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## Optimization of Spectrum Allocation in Cognitive Radio and Dynamic Spectrum Access Networks

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# Optimization of Spectrum Allocation in Cognitive Radio and Dynamic Spectrum Access Networks

A dissertation submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy

By

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2012  
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January 14, 2013

I HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER MY SUPERVISION BY Tao Zhang ENTITLED Optimization of Spectrum Allocation in Cognitive Radio and Dynamic Spectrum Access Networks BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Doctor of Philosophy .

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## ABSTRACT

Tao Zhang, Ph.D., Department of Computer Science and Engineering, Wright State University, 2012. Optimization of Spectrum Allocation in Cognitive Radio and Dynamic Spectrum Access Networks.

Spectrum has become a treasured commodity. However, many licensed frequency bands exclusively assigned to the primary license holders (also called primary users) remain relatively unused or under-utilized for most of the time. Allowing other users (also called secondary users) without a license to operate in these bands with no interference becomes a promising way to satisfy the fast growing needs for frequency spectrum resources. A cognitive radio adapts to the environment it operates in by sensing the spectrum and quickly decides on appropriate frequency bands and transmission parameters to use in order to achieve certain performance goals.

One of the most important issues in cognitive radio networks (CRNs) is intelligent channel allocation which will improve the performance of the network and spectrum utilization. The objective of this dissertation is to address the channel allocation optimization problem in cognitive radio and DSA networks under both centralized architecture and distributed architecture. By centralized architecture we mean the

cognitive radio and DSA networks are infrastructure based. That is, there is a centralized device which collects all information from other cognitive radios and produces a channel allocation scheme. Then each secondary user follows the spectrum allocation and accesses the corresponding piece of spectrum. By distributed architecture we mean that each secondary user inside the cognitive radio and DSA networks makes its own decision based on local information on the spectrum usage. Each secondary user only considers the spectrum usage around itself.

We studied three common objectives of the channel allocation optimization problem, including maximum network throughput (MNT), max-min fairness (MMF), and proportional fairness (PF). Given different optimization objectives, we developed mathematical models in terms of linear programming and non-linear programming formulations, under the centralized architecture. We also designed a unified framework with different heuristic algorithms for different optimization objectives and the best results from different algorithms can be automatically chosen without manual intervention. We also conducted additional work on spectrum allocation under distributed architecture. First, we studied the channel availability prediction problem. Since there is a lot of usable statistic information on spectrum usage from national and regional agencies, we presented a Bayesian inference based prediction method, which utilizes prior information to make better prediction on channel availability. Finally a distributed channel allocation algorithm is designed based on the channel prediction results. We illustrated that the interaction behavior between different secondary users can be modeled as a game, in which the secondary users are denoted as players and the channels are denoted as resources. We proved that our distributed spectrum

allocation algorithm can achieve to Nash Equilibrium, and is Pareto optimal.

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# Chapter 1

## Introduction

### 1.1 Cognitive Radio Networks

A software-defined radio (SDR) [1], is a radio in which the communication properties such as carrier frequency, signal bandwidth, modulation, and network access are implemented by software [2]. The term SDR was coined by Joe Mitola in 1991 [3]. A common architecture of SDR is illustrated in Figure 1.1.1.

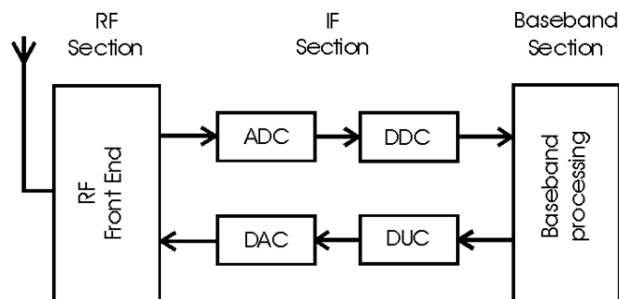


Figure 1.1.1: SDR Architecture.

Cognitive radio is considered as a more intelligent SDR, which has the ability to detect and exploit unused spectrum. A cognitive radio system is a radio system employing technology that allows the system to obtain knowledge of its operational and geographical environment, established policies and its internal state, and then

to dynamically and autonomously adjust its operational parameters and protocols according to its obtained knowledge in order to achieve predefined objectives. In [4], cognitive radio is defined as a radio that can change its transmitter parameters based on interaction with the environment in which it operates. Cognitive radio devices are capable of accessing the spectrum dynamically with the following capabilities: flexibility, agility, RF sensing, and networking [2].

- Flexibility: Cognitive radio devices have the ability to reconfigure its transmission or reception parameters and to change the waveforms;
- Agility: Cognitive radio devices have the ability to operate in two or more bands;
- Sensing: Cognitive radio devices have the ability to observe spectrum usage nearby;
- Networking: Cognitive radio devices have the ability for sharing information among all devices.

Formally, based on [5], the main functions of cognitive radio include

1. Spectrum sensing: determine which portions of the spectrum is available and detect the presence of licensed users using a licensed band nearby;
2. Spectrum decision: select the best available channels;
3. Spectrum sharing: coordinate access to spectrum with other users;
4. Spectrum mobility: vacate the channel when a licensed user is detected.

A basic cognitive cycle is shown in Fig. 1.1.2.

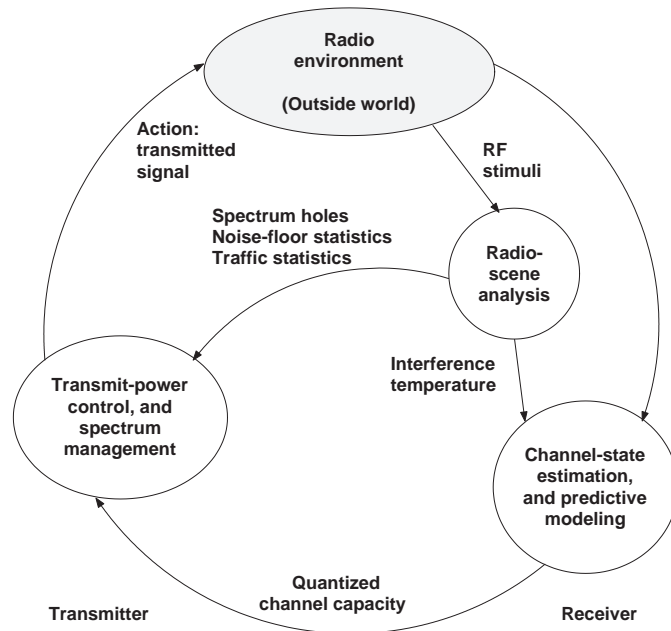


Figure 1.1.2: Cognitive Cycle.

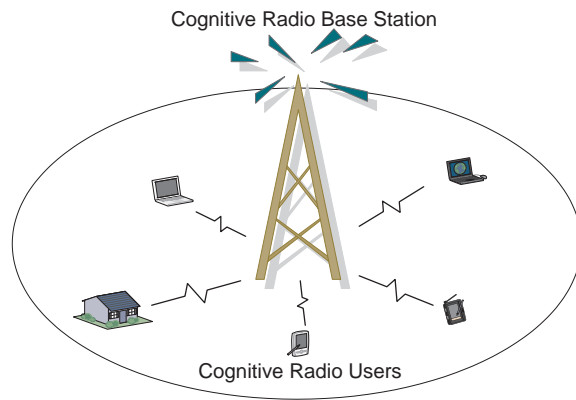
### 1.1.1 Architecture of Cognitive Radio Networks

Two types of architectures for cognitive radio networks exist, i.e., secondary users in a CRN are set up to either communicate indirectly through a central node, an secondary access point or one to the other directly. The first type is infrastructure based CRNs shown in Figure 1.1.3 and the other is ad-hoc CRNs shown in Fig 1.1.4.

### 1.1.2 Spectrum Sharing in CRNs

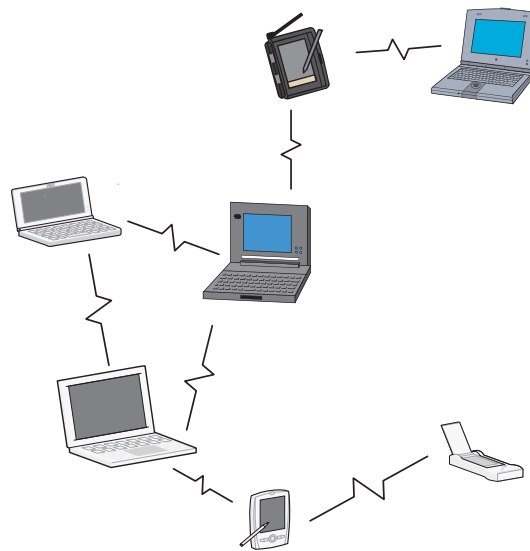
Dynamic spectrum access technologies can enable cognitive radio wireless systems to select the best available frequency spectrum at a given location and over a given period of time to optimize the utilization of available spectrum and avoid interference with other radios or other systems.

In [5], the spectrum sharing process is divided into five major steps:



---

Figure 1.1.3: An Infrastructure-Based CR and DSA Network.



---

Figure 1.1.4: An Ad-Hoc CR and DSA Network.



1. Spectrum sensing: if a secondary user wants to communicate, firstly it needs to be aware of the spectrum usage nearby its area, and can only choose spectrum bands from the unused portion of spectrum.
2. Spectrum allocation: based on the spectrum availability, secondary users can access an allocated channel. In fact this allocation depends on not only spectrum availability, but also internal (and possibly external) policies.
3. Spectrum access: it is very possible that there may be multiple secondary users trying to access the spectrum. This access should be coordinated to prevent multiple users from interfering with each other eg., colliding in a certain portion of the spectrum.
4. Transmitter-receiver handshake: once a portion of the spectrum is determined by sender for communication, we should also notify the receiver of this communication of the selected spectrum. Hence, a handshake protocol is very important for efficient communication.
5. Spectrum mobility: secondary users are seen as “visitors” to the spectrum they are allocated. Therefore, if a licensed user requires the specific portion of the spectrum in use, secondary users needs to communicate in another vacant portion of the spectrum.

## 1.2 Dynamic Spectrum Access

Dynamic spectrum access (DSA) is a concept that unlicensed users may borrow spectrum from spectrum licensees. Cognitive radio technology is especially suitable for

DSA. Since the US Radio Act of 1934, the most popular spectrum assignment is *command and control* approach. In United States, the Federal Communications Commission (FCC) has the regulatory authority to review spectrum licensing applications and issue licences to applicants. The advantage of the command and control approach is that the interference between wireless systems is successfully avoided. The licensee has the exclusive use for some specific spectrum bands. However, the command and control approach has led to low utilization of spectrum resources and caused the spectrum congestion problem.

In the past decade several more flexible approaches have been presented to mitigate the shortcomings of the command and control approach:

1. Exclusive spectrum usage rights
2. Spectrum commons (open sharing)
3. Opportunistic use

Essentially, the *exclusive spectrum usage rights* approach allows selling and trading spectrum and freely choosing technology. Since it is still exclusive spectrum usage, the interference is avoided.

The *spectrum commons* approach comes from the remarkable success of unlicensed systems within the industrial, scientific, and medical (ISM) spectrum bands. The approach provides users shared access to available spectrum resources. Interference avoidance and confliction resolving are based on the agreed upon protocols.

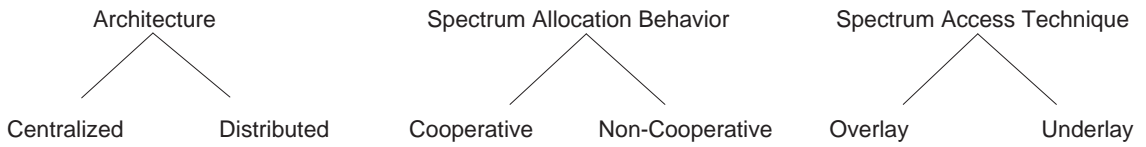
The opportunistic use approach is also based on cognitive radio technology. Cognitive networks are permitted to scan the nearby spectrum usage and determine the

available spectrum resources at a given time and place. There are two forms of opportunistic spectrum access: underlay access and overlay access. Underlay access approach restricts the signal powers to below that of the noise floor experienced by existing primary users. An example of underlay access is to use ultra-wideband (UWB). The other approach, overlay access, detects the spectrum white spaces and utilizes them. These unutilized spectrum resources can be used as long as there are no primary user.

The latter two approaches are heavily based on cognitive radio networks as cognitive radio technology can provide the necessary abilities for the implementation of the two approaches, such as awareness and adaptability. In the following section, we outline the challenge of cognitive radio networks.

### 1.3 Spectrum Allocation

In [6, 7], the term, channel, is defined as a basic slice of spectrum allocated to users. It is assumed in cognitive radio networks that a secondary user may use any number of available channels simultaneously [8]. Channels are completely orthogonal so that two users will not interfere with each other if they use different channels. Moreover, a channel may be different from other channels in terms of bandwidth and transmission. Users are assumed to be able to sense available channels, evaluate the channel characteristics (e.g., bandwidth). Without loss of generality, we assume each channel has the same physical characteristics for all secondary users. In [5], the existing solutions for spectrum allocation are classified in three aspects as shown in Fig 1.3.5, *architecture assumption*, *spectrum allocation behavior*, and *spectrum access technique*.




---

Figure 1.3.5: Classification of spectrum allocation in CR networks.

Based on the architecture of cognitive radio networks, the spectrum allocation approaches can be divided into centralized approaches and distributed approaches. Based on the spectrum allocation behavior, the approaches can be divided into cooperative and non-cooperative approaches. Cooperative approaches consider the interference measurements of all secondary users [9–12]. In other words, the interference information of each secondary user is shared among the others. On the contrary, the non-cooperative approaches consider only the node at hand [13–15]. These approaches are commonly called selfish approaches. The advantage of non-cooperative approaches is that a secondary user only needs minimal communication with other secondary users. The third classification for spectrum allocation in cognitive radio networks is based on the access technology. Overlay spectrum sharing means the secondary users can only access the spectrum channels that have not been used by primary users [9, 10, 12, 13]. While underlay spectrum sharing means the secondary users can co-exist with the primary users in a certain channel and its transmit power is regarded as noise by the primary users [11]. This technique is not considered in our work.

### **Centralized Spectrum Allocation Approach**

In cognitive radio networks with centralized architecture, a secondary central device collects the certain information from all secondary users and then makes decisions on

spectrum allocation. The information includes the location, power, available channels, interference and the appearance of primary users. Then the central device decides the spectrum allocation and broadcasts the assignments to all secondary users.

There are two main requirements in this approach. First, this approach requires a communication path between the central device and all secondary users, i.e., all secondary users must have the interference-free access to a pre-defined control channel. Second, the central device must have the ability execute a complex algorithm. The centralized spectrum allocation approaches are naturally cooperative approaches.

### **Distributed Spectrum Allocation Approach**

Distributed spectrum allocation approaches are perfectly suited for CRNs with the distributed architecture. In this architecture, every secondary user must use a distributed algorithm to determine its own spectrum assignment. Each secondary user only has locally available information and must make the best decision for itself. Two types of approaches have been proposed. The first approach is for each secondary user to detect its nearby spectrum usage and then coordinate with its neighbors to determine channel assignments. The second approach is for each secondary user to detect its nearby spectrum usage and make a channel assignment decision without any coordination with its neighbors. The first approach is easier to implement, but it also needs a common channel for each secondary user to share the information with neighbors. In fact, the two approaches correspond to cooperative and non-cooperative approaches, respectively.

## 1.4 Spectrum Availability Prediction

Since the spectrum usage changes over time and in different locations, secondary users have to sense the nearby spectrum usage from time to time to avoid the interference with primary users. With the historical sensing results, a lot of spectrum availability prediction approaches have been proposed, which are highly related to the spectrum allocation in CRNs. In other words, an effective spectrum availability prediction algorithm will help the secondary users to make right decisions in spectrum allocation and minimize the interference to the primary users. Statistical methods are widely used in spectrum occupancy and spectrum availability prediction. For instance, a statistical spectrum occupancy model [16] based on a combination of several different probability density functions was designed to generate accurate temporal and frequency behavior of various wireless transmission. In [17], the use of binary time series for spectrum occupancy characterization and prediction was proposed. Markov models and hidden Markov models (HMM) are also commonly used in spectrum availability prediction [18–20]. However, not all frequency channels were validated to fit the property of Markov chains and hidden Markov chains. In addition, the initial parameters needed in the Markov chain approach are hard to choose. Another limitation is the hidden Markov prediction model requires more memory to predict the channel availability. Even though the memory is sufficient, the limited past sensing results may mislead the prediction.

Bayesian analysis is a widely used method of statistical inference applied to many real-world problems. In this dissertation, we propose a Bayesian estimation based

prediction approach. Our approach is designed to overcome the limitations of the above approaches. The details of Bayesian estimation based prediction approach will be introduced in Chapter 5.

## 1.5 Dissertation Focus

In this dissertation, we will explore several important and challenging issues in CR networks in order to improve the spectrum utilization:

- Centralized Spectrum Allocation in CRNs

In a centralized spectrum allocation scenario, secondary users are assumed to be able to sense available channels, evaluate the channel bandwidth, and send the channel information to a central controller. The central controller makes decisions on channel allocation, and releases the allocation scheme to all users. We consider static users or users with low mobility in which spectrum sensing.

In this circumstance, we study three optimization objectives in spectrum allocation problems in CRNs: (1) maximum network throughput, (2) max-min fairness and (3) proportional fairness.

These problems are formulated as linear programs. Moreover, to balance the spectrum utilization and the fairness among all secondary users, a new model is presented to strike a balance between these two objectives.

- Spectrum Allocations in Infrastructure Based CRNs

We develop a spectrum allocation algorithm for infrastructure based CRNs.

This algorithm aims at maximizing the average bandwidth per flow, and avoiding spectrum wasting by the idle secondary base station.

- Spectrum Availability Prediction

In CRNs, since the spectrum usage changes over time and space, secondary users need to keep track of the changes of the radio environment, such as primary users' spectrum activities [6, 7]. In order to improve the spectrum utilization, accurate prediction on spectrum availability is needed. We study the spectrum availability prediction problem and propose a Bayesian inference based algorithm in CRNs. The proposed algorithm utilizes the prior information of spectrum usage of primary users.

- Distributed Spectrum Allocation Algorithm in CRNs

Development of distributed spectrum allocation algorithms is an important topic in CRNs. Game theory is an effective way to process complex distributed decision making problems and is a suitable approach for distributed spectrum allocation. In the context of game theory based approaches, we can map the spectrum allocation problem into a cooperative game with the assumption that they can enforce collaboration among themselves so as to jointly improve their benefits. Different from the other game theory approaches, the payoff function of our game mode is actually based on spectrum availability prediction which can be archived from our Bayesian inference based prediction algorithm. We describe this game mode and prove it can reach a Nash Equilibrium and Pareto optimal solution.



## 1.6 Organization of Dissertation

The rest of the dissertation is organized as follows. Chapter 2 examines the current development in research on spectrum allocation in CR networks, including centralized spectrum allocation, prediction on spectrum availability and distributed spectrum allocation. In Chapter 3 we present a heuristic spectrum allocation algorithm on infrastructure based CR networks. Chapter 4 explains in detail the development of mathematical optimization models and the design of efficient centralized algorithms for spectrum allocation in cognitive radio networks. Chapter 5 addresses the prediction on spectrum availability prediction problem in CRNs with a Bayesian inference based prediction algorithm. Chapter 6 tackles distributed spectrum allocation algorithm in CRNs, which is based on a cooperative game model. Finally, Chapter 7 concludes this dissertation.

# Chapter 2

## Spectrum Allocation in CRNs

### 2.1 Problem Statement

As discussed in the previous section, a naive spectrum allocation approach may result in low spectrum utilization. Improving spectrum utilization is a big challenge in spectrum allocation in CRNs. On the other hand, since multiple secondary users compete for a limited number of available channels, fairness issue in spectrum allocation becomes another challenge.

Therefore, in this dissertation, we are primarily addressing the two challenges in spectrum allocation in CRNs. During the past decade, a great deal of research work has been conducted in this area. In [21, 22], the effectiveness of using conflict graphs to model interference has been demonstrated. In [6, 7], the authors reduce the optimization problem into a variant of the graph coloring problem. They present a heuristic algorithm in which each vertex is assigned a unique label according a specific labeling rule. The algorithm picks the vertex with the highest valued label and assigns the color associated with the label to the vertex, and then the algorithm updates the label of each vertex and goes to the next iteration. Therefore, the label

policy is very important in the algorithm. In fact, the label policy is based on different optimization objectives. They propose three different label formula for the three well-known optimization objectives. However, the previous work is based on heuristic and has no strong theoretical support. In addition, only a handful of previous work considered the combination of the channel prediction and the spectrum allocation.

In this dissertation, we develop optimization models to solve the spectrum allocation problem under both the centralized and distributed architectures. For centralized architecture, we develop mathematical models including non-linear and integer linear program formulations, in terms of different objectives of optimization, and design efficient heuristic algorithms for the different optimization models. We develop a framework which integrates multiple algorithms and is extensible for more algorithms. For distributed architecture, we develop a distributed approach that considers the spectrum prediction, which is based on a *Bayesian* estimation based prediction approach.

## 2.2 Centralized Spectrum Allocation in CRNs

Given a cognitive radio network with  $N$  secondary users and  $M$  available channels, the objectives of our research work on the spectrum allocation optimization problem in cognitive radio networks include optimization model development, heuristic algorithm design, and spectrum availability prediction.

- *Optimization models:* We will develop linear programming and non-linear programming models for spectrum allocation with different objectives of optimization in cognitive radio networks. To solve large problems, we will develop a unified binary linear programming (UBLP) model which is then solved by the

simplex method and branch-and-bound search. We will prove that given different per-user bandwidth minimums, the optimal solution to the UBLP model would achieve specific optimization objectives, such as the maximum network throughput and the max-min fairness. For the proportional fairness objective, the solution to the UBLP model would be within a bound of the optimal solution.

- *Heuristic algorithm for UBLP model:* We will show that the original optimization problem of UBLP model is a combination of a maximum independent set problem and a maximum graph coloring problem. From the graph theory perspective, we present a spectrum allocation algorithm which is shown to be able to obtain an approximate solution to the UBLP model.
- *Heuristic algorithms for infrastructure based cognitive radio networks:* We will propose an efficient heuristic algorithm to solve the spectrum allocation problem for the infrastructure based cognitive radio networks by processing demands sequentially. This algorithm achieves good spectrum utilization and fairness as well.

## 2.3 Distributed Spectrum Allocation in CRNs

Spectrum allocation optimization for CRNs under distributed architecture is more complicated than the one under centralized architecture. However CRNs under distributed architecture are more common in the real world. Under this circumstance, every secondary user has to be sensing the spectrum usage and makes channel access

decisions independently. They also need to share their decision with nearby secondary users. Therefore, game theory as a tool for mathematical models of conflict and cooperation between intelligent rational decision-makers, applies very well to this case. In the work of [23–25], the authors proposed a novel design framework that ensures that cognitive radio interactions are beneficial and reduce network interference with each adaptation based on game theory. However, the framework is mainly focused on the physical layer, such as the power adaptation. In our work, we will focus on the distributed spectrum allocation optimization based on game theory. Moreover, since the spectrum availability is highly related with spectrum allocation optimization, we propose to use a *Bayesian* estimation based approach to forecast spectrum availability, and then utilize the prediction result as a utility function in this game. We will prove in this game, the final spectrum allocation result is *Nash equilibrium*. With one more assumption, we also prove that the spectrum allocation result could reach *Pareto optimality*.

## Chapter 3

# Heuristic Spectrum Allocation Algorithm in Infrastructure Based CRNs

In this Chapter, we consider the spectrum allocation problem for some special occasions in infrastructure based cognitive radio networks [26, 27]. In Fig 3.0.1, wireless end-users (or clients) access the spectrum through secondary basestations. For every secondary basestation, its bandwidth is shared by all traffic flows established between the end-users and the secondary base station. Under the circumstances, all end-users are assumed to be static. Hence, the spectrum utilization depends on the bandwidth used by each traffic flow. So only considering the bandwidth of each secondary base station in spectrum assignment is not enough and might result in inferior results. A persuasive example is allocating more spectrum to an idle secondary base station with no end-user, which will cause low spectrum utilization no matter how much bandwidth the secondary base station has.

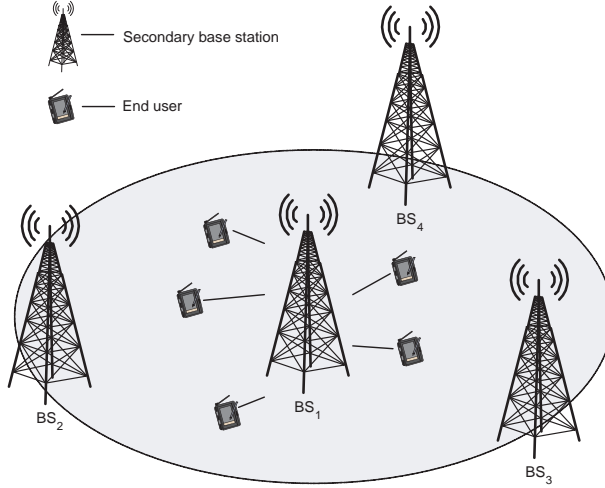


Figure 3.0.1: An infrastructure based cognitive radio network.

### 3.1 Notations in Infrastructure Based CRNs

Given an infrastructure based cognitive radio network with  $N$  secondary basestations and  $M$  available channels (Figure 3.0.1), we present notations used in the spectrum assignment model as follows:

- $\mathcal{C} = \{c_1, \dots, c_M\}$  is a set comprising  $M$  channels.
- $\mathcal{U} = \{u_1, \dots, u_N\}$  is a set comprising  $N$  secondary basestations.
- $\mathcal{F}$  is an  $N \times N$  binary matrix, and  $f_{i,j} = 1$  if  $u_i$  and  $u_j$  interfere with each other when they use the same channel, and  $f_{i,j} = 0$  otherwise.
- $X$  is an  $N \times M$  binary matrix, and  $x_{i,k} = 1$  if user  $i$  is assigned the channel  $c_k$ , and  $x_{i,k} = 0$  otherwise.
- Matrix  $\mathcal{B} = \{b_{i,k}\}_{N \times M}$  represents the channel bandwidth for secondary basestations, i.e.,  $b_{i,k}$  denotes the bandwidth that can be acquired by  $u_i$  using channel  $c_k$ . We also call  $b_{i,k}$  as the channel reward for  $u_i$ .

- $n_i$  is the number of end-users associated with  $u_i$ ;

## 3.2 Optimization Objective

In infrastructure based cognitive radio networks, a secondary base station is able to sense available spectrum holes, evaluate characteristics (e.g., bandwidth) of available spectrum and share information with other secondary base stations through a backbone network. At the network deployment stage, we assume every secondary base station uses a common channel and broadcasts the beacon packet to inform the neighbors of its existence, which can be used to build an interference matrix. We also assume the locations of secondary base stations and end-users are static during the spectrum assignment procedure. In other words, we consider static users or users with low mobility in which spectrum sensing and spectrum assignment work on a relatively large time scale. All of end-users are assumed to have the same access priority.

Without loss of generality, each end-user is able to choose and be associated with a secondary basestation based on some criteria, such as distance and power. Thus, it is common that different secondary basestations are associated with different number of end-users. Given the circumstances, the spectrum utilization and the fairness do not merely rely on the secondary base stations themselves. In fact, the spectrum utilization depends on the practical throughput of each traffic flow between the end-user and its associated basestation. If we only consider the potential reward of a channel to secondary basestations in spectrum assignment, the spectrum utilization will not be as good as expected. A convincing example is already shown in Figure 3.0.1. On



the other hand, the practical throughput of each traffic flow is unpredictable. However, the average bandwidth of flows (also termed as the average per flow bandwidth) could be obtained by dividing the total rewards of a secondary basestation by the number of its associated end-users. Therefore, we propose an end-user based optimization objective that maximizes the minimum average per flow bandwidth. The corresponding objective function is:

$$\arg \max U, \quad (3.1)$$

where the utility function is expressed as:

$$U = \min_{0 < n < N} \beta_n, \quad (3.2)$$

where  $\beta_n$  is the average per flow bandwidth of end-users associated with basestation  $n$ .

As a result of Eq. (3.1) and Eq. (3.2), we can improve the spectrum utilization by allocating more channels to the secondary basestation with more end-users.

### 3.3 Heuristic Algorithm

In this section, we propose an end-user based vertex-coloring (EBVC) algorithm for infrastructure based cognitive radio networks. Based on the definition in section 3.2, we build a graph in which a vertex corresponds to a secondary basestation and an edge connecting two vertices  $u_i$  and  $u_j$  if  $f_{i,j} = 1$ . Different from the CSGC algorithm which changes the graph into a graph with multiple colored edges, our approach preserves the original graph, and uses vertex labeling and coloring. The distinct feature in

vertex labeling is that we use average per flow bandwidth as the label value. For a vertex  $u_i$ , the label is obtained by:

$$label_i = \frac{\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k}}{n_i}. \quad (3.3)$$

Eq. (3.3) is self-explanatory, i.e., the value of  $label_i$  represents the maximum average bandwidth that an end-user can obtain from its associated secondary basestation.

In two special cases, we label the vertex  $u_i$  as follows:

$$label_i = 0, \text{ if } \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} = 0$$

$$\text{and } n_i = 0 \quad (3.4)$$

$$\forall u_i \in \mathcal{U}, \forall c_k \in \mathcal{C};$$

$$label_i = \infty, \text{ if } \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} > 0$$

$$\text{and } n_i = 0 \quad (3.5)$$

$$\forall u_i \in \mathcal{U}, \forall c_k \in \mathcal{C}.$$

Because of the mobility of end-users, even a secondary basestation with no end-user should be allocated at least one channel if conditions permit. Eq. (3.4) and Eq. (3.5) allow as many of the secondary basestations as possible to obtain at least one channel. If two vertices are tied for the label value, the vertex with more end-users has a higher priority in spectrum assignment. The formal specification of the EBVC algorithm is given in Algorithm 1. For convenience, we will use color instead of channel in the algorithm description.

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**Algorithm 1:** *EBVC*

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**Data:** A set of vertices  $\mathcal{U}$ , interference matrix  $\mathcal{F}$ , channel reward matrix  $\mathcal{B}$ ,

the number of end-users for each vertex, and the set of colors  $\mathcal{C}$

**Output:** A spectrum assignment scheme  $X$

**while** *not all available color sets of vertices are empty* **do**

**for**  $u \in \mathcal{U}$  **do**

        LabelingVertex ( $u$ );

**end**

    Pick vertex  $u_i \in \mathcal{U}$  with the lowest labeled value;

$k \leftarrow \text{ChooseColor}(u_i)$ ;

$x_{i,k} = 1$ ;

**for**  $i \leftarrow 1$  **to**  $N$  **do**

**if**  $f_{i,j} == 1$  **then**

            Delete  $c_k$  from the available color set of  $u_j$ ,

**end**

**end**

**end**

---

The iterative algorithm preferentially allocates color to the vertex with the lowest label value. At the beginning of each stage, all vertices are given a label value. Then a vertex with the lowest label value is picked and function *ChooseColor()* allocates the color with the maximum reward from the vertex's available color set to this vertex. Finally this channel is removed from the available color sets of all adjacent vertices. The EBVC algorithm stops if the available channel set  $\mathcal{C}$  of every vertex is empty.

Also, because of the mobility of end-users, this algorithm should be run periodically. The frequency of running this algorithm depends on the degree of mobility of end-users.

### 3.4 Simulation Results

The objective of the EBVC algorithm is to maximize the average per flow bandwidth, therefore improving the spectrum utilization. We compare the average network performance among three algorithms. The first one is the EBVC algorithm, the second and the third algorithms are CSGC algorithms which respectively use two utilization objectives, MMF and PF [6, 7]. Simulation based studies were performed using NS2 (version 2.32) with sufficient randomly generated scenarios including varying number of end-users and available channels. We use different metrics, such as average throughput per flow, average end-to-end delay per packet at the application layer, and packet collision ratio, to characterize the simulation results.

*Simulation Parameters:* The network topologies consist of 10 secondary basestations in a  $500 \times 500$  region of coverage. The number of end-users is varied from 20 to 80. We generated UDP flows using a constant bit rate (CBR) traffic generator with data rate 512Kb. The packet size for all traffic was set at 1024 bytes and the bandwidth of each channel was set at 11Mbps. The  $CSThresh$  was set to  $2.78869e-09$  and the  $RXThresh$  was set to  $5.76175e-10$ . The power parameter  $Pt$  was set to 0.2818. Based on the three parameters above, the transmission range of all secondary basestations was  $100m$  and any two basestations were interference adjacent only if they were in the transmission range of each other. In Figure 4.3.2, we present the comparison of

average throughput per flow among the three algorithms. As the number of end-users increases, the average throughput per flow obtained by the EBVC algorithm increases dramatically, compared with the results of the other two algorithms. The reason is that the EBVC algorithm always assigns the channels to secondary basestations with a large number of end-users. That is, the secondary basestations with more end-users capture more channels. As a result, spectrum utilization improves as well.

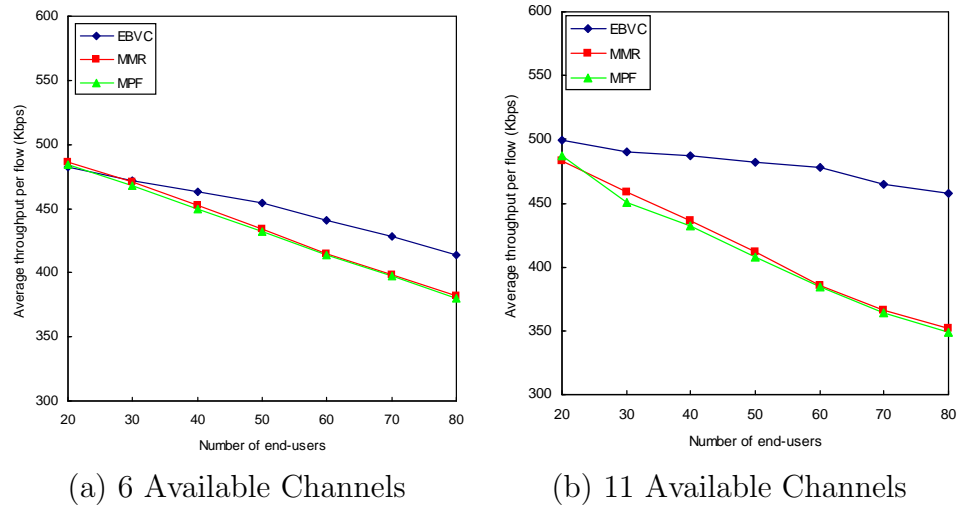


Figure 3.4.2: Comparison of average throughput per flow for the three algorithms.

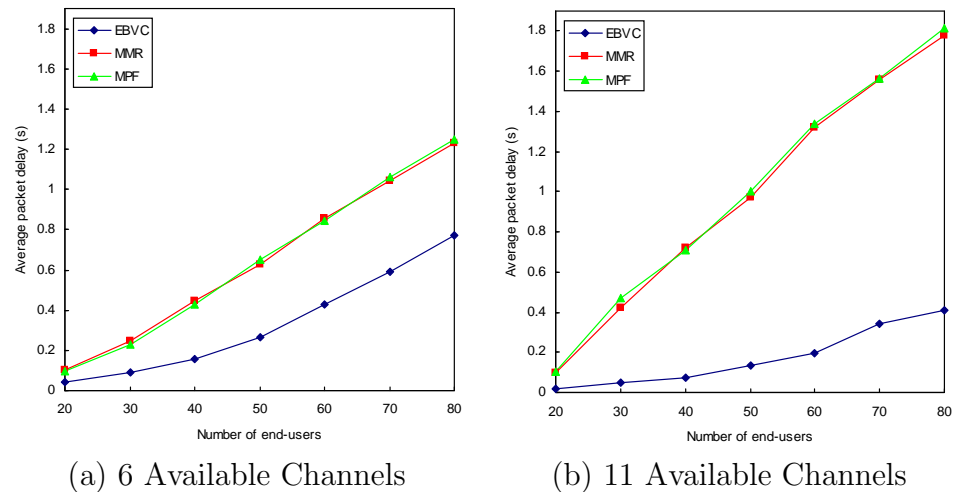


Figure 3.4.3: Comparison of average packet delay for the three algorithms.

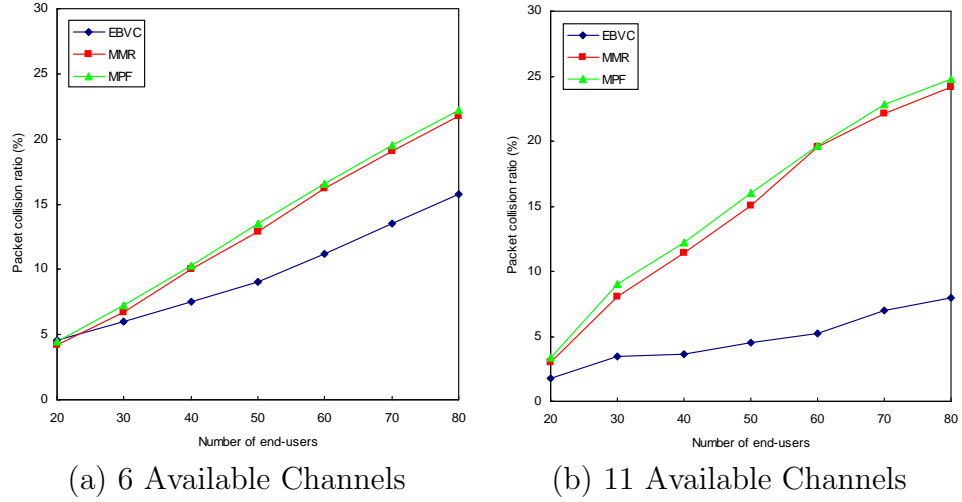


Figure 3.4.4: Comparison of packet collision ratio for the three algorithms.

When the number of available channels was 6 and the number of end-users was 20, according to Figure 3.4.2(a), the three algorithms almost had the same performance. The reason is that each secondary basestation only had a few end-users. The number of available channels assigned to a secondary basestation had little or no effect on the performance. With the increase of the number of end-users, average throughput per flow obtained by the three algorithms declined steadily. But the EBVC algorithm still maintained a larger average throughput per flow than the other two algorithms. As shown in Figure 3.4.2(b), a larger performance gap between the EBVC algorithm and the other two algorithms appears compared with that shown in Figure 3.4.2(a). The reason is that the EBVC algorithm can grant more available channels to secondary basestations with more end-users. Thus, a larger average throughput per flow could be obtained in this case. From Figure 3.4.3 and Figure 3.4.4, it is also obvious that when the number of available channels was large (11 channels), the performance of the EBVC algorithm was far better than that when the number was small (6 channels).

Another interesting finding demonstrated by Figure 3.4.2 and Figure 3.4.3 is that

the performance of the MMF and the PF algorithms in Figure 3.4.2(a) and Figure 3.4.3(a) was better than that in Figure 3.4.2(b) and Figure 3.4.3(b), though in Figure 3.4.2(b) and Figure 3.4.3(b) the MMF and PF algorithms had more available channels to allocate to secondary basestations. The basic reason is that the above two algorithms only used the bandwidth of secondary basestations as the reference factor in channel allocation so that many channels were allocated to secondary basestations with few end-users. Therefore, the secondary basestations with more end-users could not acquire the benefit of more available channels, and on the contrary, they could face a more serious hidden nodes problem with the large number of available channels. It was clearly indicated in Figure 3.4.4 that for the MMF and the PF algorithms there were more packet collisions when the number of available channels was 11. That is, for the MMF and PF algorithms, when the number of end-users is kept the same, the larger the number of available channels, the worse the performance was. On the contrary, the EBVC algorithm always allocated the most channels to the secondary basestations with more end-users. Thus, other basestations would hold fewer channels, which mitigated the hidden nodes problem to some extent.

## 3.5 Summary

We have proposed the EVBC algorithm in spectrum allocation for infrastructure based cognitive radio networks. Our algorithm aims to assign available channels to secondary base stations with more end-users and also maintain a fairness among all secondary base stations as well, which can effectively avoid spectrum wasting and improve the network performance. The simulation results show that our algorithm

can drastically improve network performance compared with MMR algorithm and MPF algorithm, meanwhile greatly decrease the interference cause by exposed nodes problem.



# Chapter 4

## Spectrum Allocation in CRNs under Centralized Architecture

### 4.1 Optimization Models for CRNs under Centralized Architecture

In this section, we develop mathematical models for different objectives of optimization in cognitive radio networks under centralized architecture, respectively. Three optimization objectives are investigated and they are briefly introduced as follows:

- **MNT**: the objective of optimization is maximum network throughput. In other words, the optimization objective maximizes the total spectrum utilization in the network regardless of fairness;
- **MMF**: the objective of optimization is max-min fairness. In other words, the optimization objective maximizes the spectrum utilization of the bottleneck user;
- **PF**: the objective of optimization is proportional fairness. In other words, the optimization objective strikes a compromise between the above two.

Maximum network throughput is an easily understood optimization objective in open spectrum networks. Without considering the fairness, the problem is formulated as follows:

Objective Function (MNT):

$$\max \sum_{i=1}^{|\mathcal{U}|} \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \quad (4.1)$$

Subject to:

$$x_{i,k} + x_{j,k} \leq 1, \text{ if } f_{i,j} = 1, \quad (4.2)$$

$$\forall i, j \in \mathcal{U},$$

$$\forall k \in \mathcal{C}$$

$$x_{i,k} \in \{0, 1\}, \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \quad (4.3)$$

Note that the objective function (4.1) is linear, constraints (4.2) prevent two users from using the same channel if they interfere with each other, and constraints (4.3) guarantee that the decision variables be 1 or 0 so that the formulation is a binary linear programming problem.

In many situations, the goal of optimization not only includes the network throughput maximization, but also a decent bandwidth minimum for every user. Max-min fairness [28] is a fairness criterion that ensures that the user with the smallest throughput gets the priority for channels. In this case, the definition of max-min fairness can be described as:

**Definition 4.1.1.** *A channel allocation scheme  $\bar{a}$  is max-min fair if and only if an increase in bandwidth  $t_i$  of user  $i$  within the domain of feasible allocations must be at the cost of a decrease of some user whose bandwidth is already smaller.*

The bandwidth of user  $i$  is given as follows:

$$t_i = \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k}, i \in \mathcal{U}, k \in \mathcal{C}.$$

That is, for any other feasible allocation of channels  $\bar{b}$ , if there is a user  $i$  with  $t'_i > t_i$ , then there must exist some user  $j$  such that  $t_j \leq t'_i$  and  $t'_j < t_j$ .

The max-min fairness optimization problem is defined as follows:

Objective Function (MMF):

$$\max w \tag{4.4}$$

Subject to:

$$\begin{aligned} x_{i,k} + x_{j,k} &\leq 1, \text{ if } f_{i,j} = 1, \\ &\forall i, j \in \mathcal{U}, \end{aligned} \tag{4.5}$$

$$\forall k \in \mathcal{C}$$

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq w, \forall i \in \mathcal{U} \tag{4.6}$$

$$x_{i,k} \in \{0, 1\}, \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \tag{4.7}$$

Constraints (4.5) avoid the interference between adjacent users. The objective function and constraints (4.6) guarantee that the objective of optimization maximizes the bandwidth of the bottleneck user.

Another very important fairness criterion in channel allocation is proportional fairness (PF) [29–31]. The definition of proportional fairness is given as follows:

**Definition 4.1.2.** Suppose  $t_i$  is the bandwidth of user  $i$  for a feasible channel allocation scheme  $\bar{a}$ , and  $t'_i$  is the bandwidth of the same user for scheme  $\bar{b}$ . The channel

allocation scheme  $\bar{a}$  is proportional fair if and only if for any other feasible allocation scheme  $\bar{b}$ ,

$$\sum_{i=1}^{|\mathcal{N}|} \frac{t'_i - t_i}{t_i} \leq 0.$$

Practically, the function used to reach a proportional fairness condition is [29–31]:

$$\sum_{i=1}^{|\mathcal{N}|} \ln \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k}. \quad (4.8)$$

Based on Eq. (4.8), the PF optimization model is formulated as:

Objective Function (PF):

$$\max \sum_{i=1}^{|\mathcal{N}|} \ln \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \quad (4.9)$$

Subject to:

$$\begin{aligned} x_{i,k} + x_{j,k} &\leq 1, \text{ if } f_{i,j} = 1, \\ &\forall i, j \in \mathcal{U}, \\ &\forall k \in \mathcal{C} \end{aligned} \quad (4.10)$$

$$x_{i,k} \in \{0, 1\}, \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \quad (4.11)$$

As explained above, constraints (4.10) depict the interference relationships and Eq. (4.11) shows the binary constraints. Obviously the objective function (4.9) of this model is nonlinear. Therefore, it is not a linear programming problem, rather a binary nonlinear programming problem. The objective function (4.9) is differentiable and strictly concave. In addition, constraints (4.10) are linear. Therefore, the optimization can in theory be solved by Lagrangian methods [32]. However, in practice a nonlinear

programming problem is harder to solve than a linear programming problem. For large problems, it is unlikely that exact optimal solutions can be determined, or even a solution that is close to the optimal.

## 4.2 An Unified Model and Algorithm Based on Binary Linear Programming

In this section, we present a new optimization objective called fairness constrained maximum throughput (FCMT). A unified binary linear programming (UBLP) model is constructed based on this objective. We show that this model captures the optimization objectives of Section 4.1. The UBLP model is illustrated as follows:

Objective Function (FCMT):

$$\max \sum_{i=1}^{|\mathcal{U}|} \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \quad (4.12)$$

Subject to:

$$\begin{aligned} x_{i,k} + x_{j,k} &\leq 1, \text{ if } f_{i,j} = 1, \\ &\forall i, j \in \mathcal{U}, \\ &\forall k \in \mathcal{C} \end{aligned} \quad (4.13)$$

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq \xi, \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \quad (4.14)$$

$$x_{i,k} \in \{0, 1\}, \forall i \in \mathcal{U}, \forall k \in \mathcal{C} \quad (4.15)$$

In this model, we add additional constraints (4.14) and set  $\xi$  as the bandwidth minimum for each user. In other words, this optimization objective maximizes the

network throughput, given that the bandwidth of each user is larger than or equal to  $\xi$  which is the per-user bandwidth minimum. The objective is formally called *fairness constrained maximum throughput*.

Based on the UBLP model, we propose an iterative algorithm (Algorithm 2) to obtain solutions with different per-user bandwidth minimums. Given a network topology, a graph  $G = (V, E)$ , in terms of users (nodes) and interference relationships (edges) between users, can be constructed. Note that all the connected components [33] of  $G$  should be determined before running the iterative algorithm. The algorithm is run separately on every connected component of  $G$  because the channel allocation for each component is independent of others. At the beginning of the algorithm, set

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**Algorithm 2:** *UBLP-ALG*( $\mathcal{V}, \mathcal{K}$ )

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1:  $G_i$  is the  $i$ th connected component of graph  $G$ 
2: for every component  $G_i$  of  $G$  do
3:    $\xi = 0$ ;
4:   while TRUE do
5:     if (BLP( $G_i$ ) cannot obtain a solution ) then
6:       stop;
7:     else
8:       save the result  $r_\xi = \text{BLP}(G_i)$ ;
9:        $\xi = \xi + 1$ ;
10:    end if;
11:  end while;
12:  if (the objective is MNT) then
13:    return  $r_0$ ;
14:  else
15:    if (the objective is MMF) then
16:      return  $r_{\xi-1}$ ;
17:    else
18:      if (the objective is PF) then
19:        return  $r_j$  with the maximum value of Eq. (4.8),  $j \in \{0 \cdots \xi - 1\}$ ;
20:      end if;
21:    end if;
22:  end if;
23: end for;

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the per-user bandwidth minimum  $\xi = 0$ , and then, call the binary linear programming function  $\text{BLP}(G_i)$  to obtain an optimal solution. The algorithm then repeats the function calling while increasing the value of  $\xi$  by 1 every time until the function  $\text{BLP}(G_i)$  cannot obtain an optimal solution.

Suppose the per-user bandwidth minimum in a connected component  $G_i$  of  $G$  is  $\delta$ , e.g.,  $\xi = \delta$ . The maximum bandwidth of a user is denoted by  $t_{max}$ , and assume that the number of total available channels is  $M$ .

**Lemma 1.** *In each connected component  $G_i$  of  $G$ ,  $t_{max}$  is  $M - \delta$ .*

*Proof.* Because  $\xi = \delta$ , the degree of each node of  $G_i$  is at least 1, and  $b_{i,k} = 1, \forall i \in \mathcal{U}, \forall k \in \mathcal{C}$ , the maximum number of channels that one user can use is  $M - \delta$ , thus,

$$\begin{aligned} t_{max} &= \sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \\ &= M - \delta. \end{aligned}$$

□

**Lemma 2.** *In each connected component  $G_i$  of  $G$ ,  $M \geq 2\delta$ .*

*Proof.* Apparently  $t_{max} \geq \xi$ , since  $\xi = \delta$  and  $t_{max} = M - \delta$ , we have  $M \geq 2\delta$ . □

In connected component  $G_i$  of  $G$ , suppose the optimization objective is PF, let  $v_{UBLP}$  denote the value of Eq. (4.8) obtained from the optimal solution to the UBLP model and let  $v_{PF}$  denote the corresponding value of Eq. (4.8) obtained from the PF model. Denote the difference between the two values by

$$\text{diff} = v_{PF} - v_{UBLP}. \quad (4.16)$$

**Lemma 3.** *In connected component  $G_i$  of  $G$ , if the optimization objective is PF, the lower bound of  $\text{diff}$  (4.16) is 0.*

*Proof.* Because the solution of the PF model has the optimal value, the value of (4.8) obtained from the UBLP model must be equal to or less than that of the PF model. Thus the lower bound of the difference between the above two values is 0.  $\square$

In connected component  $G_i$  of  $G$ , suppose the number of users is  $n_i$  and the per-user bandwidth minimum for the optimal solution to the PF model is  $\delta$ , the following lemma holds.

**Lemma 4.** *In connected component  $G_i$  of  $G$ , the upper bound of the difference (diff (4.16)) between the value ((4.8)) obtained by the UBLP model with  $\xi = \delta$  and that ((4.8)) of the PF model is  $\ln\left(\frac{M-\delta}{\delta}\right)^{n_i-1}$ .*

*Proof.* Because the minimum bandwidth of a user is  $\delta$  for both solutions, and given Lemmas 1 and 2, the maximum value of the two solutions is:

$$\ln(\delta \times (t_{max})^{n_i-1}) \quad (4.17)$$

and the minimum value of the two solutions is:

$$\ln \delta^{n_i}. \quad (4.18)$$

On the one hand, due to the PF model, the optimal value (4.8) obtained from the PF model is definitely larger than or equal to the corresponding value obtained by the UBLP model. On the other hand, again due to the PF model, the value (4.8) obtained by the UBLP model cannot be (4.18) if the value (4.8) by the PF model is (4.17). Therefore, the upper bound of the difference, diff, is:

$$\begin{aligned} \text{diff} &< \ln(\delta(t_{max})^{n_i-1}) - \ln \delta^{n_i} \\ &< \ln\left(\frac{\delta(t_{max})^{n_i-1}}{\delta^{n_i}}\right) \\ &< \ln\left(\frac{(M-\delta)}{\delta}\right)^{n_i-1} \\ &= \text{diff}_{sup}. \end{aligned}$$

$\square$

In summary, we have the following propositions:

**Proposition 1.** *An optimal solution  $\bar{a}$  to the UBLP model with  $\xi = 0$  is also an optimal solution to the MNT model.*



*Proof.* If  $\xi = 0$ , constraint (4.14) of the UBLP model becomes

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq 0, \quad \forall i \in \mathcal{U}, \forall k \in \mathcal{C}.$$

Because  $\forall i \in \mathcal{U}, \forall k \in \mathcal{C}, x_{i,k} \geq 0$  and  $b_{i,k} \geq 0$ , constraint (4.14) become unnecessary. Removing constraint (4.14), the UBLP model is the same as the MNT model. Therefore, the solution  $\bar{a}$  is also an optimal solution to the MNT model.  $\square$

**Proposition 2.** *An optimal solution  $\bar{a}$  to the UBLP model with the maximum value of  $\xi$  is also an optimal solution to the MMF model.*

*Proof.* Set the maximum value of  $\xi$  as  $\delta_{max}$  which is less than or equal to  $t_{max}$ . Suppose the solution  $\bar{a}$  of the UBLP model with  $\xi = \delta_{max}$  is not one of the solutions to the MMF model, which means that there must have a solution  $\bar{b}$  to the MMF model in which the per-user bandwidth minimum is at least  $\delta_{max} + 1$ . However, according to Algorithm 2, function  $\text{BLP}(G_i)$  cannot obtain a solution given the constraints

$$\sum_{k=1}^{|\mathcal{C}|} x_{i,k} \cdot b_{i,k} \geq \delta_{max} + 1, \quad \forall i \in \mathcal{U}, \forall k \in \mathcal{C}.$$

Therefore,  $\bar{b}$  does not exist and  $\bar{a}$  is also an optimal solution to the MMF model.  $\square$

**Proposition 3.** For connected component  $G_i$  of  $G$ ,  $\text{diff}((4.16))$  is bounded by:

$$0 \leq \text{diff} < \ln(M - 1)^{n_i - 1},$$

where  $n_i$  is the number of users in  $G_i$ .

*Proof.* According to Lemma 3, the lower bound is,  $\text{diff} \geq 0$ . Now we prove the upper bound. Assume  $n_i \geq 2$ . The derivative of  $\text{diff}_{sup}$  in Lemma 4 is:

$$\begin{aligned} \text{diff}_{sup}' &= \left( \ln\left(\frac{M - \delta}{\delta}\right)^{n_i - 1} \right)' \\ &= (n_i - 1) \times \left( \frac{\delta}{M - \delta} \right) \times \left( -\frac{M}{\delta^2} \right). \end{aligned}$$

Note that

$$-\frac{M}{\delta^2} < 0,$$

and  $M > \delta$ ,  $\text{diff}_{sup}$  is a monotonically decreasing function of  $\delta$ . Hence when  $\delta = 1$ , the upper bound is:

$$\text{diff} < \ln(M - 1)^{n_i - 1}.$$

### 4.3 Numerical and Simulation Results

To demonstrate the effectiveness of the UBLP model, simulation studies are performed under the assumptions that all users are static. All available channels are also assumed to have the same bandwidth. Two fixed topologies in Figure 4.3.1 are used in the studies. In Figure 4.3.1, a node represents a user and an edge represents the interference relationship between two users. In the two topologies, there are 6 users and the total number of available channels is set to 5.

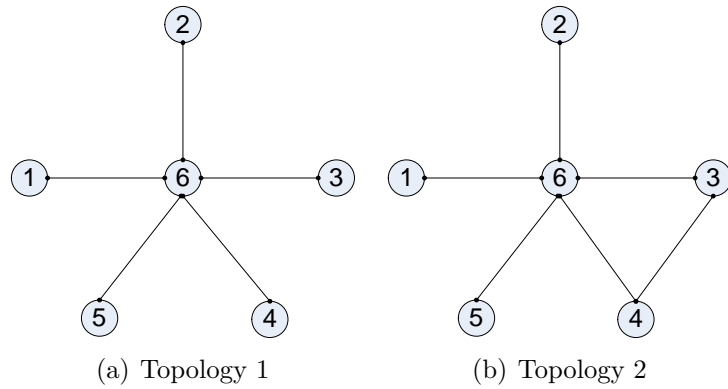


Figure 4.3.1: Two network topologies used in studies.

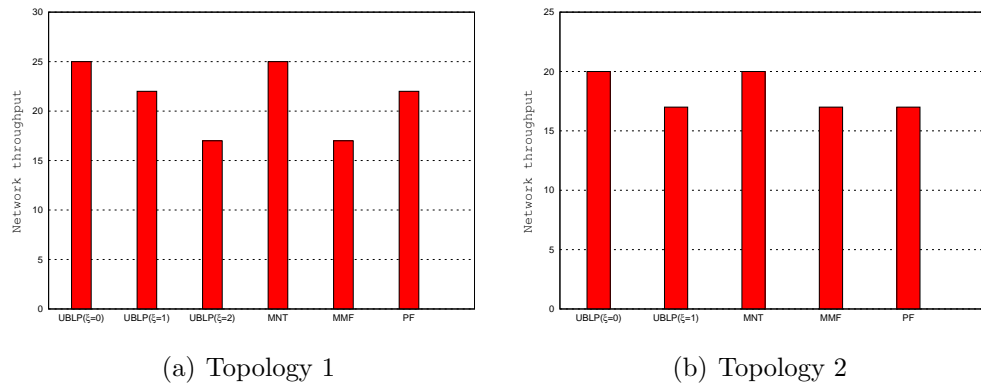


Figure 4.3.2: Network throughput.

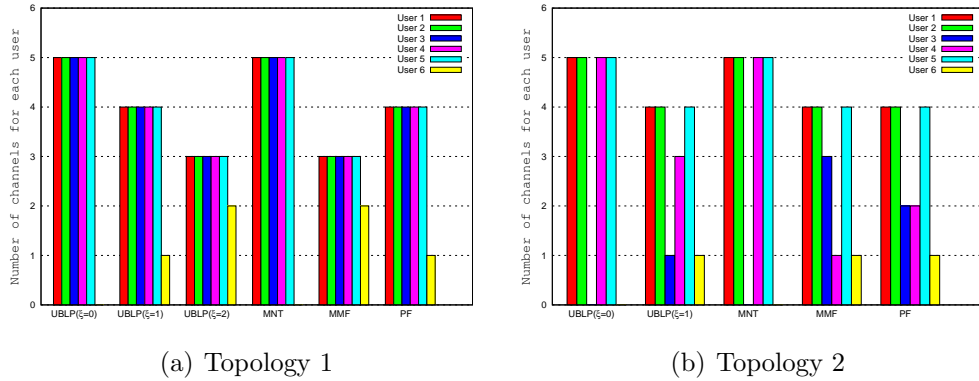


Figure 4.3.3: Number of channels for each user.

For each topology, we run the iterative algorithm (Algorithm 1) with different values of  $\xi$ . In Figure 4.3.2, the network throughput decreases with increasing  $\xi$  in both cases. The total network throughput with  $\xi = 0$  is equal to the optimal value of the MNT model. In Figure 4.3.3(a), the solution to the UBLP model with the maximum  $\xi$  is the same as the solution to the MMF model, and in Figure 4.3.3(b), the solution to the UBLP model with the maximum  $\xi$  satisfies the optimization constraints of the MMF model although it is not entirely the same as the solution to the MMF model.

Figure 4.3.3 also shows an obvious fact that the essence of max-min fairness is to decrease the difference of bandwidth between any two users. In Table 4.1, the values of logarithmic utility function Eq. (4.8) for the two topologies are presented. The value of the logarithmic utility function of the UBLP model for proportional fairness and the PF model are exactly the same in Figure 4.3.1(a). However, in some scenarios, for instance in Figure 4.3.1(b), the value of the logarithmic utility function of the UBLP model for proportional fairness is smaller (or less fair) than the one obtained from the PF model. In this case, the value of the PF model is 5.545 while

Proportional Fairness		
Topology	Model	Value
Topology 1	UBLP( $\xi = 0$ )	$-\infty$
	UBLP( $\xi = 1$ )	6.931
	UBLP( $\xi = 2$ )	6.186
	PF	6.931
Topology 2	UBLP( $\xi = 0$ )	$-\infty$
	UBLP( $\xi = 1$ )	5.257
	PF	5.545

Table 4.1: Comparison of proportional fairness.

the value is 5.257 under the UBLP model.

Just as it has been proved in the previous section, the solution to the UBLP model with the maximum  $\xi$  always satisfies the constraints and is a solution of the MMF model. In addition the UBLP model with  $\xi = 0$  is clearly the MNT model. According to the results shown in Table 4.2, solutions for proportional fairness obtained under the UBLP model are a compromise between the MNT model and the MMF model. However the objective of the UBLP model is to maximize the total network throughput while providing bandwidth satisfaction for each user, whereas the optimization objective of the PF model focuses on improving the bandwidth of the bottleneck user. In this respect the UBLP model and the PF model are different.

## 4.4 Graph Theory based Algorithms

In [34], we showed that when  $\xi = 0$ , the UBLP model returns an optimal solution for MNT. When  $\xi$  is increased gradually by 1 each time until the UBLP model returns no solution, the solution with the maximum  $\xi$  is exactly an optimal solution for MMF. The PF objective strikes a compromise between MNT and MMF. The corresponding solution has the maximum fairness value among all returned solutions with different

$\xi$ . With the increase of problem size, the UBLP model does not scale in terms of computational time. Therefore, finding an effective approximation algorithm becomes the most important focus.

From graph theory perspective, the UBLP model can be mapped into a graph  $G = (V, E)$ , where  $V = \mathcal{U}$  and  $E = \mathcal{F}$ . Here the channel set  $\mathcal{C}$  is the color set, where each channel is a certain color. We introduce some related terms:

- An independent set [35] is a set of vertices in a graph, no two of which are adjacent.
- *Maximal independent set* [35] is an independent set that is not a subset of any other independent set.
- *Maximum independent set* (MIS) [35, 36] is the largest maximal independent set in graph  $G$ .
- *Minimum graph coloring* (MGC) problem is to determine a partition of  $V$ ,  $V_0, V_1, \dots, V_k$ , such that each  $V_i$  is an independent set for  $G$  and no other partition in which the number of disjoint independent sets is less than  $k$  exists.

## 4.5 Graph Theory Perspective on UBLP model

Before presenting our approximation algorithm based on MIS and MGC, two lemmas are presented as follows.

**Lemma 5.** *The UBLP model based optimization problem when  $\xi = 0$  is equivalent to a problem of determining the maximum independent set in graph  $G$ .*

*Proof.* Given the assumption that  $b_{i,k} = b_{j,k}, \forall i, j \in \mathcal{U}, \forall k \in \mathcal{C}$ , the objective function in the UBLP model can be simplified as  $\max \sum_{i=1}^{|\mathcal{U}|} \sum_{k=1}^{|\mathcal{C}|} x_{i,k}$ . Regardless of Constraint (4.14), it is straightforward that the simplified objective function means, for each color, we only need to allocate it to a maximal independent set of  $V$ . On the other hand, according to the definition of MIS, there cannot exist another maximal independent set of vertices with a larger cardinality. Therefore, allocating every color to all vertices in MIS is the optimal solution to the UBLP model with the simplified objective.  $\square$

We denote the maximum independent set of graph  $G$  by  $V_{MIS}$ . If  $\xi$  is positive, we have Lemma (6).

**Lemma 6.** *For any channel allocation solution that satisfies Constraint (4.14) with any positive  $\xi \neq 0$ , the corresponding value of the objective function (4.2) is always less than or equal to that obtained when  $\xi$  is 0.*

*Proof.* For any allocation solution and a color  $c_m$ , where  $m = \{1, \dots, M\}$ ,  $V_{c_m}$  denotes the independent set of vertices that have color  $c_m$ . It is obvious that the cardinality of  $V_{c_m}$  is always less than or equal to the cardinality of  $V_{MIS}$ . While the value of objective function (4.2) is  $\sum_{m=0}^M |V_{c_m}|$ , Lemma (6) is hence proved.  $\square$

When  $\xi = 0$ , according to Lemma 5, each  $V_{c_m}$  is a maximum independent set. When  $\xi > 0$ , we need to use a certain number of colors (maybe all) to satisfy Constraint (4.14). From the proof of Lemma 6, for each color  $c_m$ , the cardinality of each  $V_{c_m}$  is less than or equal to that of  $V_{MIS}$ . Therefore, it is natural that we must use the least number of colors to satisfy Constraint (4.14) and allocate the other colors to the maximum independent set  $V_{MIS}$  of  $G$ , which is related to the minimum graph coloring problem. Suppose we have the optimal solutions to the MIS problem and the MGC problem for graph  $G$ , our algorithm that solves the UBLP model based optimization problem is given as follows:

---

**Algorithm 3:** *Channel-Allocation-ALG*( $P, \mathcal{C}$ )

---

- 1:  $P$ : the partition with the minimum cardinality  $K$  of the graph coloring problem. That is, the disjointed sets  $V_1, V_2, \dots, V_K$ ;
  - 2:  $V_{MIS}$ : the maximum independent set of  $G$ ;
  - 3:  $\mathcal{C} = \{c_1, \dots, c_M\}$ : the set of colors (channels);
  - 4: **for** every  $V_i \in P, i \in \{1, \dots, K\}$  **do**
  - 5:     allocate  $c_{(i-1)*\xi+1}, \dots, c_{(i-1)*\xi+\xi}$  to every vertex in  $V_i$ ;
  - 6: **end for**
  - 7: allocate  $c_{K*\xi+1}, \dots, c_M$  to every vertex in  $V_{MIS}$ ;
  - 8: **for** every  $V_i, i \in \{1, \dots, K\}$  **do**
  - 9:     **for** every vertex  $v$  in  $V_i$  **do**
  - 10:         **if**  $v$  is not joined to any vertex in  $V_j, j \in 1, \dots, K$  and  $j \neq i$  **then**
  - 11:             allocate  $c_{(j-1)*\xi+1}, \dots, c_{(j-1)*\xi+\xi}$  to  $v$ ;
  - 12:         **end if**
  - 13:     **end for**
  - 14: **end for**
- 

The for-loop in lines 4 – 6 allocates different  $\xi$  colors to each disjointed sets  $V_1, V_2, \dots, V_K$ . After that, in line 7 we allocate the remaining colors to the maximum independent set  $V_{MIS}$ . It is possible that there exists a vertex which is not joined to any vertex in more than one disjointed independent set. Thus we use the for-loop in lines 8 – 14 to check every vertex in set  $V_i, i \in 1, \dots, K$ , if find a vertex that is not joined to any vertex in another set  $V_j, j \in 1, \dots, K$  and  $j \neq i$ , then allocate the  $\xi$  colors belonging to  $V_j$  to this vertex. Since  $\sum_{k=1}^K |V_k| = |V|$ , it is clear that

$$\sum_{m=1}^M |V_{c_m}| \geq |V|.$$

We have Theorem 4.5.0.1 for Algorithm3 as follows.

**Theorem 4.5.0.1.** *For a graph  $G$ , let  $M$  denote the number of available channels and  $K$  denote the cardinality of MGC. Algorithm3 is an approximation algorithm for the UBLP model based optimization problem, and the approximation ratio is  $\frac{\xi}{M}$ .*

*Proof.* Let  $A(G)$  denote the solution of our algorithm and  $OPT(G)$  represent the optimal solution, the approximation ratio is  $\frac{A(G)}{OPT(G)}$ . As we know,  $OPT(G) \leq V_{MIS} \times M$ , and  $A(G) = \xi \times \sum_{m=1}^M |V_{c_m}| + V_{MIS} \times (M - K \times \xi)$ . Therefore, the approximation ratio is

$$\begin{aligned}
\frac{A(G)}{OPT(G)} &> \frac{\xi \times \sum_{m=1}^M |V_{c_m}| + V_{MIS} \times (M - K \times \xi)}{V_{MIS} \times M} \\
&> \frac{\xi \times \sum_{m=1}^M |V_{c_m}|}{V_{MIS} \times M} \\
&> \frac{\xi \times |V|}{V_{MIS} \times M} \\
&> \frac{\xi}{M}.
\end{aligned}$$

□

An example is shown in Figure 4.5.4. The partition of MGC is  $V_1, V_2, V_3$ . Therefore  $K = 3$  and the cardinality of  $V_{MIS}$  is 3.

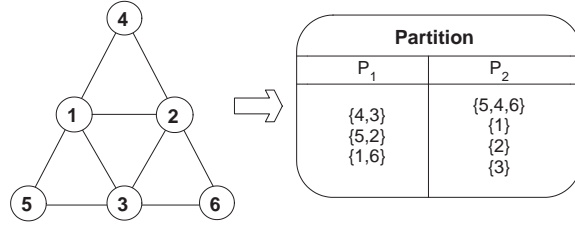


Figure 4.5.4: An example of finding MIS and MGC.

Both the MIS and MGC problems are  $\mathcal{NP}$ -complete. Moreover, we notice that current algorithms that solve the MGC problem actually rely heavily on algorithms that solve the MIS problem. In [37], Johnson presented an MGC algorithm based on the minimum degree greedy (*GMIN*) algorithm and obtained the performance guarantee  $O(n/\log n)$ . In [38], Wigderson proposed an MGC algorithm based on the maximum degree greedy (*GMAX*) algorithm and obtained the performance guarantee  $O(n(\log \log n/\log n)^2)$ . A new graph coloring algorithm [39, 40] was proposed by Boppana and Halldórsson based on a clique removal (*CR*) algorithm, resulting in



the performance guarantee  $O(n/(\log n)^2)$ . The motivation of these algorithms is the same, that is, finding a maximal independent set in the graph. Hence a better MIS algorithm becomes the primary concern. In the next section, a genetic algorithm [41] based approach for solving the MIS problem is presented.

## 4.6 Genetic Algorithm Based MIS Seeking Approach

Each chromosome has a length equal to  $|V|$ , the total number of vertices in the graph. Note that each chromosome represents a maximal independent set of the graph which can be represented by a binary vector  $\mathcal{S} = \{S_0, S_1, \dots, S_{N-1}\}$  where  $S_i = 1$  if node  $i$  belongs to MIS and 0 otherwise.

### 4.6.1 Fitness Function

The fitness function is quite simple and is equal to the cardinality of the maximal independent set. The larger the maximal independent set, the better the fitness is.

### 4.6.2 Population Initialization

To create a solution, we first put all nodes into a candidate set, then we randomly choose an undecided node as an element of a maximal independent set, and remove this node and its neighboring nodes from the candidate set. We continue this procedure until the candidate set is empty. The created solution will be put into the initial population if it is different from those solutions which have already existed in the initial population. In this way, we guarantee the diversity of the initial population.

### 4.6.3 Selection

A selection or reproduction method is used to select good solutions in a population and to form the mating pool. Two of the most commonly used selection methods are *Roulette Wheel* and *Tournament selection* [41]. In this paper, we use *Tournament selection* as our selection method. Every time we randomly choose %1 solutions from the entire population and choose the one with the maximum fitness as the winner to reproduce the offspring. Moreover, we put the top %5 best solutions into the next generation automatically, which guarantees that the current best solutions do not disappear during the reproduction procedure.

### 4.6.4 Crossover

In crossover, new chromosomes are created by exchanging information among a pair of solutions called parents from the mating pool. Here we use two point crossover. A crossing site among the parents are chosen at random and all the bits of the parent chromosomes on the right side of the crossing site are exchanged.

### 4.6.5 Mutation

The mutation operator changes a bit from 1 to 0 and vice versa with the mutation probability  $P_m$ . Bitwise mutation is done by flipping a coin with probability  $P_m$  for each bit i.e., in simulation a random number between 0 and 1 is generated and if it is less than  $P_m$  then alteration at that bit is made and is repeated for every bit.

### 4.6.6 Remove Infeasibility

After the selection, crossover and mutation, a new chromosome may not be a feasible solution because there may have two nodes  $i$  and  $j$ ,  $b_i = b_j = 1$  where  $e_{i,j} = 1$ . To remove the infeasibility, we iteratively execute a checking procedure until the chromosome is a feasible solution.

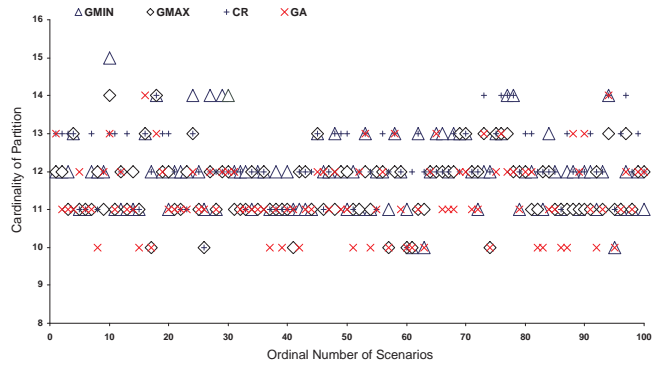
At the beginning of the evolution, the average fitness value of the initial population is very low, which reflects the diversity of all solutions. As the fitness value of the best solution increases, solutions (chromosomes) in the population become more and more similar as a result of the principle of natural selection. Therefore, the average fitness value of all solutions is close to the maximum fitness value at the end of evolution.

## 4.7 Numerical and Simulation Results

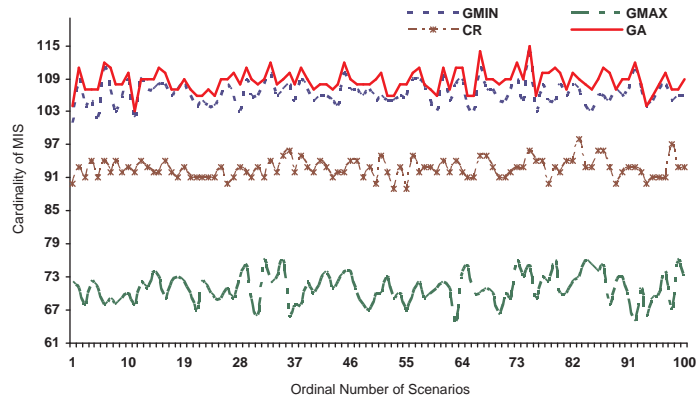
To verify the performance of different MIS algorithms, we design the following simulation. In this simulation, 500 vertices are randomly distributed on a  $1000 \times 1000$  area. In our genetic algorithm based approach, the population size and the maximum generation size are set to be proportional to the size of graph. The crossover probability and the mutation probability are set to 0.7 and 0.02, respectively. We also simulate *GMIN*, *GMAX* and *CR* for comparison. In order to test the performance of these algorithms in different situations, we set the interference distance for each pair of vertices to two values, which would produce two types of graphs, the dense graphs and sparse graphs. For each type of graphs, we randomly create 100 different topologies respectively. The results of dense graphs are shown in Figure 4.7.5. It is clear from

Figure 4.7.5(a) that the partition of  $GA$  has least cardinality in most of the scenarios. In Figure 4.7.5(b), the independent set acquired by  $GA$  always has the largest cardinality, and only the result of  $GMIN$  may be close to it. The independent set acquired by  $GMAX$  always has the least cardinality. The results of sparse graphs are shown in Figure 4.7.6. From Figure 4.7.6(a), the partition results of the four algorithms are quite similar. Our genetic algorithm based approach still obtains the best or decent results compared with the results of the other three algorithms though  $GMAX$  also performs well. When comparing the cardinality of MIS, from Figure 4.7.6(b), we can draw similar conclusion in dense graphs, that is,  $GA > GMIN > CR > GMAX$ . In summary, based on Figure 4.7.5 and Figure 4.7.6,  $GA$  always returns the largest maximal independent set and returns the partition with the smallest cardinality in most of general graphs.

Table 4.2 shows the cardinality statistics of partitions, such as mean and standard deviation. Table 4.3 depicts the cardinality statistics of MIS. For dense graphs, the average cardinality of partitions obtained by  $GA$  is 11.33, which is the best among the four algorithms. The second best is 11.61, which is obtained by  $GMAX$ . This is quite interesting because  $GMAX$ , according to simulation results, performs the worst in terms of the cardinality of MIS obtained. In the next section a more detailed discussion is presented. The standard deviations of the four algorithms are around 0.9 to 1, which provides the confidence of our conclusion. In the case of cardinality of MIS, the result of the  $GA$  approach is the best with an average of 108.63, while the means of  $GMIN$ ,  $CR$ , and  $GMAX$  are 106.38, 92.61 and 70.91, respectively. The standard deviation of the  $GA$  approach is 2.02835954, which is only slightly larger than that

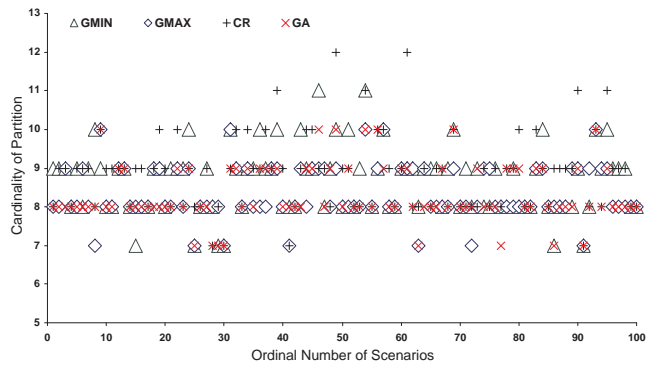


(a) Cardinality of Partitions

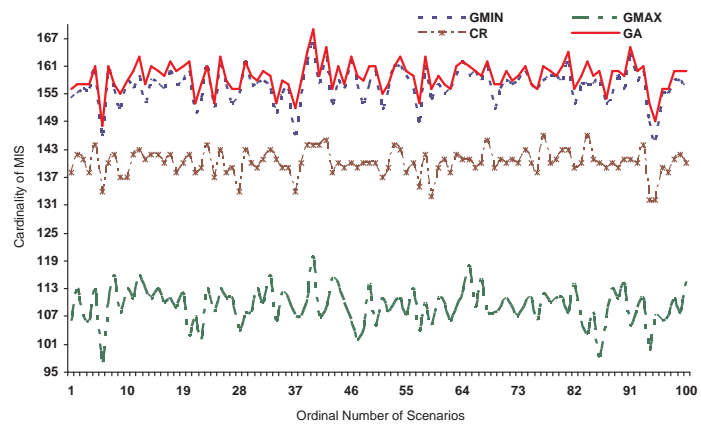


(b) Cardinality of MIS

Figure 4.7.5: Results of algorithm comparison in dense graphs.



(a) Cardinality of Partitions



(b) Cardinality of MIS

Figure 4.7.6: Results of algorithm comparison in sparse graphs.

of  $CR$ , whereas the mean of  $CR$  is much smaller than that of  $GA$ . Therefore, the  $GA$  approach performs much better. For sparse graphs, in the case of the cardinality of partitions, the four algorithms are quite close to each other. However it is still clear that  $GA$  and  $GMAX$  perform better. In the case of cardinality of MIS,  $GA$  and  $GMIN$  outperform others. Broadly speaking, the  $GA$  based approach results in the best performance in most of the cases.

Cardinality Statistics of Partitions			
Graph	Algorithm	Mean	Standard Deviation
Dense Graphs	GMIN	11.96	1.004233463
	GMAX	11.61	0.863338598
	CR	12.21	0.890976085
	GA	11.33	0.964574539
Sparse Graph	GMIN	8.67	0.853454063
	GMAX	8.35	0.687184271
	CR	8.86	0.984937059
	GA	8.33	0.725509226

Table 4.2: Statistics of graph partitions.

Cardinality Statistics of MIS			
Graph	Algorithm	Mean	Standard Deviation
Dense Graphs	GMIN	106.38	2.200918082
	GMAX	70.91	2.697155254
	CR	92.61	1.704983782
	GA	108.63	2.02835954
Sparse Graph	GMIN	156.77	3.695082728
	GMAX	109.35	3.99336571
	CR	140.15	2.768418576
	GA	158.81	3.308070938

Table 4.3: Statistics of MIS.

## 4.8 Integrated Framework

From the simulation results presented in Section 4.7, we find that even though all the algorithms for the MGC problem are based on the algorithms that solve the MIS

problem, it is not necessarily true that an MIS algorithm with better performance would produce a better solution for the MGC problem. A simple example is shown in Figure 4.5.4. The cardinality of the maximum independent set is 3, and the corresponding number of graph partitions is 4. The cardinality of the minimum graph coloring problem is 3 and each maximal independent set has 2 vertices. This example clearly shows that a better MIS algorithm does not necessarily result in a better solution for the MGC problem.

Therefore we present an integrated framework (Figure ??) in which the results of *GMIN*, *GMAX*, *CR*, and *GA* algorithms are compared, and only the best results for the MGC and MIS problems would be used to obtain the solution to the UBLP model. In Figure 4.5.4, the integrated framework returns partitions with cardinality 3 and the maximum independent sets  $V_{MIS} = 4, 5, 6$ , which avoids the weakness of using a single algorithm.

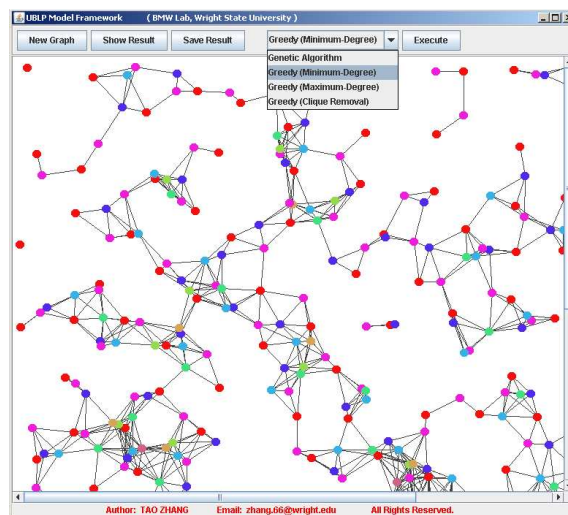


Figure 4.8.7: An integrated framework for spectrum allocation.

Then we change Algorithm 3 a bit to Algorithm 4 for the integrated framework.



---

**Algorithm 4:** *Modified-Channel-Allocation-ALG*( $\mathcal{C}$ )

---

```
1: for every  $alg \in \{GMIN, GMAX, CR, GA\}$  do
2:   run MGC-ALG( $\mathcal{V}, \mathcal{E}, alg()$ )
3:   save the corresponding result of the partition and maximal independent set
4: end for
5: choose the partition with minimum cardinality  $K$  as  $P_{min}$ 
6: choose the of independent set with maximum cardinality and name it as  $V_{max}$ 
7:  $\mathcal{C} = \{c_0, \dots, c_M\}$  is the set of colors(channels)
8: for every  $V_i \in P_{min}, i \in \{0, \dots, K - 1\}$  do
9:   allocate  $c_{i*\xi+1}, \dots, c_{i+\xi}$  to every vertex in  $V_i$ 
10: end for
11: allocate  $c_{K*\xi+1}, \dots, c_M$  to every vertex in  $V_{max}$ 
12: for every vertex  $v$  in  $V_{max}$  do
13:   for every  $V_i, i \in \{0, \dots, K - 1\}$  do
14:     if  $v$  has no connection with any vertex in  $V_i$  then
15:       allocate  $c_{i*\xi+1}, \dots, c_{i+\xi}$  to  $v$ 
16:     end if
17:   end for
18: end for
```

---

## 4.9 Summary

Channel allocation in CRNs is essential in ensuring a high network throughput and user fairness. In this Chapter, we study the optimization of spectrum allocation, considering multiple objectives. For each objective, a binary programming model is described. Then we propose a new optimization objective called fairness constrained maximum throughput. To achieve this optimization objective, a unified binary linear programming (UBLP) model is constructed which is then solved by the simplex method and branch-and-bound search. The solution to this model satisfies a bandwidth requirement for each user, e.g., the bandwidth for each user is equal to or larger than a per-user bandwidth minimum, and the solution also maximizes the network throughput. We prove that given different per-user bandwidth minimum, the optimal solution to the UBLP model achieves specific optimization objectives, such as

the maximum network throughput and the max-min fairness. For the proportional fairness objective, the solution to the UBLP model proves to be within a bound of the optimal solution. After that, we study the optimization problem from the graph theory perspective. We show that the UBLP model based optimization problem can be divided into two  $\mathcal{NP}$ -complete problems, the maximum independent set problem, and the minimum graph coloring problem. After analyzing the close relationship between these two problems, we present a channel allocation algorithm based on solving MIS and MGC. We also propose a genetic algorithm based approach to find a better solution of MIS. The simulation results show that our genetic algorithm based approach outperforms *GMIN*, *GMAX*, and *CR*. As for the MGC problem, our approach appears to be the best in most cases. Simulation results also indicate that the best algorithm for the MIS problem does not necessarily result in the best solution to the MGC problem. To obtain the best solution for channel allocation, we present an integrated framework which combines the four algorithms and always returns the best solution. Furthermore, it is also easy to integrate new algorithms into our framework. Our future work will look into the optimization of channel allocation that considers additional network dynamics, such as node mobility.

# Chapter 5

## Spectrum Availability Prediction

### 5.1 Introduction

As illustrated in Fig 5.1.1, white spaces denote frequencies allocated to a primary user but not used locally. In cognitive radio networks, by using white spaces, secondary users are capable of communicating with intended receivers, and the utilization of spectrum can be improved significantly.

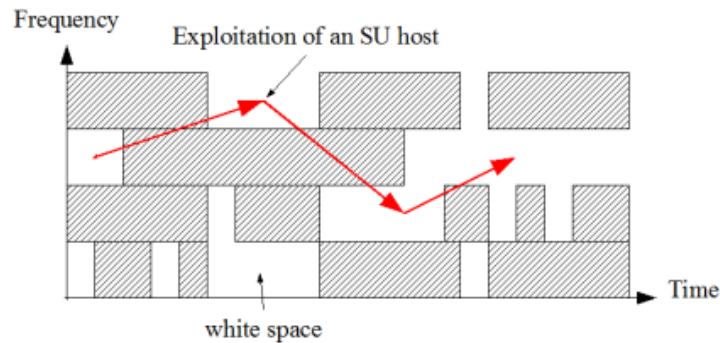


Figure 5.1.1: White spaces (spectrum hole) in cognitive radio networks.

However, since the spectrum usage changes over time and space, secondary users have to keep track of the changes of the radio environment, such as primary users' spectrum activities [6, 7]. For secondary users efficient utilization of spectrum heavily depends on the spectrum availability. Therefore, the prediction on spectrum avail-

ability is beneficial to spectrum allocation in CRNs. To avoid the interference to primary users, secondary users have to sense the nearby spectrum usage of primary users. On the other hand, secondary users may predict the availability of a specific spectrum channel based on the past sensing information. Thus, an effective spectrum availability prediction algorithm which can help secondary users to improve spectrum utilization and minimize the interference to primary users becomes important. Statistical methods are widely used in spectrum occupancy and spectrum availability prediction areas. For instance, to obtain a novel spectrum occupancy model, a statistical spectrum occupancy model [16] based on a combination of several different probability density functions was designed to generate accurate temporal and frequency behavior of various wireless transmissions.

Numerous spectrum availability prediction approaches have been proposed and studied. They are mostly based on statistics of the previous spectrum usage of primary users. In [17], the use of binary time series for spectrum occupancy characterization and prediction was proposed. The performance of the predictor suffered due to the non-deterministic nature of the binary series. In [42], an autoregressive model using Kalman filter was used to predict the status of the licensed channel. However, this model requires knowledge of the primary user's traffic characteristics which may not be known.

In [43], the authors validated that the Beta distribution is a good fit to the spectrum occupancy patterns in the 1500 MHz band observed inside a modern office building at Aachen, Germany. Therefore, they presented a Poisson-normal approximation method for spectrum availability modeling, and the classification of available

channels based on occupancy of its adjacent channels and proposition of analytical models of such available channels. The contributions of their work are based on the accuracy of the probability of availability for each channel.

Markov models and hidden Markov models (HMM) are also commonly used in spectrum availability prediction. In [18], the existence of Markov chain for channel utilization by primary users was validated using real-time measurements collected in the paging band (928 – 948 MHz). Moreover, the sensing errors were probabilistically modeled and then a Hidden Markov prediction model was presented. In [19], another hidden Markov model based spectrum prediction was proposed. In [20] the latency between spectrum sensing and data transmission was studied, and an approach for prediction of channel state based on spectrum sensing slots using higher-order HMM was proposed. However, to our best knowledge, not all frequency channels were validated to fit the property of Markov chain and hidden Markov chain. In addition, in Markov chain model, different frequency channels have different transition matrix and initial probabilities. These parameters need to be stored for efficient Markov chain parameter estimation of channel availability, whereas most of the secondary users (devices) only have limited memory. By the same token, the Hidden Markov prediction model requires more memory space to predict the channel availability. Even though the memory constraints suffice, to guarantee the accuracy of the prediction of channel availability, a relatively long time is still needed to obtain the transition matrix and other parameters based on the statistical data from the previous sensing results. A neural network approach using the multi-layer perceptron network to predict the channel availability was proposed in [44]. The approach is essentially based on the

previous sensing data set as well, which means a long time sensing phase is needed to guarantee a decent channel availability prediction. If the previous sensing data set are not large enough, the limited previous sensing investigation may mislead the prediction to some extent.

On the other hand, from the Bayesian point of view, inferences are always conditioned on prior information. Bayesian analysis is a widely used method of statistical inference applied to many real-world problems. In order to overcome the deficiency of [18, 19, 44] caused by limited sensing data set, we will present a prediction approach based on Bayesian estimation. For each channel  $i$ , its availability can be denoted by a random variable  $X \in (0, 1)$ . Given a statistic model, Bayesian estimation provides estimates for the model's parameters. Note that every spectrum channel will be assigned a prior probability for channels' availability. With the advent of upcoming sensing data, our approach can continually modify it to obtain the posterior distribution, and improve our prediction as a result.

In the following part of this chapter, we compare our prediction approach with Markov chain model based and maximum likelihood estimation based prediction approaches, which will be introduced in Section 5.2 and Section 5.3. In Section 5.4, we will describe Bayesian estimation in detail.

## 5.2 Markov Chain Model Based Estimation

Markov chain is a classical concept in stochastic process. A Markov chain is a sequence of random variables  $X_1, X_2, X_3, \dots, X_n$  with the Markov property. That is, given the present state  $X_i$ , the future state  $X_{i+1}$  only depends on  $X_i$ , and independent from

past states  $X_1, X_2, X_3, \dots, X_i$ . Formally,

$$P(X_{n+1} = x | X_1 = x_1, X_2 = x_2, X_3 = x_3, \dots, X_n = x_n) = P(X_{n+1} = x | X_n = x_n) \quad (5.1)$$

A Markov chain with memory  $m$  (or a Markov chain with order  $m$ ), is a process defined as:

$$\begin{aligned} P(X_n = x_n | X_1 = x_1, X_2 = x_2, X_3 = x_3, \dots, X_{n-1} = x_{n-1}) = \\ P(X_n = x_n | X_{n-m} = x_{n-m}, \dots, X_{n-2} = x_{n-2}, X_{n-1} = x_{n-1}) \end{aligned} \quad (5.2)$$

where  $m < n$ . Briefly speaking, a Markov chain with memory  $m$  is a process in which the future state depends on the past  $m$  states.

Under the circumstance of spectrum sensing, we denote the channel availability in  $n$  sequential time slots as  $X_1, X_2, \dots, X_n$  which are random variables. It is apparent that we can use a Markov chain model to describe spectrum availability prediction problem, in which the spectrum availability in the current time slot  $X_i$  depends on the past  $m$  states. The choice of order  $m$  is subjective. In this dissertation, for comparison purpose, we set  $m$  as 1, 2 and 3 respectively.

### 5.3 Maximum Likelihood Estimation

Maximum likelihood estimation (MLE) is an approach of estimating the parameters of a statistical model. In the situation of channel availability prediction, let  $X$  be a random variable with:

$$X = \begin{cases} 1 & \text{channel is in idle state,} \\ 0 & \text{otherwise.} \end{cases}$$

We set probability  $p(x = 1) = \theta$  and  $p(x = 0) = 1 - \theta$ . Apparently, it is a

Bernoulli distribution. Then, we assume in  $n$  sequential time slots,  $X_1, X_2, \dots, X_n$  are independent, identically distributed (i.i.d.) random variables, and they are all Bernoulli distributed with probability  $\theta$  to be idle. So the joint density function is:

$$p(x_1, x_2, \dots, x_n | \theta) = p(x_1 | \theta) \cdot p(x_2 | \theta) \cdots p(x_n | \theta)$$

The maximum likelihood estimate of  $\theta$  is the value that maximizes the likelihood  $p(x_1, x_2, \dots, x_n | \theta)$ :

$$\hat{\theta} = \arg \max \prod_{k=1}^{k=n} p(x_k | \theta)$$

This corresponds to the intuitive idea of choosing the value of  $\theta$  that is most likely given the past observed data. Considering the spectrum availability prediction, the MLE approach calculates  $\theta$  ( as the probability of idle state ) based on the sensing results in the past time slots, and then choose the channel with biggest  $\theta$ .

## 5.4 Bayesian Estimation

The classical or frequentist's view of probability is defined as the limiting frequency of occurrence of this event in an infinite number of trials [45]. For example, the probability of heads in a single coin toss is the proportion of heads in an infinite number of coin tosses. However, from Bayesian perspective, probability is related to a quantification of uncertainty, or a short definition: degree of belief [45, 46]. Bayesian view on probabilities is based on Bayes' theorem which is described in Section 5.4.1.

### 5.4.1 Bayes' Theorem

For observable scientific data, scientific hypotheses are expressed through probability distributions. The unknown quantities on which these probability distributions



depend are called parameters. In the Bayesian perspective, current knowledge about the parameters is expressed by placing a probability distribution on the parameters, which is called the *prior distribution*. When new data become available, the information they contain regarding the model parameters is expressed in the likelihood which is proportional to the distribution of the observed data given the model parameters. Then we combine the new information with the prior to produce an updated probability distribution called the *posterior distribution*. Bayes' theorem [46] provides a perfect method to describe the relation between the new information and updating posterior distribution.

Let  $H$  be any hypothesis and  $E$  be the observed data. Conditional probability implies that  $P(H|E)$  must rationally exist in a fixed relationship with beliefs known before the evidence is observed. Bayes' theorem, which takes the form

$$P(H|E) = \frac{P(E|H)}{P(E)} \times P(H) \tag{5.3}$$

providing a quantitative measure to evaluate the uncertainty in  $H$  based on the observed data  $E$  in the form of posterior probability  $P(H|E)$ . The quantity  $P(E|H)$  on the right-hand side (5.3) is evaluated for the observed data set  $E$  and can be viewed as a function of the parameter  $H$ , in which case it is called the *likelihood* function. Likelihood function represents the probability that some data is produced under the assumption of this certain assumption  $H$ .

According to the above definition of likelihood, in [45, 46] Bayes' theorem can be stated mathematically as: the posterior is proportional to the prior times the

likelihood, or more precisely,

$$posterior \propto likelihood \times prior \quad (5.4)$$

### 5.4.2 Differences Between Bayesians and Frequentists

From Bayesian's point of view, a probability is a measure of the degree of belief in an event, given the current information available. Thus, probabilities refer to a state of knowledge held by an individual.

Frequentists consider probability as a long-run frequency of a repeatable event and presented a notion of confidence intervals. From repeated experiments, probability could be a measurable frequency of events.

In Bayesians' view:

- Probability is a measure of uncertainty
- Data are observed from the realized samples.
- Parameters are unknown and described probabilistically.

On the contrary, in frequentists view:

- Probability is a long-term frequency statement about the data.
- Underlying parameters remain constant during this repeatable process.

In general, frequentist approaches assume a parameter has one particular value, and try to express uncertainty in people's knowledge after an experiment with a *confidence interval*, which shows the range of values of the parameter with at least

some minimum probability. Whereas from Bayesian Point of View, the value of parameter is not fixed, but chosen from some probability distribution, which is known as the prior probability distribution. It is obvious that maximum likelihood estimation in Section 5.3 is a frequentist approach.

## 5.5 Spectrum Availability Prediction Based on Bayesian Estimation

The spectrum availability is the deciding factor in dynamic spectrum access. However, the availability of spectrum channels is usually unknown and varies temporally and geographically. A bad prediction can lead to low spectrum utilization for secondary users. In this section we present a Bayesian estimation based prediction approach, which predicts the spectrum availability based on prior information and current observations. To evaluate the performance of the Bayesian estimation based prediction approach, a simulation is conducted in Section 5.6.

### 5.5.1 Prior Information in Spectrum Availability Prediction

In essence, spectrum availability is a statistical process. The prediction of spectrum availability is based on the previous investigation. In Section 5.3 a random variable  $X$  is defined as:

$$X = \begin{cases} 1 & \text{channel is in idle state,} \\ 0 & \text{otherwise.} \end{cases}$$

Theoretically,  $p(x = 1) = \theta$  is the limit of its relative frequency in a large number of trials. However, in the real world we cannot get the infinite trials. Moreover, in most cases, secondary users would move from one place to another, which implies that

there only exists a small sample data set. However the traditional relative frequency based predictions need a long run trials to collect sample, and usually in the beginning stage the prediction result is misleading due to the limited sample data.

On the other hand, we already have useful spectrum usage information from the authorities or specific organizations. For instance, Fig 5.5.2 illustrates the spectrum usage in the TV bands. The number of available channels and spectrum will necessarily vary by geographic location and may change over time.

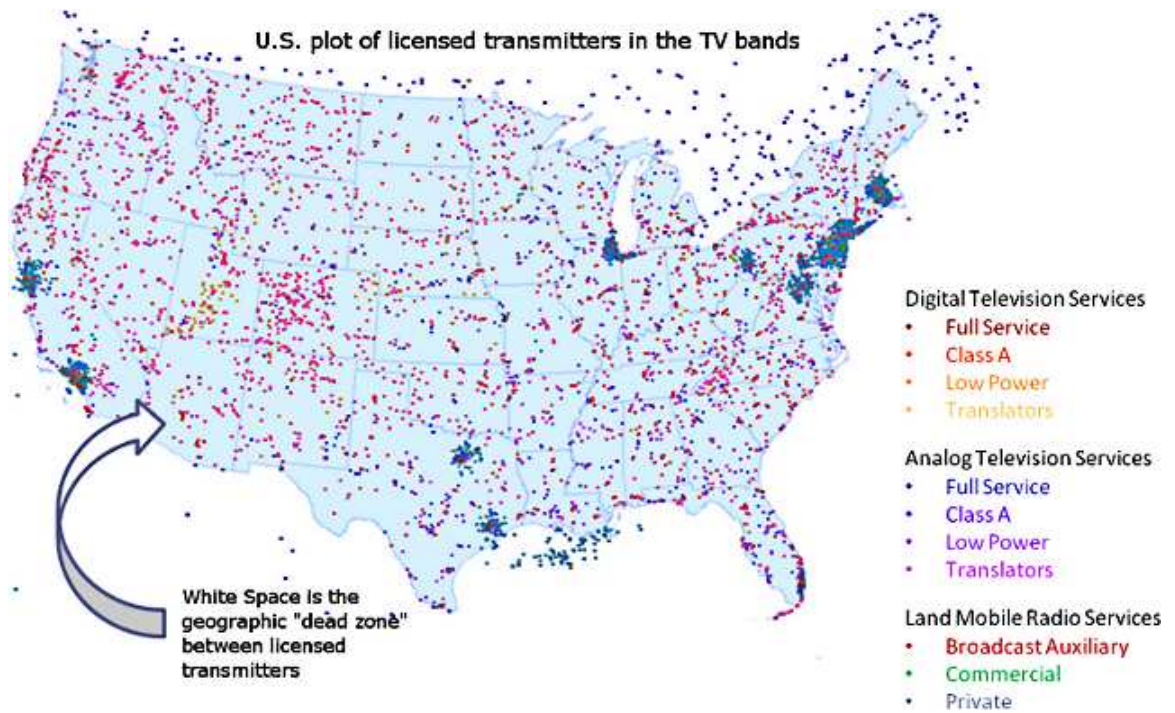


Figure 5.5.2: Spectrum Usage in the TV Bands.

In the Bayesian estimation approach, the uncertainty on the parameters  $\theta$  is represented by a probability density function. Before we observe the data, the parameters are described by a prior density  $\theta$ , which comes from the existing spectrum usage statistic information. Once we obtain observation data, we make use of Bayes' Theorem to find the posterior.

## 5.5.2 Prediction Model

The Bayesian approach is to treat all unknown parameters as random variables and assign a prior probability distribution to each. Suppose  $p(\theta)$  is the prior probability distribution for unknown parameter  $\theta$ . We define the observed data  $y$ . In Section 5.4, Bayes' theorem is described as:

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

Here we replace  $E$  with observations  $y$ ,  $H$  with belief  $\theta$ , and probabilities  $P$  with density function  $p$ , we have:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}$$

in which  $p(y)$  is the prior predictive distribution which indicates the distribution of  $y$  before  $y$  has been observed, and  $p(y|\theta)$  is the likelihood or likelihood function derived from a probability model.  $p(\theta|y)$  is called the posterior distribution of  $\theta$  that expresses uncertainty about belief set  $\theta$  after taking into account both the prior and observed data  $y$ .  $p(y)$  is the same for all possible  $\theta$  being considered, and can be set as an unknown constant. Thus by getting rid of the denominator from the above equation, we can state that the posterior  $p(\theta|y)$  is proportional to the likelihood  $p(y|\theta)$  multiplied by the prior  $p(\theta)$ . In summary, our prediction model has three components:

- Prior distribution  $p(\theta)$ , which is based on the statistic data for spectrum channels' availability.
- $p(y|\theta)$  is the likelihood function.
- $p(\theta|y)$  is the posterior distribution that expresses uncertainty about  $\theta$ .

### 5.5.3 Conjugate Distributions

In Bayesian probability theory, if the posterior distributions  $p(\theta|y)$  is in the same family as the prior probability distribution  $p(\theta)$ , the prior and posterior are then called conjugate distributions.

The formal definition of conjugate distribution is shown in the following [47]:

*Suppose a prior density  $p(\theta)$  belongs to a class of parametric densities,  $\mathcal{F}$ . Then the prior density is said to be conjugate with respect to a likelihood  $p(y|\theta)$  if the posterior density  $p(\theta|y)$  is also in  $\mathcal{F}$ .*

So the prior in conjugate distribution is called conjugate prior. The advantage of using conjugate prior is that conjugate prior has algebraic convenience that yields closed-form posteriors. Otherwise, to get the posterior, a difficult numerical integration may be necessary. Further, conjugate priors can transparently show us how a likelihood function updates a distribution.

### 5.5.4 Prior and Posterior Distribution of $\theta$

Since  $\theta \in [0, 1]$ , a prior density for  $\theta$  must have the properties:

1.  $p(\theta) \geq 0$
2.  $\int_0^1 p(\theta) d\theta = 1$

On the other hand, according to [46, 47], we prefer to make the prior distribution as a conjugate distribution. Here we choose the Beta distribution [47] as the prior for  $\theta$ .

The probability density function of the Beta distribution is

$$\begin{aligned}
 f(x|\alpha, \beta) &= \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} du} \\
 &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \\
 &= \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}
 \end{aligned}$$

in which  $\Gamma$  is the gamma function. The beta function,  $B$ , appears as a normalization constant to ensure that the total probability integrates to unity. The expectation and variance of Beta distribution are:

$$E[X] = \frac{\alpha}{\alpha + \beta} \tag{5.5}$$

$$var(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \tag{5.6}$$

In the scenario of CRNs, we assume in each time slot every channel has two status, idle or busy. Given a prior  $p(\theta|\alpha, \beta) = Beta(\alpha, \beta)$  for unknown parameter  $\theta$ , and observation data  $D = (H, T)$ , where  $H$  represents the number of idle time slots and  $T$  represents the number of busy time slots. We have:

$$p(\theta|\alpha, \beta, H, T) \propto \theta^H (1 - \theta)^T \theta^{\alpha-1} (1 - \theta)^{\beta-1}$$

With normalization, we get

$$\begin{aligned}
p(\theta|\alpha, \beta, H, T) &= \frac{\Gamma(\alpha + \beta + H + T)}{\Gamma(\alpha + H)\Gamma(\beta + T)} \theta^{H+\alpha-1}(1 - \theta)^{T+\beta-1} \\
&= \text{Beta}(\alpha + H, \beta + T)
\end{aligned}$$

So Bayesian estimate for posterior expectation of  $\theta$  is

$$\hat{\theta} = \frac{\alpha + H}{\alpha + H + \beta + T}$$

In brief, we update unknown parameter  $\theta$  based on the new observation we see.

Now the problem is how to determine  $\alpha$  and  $\beta$ . For a specific channel we have a prior expectation  $\bar{\theta}$ , and obviously based on Eq.(5.5)  $\bar{\theta}$  is equal to  $\frac{\alpha}{\alpha+\beta}$ . Then we can arbitrarily determine the values of  $\alpha$  and  $\beta$  as long as they satisfy  $\bar{\theta} = \frac{\alpha}{\alpha+\beta}$ . Note that according to Eq.(5.6), the variance of prior distribution decreases as  $\alpha + \beta$  increases. We should choose the values of  $\alpha$  and  $\beta$  discretely to avoid a big variance.

For a specific channel, the estimation process is summarized in Algorithm 5:

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**Algorithm 5:** *Bayesian-ALG*

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- 1:  $\bar{\theta}$  is the expectation of prior distribution of  $\theta$
  - 2: choose  $\alpha$  and  $\beta$  which suffice that  $\bar{\theta} = \frac{\alpha}{\alpha+\beta}$
  - 3:  $h = 0, t = 0$ ,  $h$  represents the number of idle time slots and  $t$  represents the number of busy time slots
  - 4: **while** TRUE **do**
  - 5:   **if** current time slot is busy **then**
  - 6:      $t++$ ;
  - 7:   **else**
  - 8:      $h++$ ;
  - 9:   **end if**;
  - 10:    $E[\theta] = \frac{\alpha+h}{\alpha+h+\beta+t}$
  - 11: **end while**;
-



## 5.6 Performance Evaluation

To verify the effectiveness of our proposed approach for spectrum availability prediction, we carried out a well-designed simulation experiment, in which we compare our approach with MLE and Markov chain approaches.

### 5.6.1 Prediction Approaches

Let's assume in each time slot secondary users will access a channel, and they can save the observations for every channel. MLE approach is a classical frequentist method in which the prediction is exactly based on the observations from the beginning to the current time slot. The availability of each channel is approximately equal to the frequency of idle time slots. As for Markov chain approach, we assume the channel availability in the next time slot is relative to the channel availability state of the current time slot, which is actually a Markov chain with order 1 (MC-1). We also apply another two Markov chain approaches which are with order 2 (MC-2) and with order 3 (MC-3) respectively. According to Section 5.2, in these two approaches the future availability state for a certain channel depends on the past 2 or 3 states.

Our Bayesian estimation based approach takes advantages of the existing spectrum usage information. The posterior distribution of the hypothesis is changed by the continuously incoming observations since the real channel availability is varied temporally and spatially.

## 5.6.2 Simulation Setup

In our simulation, let  $\mathcal{C} = \{c_1, \dots, c_{20}\}$  be a set comprising 20 channels. Any of them could be accessed by primary users. In every time slot, the secondary user must scan the nearby spectrum usage and thereafter access one and only one channel. During the access period, if any primary user accesses the same channel, the secondary user must quit without any delay. We assume the secondary user has enough memory to store the previous observations.

To compare the performance of our approach with MLE, MC-1, MC-2 and MC-3 approaches, we design two scenarios in our simulation. In scenario I, the value of  $\theta$  is randomly chosen from  $(0, 1)$ . Then with  $\theta$ , we generate a Gaussian idle/busy (0/1) series in 10000 time slots. Here the prior expectation of  $\theta$  and the real expectation of  $\theta$  are exactly the same. While in scenario II, the idle/busy (0/1) series is still Gaussian distributed. But the prior expectation of  $\theta$  and the real expectation of  $\theta$  are not identical. For each scenario, we run our simulation 20 rounds, each round includes 10000 continuous time slots. In terms of our Bayesian estimation based approach, in every time slot, secondary users have to re-calculate the posterior expectation of  $\theta$  on every channel based on previous observations. Then secondary users select a channel with the maximum  $\theta$  to access.

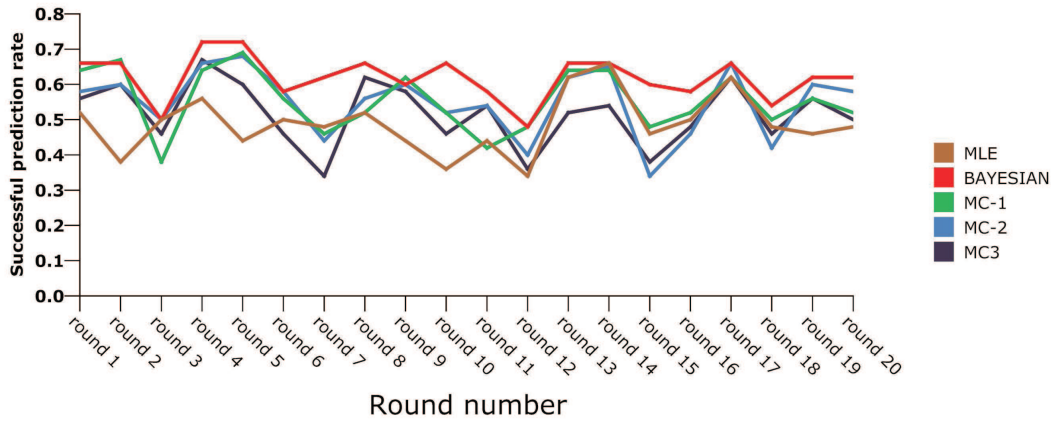
The performance of the prediction result of all approaches is shown as follows. In Figure 5.6.3, we show the comparison of prediction accuracy for scenario I. We can tell from Figure 5.6.3(a) that in the first 50 time slots, our approach almost always has the lowest collision rate from each round. In the first 100 and 500 time slots, our

Bayesian estimation based approach still has the best prediction accuracy in most of rounds, given Figure 5.6.3(b) and Figure 5.6.3(c). As for scenario II, we can tell from Figure 5.6.4 that the prediction accuracy curve of all approaches is quite similar. As for our Bayesian estimation based approach, even though the hypothesis on  $\theta$  does not follow the real value of  $\theta$ , we can tell from Figure 5.6.4(a), Figure 5.6.4(b) and Figure 5.6.4(c) that the prediction accuracy of Bayesian approach is not worse than the results of other approaches.

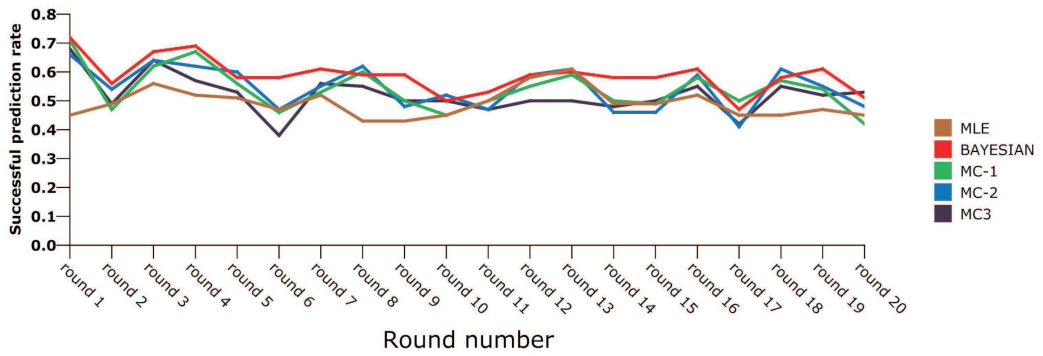
At last, with the increasing of observations, the average accuracy rate of all approaches will converge to a relatively similar value, which can be seen from Table 5.1. In both scenarios, the average accuracy rate in 10000 rounds for the four approaches is close. Especially in scenario II, the average accuracy rate in 10000 rounds for Bayesian estimation based approach is nearly equal to that of MC-1 and MC-2. It is reasonable because the long-run empirical probability will exactly converge to the true probability, no matter which approach is concerned.

Average Accuracy Rate				
Scenario	Algorithm	50 rounds	100 rounds	10000 rounds
Scenario I	MLE	0.488	0.492	0.5646
	Bayesian	0.619	0.5875	0.5836
	MC-1	0.554	0.5405	0.5751
	MC-2	0.5495	0.5465	0.5757
	MC-3	0.5155	0.521	0.5587
Scenario II	MLE	0.455	0.4845	0.561735
	Bayesian	0.53415	0.54455	0.58229
	MC-1	0.525	0.53655	0.581935
	MC-2	0.51	0.5165	0.581155
	MC-3	0.48	0.4985	0.578675

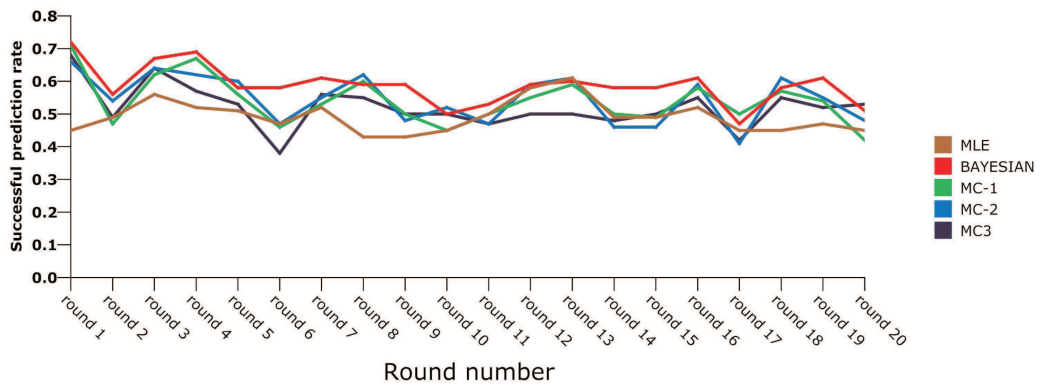
Table 5.1: Average Accuracy Rate of Prediction in 10000 Time Slots.



(a) Accuracy rate of prediction in the first 50 time slots

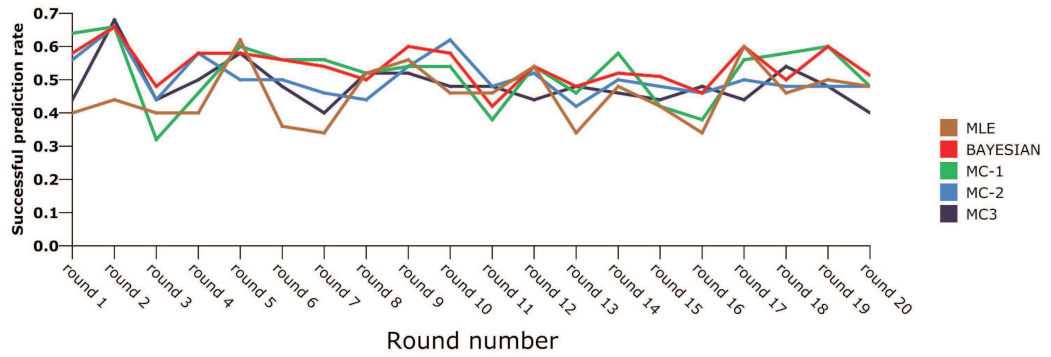


(b) Accuracy rate of prediction in the first 100 time slots

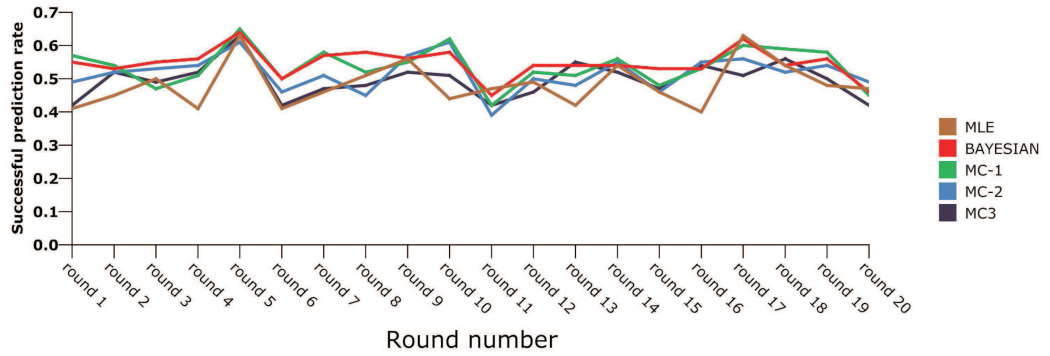


(c) Accuracy rate of prediction in the first 500 time slots

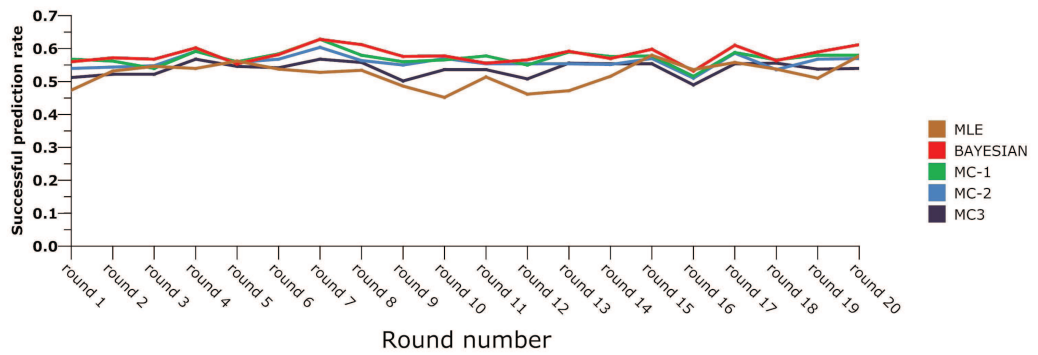
Figure 5.6.3: Performance comparison in scenario I.



(a) Accuracy rate of prediction in the first 50 time slots



(b) Accuracy rate of prediction in the first 100 time slots



(c) Accuracy rate of prediction in the first 500 time slots

Figure 5.6.4: Performance comparison in scenario II.

## 5.7 Summary

In our proposed work, we present *Bayesian* estimation based approach for channel availability prediction. We compare the prediction results of our approach with MLE and the Markov chain approaches. The results show that, given a correct prior information on spectrum usage, the *Bayesian* estimation based approach can achieve a more accurate prediction on spectrum availability. Even though prior information on spectrum usage is not correct, the accuracy rate of *Bayesian* estimation based prediction is not worse than others.

# Chapter 6

## Spectrum Allocation in CRNs under Distributed Architecture

Game theory provides a method for analyzing interactive decision problems that involve multiple rational decision makers. In cognitive radio networks, secondary users have to compete for spectrum usage. Game theory can help to model, analyze, design competition behavior for spectrum access. In this section, we present a game theoretic modeling technique and study the interactions of secondary users. This chapter is organized as follows. Section 6.1 introduces the basics of game theory. In Section 6.2 we show the related work of game theory in cognitive radio networks. Section 6.3 defines the game model in cognitive radio networks. Section 6.4 presents a game theoretic algorithm for spectrum allocation. In section 6.5 we run a simulation to show that our approach significantly decreased collision rate and improved the spectrum utilization. This chapter concludes in Section 6.6.

### 6.1 Introduction to Game Theory

Game theory provides a framework for studying strategic decision making. Game theory models an interactive decision process. Briefly speaking, in the decision making

process, the player chooses the best strategy whose outcome is a function of the other players' decisions. In general, each game includes the following elements:

- A set of users  $N$  comprising of  $n$  players,  $\{1, 2, \dots, n\}$ ;
- Each player  $i$  has its own set of strategies (actions)  $S_i$ ;
- The vector of strategies (actions) selected by all players, represented by  $s = (s_1, s_2, \dots, s_n)$ . We use  $\prod_{i=1}^n S_i$  to denote the set of all possible strategy selections of all players;
- An outcome (payoff) function with the input of all actions chosen by the players, which determines the outcomes;
- Preferences for each of the action. That is the preference ordering on these outcomes by giving a complete, transitive, reflexive binary relation on the set of all strategy vectors  $S$  [48];
- The rules in the game, such as the play order in the game.

Generally speaking, the outcome for each player will be different since the vector of selected strategies  $s \in S$  by all the players determines each player's outcome. We define the outcome function as  $u_i : S \rightarrow R$ , which is also called players' utility functions, payoff functions or objective functions. In other words,  $u_i$  is a function of  $s_i$  and  $s_{-i}$  where the strategy chosen by player  $i$  is  $s_i$  and  $s_{-i}$  is the  $(n - 1)$ -dimensional vector of the strategies chosen by all other  $n - 1$  players. Therefore, in a game, each player's outcome depends not only on the strategy chosen by himself but also on the strategies chosen by all other players.



In game theory a big common assumption is rationality, which implies that every player is motivated by maximizing his own payoff. For instance, a player  $i$  will always choose a strategy  $s_i$ , rather than another strategy  $s'_i$ , if and only if  $u_i(s_i) \geq u_i(s'_i)$ , while  $s_{-i}$ , the vector of strategies chosen by all the other players remains unchanged.

### 6.1.1 Extensive Form Game

In this chapter, we will study extensive form games which model multi-agent sequential decision making. Our focus will be on multi-stage games with observed actions where:

- There is a list of players in this game;
- There is a set of allowable actions at each node;
- All previous actions are observed, i.e., each player is perfectly informed of all previous events;
- Payoffs specified at each node Unlike normal form games, it is easy to depict sequential moves by players in extensive form games.

The extensive form game can be represented by a game tree. An example of game tree is shown in Figure 6.1.1.

In this game, every player makes a choice according to a certain order. In other words, the decision making process among all players is asynchronous, which implies every time only one player can make a decision. In Figure 6.1.1, player 1 makes the first choice. The choices available to player 1 are represented by branches emanating

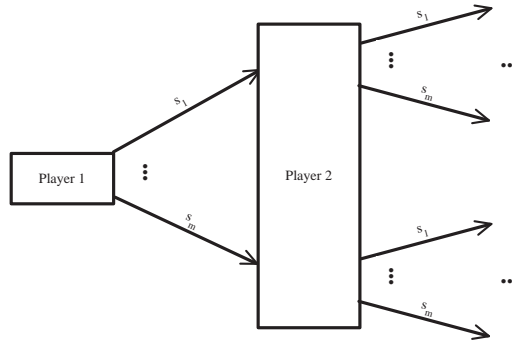


Figure 6.1.1: The Extensive Form Game

from the rectangle of player 1 which correspond to the strategies  $s_1, \dots, s_m$ . After then player 2 starts to make a choice, which splits into further branches.

### 6.1.2 Normal Form Game

The normal form game can be represented in a table. An example is shown in Figure 6.1.2 in which the two rows correspond to the strategies of player  $a$  and the two columns correspond to the strategies of player 2. In every cell of the table, the payoffs associated with the pair of strategies are listed. Actually the game in Figure 6.1.2 is a famous game named Prisoners' Dilemma which we will introduce in Section 6.1.3. In normal form game all players make their choices simultaneously, which is a key difference from extensive form game.

### 6.1.3 Nash Equilibrium

Nash equilibrium is one of the most important concepts in game theory. The formal definition of Nash Equilibrium is as follows [48]:

**Theorem 6.1.3.1.** *A strategy vector  $s \in S$  is said to be a Nash Equilibrium if for all players  $i$  and each alternate strategy  $s'_i \in S_i$ , we have*

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}).$$

That is, no player  $i$  can improve his payoff by choosing strategy from  $s_i$  to  $s'_i$  while all other players keep the strategies they have chosen in  $s$ .

In a game with multiple players, the Nash equilibrium may not be unique. A famous example is the game named prisoner's dilemma. Suppose two friends, A and B, are suspected of committing a crime and are being interrogated in separate rooms. Both of them want to minimize their jail sentence and face the following scenario: If A and B confess, they will serve six years each. If neither confesses, the prosecutor will find a lesser charge, and each will serve two years. If A confesses and B does not, A is released and B serves an aggravated ten years. If B confesses and A does not, B is released, and A serves an aggravated ten years. The matrix is shown in the Figure 6.1.2. In Figure 6.1.2 the strategy set (confess, confess) and (not confess, not confess) are two different equilibria. If any player wants to change the current strategy individually, his payoff will become lower.

		A	
		confess	not confess
B	confess	6, 6	0, 10
	not confess	10, 0	2, 2

---

Figure 6.1.2: Outcome Matrix of A and B.

If there are multiple Nash equilibrium in a game, different equilibria may have

different payoffs for the players. In Figure 6.1.2, A and B will get different payoffs in (confess,confess) and (not confess, not confess). However, any Nash equilibrium is stable, which means that no player wants to individually deviate.

### 6.1.4 Pareto Optimality

Pareto optimality is a measure of efficiency. An outcome of a game is Pareto optimal if the outcome cannot be improved upon without hurting at least one player. A Nash Equilibrium may or may not be Pareto optimal. The formal definition of Pareto optimal is shown as follows:

**Theorem 6.1.4.1.** *A strategy vector  $s^*$  is Pareto optimal if there doesn't exist another strategy vector  $s \in S$  such that  $u_i(s) \geq u_i(s^*)$ ,  $\forall i \in N$  and  $u_j(s) > u_j(s^*)$  for at least one player  $j \in N$ .*

In Figure 6.1.2, this game has two NE outcomes: (confess,confess) and (not confess, not confess). However, only (not confess, not confess) is Pareto optimal.

## 6.2 Game Theoretic Applications in CRNs

Game theory [48,49], is an effective way to process complex distributed decision making problem and was used for modeling, analyzing and designing cognitive radio networks [50].

Since a radio has a clearly defined object in terms of quantitative information, e.g., SINR, BER, and latency, secondary users can quantify literally how much better one allocation is than another. A lot of work [50–52] has been carried out in CRNs regarding game theory, which mostly focuses on analyzing the interactions of secondary users to reduce interference. In [51] the authors presented a game model named *bilateral symmetric interaction* (BSI) game in which an exact potential function definitely

existed. This function, in turn, verifies that this game is a *potential game* [53]. Therefore, the set of pure Nash equilibria in this game can be reached through locating the local optima of the potential function. In [53] the authors showed how to establish BSI in infrastructure and ad-hoc networks which employ power control. All these above game models are essentially designed to reduce sum network interference through adjusting the transmission power and adaptable waveform parameters of each secondary user.

On the other hand, some research work utilized game theoretic techniques for spectrum allocation optimality. For instance, in [54], a cooperative game theory model to analyze spectrum allocation was developed. The model has a strong assumption that the node pair within two-hop range can cooperate. A graphical game model was presented in [53], in which the payoff function is based on the expected throughput of different channels for secondary users. To compute Nash equilibrium in this game, a distributed subgradient algorithm was presented. However, the shortcoming of [55] is that the payoff of a certain channel not only depends on the theoretical throughput of this channel, but also the availability of this channel. Therefore, in Section 6.3, we present a game model in which the payoff function depends on both the expected throughput and the availability of channels.

### **6.3 Game Modeling in CRNs**

In cognitive radio networks, the competing behaviors of secondary users in spectrum allocation can be naturally modeled as a game.

### 6.3.1 Game Model

In this chapter the spectrum allocation problem in CRNs can be modeled as an extensive form game, in which all players choose their strategies according a certain order. In other words, they make their choice in sequence. We assume in this game the communication is time slotted. Each time slot includes a spectrum sensing period and a communication period. In a specific time slot a secondary user can sense all channels but access only one channel. We also assume that secondary users know the theoretical throughput of all available channels.

- **Players:** In CRNs, the players are obviously the secondary users that make decision in the interactive spectrum allocation process. We define the set of players (secondary users) as  $N$  and the set of available channels as  $M$ .
- **Strategies:** The strategies are actually the set of access decision for channels, denoted by  $C$ , and strategy  $c_i$  represents to access channel  $i$ , where  $i \in M$ .
- **Payoff function:** In Chapter 5.1, Section 5.5.1, the probability that a certain channel is idle in one time slot is denoted by  $\theta$ . Our Bayesian inference based approach provides a probability distribution for  $\theta$ , which is a key index for secondary users to evaluate channels. Different from [55] in which the payoff function is merely based on the *theoretical throughput*  $t_{ic_j}$  for secondary user  $i$  and channel  $c_j$ , in our game model the payoff function is defined as:

$$u_i(c_j) = \hat{\theta} \times t_{ic_j}.$$

where  $\hat{\theta}$  is the posterior expectation of  $\theta$ . The reason is straightforward because

the bigger the  $\hat{\theta}$ , the bigger the probability that the certain channel will be idle in the next time slot. So we defined  $u_i(c_j)$  as the *expected throughput* of channel  $c_j$  for secondary user  $i$ .

- Perfect information: The players have perfect information of other players. They know not only their own utility functions as functions of the strategies space but also the utility functions for all the other players.
- Preferences for Strategies: Players always have preferences with respect to all the possible outcomes from different strategies. If we assume all players are rational, a player always picks up a strategy which is believed to increase its payoff.

In short, the secondary users in our game model have an incentive to maximize its spectrum utilization. Combining with the spectrum availability prediction, the payoff function in our game model can reflect the spectrum utilization better than the one that only depends on theoretical throughput.

## 6.4 Game Theoretic Algorithm for Spectrum Allocation

As mentioned in Section 6.3, all players have perfect information. This game permits only one player to move at a time. As long as a player makes its choice, the decision should be broadcast to its neighbors. In Section 6.3 we also assume all players are rational. This means when it is this player's turn to make a choice, it always chooses a strategy with maximum payoff. In the context of spectrum allocation in CRNs, the

players will always choose to access a certain channel which has maximum  $\hat{\theta} \times t_{ic_j}$ . It also implies that the player will avoid accessing the channels which are already chosen by another player. So we present Algorithm 6 as follows:

---

**Algorithm 6:** *Dynamic-Channel-Allocation-ALG( $P, \mathcal{C}$ )*

---

- 1: Players set  $N$ , we assume every player has a sequence number  $i, i \in \{1, 2, \dots, n\}$ ;
  - 2:  $S_i$ : the strategy set of player  $i, i \in \{1, 2, \dots, m\}$ ;
  - 3: **for**  $i = 1 \rightarrow n$  **do**
  - 4:   player  $i$  chooses  $s_i$  which has the maximum payoff;
  - 5:   player  $i$  broadcasts its choices to neighbor;
  - 6: **end for**
  - 7: **for**  $i = 1 \rightarrow n$  **do**
  - 8:   player  $i$  changes his sequence number from  $i$  to  $(i + 1) \bmod n$ ;
  - 9: **end for**
- 

**Proposition 4.** *The final strategy vector by all players from Algorithm 6 is a Nash Equilibrium.*

The proof is as follows:

*Proof.* In this vector, any player cannot improve their payoff through changing its strategy unilaterally. Otherwise, its first choice cannot bring back the maximum payoff for this player, which is a contradiction with our assumption for players' rationality.  $\square$

The strategy vector in this game is Nash Equilibrium, but is not Pareto Optimal. For instance, in Figure 6.4.3, no matter which strategy player 1 makes, he always gets the same payoff. However, if player 1 moves to the strategy  $c_2$ , player 2 has to move to  $c_1$ . The payoff of player 2 is 1. The strategy pair  $(c_2, c_1)$  is Nash Equilibrium, but not Pareto optimal since the strategy pair  $(c_1, c_2)$  exists.

Without any loss of generality, we assume that all secondary users are affected by the same interruptions from primary users so that they share the same state of interruptions. This is reasonable because the interruption range of a primary user



		Player 2	
		$c_1$	$c_2$
Player 1	$c_1$	0,0	2, 3
	$c_2$	2, 1	0, 0

---

Figure 6.4.3: An example of two players game.

is normally quite larger than the interference ranges of secondary users. Moreover, we can assume all secondary devices are designed and manufactured by the same company. Therefore, they can obtain the same theoretical throughput for a specific channel. Given the above two assumptions, we can get:

**Proposition 5.** *Assume all players have the same payoff  $\hat{\theta} \times t_{ic_j}$  from the same channel, where  $\hat{\theta} \times t_{ic_j}$  is expected throughput of channel  $c_j$ , the game can reach Pareto optimality.*

The proof is as follows:

*Proof.* Assume we have a two-player and two-strategy game. The payoff matrix is shown in Figure 6.4.4. Both players ( play 1 and player 2 ) have the same payoff function for the same strategy, let's say  $u_1(c_i) = u_2(c_i)$ , where  $i \in \{1, 2\}$ . If player 1 firstly makes the choice  $s_i, i \in \{1, 2\}$ , that is, to access channel  $c_i$ , his payoff is  $u_1(c_i)$ . Then player 2 has to choose the other strategy  $j, j \in \{1, 2\}$ . The payoff of player 2 is  $u_2(c_j)$ . Since players are always rational,  $u_1(c_i) \geq u_1(c_j)$ . Remember that  $u_1(c_i) = u_2(c_i)$ , so  $u_2(c_i) \geq u_1(c_j)$  and  $u_2(c_i) \geq u_2(c_j)$ . Player 1 chooses  $c_j$  only under certain circumstances that  $u_1(c_i) = u_1(c_j)$ . In that situation, player 2 will choose  $c_i$  and his payoff  $u_2(c_i) = u_1(c_j) = u_2(c_j)$ . Player 2 still get the same payoff. According to the definition of Pareto Optimal, in any case, this game will reach Pareto Optimality.  $\square$

The problem of sequential decision making is that a player will always have fewer choices than the players prior to him. To mitigate this problem and maintain fairness

		Player 2	
		$c_1$	$c_2$
Player 1	$c_1$	0,0	2, 2
	$c_2$	2, 2	0, 0

---

Figure 6.4.4: An example of two-player game that reaches Pareto optimality.

among all secondary users, we propose a concept named validity period  $T$ . The spectrum allocation scheme can only be effective during  $T$ . After that, all players should replay this game. In every time period  $T$ , we move the last player in the previous decision making sequence to the head in the line. Thus we modify Algorithm 6 to Algorithm 7:

---

**Algorithm 7:** *Modified-Dynamic-Channel-Allocation-ALG( $P, \mathcal{C}$ )*

---

- 1: Players set  $N$ , we assume every player has a sequence number  $i, i \in \{1, 2, \dots, n\}$ ;
  - 2:  $S_i$ : the strategy set of player  $i, i \in \{1, 2, \dots, m\}$ ;
  - 3: Validity period  $T$
  - 4: **while** 1 **do**
  - 5:   **for**  $i = 1 \rightarrow n$  **do**
  - 6:     player  $i$  chooses  $s_i$  which has the maximum payoff;
  - 7:     player  $i$  broadcasts its choice to neighbors;
  - 8:   **end for**
  - 9:   **for**  $i = 1 \rightarrow n$  **do**
  - 10:     player  $i$  changes his sequence number from  $i$  to  $(i + 1) \bmod n$ ;
  - 11:   **end for**
  - 12:   Sleep  $T$
  - 13: **end while**
- 

In Algorithm 7, the for-loop in lines 9–11 adjust each player’s sequence number to guarantee the fairness among them. In the next round, the last player in the sequence

from the previous round will become the first play to make decision. Therefore, our game model is proposed not only for maximum spectrum utilization, but also the fairness among all players.

## 6.5 Experimental Results

In this section, numerical simulations are used to evaluate the performance of our game model proposed in Section 6.4. We simulate an ad-hoc cognitive network by randomly placing 20 secondary users on a 100 by 100 area. These users are numbered from 1 to 20. Every secondary user chooses its desired channel based on an ascending sequence. This simulation will run 10000 time slots. In order to keep up with the fairness among secondary users, after every 50 time slots we increase the order of secondary users, i.e., user 1 becomes user 2, user 2 becomes user 3,  $\dots$ , user 20 becomes user 1. The network topology is abstracted into a conflict graph Fig 6.5.5, in which each vertex represents a secondary user. Any two secondary users interfere with each other if they are within distance of 20. These neighbors are connected in the corresponding conflict graph. Every secondary user communicate with its neighbors through a common control channel. The interference range of primary users is set to 50. The number of available channels is 10. Each available channel  $i$  has a probability  $\theta_{c_i}$  to be in idle state, i.e., no any primary user access this channel at that time. In table 6.1, we assign different theoretical throughput to these 10 channels. For the sake of convenience, we use scalar value from 1 to 6.

For these 10 channels, the expectation of prior distribution of  $\theta$  is listed in table 6.2.

$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
5	2	3	6	5	3	6	1	5	5

Table 6.1: Theoretical Throughput of 10 Channels.

We compare the proposed payoff function to the payoff function only based on channels' theoretical throughput. We refer to these two as *with prediction* and *without prediction*, respectively.

Fig 6.5.6 illustrates the collision rate during 10000 time slots using both payoff functions. With spectrum availability prediction, our approach achieved a much lower collision rate than the approach without spectrum availability prediction for all 20 secondary users. Fig 6.5.7 compares the average throughput per time slot. The comparison confirmed that our approach not only effectively improved individual spectrum utilization of each secondary user, but also the total spectrum utilization of the network.

## 6.6 Summary

In this chapter, we introduce important concepts in game theory, such as extensive form game, normal form game, Nash equilibrium and Pareto optimality. Then we present a game model for CRNs to implement dynamic spectrum allocation algorithm, in which the payoff function combines the spectrum availability prediction with the theoretical throughput of channels. We prove that, this game can definitely reach a Nash equilibrium and even Pareto optimal if we assume all players share the same payoff for a specific channel. At last, we present a distributed spectrum allocation algorithm, in which we circulate the players' sequence number to maintain the fairness

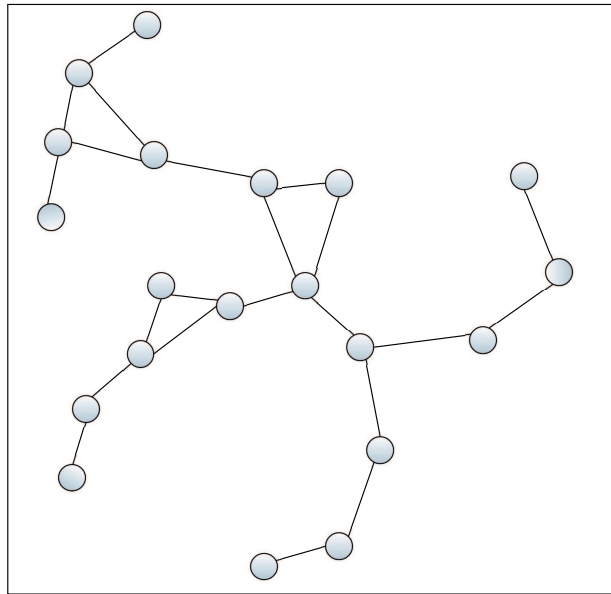


Figure 6.5.5: Conflict Graph in Simulation.

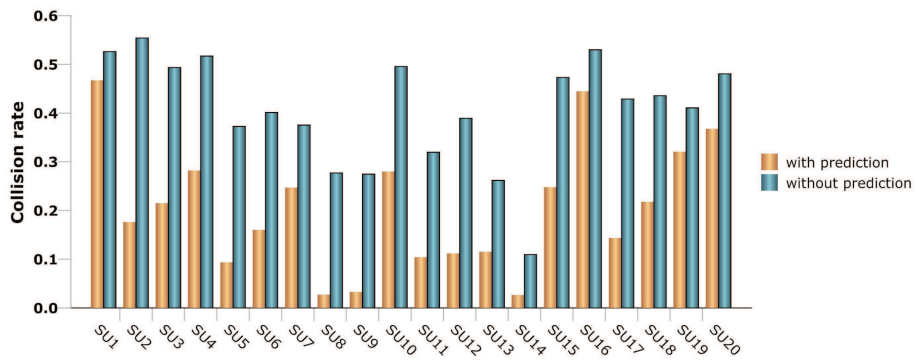


Figure 6.5.6: Collision Rate in 10000 Time Slots.

$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
0.52	0.26	0.77	0.52	0.26	0.54	0.26	0.65	0.42	0.52

Table 6.2: Expectation of Prior Distribution of  $\theta$  for 10 Channels.

in the game. The simulation results show that our approach significantly decreased collision rate and improved the spectrum utilization as well.

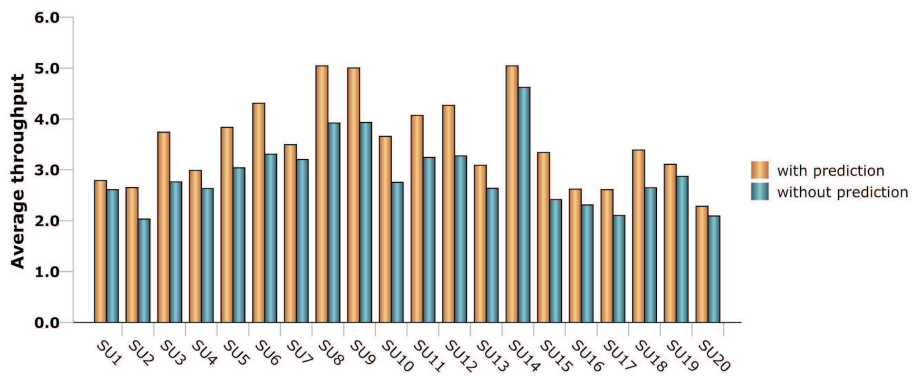


Figure 6.5.7: Average Throughput per Time Slot.

# Chapter 7

## Conclusion and Future Work

We conclude this dissertation by summarizing our contributions and identifying several new directions for future research in the area of spectrum allocation in cognitive radio networks.

### 7.1 Major Contributions

We have studied optimization models of spectrum allocation in cognitive radio networks. The optimization models are mainly based on two models: (1) under centralized architecture and (2) under distributed architecture.

- Optimization model for CRNs under centralized architecture:

For centralized architecture, we developed mathematical models including non-linear and integer linear programming formulations for different objectives of optimization. In order to simplify the solution process, we developed a unified binary linear programming (UBLP) model which can be solved by the simplex method and branch-and-bound search. We prove that given different per-user bandwidth minimum, the optimal solution to the UBLP model would achieve special objectives optimization, such as the maximum network throughput and

the max-min fairness. For the proportional fairness objective, the solution to the UBLP model would be within a bound of the optimal solution.

- Heuristic algorithm for UBLP model:

We showed that the original optimization problem of UBLP model is a combination of a maximum independent set problem and a maximum graph coloring problem. We analyze the close relationship between the two problems, and propose a channel allocation algorithm. In addition, we present a genetic algorithm based approach to obtain good solutions to the two problems. At last, we put all algorithms into an integrated framework which can go through all algorithms and always choose the best one for different optimization problems.

- Heuristic algorithms for infrastructure based CRNs:

We have proposed an end-user based algorithm for spectrum allocation in infrastructure based cognitive radio networks. Our algorithm aims at assigning available channels to secondary base-stations with more associated end-users, and also maintains fairness among all secondary base-stations as well, and as a result, effectively avoids spectrum waste and improves the network performance. Simulation results show that our algorithm can drastically improve network performance compared with the common algorithms.

- Spectrum availability prediction:

Since secondary users always prefer the channels with high availability, the prediction on spectrum availability becomes an important index in spectrum



allocation. We presented a Bayesian inference based prediction algorithm for spectrum availability which takes into account the current statistic information of spectrum usage issued by different authorities. Compared with several common prediction algorithms including MLE and Markov chain method, our prediction algorithm is proved to be the one with better performance.

- Game theory based distributed spectrum allocation:

We presented a game model for distributed spectrum allocation in CRNs. In this model, the players (secondary users) evaluate channels based on a payoff function combining theoretical throughput of channels and spectrum availability prediction. Every player assumes the other players have the same payoff vector for different channels. Then based on a certain sequence, each player chooses the best channel to access. We prove that, this game can definitely reach Nash equilibrium, and even Pareto optimality if we assume all players share the same payoff for a specific channel. Finally, we present a distributed spectrum allocation algorithm, in which we circulate the players' sequence number to maintain the fairness in the game.

## 7.2 Future Work

We have addressed several important issues in spectrum allocation in cognitive radio networks. Future work includes:

### **7.2.1 Spectrum Allocation in CRNs under Centralized Architecture**

Our future work is to improve our integrated framework into an open, expendable, convenient and efficient system. The future improvements include:

- The GUI of the integrated framework will be improved to provide the more intuitive grasp for users.
- The framework will provide an adaptable interface through which we can add more algorithms easily.
- The framework will execute multiple algorithms concurrently instead of sequentially, which will improve the efficiency greatly.

### **7.2.2 Spectrum Allocation in CRNs under Distributed Architecture**

In Chapter 6 we presented a cooperative game model to implement spectrum allocation algorithm in CRNs under distributed architecture. The key point in this game is that all secondary users must follow certain game rules and achieve a common collective payoff. Our future work includes:

- More complicated simulations will be designed to evaluate the dynamic spectrum allocation in our game model.
- A supervision mechanism which can detect any violation from secondary users will be proposed.

- We will establish a mechanism of pungent punishment for selfish behaviors from secondary users.

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1. Tao Zhang, Bin Wang and Zhiqiang Wu. Channel Allocation in Dynamic Spectrum Access Networks: A Graph Theory Perspective. *Submitted to IEEE ICC'11*.