1996

Lot Splitting in Stochastic Flow Shop and Job Shop Environments

Smunt, Timothy L.

Decision Sciences, Volume 27, Number 2, Spring 1996, pp. 215-238
http://hdl.handle.net/10945/40139

Decision Sciences
Volume 27 Number 2
Spring 1996
Printed in the U.S.A.

Lot Splitting in Stochastic Flow Shop and Job Shop Environments*

Timothy L. Smunt
Babcock Graduate School of Management, Wake Forest University, Box 7659 Reynolda Station, Winston-Salem, NC 27109

Arnold H. Buss
Operations Research Department, Naval Postgraduate School, Monterey, CA 93943-5000

Dean H. Kropp
John M. Olin School of Business, Washington University, One Brookings Drive, St. Louis, MO 63130

ABSTRACT

In recent years many firms have been implementing small lot size production. Lot splitting breaks large orders into smaller transfer lots and offers the ability to move parts more quickly through the production process. This paper extends the deterministic studies by investigating various lot splitting policies in both stochastic job shop and stochastic flow shop settings using performance measures of mean flow time and the standard deviation of flow time. Using a computer simulation experiment, we found that in stochastic dynamic job shops, the number of lot splits is more important than the exact form of splitting. However, when optimal job sizes are determined for each scenario, we found a few circumstances where the implementation of a small initial split, called a "flag," can provide measurable improvement in flow time performance. Interestingly, the vast majority of previous research indicates that methods other than equal lot splitting typically improves makespan performance. The earlier research, however, has been set in the static, deterministic flow shop environment. Thus, our results are of practical interest since they show that the specific method of lot splitting is important in only a small set of realistic environments while the choice of an appropriate number of splits is typically more important.

Subject Areas: Job Shop Scheduling, Lot Splitting, Machine Scheduling and Sequencing, Process Design, and Simulation.

INTRODUCTION

Lately, a number of papers have been published on the topic of lot splitting, sometimes called "lot streaming." In the spirit of the just-in-time production approach,

*The authors wish to thank Kenneth Baker, F. Robert Jacobs, Alex Zhang, and Paul Zipkin for their reviews of an earlier draft of this paper and for their insightful comments. This research was supported by the Babcock Graduate School of Management, Wake Forest University, Research Fellowship Program and by a Research Initiation Grant from the Naval Postgraduate School. This support is gratefully acknowledged by the authors.

lot splitting breaks large orders into smaller transfer lots and offers the ability to move parts more quickly through the production process. While the vast majority of this previous research indicates that methods other than equal lot splitting typically improve makespan performance, this research has been set in the static, deterministic flow shop environment and has not addressed the efficacy of lot splitting in the more typical environment characterized by jumbled flows and high levels of variance. In this paper, we extend the deterministic studies by studying various lot splitting policies in both stochastic job shop and stochastic flow shop settings using performance measures of mean flow time and the standard deviation of flow time.

We recently encountered the lot splitting approach at two medium-size manufacturing facilities—one is a fabricator of integrated circuit materials and the other assembles printed circuit boards. In both cases, the manufacturers were also evaluating changes to a number of process factors. While previous research in lot splitting provided some insight into the problem for the deterministic, static problem for flow shops, we found that it did not adequately address how an order should be split in a stochastic process environment, especially one with jumbled routings and with multiple job types as seen in job shops. As Dudek, Panwalkar, and Smith [4] point out, real flow shop situations usually are dynamic rather than static, and there is questionable value to flow shop research unless it can be related to practical settings.

This study extends the previous literature on lot splitting to settings that are likely to be found in industry. In contrast to most prior studies, our shop environment is stochastic and dynamic. In contrast to the results of the deterministic studies in this area, our results indicate that in such stochastic environments, the particular form of lot splitting makes little difference, although the choice of the number of splits is important.

There are many ways to split an order: the splits may be equal or unequal, with the number of splits ranging from one to the number of units in the order. We thus consider various forms of lot splitting in stochastic environments for the two extreme flow dominance conditions, flow shops and job shops, and for various levels of setup times, operation time variance, job size and shop load. Unlike previous research in this area, we vary the job size so that we can examine the best combination of both job and transfer batch size. We investigate the situation of a closed job shop where we have a limited number of product types that can be produced in lot sizes determined by management. The closed job shop represents the manufacturing environment known in industrial practice as repetitive manufacturing. In typical repetitive manufacturing environments, a fixed product mix will be produced on a dynamic basis, depending upon actual demands and forecasts, during the short- to medium-term planning horizon. See Wagner and Ragatz [18] for a more thorough discussion of open versus closed job shops.

We accomplish our research objective in two steps. First, we examine lot splitting forms that have been shown to be optimal for deterministic flow shops (Kropp and Smunt [13]). We do this to determine if these lot splitting forms can also improve performance in stochastic scenarios and in environments with jumbled flow dominance and to estimate differences, if any, between the various forms. Second, we test the effect of increasing the number of splits in order to better understand the tradeoff between reduced flow times offered by further splitting and the increased complexity of job tracking caused by this larger number of splits on the shop floor. Due to the complex nature of this research problem, we use computer simulation methodology to examine the above issues.

Our results provide managers with a better understanding as to which forms of lot splitting are best across a wide range of realistic scenarios. We found that differences between forms diminish with increased variability and capacity utilization in flow shops and in all job shop scenarios. Nevertheless, we identify one form, equal splits with "flag" (RL4F) that is more often the best form when such differences do exist. Our experiments further validate the benefit of lot splitting across a broad range of operating environments regardless of the specific form used. All lot splitting forms we considered reduced both mean flow time (MFT) and standard deviation of flow time (SDFT).

The remainder of this paper is organized as follows. In the following section we review the relevant literature. Next, we describe the research model, the factors we varied, and the parameters considered. Then we describe both of the experiments to determine the effect of lot splitting forms and present results and an experiment that tests the effect of the number of splits. Our conclusions and applications of the results follow. Finally, we summarize the paper and suggest further research.

LITERATURE REVIEW

We know of no research that compares different forms of lot splitting in stochastic environments. Karmarkar, Kekre, and Kekre [11], and Karmarkar, Kekre, Kekre, and Freeman [12] use both a simulation model of a job shop and Q-LOTS, an analytical procedure based on queueing theory, to examine the impact of lot sizes on flow times. Other authors consider the relationship between lot sizing and job flow times (Szendrovits [15], Santos and Magazine [14], and Dobson, Karmarkar, and Rummel [3]). However, none of these papers compare different lot splitting forms in stochastic environments, as we have done in this paper.

Those papers that have considered different lot splitting forms have done so under deterministic conditions. Graves and Kostreva [7] derived an expression for the optimal number of sublots under the conditions of constant demand, identical machine production rates, and equal sublot sizes. Baker and Pyke [2], Trietsch [16], and Trietsch and Baker [17] develop algorithms for minimizing makespan of a single job in a flow shop, with unequal sublot sizes permitted. Baker [1] proposed a geometric lot splitting form, which performs well in deterministic flow shops. Finally, Kropp and Smunt [13] developed both optimal and heuristic procedures for minimizing either makespan or mean flow time for a single job in a flow shop. They suggested using equal size sublots when machine setup times were small and a "flag" heuristic to deal with situations in which setup times were large. With the flag heuristic, the first sublot has the smallest feasible nonzero size and all other sublots are equal in size. In their deterministic tests they found that these heuristic approaches had excellent performance when compared to the optimal procedures.

Three more closely related papers have focused explicitly on lot splitting in stochastic environments. Jacobs and Bragg [10] use a simulation model to examine the number of lot splits and resulting flow times in a stochastic job shop. They were among the first to use the concept of repetitive lots, in which jobs can be split into several transfer batches or sublots. Using the repetitive lots approach, when a work center finishes processing on a sublot, priority is given to another sublot of the same product. In this way, the number of setups is decreased, thus increasing the effective capacity of the system and reducing flow times. Jacobs and Bragg demonstrated that repetitive lots can indeed substantially reduce mean flow times, but they did not consider methods other than equal splits. In another paper that studied lot splitting in a stochastic job shop, Hancock [9] examined a simple lot splitting heuristic and found it to improve job timeliness under the three different routing strategies he tested. His paper only considered one lot splitting form and focused primarily on the impact of routing strategies. The most recent work in the stochastic environment area, by Wagner and Ragatz [18], focused on the open job shop problem, that is, where every job is assumed to be a custom order. In this problem, the use of the repetitive lots dispatching rule does not offer any benefits in setup time reduction since every job is unique. The benefits of lot splitting result from the ability to overlap operations. They show that in an open job shop setting due date oriented dispatching rules, when used in conjunction with lot splitting, can improve a due date performance measure like tardiness.

THE MODEL

As mentioned in the Introduction, we took MFT as our primary measure of effectiveness and SDFT as a secondary measure, for evaluating different lot splitting heuristics. More precisely, we measure the long-run mean flow time for the shop in steady-state. That is, we assume that the shop has been in operation for a long time, so that steady-state is reached. We measure flow time for a particular job as the time between its release to the shop and its completion. Note that with lot splitting in effect, a job is not considered complete until all the sublots have been finished. Formally, we can express MFT as the expectation of the (stochastic) steady-state flowtime T for an arbitrary job arriving to the system in steady-state, and T is given by:

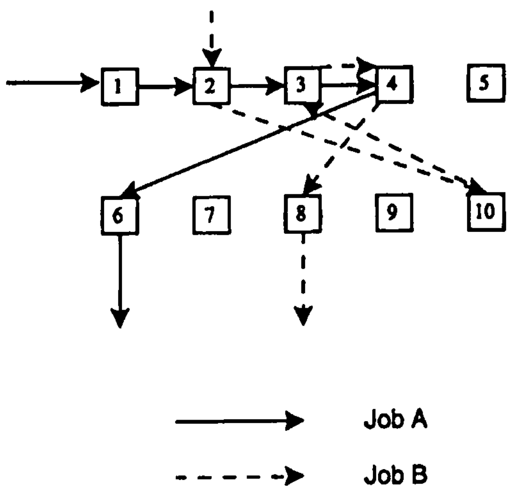
$$T = f(S, M, U, SU, CV, JS, INTER) + \varepsilon, \quad (1)$$

where S is the type of shop, M is the lot splitting method used, U is the process utilization, SU is the ratio of setup time to processing time, CV is the coefficient of variation for processing time, JS is the size of the job, $INTER$ is the job interarrival pattern, and ε is a zero-mean random variable. The objective is to minimize MFT=$E[T]$ with respect to M. Our secondary performance measure SDFT may be similarly expressed as SDFT=Var[T].

It is not possible to evaluate MFT as the expected value to T in (1) in closed form. Consequently, it must be estimated using simulation. A simulation model was implemented in SIMSCRIPT II.5. We used this simulation software primarily due to its flexibility and its ability to implement complex scheduling heuristics. Examples of material flows for our two shop structures are shown in Figures 1 and 2. In our model, entering jobs are split into smaller transfer batches, which are then independently processed through their assigned task routing. Using the repetitive lots sequencing rule (RL), a transfer batch of the same job type as the current setup at a machine is always to be processed next. If no transfer batch with the current machine setup is in the machine queue, then the first-come, first-served rule (FCFS) is used for

Figure 1: The five workcenter flowshop: Ten distinct job types, all with identical routing.

Figure 2: The ten workcenter jobshop: Ten distinct job types, all with identical routing (two are shown as examples).



sequencing. The use of FCFS as the secondary dispatch rule is consistent with Jacobs and Bragg's experiment. Other secondary dispatching rules, such as shortest operating time (SOT), can be incorporated into the RL approach to attempt to further reduce MFT or improve customer service level measures such as tardiness and lateness. We discuss the impact of SOT as a secondary dispatch rule in a later section.

Experiment Approach

We conducted two sets of experiments on the model. The first was to determine the impact of different forms of lot splitting on MFT performance and may be seen as a continuation of the work of Kropp and Smunt [13]. We tested forms that were either optimal or performed well in deterministic environments. A comparison of our results with theirs will help determine the effect of randomness on the performance of their forms and measure the robustness of their results. The second set of experiments focused on the number of equal splits. The results of these experiments will be presented in the following two sections. The factors and their levels used in this experiment are summarized in Table 1. The experimental design was full factorial with 6912(6×2×2×6×4×3×4) combinations of factor settings.

Table 1: Factors and levels for first experiment.

Factor	Levels
Lot splitting form	RL0, RL3E, RL4F, RLU1, RLU2, RLU3
Flow dominance	Job Shop, Flow Shop
Job arrival pattern	Deterministic, Stochastic
Mean job size	75, 105, 135, 165, 195, 225
Operation *CV*	0.01, 0.50, 1.00, 1.50
Processing utilization	57%, 72%, 87%
Setup ratio	0.1, 0.5, 1.0, 1.5

Lot Splitting Forms

We classified the lot splitting forms into three categories, (1) equal splits (RL3E), (2) equal splits preceded by a flag split (RL4F), and (3) unequal splits (RLU1, RLU2, RLU3) (see Table 2). In Kropp and Smunt [13], it was shown that for deterministic flow shops, a flag heuristic (one that initially sends a batch of one unit through the system) tends to work well if setup times are high. This result is due to the fact that the contribution of the setup to flowtime is mitigated by the overlap with processing of the following batches. Thus, the overlapping processing was extended to setups, and the subsequent batches spent less time in queue waiting for a setup. In Kropp and Smunt's study [13], the optimal splits following the flag often turned out to be nearly equal. As setup times approach zero, however, the optimal lot splitting strategy required that all lot splits be of equal size, without a preceding flag split. We chose RLU1, RLU2, and RLU3 to test for robustness of lot splitting distributions, since these unequal forms are similar to those found optimal in a number of cases tested by Kropp and Smunt [13].

Flow Dominance

Our next experimental factor was flow dominance, with levels at the two extremes of flow shop and job shop. We modeled our job shop to have the same structure as Jacobs and Bragg [10], that is, with 10 departments, each with a single machine, and 10 job types (see Figure 2). In general, we used parameter settings similar to those used in [10]. Each job type had an equally likely chance of arriving into the system and required five departments to complete its processing. Each department was utilized equally (no long-term bottlenecks at any machine) and was the first or last operation by any job an equal number of times. The flow shop scenario had five single-machine departments and 10 job types (see Figure 1). Each job type had the same sequence over the five departments, and was distinguished from the other job types by virtue of the required setup to change a machine from one job type to another. A five department flow shop was used in order to have the same number of tasks in the routing sequence for each job in both settings. The interarrival rates were adjusted to give identical utilizations with the job shop scenarios.

Table 2: Definition of lot splitting forms.

Lot splitting form	Definition
RL0	Repetitive Lots, No Splitting
RL3E	Repetitive Lots, 3 Equal Splits
RL4F	Repetitive Lots, 3 Equal Splits plus Flag
RLU1	Repetitive Lots, 3 splits of 20%, 40%, 40%
RLU2	Repetitive Lots, 3 splits of 25%, 35%, 40%
RLU3	Repetitive Lots, 3 splits of 30%, 35%, 35%

Job Arrival Patterns

Jobs arrived into the system having either deterministic or stochastic interarrival times (INTER). Deterministic interarrival times were used in order to mimic a steady release of work to the shop floor. Note that even though jobs arrived into the system on a regular pattern, the product type associated with the job was chosen randomly using a uniform distribution. The stochastic interarrival time scenario approximates the condition in which it is difficult to maintain a steady release of products to the shop floor, perhaps due to a high cost of holding inventory or a dynamic demand environment. Stochastic interarrival times were gamma distributed with a coefficient of variation of 0.50. This produced a distinctly random pattern of job arrivals with a moderate degree of variability. Orders were released into the shop as they arrived.

Mean Job Sizes

Each arriving job had a size (number of units) that varied uniformly by ±67% of its mean job size (*JS*). Six levels of mean job size were chosen, ranging from 75 to 225 in increments of 30, to represent typical lot sizes in a repetitive batch manufacturing environment. Interarrival times were increased on a relative basis with mean job size in order to maintain the desired three levels of process utilization. These six levels were chosen since they represent values typically used in prior research and in practice for repetitive manufacturing. Note that with three splits, the transfer batch size is as low as 25 units and as high as 75.

Operation Time Variance

We used operation time variance as our surrogate for system variance, as is common in simulation studies of job shops. Variable operation times were modeled using a gamma distribution with a coefficient of variation (*CV*) level specified by the experiment design. Empirical studies of task time distributions (e.g., [5]) indicate that such distributions tend to be unimodal and skewed to the right, making the gamma an appropriate distribution.

It is difficult to predict a priori what the effect of variability will have (although a low variance flow shop should behave similarly to a deterministic flow shop). We tested four levels of operation time coefficient of variation (Table 1, ranging from nearly deterministic (*CV*=0.01) to extremely high (*CV*=1.50)). Based on empirical evidence (e.g., Dudley [5]), *CV*=0.50 is probably the closest to actual operation task times. Nevertheless, there are situations in which a high *CV* may be appropriate: unreliable machines with many machine breakdowns, excessive amounts of rework, etc.

Shop Load

The shop load, defined by processing utilization percentage (*U*), can be an important factor in the impact of lot splitting forms. Highly congested shops could be improved by increasing the amount of overlapping processing induced by lot splitting. On the other hand, with high congestion there is increased possibility that the flow time for some jobs would increase due to a "straggling" split being caught in a queue. Note that the total utilization level of the shop will be greater than the processing utilization level due to the effect of setup times.

We varied the mean operation task times in order to obtain different levels of shop load. Within each utilization level, mean operation task times were identical in each department in order to have a balanced shop (i.e., to avoid long-run bottlenecking). The levels for this factor were set, respectively, at 0.0456, 0.0576, and 0.0696 hours per unit to achieve three different process utilization scenarios of 57%, 72%, and 87%, respectively. The extreme levels are in a range of ±15% of the processing utilization of 72% used by Jacobs and Bragg [10]. We tested these utilization levels since we hypothesize that shop load has an effect on the performance of the different lot splitting forms. We insured that total utilization levels never exceeded 100% in any of our experiments.

Setup Times

Since the repetitive lots rule has the potential to reduce MFT by decreasing setups, the proportion of setup time to processing time becomes an important consideration. We used four levels of this setup ratio (*SU*) in our experiments: 0.1, 0.5, 1.0, and 1.5. By increasing or decreasing the levels of setup ratio, total utilization also increased or decreased and ranged from approximately 60% to 95% in our experiments. The observed ratio of setup time to total processing time (including setup) ranged from 22% to 57% per job, on the average.

Performance Measures

Our primary performance measure was mean flow time (MFT) rather than makespan, which is used in most of the previous deterministic studies. While makespan is a suitable criterion for static scenarios, MFT is a more appropriate measure of performance in a dynamic setting. Another measure of interest to studies of this type is the average amount of work in process inventory (WIP), particularly in light of the recent focus in manufacturing toward reducing levels of WIP. However, in steady-state, MFT will be proportional to WIP by Little's formula (see [8]). Thus, results for MFT will translate into comparable results for WIP. In the context of this study, this fact means that lot splitting forms that reduce MFT will also reduce WIP proportionally, and consequently we need only consider MFT. This relationship was verified by our simulation experiments. We also computed the standard deviation of flow time (SDFT) as a measure of the variability of flowtime. We do not measure due date performance since we deal mostly with the repetitive manufacturing environment in which we assume that most production is "to stock." In addition, flow-time performance is a more generalizable measure since any due date setting mechanism may be correlated with dispatching rules. For example, total work content dispatching rules will typically provide good tardiness performance when the due date in the simulation was originally set by some function of total work content.

We estimated MFT and SDFT for each experimental setting by first "warming up" the system for 10,000 hours of operation, followed by the data collection portion of the run. Plots of the output for several combinations of factor settings, including those with the highest processing utilization, coefficient of variation and setup time to processing time ratio, indicated that 10,000 hours of transient observation was sufficient for each scenario to be in steady-state. Flow times were then collected in blocks of 5000 hours separated by periods of 1000 hours with no data collection. Thus, when repeated observations were desired, the resulting block means were taken as the data points. This procedure is similar to that of spaced batch means (see Fox, Goldsman, and Swain [6]). We verified that there was no significant serial correlation in the block means by performing lagged regression analyses of these means.

THE EFFECT OF LOT SPLITTING FORM

In this section we present our experiments to test different lot splitting forms. First, we will describe the experimental design, then present the results of ANOVA and multiple comparison tests for the lot splitting forms.

ANOVA Results

We conducted three different Analyses of Variance (ANOVA) to test the significance of the main and interaction effects for the different factors on mean flow time (MFT) and the standard deviation of flow time (SDFT). The ANOVAs were run for main effects only, for main and two-way interactions, and for main, two-way and three-way interactions. We found that by adding two-way interactions to the ANOVA, the R^2s for both MFT and SDFT increased from approximately .76 to .95. The incremental explanation of variance from the further addition of three-way interactions was negligible, however, adding only .02 to .03 to the R^2s. For conciseness, we show the ANOVAs with two-way interactions for both MFT and SDFT in Tables 3 and 4. The R^2 of .95 for MFT and SDFT indicates a reasonably good fit of the model and that the main effects and interactions explain most of the variance.

All main and two-way interaction effects are significant at the .05 level or better, except for lot splitting form × setup interaction for both MFT and SDFT and for setup × operation time variance interaction for SDFT. Referring back to our performance model in (1), we can see from both ANOVA tables that the process utilization factor, U, provides the greatest contribution to the explained variance. The two factors that provide the least contribution are job size, JS, and the lot splitting form, M. While we see that the form of lot splitting does have a significant effect, other factors contribute much more to explaining performance. Therefore, in order to better understand the implications of lot splitting forms, we provide more detailed statistical comparisons of various interaction effects through the use of multiple comparison tests.

Table 3: ANOVA, linear and two-way interactions, MFT dependent variable.

Source	d.f.	ANOVA SS	Mean Square	*F* Value	*P*
S	1	6383374.52	6383374.52	4536.01	.0001
U	2	98794970.67	49397485.34	35101.74	.0001
M	5	7444096.47	1488819.29	1057.95	.0001
SU	3	18020869.26	6006956.42	4268.53	.0001
CV	3	24805872.61	8268624.20	5875.67	.0001
JS	5	5601346.17	1120269.23	796.06	.0001
INTER	1	1525284.54	1525284.54	1083.86	.0001
S×*U*	2	3856644.06	1928322.03	1370.26	.0001
S×*M*	5	24344.21	4868.84	3.46	.0040
S×*SU*	3	145163.62	48387.87	34.38	.0001
S×*CV*	3	564881.11	188293.70	133.80	.0001
S×*JS*	5	1222460.86	244492.17	173.74	.0001
S×*INTER*	1	400455.96	400455.96	284.56	.0001
U×*M*	10	2400861.77	240086.18	170.60	.0001
U×*SU*	6	6930988.48	1155164.75	820.86	.0001
U×*CV*	6	13977018.02	2329503.00	1655.34	.0001
U×*JS*	10	2581995.92	258199.59	183.48	.0001
U×*INTER*	2	1690746.91	845373.46	600.72	.0001
M×*SU*	15	10800.70	720.05	0.51	.9360
M×*CV*	15	2531888.57	168792.57	119.94	.0001
M×*JS*	25	759242.76	30369.71	21.58	.0001
M×*INTER*	5	203201.85	40640.37	28.88	.0001
SU×*CV*	9	576839.97	64093.33	45.54	.0001
SU×*JS*	15	265697.99	17713.20	12.59	.0001
SU×*INTER*	3	320893.37	106964.46	76.01	.0001
CV×*JS*	15	1234158.22	82277.21	58.47	.0001
CV×*INTER*	3	636277.34	212092.45	150.71	.0001
JS×*INTER*	5	41530.10	8306.02	5.90	.0001

$R^2 = .955428$

Multiple Comparison Tests

A large number of multiple comparison tests are available to determine if significant differences exist between factor levels. We performed two widely used Multiple *F* tests, the Duncan and the Ryan-Einot-Gabriel-Welsch tests, on our data at various levels of interactions. We found that the results of these two comparison tests were almost identical in identifying where interaction effects are significant. For conciseness, we present only results of the Duncan tests in Tables 5 through 8. We show results for interaction effects with operation time and setup time separately. We also separate results for the deterministic and the stochastic interarrival times. Note that the performance measures shown in the following tables are for the job size with the lowest MFT for each Shop/Form/Utilization combination.

MFT—Operation Time Variance

Tables 5 and 6 show multiple comparisons for the MFT measure for interactions with operation time variance. As one can see from Table 5, with deterministic

Table 4: ANOVA, linear and two-way interactions, SDFT dependent variable.

Source	d.f.	ANOVA SS	Mean Square	*F* Value	*P*
S	1	2049568.21	2049568.21	12291.57	.0001
U	2	9643136.37	4821568.18	28915.68	.0001
M	5	803863.36	160772.67	964.18	.0001
SU	3	925282.80	308427.60	1849.69	.0001
CV	3	3127547.16	1042515.72	6252.12	.0001
JS	5	847644.70	169528.94	1016.69	.0001
INTER	1	217373.57	217373.57	1303.62	.0001
S×*U*	2	1084018.76	542009.38	3250.51	.0001
S×*M*	5	3023.93	604.79	3.63	.0028
S×*SU*	3	4780.04	1593.35	9.56	.0001
S×*CV*	3	13722.04	4574.01	27.43	.0001
S×*JS*	5	191814.53	38362.91	230.07	.0001
S×*INTER*	1	19440.77	19440.77	116.59	.0001
U×*M*	10	225793.01	22579.30	135.41	.0001
U×*SU*	6	418666.77	69777.79	418.47	.0001
U×*CV*	6	1159392.07	193232.01	1158.84	.0001
U×*JS*	10	273386.02	27338.60	163.95	.0001
U×*INTER*	2	101829.00	50914.50	305.34	.0001
M×*SU*	15	1271.55	84.77	0.51	.9378
M×*CV*	15	576226.78	38415.12	230.38	.0001
M×*JS*	25	73436.19	2937.45	17.62	.0001
M×*INTER*	5	20962.46	4192.49	25.14	.0001
SU×*CV*	9	2421.66	269.07	1.61	.1052
SU×*JS*	15	12668.00	844.53	5.06	.0001
SU×*INTER*	3	8004.92	2668.31	16.00	.0001
CV×*JS*	15	226587.28	15105.82	90.59	.0001
CV×*INTER*	3	101595.04	33865.01	203.09	.0001
JS×*INTER*	5	5608.66	1121.73	6.73	.0001

R^2 = .951770

interarrival times each of the six forms of lot splitting are significantly different from each other (at the .05 level) in a flow shop environment when *CV*=0.01 and the processing time utilization level is 57%. RL4F is clearly the best with a MFT of 21.7, less than half that of RL0 and at least 15% lower than any other lot splitting form's MFT. These results validate the results obtained by Kropp and Smunt [13] in that they clearly show the superior performance of the "flag" heuristic in flow shops operating with little variance in either input rates or operation times. Further examination of Table 5, however, illustrates that for a flow shop, statistical differences of the RL4F form dissipate as the utilization percentage or the *CV* increases. While RL4F performs significantly better in the flow shop even for higher utilizations, as long as *CV*=0.01, it performs better in the job shop only for the lowest utilization level of 57%.

Further review of Table 5 shows that RL4F gives the best MFT in 22 out of 24 Shop/*CV*/Utilization settings. However, according to the Duncan tests, RL4F is

Table 5: MFT comparison across *CV* levels using optimal job sizes for each Shop/*CV*/Form/Utilization combination, deterministic interarrival times.

		Utilization					
		Flow Shop			Job Shop		
CV	Form	57%	72%	87%	57%	72%	87%
0.01	RL0	45.4	68.8	130.1	61.3	104.1	250.1
	RLU3	27.6	43.9	98.2	50.3	89.2	239.3
	RLU2	27.0	42.9	100.4	50.3	89.5	238.9
	RLU1	25.8	41.7	97.4	49.9	88.9	234.3
	RL3E	28.6	45.1	101.7	50.8	89.8	**228.8**
	RL4F	**21.7**	**36.2**	**90.9**	**48.4**	**88.2**	234.4
0.50	RL0	62.2	121.3	292.5	66.3	115.8	294.0
	RLU3	34.1	68.0	210.8	53.9	96.8	260.6
	RLU2	33.7	67.9	205.6	53.4	97.8	254.7
	RLU1	33.3	68.8	202.0	54.0	98.2	253.4
	RL3E	34.7	67.6	203.1	53.2	96.8	252.1
	RL4F	**30.2**	**64.2**	**196.2**	**53.0**	**96.0**	**251.4**
1.00	RL0	84.8	163.7	409.9	80.4	153.3	418.9
	RLU3	49.7	109.3	287.5	60.3	112.1	313.5
	RLU2	49.3	110.2	321.2	60.8	113.1	313.6
	RLU1	50.6	111.6	332.0	61.1	114.0	318.4
	RL3E	50.2	110.4	305.0	60.5	112.7	312.4
	RL4F	**47.0**	**107.2**	**295.0**	**59.5**	**110.0**	**304.5**
1.50	RL0	109.9	222.0	576.4	100.8	209.6	645.6
	RLU3	68.8	146.9	**364.2**	70.2	137.1	400.8
	RLU2	69.1	147.8	408.1	71.0	138.1	415.1
	RLU1	70.0	148.6	393.3	71.8	139.9	406.8
	RL3E	68.8	143.6	366.7	71.2	137.8	414.8
	RL4F	**65.8**	**143.5**	404.8	**68.5**	**135.0**	**397.8**

Note: Numbers in bold are the best for each Shop/*CV*/Utilization combination. Numbers with lines in the same vertical column are not significantly different.

significantly different from the other forms in only four cases, three of which have *CV*=0.01, nearly deterministic processing times. When *CV* is moderate but realistic (i.e., 0.50), RL4F is significantly better than other forms only for the lowest utilization level in the flowshop. For the job shop, RL4F gives the smallest MFT in all but one setting, although the difference is significant only for low *CV* and low utilization.

Table 5 also demonstrates the superiority of all forms of lot splitting over no splitting (RL0). Not only did RL0 have significantly higher MFTs in 23 out of 24 settings, but the magnitude of the differences between RL0 and the worst lot splitting forms were substantial, typically far greater than differences between the various lot splitting forms.

Table 6: MFT comparison across *CV* levels using optimal job sizes for each Shop/*CV*/Form/Utilization combination, stochastic interarrival times.

		Utilization					
		Flow Shop			Job Shop		
CV	Form	57%	72%	87%	57%	72%	87%
0.01	RL0	46.9	68.3	93.9	56.7	90.2	209.8
	RLU3	30.4	44.8	68.6	47.8	79.1	186.9
	RLU2	29.8	45.3	69.8	46.8	77.4	194.4
	RLU1	28.5	42.5	67.0	46.1	79.1	179.0
	RL3E	30.7	47.8	71.8	45.5	78.6	**183.3**
	RL4F	**24.7**	**37.2**	**62.5**	**45.3**	**76.1**	186.1
0.50	RL0	64.0	113.7	222.5	61.1	100.6	241.2
	RLU3	35.7	69.6	189.7	47.6	81.0	200.2
	RLU2	35.4	66.7	184.6	50.9	86.2	199.0
	RLU1	34.4	67.5	183.6	49.6	**79.7**	200.0
	RL3E	36.0	69.6	171.6	**47.1**	80.8	**196.8**
	RL4F	**33.3**	**63.9**	**167.5**	47.2	81.5	197.7
1.00	RL0	89.5	145.4	310.0	68.8	131.9	339.2
	RLU3	49.0	95.2	249.4	54.3	94.8	236.0
	RLU2	50.9	**94.9**	250.0	54.7	92.9	240.4
	RLU1	**47.6**	95.8	**215.4**	53.6	95.5	**233.9**
	RL3E	48.8	97.6	232.1	55.8	99.0	234.0
	RL4F	48.5	97.6	244.6	**52.3**	**92.3**	234.7
1.50	RL0	96.2	162.0	386.7	80.0	150.3	447.6
	RLU3	**59.2**	127.9	294.4	56.0	103.1	322.7
	RLU2	59.3	129.8	296.1	**55.4**	100.5	318.6
	RLU1	63.6	136.8	296.4	56.1	99.0	316.9
	RL3E	60.8	124.0	297.5	56.6	**98.1**	308.5
	RL4F	59.9	**119.2**	**289.9**	60.9	109.6	**287.5**

Note: Numbers in bold are the best for each Shop/*CV*/Utilization combination.

Table 6, stochastic interarrival times, further illustrates the deleterious effects of variance on the superiority of the "flag" heuristic. RL4F generally remains statistically better than the other forms for flow shop settings with *CV*=0.01, but for all other scenarios in the flow shop and for all scenarios in the job shop it is not significantly different than the other lot splitting forms. As with deterministic interarrival times, RL0 is significantly worse than all lot splitting forms, in this case, over all factor settings.

MFT—Stepup Time

Tables 7 and 8 show multiple comparisons for the MFT measure for interactions with setup time. Note that when interarrival times are deterministic (Table 7), RL4F

Table 7: MFT comparison across *SU* levels using optimal job sizes for each Shop/*SU*/Form/Utilization combination, deterministic interarrival times.

		Utilization					
		Flow Shop			Job Shop		
CV	Form	57%	72%	87%	57%	72%	87%
0.10	RL0	36.8	69.8	179.5	44.1	89.0	263.3
	RLU3	16.5	30.3	75.6	25.8	53.3	174.8
	RLU2	16.7	31.1	82.8	26.1	54.0	177.5
	RLU1	16.9	31.3	83.9	26.0	54.1	173.4
	RL3E	16.7	29.9	75.6	25.6	**53.1**	174.1
	RL4F	**15.9**	**30.1**	**79.0**	**25.2**	54.0	**164.3**
0.50	RL0	51.0	100.8	276.0	60.9	122.5	353.9
	RLU3	25.8	54.0	172.0	42.1	**81.8**	242.9
	RLU2	25.2	53.3	178.5	42.0	83.3	**242.3**
	RLU1	24.8	54.8	179.7	42.6	83.7	252.1
	RL3E	26.0	54.0	**169.0**	42.1	82.5	251.8
	RL4F	**22.8**	**49.6**	175.1	**41.0**	82.2	249.8
1.00	RL0	82.6	162.1	400.3	86.9	162.3	448.1
	RLU3	46.5	99.3	297.6	68.5	125.5	348.1
	RLU2	46.6	97.9	313.6	68.1	126.3	353.3
	RLU1	46.5	98.7	312.7	68.8	127.4	343.9
	RL3E	47.6	100.2	**295.6**	68.5	126.9	347.3
	RL4F	**42.0**	**95.2**	300.0	**66.2**	**122.6**	**336.4**
1.50	RL0	118.2	243.5	553.1	116.9	209.0	543.3
	RLU3	67.9	144.2	436.1	98.3	174.6	448.3
	RLU2	68.1	139.5	436.3	99.1	175.0	449.3
	RLU1	68.1	144.7	**418.8**	99.3	175.8	443.6
	RL3E	69.7	143.6	439.5	99.4	174.7	**434.9**
	RL4F	**62.4**	**136.0**	436.5	**96.9**	**170.4**	437.6

Note: Numbers in bold are the best for each Shop/*SU*/Utilization combination.

performs statistically better for a wider set of scenarios. Interestingly, RL4F is best for all setup ratios for the flow shop when utilization was low. RL4F also performs best for all setup ratios over 0.10 for the job shop, again when the utilization is low. For the medium and high utilization levels, RL4F is not significantly different than the other lot splitting forms in either the flow shop or job shop. With the additional variance caused by stochastic interarrival times (Table 8), there is no indication of the flag heuristic performing any better than other lot splitting forms.

As with the comparisons across *CV* in Tables 5 and 6, all lot splitting forms perform significantly better than RL0. Furthermore, the magnitude of the improvement from RL0 to the worst lot splitting form is much greater than any differences between lot splitting forms.

Table 8: MFT comparison across *SU* levels using optimal job sizes for each Shop/*SU*/Form/Utilization combination, stochastic interarrival times.

		Utilization					
		Flow Shop			Job Shop		
CV	Form	57%	72%	87%	57%	72%	87%
0.10	RL0	36.0	69.1	152.2	40.9	74.6	200.1
	RLU3	17.1	35.8	80.7	24.7	47.1	136.3
	RLU2	19.0	37.9	83.9	25.2	**46.3**	129.5
	RLU1	18.3	38.8	85.4	24.7	47.1	138.0
	RL3E	17.5	34.0	**80.5**	24.4	46.8	**126.6**
	RL4F	**16.5**	**31.8**	85.6	**23.9**	47.6	132.9
0.50	RL0	52.2	98.0	213.3	53.9	99.3	264.6
	RLU3	31.3	62.5	150.1	38.0	**70.2**	189.6
	RLU2	32.2	64.3	145.7	37.8	70.5	197.5
	RLU1	29.8	62.1	**143.6**	37.5	67.8	**189.1**
	RL3E	31.1	61.7	150.4	38.2	68.5	192.3
	RL4F	**26.9**	**59.6**	147.7	**36.9**	66.7	174.2
1.00	RL0	88.5	132.3	278.3	73.4	133.1	345.1
	RLU3	49.0	93.3	245.2	59.6	**97.8**	272.6
	RLU2	47.0	87.2	243.2	61.2	102.3	282.5
	RLU1	47.8	94.0	**223.0**	59.6	98.7	264.9
	RL3E	50.1	99.0	236.6	59.2	103.6	269.9
	RL4F	**42.2**	**89.2**	227.5	**57.2**	102.8	**262.2**
1.50	RL0	113.9	189.9	369.2	98.5	166.1	427.9
	RLU3	60.8	131.9	326.1	83.4	142.9	347.3
	RLU2	**63.7**	131.5	328.0	83.6	138.0	42.9
	RLU1	66.6	132.5	**310.6**	83.6	139.7	337.9
	RL3E	65.2	**130.0**	320.5	**83.1**	**137.7**	**333.9**
	RL4F	63.8	131.2	328.9	87.6	142.5	336.7

Note: Numbers in bold are the best for each Shop/*SU*/Utilization combination.

SDFT

The standard deviation measure of flow time (SDFT) can also be a good measure of system performance since it represents the consistency of output, and, thereby, the ability to accurately predict job completion. Table 9 shows the multiple comparisons for the SDFT measure for the setup time interactions with deterministic interarrival times. The other scenarios showed similar results so we present only this one table of SDFT results for conciseness.

As expected, SDFT is higher in job shops than in flow shops for identical factor settings. Somewhat surprisingly, SDFT behaves in a manner similar to MFT. In each setting, there is substantial and significant reduction in SDFT from RL0 to the worst lot splitting form. The improvement is greater in the flow shop (30-50%) than the job shop (20-30%). The differences between lot splitting forms are much smaller

Table 9: SDFT comparison across *SU* levels using optimal job sizes for each Shop/*SU*/Form/Utilization combination, deterministic interarrival times.

		Utilization					
		Flow Shop			Job Shop		
CV	Form	57%	72%	87%	57%	72%	87%
0.10	RLO	14.6	25.6	63.7	20.7	39.9	102.2
	RLU3	6.5	11.5	**27.4**	11.6	24.0	71.6
	RLU2	**6.4**	11.9	29.3	11.8	24.4	70.8
	RLU1	6.6	11.5	30.2	11.8	24.5	70.5
	RL3E	6.6	**11.1**	27.8	**11.5**	**23.6**	71.3
	RL4F	6.5	11.7	29.0	**11.5**	25.0	**67.5**
0.50	RLO	16.2	33.4	78.8	24.0	49.7	126.7
	RLU3	7.7	16.8	48.6	15.9	**31.6**	87.6
	RLU2	**7.6**	17.3	51.6	15.9	32.3	**87.0**
	RLU1	7.7	17.5	53.1	16.4	32.5	89.7
	RL3E	7.7	17.4	**45.3**	15.9	32.2	90.8
	RL4F	7.8	**16.3**	50.0	**15.8**	32.4	90.2
1.00	RLO	21.7	42.6	104.1	30.3	58.2	145.3
	RLU3	**12.0**	**28.0**	**75.0**	22.6	43.7	110.6
	RLU2	12.9	**28.0**	88.4	22.4	43.3	113.8
	RLU1	13.6	30.2	88.3	22.6	43.6	108.8
	RL3E	12.3	31.1	85.2	22.3	43.4	110.0
	RL4F	13.3	29.4	81.9	**22.1**	**41.6**	**108.5**
1.50	RLO	31.2	62.5	141.8	36.5	69.0	164.6
	RLU3	**17.0**	39.6	**112.8**	**29.1**	54.5	131.5
	RLU2	18.1	40.2	117.1	29.4	54.3	136.0
	RLU1	18.7	42.6	120.2	29.7	56.0	130.2
	RL3E	18.5	**38.6**	109.5	29.6	55.2	**128.0**
	RL4F	20.2	41.4	118.6	**29.1**	**53.0**	130.9

Note: Numbers in bold are the best for each Shop/*SU*/Utilization combination.

and are mostly statistically insignificant according to Duncan's test. Unlike using the shortest processing time dispatching rule (without lot splitting), we see that lot splitting by itself reduces both MFT and SDFT.

Secondary Dispatching Rules

In all our scenarios, the primary dispatching rule at each workcenter is repetitive lots. In the above experiments, we used FCFS as the secondary rule when repetitive lots is not invoked. However, other secondary rules may be considered. For example, shortest operation time (SOT) is known to minimize MFT in certain cases. We investigated the impact of SOT as a secondary criterion for *CV*=0.50, *SU*=0.50, and show the results in Tables 10 and 11.

Table 10: Mean flow times with FCFS and SOT as secondary dispatching rule, deterministic interarrival times: *CV*=0.50, *SU*=0.50.

		Flow Shop		Job Shop	
U	Form	FCFS	SOT	FCFS	SOT
57%	RL0	37.3	36.3	41.7	40.4
	RLU3	19.7	18.9	31.9	32.3
	RLU2	18.8	19.5	29.8	30.4
	RLU1	18.3	19.5	32.9	31.1
	RL3E	19.4	19.9	32.8	31.3
	RL4F	15.7	16.7	32.5	36.7
72%	RL0	65.9	57.2	71.8	83.3
	RLU3	35.4	33.0	60.3	62.7
	RLU2	34.7	31.3	54.2	60.2
	RLU1	35.2	36.1	56.4	55.3
	RL3E	35.5	32.2	56.3	62.7
	RL4F	28.9	28.9	54.7	66.0
87%	RL0	165.9	147.6	192.1	240.3
	RLU3	108.2	86.2	154.1	194.7
	RLU2	101.5	89.9	145.5	208.4
	RLU1	102.5	91.6	154.2	259.0
	RL3E	105.9	93.6	153.0	192.9
	RL4F	98.6	90.6	152.0	204.8

Note: Numbers with lines in the same vertical column are not significantly different.

Observe that, as expected, SOT reduces MFT for RL0 in both flow shops and job shops. However, its impact under lot splitting is minimal. Only in the flow shop with high utilization did SOT substantially reduce the minimum possible MFT. In many other cases, especially in the job shop, the MFT actually increased when SOT was used as a secondary dispatch rule. Notice further that in every case, lot splitting by itself provided more improvements in MFT than using SOT without splitting. Thus, a shop currently using repetitive lots but not lot splitting (RL0) could benefit more by utilizing any of the lot splitting forms than by changing the FCFS rule to SOT as a secondary dispatching criterion.

Note that the results presented by Wagner and Ragatz [18] are somewhat different than ours in that they indicate SOT always provides lower MFT than does FCFS. We suspect that either the fact that all jobs were unique in their simulation model (an open job shop) or that they allowed a wide range of task times within a job to be the reasons why SOT dominated FCFS in their study. In our experiment of a closed job shop, we tested a variety of utilization levels and setup times so as to illustrate the conditions where FCFS or SOT would provide better MFT performance in the repetitive batch manufacturing environment.

Table 11: Mean flow times with FCFS and SOT as secondary dispatching rule, stochastic interarrival times: *CV*=0.50, *SU*=0.50.

		Flow Shop		Job Shop	
U	Form	FCFS	SOT	FCFS	SOT
57%	RL0	42.2	40.5	50.4	46.1
	RLU3	22.3	24.5	35.9	37.2
	RLU2	23.2	20.5	34.5	35.4
	RLU1	20.6	21.0	37.0	39.9
	RL3E	22.3	22.4	37.7	39.7
	RL4F	17.1	18.9	37.7	39.0
72%	RL0	83.6	78.5	81.6	91.4
	RLU3	48.6	42.8	68.3	64.0
	RLU2	44.2	43.9	58.9	62.3
	RLU1	42.3	47.1	63.7	67.3
	RL3E	43.5	45.4	54.4	70.8
	RL4F	44.0	41.0	61.3	70.0
87%	RL0	177.2	146.8	200.6	242.2
	RLU3	117.4	102.8	146.0	208.8
	RLU2	111.0	101.1	159.6	256.0
	RLU1	111.2	100.9	149.7	189.8
	RL3E	117.0	105.4	140.9	216.3
	RL4F	122.2	98.8	150.6	232.6

Note: Numbers with lines in the same vertical column are not significantly different.

Discussion

The preceding results address our objectives of this study. As had been pointed out elsewhere ([2], [9], and [13]), lot splitting can reduce MFT in deterministic and stochastic flow shop environments by use of overlapping processing of items from the same job. However, the differences between various forms of lot splitting diminish as the environment moves further from the deterministic flow shop. The only job shop scenarios in which there were any significant differences had very low processing utilizations, moderate setup levels, and deterministic interarrival times; in other scenarios the exact method used does not seem to matter.

The results are similar in the flow shop scenarios, although RL4F still performs well under high utilization levels. However, as the flow shop parameters approached more realistic levels of *CV*, this advantage disappears. Clearly, extending conclusions regarding lot splitting policies that perform well in deterministic settings to more realistic stochastic situations may not be justified. Whereas splitting the lots still has a beneficial effect on MFT in the stochastic environments, the exact method used matters in a relatively small subset of scenarios. The number of splits might matter most in stochastic environments. In the following section, we describe experiments that determine the effect of further splitting the lot on flowtime performance.

THE EFFECT OF THE NUMBER OF EQUAL SPLITS

We expected that the impact of increasing the number of transfer batches will be more dramatic for higher levels of the setup ratio, for higher processing utilization levels, and higher levels of variability (*CV*). We also expected that as the number of transfer batches gets very large, so the expected transfer batch size approaches 1, deleterious effects of lot splitting will appear in the job shop. Conceivably, with many small splits in the job shop, and no dominant flow to coordinate the sequencing, there is increased likelihood that at least one straggler will arrive at a machine with the "wrong" setup and get delayed due to repetitive lots working against that small batch. As we shall see, however, this delay does not seem occur.

The Experiment

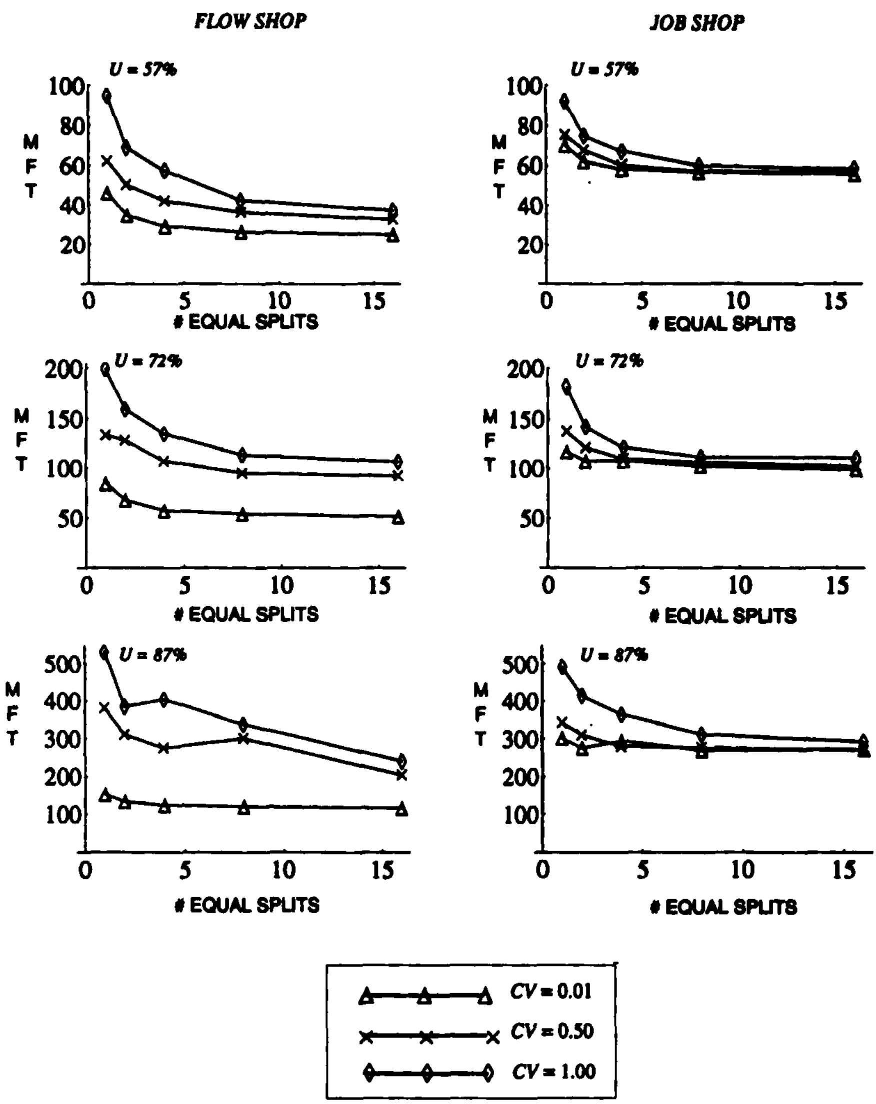
Using the same simulation model, we ran a series of experiments varying the number of equal splits. Except for the lot splitting form now being "RLnE," where "*n*" is the number of equal splits, the other parameters are identical to the experiments above in which we studied the lot splitting forms. The number of splits was run at 1 (RL0), 2, 4, 8, 16, 32, 48, and 64. Due to the lengthy computer time required, especially in the 32-64 split cases, and with the results from the previous section in mind, we only considered a subset of the other parameters. We used high, medium, and low processing utilization levels as before (57%, 72%, and 87%) and *CV* levels of 0.01, 0.5, and 1.0, respectively. We fixed the mean size of incoming jobs at 75 units in order to allow the larger number of splits to result in transfer batch sizes of near 1. The setup ratio factor was kept at 1.00 for all experiments.

Results

The results for the *CV* × number of splits interaction are shown in Figure 3 for processing utilization levels of 57%, 72%, and 87%. We only show up to 16 equal splits since MFT stayed pretty much constant for 32-64 splits. There is a dramatic difference in the flow shop results between low *CV* (0.01) and the higher *CV*s. For near-deterministic *CV* (0.01), the MFT is substantially lower for all number of splits. However, in the job shop, MFT converges for all *CV* levels as the number of splits increases. Also, the greatest reduction in MFT due to additional splitting occurs in the high *CV* situations for both the flow shop and the job shop. As the flow shop utilization increases to 87%, the difference between the low-*CV* flow shop and the higher-*CV* flow shops is even more dramatic, suggesting that lot splitting is more valuable in flow shops with highly variable environments.

In all scenarios, there was considerable improvement to MFT due to lot splitting. However, the incremental benefits of lot splitting become negligible after the first few splits. For example, for the job shop with utilization of 72% and *CV* (1.0), splitting from one lot to two lots decreased MFT by 22%, splitting from two to four decreased MFT by 14% more, and splitting from four to eight decreased MFT by 8% more. Only in the flow shop with 87% utilization and the higher two *CV* levels was there any appreciable improvement between 8 and 16 splits. As mentioned above, increasing the number of splits higher still resulted in virtually no further improvement. The amount of improvement increased with higher utilization and

Figure 3: Mean flow time versus number of equal splits for different levels of processing utilization (U) and operation CV.



with greater variability (higher CV). The overall benefits of lot splitting are greater in flow shops than job shops. It is also interesting to note that the MFT for job shops tended to converge with increased lot splitting, whereas in flow shops they stayed distinct for each CV level.

CONCLUSIONS

Flow Dominance. Unlike previous research, we found that the form of lot splitting rarely matters in situations more likely to be found in practice. We did find that the flag lot splitting form worked generally well in a flow shop environment when the variability is at low to moderate levels and the system is not highly congested. In contrast, for a purely random job shop, significant differences between the lot splitting forms do not exist except for the low utilization/low variance case. Consider the managerial implications in a job shop that evolves into a more line-oriented flow (perhaps due to product maturation or standardization). Until there is clear flow dominance, the use of unequal lot splitting forms offers little benefit over equal splits. Once there is clear flow dominance, the variability, congestion, and setup times should be considered to determine if there are possible improvements due to unequal lot splitting forms. In these circumstances, our results indicate that the use of the flag heuristic tends to best improve MFT.

Deterministic/Static vs. Stochastic/Dynamic Settings. Our results indicate there is a large discrepancy between the orderly world of the deterministic, static flow shop models and the chaotic world of the stochastic, dynamic flow shop and job shop models. Since many production facilities processing in batch mode resemble the latter more than the former, lot splitting forms that work well *only* in deterministic settings should be used with caution. As we have demonstrated, forms such as the flag heuristic, which performed well in a deterministic flow shop, seem to have little or no advantage when there is even a moderate amount of variability or congestion.

JIT/Kanban Issues. Current views of manufacturing, influenced by some Japanese companies, advocate smaller lot sizes, reduced WIP inventory, use of "pull" systems and the related production triggering mechanisms, such as kanban. Our results on the number of splits indicate that perhaps many of the benefits of such systems may be simply due to reduced lot sizes. If splitting orders into small transfer batches reduces mean flow time as indicated above, then WIP will be correspondingly reduced as well. Note that in our models we used a classic "push" system with batch processing, yet were able to reduce the flow time substantially by splitting. Furthermore, this worked well in both flow shops and in job shops. As Zipkin [19] points out, the application of Just-in-Time (JIT) methods can be problematic in a number of real settings. Therefore, firms that are not able to easily implement many aspects of JIT, including the use of kanban, may find that a large proportion of flow time benefits can be attained through lot splitting alone. Clearly, the above comments are speculative, since we do not explicitly test a number of important JIT issues in this study.

When Lot Splitting Is Not Beneficial. Lot splitting is not necessarily always beneficial. One consequence of many smaller batches in the shop is that material handling costs could skyrocket. Furthermore, the likelihood of a batch getting misplaced in a job shop increases dramatically with the number of such batches in the shop. Thus, a facility that implemented lot splitting would be wise to rationalize their layout, routing and tracking mechanisms, and make sure their material handling capabilities were sufficiently flexible to handle the resulting load. On the other hand, a flow shop already has a layout that is matched with the routing of its parts. Consequently, lot splitting may be more desirable in flow shops than job shops.

In a similar vein, the existence of minor setups may also counteract the potential benefits of lot splitting. In this case, we would consider a minor setup to be one associated with the processing of any new batch on a machine, even if it is of the same type. With more splits, the effect of such setups on MFT would increase. However, in Wagner and Ragatz's "open" shop ([18]) each job is unique, thereby always incurring a setup. Since Wagner and Ragatz demonstrated improvements in system performance due to lot splitting in open shops, the benefits of lot splitting may indeed be persistent even in the face of minor setups.

SUMMARY AND FURTHER RESEARCH

We have extensively tested various forms of lot splitting in a closed job shop and flow shop environments with different levels of setup times, processing time variability, processing utilization, job size, and type of shop. In nearly every case, lot splitting substantially improved both MFT and SDFT over no lot splitting. We also found that as the environment moves away from a deterministic flow shop, the differential impact of lot splitting forms diminishes, and there is virtually no difference in most job shop settings. As the number of splits increases, MFT tends to keep improving, but with decreasing returns. The repetitive lots rule and lot splitting appear to work together in a complementary way. The benefits of lot splitting in the environments we have considered (stochastic, job shop and flow shop) may be even greater than in the simpler deterministic, flow shop environments previously studied. Lot splitting thus provides a relatively easy way to obtain some of the benefits of smaller batches under the classic push system still employed by most batch production facilities without the need to radically change procedures. As discussed above, improved mean flow time goes hand-in-hand with decreased WIP, another practice that is currently advocated.

Our initial tests constrained the number of machines per department to one, but this simulation model could be easily modified to allow multiple machines per department. We plan to test this environment in future research, since we hypothesize that the repetitive lots rule will mimic a cellular manufacturing environment, given a sufficient number of like machines per department. Since the repetitive lots rule scans the queue of jobs waiting to use a machine in a department for one that could be processed without requiring a setup, the availability of multiple, like machines should cause dedication of machines to similar job types.

Even though we found that increasing the number of equal splits did not degrade MFT performance, a number of situations in which MFT would increase with more splits can be envisioned. We have discussed some issues regarding layout, minor setups, and material handling above. In certain settings there may indeed be an optimal number of splits that would minimize MFT, and in others, a transfer batch size of 1 may be optimal.

Finally, we have only considered flow shops and pure job shops, which are extreme cases of flow dominance. It would be interesting to determine how our results would change for intermediate cases of flow dominance (i.e., between flow shops and pure job shops). We leave the exploration of these important issues to further study. [Received: June 24, 1994. Accepted: December 6, 1995.]

REFERENCES

[1] Baker, K.R. *Elements of sequencing and scheduling* (revised edition). Dartmouth College, NH, 1995.

[2] Baker, K.R., & Pyke, D.F. Solution procedures for the lot streaming problem. *Decision Sciences, 21*, 1990, 475-491.

[3] Dobson, G., Karmarkar, U.S., & Rummel, J.L. Single machine sequencing with lot sizing. Working Paper Series No. QM8419, Graduate School of Management, University of Rochester, March 1985.

[4] Dudek, R.A., Panwalkar, S.S., & Smith, M.L. The lessons of flowshop scheduling research. *Operations Research*, 1992, *40*(1), 7-13.

[5] Dudley, N.A. Work-time distributions. *International Journal of Production Research*, 1963, *2*(2), 137-144.

[6] Fox, B., Goldsman, D., & Swain, J. Spaced batch means. *Operations Research Letters*, 1991, *10*, 255-263.

[7] Graves, S.C., & Kostreva, M.M. Overlapping operations in material requirements planning. *Journal of Operations Management*, 1986, *3*(2), 283-294.

[8] Gross, D., & Harris, C.M. *Fundamental of queueing theory*. New York: John Wiley, 1985.

[9] Hancock, T.M. Effects of lot splitting under various routing strategies. *International Journal of Operations & Production Management*, 1991, *11*, 68-74.

[10] Jacobs, F.R., & Bragg, D.J. Repetitive lots: Flow-time reductions through sequencing and dynamic batch sizing. *Decision Sciences*, 1988, *19*(2), 281-294.

[11] Karmarkar, U.S., Kekre, S., & Kekre, S. Lotsizing in multi-item multi-machine job shops. *IIE Transactions*, 1985, *17*(3), 290-297.

[12] Karmarkar, U.S., Kekre, S., Kekre, S., & Freeman, S. Lot-sizing and lead-time performance in a manufacturing cell. *Interfaces*, 1985, *15*(2), 1-9.

[13] Kropp, D.H., & Smunt, T. L. Optimal and heuristic models for lot splitting in a flow shop. *Decision Sciences*, 1990, *21*(4), 691-709.

[14] Santos, C., & Magazine, M.J. Batching in single operation manufacturing systems. *Operations Research Letters*, 1985, *4*(3), 99-102.

[15] Szendrovits, A.Z. Manufacturing cycle time determination for a multi-stage economic production quantity model. *Management Science*, 1975, 22(3), 298-308.

[16] Trietsch, D. An efficient transfer lot sizing procedure for batch processing on several machines. Working paper, Naval Postgraduate School, 1987.

[17] Trietsch, D., & Baker, K. Basic techniques for lot streaming. *Operations Research*, 1993, *41*(6), 1065-1076.

[18] Wagner, B., & Ragatz, G. The impact on lot splitting on due date performance. *Journal of Operations Management*, 1994, *12*(1), 13-26.

[19] Zipkin, P.H. Does manufacturing need a JIT revolution? *Harvard Business Review*, 1991, *69*(1), 40-48.

Timothy L. Smunt is Babcock Research Professor and an associate professor of management at the Babcock Graduate School of Management of Wake Forest University. He received his B.S. in industrial management from Purdue University, MBA from the University of Missouri-St Louis, and DBA from Indiana University. Dr. Smunt has published widely in manufacturing technology evaluation, operations planning, cellular manufacturing, and learning curve analysis. He has published articles in a number of journals, including *Operations Research, Management Science, Journal of Operations Management, Institute of Industrial Engineering Transactions, Decision Sciences* and *Interfaces*. He is a senior member of IIE and an active member of the Decision Sciences Institute, INFORMS, and APICS, where he is a Certified Fellow in Production and Inventory Management (CPFIM).

Arnold Buss is currently in the Operations Research Department at the Naval Postgraduate School. He received his B.A. in psychology from Rutgers University, M.S. in systems engineering from the University of Arizona, and Ph.D. in operations research from Cornell University. He has published articles in *Operations Research, IIE Transactions, Mathematical Problems in Engineering, Production and Operations Management,* and the *Journal of Wind Engineering and Industrial Aerodynamics.* In addition to his work in lot splitting, he is engaged in research on stochastic PERT networks, management of bottlenecks, and the simulation of radar systems. He is a member of INFORMS, IIE, and POMS.

Dean H. Kropp is an associate dean and the Dan Broida Professor of Operations and Manufacturing Management at the John M. Olin School of Business, Washington University, St. Louis, Missouri. His teaching, research, and consulting activities are in the areas of production planning and inventory control, plant location, quality management, and manufacturing strategy. He has published articles in numerous journals, including *Management Science, Decision Sciences, IIE Transactions,* and *Journal of Operations Management.* Much of his research has examined ways to improve the effectiveness of production scheduling in a rapidly changing environment.