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A New Fuzzy Logic Approach to Capacitated Dynamic Dial-a-Ride Problem

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Abstract

Almost all Dial-a-Ride problems (DARP) described in the literature pertain to the design of optimal routes and schedules for n customers who specify pick-up and drop-off times. In this article we assume that the customer is mainly concerned with the drop-off time because it is the most important to the customer. Based on the drop-off time specified by the customer and the customer's location, a pick-up time is calculated and given to the customer by the dispatching office. We base our formulation on a dynamic fuzzy logic approach in which a new request is assigned to a vehicle. The fuzzy logic algorithm chooses the vehicle to transport the customer by seeking to satisfy two objectives. The first reflects the customer's preference and minimizes the time a customer spends in the vehicle, and the second reflects the company's preference and minimizes the distance a vehicle needs to travel to transport the customer. The proposed heuristic algorithm is relatively simple and computationally efficient in comparison with most deterministic algorithms for solving both small and large sized problems.

Keywords: Dial-a-Ride problem, fuzzy logic, transportation

1. Introduction

Dial-a-Ride transportation services are employed in various areas, either to serve elderly people or people with disabilities. People call Dial-a-Ride services to request transportation to and from a location, and the Dial-a-Ride company tells the customer when a van or car will come to pick up the customer. The request can be made a few days in advance, or it could be made just a few hours before the customer would like to be picked up. The objective then for the Dial-a-Ride company is to design a set of minimum cost vehicle routes capable of accommodating all requests under constraints. In addition, the company must decide which vehicle should pick up a customer and the pick-up and drop-off time for each customer. Thus, both the transportation costs and customer convenience must be weighted when making the routing decisions.

Dial-a-Ride problems (DARP) are classified into static DARP and dynamic DARP. Static DARP are concerned with finding optimal routes for vehicles based on requests made in advance by customers, and static DARP make decisions about vehicles and pick-up and drop-off times with full knowledge about customers' requests for a particular day. Dynamic DARP are concerned with requests made in real time when vehicles already have scheduled pick-ups and drop-offs. Both the static and dynamic types are employed in either a single-vehicle or a multi-vehicle DARP systems [1]. Although many DARP solutions assume a deterministic problem, uncertainty or imprecision is an important consideration. There may be uncertainty (or imprecision) over when vehicles arrive at their stops, and customers may express flexibility about when they want to be picked up and dropped off.

For the static multi-vehicle case, Jaw et al. [2] propose one of the first insertion heuristic algorithms to deal with multi-vehicle DARP using convex optimization techniques. The algorithm considers two types of customers:

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20 desired drop off time (DDT) specified by customers, and desired pick-up time (DPT) specified by customers. Cordaeu
21 [3] proposes a Branch-and-Cut algorithm for small to medium size instances. Cordeau and Laporte [4] propose a
22 Tabu search algorithm for the static DARP. Diana and Dessouky [5] present a parallel regret insertion heuristic to
23 solve DARP for large-scale systems with time windows. The computational results show the effectiveness of this
24 approach by trading off solution quality and computational times. Zidi et al. [6] propose a mathematical model for
25 multi-objective static DARP.

26 For the dynamic multi-vehicle case, Madsen et al. [7] solve a real-life problem in which customers can specify
27 either a pick-up or drop-off time window. The algorithm is based on that of Jaw et al. [2] and dynamically inserts
28 new requests into vehicle routes. Hyytiä et. al [8] develop an efficient and modular simulation to study the perfor-
29 mance of large scale systems under specified trip arrival processes with a number of policies. In addition to that,
30 they demonstrate the viability of dynamic DARP and the trade-off between work conducted and quality of service.
31 Muñoz-Carpintero et. al [9] solve a dynamic vehicle routing problem using a hybrid predictive control strategy. They
32 use genetic algorithms to efficiently solve the problem and fuzzy logic for demand prediction. Nunez et. al [10]
33 presents a hierarchical multi-objective model based on predictive control approach to solve a dynamic pickup and
34 delivery problem. The claim is that the hierarchical nature of the system breaks down the optimization problem into
35 manageable sub-problems. Kergosien et. al [11] employ a Tabu search algorithm iteratively to solve the dynamic
36 version of assigning ambulance vehicles to move patients between care centers in a hospital complex given certain
37 constraints.

38 Colorni and Righni [12] consider solving an objective function consisting of maximizing the number of customers
39 served, optimizing the perceived service level, and minimizing the total distance traveled by the vehicles. Attanasio
40 et. al [13] use Tabu search algorithm for both static and dynamic multi-vehicle DARP. Melachrinoudis et. al [14]
41 propose a multi-objective optimization technique to solve DARP involving a health center. Bergvinsdottir et. al [15],
42 Jorgensen et. al [16], and Cubillos et. al [17] use genetic algorithm (GA) to solve DARP. Herbawi et al. [18] also
43 propose a genetic and insertion heuristic algorithm to solve dynamic DARP. De Paepe et. al [19] design a computer
44 program (DARCLASS) which takes lists of problems with known complexity as inputs and generates a classification
45 of all problems into three classes as output: solvable in polynomial time, NP-Hard, or open. Coja-Oghlan et al. [20]
46 show that even though the DARP is NP-hard when the network forms a caterpillar (worst-case), it can be solved
47 efficiently (on average). Jun [21] proposes an insertion heuristic-based hybrid genetic algorithm to solve the vehicle
48 sharing problem as a fuzzy vehicle routing problem. Van Lon et al. [22] create a decentralized multi-agent system to
49 solve the dynamic dial-a-ride problem.

50 Fuzzy numbers can express not only uncertainty and imprecision, but flexible constraints as well [23]. Fuzzy
51 modeling has the advantage that it provides an easy way to determine which vehicle is optimal for picking up a
52 customer that reflects the realities of how dispatchers naturally think about the problem. Kikuchi [24] implements
53 a fuzzy modeling approach to DARP by considering the desired time of vehicle arrival and travel time between two
54 nodes as fuzzy numbers. Teodorovic and Radivojevic [25] use fuzzy logic to minimize the total traveling distance,
55 vehicle waiting times, and passenger ride times by developing two approximating algorithms, one for vehicle selection
56 and the other for the optimal insertion of DDT.

57 In this paper, we extend the uncapacitated FDARP algorithm, proposed by Maalouf et al. [26] to include a
58 capacitated dynamic fuzzy modeling algorithm for DARP, similar to the one proposed by Teodorovic and Radivojevic
59 [25]. Unlike the one proposed by Teodorovic and Radivojevic [25], the FDARP in this paper assumes that customers
60 are mainly concerned with the drop-off time instead of the pick-up time, and hence the system specifies the pick-up
61 time. In addition, instead of using two approximate reasoning algorithms in the fuzzy optimization step, only one
62 approximate reasoning algorithm is needed in the proposed method. Another important modification is that we allow
63 a vehicle to pick up a customer and make another stop to pick up or drop off a different customer before dropping off
64 the first customer.

65 The organization of the paper is as follows. Section 2 states the problem and Section 3 describes the algorithm.
66 Details of the method and results are presented in Section 4. The conclusion is stated in Section 5.

67 **2. Problem Statement**

68 The dynamic Dial-a-Ride system for which FDARP is designed assumes that customers specify DDTs. These
69 customers rely on the Dial-a-Ride company to provide them with the best possible pick-up time. Each customer calls

and expresses a DDT, the time he or she would like to be dropped off at a given location. Although the customer-provided DDT may be exact, the FDARP algorithm assumes the customer is willing to be dropped off earlier than DDT but no earlier than the earliest drop-off time (EDT). The customer's desired drop-off time is the latest drop-off time at which the customer will be dropped off.

In a dynamic algorithm, vehicles have already been assigned routes for picking up and dropping off customers. The fundamental problem is to assign a vehicle for the new request and indicate the pick-up time for the customer. In addition to satisfying the customer's request by dropping the customer off within the time window of what the customer desires, the algorithm seeks to minimize the time the customer spends in the vehicle (dwell time) and the distance traveled by a vehicle. These two objectives may conflict with each other, and as detailed below, we use fuzzy logic to trade off between these two objectives. This bi-objective problem incorporates both the preferences of the customers, who want to ride in the vehicle as short a time as possible, and the preferences of the company, which desires efficient use of its vehicles. Many DARP solutions (see [4, 25]) exclusively focus on vehicle distance, time, or cost, but companies also want to provide good customer service [7]. One way to reflect good customer service is to minimize the time customers spend in a vehicle. We assume that all vehicles originate from one depot and that all vehicles have the same capacity and are of the same type. Once a request is assigned, it is never modified.

The algorithm uses this objective to determine the actual pick-up and drop-off times for each customer. Given a list of N customers, each specifying EDT_i and DDT_i ($i = 1, 2, \dots, N$), the following relations must hold when determining the actual drop-off time, ADT_i , and the actual pick-up time, APT_i , where DRT_i is the direct ride-time between pick-up and drop-off and MDT_i is maximum dwell time that the customer is willing to spend in the vehicle:

$$DDT_i \geq ADT_i \geq EDT_i \tag{1}$$

$$MDT_i \geq ADT_i - APT_i \geq DRT_i \tag{2}$$

Relation (1) states that the actual drop-off time must be between the earliest drop-off time and the desired drop-off time. Relation (2) indicates that the actual pick-up time subtracted from actual drop-off time must be at least as large as the direct time it takes to go from pick-up to drop-off, and that difference must not be greater than the maximum time the customer is willing to spend in the vehicle. The maximum dwell-time MDT_i could either be stated by the customer, but more likely it will be predefined by the Dial-a-Ride company that a customer will not spend more than a certain amount of time in the vehicle. This allows the company to pick up a passenger and make another stop before dropping off that same customer and differs from the model proposed by Teodorovic and Radivojevic [25].

3. Proposed Solution

The proposed solution is composed of two major parts. The first part utilizes fuzzy numbers to determine when a vehicle could pick up and drop off a potential customer. The second part deploys a fuzzy logic system or an approximate reasoning algorithm to determine the optimal vehicle to pick up the customer. As mentioned earlier, fuzzy numbers express imprecision. While most engineering design problems depend on *objective knowledge*, representing mathematical models and equations, many real-life problems involve *subjective knowledge*, representing linguistic information that is difficult to quantify precisely [27]. In traffic and transportation engineering, subjective knowledge and linguistic information are used on a daily basis by drivers, dispatchers, and passengers [28, 29]. Subjective statements such as "reservations are made with a forty-minute pickup window" or "travel time is approximately twenty five minutes" are often used. Thus, the use of fuzzy models to solve such problems is very suitable.

Fuzzy logic systems are composed of the following three steps: fuzzification, rule evaluation, and defuzzification. The fuzzification step is based on converting inputs into membership degree (between 0 and 1) based on the membership functions. Usually membership functions are triangular (as the one used in this paper), trapezoidal, or Gaussian. Rule evaluation is based on 'If-Then' statements, combining membership functions to derive another fuzzy output. Defuzzification then calculates a single numerical value based on the inferred results from the rules, and the center-of-gravity method (centroid) is usually used. Interested readers are referred to Kaufmann and Gupta [30], Mendel [27], and H-J Zimmerman [32] for rigorous mathematical details.

Teodorovic and Radivojevic [25] solve a dynamic DARP using fuzzy logic with two approximate reasoning algorithms, one for vehicle selection, and the other for the optimal insertion of DDT . Their approximating algorithms are

115 aimed at customers with a specified pick-up time, DPT . Unlike their algorithm, the proposed algorithm for this paper
 116 is designed for customers with a specified drop-off time, DDT , because we believe more emphasis should be given to
 117 DDT .

118 Customers are usually more concerned with the drop-off time than with the pick-up time. In addition, rather than
 119 allowing the dispatcher's subjective perception as to which vehicle to dispatch for a certain customer, FADRP can
 120 automatically provide such information. Furthermore, instead of using two approximate reasoning algorithms in the
 121 fuzzy optimization step, only one fuzzy approximate reasoning algorithm is needed in the proposed method.

122 The methodology in this paper consists of two parts. A request for a Dial-a-Ride is made. In the first part, an
 123 algorithm determines the best pick-up and drop-off time for each vehicle in the fleet. Certain operational constraints
 124 must be satisfied, including vehicle capacity, the maximum ride time, and the customer's drop-off time. After a pick-
 125 up and drop-off time for the request is determined for each vehicle, an optimization algorithm in the second part
 126 based on fuzzy logic determines the best vehicle to satisfy the customer's request. The fuzzy optimization algorithm
 127 seeks to minimize both the amount of time a customer stays in the vehicle and the distance traveled by the vehicle.
 128 The proposed solution is heuristic and generally does not generate an optimal solution in the deterministic operations
 129 research sense. In addition, it is assumed here that once a customer node is inserted into the schedule, it remains
 130 unchanged.

131 3.1. Part 1: Pick-up and Drop-off Times for Each Vehicle

132 The first step determines the best time that each of the M vehicles in the fleet can pick up and drop off customer
 133 i , who has specified DDT_i and EDT_i . Following Teodorovic and Radivojevic [25], we model the desired drop-off
 134 time as a triangular fuzzy number to represent the uncertainty in the customer's request. For example, we believe a
 135 customer will say that he or she would like to be dropped off around 8:00, as opposed to exactly at 8:00. A fuzzy
 136 number $DDT_i = (LDT_i, DDT_i, UDT_i)$, where LDT_i and UDT_i are the lower and upper bounds on the fuzzy number, is
 137 shown in Figure 1.

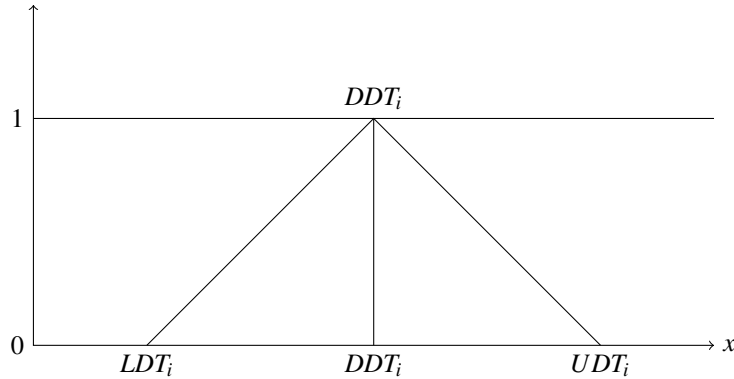


Figure 1: Desired drop-off time represented as a triangular fuzzy number.

138 FDARP sorts all pick-up or drop-off times for a given vehicle in ascending order. Since DDT_i is the input to
 139 FDARP, the insertion of the drop-off node i^- occurs before the insertion of pick-up node i^+ . A non-trivial problem
 140 emerges when these two nodes must be separated by another node, where each node represents a stop for a vehicle.

141 The following constraints must be satisfied for the actual drop-off time ADT_i :

$$ADT_i \geq AA_e + TT_{ei^-}, \quad (3)$$

$$AA_f \geq ADT_i + TT_{i^-f}, \quad (4)$$

142 where AA_e is the actual arrival time at node e (the node that precedes the drop-off node of customer i), TT_{ei^-} is the
 143 travel time from node e to node i^- , TT_{i^-f} is the travel time from node i^- to node f (the node that succeeds the
 144 drop-off node of customer i), and AA_f is the actual arrival time at node f .

145 The FDARP algorithm for this first part is outlined in Algorithm 1. The algorithm assumes that all times are
 146 expressed as fuzzy numbers, to include drop-off and pick-up times, travel times, and arrival times, and we use the
 147 fuzzy arithmetic methods described by Teodorovic and Vukadinovic [29] and Kaufmann and Gupta [31] to ensure
 148 that these operational constraints are satisfied.¹ (The algorithm can also work assuming that all numbers are precise.)
 149 When one fuzzy number is subtracted from another fuzzy number in the algorithm, the fuzzy numbers are independent
 150 of each other [23]. The algorithm repeats for each of the M vehicles, with a computational complexity of $O(M)$.

Algorithm 1: Algorithm to determine pick-up and drop-off time for customer i

Data: M, DDT_i
Result: dw, dis

```

1 begin
2   for  $j \leftarrow 1$  to  $M$  do
3     node  $e$  = stop immediately preceding  $i^-$ 
4     node  $f$  = stop immediately succeeding  $i^-$ 
5     while drop-off time  $_j$  unassigned do
6       if  $AA_f - AA_e > TT_{ei^-} + TT_{i^-f}$  then
7         drop-off time  $_j = \min(DDT_i, AA_f - TT_{i^-f})$ 
8         if  $AA_f - AA_e > TT_{ei^+} + TT_{i^+i^-} + TT_{i^-f}$  then
9           pick-up time  $_j = \text{drop-off time }_j - TT_{i^+i^-}$ 
10        else
11          node  $\ell$  = node  $e$ 
12          node  $k$  = node preceding  $\ell$ 
13          while pick-up time  $_j$  unassigned do
14            if  $AA_\ell - AA_k > TT_{ki^+} + TT_{i^+\ell}$  then
15              pick-up time  $_j = AA_\ell - TT_{i^+\ell}$ 
16              if capacity with customer  $i$  exceeds maximum capacity of vehicle  $j$  then
17                unassign drop-off time  $_j$  and pick-up time  $_j$ 
18                node  $f$  = node  $e$ 
19                node  $e$  = node preceding  $f$ 
20              else
21                node  $\ell$  = node  $k$ 
22                node  $k$  = node preceding  $\ell$ 
23          else
24            node  $f$  = node  $e$ 
25            node  $e$  = node preceding  $f$ 
26        max dwell time =  $\min(\text{pick-up time }_j \text{ for all } M \text{ vehicles})$ 
27        min dwell time =  $\max(\text{pick-up time }_j \text{ for all } M \text{ vehicles})$ 
28        max distance = maximum additional distance a vehicle travels to pick up and drop off customer  $i$ 
29        min distance = minimum additional distance a vehicle travels to pick up and drop off customer  $i$ 
30        dwell score  $dw_j$  = function of pick-up time  $_j$ , min dwell time, and max dwell time; dwell score closer to 0 indicates that
        passenger's dwell time is small
31        distance score  $dis_j$  =  $(\text{additional distance vehicle } j \text{ travels to pick-up and drop-off customer } i - \text{min distance}) / (\text{max}$ 
        distance - min distance)

```

151 Ideally, a vehicle should drop the customer off at the customer's desired drop-off time (DDT_i). If assigning the
 152 actual drop-off time (ADT_i) to be equivalent to DDT_i violates Equation (3) or (4), the operator can change ADT_i to an
 153 earlier time. The actual drop-off time may need to be readjusted so that the customer is picked up before the current

¹The greater-than inequality $>$ in the algorithm is defined as follows. Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be fuzzy numbers. $A > B$ if $(a_1 + 2a_2 + a_3)/4 > (b_1 + 2b_2 + b_3)/4$; or $(a_1 + 2a_2 + a_3)/4 = (b_1 + 2b_2 + b_3)/4$ and $a_2 > b_2$; or $(a_1 + 2a_2 + a_3)/4 = (b_1 + 2b_2 + b_3)/4$ and $a_2 = b_2$ and $a_3 - a_1 > b_3 - b_1$ [29, 31].

154 node e . If this occurs, the current node e becomes node f and the new node e immediately precedes the current node
 155 e .

156 After ADT_i is determined, the next step is the insertion of node i^+ , which requires the calculation of the actual
 157 pick-up time APT_i :

$$AA_k + TT_{ki^+} \leq APT_i, \quad (5)$$

$$APT_i + TT_{i^+\ell} \leq AA_\ell, \quad (6)$$

158 where AA_k is the actual arrival time at node k (the node preceding the pick-up node of customer i), TT_{ki^+} is the travel
 159 time from node k to node i^+ , $TT_{i^+\ell}$ is the travel time from node i^+ to node ℓ (the node that succeeds the pick-up node
 160 of customer i), and AA_ℓ is the actual arrival time at node ℓ .

161 The FDARP algorithm first assigns node i^+ such that $APT_i = ADT_i - DRT_i$. If this assignment violates Equation
 162 (5) or (6), FDARP moves node i^+ to an earlier time, up to the MDT_i (Figure 2). Regardless of whether a feasible
 163 solution is found for vehicle j , FDARP tries to find a drop-off and pick-up time for customer i for the next vehicle.

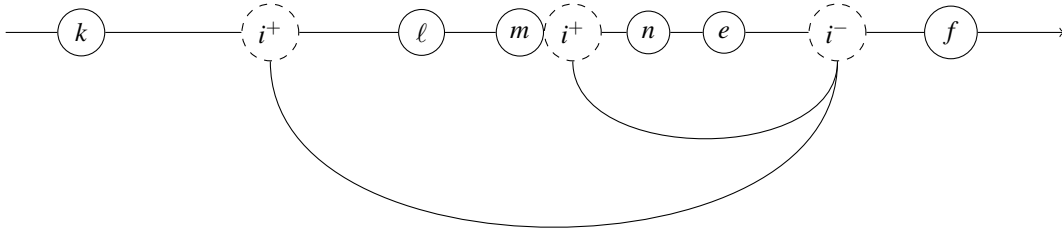


Figure 2: Assignment of new a request i to a route of a vehicle j .

164 FDARP calculates a dwell score (dw_j) and a distance score (dis_j) for vehicle j . Both scores are functions of the
 165 assigned ADT_i and APT_i for each vehicle, and the scores are scaled so that each score ranges between 0 and 1. The
 166 dwell score is based on the length of time the customer spends in vehicle j . If the time spent in vehicle j is close to
 167 DRT_i , the dwell score is near 0. As the pick-up time becomes earlier for vehicle j , the dwell time increases, which
 168 means that dw_j moves towards 1. The distance score also ranges between 0 and 1 and is based on the distance vehicle
 169 j travels. The shorter the distance vehicle j travels, the closer the distance score is to 0. The FDARP algorithm ends
 170 by returning two vectors of length M : a vector of dwell scores and a vector of distance scores.

171 3.2. Part 2: Selecting the Optimal Vehicle

172 Once feasible insertions for nodes i^- and i^+ are found, we use an approximate reasoning algorithm to select the
 173 optimal feasible solution. As mentioned earlier, optimality is defined as the strongest preference (very weak, weak,
 174 medium, strong, and very strong) based on the customer's dwell time and distance score for each vehicle. After the
 175 FDARP algorithm generates a dwell time dw_j and a distance score dis_j for each feasible vehicle j for one individual
 176 customer, determining which vehicle will carry the passenger is the next step.

177 A multiobjective value function, where the decision maker explicitly determines how much to weigh the distance
 178 a vehicle travels vis-a-vis the time a customer sits in the vehicle, could be used to to determine the optimal vehicle.
 179 However, it may be unrealistic to expect that the decision maker will assess the weights and follow the normative
 180 rules of multicriteria decision making [33, 34]. As an alternative, we deploy an approximate reasoning algorithm to
 181 trade off between the dwell score and the distance score. The approximate reasoning algorithm is based on Mamdani
 182 [35, 36] fuzzy inference system as implemented by the Matlab [37] fuzzy logic toolbox. The Mamdani fuzzy inference
 183 system uses membership functions to fuzzify inputs; calculates an output based on rules that combine the fuzzified
 184 inputs; and classifies that output in a discrete category.

185 The membership functions in this paper are identical to those in Teodorovic and Radivojevic [25], and they trans-
 186 form the crisp scores dw_j and dis_j into a combination of "small", "medium", or "big" as depicted in Figures 3 and 4.

187 The triangular membership functions are designed to reflect a dispatcher's subjective preference for a smaller dwell
 188 time and a shorter distance traveled. Each membership function assumes that a score of 0 is completely small (e.g.,
 189 small dwell time or short distance traveled) and a score of 1 is completely big (e.g., large dwell time or long distance
 190 traveled). If the score equals 0.5, the resulting fuzzification is mostly medium with small and big at equal but lower
 191 levels. Although the numerical results use the membership functions depicted in Figures 3 and 4, a user can modify
 192 these membership functions to better reflect the preferences of the dispatcher.

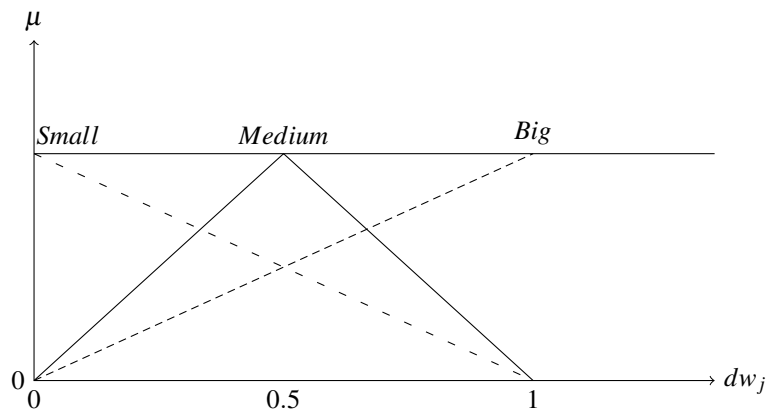


Figure 3: Membership function that fuzzifies the dwell time of a new passenger.

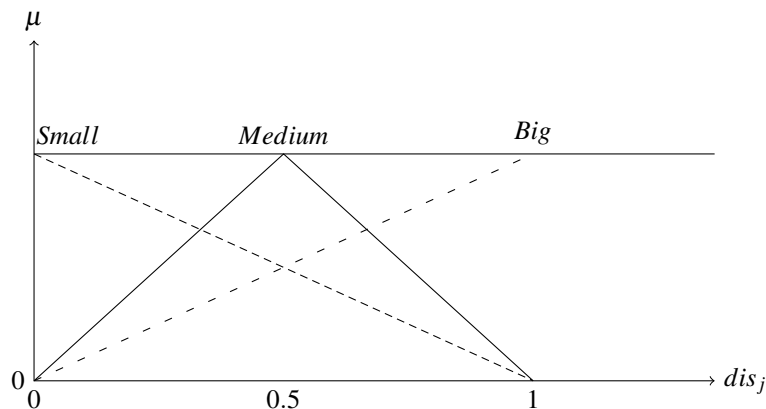


Figure 4: Membership function that fuzzifies the distance traveled to pick up a new passenger.

193 Because each score is partly small, partly medium, and partly big due to the membership function, the output
 194 is a distribution based on $3 \times 3 = 9$ relationships. Each of these relationships is depicted in Table 1. For example,
 195 **IF** the distance to pick up a new customer is *small* **AND** the dwell time is *small*, **THEN** the preference is VERY
 196 STRONG. Given these preference relationships and the previously described membership functions, the Matlab fuzzy
 197 toolbox calculates a numerical vehicle preference score ranging between 0 and 100. The vehicle preference score is
 198 categorized into one of five categories according to the structure in Figure 5: very weak, weak, medium, strong, and
 199 very strong.

200 For example, if $dw_j = 0.2$ and $dis_j = 0.5$, the distance traveled by vehicle j to pick up a customer is mostly
 201 medium, and the customer is dropped off pretty close to his or her DDT, so the fuzzification of dw_j is mostly small.
 202 The Matlab fuzzy toolbox returns a numerical output of 50.9, which corresponds to a medium preference for that
 203 vehicle (Figure 5). Although a small dwell time and a medium distance correspond to a strong vehicle preference, the
 204 membership function for dw_j classifies a significant proportion of $dw_j = 0.2$ in medium and even big (Figure 3). If
 205 the distance score is also 0.2, the numerical output is 61.5, and the vehicle preference is strong.

		Dwell Time		
		<i>Small</i>	<i>Medium</i>	<i>Big</i>
Distance to Pick up	<i>Small</i>	Very Strong (VS)	Strong (S)	Medium (M)
	<i>Medium</i>	Medium (M)	Medium (M)	Weak (W)
	<i>Big</i>	Weak (W)	Weak (W)	Very Weak (VW)

Table 1: **Algorithm 2:** Fuzzy rules for choosing optimal vehicle.

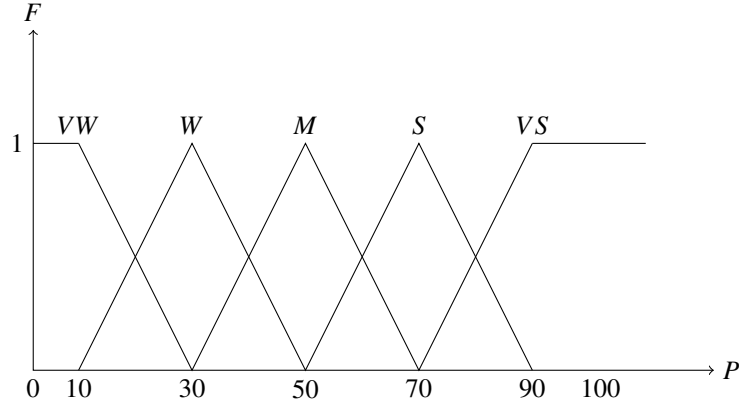


Figure 5: Preference strength scale to assign a new request to a certain vehicle.

206 4. Numerical Example

207 To the authors' knowledge, no benchmark data sets are available in the literature for the version of DARP studied
208 in this paper. Therefore, the behavior of the proposed method cannot be evaluated using data instances that have been
209 widely tested and the results cannot be compared to other optimum ones obtained in previous publications. Thus, in
210 order to analyze the efficacy of the FDARP algorithm, we have simulated instances according to realistic assumptions.
211 The performance measures used in this study, however, are very close to the ones used by Teodorovic and Radivojevic
212 [25]. The developed algorithms above were tested using two different simulations for a single day. Both simulations
213 start with a completely empty system. Customers call one at a time with a DDT_i , EDT_i , and LDT_i , and the algorithms
214 determine the optimal vehicle to pick up and drop off customer i . Each DDT_i is randomly generated from a uniform
215 distribution between 8 AM and 10 PM and $EDT_i = DDT_i - 10$ min and $LDT_i = DDT_i + 10$ min. The time it takes to
216 go directly from picking the customer up to dropping him or her off, DRT_i , is uniformly distributed between 5 and
217 45 minutes. Each customer is willing to spend at most 1 hour in the vehicle, and each will also accept being dropped
218 off at most 20 minutes earlier than his or her DDT_i . Consequently, a customer will be picked up at most 1 hour 20
219 minutes before DDT_i .

220 Although the dwell score is a fuzzy number, "small" dwell time corresponds approximately to less than 25 min-
221 utes, "medium" dwell time is more than 25 minutes but less than 50 minutes, and "big" dwell time is more than 50
222 minutes. The maximum distance a vehicle travels to pick up a customer is 25 miles, and small distance corresponds
223 approximately to 0-8 miles, medium distance is 8-16 miles, and big distance is 16-25 miles. Each vehicle can hold
224 a maximum of 10 people. All of the computations are carried out using MATLAB version 2012b on a 4 GB RAM
225 computer.

226 In the first simulation, the Dial-a-Ride company has a fleet of 6 vehicles, and 100 customers request a ride.
227 Table 2 displays the results of this simulation. Most customers arrive at their destination close to their DDT_i as they
228 wait on average about 2 minutes although 7 customers are not given a ride because the operational constraints are
229 not met. Customers being dropped off an average of only 2 minutes before their desired drop-off time is excellent
230 customer service, and many customers in this simulation go directly from their pick-up to drop-off locations. However,
231 rejecting 7 out of 100 customers because a vehicle cannot get them to their destination within the time window is a
232 deficiency. Relaxing some constraints such as dropping customers off more than 20 minutes before their DDT_i or

233 having customers spend more than 1 hour in the vehicle would allow the company to service more requests.

Table 2: Results for simulation with 6 vehicles and 100 customers

Customers rejected because constraints not met any vehicle	7
Average time customer spends in vehicle	26.81 min
Average amount of time customer dropped off before DDT	2.32 min
Average number of miles traveled by a vehicle	305.05 miles
Total number of vehicles used	6
Average ratio of actual travel time to direct travel time	1.08-1.78
Average ratio of actual traveling distance to direct traveling distance	1.09-1.89
CPU elapsed time	4.37 seconds

234 In the second simulation, the company has a fleet of 30 vehicles, and 900 customers request a ride. Table 3
 235 displays the results of this simulation with a larger fleet of vehicles. Twenty-six customers are rejected, giving the
 236 company an ability to satisfy almost 98% of all requests. The service is a little less efficient than the smaller fleet
 237 with fewer customers, and customers arrive on average 4.5 minutes before their DDT_i , compared with 2 minutes in
 238 the first simulation, and the average customer spends 34 minutes in the vehicle, compared with 27 minutes in the first
 239 simulation. The average ratio of actual travel time to direct travel time is 1.12-1.99 and the average ratio of actual
 240 traveling distance to direct traveling distance is 1.20-2.17. These ratios are close to those obtained by Teodorovic and
 241 Radivojevic [25] who depicted values of 1.25-1.36 for actual travel time to direct travel time and 1.16-1.23 for actual
 242 traveling distance to direct traveling distance.

Table 3: Results for simulation with 30 vehicles and 900 customers

Customers rejected because constraints not met any vehicle	26
Average time customer spends in vehicle	33.7 min
Average amount of time customer dropped off before DDT	4.47 min
Average number of miles traveled by a vehicle	414.63 miles
Total number of vehicles used	30
Average ratio of actual travel time to direct travel time	1.12-1.99
Average ratio of actual traveling distance to direct traveling distance	1.20-2.17
CPU elapsed time	213.3 seconds

243 As can be observed from Tables 2 and 3, the CPU elapsed time for the algorithm to assign customers to vehicles is
 244 about 4 seconds for 6 vehicles and 100 customers and about 3.5 minutes for 30 vehicles and 900 customers. The time
 245 to process and find a vehicle for the next customer is also very short. When 100 customers are previously assigned to
 246 6 vehicles, the algorithm assigns the 101st customer to a vehicle in 0.05 seconds on average. When 900 customers are
 247 previously assigned to 30 vehicles, the algorithm assigns the 901st customer to a vehicle in 0.25 seconds on average.
 248 This numerical study suggests that FDARP can find a solution in a short enough time period that a Dial-a-Ride
 249 company could rely on it in a real-time setting.

250 Varying the number of vehicles available for transporting customers can provide insight into how the size of the
 251 vehicle fleet impacts the performance characteristics. Figure 6 depicts how the number of customers that can be served
 252 (out of a total of 900 customers) and the average miles traveled by each vehicle change as the number of vehicles in
 253 the fleet changes. The results are based on 5 simulation runs. As the vehicle fleet increases in size, the Dial-a-Ride
 254 company can serve more customers, and average distance traveled by each vehicle decreases. This type of comparison
 255 can be useful for a decision maker determining the size of the company's fleet. For example, if the company currently
 256 has 30 vehicles and is considering buying a 31st vehicle, a manager can use this comparison to understand that having
 257 a 31st vehicle will allow the company to transport about 9 more passengers each day and that each vehicle will travel
 258 on average 8 fewer miles per day. The manager can weigh these additional benefits with the extra cost of having an
 259 additional vehicle and driver.

260 If the company has 32 vehicles or more, the company is able to meet almost all of the customer requests. Depend-
 261 ing on the cost of buying additional vehicles and hiring additional drivers, a company with 30 vehicles who fields 900

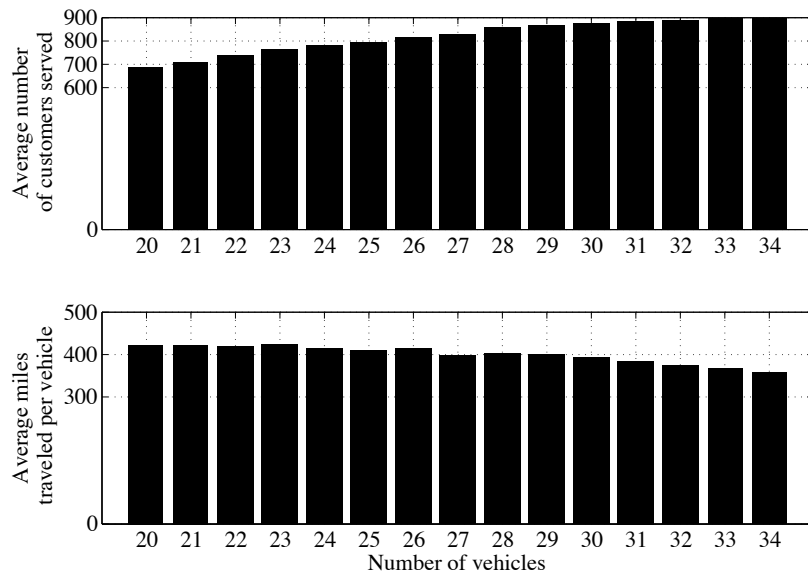


Figure 6: Performance metrics for different vehicle fleet sizes with 900 customers

262 requests per day may want to consider investing in additional vehicles.

263 5. Conclusion

264 We have presented the FDARP algorithm and demonstrated its effectiveness for DARP. Basing the pick-up and
 265 drop-off times on the time a customer wishes to arrive at a location provides a realistic way of handling each customer's
 266 request. Allowing the vehicle to make multiple stops in between picking up and dropping off a customer gives a Dial-
 267 a-Ride greater flexibility. The fuzzy logic rules enables the Dial-a-Ride company to trade off between two objectives–
 268 one of which focuses on making the company more efficient and the other which focuses on customer service. By
 269 minimizing the distance traveled by each vehicle and attempting to drop off the customer at a time closest to his or her
 270 desired drop-off time, the algorithm balances between a customer's preferences and a company's desire for efficiency
 271 and low cost.

272 FDARP takes advantage of the speed and flexibility of fuzzy rules to insert requests into a system, and it is less
 273 complex than algorithms that rely on constrained optimization methods. The numerical study provides insight into
 274 how simulation can help a company make decisions about the size of its vehicle fleet and which vehicle should pick
 275 up each customer. Using real-life data in future studies can demonstrate FDARP's usefulness especially if it is linked
 276 to a real-time GPS systems to maximize the algorithm's performance and accuracy. Although the algorithm is mainly
 277 concerned with dynamic DARP, it could be extended to solve static DARP and potentially other assignment problems.

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