1975-08

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Journal of Applied Physics, Vol. 46, No.8, August 1975
http://hdl.handle.net/10945/44941

Anode phenomena in a collision-dominated plasma

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(Received 10 February 1975)

Anodes display either a glow mode or a constricted mode in plasmas of interest for MHD generator applications. The purpose of this paper is to outline the conditions underlying the existence of anode constrictions or anode spots in conjunction with criteria governing the anode glow. A steady current flowing through velocity and thermal boundary layers is investigated. The sheath and the ambipolar region are considered from an approximation theory viewpoint, and then the nonexistence of a one-dimensional Cartesian or diffuse mode for a nonreacting anode region is shown using the continuum equations for electrostatic probes.

PACS numbers: 52.40.K, 52.70.D, 52.75.F, 52.80.H

I. INTRODUCTION

There is great interest in collision-dominated plasmas for MHD power generation, flow lasers, and many other applications. As pointed out by Cobine,[1] electrical discharge phenomena are evident in a wide range of MHD applications. The nonequilibrium or two-temperature plasma[2] is one example, and plasma behavior at the electrode regions is another. The anode electrode region displays two modes of operation which are also evident in gas discharges, namely, a glow mode and a spot mode. It will be expedient to exclude surface charge emission and hence to exclude the cathode from consideration here. The region affected by the anode comprises the sheath and the ambipolar region, and may go beyond these depending on the nature of the boundary layers and factors relating to the plasma proper.[3]

The understanding of phenomena which occur within the anode region is important in MHD devices because of the large voltage drops which are attributable to the electrodes[4,5] and because of constraints on the stability of diffuse currents that this region may impose. These constraints are independent of emission requirements (which prevail at the cathode) and can play a significant role in determining the formation of arc spots and thus the lifetime of electrode materials; moreover, in devices such as the MHD laser, nonuniformities in current are very detrimental to the operation of the laser.

In the conventional glow discharge, the anode-glow mode is associated with an ionization/recombination region adjacent to the anode which is necessary for current continuity.[6] As the current density increases, the glow "splits up" into a series of spots.[7] The existence of these spots depends not only on the level of current but also on electrode materials and on impurities in the gas. In the arc discharge, anode spots are also observed and are associated with an oxidizing environment.[8] In inert gases, including nitrogen, the anode of the arc discharge may operate in a diffuse mode. In metal vapor arcs,[9] the presence of the vapor is critical for anode spot formation.

The principal difference between the conventional gas discharge and the type of plasma discussed in this paper is that the existence of the latter is independent of the electrical power which is being dissipated at the electrodes. In this respect, the partial uncoupling of the power necessary to generate the plasma and that needed to flow the current is a simplification of the phenomena involved and suggests that the description of the nonemitting electrode is the same as that of the electrostatic probe. Cohen,[10] Su,[11] Lam,[11,12] and others have studied the continuum equations that describe electrostatic probes. Most descriptions have been limited to spherical probes with no volume ionization or recombination, and results predict electron temperatures and plasma potentials with some degree of success.[13] The calculated current-voltage characteristics of a highly positive probe, however, have little resemblance to anode drop data. We believe that the presence of anode spots may be a major source of discrepancy because results derived from the continuum probe equations are very sensitive to the configuration of the current distribution in the neighborhood of the electrode. This is discussed by Lam[12] and further elaborated upon in this paper. Positive ion emission from alkali-covered anodes may be another source for the discrepancy.[14,15]

In this paper, experimental observations are reported before the formulation of the problem is introduced. This is done in order to show the role that the degree of equilibrium of charges moving through a thermal boundary layer plays in the existence of anode spots. Of course, the degree of equilibrium governs the intensity of an existing glow, but the nonreacting or frozen-flow limit will be of particular interest here.

II. EXPERIMENTAL OBSERVATIONS

While there exists a wealth of information relating to observations in gas discharges, there is comparatively little that pertains to high-pressure flowing plasmas. In this section we present some data[16] which are remarkable because both a glow and a spot mode were evident at the anode. The primary difference between the two types of behavior was the effect of impurities (air) in the gas although the contribution of surface changes cannot be completely ruled out.

Argon seeded with 0.1% potassium at 1 atm and 2100 °K was observed in steady flow between a single pair of electrodes. Convection effects were not dominant in these results since the transit time of the gas ($\sim 10^{-4}$ sec) was considerably longer than other important times. The Mach number was about 0.1. The electrode

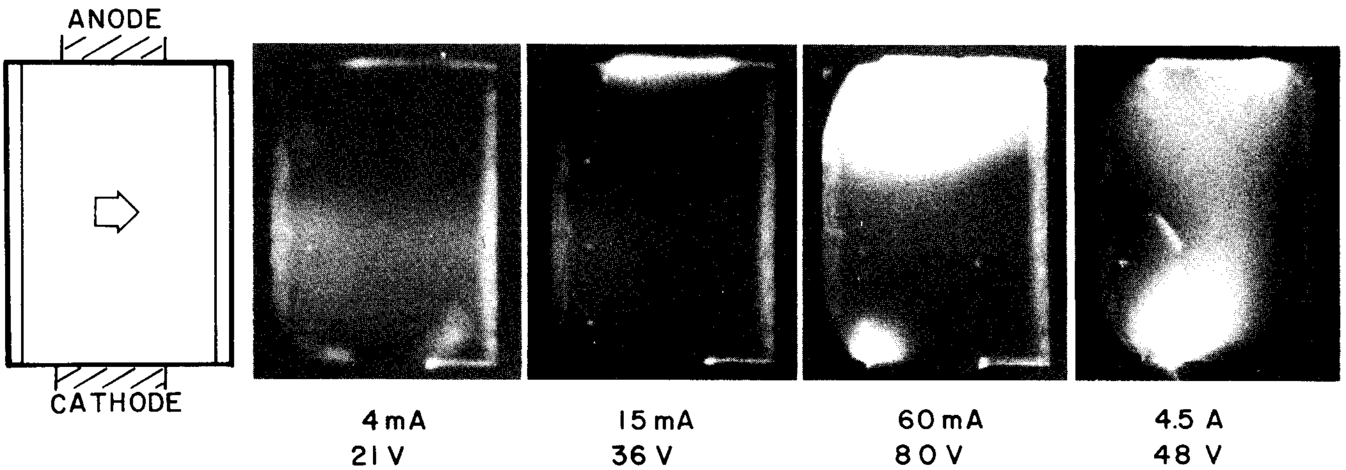

FIG. 1. Glow mode.

spacing was 5 cm, the electrode area 6 cm^2, and the anode temperature 860 °K. Figure 1 shows photographs of the anode glow, while Fig. 2 shows a photograph of anode spots.

Because the plasma itself can be quite bright, the anode glow may not always be conspicuous. In Figs. 1 and 2 most of the currents are relatively low; this permitted maximum visibility of the electrode regions. Once the cathode spot forms, the current jumps in magnitude but either the glow or spot mode prevails at the anode. Probe data in the absence of seed indicate that the difference in the anode drop between the glow and the spot mode is upwards of 15 V. This is evidence that the main species ionizing[1] in the glow is argon. The anode drop associated with these data is of the order of 50 V; other MHD data, dealing in particular with seeded combustion products, indicate anode electrode region drops of about 100 V. These anode drops are somewhat independent of the current[5] and may range between 20 and 250 V.

In the ensuing discussion, we will be referring to an MHD plasma as an equilibrium plasma which has the following properties: a free-electron concentration of 10^{18}–10^{20} m^{-3}, a temperature between 2000 and 3000 °K, and a total pressure of about 1 atm.

III. PROBLEM FORMULATION

In the formulation of the anode region, the ion inertia term will be ignored (unlike the work of Ref. 17). This is because we assume the existence of positive ions only (the movement of a small quantity of negative ions would be overshadowed by the free-electron motion). The effects of a magnetic field will not be included. In contrast with diagnostic probes, an electron space charge will be assumed to exist at the anode at all times. The equations that describe the problem are two species conservation equations, Gauss's equation (or Poisson's equation), and the electron energy equation.[18–20] The sheath will be considered first and then the ambipolar region before dealing with a subset of the original equations.

A. Sheath considerations

In the data previously referred to, it was stated that the anode drop has a magnitude of about 100 V. This voltage drop must be accountable through the sheath, the ambipolar region, and any Ohmic region which is relevant. Now, if the extent of the sheath is of the order of the Debye length, then the associated voltage drop must be kT_e/e which is of the order of a few tenths of a volt for MHD plasmas. As will be evident in Sec. III B, it is not reasonable to assign a 100-V drop to the ambipolar region. We shall assume that the Ohmic region can be only responsible for a portion of this drop since the current-voltage characteristics are typically not Ohmic and the MHD plasma is a good conductor. It is reasonable then to assign a significant part of the voltage drop to the sheath because of the strong electrostatic forces associated with a non-neutral collisional region.[21] In this case, the extent of the sheath becomes greater than a Debye length and will be seen to depend on the associated voltage drop across it.

In mks units, Gauss's equation is written

$$\nabla \cdot \mathbf{E} = (e/\epsilon_0)(n_i - n_e), \qquad (1)$$

where $\mathbf{E}$ is the electric field, e the charge of an electron, ϵ_0 the permittivity of free space, and $n_{i,e}$ the

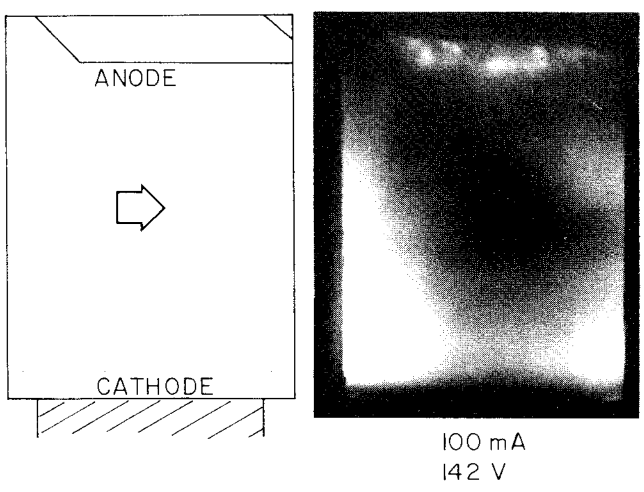

FIG. 2. Spot mode.

number density for ions and electrons. The above equation is of the boundary-layer type[19] and can be used to establish the characteristic length of the nonneutral region. When $\hat{\mathbf{E}}$ is defined as

$$\hat{\mathbf{E}} = \frac{\mathbf{E} - \mathbf{E}_a}{-\Phi_a/\lambda_s}, \tag{2}$$

where $\mathbf{E}_a$ is the field at the anode, Φ_a the anode potential with respect to the undisturbed plasma (or, more precisely, the sheath drop), and λ_s the characteristic length of the sheath, Eq. (1) becomes

$$\hat{\nabla} \cdot \hat{\mathbf{E}} = -(en_\infty \lambda_s^2/\epsilon_0 \Phi_a)(\hat{n}_i - \hat{n}_e). \tag{3}$$

In Eq. (3) all lengths have been nondimensionalized with respect to λ_s and all number densities with respect to n_∞—the corresponding equilibrium value for the undisturbed plasma. As defined, $\hat{\mathbf{E}}$ is a monotonic positive function which is suitable for approximation theory.[22] Note that the nature of $\hat{\mathbf{E}}$ is the same whether or not the electron random current is exceeded at the anode. Because each variable in Eq. (3) is of order unity, the combination of parameters must also be of order unity over the characteristic length of the nonneutral region,[18] i. e.,

$$\lambda_s \approx (\epsilon_0 \Phi_a / e n_\infty)^{1/2} = \lambda_D (e\Phi_a/kT_e)^{1/2}, \tag{4}$$

where λ_D is the Debye length based on the electron temperature. Clearly, depending on the value of Φ_a in relation to kT_e/e the extent of the sheath may be significantly greater than the Debye length. It is useful to estimate some typical cases; with $\Phi_a = 100$ V and n_∞ between 10^{18} and 10^{20} m^{-3}, λ_s has values between 2×10^{-5} and 7×10^{-6} m. Now, if the plasma is below 3000 °K, the sheath thickness is more than an order of magnitude greater than the Debye length. This shielding length λ_s is still quite small, even when compared to the boundary layer. Several conclusions are evident here: For a typical anode size of 1 cm, the problem appears to be one dimensional and the effects of convection should be negligible (and so would the magnetic field except for a reduction of the charged particle mobility by a Hall effect factor); moreover, noting that Eq. (3) applies to either a collisional or collisionless sheath, the value of λ_s somewhat exceeds the anticipated value for the electron mean free path which is about 10^{-6} m. Under the above conditions, it is anticipated that the electron transit time is so short that ionization/recombination processes will be unimportant in the sheath.

Returning now to the assumption about the Ohmic region, it is argued[23] that the Ohmic region is the predominant contributor to the anode drop. If this were the case, the anode sheath would lie more in the collisionless regime and the continuum equations would be less applicable which is consistent with a negligible sheath drop; however, nonequilibrium MHD generators should be virtually free of anode drops, but they are not. The assumptions made in this section need further refinements regarding the kinetics of charge recombination but these will not be attempted here.

B. Ambipolar considerations

While it is not correct to use approximation theory on the charge density profiles because these display several inflection points, it is possible to consider the profiles beyond the sheath, i. e., in the ambipolar region. Here the equation is of the boundary-layer type, under some restrictions, and it is proper to define a characteristic length. It is convenient to adopt here the simplified analysis shown in Ref. 24, Sec. III-7 which, although restrictive, offers some rather basic insights into the problem. The problem treated in Ref. 24 is one of a homogeneous static plasma with constant properties. A significant feature of this treatment is that the flux equations are formulated such that the net rate of charge production is of the three-body (electron) type and thus the Hinnov-Hirschberg[25] formula applies. The validity of this formula is not universal and depends strongly on the species present.

In the notation previously defined, the governing equation for the charge density profile becomes

$$(l_R/L)^2 \hat{\nabla}^2 \hat{n} = -\hat{n}(1 - \hat{n}^2), \tag{5}$$

where

$$\hat{n} \equiv n_e/n_\infty, \quad \text{or } n_i/n_\infty,$$
$$l_R \equiv (D_a/n_\infty^2 \beta)^{1/2}, \tag{6}$$

where L is the characteristic geometric dimension, l_R is a dimension characteristic of recombination, and L could be either a cold boundary layer or the interelectrode distance depending on the significance of convection. L may range from 1 mm to several cm in practical situations. D_a is the ambipolar diffusion coefficient and β is the temperature-dependent coefficient of the Hinnov-Hirschberg formula. Clearly, if $l_R/L \ll 1$ we have the equilibrium limit, and if $l_R/L \gg 1$ we have the frozen limit.

The voltage drop associated with the ambipolar region alone is simply[24]

$$\Phi_{\mathrm{amb}} \approx (kT/e)\ln(n_{e\infty}/n_{eb}),$$

where n_{eb} is the value of the charge density at the edge of the sheath. The ratio of $n_{e\infty}/n_{eb}$ may be as high as 10^6 but Φ_{amb} is not expected to exceed 4 V.

The value of l_R defined above can be easily calculated.[26] For potassium at a partial pressure of 1×10^{-3} atm, l_R is 1 mm at 2000 °K and 3 cm at 1500 °K (a typical average value in the thermal boundary layer). For cesium, l_R is 0.2 mm at 2000 °K and 2 mm at 1500 °K and also 1×10^{-3} atm. Under these conditions, partially ionized gases which utilize a cesium seed can be at the equilibrium limit and the anode glow mode of operation should prevail. As stated earlier the presence of impurities would complicate the analysis, perhaps obviating the utility[27] of Eq. (5), but for a reduction of n_∞ in Eq. (6) by 2 orders of magnitude the above cesium- or potassium-seeded plasma could be at the frozen limit. This reduction would be caused by a change of constituents in the plasma. The remainder of our discussion will center on consideration of this frozen limit where no glow may be anticipated.

C. Combined formulation

The above arguments suggest that the anode region could be described one dimensionally to a good approximation in many cases of interest. We will show here that no such one-dimensional Cartesian solution exists for the frozen limit, thus inferring the existence of anode spots.

Assume a coordinate system where y increases from the anode into the plasma. We write the two flux equations and accompanying Gauss's equation as

$$j_i = \frac{e^2 D_i}{k} \frac{n_i E}{T_i} - eD_i \frac{dn_i}{dy}, \quad (7)$$

$$j_e = \frac{e^2 D_e}{k} \frac{n_e E}{T_e} + eD_e \frac{dn_e}{dy}, \quad (8)$$

$$\frac{dE}{dy} = \frac{e}{\epsilon_0}(n_i - n_e). \quad (9)$$

In Eq. (8), j_e is the contribution of the electrons to the total current J so that $J = j_i + j_e$, and the Einstein relation has been used to write the mobility in terms of the diffusion coefficients (which will be assumed constant). At the anode $E > 0$ and $n_e \geq n_i$, hence, from Eq. (9), E decreases with increasing y. For a fixed total current J, j_i and j_e must be constant in one dimension; moreover, any ion current present must originate at the anode surface. We manipulate Eqs. (7) and (8) by taking the sum and difference, and obtain

$$K^+ = \frac{eE}{kT_e}(n_i + n_e) - \frac{d}{dy}(n_i - n_e) + \frac{en_i E}{k}\left(\frac{T_e - T_i}{T_e T_i}\right), \quad (10)$$

$$-K^- = \frac{eE}{kT_e}(n_i - n_e) - \frac{d}{dy}(n_i + n_e) + \frac{en_i E}{k}\left(\frac{T_e - T_i}{T_e T_i}\right). \quad (11)$$

Although the present proof is valid as long as K^+ and K^- are constants, we are going to assume that the anode is highly positive and nonemitting; thus,

$$j_e > j_i D_e / D_i.$$

Hence, we have defined each K to be positive as shown below:

$$K^+ \equiv j_i/eD_i + j_e/eD_e, \quad K^+ > 0 \quad (12)$$

$$-K^- \equiv j_i/eD_i - j_e/eD_e, \quad K^- > 0. \quad (13)$$

In the absence of any ion current, it is seen that K represents the magnitude j_e/eD_e, and because all j's are constant, K will be constant and nonzero for the one-dimensional case. The definition of K^+ and K^- as positive is merely for convenience and should not cloud the physical arguments represented by Eqs. (10) and (11).

The last term in Eqs. (10) and (11) arises from the need to keep T_e and T_i separate as they are usually not equal. This term, however, will be positive or zero since E is positive, and in a nonequilibrium situation $T_e \geq T_i$.

Defining new variables, $\delta \equiv n_i - n_e$ and $\sigma \equiv n_i + n_e$, Eqs. (1), (10), and (11) become

$$\frac{dE}{dy} = \frac{e}{\epsilon_0}\delta, \quad (14)$$

$$\frac{d\sigma}{dy} \geq \frac{e}{k}\frac{E\delta}{T_e} + K^-, \quad (15)$$

$$\frac{d\delta}{dy} \geq \frac{e}{k}\frac{E\sigma}{T_e} - K^+. \quad (16)$$

Since the ambipolar region is quasineutral, we know that $\delta \to 0$ as we move from the sheath to the plasma proper where, of course, the net charge density is zero. Thus, as y increases, the dependent variables must approach the following constant values

$$\delta \to 0, \quad \sigma \to \sigma_\infty, \quad E \to E_\infty.$$

Eventually a distance $y = y_0$ would be reached such that for every $y > y_0$

$$\left|\frac{e}{k}\frac{E\delta}{T_e}\right| < \frac{K^-}{2},$$

since $\delta \to 0$, and both E and T_e approach constant nonzero values. Then, remembering that $\delta \leq 0$ near the anode,

$$\frac{d\sigma}{dy} \geq \frac{K^-}{2} \quad \text{for all } y > y_0;$$

so that

$$\sigma(y) \geq \sigma(y_0) + (\tfrac{1}{2}K^-)(y - y_0),$$

which does not approach a constant. Furthermore, since

$$\frac{d\delta}{dy} \geq \frac{e}{k}\frac{E}{T_e}\left(\sigma(y_0) + \frac{K^-}{2}(y - y_0)\right) - K^+,$$

δ eventually grows at least as fast as $(eE_\infty K^-/4kT_{e\infty}) \times (y - y_0)^2$ and hence does not approach zero. These obvious inconsistencies arise because of the inflexibility of K^+ and K^- to change in order to accomodate other changing variables. The necessary requirement for a solution is that the current density should decrease away from the anode, giving rise to current concentrations at the surface. Under this condition, K^+ and K^- would diminish at the proper rate so that $\delta \to 0$ and $\sigma \to \sigma_\infty$. Mechanisms which could allow the reduction of K^+ and K^- and still retain continuity of charge are (i) diffusion in two or three dimensions and (ii) ionization/recombination. The former negates the applicability of a one-dimensional solution while the latter eliminates frozen flow.

IV. CONCLUSIONS

Within a restricted framework, it has been shown that plasmas typical of MHD generators may have the anode operating in either a glow mode or a constricted mode depending on the degree of equilibrium of the charges as they approach the surface. The level of current density and the mechanism of ionization/recombination effective in the gas mixture play a primary role in determining which mode is operative. The role of convection and of a magnetic field have been left undefined except for mentioning that they may be secondary effects when compared with the above.

In the data represented by Figs. 1 and 2, the potassium-seeded argon plasma shows both modes of operation at the anode. The changeover from anode glow to anode spots is attributed to the presence of impurities that reduced the free-electron concentration. Surface changes are not likely to affect the flow mode if charge emission or charge reflections are unimportant. There is, of course, the boundary condition at the surface which is usually taken as a catalytic condition but note here that our arguments are based on conditions removed from the anode surface (i. e., $y > y_0$). There is, moreover, evidence that the catalytic condition is inappropriate.[28]

Various "probe" solutions are available to the subset of equations treated in this paper (i. e., the frozen-chemistry constant-property plasma).[18] Interestingly enough, the geometries treated are either cylindrical or spherical. It is recognized that the one-dimensional solution does not exist without ionization and recombination.[10,19] However, the fact that a plasma may constrict, even on a spherical probe exercising a degree of freedom not built into conventional probe solutions, has not been properly accounted for. Recognition of this condition will remove the upper limit on the current which presently applies to probe solutions. Also, when it is realized that the spot mode of operation is a source of current inhomogeneity, which is independent of cold boundary layers but may lead to an intense anode spot if the system is thermally unstable,[29,30] then the design of MHD devices may be approached with the appropriate care.

ACKNOWLEDGMENT

This work is supported by the Air Force Office of Scientific Research.

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