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Elastic Electron Scattering from ^{6}Li and ^{7}Li at Low Momentum Transfer*

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Elastic electron scattering experiments were performed on ^{6}Li and ^{7}Li at momentum transfers less than 1 F^{-1}. Charge form factors are reported, and model-independent as well as model-dependent rms radii are calculated. The model-independent radii for ^{6}Li and ^{7}Li are 2.51 ± 0.10 and 2.35 ± 0.10 F, respectively.

INTRODUCTION

Elastic scattering of high-energy electrons may be used to determine the radial distribution of charge within the nucleus.[1,2] The present experiments with ^{6}Li and ^{7}Li were undertaken to resolve disagreement among the published values of the rms charge radii for these nuclei. In the case of ^{6}Li, values of 2.80, 2.41, and 2.54 F have been reported, respectively, by Burleson and Hofstadter,[3] Bernheim,[4] and Suelzle.[5,6] For ^{7}Li, values of 2.33 and 2.39 F have been reported by Bernheim and Suelzle, respectively. Suelzle also reports why the early result of 2.80 F for ^{6}Li was in error.

In principle, scattering experiments carried out at low values of the momentum transfer q determine the mean square radius of the nuclear-charge distribution, as we show below (see Hofstadter,[1] Sec. II). If we assume the Born approximation to be valid, the observed differential scattering cross section σ_E may be written as

$$\sigma_E = F^2(q)\sigma_M, \tag{1}$$

where $F(q)$ is called the form factor for the charge distribution, and σ_M is the Mott scattering cross section. Further, we have

$$F(q) = \frac{1}{Ze}\iiint e^{i\mathbf{q}\cdot\mathbf{r}}\rho(r)d^3r, \tag{2}$$

where q is the momentum transfer vector ($\hbar = 1$), $\rho(r)$ is the nuclear-charge density, e is the charge of the proton, and Z is the atomic number. If the charge distribution is spherically symmetric, it is easily shown that

$$F(q) = \frac{4\pi}{Zeq}\int_0^\infty \sin(qr)\rho(r)r\,dr. \tag{3}$$

For small q we may substitute the first few terms of the power-series expansion for $\sin x$ and obtain

$$F(q) = 1 - \tfrac{1}{6}q^2\langle r^2\rangle + \cdots, \tag{4}$$

where $\langle r^2\rangle$ is the mean square charge radius. If $F(q)$ is plotted as a function of q^2, the slope of the curve near $q^2 = 0$ will determine $\langle r^2\rangle$. This method for finding $\langle r^2\rangle$ is said to be model-independent because no particular nuclear model is assumed in advance. Also, at low values of q, various corrections to the simple scattering theory (such as the effects of magnetic and higher-order electric moments for nuclei with spin) are small.

Experiments performed at higher values of q determine the radius by assuming a general form for $F(q)$ in which several parameters may be adjusted to fit the experimental data. The behavior of $F(q)$ near $q = 0$ will depend on the model, so that a radius determined in this manner is designated model-dependent. See Suelzle[5,6] for an example of this approach.

In this paper we present values of $F(q)$ for both ^{6}Li and ^{7}Li with q^2 in the range from 0.008 to 0.8 F^{-2}, as well as the radii determined from both model-independent and model-dependent calculations.

EXPERIMENT

The electron linear accelerator at the U. S. Naval Postgraduate School is capable of producing an electron beam of energy up to 100 MeV, and a range $0 \leq q \leq 1$ F^{-1} can be obtained. Targets of ^{6}Li, ^{7}Li, and C (with thickness values of 51.3, 80.1 and 27.9 mg/cm^2, respectively) were mounted in a movable target ladder, and the scattered electrons were counted from all three targets in succession at a single spectrometer momentum setting. Thin targets were chosen to minimize energy loss, straggling, and multiple scattering, but

this allowed errors of up to 5% in the cross section owing to uncertainty in thickness.

The counting system was a telescope consisting of two plastic scintillators separated by about 4 cm. Four scalers were employed, one for coincidence counts, one for each individual counter, and one for accidental coincidence by using a delay cable on one channel. The coincidence counts were corrected for background, scaler losses, and accidentals, and except at higher energies (where there was high background), the total correction was less than 3%. To avoid contamination of the lithium targets, thin Mylar windows isolated the targets chamber from the spectrometer, so that the spectrometer could be rotated without letting air into the target chamber. Also, a thin aluminum window isolated the accelerator from the target chamber. These windows degraded the energy resolution of the system to about 1%. The absence of elastic peaks where carbon, nitrogen, and oxygen should appear indicated that there was no significant target contamination.

CALCULATIONS

The lithium form factors were calculated from the observed lithium and carbon elastic peaks and the known carbon cross section. The cross sections were assumed to be given by

$$\sigma_E = P^2 F^2(q)\sigma_M , \tag{5}$$

where P corrects Eq. (1) for the error from assuming the Born approximation.

For carbon, the form factor corresponding to the charge distribution reported by Bentz[7] is

$$F_C = (1 - 0.2775q^2)\exp(-0.679q^2), \tag{6}$$

where q is in F^{-1}. The correction factor P was calculated by comparing a phase-shift calculation of the cross section[8,9] with the Born-approximation result.

In the case of carbon, the charge distribution corresponding to the form factor given by Eq. (6) was used. In the case of lithium, the charge distribution was not known in advance, so the value of P was calculated by assuming the charge distribution obtained from a previous experiment.[6] Then it was shown that P was very insensitive to the charge distribution used, so that P would not change significantly if our data gave a charge distribution slightly different from the result of the previous experiment. Specifically, it was found that for lithium at 80 MeV and 135°, a 10% change in the radius of the charge distribution produced a change in P of only 0.1%. The phase-shift-correction factors listed in Tables I and II (containing the phase-shift corrections for both carbon and lithium) were multiplied by the uncorrected form factors to obtain the phase-shift-corrected form factors listed in the last column.

The scattering data were corrected for the effects of radiation in the target (Bethe-Ashkin correction[10]), and for radiation during the main scattering event. (The Schwinger correction in the form reported by Tsai[11] was used.)

To obtain the charge form factors, corrections were made for the magnetic scattering, and in the case of ^{7}Li, the contribution of the unresolved

TABLE I. ^{6}Li charge form factors.

E (MeV)	θ (deg)	q^2 (F^{-2})	Phase-shift correction	$F_{ch}(q^2)$
23.63	45	0.0088	1.0129	1.0124 ± 0.026
36.49	45	0.0206	1.0065	0.9855 ± 0.031
36.17	75	0.0510	0.9884	0.9152 ± 0.028
37.04	90	0.0720	1.0065	0.9220 ± 0.033
37.04	90	0.0720	1.0065	0.9110 ± 0.026
42.99	75	0.0716	1.0055	0.9279 ± 0.024
52.33	60	0.0714	1.0053	0.9205 ± 0.024
42.99	135	0.1638	1.0025	0.7853 ± 0.032
50.02	105	0.1633	0.9980	0.8620 ± 0.026
79.20	60	0.1621	0.9998	0.8152 ± 0.022
52.73	135	0.2446	0.9910	0.7769 ± 0.028
79.81	75	0.2431	0.9983	0.7547 ± 0.021
79.34	105	0.4058	1.0024	0.6605 ± 0.017
88.94	90	0.4046	0.9977	0.6404 ± 0.017
79.41	135	0.5469	1.0039	0.5798 ± 0.025
92.32	105	0.5460	0.9947	0.5660 ± 0.016
96.87	120	0.7123	0.9832	0.5288 ± 0.018
96.68	135	0.8047	0.9600	0.4319 ± 0.015

TABLE II. ^{7}Li charge form factors.

E (MeV)	θ (deg)	q^2 (F^{-2})	Phase-shift correction	$F_{ch}(q^2)$
24.14	45	0.0088	1.0129	0.9696 ± 0.025
37.00	45	0.0206	1.0065	0.9765 ± 0.030
36.68	75	0.0510	0.9884	0.8593 ± 0.026
37.55	90	0.0720	1.0065	0.8919 ± 0.025
37.55	90	0.0720	1.0065	0.9014 ± 0.031
43.50	75	0.0717	1.0055	0.9269 ± 0.027
52.84	60	0.0714	1.0053	0.8270 ± 0.023
43.50	135	0.1641	1.0025	0.7915 ± 0.026
50.53	105	0.1635	0.9980	0.8481 ± 0.024
79.71	60	0.1622	0.9998	0.8165 ± 0.022
53.24	135	0.2452	0.9911	0.7598 ± 0.023
80.32	75	0.2435	0.9983	0.7679 ± 0.021
79.85	105	0.4068	1.0024	0.6595 ± 0.018
89.45	90	0.4056	0.9977	0.6599 ± 0.018
79.92	135	0.5488	1.0039	0.5671 ± 0.017
92.83	105	0.5475	0.9947	0.5870 ± 0.016
97.38	120	0.7149	0.9832	0.5295 ± 0.017
97.19	135	0.8080	0.9600	0.4553 ± 0.015

0.478-MeV excited state was subtracted. These corrections, as described by Suelzle,[5,6] were negligible for ^{6}Li and generally less than 2% for ^{7}Li.

ERRORS

The lithium form factors were calculated from the ratios of the observed lithium and carbon cross sections. Many errors will tend to cancel in such a relative rather than absolute cross-section experiment. Beam-monitor calibration, spectrometer solid angle, and the energy calibration of the spectrometer and deflection magnets need to be stable and reproducible, but do not need to be known absolutely. Even errors in scattering angle almost cancel because the Mott cross section, which is very sensitive to the angle, appears as a factor in both the lithium and carbon cross sections.

Statistical counting errors in the experimental lithium and carbon cross-section measurements varied from 0.7 to 1.7%. The error associated with a lithium form factor was calculated from the lithium and carbon counting errors, and an additional 1% error was included in an attempt to represent other experimental errors such as random changes in the beam monitor efficiency. The errors calculated in this manner are quoted in the tables and shown in the figures.

The remaining and principal sources of errors in this experiment were: (a) a large background counting rate which was significant for the high-energy ($E \geq 60$ MeV) points; (b) multiple scattering in the target and the windows between the target and the spectrometer, which was significant for the low-energy ($E \geq 30$ MeV) points; and (c) errors in the target thickness. Also a change in the form factor for carbon would change our lithium results, since this was a relative experiment.

RESULTS

Significant results of this experiment consist of the values of the charge form factors listed in Tables I and II, and the values of the rms radii discussed below and listed in Table III. To calculate the rms radius, the charge form factors were corrected for the effects of the quadrupole moment to obtain the monopole form factors. For ^{6}Li the correction was insignificant, but for ^{7}Li the monopole form factor was smaller than the charge form factor by 0.47, 1.12, and 2.01% at q^2 values of 0.4, 0.6, and 0.8 F^{-2}, respectively. The corrections used were from Ref. 6, Eqs. (33) and (38). Even though the quadrupole correction to the form factor was as high as 2%, the correction changed the radius by less than 0.1%.

The "model-independent" radius was calculated by a least-squares fit of a polynomial expression $(a+bq^2+cq^4)$ to the experimental data points. For ^{6}Li the results were $a=0.992\pm0.018$, $b=-1.042\pm0.082$ F, $c=0.481\pm0.11$ F^2, and $\langle r^2\rangle^{1/2}=2.51\pm0.10$ F. From Eq. (2) it may be seen that we should have $F(q)=1$ for $q=0$, and thus the value of a should be unity; however, an error in the measurement of the target thickness would multipy all values of the cross section by a constant factor and result in $a\neq1$. For ^{7}Li, the corresponding values were $a=0.954\pm0.018$, $b=-0.872\pm0.082$ F, $c=0.342\pm0.11$ F^2, and $\langle r^2\rangle^{1/2}=2.35\pm0.10$ F. In both cases, $\langle r^2\rangle^{1/2}=-6b/a$.

The rms radius may also be found by choosing a form factor corresponding to a specific nuclear model and adjusting parameters to fit the data. If there are few free parameters, it is possible to have greater statistical accuracy in the resulting rms radius, but of course there are assumptions implicit in the model. Various calculations using only the low-q results of this experiment showed that several models fit these data equally well, but predicted different rms radii. An empirical procedure employed was to assume a model which would fit the high-q data of other experiments reasonably well, and vary only one or two parameters to fit the present data. For ^{6}Li Suelzle was unable to fit the high-q data with either the simple shell model or a modified shell model with unequal S and P wells. However, a form

$$F=d(e^{-a^2q^2}-c^2q^2e^{-b^2q^2}), \tag{7}$$

was successful, with $a^2=0.87$ F^2, $b^2=1.7$ F^2, $c^2=0.205$ F^2, $d=1$. Keeping Suelzle's values of b and c, a least-squares fit to the present low-q data yielded $a^2=0.859$ F^2, $d=0.988$, and $\langle r^2\rangle^{1/2}=2.53\pm0.03$ F. Again, $d\neq1$ could be a result of an error in the measurement of the target thickness. Our data points for ^{6}Li and the least-squares curves are shown in Fig. 1.

For ^{7}Li, Suelzle, Yearian, and Crannel[6] [Eq. (41)] were able to fit the data with a form factor

TABLE III. rms radii in F.

^{6}Li:		
	This experiment, model-independent	2.51 ± 0.10
	This experiment, Suelzle-type model	2.53 ± 0.03
	Li *et al.*	2.56 ± 0.05
	Suelzle, Yearian, and Crannell	2.54 ± 0.05
	Bernheim, simple shell model	2.41
^{7}Li:		
	This experiment, model-independent	2.35 ± 0.10
	This experiment, shell model	2.29 ± 0.04
	Suelzle, Yearian, and Crannell, shell model	2.39 ± 0.03
	Bernheim, simple shell model	2.33 ± 0.06

corresponding to the harmonic-oscillator shell model,

$$F_{c0} = (1 - \tfrac{1}{18} q^2 a_0^2) \exp\left[-\frac{1}{4} q^2 a_0^2 \left(1 - \frac{1}{A}\right)\right] F_n, \quad (8)$$

where F_{c0} is the monopole form factor, A is the mass number, F_n is the nucleon form factor, and $a_0 = 1.745$ F. We fit our data to the simplified form,

$$F_{c0} = d(1 - cq^2) e^{-aq^2}, \quad (9)$$

where $c = 0.169$ F^2 was obtained from Eq. (8), a and d were fitting parameters, and the nucleon form factor was absorbed in the Gaussian factor. The least-squares fit yielded $d = 0.946$, $a = 0.713$, and $\langle r^2 \rangle^{1/2} = 2.29 \pm 0.04$ F. To illustrate the model dependence of the result, a fit was also attempted for a modified shell model with an enlarged p-shell radius (as Suelzle attempted), resulting in $\langle r^2 \rangle^{1/2} = 2.35$ F. However, this modified shell model does not agree with the high-q or magnetic scattering data, so there is no reason to consider it further. Our data points for ^{7}Li and the least-squares curves are shown in Fig. 2.

A comparison of the ^{7}Li radii obtained in the various experiments shows some disagreement. Our model-independent radius agrees with either experiment because of the larger error, but the radius obtained with the simple harmonic-oscillator shell model favors the lower radius of Bernheim. In the case of ^{6}Li, our radius agrees with Suelzle's value, but appears to disagree with Bernheim's result. However, Bernheim's form factors agree very well with our results, and if we fit our data to Bernheim's model, we agree with her radius. Bernheim's experiment did not include q values low enough for a model-independent radius, nor q values high enough to justify the assumed model (simple harmonic-oscillator shell model). So we conclude that as long as the high-q experiments justify the general form of the Suelzle model for ^{6}Li, all the data (low- and high-q) are consistent with the larger radius of 2.53 or 2.54 F. This result also agrees with the recently published value of 2.56 ± 0.05 F obtained from higher-q values by Li *et al.*[12]

A possible remaining source of error in our lithium form factors and radii is associated with the carbon form factor which is required for our calculations, since we measured lithium cross sections relative to carbon. The recent high-q experiments at Stanford University[13] report a radius for carbon of 2.46 F, compared to the value of 2.395 F obtained by Bentz.[7] Using the form factors associated with this larger carbon radius increases all of our lithium radii by 0.06 F. We believe the measurements by Bentz are appropriate for our calculations, since the range of q values is similar.

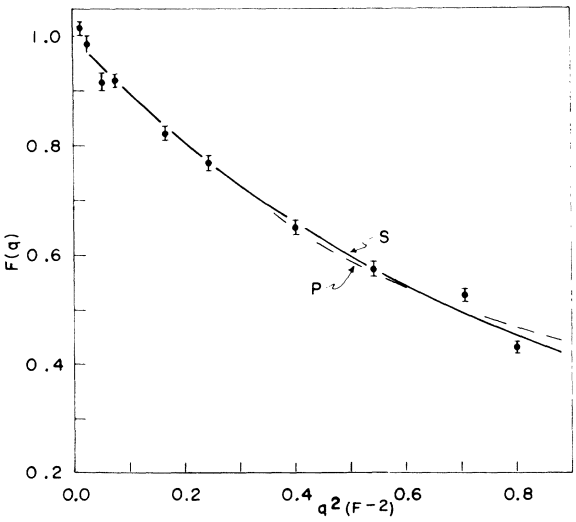

FIG. 1. Form factors for ^{6}Li. The experimental results (points), the least-squares fit of a Suelzle-type model (S), and model-independent (polynomial) fit (P), with rms radii of 2.53 and 2.51 F, respectively.

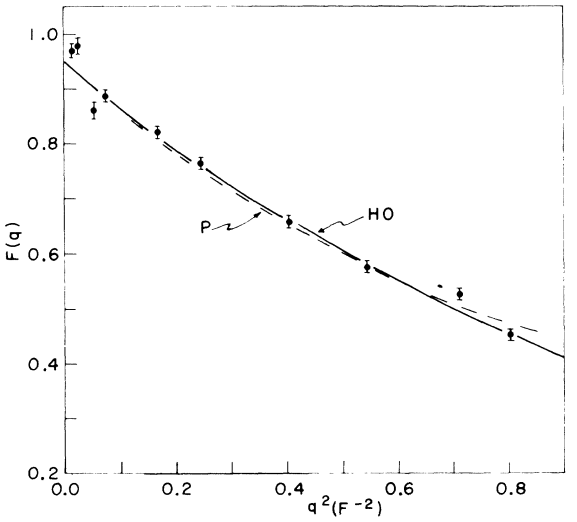

FIG. 2. Form factors for ^{7}Li. The experimental results (points), the least-squares fit of a harmonic-oscillator shell model (HO), and model-independent (polynomial) fit (P), with rms radii of 2.29 and 2.35 F, respectively.

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Scattering of Polarized 3-MeV Neutrons from ^{3}He

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Partially polarized 3-MeV neutrons have been scattered from ^{3}He contained in a high-pressure gas scintillation cell. The resulting asymmetry in the elastic scattering was determined for seven scattering angles from 40 to 120° (lab). Time-of-flight techniques and neutron–γ-ray discrimination in the side detectors were used for background reduction. The measured asymmetry is small for angles less than 60° (lab) and reaches a maximum value at approximately 110° (c.m.). The maximum asymmetry determined from the present experiment corresponds to a polarization of +0.50. The differential polarization can be fitted within experimental uncertainties by assuming that D-wave effects are negligible. The principal features of the data are qualitatively reproduced by Dodder's reaction-matrix calculations which use the Werntz-Meyerhof level parameters for ^{4}He.

I. INTRODUCTION

Among the many ways of obtaining information on the unbound energy levels of ^{4}He is the study of interactions in the $p+$T and $n+^3$He channels for which ^{4}He is the compound system. Previous measurements by many experimenters of the cross sections and neutron polarizations for the T(p,n)-^{3}He reaction and recent experiments at Los Alamos with polarized tritons have provided considerable information on excited states of ^{4}He in the region between 20 MeV and the $d+$D threshold at 23.8 MeV in ^{4}He. The data are more sparse for reactions initiated by neutrons.

Differential cross sections for n-^{3}He scattering have been measured by Seagrave, Cranberg, and Simmons[1] and by Sayers, Jones, and Wu,[2] but few polarization measurements exist. Seagrave, Cranberg, and Simmons have reported polarization measurements at 94 and 121° (c.m.) for 1.1-MeV neutrons and at 137° for 2.15-MeV neutrons. Behof, Hevezi, and Spalek[3] performed polarization measurements for the scattering of 3.3-MeV neutrons in the angular range from 109 to 167° (c.m.). Angular distributions of polarizations for higher incident neutron energies have been reported by both Behof, Hevezi, and Spalek and Busser *et al.*[4] The polarization data for n-^{3}He scattering have been reviewed by Barshall[5] and, more recently, by Seagrave.[6]