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# Some Recent Advances in Numerical Weather Prediction[1]

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ABSTRACT

Recent developments in numerical weather prediction during the past several years are briefly summarized for the nonspecialist.

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## 1. Introduction

This review is intended to summarize some developments in numerical weather prediction which have occurred during approximately the last three years. This roughly covers the period since the completion of two major texts by Haltiner (1971) and Phillips (1973). Earlier research will be referenced only when required for the development of a particular topic. An indication of the intense activity in this field can be seen in the papers presented at the Second American Meteorological Society Conference on Numerical Prediction in Monterey, Calif., during 1–4 October 1973, for which the abstracts have been published in the July 1973 *Bulletin of the American Meteorological Society*.

Since the first application of numerical integration techniques to large scale weather prediction (Charney, Fjörtoft, and von Neumann, 1950), these techniques have been applied to a large variety of atmospheric and oceanic prediction problems. These include studies of the general circulation of the atmosphere and the oceans, tropical storms, fronts, sea breezes, cumulus convection, atmospheric pollution, the planetary boundary layer and turbulence. This review will focus on the developments in large scale weather prediction, although many of the results have applications to other problems. The parameterization of smaller scale phenomena will not be considered in this review because it deserves a separate comprehensive treatment.

Numerical integration of the atmospheric equations as an initial value problem is the primary basis for the prediction of synoptic-scale disturbances for periods between 12 hours and perhaps five days and, in addition, to some extent for smaller scales and much longer periods. The sources of error in such prediction are a consequence of a) gaps and errors in the data which make up the initial state, b) limitations in the objective analysis-initialization schemes which are applied to the data, c) truncation errors in numerical integration schemes, d) incomplete representation of the many complicated dynamical processes at work in the atmosphere and finally, e) limitations imposed by the predictability of the atmosphere.

## 2. Initialization and data assimilation

A standard method of objective analysis which has been in use for nearly 15 years or so begins with an initial guess based on a prognosis of the mass field from the previous analysis or, perhaps simply a persistence forecast. The initial gridpoint values are then modified with actual observations weighted according to the distance between the observation and the gridpoint.

Another common method of objective analysis interpolates between observations to gridpoints by requiring the gridpoint values to satisfy a Poisson equation. Starting with an initial guess, the gridpoint values are then modified by the use of observed values which are treated as internal boundary points. This scheme was recently evaluated by Leary and Thompson (1973) for accuracy with known spherical harmonics. Wavenumber 2 was reasonably well represented with 87% of its amplitude squared appearing in the analysis field, while only 13% of the input amplitude squared of wavenumber 12 survived the analysis. Aside from demonstrating a rather severe deficiency of this analysis

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scheme, the study suggested that there is a less steep drop-off in the kinetic energy of the wind field, perhaps a "$-2$" rather than the "$-3$" power law obtained from spectra of such objectively analyzed data.

With either method of analysis of the geopotential field the winds have been usually obtained geostrophically or by means of a balance equation, at least in middle latitudes.

In the tropics where the observational errors in the pressure field may be comparable to the pressure variations associated with synoptic disturbances (except for tropical storms), it is generally unwise to attempt to obtain the streamfunction from the pressure field by solving the balance equation, which, in fact, is singular at the equator. A recommended procedure is to calculate the vorticity directly from the observed *wind field* and then obtain the streamfunction by solving the Poisson equation $\nabla^2\psi=\zeta$. Finally, the geopotential field is calculated from the streamfunction with some form of the balance equation. Saha and Suryanarayana (1971) made a series of calculations of the geopotential in this manner from the quasi-geostrophic relation, the linear balance equation, the balance equation and the so-called vorticity form of the balance equation which are, respectively,

$$
\begin{aligned}
&\nabla^2\phi=f\nabla^2\psi\\
&\nabla^2\phi=\nabla\cdot(f\nabla\psi)\\
&\nabla^2\phi=2J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right)+\nabla\cdot(f\nabla\psi) \qquad (1)\\
&\nabla^2\phi=\eta\zeta+\mathbf{k}\cdot\nabla\eta\times\mathbf{V}-\nabla^2(V^2/2)
\end{aligned}
$$

Observed winds were used to evaluate the vorticity, $\zeta=\mathbf{k}\cdot\nabla\times\mathbf{V}$, and $\psi$ was obtained from $\nabla^2\psi=\zeta$. The geopotential fields obtained from the last three forms were very similar and compared favorably with the analyzed geopotential fields at the 850, 700, 500, and 300 mb levels. However, the last equation, the vorticity form, gave the least rms error.

With care to insure vertical consistency and avoid superadiabatic lapse rates the foregoing methods of objective analysis have been quite adequate for initializing the vorticity (filtered) type of prediction models. However, the primitive equation or P.E. models introduced operationally about a half dozen years ago are much more sensitive to the initial conditions than are the filtered models from which gravity waves have been eliminated. As a consequence, even second order differences between the objectively-analyzed wind and pressure fields and those existing in nature can give rise to quite large, false inertial-gravity waves with periods ranging from a few time increments, $\Delta t$, to a day or more. Through the "geostrophic adjustment" process which is inherent in the P.E. models, these spurious gravity modes are eventually dispersed and suppressed to acceptable magnitudes, usually within 12 to 24 hours if no new data is added. However, if, for example, non-synoptic data were to be added progressively to the model, the period of trauma would naturally be extended.

To reduce these spurious gravity waves greater care must be taken to bring the initial fields of pressure and wind to a state of better balance before beginning the forecast. This procedure is usually referred to as *initialization*, as contrasted to simply objective analysis, and it involves the expenditure of more computer time.

It should be mentioned at this point that several experiments (for example, Houghton *et al.*, 1971) indicate that meteorological forecasts of a day and longer do not appear to be effected in any major way by the presence of the initial gravity noise as measured by standard verification methods. On the other hand, for shorter periods, especially during the first 12 hours, the spurious gravity waves can generate sizable pressure changes and false vertical velocities. The latter, in turn, may produce unrealistic moisture patterns, precipitation and associated heating effects. The latter may be of relatively small scale and subsequently lost in coarse grid predictions.

Perhaps the most common method of obtaining the initial rotational part of the wind is by solution of the balance equation using objectively-analyzed geopotential fields on constant pressure surfaces. Since the nonlinear balance equation allows for curved motion to some extent, the resulting winds are in better balance with the pressure force than geostrophic winds. But, as shown by Phillips (1960), an appropriate divergent wind component is necessary if the inertial-gravity waves are to be excluded or negligible.

Sundqvist (1973), on the basis of limited tests, suggested that the solution of a balance equation with zero mass divergence ($\nabla_\sigma\cdot\pi\mathbf{V}=0$) on sigma surfaces would lead to initial winds which give less gravitational noise than the winds from the conventional balance equation on pressure surfaces followed by interpolation to sigma surfaces. Further comment on this will be made later.

In another approach, Temperaton (1973) verified that gravity oscillations were indeed present after initialization with the exact rotational part of the wind. The latter was obtained from an idealized experiment where "control" data was first generated by first running a numerical prediction model for two days using the Euler-backward scheme to dampen the high frequency modes leaving essentially only the slowly evolving meteorological modes. Then the geopotential field was treated as the observed field and a series of experiments were run for an additional day with different initialization methods. Specifically, the rotational wind component was extracted from the control field by solving the Poisson equation, $\nabla^2\psi=\mathbf{k}\cdot\nabla\times\mathbf{V}$, for the streamfunction $\psi$. Initializing with this wind resulted in gravity noise comparable to that resulting from winds obtained from solution of the balance equation.

The divergent component of the wind implied by the quasi-geostrophic theory can be obtained by solving the corresponding $\omega$-equation and then using the continuity equation to obtain the divergent wind potential $\chi$ as follows:

$$\nabla^2\chi = -\frac{\partial \omega}{\partial p}. \tag{2}$$

This is a rather lengthy task, however; and in a study of initialization with the NCAR global model Houghton, Baumhefner, and Washington (1971) found that inclusion of the vertical velocity from an $\omega$-equation and the corresponding divergent wind component in the initial winds did not significantly reduce inertial gravity oscillations.

At the National Meteorological Center (NMC) solution of the balance equation was abandoned several years ago in favor of extracting the rotational wind component directly from objectively analyzed winds. The resulting initial wind field gives better agreement with the observations without significantly increasing the noise level in the prediction model. In addition, the 12 h forecast divergent wind component has been added. However, in a very recent GARP report (No. 8, Jan 1975) Clifford, Brown, and McPherson found with the NMC global model the noise was independent of the initial divergence obtained by a variety of techniques, and further none consistently improved forecasts.

Although they are usually treated as separate steps, objective analysis and initialization have the same goal, namely, to provide an accurate, properly balanced state from which the hydrodynamical equations can be numerically integrated forward in time. Obviously, the two steps are not dynamically independent and need not be separated, which is the view taken by a number of investigators. A specific example is Sasaki's (1958) initialization by variational methods which incorporates dynamical constraints beyond the usual geostrophic wind law used in the objective analysis. These constraints have included the use of a generalized wind equation, suppression of high-frequency oscillations, as well as minimizing rms differences between observations and a first guess field (or the objective analysis). Lewis (1972) applied Sasaki's technique to develop an operational analysis of wind and temperature for the global band 40°S to 60°N from the surface to 250 mb. At the Monterey NWP meeting in October 1973, Sasaki applied the variational technique to the problem of initialization using a dynamical constraint on the kinetic energy which reduced the gravity wave noise considerably.

A spectral objective analysis has been developed by Flattery (1970) at NMC which fits Hough functions (the eigenfunctions of Laplace's tidal equation) to the observed data by a least squares method. The procedure is currently used operationally at NMC.

Several other approaches have been utilized to suppress the inertial-gravity oscillations usually present in the early stages of a P.E. forecast. One procedure consists of integrating forward and backward about the initial time starting with the objectively analyzed data and periodically restoring either the mass or the wind field at the initial time, usually the mass field in middle latitudes and perhaps the wind field in tropical latitudes. For this purpose it would appear advantageous to use an integration method which selectively damps high-frequency oscillations such as the Euler-backward scheme introduced by Matsuno (1966). However, as shown by Okland (1972), this scheme does not damp gravity waves well in low latitudes, nor waves of large horizontal extent or the high vertical modes. Thus considerable computing time may be required by this two-step scheme to accomplish the desired goal of suppressing the spurious gravitational noise, perhaps the equivalent of a 24- or 48-hour forecast which is operationally undesirable if not infeasible.

The presence of the gravity modes is usually reflected in the horizontal divergence, the vertical velocity and the surface pressure tendency. Recognizing that the divergent part of the wind is intimately related to the propagation of the gravity oscillations, McPherson (1973) tested a special viscosity term to dampen the divergent wind component based on the following concept:

$$\begin{aligned} &\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\cdots\cdots\cdots\cdots-\mu\frac{\partial D}{\partial x}+\nu\frac{\partial \zeta}{\partial y}=0 \\ &\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+\cdots\cdots\cdots\cdots-\mu\frac{\partial D}{\partial y}-\nu\frac{\partial \zeta}{\partial x}=0. \end{aligned} \tag{3}$$

Here $D$ is the horizontal velocity divergence and $\zeta$, the relative vorticity. Now when the divergence and vorticity equations are formed, one obtains

$$\begin{aligned} &\frac{\partial D}{\partial t}+\cdots\cdots\cdots\cdots-\mu\nabla^2 D=0 \\ &\frac{\partial \zeta}{\partial t}+\cdots\cdots\cdots\cdots-\nu\nabla^2\zeta=0. \end{aligned} \tag{4}$$

It is clear that with $\nu=0$ and $\mu\neq 0$, divergence but not vorticity will be suppressed. McPherson evaluated this novel concept with the NMC 8-layer global primitive equation model using Shuman's semi-momentum differencing. With $\mu=0$, oscillations primarily in the form of external gravity modes appeared in middle latitudes as follows:

| Amplitude | Period |
|---|---|
| 5–10 m | 3–4 h |
| 20–25 m | 6–7 h |
| 20–60 m | 10 h |

These waves were largely eliminated with a viscosity coefficient of $\mu=2.5\times10^8$ m²/s in about 7 hours time. Several unfortunate side effects occurred however; there was abnormally high surface pressure at the end of 12 hours in the vicinity of mountains together with a downstream trough. Also values of $\mu$ in excess of $10^7$ m²/s eliminated all precipitation. Clearly further evaluation is necessary. McPherson speculated that the slope of the sigma surfaces in the vicinity of mountains may be responsible. We would not find this surprising since two-dimensional diffusion on steep-sloping sigma surfaces near mountains may involve large vertical wind shear and temperature differences, with undesirable consequences. Perhaps the diffusion technique would be more successful if carried out on quasi-horizontal surfaces, or in such a way as to avoid changing the selective vertical diffusion.

At this point we would like to return to Temperaton's experiment on dynamical initialization. He used the so-called "shallow water" equations in flux form with spherical coordinates. In previous experiments several years ago Nitta and Hovermale (1967) had initialized by integrating forward $N$ time steps, backward $2N$ steps, and then returning to the initial time with $N$ steps forward; however, $N$ was taken to be just 1. The Euler-backward scheme gave somewhat faster convergence than the leapfrog scheme. Mesinger (1972) recommended a larger amplitude $N\Delta t$ and partial restoration of mass more frequently. The inclusion of heating and friction in this initialization procedure could lead to undesirable results if these processes are not computationally reversible.

Temperaton (1973) tried restoring mass only partially at the central time but was not successful. Using a time amplitude of $4\Delta t$ and $\frac{2}{3}$ restoration of the mass field at every other time step gave quite rapid convergence with respect to the total rms wind error. However, gravity wave activity, as measured by the rms divergence, was not diminished. Analysis of the winds showed that the rotational wind component converged rapidly toward the correct value but the divergent wind component was more in error than with the case $N=1$. He tried another approach of integrating forward $N$ time steps from the initial time, then $N$ steps backwards from the initial time and finally averaging the end results as follows:

$$u(t_0)=\tfrac{1}{2}[u(t_0+N\Delta t)+u(t_0-N\Delta t)]. \tag{5}$$

The mass is now restored and the procedure repeated as many times as is needed to bring about convergence. The leapfrog method was used except for a first forward time step. The averaging tends to remove high-frequency modes but generally leaves low-frequency waves relatively unaffected. For example, the $4N\Delta t$ period would be removed by such averaging while the $6N\Delta t$ wave would be reduced by about one-half. Iteration cycles of $3\Delta t$, $6\Delta t$ and $9\Delta t$ were tried with the best results obtained from the $6\Delta t$ (1 h) case. The Euler-backward method was tried for starting in each direction but showed no improvement over a simple forward step. The principal result was that considerably faster convergence was achieved using the averaging technique than with the Euler backward scheme and $N=1$. The high frequency oscillations in the rms divergence were markedly reduced to about one-tenth the amplitude occurring with the use of the exact rotational wind component without the averaging scheme. Nevertheless, there remained an easily recognizable oscillation with a period of about four hours. Elimination of this oscillation and further reduction of the noise was achieved by first adjusting the wind field while restoring the mass field at the initializing time and then repeating the forward and backward averaging procedure except that the mass field was averaged while the newly computed winds were restored after each cycle; thus both mass and winds were adjusted. In all, 20 cycles, equivalent to a 40-hour forecast, were completed, which is again too long for operational forecasting.

Considerable suppression of gravity noise can be achieved by using a running time average of the dependent variables in connection with the leapfrog scheme due to A. Robert (1966). Specifically, when a new value of a predicted variable, say $A(t+\Delta t)$, has been calculated using the leapfrog time differencing, a weighted average of the last three time values is calculated as follows:

$$A^*(t)=A(t)+\nu[A^*(t-\Delta t)-2A(t)+A(t+\Delta t)]. \tag{6}$$

The new value $A^*(t)$ is now used in the next leapfrog step to $A(t+2\Delta t)$, that is

$$A(t+2\Delta t)=A^*(t)+2\Delta t\left(\frac{\partial A}{\partial t}\right)_{t+\Delta t}. \tag{7}$$

The weight factor $\nu$ controls the amount of smoothing; $\nu=0.25$ gives the familiar 1-2-1 averaging. The simpler averaging function with $A(t-\Delta t)$ instead of $A^*(t-\Delta t)$ in Eq. (6) would completely remove waves of period $2\Delta t$ and dampen a $4\Delta t$ wave by about one half, while much longer periods are relatively unaffected. Recall here that the spurious computational mode generated in the leapfrog scheme has very nearly a period of $2\Delta t$ if a corresponding physical mode is of a relatively lower frequency. As a consequence, Robert's averaging procedure is effective in removing the computational mode. Moreover, repetition of the averaging at every time step tends to damp somewhat lower frequencies, including a considerable part of the spurious gravity noise generated through the early hours of a P.E. forecast. Asselin (1972) has computed the damping and phase shift characteristics as a function of frequency for a time filter of this form. Low frequency meteorological waves show negligible amplitude change and phase shift, but as expected there is very effective damping of computational modes.

In an initialization experiment with a five-level, global P.E. model, Haltiner and McCullough (1975) did not find any significant reduction of gravity noise with initial rotational winds from a balance equation solved on $\sigma$-surfaces as compared to similar winds from a $p$-surface balance equation. This may be somewhat surprising since from theoretical considerations, it might be expected that at least external gravity waves would be suppressed. Sundqvist assumed zero mass divergence, $\nabla_\sigma \cdot \pi \mathbf{V}=0$, where $\pi$ is surface pressure, to obtain the balance equation on sigma surfaces. It follows from the integrated continuity equation, namely,

$$\partial\pi/\partial t = -\int_0^1 \nabla\cdot(\pi\mathbf{V})d\sigma,$$

that if $\nabla\cdot\pi\mathbf{V}\equiv 0$, the surface pressure tendency will vanish initially which should tend to suppress external gravity waves. In any event, further initialization experiments with the $\sigma$-balanced winds should be carried out.

Haltiner and McCullough also combined the Robert time filter with the averaging scheme of Temperaton to substantially reduce the noise in the equivalent computer time of a 12 h forecast. The latter was accomplished by two cycles of averaging winds from a 3 h forecast and a 3 h hindcast and restoring the mass field to the initial values each time. The winds thus obtained together with the original mass field constituted the initial conditions for prediction, which resulted in a substantial decrease in high frequency surface pressure oscillations compared to the immediate use of the balanced winds for initialization.

To supplement the initial rotational wind component, estimates of the divergent wind component from 12 h forecasts or calculated from diagnostic vertical velocities have been used to initialize forecasts. A recent report by C. H. Dey, J. A. Brown, and R. D. McPherson (1975) revealed that use of the 12-h forecast wind divergence did not consistently improve forecast skill nor the suppression of gravity noise. Similarly, Houghton, Baumhefner, and Washington (1971) found that the addition of an initial divergent wind component calculated from diagnostic $\omega$-equations together with the vertical velocities did not reduce the gravity noise nor significantly affect the predictions *after* 12 hours.

*a. Non-synoptic data assimilation*

The advent of the satellite infrared spectrometers (e.g., SIRS), which can sound the atmosphere from space, has provided a new and potentially important source of data which could be very valuable over sparse-data areas. An analysis of the usefulness of satellite radiation data has been given by Bengtsson and Morel (1974). An important question is how to utilize most effectively data which do not occur at or very near the regular synoptic observation times, a problem already existing in connection with aircraft observations. The procedure for incorporating off-time reports into an analysis-prediction scheme is referred to as *four-dimensional assimilation.* Clearly the trauma of spurious inertial gravity waves associated with initialization must be controlled if new observational data is to be more or less continuously incorporated into a prediction procedure.

Some of the early idealized experiments dealing with the assimilation of SIRS data by Charney *et al.* (1969), Jastrow and Halem (1970) and Williamson and Kasahara (1971) showed that the simple insertion of temperature data can eventually determine the wind field and vice versa. The process of inserting some variables into a numerical forecast while others are unchanged has been referred to as *updating*, which seems to be a rather restrictive definition in terms of operational forecasting. In any event, the degree to which one field, for example, winds, can be obtained by updating another, perhaps temperature, depends in part on the predictability of the model which, in turn, depends on natural instabilities, nonlinear exchanges, frictional dissipation, etc. Using model data to simulate the real world, then introducing errors and updating with this "observed" data provides an indication of the best possible results that could be obtained by updating an operational prediction model with real data. In actual operational forecasts there are other sources of errors including the representation of the physical processes and numerical truncation. Naturally, the better the model, the less the predictability error, but simple insertion of data is assuredly not the best way to update. It would be quite logical to modify other variables by static or dynamic balancing and refer to the whole procedure as updating or four-dimensional data assimilation.

Mesinger (1972) suggested the following three names for methods of obtaining a proper balance between the mass and motion fields:

1) *Static balancing*—use of a wind law such as the geostrophic relation or the balance equation.
2) *Dynamic balancing*—integrating backward and forward about a central time, restoring mass or wind at $t=0$ or letting both vary. Sasaki's variational method can also be considered as a combination of static and dynamic balancing.
3) *Four-dimensional balancing* in space and time—introduction of data three-dimensionally periodically at discrete time intervals, including regular synoptic times.

Hayden (1972) used a 2-layer P.E. model with Shuman's semi-momentum differencing scheme to run a series of experiments with the insertion of temperature calculated from geopotentials which were in turn derived from the SIRS temperatures. Data were inserted six times during a 12 h forecast. Temperature tendencies over a period of one orbit were generally less than the observational error; consequently, as found by Talagrand (1971), too frequent updating may be deleterious. Hayden found that a static balancing, consisting of a geostrophic wind correction, $\Delta V$, computed from the changes in geopotential $\Delta\phi$ inferred from the temperature insertions aided the geostrophic adjustment process and reduced the shock of updating. He also attempted dynamic balancing with $N=2$, i.e., integrating two steps forward, $4\Delta t$ backward and $2\Delta t$ forward to return to the initial time.

Three criteria were used to measure the successful assimilation of data: a) the updating must not unduly shock the model which he inferred from a measure of the divergent wind component; b) does the model remember the data inserted (this was tested by reversing the forecast after 12 h and noting whether the hindcast temperatures were nearer to the inserted values than were the original temperatures that existed prior to the updating.); c) lastly, does the updating result in better mass and wind distributions when compared to the NWS analyzed fields? For this purpose the 12 h forecast was followed by a 12 h hindcast. Then the difference between the cycled data and the initial data were correlated with the difference between the NWS analysis and the initial state; a positive correlation indicates success.

The following conclusions were reached:

a) Even with poor initial conditions, temperature can be assimilated without shock if some balancing is performed to aid the geostrophic adjustment process. Dynamic balancing is sufficient but static balancing in the form of a simple geostrophic wind correction helps speed up the assimilation process considerably, at least outside the tropics.

b) Hayden anticipates that under operational conditions, where the model state is maintained similar to the observed state, four-dimensional assimilation can be accomplished without time-consuming dynamic initialization.

c) Four-dimensional data assimilation is evidently more effective than regular objective analysis of off-time reports that have been updated to regular synoptic times by Lagrangian advection. The exact details of the surface pressure appear to be relatively unimportant to the effectiveness of the four-dimensional assimilation, given sufficient time. However, the SIRS-B data by themselves are not capable of defining the circulation because the data density is not sufficient for objective analysis, nor with the model, of producing even an approximate surface pressure field.

Bengtsson and Gustavsson (1971) had previously found also that analysis prior to insertion of data, say from a satellite, leads to a more rapid reduction of error. Talagrand and Miyakoda (1971) showed that a synthesis technique of averaging forecasts made from objective analyses made at different times can reduce the random errors of measurement and analysis, sort of a Monte Carlo approach. They did some studies of inserting data into a running forecast when and where the data were available. If the difference between the predicted values and the observations is less than or equal to the observational error, *don't insert it*; there's no point to shocking the system and uselessly creating spurious inertial-gravity waves.

An important question not raised here thus far regarding SIRS data is what impact do the temperatures inferred from satellite radiances have on forecasts. As described in GARP Report No. 6, 1974 by Bengtsson and Morel, extensive studies have been conducted by GISS and NOAA personnel. The conclusion reached is that improvement in forecasts is slight, if any. But it is recognized that difficulties exist with respect to verification. There was lack of data over critical areas such as near troughs. Also the VTPR data may be able to detect small scale weather systems which are not resolved or predicted adequately on a coarse grid. In any case, it is reasonable to expect that with some improvement in temperature profiles and inferred winds from satellites, coupled with surface observations from a variety of sources including buoys, the combination will substitute to a considerable degree for the lack of regular soundings in otherwise data-void areas.

## 3. Integration methods

### *a. Semi-implicit schemes*

Although the momentum or primitive equations are simpler and involve fewer approximations than the filtered equations, the presence of gravity waves requires a much smaller time step to avoid computational instability with explicit integration schemes. Otherwise the small step is of little advantage or perhaps even harmful insofar as meteorological waves are concerned, and it is certainly expensive in terms of computer time. As a consequence, there has been considerable effort to circumvent the stability requirement. In the Soviet Union, Marchuk (1965) introduced a differencing scheme which treated the gravity modes implicitly and the low frequency meteorological modes explicitly, thus permitting a much larger time step. Kwizak and Robert (1971) successfully applied a semi-implicit differencing method similar to one suggested by Kurihara (1965) to a barotropic 500 mb forecast. To illustrate the technique we adopt the notation of Elvius and Sundstrom (1973) applied to the shallow water equations, which in differential form are

$$\begin{aligned} u_t &= -\phi_x - uu_x - vu_y + fv \\ v_t &= -\phi_y - uv_x - vv_y - fu \\ \phi_t &= -\phi(u_x+v_y) - (u\phi_x + v\phi_y). \end{aligned} \tag{8}$$

When these equations are put in difference form the pressure gradient terms in the momentum equations and the divergence term in the continuity equation, which together primarily govern the propagation of gravity waves, are evaluated semi-implicitly (i.e., with averages at times, $n-1$ and $n+1$) while the remaining terms are evaluated explicitly (i.e., at time $n$).

Then $u^{n+1}$ and $v^{n+1}$ in the differenced continuity equation are replaced by substitution from two momentum equations to form an elliptic equation in $\phi^{n+1}$ as follows:

$$[1-\Delta t^2\phi^n(D_{ox}{}^2+D_{oy}{}^2)](\phi^{n+1}-\phi^{n-1}) = F(x,y,t_n,t_{n-1}). \quad (9)$$

Here $F$ is composed of terms at times $t_n$ and $t_{n-1}$. After solving this equation for $\phi^{n+1}$, $u^{n+1}$ and $v^{n+1}$ may be calculated from the momentum equations. As a consequence of the implicit character of the difference equations with respect to gravity waves, a much larger time step is permitted without encountering linear computational instability. Of course, extra time is taken at each step to solve the elliptic equation, but overall, computer time is reduced by a factor of 3 to 4.

Elvius and Sundström (1973) suggested an efficient differencing system which is staggered in both space and time. This scheme not only permits a larger time step but also reduces the phase speed error of the low frequency or "meteorological" mode. They also developed suitable boundary conditions for use with a fine mesh model, which is undergoing further tests with realistic initial conditions. Treating the coriolis terms implicitly would also permit a slightly larger time step.

Baroclinic models are more complicated, but the semi-implicit technique is applicable in a similar manner. Gerrity, McPherson, and Scolnik (1973) have developed the semi-implicit difference equations using Shuman's semi-momentum differencing technique for the NMC 6-layer primitive equation model. The model has run stably for four days; however, it is not yet operational.

When the implicit scheme is used, the phase velocities of gravitational oscillations are reduced, hence the geostrophic adjustment process can be retarded, as will any initialization procedure for damping spurious inertial-gravity waves. Experiments by McPherson and Kistler (1973) verified the delayed damping of the gravity waves; however, because of the larger time step, dynamic initialization can be accomplished with less computer time. The net result was still a gain of 2 to 1 over the explicit integration during an initialization procedure.

A somewhat similar procedure, but saving only about $\frac{1}{3}$ computer time, evaluates only the pressure force semi-implicitly. This is accomplished by first solving the continuity equation for the surface pressure, $\pi$ ($\sigma$-coordinates), then the thermodynamic equation for $T$, then with the new $T$ and $\pi$, the hydrostatic equation for a new $\phi$. Now the pressure force can be evaluated implicitly while integrating the momentum equations.

## 4. Direct methods for Helmholtz and Poisson equations

The need to solve Helmholtz-type equations in connection with the semi-implicit methods, as well as the balance equation, has stimulated interest in the more recent "direct" methods to solving Poisson and Helmholtz equations. Leslie and McAvaney (1973) have compared the speed and accuracy of a number of methods for solving equations of the form:

$$\nabla^2\phi-\alpha(x,y)\phi=f(x,y) \quad (10)$$

where the Helmholtz coefficient $\alpha$ and the forcing function $f$ are known. When approximated by the usual finite difference analogues, the foregoing equation is expressible as a system of linear equations with a block tri-diagonal matrix of coefficients. In the past, iterative methods such as the Liebmann successive over-relaxation method (SOR) have been widely used because of their simplicity. However, faster direct methods developed in recent years are replacing the relaxation methods, particularly when a rectangular region is involved with simple boundary conditions. The direct methods may be divided into four categories as follows:

a) *Block method*, which uses the fact that $A$ is block tridiagonal.
b) *Cyclic reduction method* (*DCR*), which reduces the dimensions of the matrix to be solved in a recursive manner. This method had been restricted to certain numbers of interior points, such that with an $MXN$ grid either $M$ or $N$ must equal $2^k-1$. (This limitation has been overcome very recently as will be mentioned later.)
c) *Matrix reduction* which through coordinate transformation reduces the problem to a simpler tridiagonal form that has an easily computed solution. The dimension reduction method (DRM) is relatively simple when a fast Fourier transform is available.
d) Finally, in certain circumstances the *fast Fourier transform* can be applied directly.

Table 1 shows some comparative solution times and accuracies (RE) of the DCR, DRM, and SOR methods applied to Poisson and Helmholtz equations on a $65\times65$ grid. It is clear that with respect to Poisson type equations, the direct methods are far superior. To solve the Helmholtz equations by direct methods, the Helmholtz coefficient has to be a constant or small. On the other hand, the relaxation (SOR) methods permit a variable Helmholtz coefficient and give rapid convergence when that coefficient is large. This technique can then compare favorably with the direct methods, particularly if a little less accuracy is accept-

TABLE 1. Comparative computation times and relative accuracies (RE) for several methods of solution of Poisson and Helmholtz equations (from Leslie and McAvaney, 1973).

| | DCR | | DRM | | SOR | |
|---|---|---|---|---|---|---|
| | RE | Time | RE | Time | RE | Time |
| Poisson | $10^{-6}$ | 0.89 | $10^{-6}$ | 1.25 | $10^{-6}$ | 42.3 |
| | $10^{-13}$ | 0.90 | $10^{-13}$ | 1.27 | $10^{-10}$ | 75.9 |
| Helmholtz | | 5.3 | | 7.5 | $10^{-2}$ | 6.4 |
| | | | | | $10^{-4}$ | 11.0 |
| | | | | | $10^{-6}$ | 16.3 |

able, say, RE$\sim 10^{-4}$. Also the SOR methods, which are more readily generalized to irregular domains and mixed boundary conditions, are simple to program and require minimal storage in comparison to direct methods.

In U. S. A. Roland Sweet of NCAR has used block cyclic reduction methods to obtain fast, accurate solutions to Poisson's equation. Basically these programs were restricted to rectangles or their equivalents in other coordinate systems, such as the surface of a sphere in spherical coordinates. Except for Dirichlet boundary conditions, the number of intervals in one direction must be of the form $2^p3^q5^r$ where $p$, $q$, and $r$ are integers. The residual error is near the limit of the computer, i.e., about $10^{-13}$ when double precision is used on an IBM 360 on a 63×63 grid.

These techniques have been extended by T. E. Rosmond and F. D. Faulkner of the Navy Environmental Research Facility (EPRF) in Monterey (personal communication) to solving Helmholtz equations iteratively on rectangles, spheres and disks and for a number of three-dimensional problems. When the Helmholtz term or the coefficients of the differential operator vary with both independent variables, the direct methods can be used iteratively but are advantageous over the SOR methods only if there is not much variation in the coefficients from row to row. Refining the grid for a given region has little effect on the convergence rate of the fast Poisson solvers but decreases the convergence rate for SOR methods.

The extension of the block cyclic reduction method to three dimensions presents no significant difference from the two-dimensional application and programs have been developed at EPRF for solving separable 3-d elliptic equations for all the common boundary conditions. The programs require the number of intervals in two coordinate directions to be a power of two. The latter restriction can be relaxed by extending the techniques which have eliminated this restriction in the two dimensional algorithms. These developments will greatly facilitate implementation of the semi-implicit numerical schemes to fluid dynamics problems.

## 5. Global grids

As computing capability has improved in the last several years efforts have increased toward operational global forecasting and also in fine mesh models. The absence of lateral boundaries in a global model is an important advantage since such unnatural barriers cause errors in hemispheric models that eventually propagate from the fictitious boundaries in the tropics into middle latitudes. Numerical forecasting in the tropics, which admittedly has severe limitations at present, would be quite hopeless if artificial boundary conditions are imposed in low latitudes. On the other hand, a tropical band with middle latitude walls is not satisfactory either because these boundaries are far too active. So the only alternative appears to be global models despite the vast areas of little or no conventional data in the Southern Hemisphere, albeit the weather satellites are helping to overcome the data problem.

A significant difficulty with global models is the lack of a suitable plane projection which does not seriously distort some areas. The most natural approach is a latitude-longitude grid; however, the convergence of the meridians poses a problem, for as the distance between equal longitude spacing shrinks toward the poles, a shorter time step is needed to maintain linear computational stability. A time step short enough to maintain stability in polar regions is exceedingly wasteful in low latitudes. Various techniques have been used to overcome this difficulty with varying degrees of success. The most common approach of late has been to filter out or dampen the short waves that would lead to instability near the poles so that a relatively large time step can be used throughout, rather than simply decreasing the time step with increasing latitude. In the Arakawa-Mintz (1972) model, the procedure consists of temporarily Fourier analyzing fields which will be differenced with respect to longitude and then modifying such Fourier amplitude so that the CFL stability criterion can be satisfied without shortening the time step. This principle can be illustrated with a simple advection equation for which the stability criterion for wavenumber $k$ and maximum wave speed $c$ is typically of the form

$$\frac{c\Delta t}{d_j}\sin kd \leq \epsilon, \tag{11}$$

where $d_j = a\cos\varphi_j\Delta\lambda$ is the grid distance at latitude $\varphi_j$ and $\epsilon$ is a constant perhaps unity. By reducing the amplitude of each wave component of the longitudinal gradient at each latitude by a factor $S_{jk}$ the criterion becomes

$$(S_{jk})\frac{c\Delta t}{a\nabla\lambda\cos\varphi_j}\sin kd \leq \epsilon. \tag{12}$$

It is clear that for a fixed $\Delta\lambda$ and $\Delta t$, computational stability can be maintained by decreasing $S_{jk}$ as the latitude and wavenumber increase. This type of procedure is applied to longitudinal derivatives in the terms involving gravity wave propagation.

At the NOAA Geophysical Fluid Dynamics Laboratory Holloway, Spelman and Manabe (1973) applied space filtering to all time integrated variables at each time step. The filtering limits the east-west wavelength at all latitudes to the distance of two gridlengths at the equator. The minimum wavelength is given by $L_{\min}=4\pi aN^{-1}$, where $N$ is the number of gridpoints around a latitude circle and $a$ is the radius of the earth. The maximum number of waves at latitude $\varphi$ is

$$\frac{2\pi a \cos\varphi}{L_{\min}}=(N/2)\cos\varphi, \tag{13}$$

which is rounded to the nearest integer. This is accomplished by a Fourier analysis followed by synthesis with only the desired waves at each latitude circle included. The procedure has no significant effect on quadratic conservation properties of the differencing schemes. The components of vector variables are first transformed into polar stereographic coordinates to avoid the problems associated with averaging vectors from widely separated longitudes where the unit vectors differ substantially in direction as discussed by Shuman (1970). When tested on barotropic and baroclinic models the foregoing procedure proved to be superior to the Kurihara grid which has a poleward decrease in the number of gridpoints per latitude circle in such a way as to keep the distance between gridpoints from decreasing appreciably.

Williamson and Browning (1973) found with a grid that is uniform in a curvilinear coordinate system, the accuracy of approximations involving curvilinear velocities is less near the singularities. In order to avoid the small time step associated with a uniform grid, they tried the method of skipping points near the poles to maintain a more nearly uniform distance between gridpoints, but the skipped grid resulted in large errors. More accurate interpolations did not help this matter. However, by applying the Fourier technique to remove short wavelengths the errors were comparable to a uniform gird requiring a much shorter time step.

## 6. Fourth-order differencing

In another aspect of computation-time vs accuracy, it has been shown that a greater reduction in phase error can be achieved per unit of extra computer time by selective use of fourth-order space differencing than by reducing the grid size [see, for example, Williams (1972)]. This may be illustrated with the advection equation $\partial F/\partial t+C(\partial F/\partial x)=0$. Suppose $\Delta t$ is chosen to maintain linear computational stability in a P.E. model which permits a fast external gravity wave with a phase speed $C_g$, of perhaps 300 m/s or more, and $C_g\Delta t/\Delta x\simeq 1$. Then for the slower meteorological waves, with phase speed, say $C_m$, the ratio $C_m\Delta t/\Delta x$ will be much less than one, perhaps a tenth, or so. Some computations (unpublished) were made for a centered difference form of the above equation with both second- and fourth-order space differences and second-order time differencing (leapfrog). As an illustration of the improvement in phase speed accuracy, assume $C_m\Delta t/\Delta x=0.2$. Then the ratios of the finite difference phase speeds to the true speed for several wavelengths are as follows:

| $L$ | $4\Delta x$ | $6\Delta x$ | $8\Delta x$ | $10\Delta x$ | $12\Delta x$ |
|---|---|---|---|---|---|
| 2nd order | 0.64 | 0.84 | 0.91 | 0.94 | 0.96 |
| 4th order | 0.86 | 0.97 | 0.99 | 1.00 | 1.00 |

Although this is a much simplified illustration, improvements in phase speed accuracy of 5 to 20% or more can be expected with fourth-order space differences. Moreover, the latter need only be applied to the terms governing the propagation of meteorological waves and not the terms involving gravity wave propagation. It should be mentioned that the fourth-order approximation required a somewhat smaller time step, perhaps 20 to 25%, to maintain linear computational stability [see Haltiner and Williams (1973)]. On the other hand, halving the grid distance in the simple one-dimensional model described above would quadruple the computation time, yet the resulting improvement in phase speeds would be roughly the same as going from second to fourth order, as may be seen by comparing the $4\Delta x$ and $8\Delta x$ ratios above (0.64 and 0.91), or the $6\Delta x$ and $12\Delta x$ values, 0.84 and 0.96.

## 7. Mesh models

Because of the enormous range of scales of atmospheric phenomena and limitations of even the most modern computers, it is obviously impossible to model numerically all phenomena on a single mesh of uniformly spaced gridpoints. Scales which are too small to be represented with a specified length must be parameterized in terms of the large-scale variables if their influence is to be felt. However, this procedure may be inadequate at times. Also there are important subsynoptic and mesoscale phenomena for which prediction is highly desirable, even if only for a short range of time. Consequently, it is desirable at times to superimpose a fine mesh grid (FMG) on a coarse mesh (CMG) covering a much larger area, perhaps a hemisphere or the entire globe. Quite a few meteorological organizations, both foreign and domestic, are currently carrying out numerical integrations on such fine-mesh, limited area grids.

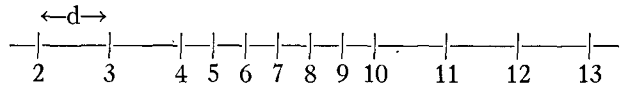
One of the most critical problems in dealing with limited area forecasts, including the superposition of different grids, is the treatment of the boundary conditions. This problem is not really new and had to be faced in the first numerical prediction experiments by Charney, Fjørtoft, and von Neumann (1950). They concluded correctly that the optimal procedure was to specify precisely as many boundary values as required by the corresponding linearized equation to have a well-behaved solution. Additional values needed for the finite difference equations should be computed by extrapolations from interior values. Apparently their method of extrapolation proved to be unstable, as later shown by Platzman (1954), and they simply specified all values on the boundary. This gave stable results which were less accurate and the conditions were more stringent than necessary. Having to maintain constant values along inflow boundaries can lead to large errors propagating into the forecast region; though this is certainly less of a problem when fine-mesh boundaries are permitted to change periodically through the use of coarse-mesh forecasts. Nevertheless, the latter situation is in a sense a special case of the limited area forecast, for although the fine-mesh boundary values are no longer constant, it is necessary to obtain them by interpolation in space and time from coarse-grid values. The objective then is to do this in such a way as to avoid any instability at the boundaries and to obtain the most accurate forecast possible.

In general, overspecification of boundary conditions leads to parasitic waves, perhaps instability; while too few conditions will leave the solution undetermined.

The proper procedure for a barotropic primitive equation model, proposed by Elvius and Sundström (1973), following Charney (1962), is to prescribe at all boundary points the quantity $V_n-\varphi/\bar{\varphi}_0^{1/2}$, which corresponds to the outgoing characteristic. This permits gravity waves to leave the region rather than be reflected. In addition, the tangential component of velocity is prescribed at inflow points. For a baroclinic system the situation is much more complicated, and the equations are no longer hyperbolic when the hydrostatic assumption is imposed. Sundström (1973) recommends an approximation which specifies the tangential velocity at inflow points as above while at outflow points the tangential velocity is extrapolated from the interior. Next a combination of normal velocity component and potential temperature corresponding to the inward external gravity wave is specified on all boundaries, and the combination corresponding to the first outward gravity wave is computed by extrapolation from interior values in such a way as to avoid instability. This would not allow the internal gravity waves to leave the region.

Returning now to actual experiments with nesting of grids, the early efforts of superimposing a fine mesh on a coarse mesh consisted of integrating the coarse mesh model, for say 24 hours, and saving the "history tape" of data at every time step. Then the boundary values for the fine mesh model are interpolated in space and time from the CMG predictions, which is an overspecification and less accurate but nevertheless is computationally stable. Here the FMG predictions are influenced by the large scales depicted on the CMG, but not vice versa. Nevertheless, the fine mesh gives better resolution of physical features and permits smaller scale disturbances to develop. Some examples of this approach are the experiments of Hill (1968), Wang and Halpern (1970).

A successful meteorological experiment which permitted interaction between the CMG and FMG through *simultaneous* integrations was carried out by Harrison and Elsberry (1972). By utilizing a movable fine mesh centered over a traveling one-dimensional gravity wave the FMG produced forecasts comparable to those obtained by a fine mesh everywhere. The boundaries of the adjacent CMG and FMG overlapped as illustrated below.




In their scheme the FMG integration determines the values at points 6 and 8; then these values are used for subsequent predictions for points 4 and 10 in the CMG. Similarly, the FMG predictions for points 5 and 9 utilize the values at points 4 and 10 determined by the CMG integrations. Thus, for example, with a simple advective equation $\partial u/\partial t+U\partial u/\partial x=0$, the interface equations are

$$\frac{\partial u_4}{\partial t}=-\frac{U}{2d}(u_6-u_3), \tag{14}$$

and

$$\frac{\partial u_5}{\partial t}=-\frac{U}{d}(u_6-u_4). \tag{15}$$

The computed changes for the larger time steps at the CMG points which form the boundaries of the FMG (i.e. points 4 and 10) are proportioned equally to supply the intermediate temporal values of $u_4$ which are needed at the smaller time intervals to integrate the adjacent FMG points (i.e., points 5 and 9). Similarly, linear spatial interpolation is performed to obtain values between CMG points as needed for the FMG integration. As a consequence of this procedure the FMG integration influenced the CMG, moreover the integration procedure is stable and did not create unacceptable noise or near-discontinuities at boundaries.

Applying this procedure to an idealized two-dimensional tropical disturbance and keeping the fine mesh grid centered over the disturbance gave results equivalent to a fine mesh everywhere in terms of forecast intensity and also for energy fluxes across the boundaries dividing the grids. However, some "noise" did develop at the boundary. This was suppressed by smoothing across the boundary and the addition of a small diffusion term.

Phillips and Shukla (1973) considered the two strategies of *one-way interaction* using the history tape and the *two-way interaction* such as the one just described. By a heuristic argument based on the method of characteristics, they inferred that the two-way interaction procedure would give a more faithful reproduction of the proper transmission of information into and out of the fine-mesh region. Some numerical tests with the shallow-water equations showed that the two-way strategy did indeed lead to less error. They also found that the Lax Wendroff two-step scheme gave a somewhat larger error at 12 hours than did the leapfrog scheme, but the reverse was true at 24 and 48 hours.

Ookochi (1972) combined a fine mesh with a coarse mesh for the integration of barotropic primitive equations in flux form on a staggered grid. The results were essentially a composite of complete fine-mesh and coarse-mesh integrations with no significant noise at the boundaries. The principal integral properties involving mass, total energy, etc. were well conserved during the 96 h experiment.

Harrison (1973) describes some further experiments with systems of two and three nested grids for the simulation of a tropical storm by integration of the primitive equation on a four-level model. His calculations demonstrated the feasibility and advantages of nested grids in savings of computer time. Presumably better phase speeds would be achieved as a consequence of the fine mesh because of smaller truncation errors. Note, however, that any wave, particularly those that are poorly represented on the coarse grid, will change phase speed when passing through the interface into the fine mesh, and again later when it leaves the fine mesh. As a consequence, near discontinuities in phase and erroneous interaction can occur with that part that remains in the coarse net.

## 8. Spectral methods

The spectral method expresses the spatial variation of the prediction fields in terms of a series of orthogonal functions. The coefficients in the series are now the forecast quantities rather than gridpoint values of the original dependent variables. We will illustrate the technique with the barotropic vorticity equation:

$$\frac{\partial}{\partial t}\nabla^2\psi+\mathbf{k}\times\nabla\psi\cdot\nabla(\nabla^2\psi)=0 \qquad (16)$$

where $\psi$ is the streamfunction and the beta term has been neglected for simplicity. We expand $\psi$ into the following series:

$$\psi(x_1,x_2,t)=\sum_{\alpha}\Psi_\alpha(t)Y_\alpha(x_1,x_2). \qquad (17)$$

The functions $Y_\alpha$ are orthogonal and normalized so that

$$\int Y_\beta{}^*Y_\alpha dS=\delta_{\alpha\beta}, \qquad (18)$$

where $Y_\beta{}^*$ is the complex conjugate of $Y_\beta$ and

$$\delta_{\alpha\beta}=\begin{cases}0 & \alpha\neq\beta\\ 1 & \alpha=\beta.\end{cases}$$

The integration in (18) is carried out over the entire forecast region. The functions $Y_\alpha$ satisfy the equation

$$\nabla^2Y_\alpha+K_\alpha Y_\alpha=0, \qquad (19)$$

where the eigenvalues $K_\alpha$ are positive and increase for the smaller scale eigenfunctions. Substitute (17) into the forecast equation (16), multiply by $Y_\gamma{}^*$, integrate over the entire region and use the relation (18) which gives

$$-K_\gamma\frac{d\Psi_\gamma}{dt}+\sum_\alpha\sum_\beta L_{\gamma,\alpha,\beta}\Psi_\alpha\Psi_\beta=0. \qquad (20)$$

The interaction coefficients, which can be computed once and for all, are given by

$$L_{\gamma,\alpha,\beta}=-K_\beta\int Y_\gamma{}^*\mathbf{k}\cdot\nabla Y_\alpha\times\nabla Y_\beta dS. \qquad (21)$$

Equation (20) represents an infinite number of ordinary differential equations when all appropriate values of $\gamma$ are considered. In practice the series given by (17) is truncated in such a way that the desired meteorological features are acceptably described. The equations for the remaining coefficients can then be integrated in time numerically.

Lorenz (1960) treated Eq. (16) with this procedure in cartesian coordinates. In that case the eigenfunctions are products of sines and cosines and the interaction coefficients take on a particularly simple form. He noted that when the series is truncated total energy will still be conserved if the excluded coefficients are set to zero in the interaction sums. He also demonstrated the usefulness of very low order systems with his study of a 3-component set.

Silberman (1954) first applied this technique in spherical coordinates although he kept the zonal mean flow fixed in time. Platzman (1960) and Baer and Platzman (1961) performed a complete treatment of the barotropic vorticity equation in spherical coordinates and also carried out some experiments. Orszag (1974) has given an excellent treatment of the spectral method in spherical coordinates. In spherical coordinates the eigenfunctions are spherical harmonics which are composed of sines and cosines of the longitude and Legendre polynomials of the sine of the latitude. With these functions the interaction coefficients are more complicated and they have more nonzero elements than in cartesian coordinates.

The spectral method has several advantages over the gridpoint method. First the spectral method computes spatial derivatives exactly so that the phase error which occurs with the finite difference method is eliminated. Also the aliasing that occurs with finite differences is excluded and as a result it is easy to conserve certain quantities which are conserved in the continuous equations. Poisson equations are easily solved without relaxation or other inversion techniques because of relation (19). Another advantage is the treatment of global motions without the presence of singularities.

The most important disadvantage of the spectral method is that it requires much more computer time than the gridpoint method if there are very many degrees of freedom. This can be seen from Eq. (20) which shows that for each degree of freedom many products must be computed in the nonlinear term. With the gridpoint method, for each degree of freedom, there are only a few products involving quantities at surrounding gridpoints. The spectral method is generally more complicated to apply and it is not convenient for complicated boundaries. The spectral method also suffers when values must be evaluated locally such as in determining condensation criteria. We shall see that some of these difficulties have been alleviated with recent developments.

Robert (1966, 1968) modified the spectral technique in two studies with the primitive equations on the sphere. In place of the spherical harmonic functions he used the following function of sines and cosines:

$$G_n{}^m(\lambda,\varphi)=e^{im\lambda}\cos^{|m|}\varphi\sin^n\varphi. \tag{22}$$

These functions are not orthogonal, but the spherical harmonic functions can be written as a sum of the functions given by (22). The nonlinear terms are easily computed with these functions, and the number of elements involved is much less than with the spherical harmonics. When the orthogonality relations are required the spherical harmonics can be formed in terms of these functions. Using this simplified procedure Robert carried out a number of spherical integrations with a relatively small number of terms.

A very important advance for the spectral method was the development of the fast Fourier transform by Cooley and Tukey (1965). Their technique allows Fourier transformation of a field with $N$ degrees of freedom with order $N \log_2 N$ operations, while the direct method requires order $N^2$ operations. This allows rapid calculation of local fields by summing the series with the fast Fourier transformation, such as might be required for condensation tests.

Orszag (1969, 1970) has used the fast Fourier technique to save computation time in the evaluation of the nonlinear terms. He carries out all differentiation of quantities while they are represented by the series, but products of fields are performed with the fields at gridpoints. This requires an inverse Fourier transform to obtain data at gridpoints and a regular Fourier transformation to return to spectral form. The fast Fourier transformation can be used in both of these operations with a great time savings. When this procedure can be used it greatly reduces prediction time, for large numbers of variables, because the sum in (20) is replaced by a much faster operation.

Merilees (1973a) has developed an algorithm for computing the sum of a series of spherical harmonics. The algorithm is approximately 10–20 times faster than the standard method, but it suffers a precision problem and it breaks down when the resolution is too high. Orszag (1974) has discussed a method for avoiding this difficulty which permits rapid computation of the nonlinear terms; thus the prediction time for a global spectral model can be greatly reduced.

Bourke (1972) formulated a global spectral model based upon the one-layer shallow water equations. He used the vorticity and divergence equations instead of the 2 components of the equation of motion. The technique for evaluating the nonlinear terms which was developed by Orszag (1970) and Eliasen *et al.* (1970) is employed. The computational efficiency of the model was found to be far superior to that of an equivalent model based on the interaction coefficients. This model, in integrations of 116 days, satisfied the principles of conservation of energy, angular momentum and square potential vorticity to a high degree.

Daley (1973) used Bourke's (1972) model to examine the possibility of using different spatial resolution for different dependent variables. He suggested that for the smaller scales it would be sufficient to predict only the wind field because for those scales the pressure field adjusts to the wind field. In a test the smaller scales were predicted with a filtered model and the larger scales with the full primitive equations. He found that this combined system gave better forecasts than a primitive equation system with intermediate resolution. The application of this procedure to baroclinic equations may be more difficult.

Recent developments have made the spectral models much more competitive with gridpoint models with respect to computational efficiency. For the same resolution the spectral models can be expected to give better forecasts, and the forecasts may be better even with somewhat poorer resolution. A series of tests are needed to determine whether or not better forecasts can be made with the spectral method with the same computational effort.

Orszag (1972) and Merilees (1973b) have proposed a forecast technique which is a combination of the finite difference method and spectral method. This technique which is known as the pseudospectral approximation employs finite differences except in the computation of spatial derivatives. Before the derivatives are computed, the gridpoint data is fast Fourier analyzed. Continuous derivatives are then computed and the series is summed with the fast Fourier transform. This procedure is used for both longitudinal and latitudinal derivatives. The latitudinal derivatives in some cases require a sign change at the poles. Merilees (1973b) found excellent results in a test prediction of Haurwitz waves with the shallow water equations.

## 9. Finite element method

The extensive success of the finite element method in solid mechanics has attracted the interest of investigators in other fields. Salinas (1973) has given a brief review of the method which proceeds as follows: At the start, an approximate solution in the form of a linear combination of specified (basis) functions is assumed. The coefficients (multipliers) of the basis functions are to be determined to yield the best approximate solutions. This is accomplished by minimizing a measure of the error (called the residual function) associated with the assumed solution. Several techniques are available for minimizing the residuals. Normally the basis functions are defined in such a way that they have a simple variation (quadratic, cubic, etc.) over an element of area (piecewise polynomials), outside of which they are zero. The "elements" can have a variety of shapes. The finite element method is a generalization of the method of weighted residuals.

Newton (1973) has successfully applied the technique to gravity waves radiating out from an oscillating ship. Gallagher and Chan (1973) have treated the steady state circulation in Lake Ontario. Triangular and otherwise shaped elements were used to accurately fit the lake's coastline. Wang *et al.* (1972) applied the method to the one-dimensional shallow water equations. They obtained better results for both the linear and nonlinear cases than with the usual finite difference methods. Price and MacPherson (1973) have used a similar method in the two-level general circulation model developed by Mintz and Arakawa (Langlois and Kwok, 1969). They arranged the elements in such a way that the area of each element is nearly constant over the entire globe. They also provided for a subregion in which the elements telescoped to a smaller size. Predictions with this model gave encouraging results.

For meteorology, the principal advantage of the method as shown by Price and MacPherson (1973), is the possibility of easily changing the element size and shape. This would be useful in situations where meshed models are now used and also on the sphere where the elements could be kept at a fixed scale.[2]

## 10. Vertical coordinates

Kasahara (1974) has presented a review of vertical coordinate systems used in numerical weather prediction. The most commonly used vertical coordinate in primitive equation prediction models is the sigma coordinate ($\sigma$) which leads to $\sigma \equiv 1$ and $d\sigma/dt = 0$ on the lower boundary. This simplification of the lower boundary conditions is accompanied by a more complicated expression for the pressure gradient force.

The use of potential temperature as a vertical coordinate [Eliassen (1962)] has received new attention in the last few years. If there is no heating the isentropic surfaces will move as material surfaces. This feature is very helpful for resolving frontal zones and sharp, upper-level jets. A possible disadvantage of the isentropic coordinates is the fact that the potential temperature shows a large variation along the lower boundary. Primitive equation integrations have been carried out by Eliassen and Raustein (1967), Gall (1972), and Shapiro (1973) with isentropic coordinates. They encountered no major difficulties, and they obtained good simulations of the jet stream structure. Gall (1972) included latent heat in his study. Bleck (1973, 1974) made forecasts with real data with the quasi-geostrophic form of the isentropic potential vorticity equation. The forecasts showed skill in predicting cyclone development. The isentropic models show considerable promise for limited area models which treat the smaller scale synoptic features. These models have not been tested in global formulations or for longer period forecasts.

## 11. Forecasting skill

With the greater sophistication in numerical modelling and the large advances in computer technology, it may be rightly asked whether there have been concomitant improvements in forecasting skill. To answer this question we may turn to verification statistics published by the National Weather Service which are probably indicative of other groups as well. Figure 1 from Shuman (1972) shows the $S_1$ scores, which are approximately a measure of the normalized rms vector error of pressure gradient, for 30 h, sea-level (upper scale) and 500 mb (lower scale) forecasts for North America from 1948 to 1971. (Shuman states that in terms of practical skill, scores of 0.30 at sea level and 0.20 at 500 mb are nearly perfect while scores of 0.80 at sea level and 0.70 at 500 mb are considered essentially worthless.) There is clearly a general downward trend over the years indicating increasing skill. The improvement in the latter half of the 1950s and early 1960s is

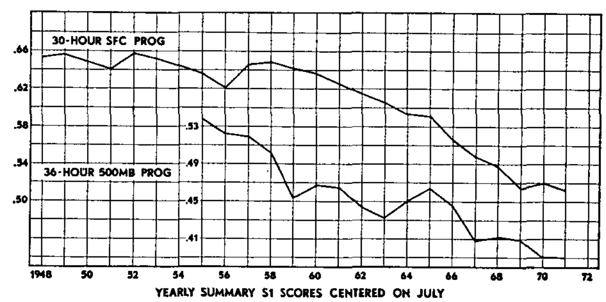



FIG. 1. (After Shuman, 1972.) Annual average $S1$ scores (Teweles and Wobus, 1951) for 30 h sea level (upper curve) and 36 h 500 mb (lower curve) forecasts. The $S1$ score is roughly a measure of normalized rms vector error of pressure gradient. The area of verification for both levels covers North America. The two curves are plotted on the different scales shown. The scale for sea level is the one labelled from 0.50 to 0.66, the scale for 500 mb from 0.39 to 0.54. To calibrate the scores in terms of practical skill, a sea-level forecast with a score of 0.30 is virtually perfect; one with a score of 0.80 is worthless. For 500 mb, 0.20 represents a virtually perfect forecast, 0.70 worthless.

[2] Further time savings can be achieved if the semi-implicit method of Section 3 is used, especially since the inversion of Eq. (9) is trivial with spectral functions.

ascribed to the introduction and continuing improvements in the barotropic numerical 500 mb forecasts, while at sea level there was increasing skill on the part of prognosticators in the use of the 500 mb forecasts.

The first successful baroclinic model, a three-level filtered version, became operational in 1962, which is not reflected clearly in the graphs because the years 1964–66 proved to be a more difficult period to forecast and barotropic forecasts suffered as well. Nevertheless, a new plateau of skill was established and there was a general downward trend of $S_1$ scores. The first surface baroclinic model was a simplified graphical type due to Dr. Richard Reed which went into operation in 1962 and drew from the independently-made numerical 500 mb predictions. The continuing improvement in surface predictions from 1962 to 1966 was largely a result of Reed's model.

The NMC 6-layer primitive equation model has been under continuous development since 1966; it is the first numerical model at NMC to produce a better sea-level prediction than manual predictions without NWP guidance. However, with the numerical product in hand, man can improve the prognosis by about five points on the average in terms of $S_1$ scores. The net result has been a continued improvement in the skill scores.

Figure 2 from Cooley and Derouin (1972) shows the NMC average 36-hour, 500 mb $S_1$ scores. The light shading shows the human improvement. Figure 3 shows the 30 h surface scores.

Considering the marked increase in accuracy of surface and 500 mb prognoses the corresponding improvement in forecasts of weather elements is somewhat disappointing. Precipitation-no precipitation forecasts for the periods 12–24 and 24–36 hours improved only about 5% between 1959 and 1970. There was somewhat greater improvement in temperature forecasts. For example, the annual number of maximum temperature forecast errors equal to 10°F or greater at Salt Lake City decreased from 60 in 1949 to 22 in 1971.

Sanders (1973) recently reported on six years of forecasting temperature and precipitation by *staff* and *students* of Massachusetts Institute of Technology for

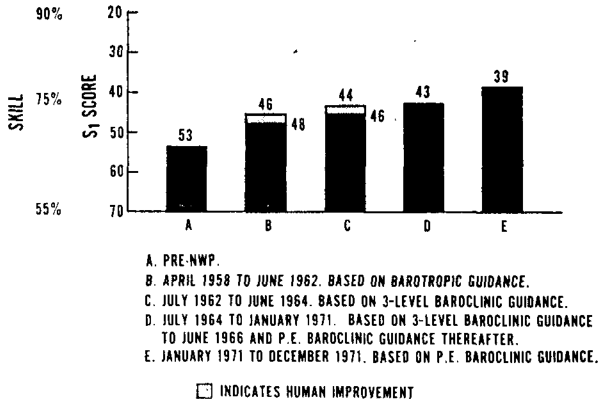



FIG. 3. NMC average 36 h 500 mb $S_1$ scores.

the local observation site at Logan Airport. Despite continuous improvement in predicted synoptic patterns at the surface and aloft, there has been no increase of skill in temperature and precipitation forecasts as measured by the incremental accuracy over the climatological mean. As may be expected, skill decreased with increasing length of forecast and more rapidly with precipitation than with temperature. Minimum skill occurred in summer, probably due to the greater influence of mesoscale phenomena. Sanders also indicated that the lack of improvement in forecasting temperature and precipitation and even a slight downward trend in the latter is also shared by NMC forecasters. He suggests that the reason for the first day forecasting difficulties may lie in the fact that the benefits of short-range synoptic-scale forecasts of the mass field have been maximized and that the source of the errors in precipitation and temperature forecasts are primarily a result of mesoscale phenomena such as fronts, land-sea circulations, convection, urban influences, etc. According to Sanders' results skill in precipitation forecasts drops to 10% in 2.5 days and in four days for temperature. He suggests that something of a breakthrough in synoptic forecasting is needed to improve significantly prediction beyond the first day.

Despite some variations in the trends of forecasting skills, it is safe to conclude that, overall, forecasting ability has shown definite improvement since the advent of numerical weather forecasts. Moreover, further improvements can be expected in the latter half of the 1970's as the current research efforts in numerical techniques, simulation of the physical processes, and initialization techniques become operational, along with better satellite data and smaller mesh sizes.

We may now ask the question is there no limit to the ultimate range and accuracy of weather forecasts if one is willing to spend enough money to provide the necessary data and enough computing power for the calculations? To answer this question the predictability of the atmosphere must be considered.

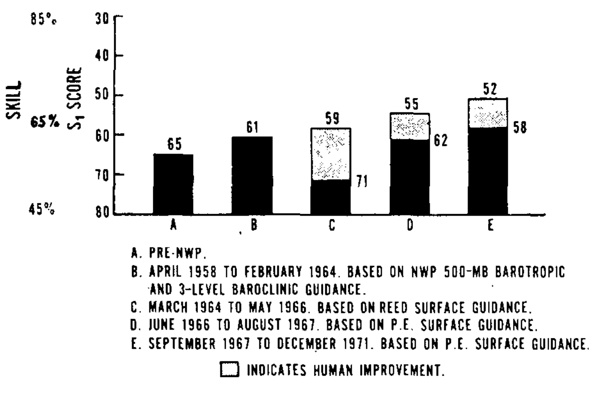



FIG. 2. NMC average 30 h surface $S_1$ scores.

## 12. Atmospheric predictability

Lorenz (1965) pointed out that perfect prediction can never be expected because a) the governing laws are not perfectly known but only approximations, b) nor is the system of equations strictly deterministic, and finally c) the initial state is not perfectly known. Moreover, he shows that even for a determinate system of equations, such as one normally used in numerical weather prediction, separate solutions which are nearly identical at the initial time do not remain nearly identical as time progresses, but eventually become as different as two solutions chosen randomly at the same time and day of the year. The atmosphere contains some periodic oscillations—principally the annual and diurnal variations and their overtones—which are predictable at essentially infinite range. However, accurate long range prediction of the remaining features is impossible because the initial state of the atmosphere will always be imperfectly known.

Lorenz (1969a) suggested three approaches that might be used to estimate the predictability of the atmosphere:

a) The dynamical approach wherein a system of equations closely resembling the atmospheric equations is integrated from slightly different initial conditions and then the rate of amplification of the differences is determined;

b) The empirical approach which utilizes similar weather types, usually referred to as "analogues" and determines their subsequent separation in statistical terms as a function of time;

c) The empirical-dynamical approach which utilizes a new set of equations (derived from the atmospheric equations) which describes a spectral distribution of forecast errors.

Most of the studies which follow (a) used general circulation models which had been integrated until they were in approximate balance. At this time another integration was begun with an initial state which consisted of the balanced field plus a small departure. As this solution departed from the control experiment the growth rate of the error field and the range of predictability were obtained. Experiments by Mintz and Arakawa, Smagorinsky and Leith have been summarized by Charney *et al.* (1966). Charney estimated the rms doubling time of small errors to be about 4 to 5 days and a predictability limit imposed by typical observational errors of about two weeks. Smagorinsky (1969) presented the results of experiments carried out at the NOAA Geophysical Fluid Dynamics Laboratory with their 10-level primitive equation general circulation model. A random temperature disturbance with a standard deviation of 0.5°C at all levels was added to the control run. The vertical average of the standard deviation dropped from 0.5°C to 0.2°C after one day, reflecting a "geostrophic adjustment" between the disturbed fields and the undisturbed fields. Thereafter, the error growth was exponential for the next seven days with a doubling time of about 2.5 days. Smagorinsky concluded that the *deterministic limit of predictability* for synoptic scale disturbances is about *three weeks*; however, the *current practical limit is about one week*. He also noted that short-wave predictability decays most rapidly.

The cause of the exponential growth of small errors in these studies was attributed to baroclinic instability [Charney *et al.* (1966)]. However, Lorenz (1972) suggested that the error growth might be due to barotropic instability of wave disturbances. In particular he investigated the stability of a finite amplitude unbounded Rossby wave. He found instability when the waves are sufficiently strong or the wavenumber is sufficiently high, and the growth rates are comparable to the growth rates obtained from predictability studies. Lilly (1973) emphasized the importance of barotropic instability in a predictability study which employed a 2-dimensional model. Recently, Lorenz (1973b) has concluded that baroclinic instability is the most important cause of lack of predictability in the atmosphere.

Lorenz (1969c) utilized approach *b* to obtain the rate of separation of two fields which were initially similar. Five years of twice-daily height values of the 200, 500, and 850 mb surfaces on a grid of 1003 points were obtained. A weighted root-mean-square height difference of the difference between two states, or the "error." For each pair of states occurring within one month of the same time of year, but in different years, the error was computed. Numerous mediocre analogues were found but there were no really good ones. The smallest errors had a doubling time of about eight days. Since larger errors grow less rapidly this is probably an over-estimate of the doubling time. Extrapolation with the aid of a quadratic hypothesis indicates that very small errors double in about 2.5 days. This compares very well with the numerical results reported by Smagorinsky (1969).

In a more recent paper Lorenz (1973a) has used the same data set to investigate the range of predictability. States of the atmosphere separated by 12 days or less are found on the average to resemble each other more closely than randomly selected states, even after an adjustment for the seasonal trend has been made. Higher correlations were obtained with a form of damped persistence. These results demonstrate the existence of partial predictability of instantaneous weather patterns at least 12 days in advance.

Lorenz (1969b) followed approach *c* with a statistical treatment of the 2-dimensional vorticity equation. He derived an equation for the "error energy" which is obtained from the difference between two solutions. Thompson (1957) considered similar equations. The linearized equations are written in spectral form and ensemble averages are taken. A further assumption is included to close the set of predictive equations. The equations require the specification of a mean energy spectrum, and Lorenz used the −5/3 law for the smaller scales. The equations were integrated numerically from an initial error distribution; the error energy of each wavenumber was not allowed to exceed the mean atmospheric energy for that wavenumber. The numerical solutions shows a rapid growth of the error for the very small scales and a very slow growth for large scales. Lorenz's solutions show that the range of predictability for cumulus scales is almost an hour, while the synoptic-scale motions can be predicted a few days ahead. Predictability for the largest scale disappears after about 17 days. In particular if the initial error is confined to the smallest scales the error in those scales rapidly reaches the maximum error. Then growth occurs in the next larger scales and so on until the error is propagated to the largest scale. Lorenz points out that if the error in the smallest scale is reduced then the range of predictability of the largest scale features will only be increased by a time interval which is less than the range of predictability of the small scales where the error was reduced. Although these results are dependent on the closure assumption and the energy spectrum which is used, the study shows clearly the importance of the nonlinear propagation of errors between different scales of motion.

Robinson (1967, 1971) assumed that the dynamic equations do not allow one to predict motions of a given scale over periods longer than the fluid elements of this scale maintain their identity against turbulent diffusion by smaller scale motions. Then based upon the dissipation characteristics of the atmosphere, he derived predictability ranges of a few days for synoptic-scale motions and about one hour for cumulus scale motions, which are roughly the same as obtained by Lorenz (1969b).

Leith (1971) and Leith and Kraichnan (1972) have also used homogeneous isotropic turbulence models to study atmospheric predictability. Leith (1971) considered two dimensions and closed his equations with the eddy-damped Markovian approximation. Leith and Kraichnan (1972) considered both two and three dimensions and they used Kraichnan's (1971) closure method which is based on the test-field model. The solutions in three dimensions showed that even where there is a strong energy cascade there is also an upscale propagation of error. The two-dimensional solutions used a "−3" power energy spectrum for the smaller scales. Charney (1971, 1973) showed that quasi-geostrophic three-dimensional flow should have a "−3" power energy spectrum under proper conditions. Leith and Kraichnan (1972) found that the two-dimensional flow was more predictable than three-dimensional flow. They concluded that an initial state determined with a horizontal resolution feasible with a satellite-based observing system would result in significant predictability of large scale motion for more than one week. They also point out that the test-field model probably underestimates rather than overestimates predictability times.

In summary, the studies of predictability suggest that specific weather patterns and events are predictable, or partially so, only for a period of at most several weeks; however, the possibility of predicting general trends of perhaps temperature and precipitation above or below normal for longer periods is not precluded as yet.

## 13. Stochastic dynamic prediction

Recent studies in stochastic dynamic prediction are closely related to atmospheric predictability. Epstein (1969) developed a method for including the effect of random errors in initial conditions in a forecast model. The forecast equations which he treated were of the spectral type with quadratic nonlinearities. He first took an ensemble average of the basic equations which leads to predictive equations for the ensemble average or the expected value of each dependent variable. These equations also include the variances and covariances between the dependent variables. Equations for the time change of the latter quantities, which are second moments, are obtained by multiplying each equation by each of the variables and carrying out the ensemble averages. These equations, however, contain the third moments. Equations for the time change of the third moments are obtained in the same way and they contain the fourth moments. This is merely the closure problem of classical turbulence theory.

Epstein closed his equations by neglecting the third moments about the instantaneous mean. He integrated the resulting equations for the Lorenz (1960) three component system. Initially he specified the expected value of each variable as well as its variance due to possible observational errors. The initial covariances were set equal to zero. A deterministic forecast was also performed with original prediction equations. Numerical integrations showed that the deterministic solution eventually diverges from the stochastic prediction of the expected value. The latter should be a better forecast in the statistical sense. The predicted variances grow in time which gives the influence of the initial error. Also a large number of deterministic forecasts were made with slightly different initial conditions. These numerical solutions were then averaged using a Monte Carlo method. The difference between the Monte Carlo average and the approximate stochastic solution is a measure of the error which is caused by the closure assumption. In some of the solutions these quantities remained close while in others they began to depart after a few days.

Fleming (1971a) has continued this work with a two-level quasi-geostrophic spectral model similar to the one used by Lorenz (1965). For interpretation he divided the energy into a "certain" energy and "uncertain" energy. He also reconsidered Epstein's closure assumption which neglected the third order moments. Fleming tested the quasi-normal approximation which computes the fourth order moments in the third order equations in terms of the second order moments. This requires the numerical solution of the third order equations in addition to the others. He found that this approximation was better up to a certain time, after which it became unstable. He then considered a modified form in which a linear damping term was added to the third order equations. This eddy-damped quasi-normal approximation was stable and gave excellent results in most cases.

Fleming (1971b) used the eddy-damped quasi-normal approximation in a stochastic formulation of the barotropic model. He used two wavenumbers in latitude and 15 in longitude in a study of predictability. The predictability times obtained are similar to those obtained by Lorenz (1969b). Predictability studies using the baroclinic model of the earlier paper showed that predictability is increased when heating and friction are present. Fleming (1972) considered the effect of random variations in the thermal forcing and the effect of errors resulting from the improper treatment of the smaller scales.

Leith (1974) has concluded that the stochastic dynamic method cannot be used for the larger numerical models because of the very excessive computer time requirements. He suggests instead the use of the Monte Carlo procedure which involves a collection of deterministic forecasts. The procedure was evaluated with a simple two-dimensional turbulence model of large-scale atmospheric motions. A study of the dependence on sample size of mean square forecast error, both with and without a final linear regression correction, showed a considerable improvement with moderate sample sizes over conventional single forecasts without regression. These results suggest that a sample size of about 10 may be adequate for producing the error variance information needed for optimal data assimilation.

## 14. Summary

Research on initialization has intensified in the last few years. Most operational forecast models employ the primitive equations which are sensitive to the initial balance between the wind and the pressure fields. Short range prediction of precipitation is also very sensitive to the initial divergence field. The development of global models requires a proper initialization in the tropics which may differ from middle latitudes. The most promising initialization techniques include the variational formulations and the averaging of backward and forward forecasts about the initial state. The assimilation of nonsynoptic data into a forecast model has received considerable attention. This has been motivated by availability of nearly continuous satellite data. An important problem here involves controlling the imbalance introduced into the forecast by the localized new data. Several groups have developed limited area models which are designed to give better forecasts in specific regions. These models use boundary conditions from hemispheric or global models and in some cases the limited area forecast is allowed to affect the exterior region. Isentropic models have been developed and applied to smaller scale synoptic features.

Operational predictions of pressure and wind have shown continuing improvement as a consequence of progressively better numerical models. However, the concomitant improvement in forecasting precipitation and temperature has not kept the same pace. It is expected that forecasts of weather elements will improve as the limited area models produce better descriptions of mesoscale systems.

Studies of predictability suggest that specific weather patterns and events are predictable, or partially so, only for about two weeks. The possibility of predicting general trends above or below normal for longer periods is not as yet precluded. Perhaps the consideration of atmosphere-ocean interaction will lead to longer range prediction of weather trends.

The stochastic dynamical prediction procedure is closely related to studies of atmospheric predictability. The procedure uses the uncertainty in the initial state to predict the uncertainty in the forecast at later times as well as the expected value. This method cannot be used for operational forecasts in the near future because of the large computer time requirements. However, an indication of the growth of uncertainty can be obtained by examining a relatively small number of deterministic forecasts with slightly different initial conditions.

The development of the semi-implicit method has potential for a significant savings in computer time. The method treats the gravity wave terms implicitly and the advection terms explicitly. This permits the use of a longer time step with the additional requirement that an elliptic equation be solved at each time step. The net effect of this is a reduction in computer time by a factor of at least 2. The recent development of direct methods for solving elliptic equations which are faster and more accurate than the traditional relaxation methods will be of great value in the semi-implicit methods. However, the latter still have some difficulties over mountains at the time of writing.

Present operational forecasts often underpredict the movement of synoptic disturbances and this error has been attributed to space truncation error in the finite difference equations. Several groups are using fourth order space differencing in order to reduce this error. Also the pseudospectral method has been put forward as a method to eliminate space truncation effects.

New interest has developed in the use of the spectral method since the introduction of the fast Fourier transform. The use of this transform greatly speeds up computation time with the spectral method. Further tests are required to determine whether the spectral method will give better forecasts than the finite difference method for the same amount of computer time.

The finite element method of solid mechanics has been recently introduced into meteorology. This method has the advantage of a very flexible element size, but it has not been widely tested on meteorological problems.

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