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AN INTEGRAL MODEL FOR THERMAL BACKSCATTERING FROM THE EXHAUST PLUME OF A SPACE-BASED HF LASER

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Abstract

The operation of a space-based HF laser may be hampered due to self contamination by corrosive exhaust products. We estimate one effect contributing to contaminating backflow: thermal backscattering from the rarefaction fans flanking the exhaust ring-jet. Our computational model is based on a first-iterate approximation to the Boltzmann equation in integral form. Results indicate that thermal backscattering of corrosive speies (HF, DF) is negligible.

Nomenclature

$B_i(S,\vec{\xi})$	Maxwellian distribution function
D	molecular diameter (for hard sphere collisions)
$f_i(S,\vec{\xi})$	distribution function
$g_i(S,\vec{\xi})$	molecule generation rate
h_i	mole fraction
k	Boltzmann constant
m_i	mass of a molecule
M	Mach number
n	number of species
N(S)	number density
Qc	flux arriving at point c
R	distance from corner of centered fan
S	coordinate parallel to $\vec{\xi}$
T(S)	temperature
U(S)	flow velocity
Wav	average molecular weight
X	Cartesian point vector
α	angle between $\vec{\Omega}$ and normal to surface
γ	specific heat ratio
ψ	characteristic angle (in centered fan)
$\vec{\xi}$	molecular velocity
ρ	density
$\eta_i(S',S,\vec{\xi})$	number of collisions expected for molecule traveling from S' to S with velocity $\vec{\xi}$
$\omega_{ij}(S,\vec{\xi})$	collision frequency
σ_{ij}	collision cross-section (hard spheres) between species i and j
$\vec{\Omega}$	unit vector from point c, pointed at the fan
$d^3\vec{\Omega}$	element of solid angle
ϕ	angle of rotation coordinate

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Indices

i,j	indices of molecular species
0	stagnation value
1	nozzle exist value
f	limiting (vacuum) characteristic value

I. INTRODUCTION

This presentation is part of a study on the contaminating backflow from the exhaust plume of a large earth-orbiting HF laser. The basic design concept is a cylindrical spacecraft with the laser beam emanating in an axial direction from one end, and a massive radial outflow of DF/HF combustion products (diluted by He, H_2, H) from a ring-nozzle located midway along the spacecraft (Figure 1). The exhaust plume is an underexpanded supersonic ring-jet, designed to stay clear of the spacecraft by maintaining a Prandt-Meyer turning angle at the nozzle lips of well below 90°.

However, it is well known from experience with rocket plumes in space that "cavity regions" (in the continuum gasdynamics sense) are filled by a free-molecular flow. Assuming that the boundary layer at the nozzle lips can be eliminated (e.g., by an expanding step design just upstream of the nozzle lip), the two mechanisms which may generate molecular flow into cavity regions, are thermal backscattering and scattering by ambient molecules traveling at orbital speed.

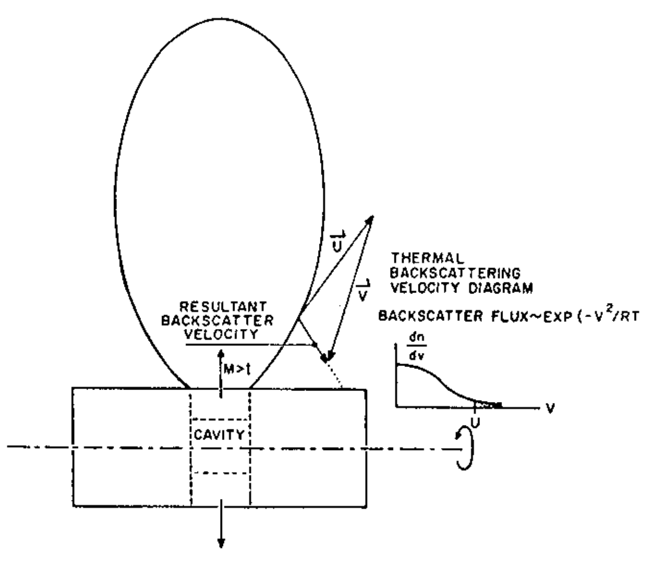

Figure 1. Thermal Backscattering from Laser Exhaust Plume

In a former publication[1], we presented a breakdown surface model for evaluating the contribution of thermal backscattering to the contaminating backflow from the ring-plume of an HF laser. In that model, the transition layer between continuum and free-molecular flow is replaced by a "breakdown surface". The flow is assumed to change abruptly upon crossing this surface, from continuum to free-molecular flow. This is a rather crude approximation, letting the transition process which ought to be governed by the Boltzmann equation, be lumped into a single "jump", thus avoiding the need to solve the Boltzmann equation.

The purpose of this paper is to present an improved integral model for evaluating the thermally backscattered flux from the lip-centered rarefaction fan of an HF laser exhaust plume (Figure 1). This model is based on a first iterate approximate solution to the Boltzmann equation[2]. Our results indicate (as in reference 1) that the contribution of thermal backscattering to the flux arriving at the spacecraft consists predominantly of light species (H, H_2, He). The flux of the heavy contaminating species (HF, DF) is negligible.

A qualitative description of the thermal backscattering effect is obtained by considering the outer boundary of the ring-symmetric exhaust plume (Figure 1). It starts out at the nozzle lip on one side as a centered rarefaction fan, and it gradually curves in a balloonlike shape to the nozzle lip on the other side. This corresponds to a transition from a centered rarefaction fan flow to a cylindrically expanding flow and back to a centered fan. Thus, the velocity vector (whose magnitude is close to the limiting speed of expansion to vacuum for the exhaust total enthalpy), rotates from the oblique Prandtl-Meyer direction at the nozzle lip to an outright radial direction at the plume apex. Consequently, the thermal velocity needed to "turn around" a molecule at the fringes of the supersonic exhaust plume so that its resultant velocity vector points to the spacecraft, is minimal near the nozzle lips and becomes progressively larger toward the plume apex. Since the probability of a molecule having a thermal speed in excess of the local speed of sound decreases sharply as the speed increases, the primary source of thermally backscattered molecules is a portion of the centered rarefaction fan adjacent to the nozzle lip. It was found out that for the typical HF laser operating conditions (see below), the effective range in this regard is only about 0.1 (m) from the nozzle lip. A further simplification is obtained by noting that since the spacecraft radius (about 2.5 (m)) is much larger than this range, the flow field in the pertinent portion of the centered rarefaction fan can be approximated by the standard (plane-symmetric) Prandtl-Meyer flow field, which permits a closed-form integral expression for the thermally backscattered flux.

Our study pertains to presumably typical operating conditions of the HF laser. These operating conditions were largely determined from a report on some HF laser tests coducted at TRW in 1971[3] (in particular, Table 5, Test III of this report). Typical values of parameters at the nozzle exist are:

Composition (Mole Fractions)	[H]=.091 [HF]=.091 [DF]=.135;[He]=.579 $[H_2]$=.104
Specific Heat Ratio	$\gamma=1.54$
Mach Number	$M_1=4.0$
Average Molecular Weight	$W_{AV}=7.27$
Stagnation Temerature	$T_0=2300$(K)
Stagnation Density	$\rho_0=0.0075$(kg m^{-3})
Molecular Diameter (Hard Sphere)	$D=2.5 \times 10^{-10}$(m)

The exit Mach number is a design variable. We chose $M_1=4$ since it results in a modest clearance angle of 41° between the limiting streamline and the spacecraft (the corresponding Prandtl-Meyer turning angle is 49°). The stagnation temperature ought to be closer to 1400 (K), which corresponds to the given composition. It is estimated that the hydrogen recombination rate is too slow to be completed during the diffuser expansion[3]. However, we chose $T_0=2300$ (K) which corresponds to complete hydrogen recombination, since it is the most pessimistic choice, resulting in higher thermally backscattered flux.

The plan of this paper is the following. The integral form of the Boltzmann equation and the first iterate approximation are developed in section II. In section III we describe the application to a centered rarefaction fan and sketch the spatial integration of flux. Results for the typical operating conditions are given and discussed in section IV. References are given in section V.

II. INTEGRAL FORMS OF BOLTZMANN EQUATION

Boltzmann's equation can be written in an integral form which has the appealing physical interpretation of "counting molecules" having some velocity ξ, along their free-flight path from their point of last collision ("creation") to a point of a diverting collision ("annihilation"). We give a step-by-step account of this development which leads to a first iterate integral approximation to the Boltzmann equation[2].

Let $f_i(\vec{X},\vec{\xi})$ denote the velocity distribution function of species i (i=1, 2,...,n), at point $\vec{X}$. The Boltzmann equation for stationary flow of hard-sphere molecules is:

$$\vec{\xi}\cdot\vec{\nabla}_X f_i(\vec{X},\vec{\xi}) = \sum_{j=1}^{n} \sigma_{ij}\int[f_i' f_{1j}' - f_i f_{1j}]|\vec{\xi}_{1j} - \vec{\xi}|d^3\vec{\xi}_{1j} \qquad (2)$$

where σ_{ij} is the (hard-spheres) collision cross-section between species i and j, and f_i', f_{1j}', f_i, f_{1j}, are related for velocity vectors $\vec{\xi}_i'$, $\vec{\xi}_{1j}'$, $\vec{\xi}$, $\vec{\xi}_{1j}$ respectively. $\vec{\xi}_i'$ and $\vec{\xi}_{1j}'$ are the post-collision velocities of molecules i,j whose pre-collision velocities are $\vec{\xi}_i$, $\vec{\xi}_{1j}$.

The next step is to rewrite the partial differential equation (2) in a characteristic form, i.e., as an ordinary differential equation along a fixed line in space. This is done by observing that for some veloctity ξ parallel to a coordinate axis S , the Boltzmann equation assumes the following ordinary differntial relation:

$$\xi\frac{d}{ds} f_i(S,\vec{\xi}) + \omega_i(S,\vec{\xi})\, f_i(S,\vec{\xi}) = g_i(S,\vec{\xi})$$

$$\omega_i(S,\vec{\xi}) = \sum_{j=1}^{n} \sigma_{ij}\int f_{1j}|\vec{\xi}_{1j} - \vec{\xi}|d^3\vec{\xi}_{1j} \qquad (3)$$

$$g_i(S,\vec{\xi}) = \sum_{j=1}^{n} \sigma_{ij}\int f_i' f_{1j}'|\vec{\xi}_{1j} - \vec{\xi}|d^3\vec{\xi}_{1j}$$

This ordinary differential relation (assuming $f_i(S_1,\vec{\xi})$ is given as boundary condition) has the following formal solution:

$$f_i(S,\vec{\xi}) = f_i(S_1,\vec{\xi})\,\mathrm{EXP}[-\eta_i(S_1,S,\vec{\xi})] + \int_{S_1}^{S} \xi^{-1} g_i(S',\vec{\xi})\,\mathrm{EXP}[-\eta_i(S',S,\vec{\xi})]dS' \qquad (4)$$

$$\eta_i(S',S,\vec{\xi}) = \int_{S'}^{S} \xi^{-1}\,\omega_i(S'',\vec{\xi})dS''$$

As pointed out by Kogan[4] (section 2.7 of reference 4) the integral form (4) is not a unique integral representation of the Boltzmann equation (2). However, (4) is a desirable form since it lends itself to an immediate physical intepretation: $\eta_i(S',S,\vec{\xi})$ is the expected number of collisions of a molecule i traveling at velocity $\vec{\xi}$ from point S' to point S . The prbability that this molecule would travel from S' to S collisionlessly is $\mathrm{EXP}[-\eta_i(S',S,\vec{\xi})]$. Thus, (4) can be interpreted as "counting molecules" that arrive collisionlessly at S from the boundary at S_1 and from every intermediate point $S_1<S'<S$ where $\vec{\xi}$ molecules are generated at the rate $g_i(S',\vec{\xi})$.

This interpretation helps in justifying the following first iterate approximation to (4), for the case of transverse thermal scattering in a supersonic flow (such as Prandtl-Meyer rarefaction fan). The collision frequency $\omega_i(S,\vec{\xi})$ is greatly simplified by assuming that a sample molecule collides with a stream of "cold" gas flowing at velocity $\vec{U}(S)$ and having number density N(S). It is further assumed that all collision cross-sections are uniform, i.e., $\sigma_{ij} = \sigma$. The first-iterate natural choice for zero-order distribution function would be the local Maxwellian obtained from the continuum compressible flow field $B_i(S,\xi)$. Invoking the principle of detailed balancing in the equilibrium case, the zero-order generation term g_i is equal to the annihilation term $\omega_i f_i$. Consequently:

$$\omega_i(S,\vec{\xi}) = \sigma|\vec{\xi} - \vec{U}(S)|N(S)$$

$$g_i(S,\vec{\xi}) = \omega_i(S,\vec{\xi})\, B_i(S,\vec{\xi}) \qquad (5)$$

$$B_i(S,\vec{\xi}) = h_i N_i(S)\left[\frac{m_i}{2\pi kT(S)}\right]^{3/2} \mathrm{EXP}\left[-\frac{m_i|\vec{\xi}-\vec{U}(S)|^2}{2kT(S)}\right]$$

Where h_i is the mole fraction (assumed uniform). We also note that this approximation to g_i is consistent with the homogeneous case where $f_i(S,\vec{\xi})$ is independent of S. In this case the S integral in (4) becomes an integral of $\mathrm{EXP}[-\eta d\eta]$, so that (4) is identically satisfied.

Combining (4) and (5) with the continuum flow solution, which determines N(S), T(S) and $\vec{U}(S)$, the distribution function is evaluated for any $\vec{\xi}$ or S by performing the S-integration:

$$f_i^1(S,\vec{\xi}) = B_i(S_1,\vec{\xi})\mathrm{EXP}[-\eta_i(S_1,S,\vec{\xi})] + \int_{S_1}^{S} \xi^{-1}\, \sigma|\vec{\xi}-\vec{U}(S')|N(S')\, B_i(S',\vec{\xi})\mathrm{EXP}[-\eta_i(S',S,\vec{\xi})]dS'$$

$$B_i(S,\vec{\xi}) = h_i N(S)\left[\frac{m_i}{2\pi kT(S)}\right]^{3/2}\mathrm{EXP}\left[-\frac{m_i|\vec{\xi}-\vec{U}(S)|^2}{2kt(S)}\right] \qquad (6)$$

$$\eta_i(S',S,\vec{\xi}) = \int_{S'}^{S} \xi^{-1}\, \sigma|\vec{\xi}-\vec{U}(S'')|N(S'')dS''$$

From this expression for the distribution function at any $(S,\vec{\xi})$, the density and flux can be computed as the appropriate $\vec{\xi}$-moments. In the next section we describe how this approximation is applied to the cavity region bordering on a Prandtl-Meyer fan.

III. APPLICATION TO A RING-JET

The general expression (6) for the first iterate approximation to a distribution function is now considered for a centered rarefaction fan as shown in Figure 2. In this case it was found convenient to introduce a slight change in the coordinate system: ξ is antiparallel to S, S_1 *denotes a point on the exit* characteristic ($M=M_1$) and the distribution *function is specifically evaluated on the* limiting (vacuum) characteristic where S=0. The resulting expressions are (dropping S=0 arguments and superscript 1):

$$f_i(\vec{\xi}) = B_i(S_1,\vec{\xi})EXP[-\eta_i(S_1,\vec{\xi})] + \int_0^{S_1} \xi^{-1}\ \sigma|\vec{\xi}-\vec{U}(S)|N(S) B_i(S,\vec{\xi})EXP[-\eta_i(S,\vec{\xi})]dS$$

$$B_i(S,\vec{\xi}) = h_i N(S)[\frac{m_i}{2\pi kT(S)}]^{3/2} EXP[-\frac{m_i|\vec{\xi}-\vec{U}(s)|^2}{2kT(S)}] \qquad (7)$$

$$\eta_i(S,\vec{\xi}) = \int_0^S \sigma N(S)|\vec{\xi}-\vec{U}(S')|\xi^{-1}dS'$$

Since S<0 (Figure 2) corresponds to points in the cavity region where N(S)=0, $f_i(\xi)$ as given by (7) is the first iterate distribution function at any point on the *negative part of the S-axis.* Consequently, in order to evaluate flux or density at a cavity point, we have to compute $\vec{\xi}$-moments of $f_i(\xi)$ as given by (7). The flux arriving at point c on the cylinder (Figure 2) is given by:

$$Q_c = \int d^3\vec{\Omega}\cos\alpha \int_0^{\infty} \xi^3 f_i(-\xi\vec{\Omega})d\xi \qquad (8)$$

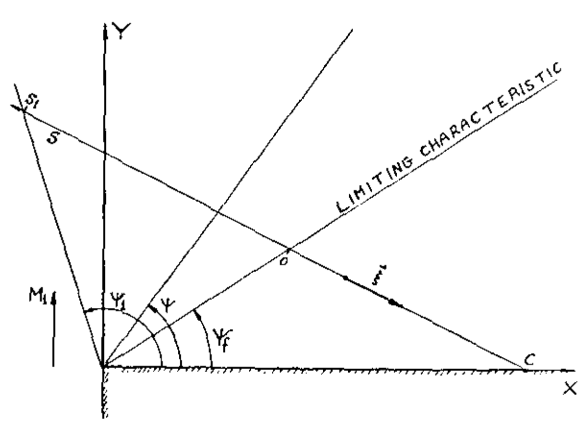

Figure 2. Prandtl-Meyer fan

where $\vec{\Omega}$ is a unit vector pointing from c to the fan, S for the integration in (7) is along $\vec{\Omega}$ and α is the angle betwen $\vec{\Omega}$ and the normal to the cylinder at c. In performing the integration prescribed by (8), account *must* be taken of the ring-symmetry of the rarefaction fan. This is done by considering area elements ($\Delta R, \Delta\phi$) on the conic surface generated by rotating the limiting characteristic about the spacecraft axis (ϕ is the angle of rotation, R is distance from nozzle lip). Each element subtends a solid angle $\Delta^3\vec{\Omega}$. The coordinate S is taken along the line *joining* c with the mid-point of the ($\Delta R, \Delta\phi$) element. The characteristic angle $\psi(S)$ is determined from appropriate three-dimensional geometrical relations, and the Prandtl-Meyer flow field is expressed as function of $\psi(S)$, as though the rarefaction fan were planar (R is much smaller than the radius of the spacecraft). The numerical integration is four dimensional (S,ξ,ϕ,R), and takes about 30 seconds computer time per point c (five species) on IBM 3033.

IV. RESULTS AND DISCUSSION

Consider the exhaust characterized by the typical operating conditions (1). The continuum flow field needed for evaluating the thermally backscattered flux (equations (6) and (7)), is given by the standard Prandtl-Meyer solution to a self-similar centered rarefaction fan, assuming the gaseous mixture is an ideal gas with homogeneous composition. The integral model was applied to this case, resulting in the molecular flux at various points on the spacecraft, for every *species.* These results are depicted in Figure 3, where they are also compared

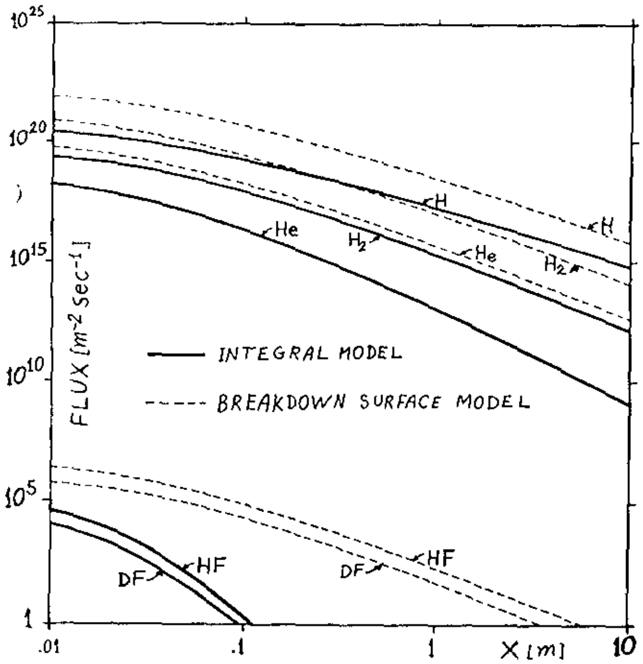

Figure 3. Thermally backscattered flux arriving at the spacecraft as function of distance from nozzle lip

with the corresponding flux evaluated from the breakdown surface model[1].

Due to the dependence of the Maxwellian distribution function on the molecular mass, the flux decreases sharply as the molecular mass increases. The five species are appropriately grouped as "light" (H, H_2, He) and "heavy" (HF, DF), due to the large disparity in their corresponding molecular weights (1,2,4 versus 19,20). Consequently, the flux of the heavy species is drastically lower than that of the light species (see Figure 3).

In fact, the thermally backscattered flux of HF and DF as evaluated by the present model is utterly negligible. We conclude that while other effects may still be found to contribute significantly to contaminating flux of HF and DF, the contribution of thermal backscattering to this flux may be neglected.

How reliable is this conclusion, considering the fact that it is based on an approximate solution to the Boltzmann equation, whose accuracy was not quantified?

We do not know of a mathermatical procedure for estimating the error introduced by the first iterate approximation to the Boltzmann equation. However, it may be helpful to compare the present flux with that evaluated from the breakdown surface model[1]. It was argued[1] that the flux predictions of that model were overestimated. Basically, the reason for that is that an "escape probability" factor, such as the factor $EXP[-\eta]$ (where η is an expected number of collisions) in the present model, was not included in the breakdown surface model. Indeed, the results shown in Figure 3 are consistent with the assertion that the breakdown surface flux is an overestimate.

We contend that the flux evaluated from the integral model is probably an underestimate of the thermally backscattered flux that would be obtained from an exact solution to the Boltzmann equation.

The reasoning leading to this assertion is based on the following interpretation of the present model. The flux arriving along a path S at the spacecraft (equations (6), (7) and Figure 2) is a sum of contributions from all points $S>0$ within the fan, multiplied by the no-collision probability factor. Under the first-iterate scheme, the continuum flow field (with its local Maxwellian distribution function) is the zero-order distribution function from which thermally backscattered flux is computed. However, it is well known from both experimental evidence and theoretical analysis[5], that each streamline in a source-like flow into vacuum, goes through a region of breakdown where the flow is no more governed by the laws of continuum gasdynamics. Specifically, this continuum breakdown process invariably entails an effective temperature higher than the temperature that would be obtained in isentropic expansion to the actual local density. As shown by Bird[6], the density in a rarefied Prandtl-Meyer fan is remarkably close to that of the isentropic (continuum) flow, up to a distance of less than one mean free path from the corner. Hence, the effect of continuum breakdown is to bring about higher temperature than that of the Prandtl-Meyer solution.

We may thus argue that an improved model would be obtained by replacing the "isentropic" Maxwellian $B_i(S,\xi)$ in (6), by a pseudo-Maxwellian distribution where the temperature has been substituted by the (higher) effective post-breakdown temperature. Obviously, this modification would give rise to significantly higher flux than that obtained from the present scheme. Hence, the present model underestimates the thermally backscattered flux.

In conclusion, we have presented plausible arguments (albeit not a comprehensive and rigorous proof) to the effect that the present integral model along with the former breakdown surface model, provide lower-bound and upper-bound estimates to the thermally backscattered flux from an HF laser exhaust plume.

V. REFERENCES

1. McCarty, S. E., Fuhs, A. E., and Falcovitz, J., "A Breakdown Surface Model for Thermal Backscattering from the Exhaust Plume of a Space-Based HF Laser", Naval Postgraduate School, Monterey, CA, NPS67085-010, 1985.
2. Patterson, N., Introduction to the Kinetic Theory of Gas Flows, University of Toronto Press, 1971.
3. Mastrup, F., Broadwell, E., Miller, J. and Jacobs, T. A., "Hydrogen Fluoride Laser Technology Study", Technial Report AFWL-TR-72-28, October 1972.
4. Kogan, M. N., Rarefied Gas Dynamics, Translated from Russian, Translation Editor Leon Trilling, Plenum Press, New York, 1969.
5. Bird, G. A, Molecular Gas Dynamics, Clarendon Press, Oxford, Section 8.3, 1976.
6. Bird, G. A., "Prandtl-Meyer Flow of a Finite Knudsen Number Gas", proceedings of the Seventh Australian Conference on Hydrodynamics and Fluid Mechanics, 1980.