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# Measurement of the viscoelastic properties of water-saturated clay sediments.

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# THESIS

MEASUREMENT OF THE VISCOELASTIC PROPERTIES
OF WATER-SATURATED CLAY SEDIMENTS

by

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# MEASUREMENT OF THE VISCOELASTIC PROPERTIES OF WATER-SATURATED CLAY SEDIMENTS

by

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ABSTRACT

The complex shear modulus of both kaolin-water and bentonite-water mixtures has been determined in the laboratory. The method involved measuring the reaction on a torsional elastic wave propagated down a metal rod when the rod was immersed in sediment. Observations were made over the frequency range two to forty-three kHz. Dispersed sediments behaved like Newtonian liquids. Undispersed sediments, however, were viscoelastic in character, and their shear moduli exhibited no dependence on frequency. For undispersed kaolin mixtures, a typical result is $(21.6 + i\ 1.2) \times 10^4$ dynes per square centimeter at 32 percent concentration. In bentonite, a modulus of $(4.6 + i\ 2.0) \times 10^5$ dynes per square centimeter at 19 percent concentration is representative.

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1. Introduction

Knowledge of the acoustic properties of ocean sediment is important for both shallow water sound propagation and the bottom bounce mode of sonar operation.

Because the exact character of the ocean bottom is largely unknown, most physical models assume it to be a multilayered liquid. Bottom loss prediction, then, requires solving the relatively simple boundary value problem of a compressional wave incident on a liquid-liquid interface.

Such liquid bottom models, however, often yield reflection loss predictions which disagree with experiment (1). In some cases, better agreement is achieved by assuming a viscoelastic model for bottom loss prediction.

Although the boundary value problem is then more complex, with both a compressional and a shear wave transmitted, it is solvable. Therefore, useful results could be obtained from a viscoelastic model if the shear modulus as well as the viscosity of the sediment were known.

However, it is difficult to determine these parameters in an independent way. Longitudinal sound propagation studies do not provide values of the shear modulus. Moreover, the only other direct propagation method, the Stonely wave technique used by Bucker (1), is limited to low frequencies.

Research conducted by Lt. Hutchins at this institution demonstrated the feasability of measuring the shear modulus and viscosity of simulated ocean sediment. Values of these quantities were obtained over the narrow frequency range, 38.3 to 38.9 kHz.

The purpose of the research here reported was twofold: first, to extend the range of measurement down to those frequencies of interest in sonar applications, and second, to study a variety of simulated sediments over this broader range.

## 2. Theory and Method of Measurement

### (a) General

The measurement method involves propagating a torsional wave down a cylindrical metal rod imbedded in sediment. Measuring the reaction to the shear wave generated in the sediment by the twisting rod surface allows determination of the sediment's shear modulus and viscosity. Two types of propagation are used, travelling waves (pulse method) and standing waves (resonance method). For both types, the shear motion is generated by a piezoelectric torsional oscillator mechanically coupled to a stainless steel rod.

Following McSkimim's hypothesis (13), plane shear waves of the form $U_x = U_0 e^{-\alpha r} e^{i(\omega t \pm kz)}$ progagating into an infinite medium are assumed. The x direction is parallel to the circumference of the rod; the z direction is vertical.

If the sediment were solid, its shear modulus would by given by: $G = \tau / S$, where $\tau$ is shear stress, and S is shear strain. In a liquid sediment, the shear viscosity coefficient would be $\eta = \tau / \dot{S}$ .

Assuming a viscoelastic sediment, both G and $\eta$ are complex (11).

$$G_c = G_1 + i G_2$$
$$\eta_c = \eta_1 - i \eta_2$$

$G_2 = 0$ in a lossless solid; $\eta_2 = 0$ in a Newtonian liquid. Because for plane simple harmonic waves, $\dot{u}_x = i \omega u_x$,

$$\eta_c = \tau / i \omega S = -i G_c / \omega$$

Hence, $G_1 = \omega \eta_2$, $G_2 = \omega \eta_1$.

The specific acoustic load impedance of the sediment on the rod, Z, is given by $Z = \tau / \dot{u}_x$, and for plane shear waves is (13).

$$Z = \sqrt{\pi f \rho_{SED} \eta} \, (1 + i) = R + i X$$

Where R and X are the specific acoustic resistance and reactance of the sediment.

Substitution $\eta_c$ for $\eta$, and separating real and imaginary parts, yields $\eta_1 = 2RX / \omega \rho_{SED}$ $\quad \eta_2 = R^2 - X^2 / \omega \rho_{SED}$

whence, $G_1 = R^2 - X^2 / \rho_{SED}$ $\quad G_2 = 2RX / \rho_{SED}$

Thus, determining shear moduli and viscosities reduces to measuring R and X.

(b) Pule Method

In this method devised by McSkimin (13), a pulse train, propagated down the rod, is reflected from the rod's free ends. In travelling the length of the rod, each pulse is attenuated and undergoes a phase shift. Imbedding the rod in sediment introduces a mechanical load impedance at the surface of the rod, and there results additional attenuation and phase shift.

Letting: $\Delta A$ = total change in attenuation in nepers.

$\Delta B$ = total change in phase in radians, it is shown (equation A3 of the appendix), that

$$Z = \frac{\rho_{ROD} V_0 a}{4 n l} (\Delta A - i \Delta B) = R + i X$$

where $V_0$ = the velocity of shear propagation in the unloaded rod.

a = the radius of the rod.

n = the number of the echo observed.

l = the length of rod covered by sediment.

$\Delta A$ may be computed from the change in attenuation in db, $\Delta\alpha$, between the unloaded and the sediment imbedded rod. Similarly, $\Delta B$ may be found from the change in propagation speed of the torsional wave as measured using an interference method. The quantity actually measured is $\Delta f$, the change in frequency necessary to bring the echo and transmitted pulse back into phase.

From equations A4 and A6 of the appendix:

$$\Delta A = \frac{\Delta\alpha}{8.68}, \quad \Delta B = \frac{4\pi n l_0}{V_0} \Delta f$$

, where $l_0$ is the length of the rod.

(c) Resonance Method

In this method, based upon the work of Mason (10), the same torsional transducer - rod combination is driven in a continuous wave mode, exciting standing waves on the rod. The rod-transducer system is represented by an equivalent electrical circuit consisting of a capacitance, C, in parallel with a resistance $R_E$ (or conductance $G_E$). Resonance, when the rod has a true standing wave, is the frequency of maximum $G_E$.

At each resonance mode, input electrical conductance, is measured, first in air, and then with sediment loading. This allows calculation of the change in equivalent parallel electrical resistance at resonance, $\Delta R_E$. The change in resonance frequency for each mode, $\Delta f$, is also recorded. Mason has shown that (10):

$$\Delta R_E = K_1 R_T \quad \text{and} \quad \Delta f = K_2 X_T$$

where $K_1$ and $K_2$ are constants which are functions of supposedly fixed physical parameters. These constants are determined from measurements on Newtonian fluids of known viscosity. (In a Newtonian fluid, $\eta_2 = 0$: therefore, $R_T = X_T$). $R_T$ and $X_T$ are the total impedance presented to the rod by the sediment.

Then: $$R = R_T/l \qquad X = X_T/l$$

where $l$ is the length of rod covered by sediment.

3. Equipment and Procedures

(a) Equipment

The driving force for torsional oscillations used in this experiment was one of four hollow barium titanate cylinders, having resonant frequencies of 14.2, 18.7, 28.9, and 38.3 kHz. (See Appendix B). Each cylinder was cemented to a stainless steel base, ½ inch long, and of the same diameter as the crystal. Each base had a ¼ inch diameter, ½ inch long exposed screw thread machined at one end (see Figure 1).

Two ½ inch stainless steel cylindrical rods were machined to accept this screw thread; therefore, a tight mechanical joint allowed a transducer - rod combination to execute torsional oscillations. Stainless steel was used because it does not corrode when immersed in sediment for long periods.

Both rods were polished on a lathe with 320 and 600A abrasive paper. This was followed by further smoothing with rouge and a buffing wheel, and finally with hand polishing.

One rod was solid, one meter in length, and squared at its free end. The other was partially hollow, nineteen centimeters in length, and tapered at one end, (see Figure 3). Because coupling between transducer and rod must be constant, one transducer was mechanically coupled to each rod for the duration of the experiment. The two inch, ½ inch diamter transducer was used with the long rod, while the short rod was driven with a 1½ inch, ½ inch diameter transducer.

The long rod was supported top and bottom by needle pivots inside a 6.8 cm inner diameter water jacket. The bottom pivot rested in a retractable slide secured to a brass plate with a locking pin. The plate was in turn fastened to the jacket with machine screws. This retractable slide facilitated placing the rod on the bottom pivot. "O" rings prevented leakage at the two metal to metal junctions (see Figure 2).

The short tapered rod, suspended from a stand by fine wire, hung within another water jacket, 21 cm long and of 15.3 cm inner diameter.

Both jackets were wrapped in fiberglass to prevent heat loss. Additionally, a removable cover was made for each to reduce convection. Water circulated in both jackets from a 24.5$^{o}$C thermally controlled bath; the larger jacket maintained a 24.5 $\pm$ .05$^{o}$C temperature and the smaller jacket one of 24.1 $\pm$ .05$^{o}$C. Jacket temperature was monitered with a thermocouple.

The pulse method equipment setup is shown in Figure 4. The sinusoidal signal from the oscillator is connected in parallel to a frequency counter and to a tone burst generator. Providing a phase reference for cancellation, this signal is also directed through the attenuator to the oscilloscope. The 600 ohm resistive attenuator is controlled in 0.1 db steps, while the five decade counter is operated on a ten second gate.

The tone burst generator directs two signals to the input gate. First is an eight or sixteen cycle sinusoidal main pulse, accompanied by a -40 db ripple during the pulse-off time. Second is a gating signal whose polarity changes with generator switching.

The input gate attenuates the ripple voltage to 84 db below the main pulse. This is sufficient attentuation to prevent masked echoes (see Appendix C).

After being driven by the amplified main pulse, the transducer receives echoes from the rod; these echoes, in turn, enter the clipping circuit. The pulse repetition rate in the tone burst generator is sufficiently low to allow echo train decay to below the noise level.

The clipping circuit clips the main pulse to the level of the echo being observed, This level is set on the variable power supply. Thus, main-pulse-caused oscilloscope saturation is prevented.

Operated with a differential amplifier plug-in unit, the dual beam oscilloscope is used to display, depending upon the switch selected, the echo train, the attenuated direct sinusoid, or the difference of these two signals. Its main sweep triggered by the tone burst generator, the oscilloscope has a delayed sweep permitting detailed observation of a single echo.

The resonance method equipment setup is shown in Figure 6. The sinusoidal signal generated by the frequency synthesizer is amplified and connected in parallel to the counter and the admittance meter. The five decade counter is again set for a ten second gate.

The transducer is connected directly to the complex admittance meter. This meter reads the input electrical conductance and susceptance of the transducer-rod system.

Regulated line voltage to all equipment is supplied by a constant voltage transformer. This regulation is necessary to reduce both overall noise level and frequency drift. The CENCO oscillator used for the pulse method drifts approximately 3 parts in $10^5$ per hour. On the other hand, the frequency synthesizer output is stable to 0.1Hz over a period of several weeks.

Ordinary sound speed in the samples was measured with a previously existing apparatus consisting of (1) two barium titanate disk-transducers, one of which operated as a transmitter, the other as a receiver, (2) an oscilloscope, (3) a pulsed oscillator, (4) a time-mark generator, and (5) an amplifier, (see Figure 7).

Time marks, used to trigger the pulsed oscillator, are displayed on one channel of a dual beam oscilloscope. On the other channel is displayed the amplified pulse received by the hydrophone.

By measuring time delays from transmitter to receiver, first with the transducers in 25°C distilled water, and then in sediment, relative longitudinal sound speed is determined. The sound frequency was 0.95MHz.

(b) Procedure.

Because travel time between echoes must exceed pulse length, only the long rod was used for the pulse method. Following Prather (14), the third, fourth, and fifth echoes were observed. Additionally, as suggested by Hutchins (7), measurements were confined to the vicinity of transducer resonance, namely 23 to 32 kHz.

Frequency and attentuation necessary for center pulse cancellation were recorded, first in air, and then with the rod immersed in each of four Newtonian fluids. Ethylene glycol, cottonseed oil, and medium and heavy lubricating oil were used. Calculations were done by computer (see Appendix D, program PLSCAL). Approximate agreement between pulse-method measured viscosities and those measured with a Brookfield viscometer insured smoothness of the rod.

However, the pulse method was found to be accurate over a narrower frequency range than originally anticipated, namely 5kHz. It was, therefore, abandoned in favor of the resonance method.

For the resonance method, both rods were driven in a standing wave mode, and resonance frequencies as well as input electrical conductances at resonances were recorded. Measurements made in air and in the four above mentioned Newtonian fluids, together with Brookfield-viscometer-values of real viscosity, permitted calculation of $K_1$ and $K_2$ for both rods. (See Appendix D, program KCAL.) The K values obtained as functions of frequency with each fluid were averaged to obtain the final values listed in Tables 1 and 2.

For both rods, $K_1$ was not frequency independent. In addition, to being a function of fixed parameters such as length and radius of the rod, $K_1$ depends upon transducer electromechanical coupling. At rod-transducer resonances away from the transducer's fundamental resonance, end effects occur. These end effects are caused by a non-uniform polarization field and slight assymmetry in electrode placement; in turn, end effects cause coupling variation.

| Frequency in kHz | $K_1$ | $K_2$ |
|---|---|---|
| 2.9 | 888. | .000643 |
| 5.8 | 58.9 | .000626 |
| 8.7 | 12.1 | .000569 |
| 10.1 | 6.86 | .000567 |
| 13.0 | 2.68 | .000567 |
| 14.3 | 2.28 | .000623 |
| 17.2 | 1.65 | .000660 |
| 20.1 | 1.28 | .000685 |
| 23.0 | .992 | .000558 |
| 25.9 | .800 | .000596 |
| 28.8 | .855 | .000554 |
| 31.7 | .952 | .000531 |
| 34.6 | 1.10 | .000542 |
| 37.5 | 1.17 | .000651 |
| 40.4 | 1.21 | .000710 |
| 43.2 | 1.93 | .000799 |
| 45.9 | 3.55 | .000645 |

TABLE 1. Resonance Method: Average Values of $K_1$ and $K_2$ for the One Meter Rod

| Frequency in kHz | $K_1$ | $K_2$ |
|---|---|---|
| 5.6 | 248. | .00550 |
| 12.8 | 17.7 | .00563 |
| 18.7 | 1.48 | .00323 |
| 24.1 | 2.20 | .00531 |
| 42.8 | 1.16 | .00374 |
| 49.5 | 3.16 | .00554 |

TABLE 2. Resonance Method: Average Values of $K_1$ and $K_2$ for the Nineteen Centimeter Rod

Thus, while constant at a specific frequency, $K_1$ was not constant over the range of interest. $K_2$ also exhibited variation with frequency, although not as severe as that of $K_1$. However, the criterion of no variation in $K_1$ and $K_2$ at a particular frequency was met.

Standing waves were excited over the entire short hollow rod at 5.6, 18.7, 31.7, and 42.8 kHz. For those resonances occurring at 12.8, 24.2, 37.9, and 46.9 kHz, standing waves appear primarily over the solid upper portion. In all, this solid-hollow-solid rod is a complicated vibratory system. The resonances at 31.7 and 37.9 kHz exhibited anomalous behavior, and were not used for further measurement.

(c) Sediment Preparation

A variety of clays were used in this experiment. Two were kaolin, N.F. (aluminum disilicate), a major constituent of shallow water sediment. Of these two, one was in colloidal powder form. Grain size analyses of these kaolin clays, performed by the Naval Civil Engineering Laboratory, Port Hueneme, California, appear as Figures 8 and 9.

The other two clays were primarily bentonite which, unlike kaolin, is expansive. Specifically, these two were "Fuller's earth", for which a grain size analysis appears as Figure 10, and "Quik-Gel", a commerical bentonite of greater purity than Fuller's earth, and marketed by Baroid division of the National Lead Company.

All sediments were prepared in the same manner. Distilled water was added to a known weight of dry material. After stirring to produce homogeneity, the mixture was boiled until it appeared outgassed. Then the mixture was weighed, and the percentage concentration of solids by weight was obtained. Finally, the mixture was transferred to one of the water-jacketed pipes while still warm.

Kaolin mixtures were transferred to the long pipe, and were poured around the rod. Pushing the long rod, with its blunt end and bottom pivot support, into already poured sediment led to unrepeatable observations. Bentonite mixtures were poured into the short jacket. The short tapered rod was then pushed into the sediment. Because of its taper and overhead suspension, the short rod could be so inserted.

As the approximate volume of the jackets was known, approximate densities and porosities could be calculated. Porosity is the percentage of voids per unit volume, and is given by:

$$P = \left[ \frac{\rho_{SED}}{\rho_{WATER}} - \frac{\text{WEIGHT OF SOLIDS}}{(\rho_{WATER}) \times (\text{VOLUME OF SEDIMENT})} \right] \times 100\%$$

The sediment was allowed to settle under gravity, and clear water, when it appeared, was removed on a daily basis. Measurements were taken on alternate days, starting one or two days after the sediment was poured. The computer was again used for all calculations (see Appendix D, programs RESCAL and MELES).

Finally, after removing the sediment from its jacket, its longitudinal sound speed was measured. Then, a sample of known volume was taken, weighed and dried to allow a more exact determination of concentration, density, and porosity.

4. Results and Conclusions

No sediment here observed was completely stable; the height of any particular column of sediment decreased with time. However, in only certain mixtures was this decrease either greater than one percent or accompanied by collection of clear water at the top. In the below discussion, those sediments where this decrease was small will be referred to as stable.

The colloidal powder kaolin and water mixtures were stable, and appeared watery. In all cases, these sediments had a negligible $G_1$; moreover, as shown in Figure 11, a typical result, $G_2$ increased linearly with frequency. As such, within experimental accuracy, colloidal powder sediments appeared to be Newtonian fluids.

As $G_2 = \omega \eta_1$, $\eta_1$ was determined by fitting a least squares line, required to pass through the origin, to a plot of $G_2$ vs $\omega$ (see Appendix D, program MELES). Viscosity values, determined by least squares fit to 17 data points over the range 2.9 to 43.2 kHz, appear in Table 3. Viscosity values obtained by direct averaging agree with those determined by fitting. The standard deviation of each set of 17 viscosity measurements used in averaging is also shown in this table.

Non-colloidal kaolin sediment was measured in pure form (kaolin and distilled water) and with additives. Results appear in Table 4. In pure sediment, values obtained for both $G_1$ and $G_2$ exhibited no trend with frequency. As such, it appears that both shear moduli are, within experimental accuracy, frequency invariant. However, as can be seen from the large standard deviations, $G_2$ values were extremely scattered; $G_1$ values were in general less spread. A typical set of data points is plotted in Figure 12.

Because this mud-like non-colloidal kaolin sediment exhibited much greater damping than the colloidal, rod resonances at frequencies

| Average Concentration % by Weight | Density gms/cc | Prosity % | $c_{SED}/c_{WATER}$ | Settling Time Days | Mean Viscosity Poise | Standard Deviation Poise |
|---|---|---|---|---|---|---|
| 45.3 | 1.28 | 70 | --- | 2 | .086 | .028 |
| 45.3 | 1.28 | 70 | .98 | 3 | .087 | .028 |
| 48.3 | 1.38 | 67 | --- | 2 | .213 | .060 |
| 48.3 | 1.38 | 67 | --- | 3 | .211 | .058 |
| 48.7 | 1.39 | 67 | --- | 4 | .194 | .052 |

TABLE 3. Measured Real Viscosities: Kaolin N.F., Colloidal Powder and Water at 24.5°C. Values are the average of 16 measurements over the frequency range 2.9 to 43.2 kHz.

| Additives | Average Concentration % by Weight | Density gms/cc | Prosity % | $c_{SED}/c_{WATER}$ |
|---|---|---|---|---|
| None | 31.4 | 1.24 | 86 | --- |
| | 31.9 | 1.24 | 86 | --- |
| | 32.1 | 1.26 | 86 | --- |
| | 32.1 | 1.28 | 86 | .99 |
| None | 39.1 | 1.31 | 81 | --- |
| None | 44.4 | 1.35 | 75 | .97 |
| Calgon: 0.8 gms/liter | 31.0 | 1.21 | 84 | --- |
| | 31.0 | 1.22 | 84 | --- |
| | 31.0 | 1.22 | 84 | .98 |
| Calgon: 0.7 gms.liter. | 31.7 | 1.26 | 85 | --- |
| Table Salt: | 32.0 | 1.27 | 85 | --- |
| 31.5 parts per thousand by weight | 33.6 | 1.27 | 85 | .99 |
| Sand: 1/3 dry | 28.8 | 1.22 | 88 | --- |
| weight; or, | 31.8 | 1.25 | 86 | --- |
| 9.7% of wet | 33.8 | 1.27 | 84 | --- |
| sediment weight | 35.0 | 1.29 | 84 | .98 |

TABLE 4(a). Measured Shear Moduli: Non-Colloidal Kaolin and water at 24.5°C. Values are the average of 14 measurements over the frequency range 8.6 to 43.2 kHz.

| Settling Time Days | $G_1$ $10^4$ Dynes per $cm^2$ | Standard Deviation | $G_2$ $10^4$ Dynes per $cm^2$ | Standard Deviation |
|---|---|---|---|---|
| 1 | 8.00 | .97 | 1.06 | .81 |
| 3 | 9.98 | 1.17 | .89 | .76 |
| 5 | 21.6 | 1.9 | 1.20 | .78 |
| 7 | 26.6 | 3.4 | 2.85 | 1.20 |
| 2 | 54.8 | 7.4 | 9.68 | 4.36 |
| 2 | 67.9 | 14.5 | 16.6 | 6.9 |
| 1 | .074* | .036¢ | | |
| 3 | .089* | .032¢ | | |
| 5 | .065* | .020¢ | | |
| 2 | 14.6 | 1.7 | .75 | .72 |
| 4 | 19.9 | 3.0 | 1.26 | .76 |
| 6 | 26.6 | 3.7 | 2.35 | 1.44 |
| 2 | 3.47 | 1.70 | 3.79 | 1.22 |
| 4 | 13.6 | 2.5 | 3.62 | 1.37 |
| 6 | 9.58 | 1.59 | 3.08 | 1.06 |
| 8 | 13.7 | 2.0 | 2.90 | 1.07 |

TABLE 4(b)

* Average measured real viscosity in poise

¢ Standard deviation of viscosity measurements

| Sediment Composition | Average Concentration % by weight | Density gms/cc | Porosity % |
|---|---|---|---|
| "Fuller's Earth" and Water | 14.0 | 1.06 | 95 |
| | 14.9 | 1.09 | 94 |
| | 14.9 | 1.10 | 94 |
| "Fuller's Earth" and Water | 19.0 | 1.09 | 92 |
| | 19.0 | 1.12 | 92 |
| | 19.0 | 1.13 | 91 |
| "Fuller's Earth" and Water | 31.8 | 1.09 | 75 |
| "Quick-Gel" | 11.1 | 1.06 | 95 |
| | 11.1 | 1.06 | 95 |
| | 11.1 | 1.07 | 95 |
| | 11.1 | 1.07 | 95 |

TABLE 5(a) Measured Shear Moduli: Bentonite Sediments at 24.1°C

| $c_{SED}/c_{Water}$ | Settling Time Days | Frequency kHz | $G_1$ $10^5$ Dynes per $cm^2$ | $G_2$ $10^5$ Dynes per $cm^2$ |
|---|---|---|---|---|
| --- | 1 | 18.7 | .08* | |
| | | 24.1 | .04* | |
| | | 42.8 | .04* | |
| --- | 3 | 18.7 | .10* | |
| | | 24.1 | .07* | |
| | | 42.8 | .07* | |
| .99 | 5 | 18.7 | .20* | |
| | | 24.1 | .10* | |
| | | 42.8 | .08* | |
| --- | 1 | 18.7 | 1.5 | .55 |
| | | 24.1 | 1.5 | .27 |
| | | 42.8 | 1.0 | .29 |
| --- | 3 | 18.7 | 4.6 | 2.0 |
| | | 24.1 | 4.7 | 1.4 |
| | | 42.8 | 5.3 | 1.9 |
| .99 | 5 | 18.7 | 3.9 | 2.5 |
| | | 24.1 | 4.8 | 1.6 |
| | | 42.8 | 5.2 | 1.4 |
| .96 | 1 | 18.7 | 24. | 10. |
| | | 24.1 | 16. | 7.7 |
| | | 42.8 | 21. | 11. |
| --- | 1 | 18.7 | 2.2 | 2.1 |
| | | 24.1 | 1.9 | 1.8 |
| | | 42.8 | 2.3 | 1.1 |
| --- | 3 | 18.7 | 2.4 | 2.6 |
| | | 24.1 | 1.9 | 1.9 |
| | | 42.8 | 2.4 | 2.2 |
| --- | 5 | 18.7 | 2.0 | 2.9 |
| | | 24.1 | 1.8 | 2.1 |
| | | 42.8 | 2.0 | 3.2 |
| .98 | 7 | 18.7 | 2.3 | 3.6 |
| | | 24.1 | 2.2 | 2.6 |
| | | 42.8 | 2.5 | 3.5 |

TABLE 5(b)

*Measured real viscosity in poise

far from transducer resonance were of too low a conductance to permit measurement. Hence, the frequency range was narrowed to 8.6 to 43.2 kHz.

When the deflocculating agent, Calgon, was added to non-colloidal sediment, in the amount 0.8 grams per liter, the mixture was dispersed. It then appeared watery and became more stable. As in the case of colloidal sediments, measurement indicated the mixture was a Newtonian fluid.

Adding 35.5 parts per thousand by weight of table salt to this deflocculated mixture again caused it to be mud-like and unstable. Shear moduli were similar in value to those obtained before dispersal, and agin did not vary with frequency.

By adding distilled water to a mixture two parts kaolin to one part sand by weight, the last sediment shown in Table 4 was formed. The sand used was uniform; 99% of the particles were between .59 and .84 millimeters in diameter. This sediment was highly unstable; data taken hours, vice days, apart showed shear moduli variation. Tabulated values are the average of several measurements taken over a three hour period. Again, this sediment was mud-like, and its moduli appeared, within experimental accuracy, frequency independent.

All bentonite sediments were comparatively stable after settling for two days. With one exception, they were extremely mud-like and formed rheopectic gels. With their larger moduli, they exerted great damping on the short rod; therefore, measurements were confined to three frequencies relatively close to transducer resonance.

Results appear in Table 5. There, shear moduli are seen to be larger than for kaolin. "Quik-Gel", at low concentration, yielded the same order of magnitude moduli as higher concentration Fuller's earth.

The one low concentration "Fuller's earth" sediment measured appeared to be watery, stable, and to be a Newtonian fluid. Thus, measured viscosity vice shear modulus is tabulated.

5. Discussion of Error and Limitations

For the long rod (kaolin sediment), error in $G_1$ was 10%, and error in $G_2$ was 25%. This error was determined from reproducibility of observed values. That is, using average K values, the viscosity of the four Newtonian liquids was measured. Values obtained, when compared to known values, yielded the above figures.

The variation leading to these errors results from several practical limitations. First, stainless steel, although non-corrosive, is relatively insensitive to loading. Moreover, it has a temperature dependent shear modulus. Use of a less dense steel alloy with a smaller temperature coefficient would improve accuracy. Sensitivity could be further improved by using a completely hollow rod, polished inside as well as outside.

Second, because the rod transducer system was of high electrical susceptance, the admittance meter often saturated at frequencies greater than 25 kHz. To prevent meter saturation, conductance values had to be read on a less than optimum scale; in turn, this led to difficulty in determining resonance frequency. This difficulty could be overcome by designing a conductance meter such that its performance would be independent of susceptance.

Fortunately, however, most sediments here observed offered sufficient damping to lower Q's to values on the order of several hundred. Therefore, $G_E$ was correctly measured in spite of frequency uncertainty. Hence, the larger sediment resistance was more precisely determined than the smaller reactance, and $G_1$ was less uncertain than $G_2$.

In addition to these practical limitations, short rod measurements were also limited by sediment settling. Because the short rod was not of uniform cross-section, the length covered by sediment was critical. During calibration, the rod was always covered to the same length, and K values were consistent. However, because sediments settle, this same length of rod was not always covered by sediment, and moduli obtained are only accurate to 35%.

This resonance method is also limited by validity of the assumption of plane waves propagating into an infinite medium.

Assuming a solid model, the shear wave length in sediment, $\lambda$, is given by:

$$\lambda \doteq \frac{1}{f}\sqrt{G/\rho_{SED}}$$

At 10 kHz, near the lower end of the range of interest, using the density of water as an approximation to sediment density, $\lambda$ is approximately $10^{-4}G^{\frac{1}{2}}$. For $G = 10^6$, the largest value here reported, $\lambda$ is approximately 0.1 cm. The radius of curvature of the rods was approximately 0.6 cm. Since for plane waves, $\lambda$ should be much less than the radius of curvature, this method is inapplicable for shear moduli greater than $10^6$. However, the plane wave assumption is more valid either at higher frequencies or for lower order of magnitude moduli.

On the other hand, the infinite medium assumption was completely valid. The attenuation coefficient, $\alpha$, for plane shear wave is given by Mason (10) as:

$$\alpha = \sqrt{\omega \rho_{SED}/2\eta}$$

At 1 kHz, using the density of water which is the lower limit of sediment density, and using 3 poise, the upper limit of measured viscosity, $\alpha$ is on the order of 30 nepers/cm. Equivalently, the shear wave is attenuated to 1/e of its initial value in .03 cm, a distance much less than the inner radius of either jacket.

Although kaolin and bentonite are major constituents, real ocean sediments are highly impure. As seen here, additives may radically change sediment acoustic properties. Therefore, the figures here reported are in no way meant to represent those found for real sediment.

For comparison, Hamilton (4) and Bucker (1) reported moduli between $10^8$ and $10^{11}$ dynes/$cm^2$ from calculations based on real sediment parameters. However, calculations of longitudinal wave attenuation, based upon the shear modulus values obtained here, consistently agree in order to magnitude with those measured in apparently similar kaolin sediments by Hampton (5).

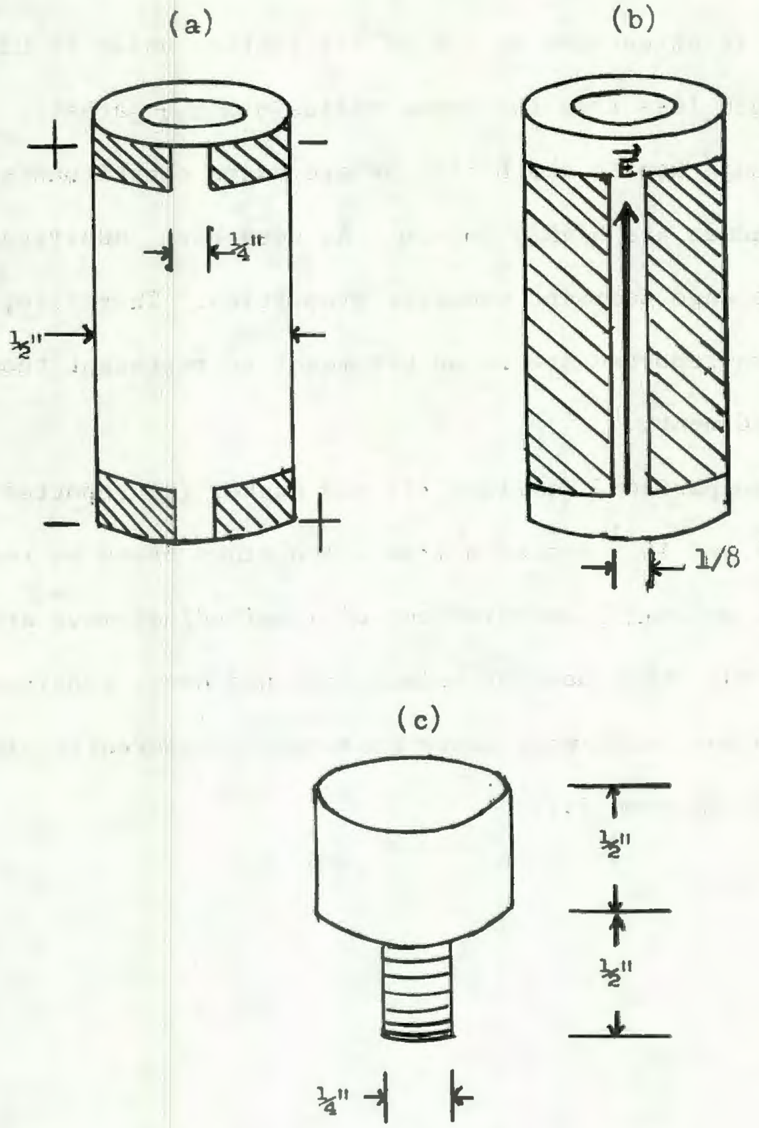


Figure 1. Barium Titanate Transducer Electrode Placement.
(a) Polarization (b) Driving
(c) Stainless Steel Threaded Support Stand

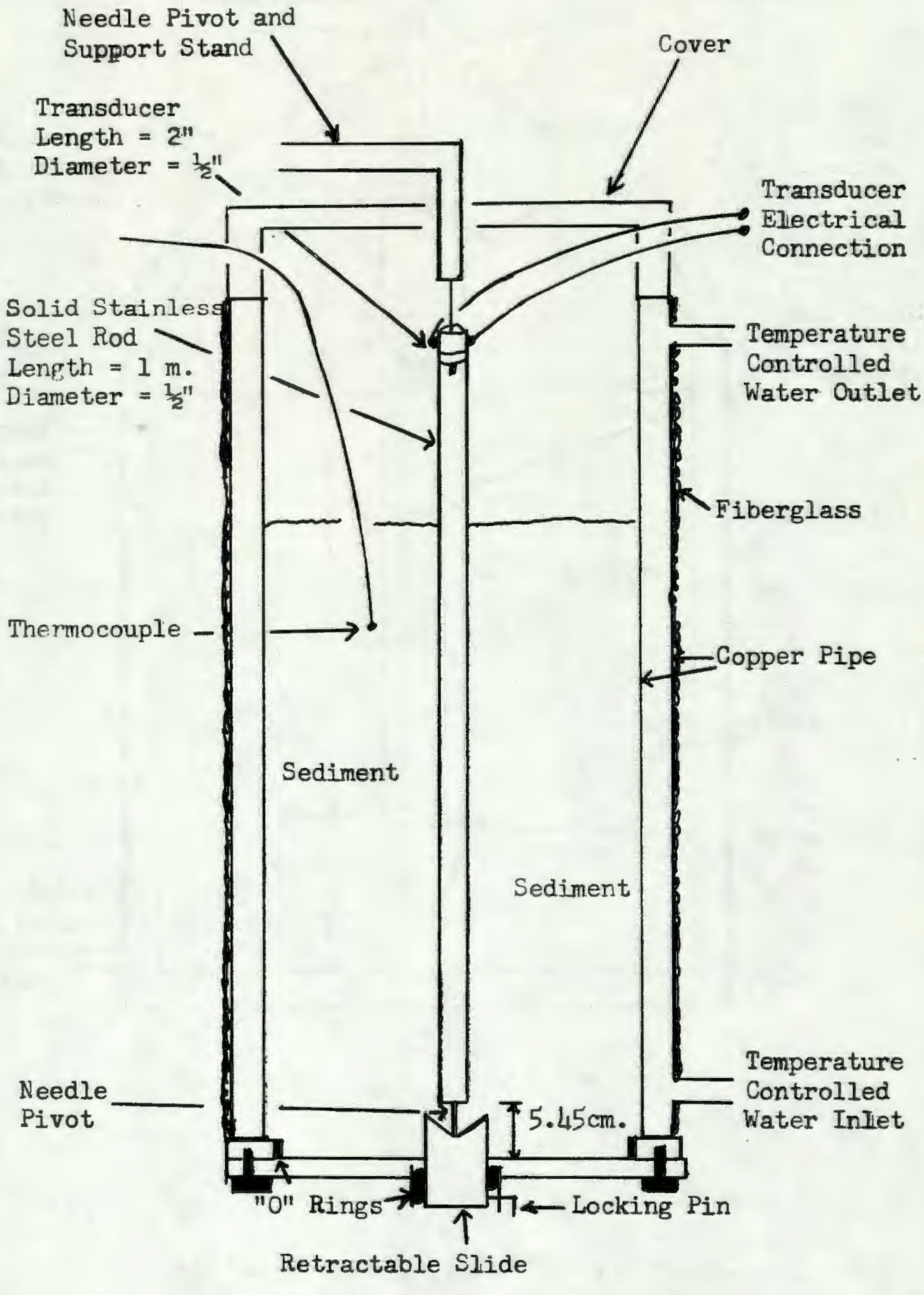


Figure 2. Cutaway View of Long Water-Jacketed Pipe and Transducer-Rod Combination

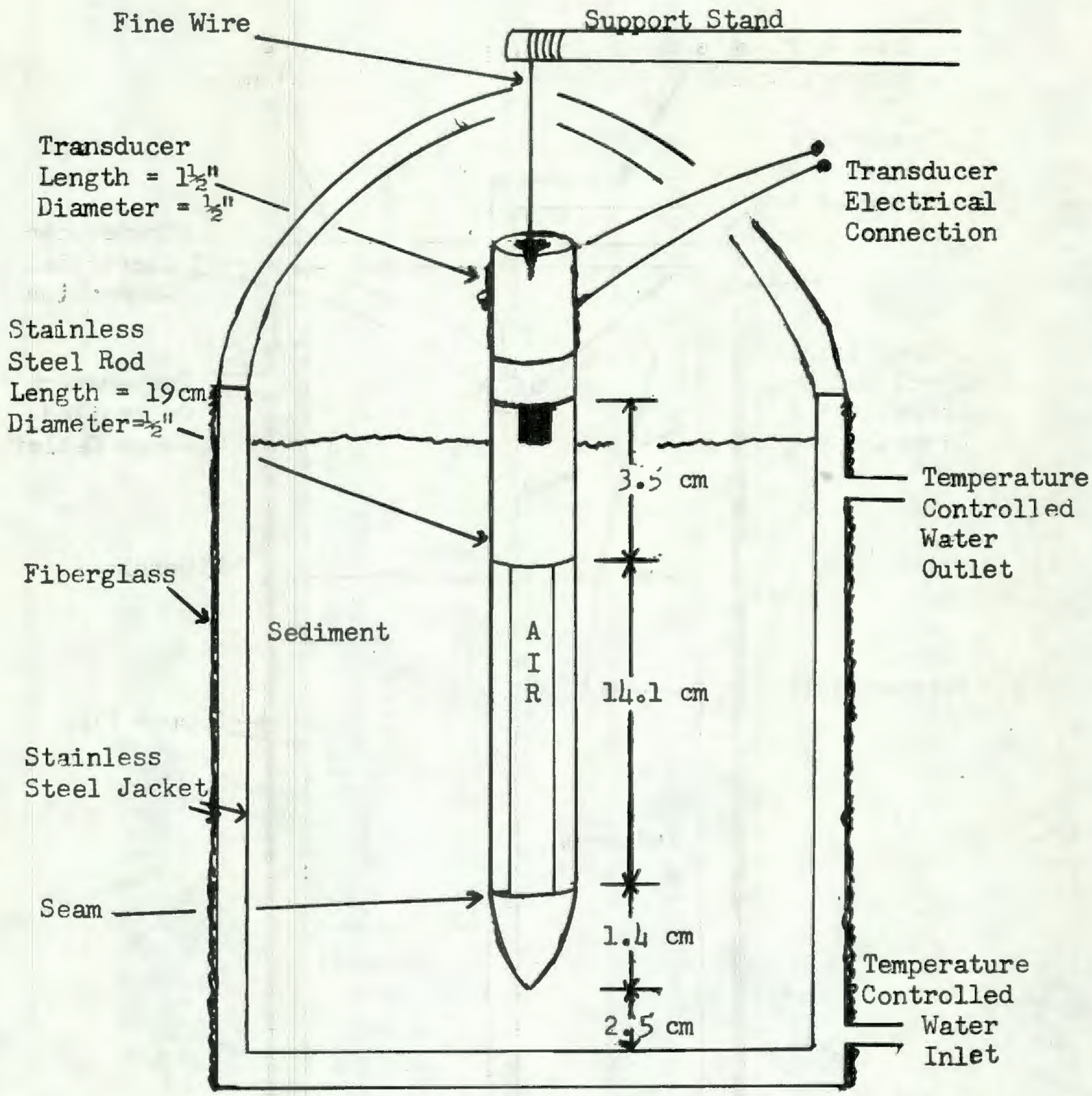


Figure 3. Cutaway View of Short Water-Jacketed Pipe and Transducer-Hollow Rod Combination

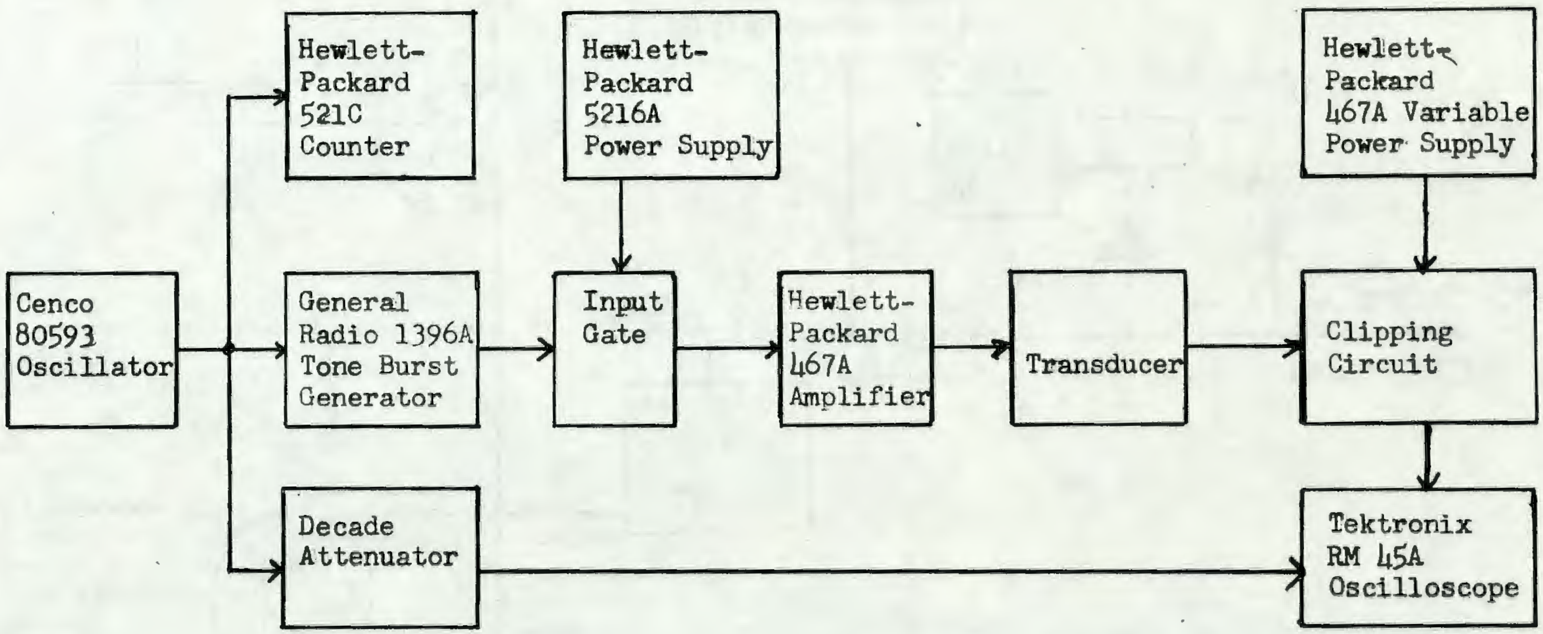


Figure 4. Block Diagram of Epuipment for the Pulse Method

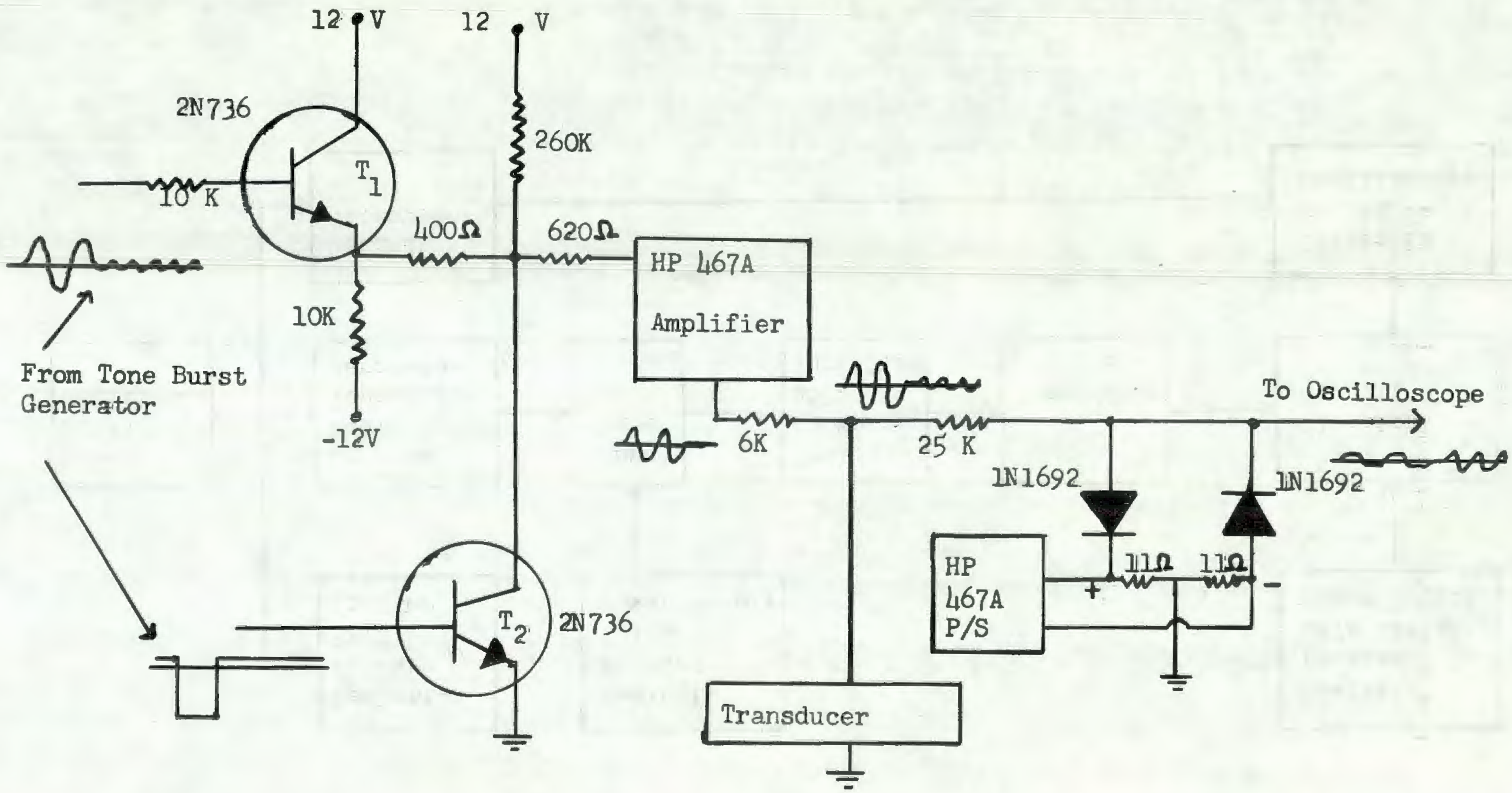


Figure 5. Pulse Method: Input Gate and Clipping Circuit: Schematic

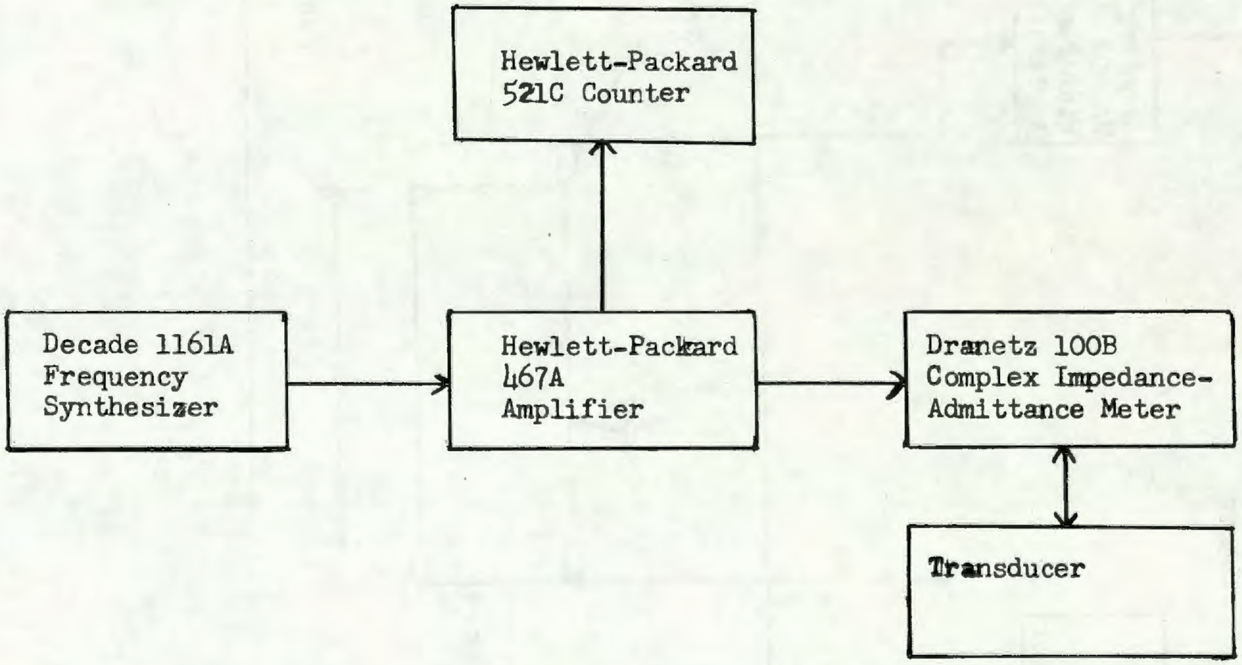


Figure 6. Block Diagram of Equipment for the Resonance Method

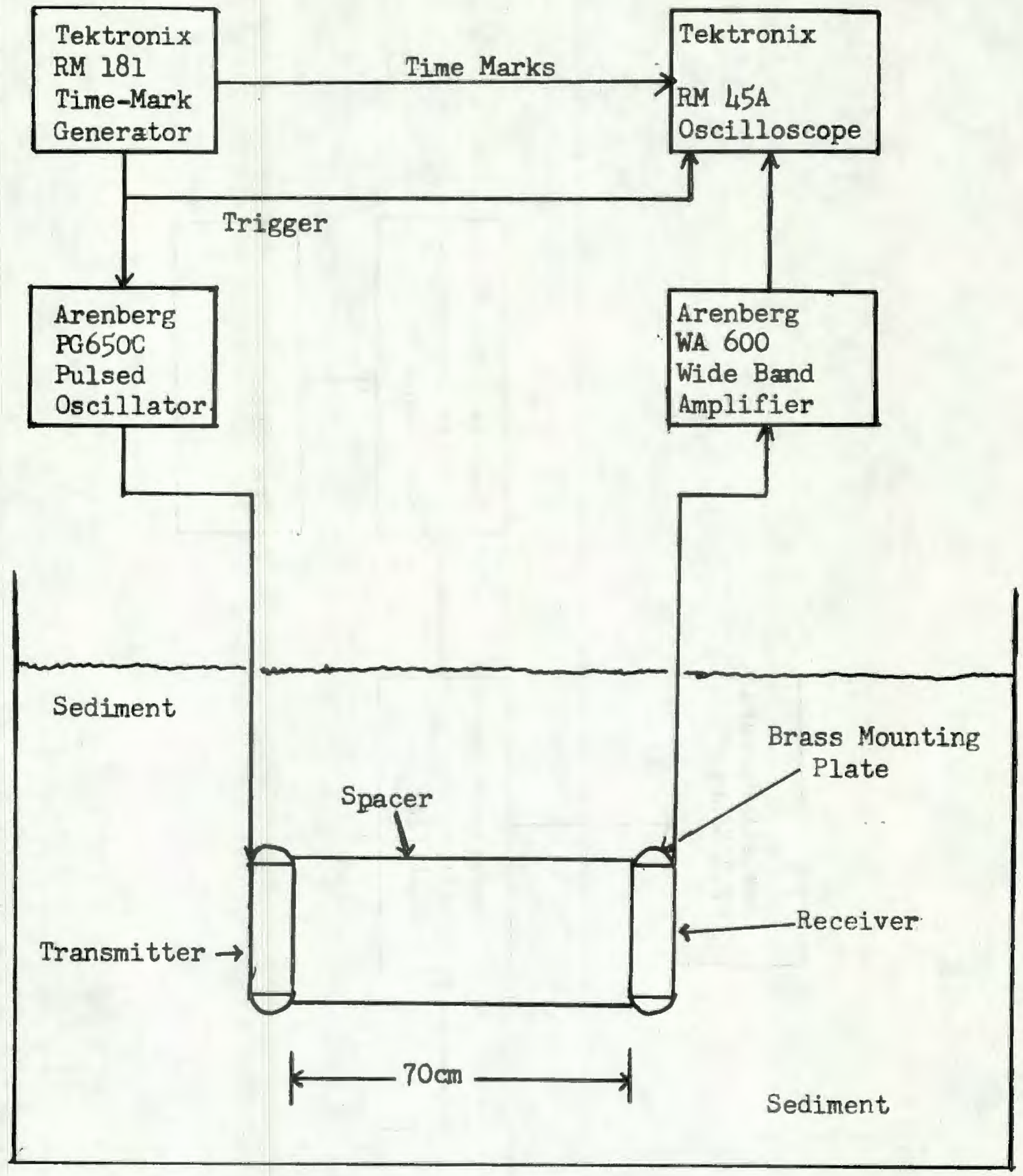


Figure 7. Apparatus for Longitudinal Sound Speed Measurement

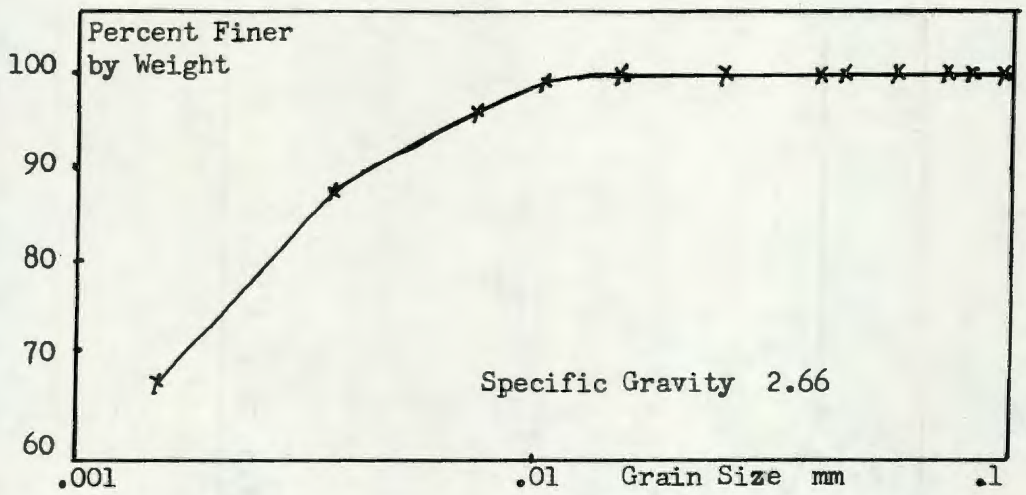


Figure 8 Grain Size Analysis of Kaolin N. F., Colloidal Powder: Performed by NCEL

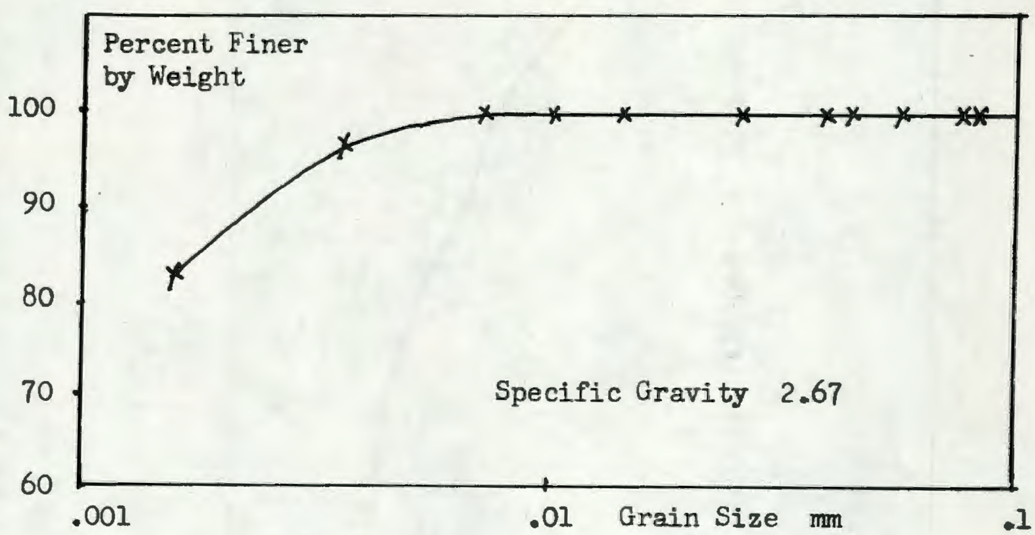


Figure 9. Grain Size Analysis of Kaolin, N. F. (non-colloidal): Performed by NCEL

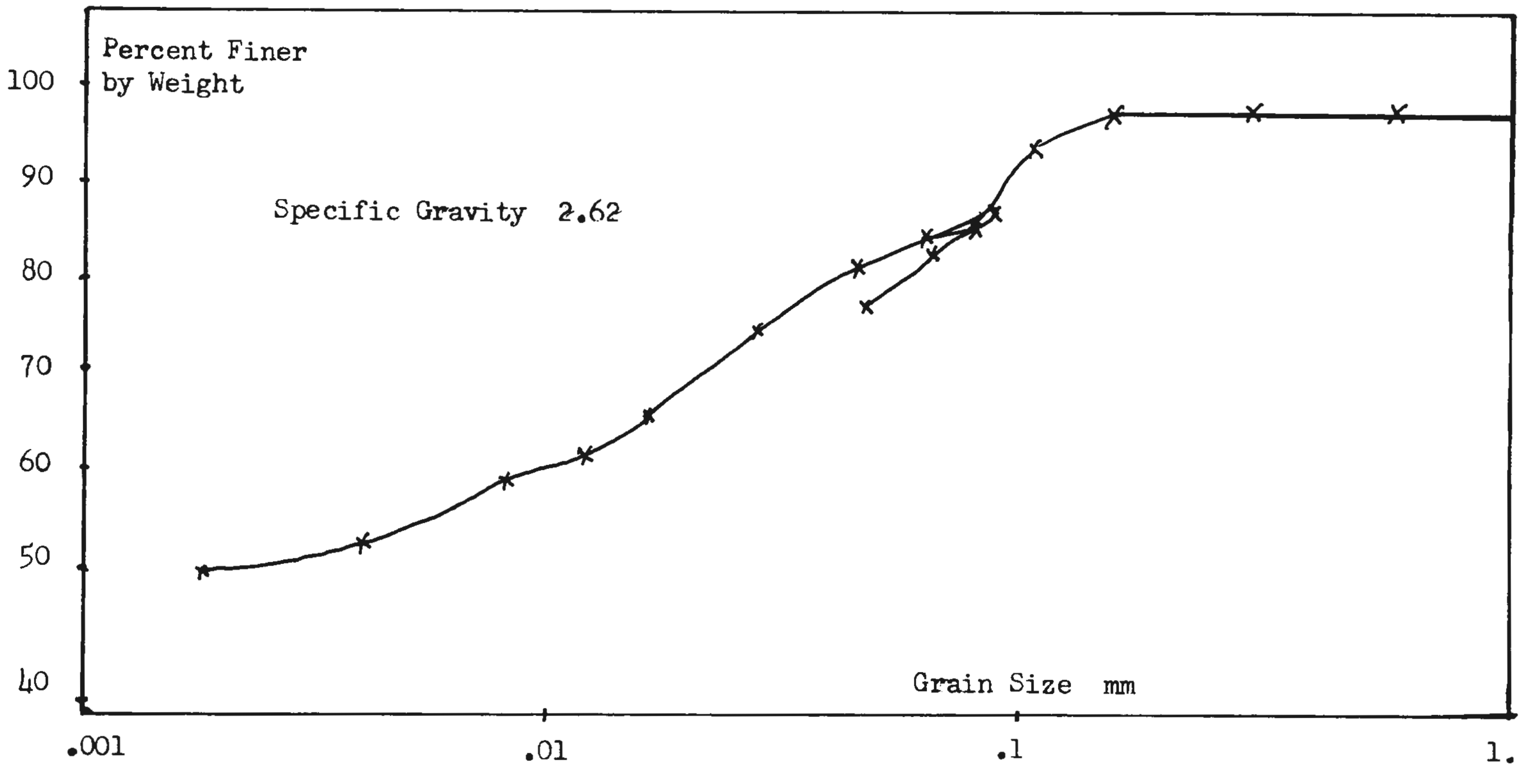


Figure 10. Grain Size Analysis of Dry "Fuller's Earth": Performed by NCEL

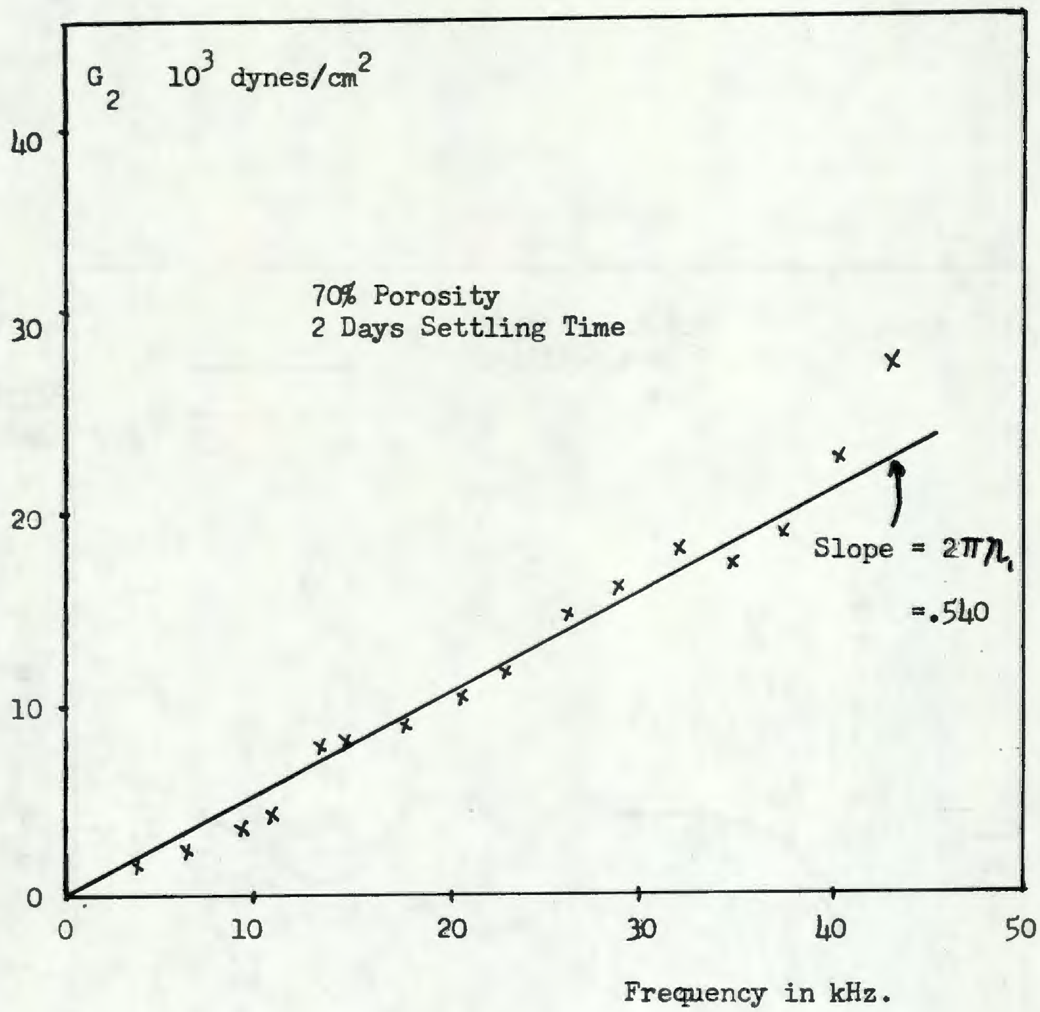


Figure 11. Measured $G_2$ vs Frequency: Colloidal Kaolin and Water

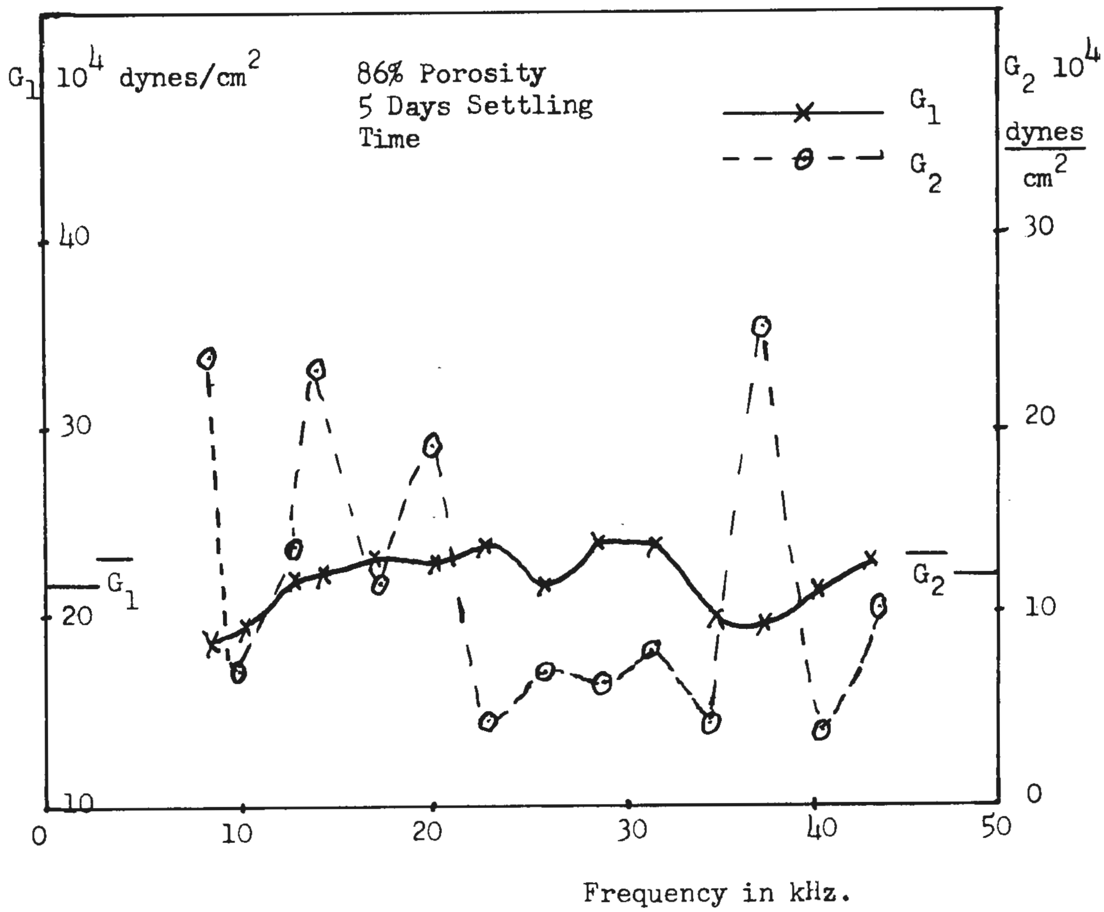


Figure 12. Measured Shear Moduli: Non-Colloidal Kaolin and Water

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APPENDIX

(A) Pulse Method: Detailed Equations

For torsional waves propagating down a radially symmetric rod, McSkimmin (13) gives the specific acoustic impedance at radial position a as:

$$Z = -\frac{i\, G a\, k^2}{4\omega} \qquad \text{(A1)}$$

where G is the shear modulus of the rod, $\gamma = A + i\,B$ is the complex propagation constant (A is attentuation in nepers/cm; B is the phase shift in rad/cm), and $k^2 = \frac{\rho_{ROD}\,\omega^2}{G} + \gamma^2$

In air, no shear stress at the boundary is assumed; therefore, both Z and $k$ are zero. Hence, in air, $\gamma_0 = -\sqrt{\frac{\rho_{ROD}}{G}}\,\omega = A_0 + i\,B_0$

In sediment, the impedance is not zero, and

$$k^2 = -(A_0 + i\,B_0)^2 + (A + i\,B)^2 \qquad \text{(A2)}$$

Letting $A^1 = A - A_o$ = change in attentuation per unit length,

and $B^1 = B - B_o$ = change in phase per unit length,

substituting (A2) into (A1), and ignoring second order terms,

yields $$Z = \frac{\rho_{ROD}\, V_0\, a}{2}\left(\Delta A^1 - i\,\Delta B^1\right)$$

where $V_o = \sqrt{G/\rho_{rod}}$ is the shear velocity in the unloaded rod.

Now, let $\Delta A$ = the total change in attenuation due to immersing the rod in sediment

and $\Delta B$ = the total change in phase due to immersing the rod in sediment.

Then, with length $l$ of rod covered by sediment, and observing echo number n, the effective length of rod covered by sediment is $2nl$.

So, $$\Delta A^1 - i\,\Delta B^1 = \frac{1}{2nl}\left(\Delta A - i\,\Delta B\right)$$ and, (A3) $$Z = \frac{\rho_{ROD} V_o a}{4nl} (\Delta A - i\Delta B) = R + iX$$

is the impedance per unit length offered by the sediment.

Calculation of $\Delta A$:

Measuring the attentuation change in db, $\Delta\alpha$, readily yields,

(A4) $$\Delta A = \frac{\Delta\alpha}{20 \log_{10} e} = \frac{\Delta\alpha}{8.68}$$

Calculation of $V_o$ and $\Delta B$:

Assuming the velocity of shear propagation is frequency independent, a pulse propagating down the rod undergoes a phase shift $\phi = \omega t$, where t is the total time delay.

$t = \frac{2nl_o}{V_o}$, where $l_o$ is the rod's length.

At a given frequency $f_o$, $\phi_o = 2\pi f_o \left(\frac{2nl}{V_o}\right)$

For the echo and independently attenuated transmitted pulse to cancel, their relative phase must be the same. The transmitted pulse is always of the same phase. If cancellation occurred at $f_0$, then cancellation also occurs at $f_m$, where $\phi_m = \phi_o + 2\pi m$, m any integer.

$V_o$ may be calculated from the total frequency change necessary to give m successive $2\pi$ radian phase changes in air.

That is, $\phi_m = 2\pi f_m \left(\frac{2nl}{V_o}\right) = 2\pi\left(f_o \frac{2nl}{V_o} + m\right)$; or,

(A5) $$V_o = \frac{2nl(f_m - f_o)}{m}$$

Again, in air, for cancellation, $\phi_o = 2\pi f_o t$. With the rod in sediment, for cancellation, $\phi_S = 2\pi f_S t = 2\pi f_o t + \Delta B$ must be met.

Hence, $\Delta B = 2\pi(f_S - f_o)\frac{2nl}{V_o}$, or

(A6) $$\Delta B = \frac{4\pi(\Delta f)nl}{V_o}$$

(B) Transducers.

The torsional transducers were made from ½ and 3/4 inch barium titanate cylinders. After placing polarizing electrodes as shown in Figure 1, the cylinders were heated in an oil bath to above the Curie temperature. A DC voltage in excess of 10,000 volts was applied and maintained as the cylinders were cooled to room temperature.

Fine wire leads were then attached with conducting epoxy resin, and silver paint was used to cover the attachment area as well as the remainder of the driving electrode location. For added strength, a thin coat of non-conducting epoxy resin was then used to cover the leads.

Resonance frequencies were checked against computed values to insure that only the torsional mode was excited. Electromechanical coupling coefficients from .12 to .27 were obtained.

(C) Pule Method: Input Gate and Clipping Circuit

A detailed diagram of the input gate and clipping circuit is shown in Figure 5.

The sinusoidal pulse from the tone burst generator passes emitter follower, $T_1$. A gating signal, also from the tone burst generator, controls switching transister, $T_2$. During the main pulse, $T_2$ is off, and the amplified main pulse is directed to the transducer. However, during the occurrance of the undesired ripple, the gating signal turns $T_2$ on, and the ripple is grounded.

The 6K resistor serves to isolate echoes from the 50 milliohm output impedance of the HP467A amplifier.

The clipping circuit consists of two normally reversed biased diodes: the bias level is variable, and is controlled by the HP467A power supply. Unable to overcome this reversed bias, echos enter the oscilloscope. The much larger main pulse, on the other hand, forward biases the diodes, and is clipped to the externally set bias level.

(d) Programs

On the following eight pages are programs PLSCAL, KCAL, RESCAL, and MELES.

```
C      PROGRAM PLSCAL
C      PROGRAM FOR PULSE METHOD CALCULATIONS
C      ETHYLENE GLYCOL, FIFTH ECHO
       DIMENSION FAIR(61),FSED(61),ATAIR(61),ATSED(61),DELTAA(61),DELTAB(
      C61),R(61),X(61),ETAONE(61),ETATWO(61),GONE(61),GTWO(61)
C      FREQUENCY TO SIX SIGNIFICANT FIGURES, ATTENUATION TO 2 DEC. PLACES
       READ (5,100) FAIR
  100  FORMAT (8F10.1)
C      FAIR, FSED, ATAIR, AND ATSED ARE READ IN AS ENTIRE ARRAYS.
C      THEREFORE, DATA CARDS CONTAINING ZEROS MUST BE INCLUDED FOR DATA
C      VALUES BETWEEN M+1 AND 61.  THESE DATA CARDS MAY CONTAIN ANY
C      NUMBERS SINCE THESE DATA WILL NOT BE PROCESSED.
       READ (5,100) FSED
       READ (5,101) ATAIR
  101  FORMAT (16F5.2)
       READ (5,101) ATSED
       ZETA=354257.
C      ZETA IS ONE FOURTH TIMES RHO OF ROD TIMES VEL. OF SOUND IN ROD
C      WITH AIR LOADING TIMES RADIUS OF ROD
       READ (5,102) N,XL,XLENTH,ROSED,M
  102  FORMAT (I1,3F10.5,I2)
C      N IS ECHO NUMBER, XL IS LENGTH OF ROD IN CM, XLENTH IS LENGTH OF
C      ROD COVERED BY SEDIMENT IN CM, ROSED IS DENSITY OF SEDIMENT IN
C      GMS PER CUBIC CM, M IS THE NUMBER OF NULLS READ.
       ZETMOD=(ZETA/N)/XLENTH
       WRITE (6,103) N,XLENTH,ROSED
  103  FORMAT (1H1,2HN=,I1,18H   LENGTH COVERED=,F5.2,10H   RHOSED=,F5.3)
       WRITE (6,104)
  104  FORMAT (1H0,1X,1HM,4X,4HFAIR,8X,4HFSED,5X,5HATAIR,5X,5HATSED)
       DO 106 I=1,M
       WRITE (6,105) I,FAIR(I),FSED(I),ATAIR(I),ATSED(I)
  105  FORMAT (I3,2F10.1,2F10.2)
  106  CONTINUE
C      NOW FORM THE FREQUENCY CHANGE(DELF) AND THE ATT CHANGE (DELATT)
C      AND PUT THESE DIFFERENCES INTO THE AIR ARRAY.
       DO 107 I=1,M
       FAIR(I)=FAIR(I)-FSED(I)
       ATAIR(I)=ATSED(I)-ATAIR(I)
C      CONVERTING DELATT FROM DB TO NEPERS, 8.6859=LOG E**2/(1/10)
       DELTAA(I)=ATAIR(I)/8.6859
C      CALCULATING REMAINING QUANTITIES
       DELTAB(I)=(12.566*N*XL*FAIR(I))/290947.
       R(I)=ZETMOD*DELTAA(I)
       X(I)=ZETMOD*DELTAB(I)
       GONE(I)=(R(I)**2-X(I)**2)/ROSED
```

```
      GTWO(I)=2.*R(I)*X(I)/ROSED
      ETAONE(I)=GTWO(I)/(6.283*FSED(I))
  107 ETATWO(I)=GONE(I)/(6.283*FSED(I))
      WRITE (6,108)
  108 FORMAT (1H1,2H M,4X,4HFSED,4X,4HDELF,3X,6HDELATT,4X,6HDELTAA,4X,6H
     CDELTAB,6X,1HR,8X,1HX,8X,5HETA 1,6X,5HETA 2,6X,3HG 1,9X,3HG 2)
      DO 110 I=1,M
      WRITE (6,109) I,FSED(I),FAIR(I),ATAIR(I),DELTAA(I),DELTAB(I),R(I),
     CX(I),ETAONE(I),ETATWO(I),GONE(I),GTWO(I)
  109 FORMAT (1H0,I2,F9.1,F7.1,F9.2,2F10.4,2F9.2,2F11.4,2F12.0)
  110 CONTINUE
      END
```

```
C     PROGRAM KCAL
C     PROGRAM FOR CALCULATING KONE AND KTWO
      DIMENSION FAIR(20),FSED(20),GAIR(20),GSED(20),DELRE(20),DELFR(20),
     CDREL(20),DFRL(20),S(20),X(20),XKONE(20),XKTWO(20)
      READ (5,100) FAIR
  100 FORMAT (8F10.2)
      READ (5,100) FSED
C     FAIR,FSED,GAIR,AND GSED ARE READ AS ENTIRE ARRAYS,AND SO,DATA
C     CARDS MUST BE SUPPLIED. EXTRA DATA,ALTHOUGH READ,WILL NOT BE
C     PROCESSED. M IS NUMBER OF DATA POINTS.
      READ (5,101) GAIR
  101 FORMAT (8F10.9)
      READ (5,101) GSED
C     ETAONE OS THE REAL NEWTONIAN VISCOSITY IN POISE, ROSED IS THE
C     DENSITY OF THE FLUID,XLENTH IS THE LNNGTH OF ROD COVERED.
      READ (5,102) ETAONE,ROSED,M,XLENTH
  102 FORMAT (2F10.6,I2,F10.5)
      WRITE (6,113)
  113 FORMAT (1H1,55HMED.  LUBE OIL,1 1/2 INCH, 1/2 INCH DIAMETER TRANSD
     *UCER)
      WRITE (6,103) ETAONE,ROSED,XLENTH
  103 FORMAT (1H0,7HETA 1 =,F6.4,5X,8HRHOSED =,F7.4,5X,16HLENGTH COVERED
     C =,F6.3)
      WRITE (6,104)
  104 FORMAT (1H0,4X,4HFAIR,6X,4HFSED,20H G AIR IN MICROMHOS ,19H G SED
     CIN MICROMHOS)
      DO 106 I=1,M
      WRITE (6,105) FAIR(I),FSED(I),GAIR(I),GSED(I)
  105 FORMAT (2F10.1,8X,6PF10.2,10X,6PF10.2)
  106 CONTINUE
      DO 107 I=1,M
C     FORM RE=1/G AND PUT IN G ARRAYS
      GAIR(I)=1/GAIR(I)
      GSED(I)=1/GSED(I)
C     FORM DIFFERENCES
      DELRE(I)=GSED(I)-GAIR(I)
      DELFR(I)=FAIR(I)-FSED(I)
C     CALCULATING REMAINING QUANTITIES
      DREL(I)=DELRE(I)/XLENTH
      DFRL(I)=DELFR(I)/XLENTH
      S(I)=(ROSED*3.1416*FSED(I)*ETAONE)
      X(I)=SQRT(S(I))
      XKONE(I)=DREL(I)/X(I)
  107 XKTWO(I)=DFRL(I)/X(I)
      WRITE (6,108)
```

```
  108 FORMAT (1H1,4X,4HFSED,5X,6HRE AIR,4X,6HRE SED,2X,8HDELTA RE,10H  -
     CDELTA F,20H DEL RE/L   -DEL FR/L,5X,5HR = X,6X,3HK 1,7X,3HK 2)
      DO 110 I=1,M
      WRITE (6,109) FSED(I),GAIR(I),GSED(I),DELRE(I),DELFR(I),DREL(I),DF
     CRL(I),X(I),XKONE(I),XKTWO(I)
  109 FORMAT (1H0,F9.1,3F10.0,F10.1,F10.2,F10.5,F10.2,F10.4,F10.7)
  110 CONTINUE
C     AVERAGE THE K VALUES
      A=0.
      B=0.
      DO 111 I=1,M
      A=A+XKONE(I)
  111 B=B+XKTWO(I)
      C=A/M
      D=B/M
      WRITE (6,112) C,D
  112 FORMAT(1H0,13HAVERAGE K 1 =,F8.6,5X,13HAVERAGE K 2 =,F8.6)
      END
```

```
C       PROGRAM RESCAL
C  PROGRAM FOR RESONANCE METHOD CALCULATION
        DIMENSION FAIR(20),FSED(20),GAIR(20),GSED(20),DELRE(20),DELFR(20),
       *DREL(20),DFRL(20),XKONE(20),XKTWO(20),R(20),X(20),GONE(20),GTWO(20
       *),ETAONE(20),ETATWO(20)
        READ(5,100) N,M
C  THIS PROGRAM CALCULATES, WITH ONE SET OF AIR READINGS,G'S AND ETA'S
C  FOR SEVERAL SETS OF MUD DATA. A RUN IS THE CALCULATION OF G'S AND
C  ETA'S FOR ONE SET OF DATA. M IS THE NUMBER OF DATA POINTS PER RUN. M
C  MUST BE THE SAME FOR EACH RUN.
   100 FORMAT(2I2)
        READ(5,101) FAIR
C  FAIR ,FSED,GAIR,K1 AND K2 ARE READ IN AS WHOLE ARRAYS,AND WHOLE
C   ARRAYS MUST BE SUPPLIED.IF THE NUMBER OF DATA POINTS IS 17,READ IN 20
C  BITS OF DATA. THE LAST THREE NUMBERS OF EACH ARRAY ARE ARBITRARY AND
C  WILL NOT BE PROCESSED.
   101 FORMAT(8F10.2)
        READ(5,102)GAIR
   102 FORMAT(8F10.9)
        READ(5,103) XKONE
   103 FORMAT(8F10.4)
        READ(5,102)XKTWO
        WRITE(6,104)
   104 FORMAT (1H1,40H1 1/2 INCH, 1/2 INCH DIAMETER TRANSDUCER)
        WRITE(6,105)
   105 FORMAT(1H0,10H M    F AIR,8X,13HG AIR IN MHOS,5X,3HK 1,9X,3HK 2)
        DO 107 I=1,M
        WRITE(6,106) I,FAIR(I),GAIR(I),XKONE(I),XKTWO(I)
   106 FORMAT(1H0,I2,F9.1,7X,F10.8,5X,F9.4,F10.7)
C  FORM REAIR=1/GAIR AND PUT INTO GAIR ARRAY
   107 GAIR(I)=1./GAIR(I)
        DO 109 I=1,N
        J=I+6
        WRITE(6,108) I,J
   108 FORMAT(1H0,4HRUN ,I2,8H IS FOR ,I2,23H MAY       KAOLIN    DATA)
   109 CONTINUE
         DO 119 J=1,N
C  DATA READ FROM HERE ON IS FOR A SINGLE RUN. THE NUMBER OF SETS OF
C  THESE DATA MUST MATCH N.
        READ(5,101) FSED
        READ(5,102)GSED
        READ(5,110) ROSED,XLENTH
C  ROSED IS SEDIMENT DENSITY IN GMS/CC. XLENTH IS LENGTH OF ROD COVERED
C  BY SEDIMENT IN CM.
   110 FORMAT(2F10.5)
```

```
      WRITE(6,111) J
  111 FORMAT(1H1,10HRUN NUMBER,I2)
      WRITE(6,112) ROSED,XLENTH
  112 FORMAT(1H0,8HRHOSED =,F7.4,5X,16HLENGTH COVERED =,F6.3)
      WRITE(6,113)
  113 FORMAT(1H0,10H M   F SED,5X,18HG SED IN MICROMHOS)
      DO 115 I=1,M
      WRITE(6,114) I,FSED(I),GSED(I)
  114 FORMAT(1H0,I2,F9.1,7X,6PF10.2)
C FORM RESED=//GSED AND PUT INTO GSED ARRAY
      GSED(I)=1./GSED(I)
C FORM DIFFERENCES
      DELRE(I)=GSED(I)-GAIR(I)
      DELFR(I)=FAIR(I)-FSED(I)
C CALCULATING REMAINING QUANTITIES
      DREL(I)=DELRE(I)/XLENTH
      DFRL(I)=DELFR(I)/XLENTH
      R(I)=DREL(I)/XKONE(I)
      X(I)=DFRL(I)/XKTWO(I)
      GONE(I)=(R(I)**2-X(I)**2)/ROSED
      GTWO(I)=2.*R(I)*X(I)/ROSED
      ETAONE(I)=GTWO(I)/(6.2832*FSED(I))
  115 ETATWO(I)=GONE(I)/(6.2832*FSED(I))
      WRITE(6,116)
  116 FORMAT (1H1,2H M,2X,4HFSED,5X,6HRE AIR,4X,6HRE SED,2X,8HDELTA RE,1
     *0H  -DELTA F,21H  DEL RE/L  -DEL FR/L,6X,1HR,7X,1HX,6X,5HETA 1,5X,
     *5HETA 2,8X,3HG 1,8X,3HG 2)
      DO 118 I=1,M
      WRITE(6,117) I,FSED(I),GAIR(I),GSED(I),DELRE(I),DELFR(I),DREL(I),D
     *FRL(I),R(I),X(I),ETAONE(I),ETATWO(I),GONE(I),GTWO(I)
  117 FORMAT(1H0,I2,F8.1,F9.0,2F10.0,F10.1,F10.2,F10.5,F10.2,F8.2,F9.4,F
     *10.4,2F11.0)
  118 CONTINUE
  119 CONTINUE
      END
```

```
C       PROGRAM MELIS
C PROGRAM FOR MEAN AND VARIANCE CALCULATION AND LEAST SQUARES FIT.
        DIMENSION X(16),Y(16),DIFX(16),DIFY(16),DIFSQX(16),DIFSQY(16),PROD
       *(16)
        READ(5,100) N,M
C N IS NUMBER OF RUNS, A RUN IS THE PROCESSING OF ONE SET OF XAND Y
C VALUES. M IS THE NUMBER OF DATA POINTS PER RUN, AND MUST BE THE SAME
C FOR EACH RUN.
  100   FORMAT (2I2)
C Y IS THE SHEAR MODULUS,X IS FREQUENCY
        DO 113 I=1,N
        READ (5,101) X
  101   FORMAT (8F10.2)
C X AND Y MUST BE READ AS ENTIRE ARRAYS. ARBITRARY DATA POINTS, WHICH WILL
C NOT BE PROCESSED, MUST BE SUPPLIED,IF NECESSARY,TO COMPLETE THE ARRAYS
        READ (5,102) Y
  102   FORMAT (8F10.0)
        WRITE (6,103) I
  103   FORMAT(1H1,11HRUN NUMBER ,I2)
        WRITE (6,104)
  104   FORMAT (1H0,2H M,2X,10H FREQUENCY,3X,13HSHEAR MODULUS)
        SUMX=0.
        SUMY=0.
        DO 106 J=1,M
        WRITE (6,105) J,X(J),Y(J)
  105   FORMAT(1H0,I2,2X,F10.1,3X,F10.0)
        V=M
        SUMX=X(J)+SUMX
  106   SUMY=Y(J)+SUMY
        XBAR=SUMX/V
        YBAR=SUMY/V
        WRITE (6,107) XBAR,YBAR
  107   FORMAT (1H0,7HXBAR = ,F7.1,5X,6HYBAR =,F10.0)
        SUMX=0.
        SUMY=0.
        SUMXY=0.
        DO 108 J=1,M
        DIFX(J)=X(J)-XBAR
        DIFY(J)=Y(J)-YBAR
        DIFSQX(J)=DIFX(J)**2
        DIFSQY(J)=DIFY(J)**2
        PROD(J)=Y(J)*DIFX(J)
        SUMX=SUMX+DIFSQX(J)
        SUMY=SUMY+DIFSQY(J)
  108   SUMXY=SUMXY+PROD(J)
```

```
      SQUARE=SUMY/(V-1.)
      S=SQRT(SQUARE)
      WRITE(6,109)SQUARE,S
  109 FORMAT(1H0,11HVARIANCE = ,F12.0,10X,21HSTANDARD DEVIATION = ,F9.0)
      RAT=S/YBAR
      WRITE (6,110) RAT
  110 FORMAT(1H0,16HSIGMA/MU OF G = ,F10.7)
      ALPHA=SUMXY/SUMX
      BETA=YBAR-ALPHA*XBAR
      WRITE (6,111) ALPHA,BETA
  111 FORMAT(1H0,8HALPHA = ,F11.7,10X,7HBETA = ,F11.0)
      ZONE=BETA+(ALPHA*1000.)
      ZTWO=BETA+(ALPHA*50000.)
      WRITE(6,112) ZONE,ZTWO
  112 FORMAT(1H0,13HG AT 1 KHZ = ,F10.0,10X,14HG AT 50 KHZ = ,F10.0)
  113 CONTINUE
      END
```

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13. ABSTRACT


The complex shear modulus of both kaolin-water and bentonite-water mixtures has been determined in the laboratory. The method involved measuring the reaction on a torsional elastic wave propagated down a metal rod when the rod was immersed in sediment. Observations were made over the frequency range two to forty-three kHz. Dispersed sediments behaved like Newtonian liquids. Undispersed sediments, however, were viscoelastic in character, and their shear moduli exhibited no dependence on frequency. For undispersed kaolin mixtures, a typical result is $(21.6 + i\ 1.2) \times 10^4$ dynes per square centimeter at 32 percent concentration. In bentonite, a modulus of $(4.6 + i\ 2.0) \times 10^5$ dynes per square centimeter at 19 percent concentration is representative.


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| Resonance | | | | | | |
| Kaolin | | | | | | |
| Bentonite | | | | | | |

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