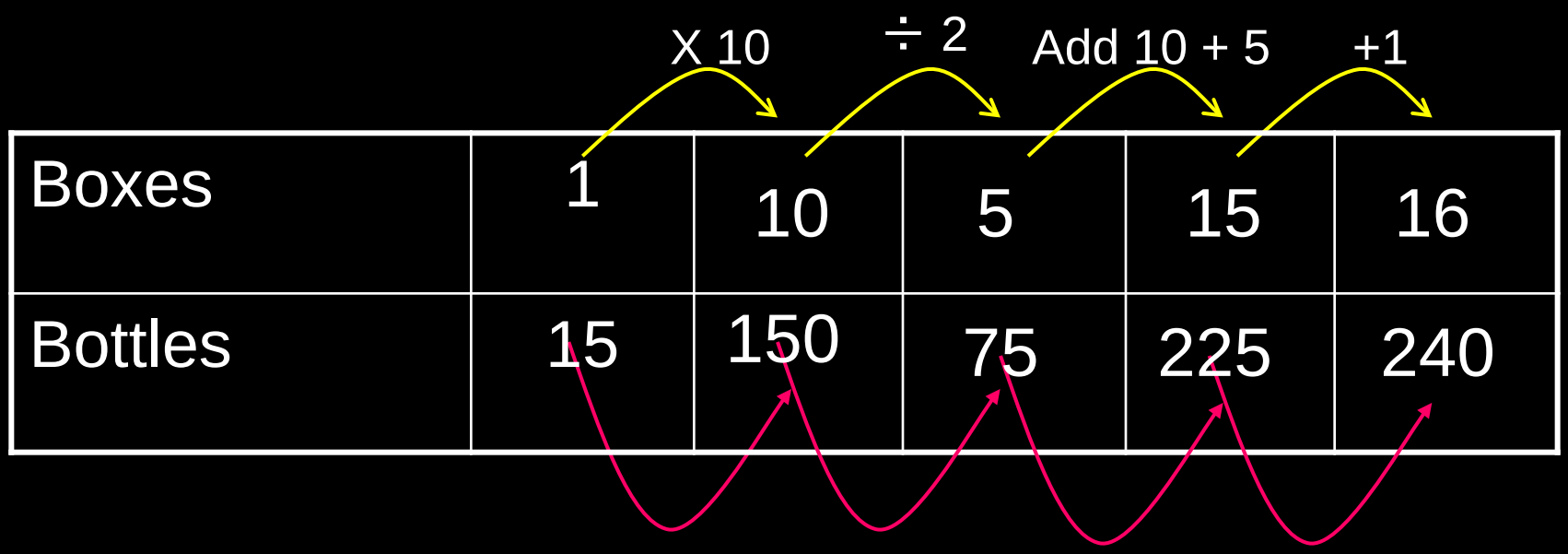
Ratio Tables

Ratio Tables

- There is 1 tent per 12 children. How many children can fit in 14 tents?



Bottles of water are packaged into 15 bottle boxes. How many bottles of water would there be in 16 packages?



Boxes	1	10	5	15	16
Bottles	15	150	75	225	240

Integrating maths and science

- DENSITY

- Density is a topic within the science curriculum
- Density compares mass and volume
- The topics of mass and volume are part of the mathematics curriculum

Sinking and Floating - Density

Why is density difficult?

Density is directly proportional to the mass.

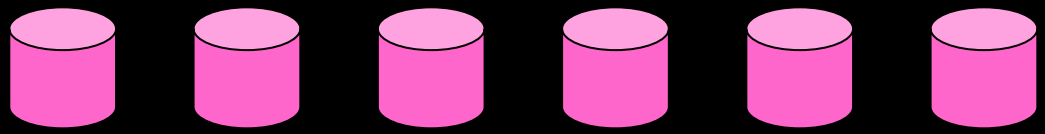
&

Density is inversely proportional to the volume.

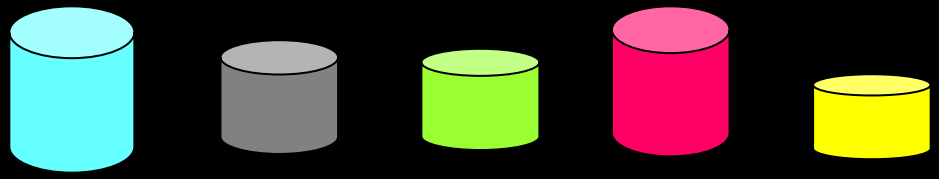
To think about both relationships at the same time is difficult.

DENSITY JARS

- Activity 1 – Constant Volume, different masses



- Activity 2 – Constant Mass, different volumes



- Activity 3 – Sinking and Floating

Jar	Mass (g)	Volume (mL)	Density (g/mL)
1	200	780	0.26
2	400	780	0.51
3	600	780	0.77
4	800	780	1.02
5	1000	780	1.28
A	500	320	1.56
B	500	400	1.25
C	500	630	0.79
D	500	710	0.70
E	500	850	0.59

Density with Year 5?

- Is it too difficult for these students?

Why the Titanic sank

First view

- The titanic is still full of air (empty), which means it is less dense than the water.

Second View

- The mass of the titanic is increasing but the volume doesn't change, this is because the water is filling the titanic up through the hole.

Third View

- The titanic is now vertical the water is ¾ of the way, its still less dense than the water but the mass is getting bigger.

Fourth View

- The titanic has now sunk this is because the titanic is more dense than the water and the mass has reached its full capacity so there is no air left in the Titanic but the volume is still the same!

Titanic 6

The crew have no fear of what is going to happen to the titanic 6.

The titanic 6 is
floating very happily!

The water is going into titanic 6 and the mass is getting bigger because there is water in the titanic, so the air bubble is getting smaller.

The titanic 6 has just run into an iceberg, so it has a hole in it.

Half air, half water…

The ratio of mass and volume, has now changed, its getting closer to the density of water.

The crew are worrying…

The titanic 6 has the same density of water, so, there is a tiny air bubble.

The crew are in life boats and are paddling away.

The titanic 6 has now sunk, so, it is more dense than water.

The crew are now safe on land.

Concluding comments

- The development of proportional reasoning is a slow process exacerbated by its nebulous nature and lack of specific prominence in school syllabus documents.
- Proportional reasoning is essential in many school subjects.
- All teachers can support the proportional reasoning skills of their students

Dave Allen's Proportional Reasoning

30% of all road accidents are caused by drunks.

That means that 70% of road accidents are caused by sober drivers.

I think that all sober drivers should get off the road because drunk drivers are safer drivers

- Thank you

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