



Calhoun: The NPS Institutional Archive

Faculty and Researcher Publications

Faculty and Researcher Publications

1998

Failure, repair and replacement analysis of a navy subsystem: case study of a pump

Gaver, Donald P.

Applied Stochastic Models and Data Analysis, Volume 13, pp. 369-376 (1998).

<http://hdl.handle.net/10945/44158>



Calhoun is a project of the Dudley Knox Library at NPS, furthering the precepts and goals of open government and government transparency. All information contained herein has been approved for release by the NPS Public Affairs Officer.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

FAILURE, REPAIR AND REPLACEMENT ANALYSIS OF A NAVY SUBSYSTEM: CASE STUDY OF A PUMP

DONALD P. GAVER, PATRICIA A. JACOBS AND DONALD D. DUDENHOEFFER*

Department of Operations Research, Naval Postgraduate School, Monterey, CA 93943, U.S.A.

SUMMARY

An age-replacement policy may be used to minimize the cost of replacement and repair of a system whose rate of repairable failures increases with age. In this paper we model the failure times before replacement of one such system—a pump used on a submarine—by a non-homogeneous Poisson process having log-linear hazard rate which is a function of age. Maximum likelihood is used to estimate model parameters. A renewal-reward process is used to obtain estimates of log-run average costs of age-replacement policies. It should be noted that the policy obtained does not recognize possible between-submarine differences but does acknowledge that noise from a submarine pump is not to be tolerated. © 1998 John Wiley & Sons Ltd.

KEY WORDS reliability; age-replacement policy; non-homogeneous Poisson process; minimal repair

1. INTRODUCTION AND PROBLEM SETTING

Many important systems, for example military submarines or aircraft, or civilian automobiles, aircraft, trucks and construction equipment are composed of various interlinked subsystems that occasionally require nearly immediate corrective maintenance or repair, or eventually total replacement, sometimes at short notice and at unforeseen times. The mean rate of occurrences of such events, *failures* for short, may tend to increase with the service age of the subsystem, actual events appearing randomly. Each time such an event occurs, a repair or replacement is needed, which temporarily disables the subsystem and handicaps the host system itself, degrading or removing its mission capability. Repair, even in the field, takes time and incurs cost, while some repairs and replacements must be conducted at special locations remote from the operating environment, further decreasing system on-site or mission availability. If repairable failures begin to accelerate with a subsystem's age (quantified by time in service, or some indicator of wear) there may be good reason for total subsystem replacement before an actual non-repairable failure; doing so may be costly, but tends to prevent an increasing crescendo of less-expensive repairs, and, more importantly, forestalls disastrous failures during an actual mission.

* Correspondence to: Lt. D. D. Dudenhoefter, Department of Operations Research, Naval Postgraduate School, Monterey, CA 93943, U.S.A.

This paper studies a specific instance of the above general situation, the case of a *cam-driven reciprocating pump* used on many U.S. Navy submarines.

The study begins by examining historical data collected on the times of operation until repair or replacement of the pump on each of 61 Navy submarines. These data are obtained from operational units, and are presumed accurate to about the monthly (discrete-time) level. Some of the data entries are censored: a particular submarine may be recorded as having experienced several repairable pump failures, followed by pump replacement, then more repairable failures, with the record terminated, possibly without the occurrence of any failure.

The above data has been summarized by a non-homogeneous Poisson process (NHPP) model as follows: the NHPP with age-dependent non-decreasing hazard rate governs the occurrence of repairable failures until the appearance of a replacement event; after replacement the age-dependent hazard is re-started, as is appropriate for a new pump.

It has been assumed that the age dependence can be adequately represented by a hazard that is log-linear in age, and that the parameters thereof are the same for all pumps on all submarines; this assumption requires further validation, and its replacement by a hierarchical model is in prospect. The above is an adaptation of the Cox–Lewis model discussed in their monograph of 1966 (pp. 45–51);¹ other parametric alternatives can also be entertained. The log-linear hazard model has been fitted by maximum likelihood, and the fits suggest a weak hazard-rate increase with age. Preliminary data analysis allows estimation of the probability that a failure event is repairable, and suggests that such events may be modelled as occurring like age-independent Bernoulli trials (preliminary data analysis suggests that age dependence is weak, and can be ignored to a first approximation). The occurrence of the complementary event signifies that replacement is required, whereupon the NHPP is restarted from age zero.

Use of the above fitted model permits an age-dependent replacement strategy to be evaluated; a maximum-age replacement strategy can be determined having this structure: if a failure requiring replacement occurs before the threshold is reached, the replacement is immediately scheduled (this may require mission termination), and a new pump is installed with the same maximum age threshold; otherwise, the pump is replaced when its age reaches the threshold. The threshold value can be optimized by cost considerations: if the threshold is small, replacements are too frequent and costs attributable to replacement are excessive, while if too large the costs associated with repairs become excessive. Application of renewal–reward theory guides the choice of a threshold that approximately optimizes the long-run cost rate. The above policy can be modified to constrain for a tolerable level of risk of failure during a mission of specified duration. Preliminary numerical results suggest that the threshold values are reasonable; they happen to be considerably larger than those in official use in the past, thus implying that significant cost savings might be realizable.

The setup described is an initial or pilot model for a general situation that occurs more generally, both in the submarine application and elsewhere (possibly when fleets of rental cars are to be managed). A number of model enhancements are envisioned: individualized assessment of age rate by submarine so that replacement can be better tailored to need, and a replacement age threshold choice that balances the needs of various subsystems that age at different rates.

1.1. Previous work on related models

There has been considerable previous work on similarly-formulated problems, beginning probably with that of Barlow and Hunter² and including work by Block *et al.*^{3,4,5} A review was

published by Valdez-Flores and Feldman⁶ (see also Whitaker and Samaniego⁷ and, for a somewhat related problem, Glazebrook *et al.*⁸). There are doubtless other relevant papers that have gone unmentioned here, but that may be referenced in some of the above. Relatively few of these actually confront the models with data as is done here. So far, the authors have seen no analyses that explicitly recognize systematic, if temporary, differences between subsystem or item failure properties resulting from conditions of usage, modelled as either deterministic or random.

2. DATA AND DATA ANALYSIS

The data used in this study were obtained from a set of sixty-one pumps that exhibited 140 failures over the observation period beginning in May 1987 and ending in December 1993. The set is assumed to be representative. The final observation for any submarine was either a failure and replacement prior to December 1993, or, otherwise, the pump was last observed still in operation. This set does not include all pumps in operation, but a subset for which there exist reliable dates for pump installation. Additionally, the pumps were carefully screened to ensure that all pump replacements resulted from non-repairable failures. The installation and failure times are rounded to the nearest month and henceforward all references to time and age will be in months. The time in service is adjusted to remove any inactivation period of two or more months in length from the pump's operating age. Failure and replacement data originate from the Maintenance and Material Management (3-M) system which reports routine maintenance.

Skilled personnel attached to the Navy Bureau of Ships conducted a 'failure modes and effects analysis' (FMEA) to identify the critical failure modes characteristic of the pump and to characterize the required repair actions. Information on the types of failure, their frequency, the repair requirements, and the trends for the different failure modes is an essential element in any analysis of equipment performance. This paper focuses specifically on the occurrence of failures and the required repair actions. Failures are not explicitly distinguished by type or cost herein. However, the life-cycle cost in terms of material and labour is included implicitly in the calculation of the age-replacement policy.

Several assumptions were made regarding the pump failure data. These assumptions are as follows.

- (1) The submarines in the sample set have relatively similar operating cycles, i.e. pumps on different submarines will face similar operating conditions and undergo roughly the same number of hours of operation for the same time period. This may well be questionable, and will be investigated in later analyses.
- (2) Every component failure causes equipment failure.
- (3) Failures are immediately evident.
- (4) Pumps are only repaired or replaced at failure and not in anticipation of failure (In actuality this is not always the case, but the data set was cleansed to ensure only replacements related to failures were counted).
- (5) Consecutive failure times on an individual pump are conditionally independent, given the previous failure time.
- (6) A repair returns the pump to full operation, but does not restore it to a 'good-as-new' condition.
- (7) Equipment repairs require times are negligible compared to times between failures.
- (8) A *replacement* constitutes the installation of a new pump or a complete overhaul of the current pump.

3. AGE-DEPENDENCY OF FAILURE RATES

Engineering intuition suggests that the failure rate, or probability that a pump fails during a given ‘small’ time period, may well increase with service age since it is a mechanical item. This supposition was tested informally by comparing the average number of failures exposed per ship-month and by performing a modified Laplace test (Cox and Lewis¹ 1966). The indication of such age-dependency (graphical plots reinforced by a Laplace test *p*-value of 0.043) led to fitting a parametric model: the non-homogeneous Poisson process with instantaneous rate parameter depending on age, *t*:

$$\lambda(t) = \exp[\alpha + \beta t] \tag{3.1}$$

and cumulative age-dependent hazard

$$A(t) = \int_0^t \lambda(t') dt' = e^\alpha(e^{\beta t} - 1)/\beta \tag{3.2}$$

where no attempt was made to adjust for discrete-time effects. Additionally, each failure can be either repairable or a cause of replacement; to model this, a Bernoulli trial was executed at each failure and with probability *p*(*t_{ij}*), *i* denoting failure number and *j* pump number, a repairable failure occurred, otherwise replacement with probability *q*(*t_{ij}*) = 1 - *p*(*t_{ij}*). A likelihood function was then written that combined all pump failure data; this simplifies to

$$L(\alpha, \beta, \underline{p}; \text{data}) = \prod_{j=1}^J \left(\prod_{i=1}^{n_j} \lambda(t_{ij}) p(t_{ij}) \right) \exp \{ - A(t_{n_j, j}) \} \left(\frac{q(t_{n_j, j})}{p(t_{n_j, j})} \right)^{\bar{I}_{n_j}} \exp \{ - [A(\tilde{t}_j) - A(t_{n_j, j})] \} I_{n_j} \tag{3.3}$$

where

$$I_{n_j} = \begin{cases} 1 & \text{if the last observed failure is repairable} \\ 0 & \text{if the last observed failure is non-repairable} \end{cases} \quad \bar{I}_{n_j} = 1 - I_{n_j}$$

and *t_{ij}* is the age of occurrence of the *i*th failure of the *j*th pump (*i* = 1, 2, ..., *n_j*), *t_{n_j, j}* is the age of occurrence of the last (*n_j*)th observed pump failure, \tilde{t}_j is the total observed age on the *j*th pump; by convention $\tilde{t}_j = t_{n_j, j}$ if the last observed failure is not repairable or there is no observed excess life. Now condition on *n_j*, the total number of observed failures for the *j*th pump for every pump; the conditional likelihood becomes proportional to

$$\prod_{j=1}^J \prod_{i=1}^{n_j} \lambda(t_{ij}) / A(\tilde{t}_j)$$

which for the specific model (3.1) and (3.2) gives the likelihood to β which is proportional to

$$L_\beta(\beta; \underline{t}) = \prod_{j=1}^J \prod_{i=1}^{n_j} \beta e^{\beta t_{ij}} / (e^{\beta \tilde{t}_j} - 1) \tag{3.4}$$

Taking logarithms, differentiating with respect to β equating the result to zero and solving produces the m.l.e. $\hat{\beta} = 0.0226$. A second derivative at $\hat{\beta}$ provides Fisher information and hence an approximate variance for $\hat{\beta}$. Return to the full likelihood and differentiate with respect to α ; this

differential supplies the m.l.e. formula

$$\hat{\alpha} = \ln \left[\frac{\hat{\beta} \sum_{j=1}^J n_j}{\sum_{j=1}^J (e^{\hat{\beta} t_{ij}} - 1)} \right] \quad (3.5)$$

and, in the present case, $\hat{\alpha} = -3.189$. Fisher information, and therefrom the approximate variance for $\hat{\alpha}$, is also obtainable. Use of these approximate values and the asymptotic normality of m.l.e.'s then give the 95% confidence limits on α and β shown in Table I.

Apparently, β in particular is significantly positive. An alternative approach to the use of Fisher information in order to obtain confidence limits could be the use of parametric bootstrapping. The fitted model is compared to the actual data shown in Table II and Figure 1. It is noted that the model-predicted 'rate' (actually, the non-homogeneous Poisson mean) and the 8-month average 'rate' agree satisfactorily until month 49, but not well beyond. However, there is relatively little pump exposure at that age so some unexplained cause for apparent failure rate drop off at late ages must be sought. It is judged to be conservative to adopt the model conclusions, and hence this option is selected.

We note that an independent likelihood analysis can be conducted of the probability that a pump failure is repairable, $p(t_{ij})$. The analysis is summarized by stating that no strong age dependence was evident. Pooling the 126 repairable failures, (out of 140 failures) resulted in an estimate of $p = 126/140 = 0.90$, which will be used in subsequent computations.

Table I. 95% confidence intervals for parameters $\hat{\alpha}$ and $\hat{\beta}$

Parameter	Lower limit	m.l.e.	Upper limit
α	-3.50	-3.189	-2.87
β	0.012	0.02258	0.033

Table II. Comparison of data failure and the fitted model failure rate

Interval (months)	Total exposure (months)	Observed failures	Interval failure rate	Model failure rate
1-8	488	23	0.047	0.045
9-16	438	30	0.068	0.054
17-24	372	29	0.078	0.065
25-32	277	23	0.083	0.078
33-40	188	15	0.080	0.093
41-48	152	14	0.092	0.112
49-56	76	5	0.066	0.134
57-64	28	1	0.036	0.160

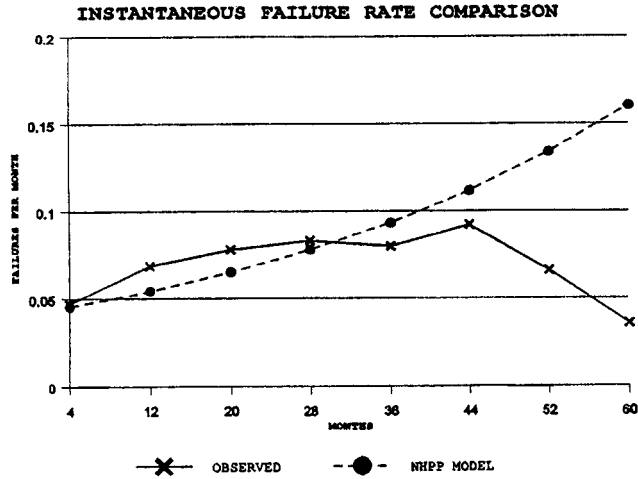


Figure 1. Observed interval failure rate versus model failure rate, $\lambda(t) = \exp(\alpha + \beta t)$

4. COST MINIMIZATION AND MAXIMUM REPLACEMENT AGE

We propose a simple and practical age replacement policy: let X denote the age at which a pump experiences a (first) non-repairable failure, and let T denote a maximum age of replacement. That is, pumps are replaced at cycle times

$$L = \begin{cases} X & \text{if } 0 \leq X < T \\ T & \text{if } T \leq X \end{cases} \tag{4.1}$$

We now appeal to renewal–reward theory (cf. Ross⁹) to show that the long-run rate of cost accrual per unit time can be written as

$$\bar{c}(T) = \frac{E[C(L(T))]}{E[L(T)]} \tag{4.2}$$

$C(L(T))$ denotes a random life-cycle cost, given decision variable T ; $L(T)$ is the corresponding random life-cycle duration. We assume that each life-cycle begins with a mean replacement cost, c_n , here taken to be US \$111 000; a random (non-homogeneous Poisson, parameters $\hat{\alpha}$ and $\hat{\beta}$) number of repairable failures occurs during that cycle, each one of which incurs mean cost c_r , here taken to be US \$4900.

In order to evaluate equation (4.2), prior to minimization the following steps are required.

$$E[C(L)] = c_n + c_r E[N(L)] \tag{4.3}$$

where $N(L)$ denotes the number of repairable failures during a random life cycle; from equations

(3.1) and (3.2) in general

$$\begin{aligned}
 E[N(L)] &= \int_0^T \left(\sum_{n=0}^{\infty} n \exp\{-\Lambda^n(x)\} \frac{\Lambda^n(x)p^n}{n!} \right) q\lambda(x) dx + \sum_{n=0}^{\infty} n \exp\{-\Lambda(T)\} \frac{\Lambda^n(T)p^n}{n!} \\
 &= pq \int_0^T \Lambda(x) \exp\{-q\Lambda(x)\} \lambda(x) dx + p\Lambda(T) \exp\{-q\Lambda(T)\} \\
 &= \frac{p}{q} [1 - (1 + q\Lambda(T)) \exp\{-q\Lambda(T)\}] + p\Lambda(T) \exp\{-q\Lambda(T)\} \\
 &= \frac{p}{q} (1 - \exp\{-q\Lambda(T)\}).
 \end{aligned}
 \tag{4.4}$$

The expectation of the life-cycle duration is seen to be

$$E(L) = \int_0^T \exp\{-q\Lambda(x)\} dx
 \tag{4.5}$$

which cannot be explicitly evaluated in closed form but can be numerically integrated (for small β a three-term Taylor's series expansion of the exponential in equation (3.2) provides an error-function approximation for equation (4.5) that could be useful, but has not been used). Expression (4.3) can thus be evaluated numerically; a graph is shown in Figure 2, which suggests that a replacement interval of about 111 months is optimum for the parameter values selected. Incidentally, this interval is far in excess of the value in current use.

5. DISCUSSION

Sensitivity studies that vary parameter values and the non-homogeneous Poisson hazard rate function form are easily conducted given the general formulas (4.3), (4.4) and (4.5); some detail appears in the Naval Postgraduate School MS thesis by D. Dudenhoeffer.¹⁰

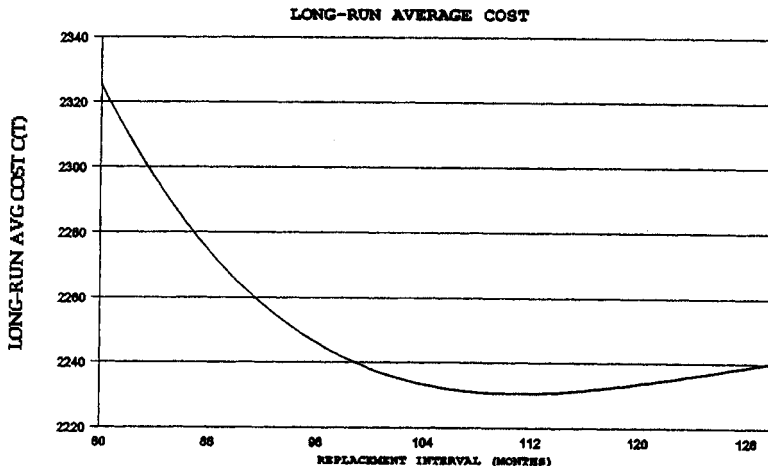


Figure 2. Long-run average cost

Finally, the cost function invoked does not reflect certain, non-monetary but real Navy-submarine-specific operational issues: in case the pump fails and can be repaired while on mission, its function is temporarily assumed by another pump, but one that is considerably noisier while in operation. The noise so generated may render the submarine more readily detectable and thus at risk. Consequently, it may be advisable to replace a failure-prone pump earlier than unconstrained simple economics might suggest.

REFERENCES

1. D. R. Cox and P. A. Lewis, *The Statistical Analysis of Series of Events*, Latimer Trend, Whitstable, U.K., 1966.
2. R. Barlow and L. C. Hunter, 'Optimum preventive maintenance policies', *Operations Research*, **8**, 90–100 (1960).
3. H. W. Block, W. S. Borges and T. H. Savits, 'Age-dependent minimal repair', *J. Appl. Probability*, **22**, 370–385 (1985).
4. H. W. Block, W. S. Borges and T. H. Savits, 'Preventive maintenance policies', in A. P. Basu (ed), *Reliability and Quality Control*, Elsevier/North-Holland, Amsterdam, 1986, pp. 101–106.
5. H. W. Block, W. S. Borges and T. H. Savits, 'A general age replacement model with minimal repair', *Naval Research Logistics*, **35**, 365–372 (1988).
6. C. Valdez-Flores and R. M. Feldman, 'A survey of preventive maintenance models for stochastically deteriorating single-unit systems', *Naval Research Logistics*, **36**, 419–446 (1989).
7. L. R. Whitaker and F. J. Samaniego, 'Estimating the reliability of systems subject to imperfect repair', *J. American Statistical Assoc.*, **84**, 301–309 (1989).
8. K. D. Glazebrook, M. P. Bailey and L. R. Whitaker, 'Cost rate heuristics for semi-Markov decision processes', *J. Appl. Probability*, **29**, 633–644 (1992).
9. S. M. Ross, *Stochastic Processes*, Wiley, New York, 1983.
10. D. D. Dudenhoefter, 'Failure analysis of a repairable system: the case study of a cam-driven reciprocating pump', MS thesis, Naval Postgraduate School, Monterey, California, U.S.A., 1994.