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# Efficient strategies for transporting mobile forces

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During a military operation, it may be necessary to move military units quickly and efficiently from one zone at the theatre of operations to another one. This need is prevalent in particular at the earlier stages of an operation when combat units are accumulating at the theatre of operations. Such mobility missions are carried out by specially designed semi-trailers, called transporters, that carry the armoured fighting vehicles (AFVs) of the military unit. In many cases the number of available transporters is smaller than the number of AFVs that are to be carried, thus several tours of the transporters may be needed to transport the entire unit to its destination. In this paper we examine three generic transportation strategies that may apply to such mobility missions: fixed unloading point, variable unloading point and a flexible strategy in which both loading and unloading points may vary from one tour to another. The efficiency of each specific transportation plan, within a given generic strategy, is evaluated with respect to the criterion of minimum accumulation time.

**Keywords:** mobility; transportation; tours, accumulation time

## Introduction

One of the important stages in military operations is that of accumulating combat units at the theatre of operations. The objective is to move these units from their home bases to their designated staging areas at the front as fast as possible, while maintaining high level of combat readiness. This stage of the operation is called *force accumulation.* Force accumulation is critical in particular in situations where the enemy has staged a surprise attack. In such cases, the defending side has to deploy its forces as quickly as possible to repel the attack. In a forward-deployment scenario, or in a power-projection one in which the theatre of operation is connected to the rear by land lines of communication, this process is usually executed by ground transportation.

Armoured fighting vehicles (AFVs) such as tanks and armoured personnel carriers are carried on-board specially designed trucks (semi-trailers) called *transporters.* A need to transport AFVs may rise also during a campaign when it is necessary to shift combat efforts from one zone of the theatre to another.

Transporting AFVs by transporters is usually faster than letting them travel by themselves on their own tracks. It also eliminates possible physical wear, or even mechanical failures, that may occur to an AFV while moving on the ground, and it saves fuel that otherwise is consumed by the AFV. There are also possible shortcomings to this mode of transportation: it may be more vulnerable to enemy attacks and less efficient in situations where roads are damaged or blocked. However, its advantages with respect to time and physical wear of the AFVs make this mode of transportation appealing to commanders and military planners.

Transporters are usually a scarce resource and in many situations the number of transporters that are allocated to a military unit (eg a division) is smaller than the number of AFVs that are to be moved. This constraint implies that the transportation process may comprise several tours of the transporters. That is, the transporters load only part of the unit, transport it to the unloading point (eg staging area), unload it and then return to carry another part of the unit in a second tour. This process repeats itself until the entire unit has been transported to its destination.

A fundamental question regarding this process is where to start a certain tour (loading the AFVs) and where to terminate it (unloading the AFVs). A common practice is to load the AFVs in each tour at the same loading point (home base) and to unload them at the same unloading point which, in some cases, is the staging area itself.

Two contradictory sets of considerations affect the choice of the unloading point. On the one hand the objective is to advance the transporters as close as possible to the staging area, thereby minimizing the travel time and the mechanical wear of the AFVs. On the other hand, if the unloading point is closer to the loading point, then each tour becomes shorter, therefore more tours may be performed in a given time period and the system as a whole may become more efficient and responsive to changes. Also, closer to the front line, traffic may be

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heavier, more chaotic and less secure. These effects may reduce the rate of advance of the transporters hence eliminating the speed advantage that is gained by using them.

In this paper we examine three generic transportation strategies: fixed unloading point, variable unloading point and a flexible strategy in which both loading and unloading points may vary from one tour to another. The efficiency of each transportation plan, within a given generic strategy, is evaluated with respect to the main criterion of minimum accumulation time. Other quantitative criteria such as fuel consumption and attrition may be considered too. Qualitative (or at least not readily measurable) criteria such as security and command and control are difficult to evaluate quantitatively and therefore are not considered in this analysis.

To the best of my knowledge, this particular type of mobility problem has not been addressed so far in the operations research literature. The reason for this void may be the special and unique nature of the problem that applies only to certain (land mobility) military situations. Such situations, however, may be typical in regional conflicts all over the world. This problem resembles, however, a well-known problem in civil transportation—the *transportation feeder* problem.

Transportation feeder problems apply to situations where more than one mode of transportation is utilized for mass transit or trans-shipment of cargo. A typical issue that is considered in this type of problem 1–5 is interstation spacing. When a transit system comprises more than one mode of transportation (eg subway and buses), synchronization may be achieved at the expense of more stations—the loading and unloading points. Thus, spacing the stations may determine the velocity of the entire transit system. This spacing problem is similar to the question of optimal loading and unloading points that is considered in this study. The velocity of the system is manifested by the accumulation time, and the objective is to maximize velocity, that is, minimize accumulation time.

In the next section we describe the situation and introduce some definitions and notation. This is followed by a section discussing three possible transportation strategies and presenting several optimization models. A fleshed out example of a practical force accumulation mission is presented and analysed in the penultimate section. The last section contains some concluding remarks.

## Definitions and notation

Let $N$ denote the number of AFVs to be transported. The transportation mission is to be accomplished by $M (M < N)$ transporters. The AFVs are loaded at some point and unloaded at another. A point $s$ on the road, where loading and unloading of AFVs are possible, is called a *potential transition point*, or in short, PTP. The distance between two successive PTPs, $s$ and $s+1$, is denoted by $d_{s,s+1}$. In particular, the origin and destination are PTPs ($s = 1$ and $s = S$, respectively).

Three types of travel are possible in this transportation situation: *tracked travel*—an AFV advances while moving on its tracks; *loaded travel*—a transporter moving loaded with an AFV; and *unloaded travel*—transporters returning unloaded to load another tour of AFVs. Arguably, the velocities of these three types of travel may depend on the number of vehicles $M$ that travel in the convoy; longer convoys tend to be slower than shorter convoys. Also, the average speed may vary from one segment of the road, $[s, s+1]$, to another since, for example movement on steep or narrow roads may be slower than on flat and wide ones. Likewise, the duration of the loading and unloading processes is dependent too on the size of the convoy $M$ and on the particular PTP, $s$.

To summarize the notation, let:

$N$ = number of AFVs to be transported;
$M$ = number of available transporters;
$n = \lceil N/M \rceil$ = number of tours;
$S$ = number of PTPs;
$d_{s,s+1}$ = distance from PTP $s$ to PTP $s+1$;
$VL_{s,s+1}(m)$ = average velocity of a loaded convoy of $m$ transporters on road segment $[s, s+1]$;
$VE_{s,s+1}(m)$ = average velocity of an empty convoy of $m$ transporters from PTP $s+1$ back to PTP $s$;
$VR_{s,s+1}(m)$ = average velocity of $m$ AFVs travelling on their tracks on (or beside) road segment $[s, s+1]$;
$U_s(m)$ = unloading time of $m$ transporters at PTP $s$;
$W_s(m)$ = loading time of $m$ transporters at PTP $s$;
$b_L/b_E/b_R$ = failure rate (mechanical breakdowns or accidents) per 1 km of loaded transporter travel, empty transporter travel and AFV tracked travel, respectively;
$\Delta$ = average total delay time of a convoy as a result of a failure in a single transporter;
$f$ = fuel consumption rate of an AFV.

## Real world situations—the discrete case

Although theoretically any point on the way between the origin and the destination may be a PTP, in real-world situations topography, landscape, urban development and enemy threat limit the number of available PTPs, and constrain their size and accessibility. Thus, only a finite number of PTPs, $S$, is to be considered. The loading and unloading times may vary from one PTP to another. These times also depend on the size of the convoy. As mentioned in the previous section, the velocity of a convoy depends, in addition to its status, loaded or unloaded, also on the number $m$ of vehicles, and on the particular road segment $[s, s+1]$.

While total accumulation time is arguably the most important criterion, it is possible that in some combat situations it may be important to accumulate as fast as possible just a (large) portion of the force in lieu of slower total accumulation time. Thus, partial accumulation time is also a criterion worth looking at. Additional criteria are mechanical wear and fuel consumption that are caused by the tracked travel of the AFVs. Since the objective is combat readiness, mechanical wear and empty fuel tanks, both as results of tracked travel, can offset possible operational benefits gained by fast accumulation time.

### *Velocity*

The average velocity of a convoy (loaded transporters, empty transporters or AFVs) is defined as the distance of a road segment divided by the travel time. The travel time is measured from the moment when the *first* vehicle in the convoy starts its movement at the starting point until the moment when the *last* vehicle arrives at the end point. Clearly, the average velocities depend on the number $m$ of vehicle in a convoy; the larger $m$, the slower the convoy. While these velocities are most likely nonlinear functions of $m$, we assume that, for the range of values that are applicable to typical tactical situations (convoys of 30 – 60 vehicles), a linear approximation will suffice to capture this dynamic feature. This assumption is without any loss of generality; any analytic function can replace it in the model with no significant effect on the analysis efforts. Thus the velocities are

$$\begin{aligned} VL(m) &= V_L^0 - v_L m \quad m \geqslant 1 \\ VE(m) &= V_E^0 - v_E m \quad m \geqslant 1 \\ VR(m) &= V_R^0 - v_R m \quad m \geqslant 1 \end{aligned} \tag{1}$$

for loaded travel, unloaded travel and tracked travel, respectively. $V_L^0$, $V_E^0$ and $V_R^0$ are the nominal velocities, while $v_L$, $v_E$ and $v_R$ are the velocity degradation factors due to the size of the convoy. For notational convenience we dropped here the indices $s, s+1$ that indicate the specific road segment.

### *Loading and unloading times*

Similarly to the travel velocities, we also assume that the loading and unloading times are approximately linear. Thus, for any PTP (the index $s$ is dropped for brevity)

$$\begin{aligned} U(m) &= U_0 + um \quad m \geqslant 1 \\ W(m) &= W_0 + wm \quad m \geqslant 1 \end{aligned} \tag{2}$$

$U_0$ and $W_0$ are the nominal unloading and loading times, respectively, and $u$ and $w$ are the corresponding scale parameters. To simplify the exposition of the models to follow, and without any loss of generality, we assume that $N = nM$. Thus, each one of the $n$ tours comprises the same number of vehicles ($M$) and therefore we may drop the index $m$ from all time expressions. Also, we define the travel times, between PTP $i$ and PTP $j$, for each one of the three types of travel as follows:

$$TL_{ij} = \sum_{l=i}^{j-1} d_{l,l+1}\left(\frac{1}{VL_{i,l+1}} + b_L\Delta\right) \tag{3}$$

$$TE_{ij} = \sum_{l=i}^{j-1} d_{l,l+1}\left(\frac{1}{VE_{l,l+1}} + b_E\Delta\right) \tag{4}$$

$$TR_{ij} = \sum_{l=i}^{j-1} d_{l,l+1}\left(\frac{1}{VE_{l,l+1}} + b_R\Delta\right) \tag{5}$$

The terms $b_L\Delta$, $b_E\Delta$ and $b_R\Delta$ represent delays that are due to mechanical failures or accidents en route.

### *Fixed PTP*

In the fixed PTP case we assume that unloading is possible only at one PTP. This assumption is reasonable in many realistic circumstances when operational or budget constraints permit setting up unloading facilities only at one PTP.

Obtaining the optimal PTP in the fixed PTP case is simple. The optimal unloading point $s$ is determined such that

$$T_n^1(s) = n[W_1 + U_s + TL_{1,s}] + (n-1)TE_{1,s} + TR_{s,S} \tag{6}$$

is minimized. That is, $S$ computations of eqn (6).

The total mechanical wear of the unit (defined as the number of stalled AFVs) is

$$O_n^1 = b_R d_{s,S} N \tag{7}$$

and the amount of fuel that is left in the AFV's fuel container upon arrival to the destination point is

$$F_n^1 = \max\left\{0, \frac{(F_0 - fd_{s,S})}{F_0} 100\right\} \tag{8}$$

### *Variable PTP*

The possibility to unload the AFVs in more than one PTP has several operational advantages compared with the fixed PTP case. First it is more flexible; unexpected changes in plans and circumstances may be dealt with more effectively when there are several potential unloading sites. Second, a situation where multiple PTPs are available is more robust than the single fixed PTP since one transfer point may replace another damaged or unsafe one. Third, it may shorten the total accumulation time, as it is analysed in the following.

Suppose that the $i$th tour is unloaded at PTP $s_i, i = 1, \ldots, n-1$. If the lift operation started at time $t = 0$, then the accumulation time of the $i$th tour is

$$\Theta_i = iW_1 + \sum_{l=1}^{i-1} (TL_{1,s_l} + TE_{1,s_l} + U_{s_l}) + TL_{1,s_i} + U_{s_i} + TR_{s_i S} \quad i = 1, \ldots, n \quad (9)$$

Notice that the total accumulation time of the first $i$ tours is not necessarily $\Theta_i$. It is possible that an earlier tour will arrive at the destination later than a later tour. Thus, the total accumulation time of $i$ tours is

$$T_i^2(s_1, \ldots s_i) = \text{Max}_{l=1,\ldots,i}\, \Theta_l\, , i = 1, \ldots, n \quad (10)$$

If we assume that for any segment $[s, s+1]$ $TR_{s,s+1} \geqslant TL_{s,s+1}$ (*tracked travel* is always slower than *loaded travel*), then it is easily seen that at optimality

$$s_1 \leqslant s_2, \leqslant \ldots \leqslant s_n \quad (11)$$

must hold. Thus, in order to find an optimal set of PTPs we need to examine only assignments that satisfy eqn (11).

The optimal PTPs $(s_1, s_2, \ldots, s_n), s_i \in \{1, \ldots, S\}$ are obtained as an optimal solution of the following $0-1$ mixed integer programming problem which arises from the MinMax principle that determines the optimal strategy. Let

$$x_{is} = \begin{cases} 1 & \text{if the } i\text{th tour is unloaded at PTP s} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

For notational convenience define

$$\begin{aligned} A_s &= TL_{1s} + TE_{1s} + U_s \\ B_s &= TL_{1s} + U_s + TR_{sS} \end{aligned} \quad (13)$$

Then, we wish to solve

$$\begin{aligned} &\text{Min } Z \\ &s.t \\ &\sum_{l=1}^{i-1}\sum_{s=1}^{S} A_s x_{ls} + \sum_{i=1}^{S} B_s x_{is} - Z \leqslant -iW_1, i = 1, \ldots, n \\ &\sum_{s=1}^{S} x_{is} = 1, \qquad i = 1, \ldots, n \\ &x_{is} = 0 \text{ or } 1, \qquad Z \geqslant 0 \end{aligned} \quad (14)$$

The property in eqn (11) may be utilized in the branch-and-bound procedure for solving eqn (14) efficiently. In particular, at optimality, if $x_{is} = 1$ then $x_{tu} = 0$ for all $t \geqslant i$ and $u \leqslant s$.

The total mechanical wear is

$$O_n^2 = b_R M \sum_{i=1}^{n} d_{s_i,S}$$

and the average percentage of fuel that is left in the AFV's fuel container upon arrival to the destination point is

$$F_n^2 = \text{Max}\left\{0, \frac{F_0 - \frac{f}{n}\sum_{i=1}^{n} d_{s_i,S}}{F_0} 100\right\}$$

*Flexible strategy*

Maximum operational efficiency and flexibility is obtained when tracked travel may be concurrent to loaded and unloaded travel at any time in the lift operation, and when each PTP may also be a possible loading point. In this transportation strategy the entire unit commences its movement towards the destination point at time 0. The first tour is transported to a certain unloading PTP, $s_1$, from which it travels independently (on its own tracks) to the destination point. Upon unloading the AFVs, the empty transporters return back and meet, at a certain PTP, the AFVs of the designated second tour. The transporters load the second tour of AFVs, carry them to their unloading PTP, $s_2$, return for the third tour, and so on. Once the AFVs, travelling on their tracks, arrive at a PTP, the decision whether to continue on their tracks to the next PTP or to stay and wait for the transporters to carry them forward depends on the relative waiting times. For example, if at PTP $s$ the AFVs will have to wait 1 h for the transporters which return from an earlier tour, while at PTP $s+1$ the transporters will have to wait 2 h for the advancing AFVs, then it will be more efficient to halt the AFVs at $s$ than to advance them, on their tracks, to $s+1$.

This strategy is the most complex and difficult to implement. Efficient $C^3$ systems, meticulous operational procedures and strict discipline are necessary conditions for a successful implementation of this flexible strategy. Notwithstanding these constraints, this strategy is the most efficient one with respect to the accumulation time criterion.

Suppose that the loading point of the $i$th tour is at PTP $l(i)$ and its unloading point is $u(i)$. The time $Q_i$ at which the transporters are available at $l(i)$ to start loading the $i$th tour is obtained recursively:

$$Q_i = R_{i-1} + TL_{l(i-1),u(i-1)} + TE_{l(i)u(i-1)} + W_{l(i-1)} + U_{u(i-1)} \quad (15)$$

where $R_{i-1}$ is the loading start time of tour $i-1$. It is easily seen that $R_{i-1} \geqslant Q_{i-1}$ must hold. Moreover, the loading start time is the maximum between $Q_i$ and the tracked travel time from the origin to $l(i)$. That is,

$$R_i = \text{Max}\{TR_{1,l(i)}, Q_i\} \quad (16)$$

Clearly, $R_1 = 0$.

Similarly, and in addition, to the definition of $x_{is}$ in eqn (12), we define

$$y_{is} = \begin{cases} 1 & \text{if round } i \text{ is loaded at PTP } s \\ 0 & \text{otherwise} \end{cases}$$

The 0–1 mixed integer-programming problem that determines the shortest total accumulation time is:

$$\text{Min } Z \tag{17}$$

s.t.

$$R_i - \sum_{s=1}^{S} TR_{1s} y_{is} \geqslant 0 \qquad i = 1, \ldots, n \tag{18}$$

$$R_i - R_{i-1} - \sum_{s=1}^{S} (TL_{1s} + U_s + TE_{1s}) x_{i-1,s} - \sum_{s=1}^{S} (W_s - TL_{1s}) y_{i-1,s} + \sum_{i=1}^{S} TE_{1s} y_{is} \geqslant 0, \quad i = 1, \ldots, n \tag{19}$$

$$R_i + \sum_{s=1}^{S} (W_s - TL_{1s}) y_{is} + \sum_{s=1}^{S} (TL_{1s} + U_s + TR_{sS}) x_{is} - Z \leqslant 0, \quad i = 1, \ldots, n \tag{20}$$

$$\sum_{s=1}^{S} x_{is} = 1, i = 1, \ldots, n \tag{21}$$

$$\sum_{s=1}^{S} y_{is} = 1, i = 1, \ldots, n \tag{22}$$

$$x_{is}, y_{is} \in \{0, 1\}; \quad R_i, Z \geqslant 0 \tag{23}$$

*A few comments on eqns (17)–(23)*

1. In the formulation of the problem we implicitly assumed that the time parameters are such that each AFV is transported, at one point or another, for a certain distance. In particular, each tour incurs loading time and unloading time. The formulation of the problem can be modified to account for situations were some AFVs may have to travel by themselves (on their tracks) all the way from the origin to the destination point. Specifically, such situations may be detected in eqns (18)–(23) if for some $i$, $y_{iS} = 1$, which means that the optimal loading point is at the destination.
2. Constraints (18) and (19) manifest the condition at eqn (16). Constraint (18) simply states that the start time, at the designated PTP, of the loaded travel of tour $i$, cannot be earlier than the arrival time at this PTP of the AFVs (travelling on their tracks). Constraint (19) states that this time cannot also precede the arrival time of the empty transporters at this PTP.
3. It is clear that for any tour $i$, if $x_{ik} = y_{is} = 1$ then $k > s$ must hold; one cannot unload something before loading it. The structure of the problem guarantees that this property holds, thus there is no need for an explicit constraint for that condition.
4. Similarly to property (11), the $x_{is}$ and $y_{is}$ variables in eqns (17)–(23) must satisfy:

   (1) if $x_{ir} = x_{i+1,q} = 1$ then $q \geqslant r$
   (2) if $y_{ir} = y_{i+1,q} = 1$ then $q \geqslant r$

   This property may be utilized in the solution of our mixed integer problem.

The total mechanical wear after $n$ tours is:

$$O_n^3 = b_R M \sum_{i=1}^{n} [d_{1,l(i)} + d_{u(i),S}]$$

**Table 1** Distance and velocity data for the example

| *Road segment* | *(1,2)* | *(2,3)* | *(3,4)* | *(4,5)* |
|---|---|---|---|---|
| Distance ($d_{s,s+1}$) | 50 km | 75 km | 40 km | 20 km |
| Nominal *loaded travel* velocity, $V_L^0$ | 40 km/h | 40 km/h | 30 km/h | 30 km/h |
| Nominal *unloaded travel* velocity, $V_E^0$ | 60 km/h | 50 km/h | 40 km/h | 40 km/h |
| Nominal *tracked travel* velocity, $V_R^0$ | 10 km/h | 10 km/h | 10 km/h | 10 km/h |
| Rate of change of $V_L^0$, $v_L$ | 0.1 | 0.1 | 0.15 | 0.2 |
| Rate of change of $V_E^0$, $v_E$ | 0.05 | 0.1 | 0.1 | 0.15 |
| Rate of change of $V_R^0$, $v_R$ | 0 | 0 | 0 | 0 |

**Table 2** Loading and unloading time parameters

| *PTP* | *1* | *2* | *3* | *4* | *5* |
|---|---|---|---|---|---|
| Nominal unloading time, $U_0$ | — | 5 min | 6 min | 7 min | 8 min |
| Nominal loading time, $W_0$ | 5 min | 5 min | 6 min | 7 min | — |
| Rate of change of $U_0$, $u$ | — | 0.5 min | 0.6 min | 0.7 min | 0.8 min |
| Rate of change of $L_0$, $w$ | 0.4 min | 0.5 min | 0.6 min | 0.7 min | — |

**Table 3** Attrition and consumption parameters

| *Parameter* | *Symbol* | *Value* |
|---|---|---|
| Mechanical failure rate of tracked travel | $B_R$ | 0.0004 |
| Failure rate of a transporter in loaded travel | $b_L$ | 0.0005 |
| Failure rate of a transporter in unloaded travel | $b_E$ | 0.0004 |
| Average total delay time due to stalled transporter | $\Delta$ | 10 min |
| Fuel tank capacity of an AFV | $F_0$ | 1000 l |
| Fuel consumption rate of an AFV | $f$ | 4 l/km |

**Table 5** Accumulation time, fuel consumption and mechanical wear—fixed PTP, $n = 2$

| *PTP* | *First tour (half of the force)* | *Second tour (the entire force)* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|
| 1 | — | 18.5 | 26 | 9.6 |
| 2 | 16.4 | 20.4 | 46 | 7.0 |
| 3 | 11.5 | 20.5 | 76 | 3.1 |
| 4 | 10.0 | 22.5 | 92 | 1.0 |
| 5 | 9.4 | 24.1 | 100 | 0 |

and the average percentage of fuel that is left in the AFVs fuel container upon arrival at the destination point is

$$F_n^2 = \mathrm{Max}\left\{0, \frac{F_0 - \frac{f}{n}\sum_{i=1}^{n}[d_{1,l(i)} + d_{u(i)S}]}{F_0} 100\right\}$$

## An example

Suppose that $N = 130$ AFVs (approximately a brigade) are to be transported for a distance of 185 km. There are $S = 5$ PTPs (including the origin and the destination point). The distances and the velocity parameters for the four road segments are shown in Table 1.

The loading and unloading parameters are shown in Table 2. The values of the other parameters are summarised in Table 3.

The number $M$ of transporters is derived from the number $n$ of planned tours. Once $n$ is set, the required number of transporters is the smallest integer larger than or equal to $N/n$. A larger number of transporters may accumulate a larger portion of the force faster, but at the expense of redundancy and possible slower travel, loading and unloading times. A smaller number will obviously be insufficient.

Thus for the cases $n = 2$ and 3 tours we need to consider, for the $N = 130$ AFVs, only $M = 65$ and 44, respectively.

### *Fixed PTP*

For the fixed PTP case the minimum total accumulation time is obtained when $s = 1$ (18.5 h). That is, all the AFVs travel the entire distance on their tracks and thus no transporters are needed. Clearly, this strategy is the least desirable with respect to attrition and fuel consumption. In the case of $n = 3$, if the objective is to accumulate as fast as possible two-thirds of the force, then the optimal PTP is $s = 3$ with accumulation time of 18.6 h. Tables 4 and 5 summarize these results for $n = 3$ and $n = 2$, respectively.

### *Variable PTP*

If each one of the five PTPs can be utilized as an unloading point, then several travel plans (in principle, there are $n^s$ such plans) may apply. However any efficient plan must be such that eqn (11) is satisfied. In particular, the last tour must unload at PTP 5. Tables 6 and 7 present possible 'nondominated' travel plans for $n = 3$ and $n = 2$, respectively.

The fastest accumulation time for $n = 3$ is obtained for the travel plan (1,3,5). This plan is obtained also as a solution of eqn (14). According to this plan, one-third of the force should travel the whole distance on its tracks, one tour is unloaded at $s = 3$, and the last tour is carried all the way on transporters to the destination point at $s = 5$. Notice that in this travel plan the transporters are needed in fact for only two tours. The average percentage of fuel that is left in the AFVs is 67% and the attrition is 4.3 vehicles. However, the AFVs that need refuelling the most, arrive last, thus withholding the entire force until they complete their refuelling (a time factor that is not considered here explicitly). The travel plan (2,3,5) is somewhat slower than the fastest one (15.9 h for 67% accumulation and 19 h for 100% accumulation, as compared to 15.7 and 18.5, respectively), however the average fuel consumption and the attrition are

**Table 4** Accumulation time, fuel consumption and mechanical wear—fixed PTP, $n = 3$

| *PTP* | *First tour (one-third of the force)* | *Second tour (two-thirds of the force)* | *Third tour (the entire force)* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|---|
| 1 | — | — | 18.5 | 26 | 9.6 |
| 2 | 15.9 | 19.3 | 22.7 | 46 | 7.0 |
| 3 | 10.9 | 18.6 | 26.3 | 76 | 3.1 |
| 4 | 8.8 | 19.8 | 30.6 | 92 | 1.0 |
| 5 | 7.9 | 20.7 | 33.2 | 100 | 0 |

**Table 6** Accumulation time, fuel consumption and mechanical wear for various travel plans—variable PTP, $n = 3$

| *Unloading PTP of first tour* | *Unloading PTP of second tour* | *Unloading PTP of third tour* | *67% accumulation* | *100% accumulation* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|---|---|
| 1 | 2 | 5 | 15.9 | 18.5 | 57 | 5.6 |
| 1 | 3 | 5 | 15.7 | 18.5 | 67 | 4.3 |
| 2 | 2 | 5 | 15.9 | 19.3 | 63 | 4.8 |
| 2 | 3 | 5 | 15.9 | 19.0 | 74 | 3.4 |
| 2 | 4 | 5 | 15.9 | 22.2 | 79 | 2.7 |
| 3 | 3 | 5 | 18.6 | 23.3 | 84 | 2.1 |
| 3 | 4 | 5 | 16.6 | 26.5 | 89 | 1.4 |
| 4 | 4 | 5 | 19.8 | 29.7 | 95 | 0.7 |

**Table 7** Accumulation time, fuel consumption and mechanical wear for various travel plans—variable PTP, $n = 2$

| *Unloading PTP of first tour* | *Unloading PTP of second tour* | *50% accumulation* | *100% accumulation* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|---|
| 1 | 5 | 9.4 | 18.5 | 63 | 4.8 |
| 2 | 5 | 13.4 | 16.5 | 73 | 3.5 |
| 3 | 5 | 11.7 | 18.2 | 88 | 1.6 |
| 4 | 5 | 10.0 | 21.9 | 96 | 0.5 |
| 5 | 5 | 9.4 | 24.1 | 100 | 0 |

reduced by more than 20%. Moreover, the AFVs that need refuelling the most arrive first and therefore their refuelling can be concurrent with the accumulation process of the later tours, thus saving additional refuelling time. For $n = 2$, it seems that either (2,5) or (3,5) are the most efficient travel plans in terms of total accumulation time, 50% accumulation time, fuel consumption and attrition.

### *Flexible strategy*

Solving problems (17)–(23) results in the following optimal travel plans for $n = 3$ and $n = 2$:

*$n = 3$ tours*

| *Tour no.* | *Loading PTP* | *Unloading PTP* | *Accumulation time* |
|---|---|---|---|
| 1 | 1 | 2 | 15.9 |
| 2 | 2 | 3 | 14.4 |
| 3 | 3 | 5 | 16.4 |

The projected attrition is 6.4 vehicles and the average remaining fuel percentage is 51%.

**Table 8** Accumulation time, fuel consumption and mechanical wear for the three strategies, $n = 3$

| *Strategy* | *33% accumulation* | *67% accumulation* | *100% accumulation* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|---|
| Fixed PTP: *no tracked travel* | 7.9 | 20.7 | 33.2 | 100 | 0 |
| Fixed PTP: *tracked travel* | — | — | 18.5 | 26 | 9.6 |
| Variable PTP: *travel plan (1,3,5)* | 10.9 | 15.7 | 18.5 | 67 | 4.3 |
| Flexible strategy: *travel plan (1–2,2–3,3–5)* | 14.4 | 15.9 | 16.4 | 51 | 6.4 |

**Table 9** Accumulation time, fuel consumption and mechanical wear for the three strategies, $n = 2$

| *Strategy* | *50% accumulation* | *100% accumulation* | *Fuel (% left in tank)* | *Wear* |
|---|---|---|---|---|
| Fixed PTP: *no tracked travel* | 9.4 | 24.1 | 100 | 0 |
| Fixed PTP: *tracked travel* | — | 18.5 | 26 | 9.6 |
| Variable PTP: *travel plan (2,5)* | 13.4 | 16.4 | 73 | 3.5 |
| Flexible strategy: *travel plan (1–3,2–5)* | 11.7 | 15.5 | 78 | 2.9 |

*n = 2 tours*

| *Tour no.* | *Loading PTP* | *Unloading PTP* | *Accumulation time* |
|---|---|---|---|
| 1 | 1 | 3 | 11.7 |
| 2 | 2 | 5 | 15.5 |

The projected attrition is 2.9 vehicles and the average remaining fuel percentage is 78%.

### *Comparison*

Table 8 compares the three transportation strategies and the case were no tracked travel is permitted (that is, fixed PTP strategy where the unloading point is at the destination). These strategies are compared with respect to (partial and total) accumulation times, mechanical wear (attrition) and fuel consumption, for the case where $n = 3$. Table 9 presents the same results for the case $n = 2$. Each strategy is represented by the travel plan that minimizes the total accumulation time.

### *Discussion*

The advantage in utilizing transporters to move a military unit from one point to another depends on the relative speeds of the tracked travel, loaded travel, unloaded travel, loading process and unloading process. When tracked travel is relatively slow, or when the travel distance is too long for the AFVs to travel on their tracks, then using transporters becomes advantageous and even necessary. In such cases efficient transportation strategies can greatly reduce accumulation times, while maintaining acceptable fuel consumption and attrition levels. In our example, utilizing transporters in a flexible manner reduced the total accumulation time by 12% when $n = 3$, and by 16% when $n = 2$. The effect of the flexible PTP strategy on the accumulation time could be even higher if we dropped the relaxing assumption in Table 1 that the nominal tracked travel is not affected by road conditions and number of vehicles.

The computational effort that is needed to solve this problem is negligible. For the fixed PTP strategy the solution is trivial. For the variable PTP and flexible PTP strategies MIP models need to be solved. However, for actual lift operations these problems are very small and therefore may be easily solved by standard MIP packages. For example, if the number of tours is $n = 3$ and the number of PTPs is $S = 5$, then the number of variables for the variable PTP case is 16 and the number of constraints is 6. In the flexible strategy case there are 34 variables and 15 constraints.

## Summary and conclusions

The situation that has motivated the analysis in this paper is typical to military operations that span over relatively large areas. During such operations, and in particular at the earlier stages where combat units are accumulated at the theatre of operations, it may be necessary to transport military units fast and efficiently. Such a transportation mission is carried out using transporters that carry the AFVs of the combat unit.

It was shown that this transportation situation, for example, where a transported entity can also travel independently on its own, at the cost of time and wear, may be modelled as an elementary scheduling problem for which simple solutions exist. The models that were described in this paper may be utilized to determine efficient schedules that optimize logistics resources, and to select loading and unloading sites that are best suited to achieve this efficiency.

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