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Pricing Contracts Under Uncertainty in a Carbon Capture and Storage Framework

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Abstract

Carbon capture and storage (CCS) has been demonstrated as a viable option for reducing carbon emissions to the atmosphere. We consider a situation where a tax on emissions is imposed on carbon dioxide (CO_2) emitters to encourage their participation in CCS. Operators of CO_2 transportation pipelines and storage sites enter into individual contracts with emitters to store CO_2. We study the problem of setting the price and volume of these contracts under cost and emission uncertainty, and show how the storage operator's profit can be optimized.

Keywords: carbon capture and storage, pricing, uncertainty quantification

1. Introduction

Most low-carbon energy technologies require well-designed policy incentives to encourage widespread deployment of the necessary infrastructure. For carbon capture and storage (CCS), large networks of capture plants, transportation pipelines, and storage reservoirs will be necessary. In parallel, a market-based incentive structure is required to encourage emitters to reduce emissions by employing CCS and participating in a network.

The potential impacts of CO_2 emissions to the atmosphere are well known and have triggered significant work on low-carbon energy technologies. In order to reduce atmospheric concentrations of greenhouse gases over the long term, multiple solutions are needed to reduce the total emission rate. No single technology has been identified that is sufficient to meet the challenge alone, given the magnitude of global dependence on carbon-intensive fuels.

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Most major studies on practical strategies to reduce global emissions have included CCS on the list of technologies that can have a significant impact on emission by 2050 (Pacala and Socolow, 2004; International Energy Agency, 2010). The basic idea behind CCS is to identify major point sources, like coal-fired power plants, and then capture the produced CO_2 before it is released to the atmosphere. The captured gas can then be compressed and piped to special storage sites where it is injected into deep subsurface reservoirs. The sequestered CO_2 is stored indefinitely and therefore does not increase atmospheric concentrations.

There are currently eight industrial scale CO_2 capture and storage projects operating around the world, with dozens more in the construction or planning stages (Global CCS Institute, 2012). The longest running project, Sleipner, has been injecting CO_2 since 1996 and so far has stored 16 Mt (1 Mt = 10^6 metric tonnes = 10^9 kilograms) in a deep reservoir beneath the Norwegian North Sea (Arts et al., 2008; Chadwick et al., 2012). Sleipner was developed in response to the passage of a Norwegian CO_2 tax in 1991, and the avoided tax burden quickly paid for the development costs.

Existing literature (Middleton and Bielicki, 2009; Middleton et al., 2012) focuses on building a network that minimizes the overall capture, transportation, and storage costs, treating all players as a unified decision maker. To date, most CCS projects have taken an integrated approach, where the capture and storage operations are developed in partnership. In the future, however, other deployment models may be more common (Esposito et al., 2011). Our work designs contracts between individual players: a CO_2 emission producer and a pipeline and storage provider. We refer to these players as the “emitter” and the “storage operator” respectively. Emitters can be any point-source of CO_2: coal and natural-gas power plants, steel and cement manufacturers, and other industries. We study how a storage operator can design a contract that specifies the amount of CO_2 to be stored at a fixed unit price (per Mt). The inputs are the storage operator’s costs, his expectations of the emission quantity and the emitter’s capture costs, and an external tax on emissions faced by the emitter. Future work will study a network of such contracts, where one storage operator can transport and store CO_2 from many sources.

Our work is based on three premises: (1) all participants are utility maximizers, (2) a fixed carbon tax has been established, and (3) some of the quantities needed to design the contract are uncertain. The first premise enables us to establish an economic model of how participants make their decisions. The second premise is essential since emitters will have little financial incentive to participate in CCS if they are free to emit CO_2 into the atmosphere. Carbon taxes are generally considered an effective economic incentive to reduce CO_2 emissions. To date, many countries and municipalities have adopted some form of carbon taxation, including: Finland (1990); Netherlands (1990); Norway (1991); Sweden (1991); Denmark (1992); United Kingdom (2001); Boulder, Colorado (2007); Quebec (2007); British Columbia (2008); and the Bay Area Air Quality Management District, California (2008) (Sumner et al., 2011). A number of other countries have carbon tax proposals under consideration.

Due to the novelty of CO_2-capture technology and the one-of-a-kind nature of new capture plants, the true cost to install and operate the capture facility can only be known to the emitter. The emitter may want to conceal these costs to keep the storage contract price low. Additionally, the emission quantity is also not constant. In the case of power plant, the uncertainty in emission quantity is a result of fluctuations in electricity demand and plant downtime for maintenance. Such uncertainties affect the optimal contract the storage operator offers to the emitter. The third premise thus allows us to construct the optimal contract for the storage operator while allowing information to be unknown. Optimizing the price and volume of the contract together can further increase the profits to the storage operator, especially when the distributions of the capture costs and emissions volume are correlated.

The models proposed in this paper encompass the unique aspects of this problem. The emitter compares the cost of paying the emission tax against the cost of engaging a storage operator to store their CO_2. Meanwhile, a storage operator has limited capacity and incurs costs for transporting and storing captured CO_2. Neither player will participate if a profit cannot be obtained. We propose a newsvendor-type solution to determine the optimal contract that maximizes profit to the storage operator, and minimizes costs to the emitter. Our results provide guidance in determining how much CO_2 should be transported and stored, and at what cost. We can also analyze the effect of different costs on the optimal contracts.

2. Model Details

In this section we describe the components of our model, which are summarized in Table 1. The storage operator faces a one-time cost K for setting up the site, a unit cost α_1 (per Mt) for transporting and storing CO_2 up to the contract amount, and a marginal cost α_2 (per Mt) to transport and store any excess CO_2 above the contract amount. Note that this is a simplified cost model, and masks many of the financing details associated with large capital projects. More complex cost structures, however, can be converted to equivalent fixed and unit costs.

There is a maximum capacity *cap* of CO_2 that the storage operator can take due to limitations in the size of the storage reservoir. Typically there is a maximum storage rate (Mt/yr) rather than a maximum volume (total Mt), but here we focus on a single-period setting and assume that the contracted amount of CO_2 will be injected at a rate below the physical limit. Additionally, there is a *tax* (per Mt) imposed on the emitter for any CO_2 that is released to the atmosphere.

The power plant has an emissions volume *em* (in Mt) during the single period which is a random variable from a known distribution. Additionally, the capture costs *cc* is the (per Mt) cost of employing capture technology to prevent CO_2 from being emitted. This cost is privately known to the emitter, but the storage operator can estimate a distribution on *cc* to help decide what price to charge the emitter.

We divide the decision process into two stages. In the contracting stage, the storage operator maximizes his expected profit over two decision variables. The first is the price, *prc*, to charge the emitter per Mt of CO_2 transported and stored. The second is the contract amount, *con* (in Mt) that is the amount the emitter can store at a given value of *prc*. Once such decisions are made, the storage operator presents a contract that specifies both the price and the contract amount. The emitter decides whether to accept the contract by comparing *prc* against the difference between the tax and the capture cost. If the emitter finds it is cheaper to capture and store the carbon than to pay the emissions tax, he accepts the contract, and the storage operator commits to build the pipeline capacity so that the contracted amount of CO_2 (*con*) can be transported via pipeline and stored. The storage operator only builds the pipeline between the emitter and the storage site after the contract is accepted and prior to the beginning of transporting CO_2.

In the execution stage, the emitter observes the actual emission quantity and has the right (but not the obligation) to store *con* Mt of CO_2 at the operator's site. If the emitter wants to store more CO_2 than the contracted amount, the storage operator may choose not to accept the excess based on either the cost of transportation or the capacity of the site. However, the storage operator is not able to charge a different price for the additional amount of CO_2.

Table 1: List of terms and variables

Data [units]	
K	One-time setup cost [$] for site s_1
α_1	Marginal cost [$/Mt] incurred by the storage operator for transporting and storing CO_2 via pipelines
α_2	Marginal cost [$/Mt] incurred by the storage operator for transporting and storing CO_2 via other means
cap	Maximum capacity for site s_1 [Mt]
tax	Fixed tax [$/Mt] that the emitter pays for CO_2 vented
Random Variables [units]	
cc	Cost [$/Mt] incurred by the emitter for CO_2 captured
em	Amount of CO_2 [Mt] emitted by the emitter
Decision Variables [units]	
prc	Price [$/Mt] that the storage operator charges the emitter for CO_2 stored at site s_1
con	Contract amount of CO_2 [Mt] transported via pipeline and stored at the operator's site

Under the framework described above, we construct two linked models for the storage operator to determine the optimal contract volume and the optimal price. In the first model in Section 3, we choose the optimal contract volume given uncertainty in the emission quantity and a pre-determined unit price. We extend the basic model to select the optimal price considering both uncertainty in the capture cost and emission quantity in Section 4. In Section 5 we present numerical results for realistic input values, and study the effect of correlation between em and cc on the optimal profit. Section 6 draws conclusions and details avenues for further research. All proofs appear in the Appendix.

3. Stochastic Emission Quantity

First, we determine the optimal contract amount in the presence of a stochastic emission quantity. Because there is uncertainty in the capture process, the demand for power varies, and the plant may need to shut down for unexpected maintenance, neither the emitter nor the storage operator can definitively predict the volume of CO_2 emissions available for storage. To address this uncertainty, we treat the emissions volume (em) as a random variable with a known distribution. Let $f(em)$ denote the continuous density distribution function of the emission quantity, and $F(em)$ denote its cumulative density function (c.d.f.). Since the emission quantity needs to be positive and can be potentially quite large, we further assume that the density function has a non-negative and continuous support. The storage operator's goal is to optimize the contract given a pre-determined prc, which, for now, we assume is known.

Because the storage operator needs to build the pipelines for transporting CO_2 from the emitter to the site after the contract has been accepted but prior to any CO_2 being transported, the optimal contract amount is an important decision. For simplicity, we assume that the emitter builds the capacity of the pipelines to match the contract amount. If the emitter asks to store more than the contract amount, the storage operator can use another method to transport the excess CO_2 at a higher marginal cost, α_2 ($> \alpha_1$). In general, building a pipeline is the most efficient way of transporting large volumes of CO_2, and other options such as trucking are more expensive in the long run (Herzog and Golomb, 2004).

Because our model assumes that the storage operator can only charge a single price for all CO_2 stored, the profit function for the storage operator varies depending on em. Recall that cap denotes the maximum capacity at the site the operator makes the injection. If emission amount is less than cap, the storage operator receives revenue $prc \cdot em$; otherwise, he receives $prc \cdot cap$. On the costs side, the operator incurs a cost of $K + \alpha_1 \cdot con$, (where K is the fixed setup cost and α_1 is the marginal cost of operating the transport and storage system) regardless of the emission quantity. When em is above con, the storage operator incurs an additional cost of $\alpha_2 \cdot (\min(em, cap) - con)$ for transporting and storing the additional CO_2. The following equation summarizes the profit function for the three cases:

$$\Pi(con, prc) = \begin{cases} prc \cdot em - \alpha_1 \cdot con - K & \text{if } em \leq con \\ prc \cdot em - \alpha_1 \cdot con - \alpha_2 \cdot (em - con) - K & \text{if } con < em < cap \\ prc \cdot cap - \alpha_1 \cdot con - \alpha_2 \cdot (cap - con) - K & \text{if } em \geq cap. \end{cases}$$

The storage operator's expected profit is:

$$\begin{aligned} E[\Pi(con, prc)] &= \int_{em=0}^{con} [prc \cdot em]\, f(em) dem \\ &+ \int_{em=con}^{cap} [prc \cdot em - \alpha_2 \cdot (em - con)] f(em) dem \\ &+ \int_{em=cap}^{\infty} [prc \cdot cap - \alpha_2 \cdot (cap - con)] f(em) dem \\ &- \alpha_1 \cdot con - K. \end{aligned} \tag{1}$$

Equation 1 shows the calculation of the expected profit by performing a probability-weighting over the possible profits the storage operator would receive under different values of em. Since the storage operator's goal is to maximize his expected profit, we can formulate an optimization problem as

$$\max_{con} \{E[\Pi(con, prc)] \mid \text{s.t. } con \leq cap\}. \tag{2}$$

The optimal solution to the problem is summarized in the proposition below, and the proof is provided in the Appendix.

Proposition 1. *Given a pre-determined price, the optimal contract that maximizes the storage operator's expected profit is* $con^* = \min \left\{ F^{-1} \left(\frac{\alpha_2 - \alpha_1}{\alpha_2} \right), cap \right\}$, *where* F^{-1} *is the inverse c.d.f. of* F.

Proposition 1 suggests that the optimal contract amount is independent of the pre-determined unit price (prc) charged for the contract (as long as the price is high enough for the storage operator to make a profit). Instead, it depends on the marginal costs (α_1 and α_2) of transporting and storing CO_2 as well as the distribution function of the emission quantity. The optimal contract amount con^* increases in α_2 but decreases in α_1. Since the storage operator commits himself to transport and store the contract amount via pipelines, he is thus incentivized to build a bigger pipeline capacity when the marginal cost of pipeline is cheaper and/or when the other transportation methods are costly. In one extreme case where the marginal cost of transporting and storing carbon is the same using pipelines as other transportation methods (i.e. $\alpha_1 = \alpha_2$), the storage operator should not build any pipeline (i.e. $con^* = 0$). In the other extreme case where other transportation is not available (i.e. α_2 is infinity), the storage operator should set the contract amount to be his total capacity. In Section 5 we explore some numerical examples showing the optimal contract amount under different distributional settings.

4. Stochastic Capture Cost

Next, we model the decision for how much the storage operator should charge the emitter per Mt of CO_2 stored given uncertainty in both the capture cost cc and the emission quantity em. Because the emitter's capture cost varies depending on the technology used, it is private information and the emitter does not have any incentive to report the cost truthfully. To address this information asymmetry, the storage operator needs to form rational expectations of the distribution of the emitter's capture cost. We allow cc to be a random variable with a known distribution. The storage operator's objective is to find the optimal price for the contract to maximize his profit while still incentivizing the emitter to participate (as long as the emitter is sufficiently efficient in capturing CO_2 and cc is low enough).

For a given price, the emitter will either accept or reject the contract based on his capture cost. If the price is set too high, the storage operator bears the risk of being turned down because the emitter is better off emitting to the atmosphere and paying the tax. On the other hand, if the price is set too low, the storage operator leaves "information rent" to the emitter which in turn lowers his profit.

Given a contract (prc, con) specified by the storage operator, the emitter determines whether he should accept the contract or vent the CO_2 and pay the tax. Since the cost to the emitter is the price to store the CO_2 plus the cost of capturing it, he will accept the contract only if $prc + cc \leq tax$. If the actual emission quantity is less than the contracted amount ($em < con$), then both parties agree to only transport and store em amount of CO_2 at the price of prc. On the other hand, if the emitter produces more CO_2 than contract amount ($em > con$), the emitter can either pay tax on the excess amount or can request the storage operator to transport more to his site. For simplicity, we assume that the storage operator is unable to charge a higher price for the additional CO_2. However, he may choose not to fulfill the request if there is a lack of capacity.

Though the capture cost is the emitter's private information, it is natural for us to assume that the storage operator can learn the distribution of the emitter's capture cost through estimation. Let $g(cc)$ denote the continuous distribution function of the capture cost (per Mt of CO_2) of the emitter. Since the capture cost should be positive and can be very high, we assume that the density function g has a non-negative and continuous support. Recall from Section 3 that the optimal contract amount does not depend on the price or the distribution of cc, and we can thus use the contract amount (con^*) as given.

If the price is set such that the emitter is willing to accept the contract, the storage operator will receive revenue from the contract. The storage operator also pays the cost of operating the site as well as the cost of transporting and storing the contracted (or requested) amount of CO_2. Otherwise, the emitter rejects the contract and the storage operator builds neither the storage site nor the pipeline connecting the emitter and the storage site, and consequently incurs no costs.

Consider the problem of designing a contract from the storage operator's point of view given the uncertainty in cc and em. For a given price prc, the storage operator's profit can be written as:

$$\Pi'(con, prc) = \begin{cases} prc \cdot em - \alpha_1 \cdot con - K & \text{if } cc \leq tax - prc \;\&\; em \leq con \\ prc \cdot em - \alpha_1 \cdot con - \alpha_2 \cdot (em - con) - K & \text{if } cc \leq tax - prc \;\&\; con < em < cap \\ prc \cdot cap - \alpha_1 \cdot con - \alpha_2 \cdot (cap - con) - K & \text{if } cc \leq tax - prc \;\&\; em \geq cap \\ 0 & \text{otherwise} \end{cases}$$

We can thus calculate the storage operator's expected profit while allowing the contract to be rejected. This is the same as the expected profit given the contract is accepted times the probability that the contract is accepted ($G(tax - prc)$):

$$E[\Pi'(con, prc)] = E[\Pi(con, prc)] \cdot G(tax - prc). \tag{3}$$

To maximize the storage operator's expected profit, we solve the following unconstrained optimization problem by determining the optimal price:

$$\max_{prc}\{E[\Pi^{'}(con^{*}, prc)]\}.$$

Proposition 2. *Given the optimal contract amount con^*, the optimal price prc^* that maximizes the storage operator's expected profit solves an implicit equation and meets two conditions. The implicit equation is*

$$G(tax - prc) \cdot \int_0^{cap} \overline{F}(em)dem = g(tax - prc) \cdot E[\Pi(con^*, prc)]. \quad (4)$$

where $\overline{F}(em)$ is $1 - F(em)$. In addition, the following conditions must hold at prc^*:

$$g'(tax - prc^*) \cdot E[\Pi(con^*, prc^*)] - 2g(tax - prc^*) \cdot \int_0^{cap} \overline{F}(em)dem < 0,$$

$$\textit{and } E[\Pi(con^*, prc^*)] \geq 0.$$

Proposition 2 suggests that the optimal price depends on $G(tax - prc)$, which is the cumulative probability that the capture cost cc is lower than the difference between tax and price, $tax - prc$. We can also interpret $G(tax - prc)$ as the estimated probability that the emitter will participate in CCS with the storage operator. As mentioned before, the storage operator wants to set the unit price to be as high as possible to maximize his expected profit, but also ensures the price is low enough to induce efficient emitters (i.e., with low capture costs) to accept the contract. This tradeoff is thus incorporated by having the distribution of cc in the optimal solution in the terms $G(tax - prc)$ and $g(tax - prc)$.

The discontinuity of the profit function at $cc = tax - prc$ results in the reliance on the density function $g(tax - prc)$. While such reliance makes a closed form solution for prc^* intractable for general distribution functions, simple line search methods can be used to find the optimal solution numerically for any arbitrary distribution of g. The two conditions stated in Proposition 2 serve the purpose of ensuring prc^* is the optimal price and the expected profit for the storage operator is non-negative (once the emitter accepts the contract). These conditions may further restrict the value of prc^*, the allowable range of tax, and the allowed shape of the density distribution $g(cc)$.

To provide more intuition for our result, let us consider a uniform distribution $U[0, b]$ for $g(cc)$. In this case, $G(tax - prc) = \dfrac{tax - prc}{b}$ and $g(tax - prc) = \dfrac{1}{b}$. The optimal price that satisfies Equation (4) is

$$prc^* = \frac{\alpha_2 \cdot \int_{con^*}^{cap} \overline{F}(em)dem + \alpha_1 \cdot con^* + K}{2 \cdot \int_0^{cap} \overline{F}(em)dem} + \frac{tax}{2}. \tag{5}$$

Since $g(cc)$ is a uniform distribution, $g'(tax - prc^*) = 0$ and the condition

$$g'(tax - prc^*) \cdot E[\Pi(con^*, prc^*)] - 2 \cdot g(tax - prc^*) \cdot \int_0^{cap} \overline{F}(em)dem$$
$$= \frac{-2}{b} \int_0^{cap} \overline{F}(em)dem < 0$$

is satisfied. The condition $E[\Pi(con^*, prc^*)] \geq 0$ holds as long as

$$tax \geq \frac{\alpha_2 \cdot \int_{con^*}^{cap} \overline{F}(em)dem + \alpha_1 \cdot con^* + K}{\int_0^{cap} \overline{F}(em)dem}. \tag{6}$$

A simplified expression for $E[\Pi(con^*, prc^*)]$ is derived in the proof of Proposition 2 that allows for the calculation of (6). Note that the optimal price does not depend on b in the uniform case. The optimal price prc^* depends on the one-time setup cost faced by the storage operator and the tax faced by the emitter. Both a higher setup cost and higher tax increase the price the storage operator can charge; a higher tax also increases the emitter's willingness to participate in CCS. In addition, the optimal price is independent of the contracted amount since

$$\frac{\partial prc^*}{\partial con^*} = \frac{-\alpha_2 \cdot \overline{F}(con^*) + \alpha_1}{2 \cdot \int_0^{cap} \overline{F}(em)dem} = \frac{-\alpha_2 \cdot (1 - F(F^{-1}(\frac{\alpha_2 - \alpha_1}{\alpha_2})) + \alpha_1}{2 \cdot \int_0^{cap} \overline{F}(em)dem} = 0.$$

That is, a higher difference between the costs of pipeline and other means of transportation (α_1 and α_2) leads to a higher contract amount the storage operator is willing to commit to, but the optimal price does not change accordingly.

5. Experimental results

In this section, we provide numerical examples to assess the effect of the distributions of *cc* and *em* on the optimal policies. The contract decisions are computed in two stages. In the first stage, the optimal contract amount is calculated independently of the price. In the second stage, we set the optimal price. While the optimal values can be calculated numerically for given distributions of *cc* and *em*, we can also use simulation to choose the contract values. Simulation is an easy way to generate expected profit values for different contract options. We demonstrate that more information about *cc* and *em* (i.e., lower variance in the distributions) can lead to higher expected profits for the storage operator.

We use the parameter values given in Table 2. These values are taken to be representative of realistic costs based on current experience (Herzog and Golomb, 2004; Rubin et al., 2007; Sumner et al., 2011). Due to the immaturity of the CCS industry and the site-specific nature of all costs, however, estimated values for each of these parameters can cover very broad ranges and are the subject of some debate.

Table 2: Parameter values used in numerical solution.

Parameter	Value
K	\$80 million
α_1	\$6 per tonne
α_2	\$15 per tonne
cap	80 Mt
tax	\$100 per tonne
Mean of em	2.5 Mt
Mean of cc	\$50 per tonne

5.1 Uniform distributions

Simulation optimization has recently become a very popular method for finding optimal solutions when objective functions can be evaluated via simulation (for an overview, see Fu et al. (2008)). In order to estimate the optimal solution, we simulate many possible values of *cc* and *em* according to their distributions g and f, and determine the optimal values of *prc* and *con* that would provide the highest average profit using those simulated values. As a first example, we consider uniform distributions for em and cc, which take the form $U(a, b)$ with a and b as the lower and upper limits of the distribution. Specifically, we use a distribution of the form $U(\mu - k\sigma, \mu + k\sigma)$, where μ is the mean of the distribution and σ is some level of variation in the distribution. We vary k (using the same k for both cc and em) to show the effect on the optimal solution when the ranges of the distributions vary.

Table 3: Optimal solutions for different values of k when the distributions of em and cc take the form $U(\mu - k\sigma, \mu + k\sigma)$ with $\mu_{em} = 2.5$ Mt, μ_{cc} =\$50/tonne, $\sigma_{em} = 0.5$ Mt, and σ_{cc} =\$10/tonne.

k	con^* [Mt]	prc^* [\$/tonne]	Optimal Profit (millions)
0.25	2.52	\$47.92	\$24.42
0.5	2.54	\$46.46	\$18.92
1	2.59	\$48.73	\$14.68

Table 3 shows that the optimal expected profit increases as the range of the uniform distributions decrease. In other words, there is a higher level of certainty available on the estimate of the mean values of cc and em, a higher profit can be obtained. Figure 1 shows graphically how the profit function narrows as the variance of the distribution decreases. However, we note that the optimal choices of con^* and prc^* do not change as drastically as the profit function, suggesting that a robust solution is available given uncertainty in the variance of the distribution.

5.2 Normal distributions

We now consider normal distributions for cc and em as $\mathcal{N}(\mu_{cc}, \sigma_{cc})$ and $\mathcal{N}(\mu_{em}, \sigma_{em})$, respectively. The μ parameters are the sample means of the distributions, while the σ values are the standard deviations. We modify the coefficient of variation (CoV), which is defined to be σ/μ for the normal distribution, and represents the relative level of dispersion given the mean. Keeping the mean constant, we modify the CoV through the standard deviation (using the same CoV for both cc and em). Table 4 displays the optimal solutions, and we again see that a higher optimal profit can be obtained when the uncertainty in the distribution is lower.

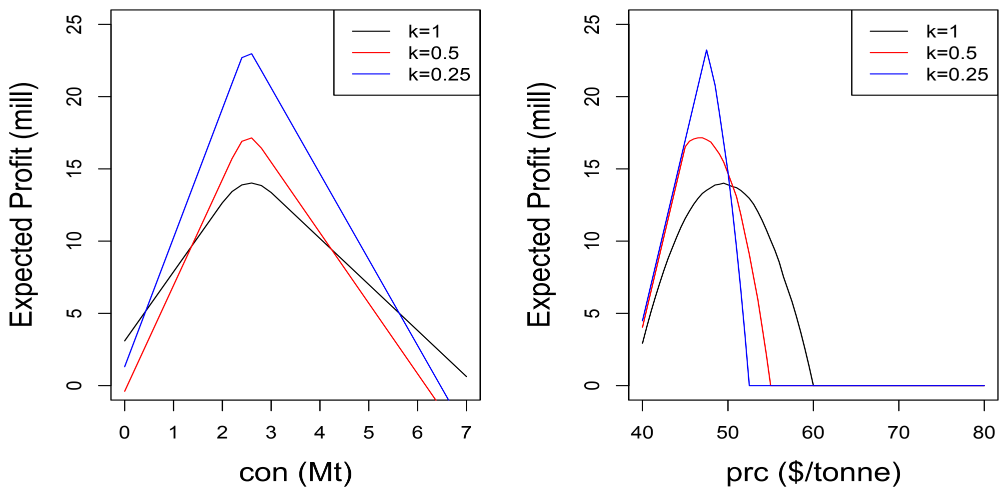

Figure 1: Profit functions calculated for uniform distributions using distribution from Table 3.

Table 4: Optimal solutions for different values of σ/μ for $\mathcal{N}(\mu, \sigma)$ with $\mu_{em} = 2.5$ Mt and μ_{cc} =\$50/tonne.

CoV	con^* [Mt]	prc^* [\$/tonne]	Optimal profit (millions)
1/6	2.61	\$49.90	\$14.21
1/5	2.62	\$50.41	\$13.74
1/4	2.64	\$53.00	\$13.60

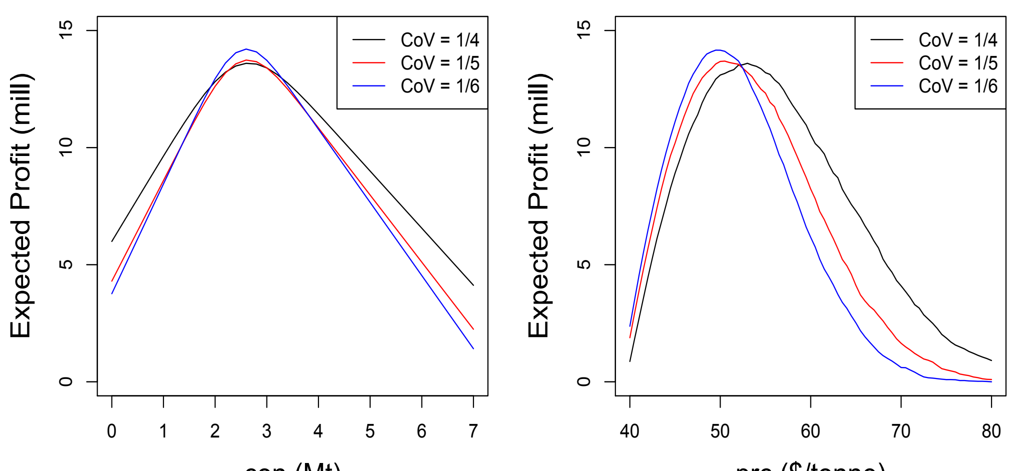

Figure 2: Profit functions calculated normal distribution from Table 4.

We see that as the uncertainty (CoV) in the distribution decreases, the expected profit increases and the profit function becomes steeper. The expected profit increases despite the fact that con^* and prc^* decrease, and this is because there is a lower probability of a loss in revenue due to the contract being denied, or the contract amount being greatly different from the emissions amount.

To better view the profit function over choices of (prc, con), we plot the expected profit over both dimensions. Figure 3 shows a contour plot of the expected profit under the different options of (prc, con). We see that for many solutions, the expected profit to the storage operator is negative. Therefore, the decision of the amount of CO_2 to store, and the best price to charge the emitter, are important for making this venture profitable.

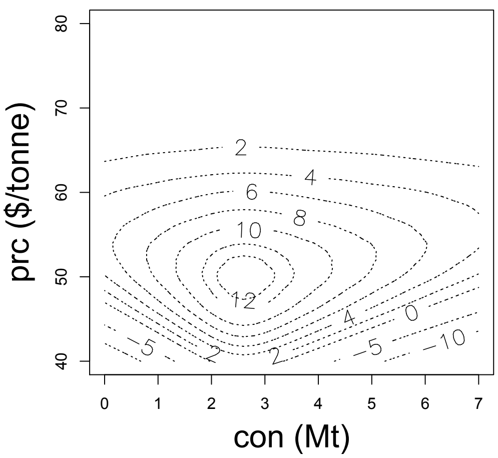

Figure 3: Contour plot of E[Π] over *prc* and *con* using normal distributions with a CoV of 1/6.

5.3 Correlated distributions

Lastly, we use simulation optimization to find optimal solutions for cases where the distributions of *em* and *cc* are correlated. For example, economies of scale may lead to a lower capture cost, suggesting *em* and *cc* might be negatively correlated.

By jointly simulating *cc* and *em* according to correlated distributions, we can capture the potential dependence in the randomness and find the appropriate optimal solution. As an example, we simulate *em* and *cc* from the same normal distributions as used in the previous section with CoV values of 1/6 as in Figure 3, but with correlations of −0.5 and 0.5. Figure 4 shows the contour plots of the expected profit function for the correlated cases.

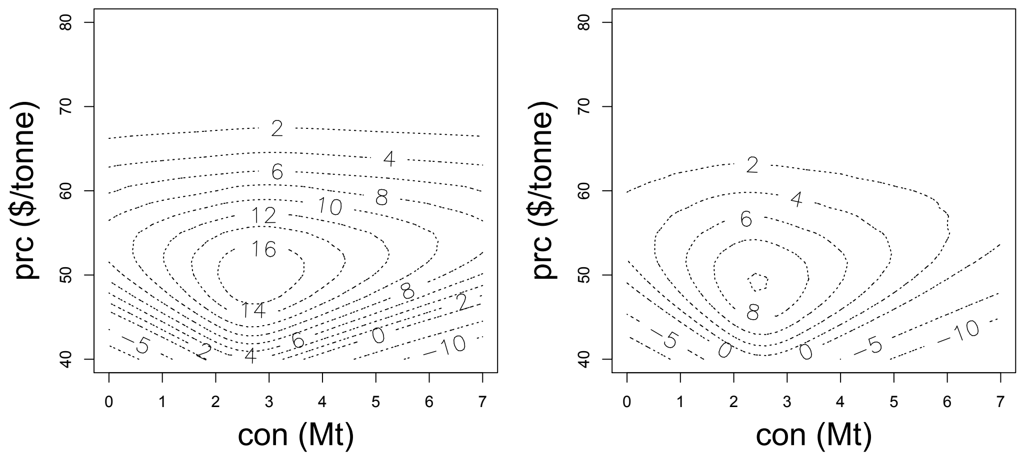

Figure 4: Contour plot of E[Π] over *prc* and *con* using normal distributions with a CoV of 1/6, and a correlation of −0.5 (left) and 0.5 (right) between *em* and *cc*.

The left plot of Figure 4 shows that a higher expected profit can be obtained when the emission quantity and capture cost are negatively correlated. This is because larger plants who emit more tend to have lower capture costs and so are more likely to accept the contract for a given price, leading to a higher revenue for the storage operator. When the emission quantity and the capture cost are positively correlated, however, larger plants tend to have very high costs and are thus less likely to accept the contract. As a result, the storage operator's expected profit is lowered. This intuition is supported by the right plot of Figure 4.

6. Conclusions

We propose a stylized model to address the problem of incentivizing both the storage operator and the emitter to participate in CCS. While a tax on emissions would encourage power plants and other emitters to reduce the level of CO_2 released into the atmosphere, in order for CCS to contribute to this reduction it must be profitable for an independent party to invest in transportation and storage technology. Profit maximization policies can be used to estimate the likely amount of CO_2 that could be stored if such a party was to take advantage of a carbon tax. Given the relative newness of such contracts and CCS technology, uncertainty in the models allow for both parties to better deal with the risk associated with such contracts.

The methods used in this paper for designing optimal contracts can be extended to more complicated models. For example, other costs or revenues, whether deterministic or stochastic, to both emitters and storage operators, could be added to the profit functions. More sophisticated pricing mechanisms can also be incorporated using the same methods, and we intend to continue to explore these methods.

Future work will design contracts over networks of emitters and storage operators. It is likely that one storage operator will attempt to enter into contracts with multiple emitters, each with different costs. This would allow the storage operator to pool the risk associated with different emitters and potentially increase the amount of CO_2 stored.

Appendix

Proof of Proposition 1. To find the optimal value of *con* that maximizes the storage operator's expected profit, we first solve the unconstrained problem and maximize the objective function (1). We take the derivative of the expected profit function with respect to *con* and set it equal to 0. Applying Lebniz rule, we obtain the following equation:

$$\begin{aligned}\frac{\partial E[\Pi(con, prc)]}{\partial con} &= (prc \cdot con) f(con) + \int_{con}^{cap} \alpha_2 \cdot f(em) dem \\ &\quad -(prc \cdot con) f(con) + \int_{cap}^{\infty} \alpha_2 \cdot f(em) dem - \alpha_1 \\ &= \int_{con}^{cap} \alpha_2 f(em) dem + \int_{cap}^{\infty} \alpha_2 f(em) dem - \alpha_1 \\ &= \alpha_2 (1 - F(con)) - \alpha_1 = 0. \end{aligned} \tag{7}$$

We thus find the stationary point as

$$con^* = F^{-1}\left(\frac{\alpha_2 - \alpha_1}{\alpha_2}\right).$$

We now verify that the second order condition is satisfied at con^*, i.e.

$$\left.\frac{\partial^2 E[\Pi(con,prc)]}{\partial con^2}\right|_{con^*} = -\alpha_2 f(con)|_{con^*} = -\alpha_2 f(con^*) < 0.$$

Thus, con^* is the optimal solution that maximizes the storage operator's expected profit. Since the storage operator cannot set the contract amount to exceed the maximum capacity of the operator, and the expected profit function is concave for all values of con for which $f(con) > 0$, we set the optimal contract amount to be $con^* = \max\{F^{-1}\left(\frac{\alpha_2-\alpha_1}{\alpha_2}\right), cap\}$. □

Proof of Proposition 2. First we re-write $E[\Pi(con^*,prc)]$ as

$$\begin{aligned}
E[\Pi(con^*,prc)] &= prc\cdot\left(\int_0^{cap} em\cdot f(em)dem + \int_{cap}^{\infty} cap\cdot f(em)dem\right) \\
&\quad -\alpha_2\cdot\left(\int_{con^*}^{cap}(em-con^*)f(em)dem + \int_{cap}^{\infty}(cap-con^*)f(em)dem\right) \\
&\quad -\alpha_1\cdot con^* - K \\
&= prc\cdot\left(em\cdot F(em)\Big|_0^{cap} - \int_0^{cap} F(em)dem + cap\cdot(1-F(cap))\right) \\
&\quad -\alpha_2\cdot\left((em-con^*)\cdot F(em)\Big|_{con^*}^{cap} - \int_{con^*}^{cap} F(em)dem\right) \\
&\quad -\alpha_2\cdot(cap-con^*)\cdot(1-F(cap)) - \alpha_1\cdot con^* - K \\
&= prc\cdot\left(cap\cdot F(cap) - \int_0^{cap} F(em)dem + cap - cap\cdot F(cap)\right) \\
&\quad -\alpha_2\cdot\left((cap-con^*)\cdot F(cap) - \int_{con^*}^{cap} F(em)dem\right) \\
&\quad -\alpha_2\cdot((cap-con^*) - (cap-con^*)\cdot F(cap)) - \alpha_1\cdot con^* - K \\
&= prc\cdot\left(cap - \int_0^{cap} F(em)dem\right) \\
&\quad -\alpha_2\cdot\left((cap-con^*) - \int_{con^*}^{cap} F(em)dem\right) - \alpha_1\cdot con^* - K \\
&= prc\cdot\int_0^{cap}\overline{F}(em)dem - \alpha_2\cdot\int_{con^*}^{cap}\overline{F}(em)dem - \alpha_1\cdot con^* - K.
\end{aligned}$$

Thus, $\frac{\partial E[\Pi(con^*,prc)]}{\partial prc} = \int_0^{cap}\overline{F}(em)dem$. To find the optimal value of prc that maximizes the storage operator's expected profit, take the derivative of the expected profit function $E[\Pi']$ with respect to prc and set it equal to 0:

$$\begin{aligned}
\frac{\partial E[\Pi^{'}(con^*, prc)]}{\partial prc} &= \frac{\partial (E[\Pi(con^*, prc)] \cdot G(tax - prc))}{\partial prc} \\
&= \frac{\partial E[\Pi(con^*, prc)]}{\partial prc} \cdot G(tax - prc) - E[\Pi(con^*, prc)] \cdot g(tax - prc) \\
&= G(tax - prc) \cdot \int_0^{cap} \overline{F}(em) dem - g(tax - prc) \cdot E[\Pi(con^*, prc)] \\
&= 0.
\end{aligned}$$

The stationary point (prc^*) solves the above equation. Next, we check second order condition at prc^*:

$$\begin{aligned}
\left.\frac{\partial^2 E[\Pi]}{\partial prc^2}\right|_{prc^*} &= -g(tax - prc^*) \cdot \int_0^{cap} \overline{F}(em) dem + g'(tax - prc^*) \cdot E[\Pi(con^*, prc^*)] \\
&\quad - g(tax - prc^*) \cdot \frac{\partial E[\Pi(con^*, prc^*)]}{\partial prc} \\
&= g'(tax - prc^*) \cdot E[\Pi(con^*, prc^*)] - 2g(tax - prc^*) \cdot \int_0^{cap} \overline{F}(em) dem.
\end{aligned}$$

When the value of the second derivative is less than 0, prc^* is the optimal solution that maximizes the storage operator's expected profit. In addition, to ensure the operator's expected profit is non-negative, we further restrict $E[\Pi(con^*, prc^*)] \geq 0$. □

Arts, R., Chadwick, A., Eiken, O., Thibeau, S., Nooner, S., 2008. Ten years' experience of monitoring CO_2 injection in the Utsira Sand at Sleipner, offshore Norway. First break 26 (1).

Chadwick, R., Williams, G., Williams, J., Noy, D., 2012. Measuring pressure performance of a large saline aquifer during industrial-scale CO_2 injection: The Utsira Sand, Norwegian North Sea. International Journal of Greenhouse Gas Control 10, 374–388.

Esposito, R. A., Monroe, L. S., Friedman, J. S., 2011. Deployment models for commercialized carbon capture and storage. Environmental science & technology 45 (1), 139–146.

Fu, M. C., Chen, C.-H., Shi, L., 2008. Some topics for simulation optimization. In: Proceedings of the 40th Conference on Winter Simulation. Winter Simulation Conference, pp. 27–38.

Global CCS Institute, 2012. Global status of large-scale integrated CCS projects, June 2012 update. Global CCS Institute, Canberra, Australia.

Herzog, H., Golomb, D., 2004. Carbon capture and storage from fossil fuel use. Encyclopedia of Energy 1, 1–11.

International Energy Agency, 2010. Energy Technology Perspectives 2010: Scenarios and Strategies to 2050. International Energy Agency, Paris, France.

Middleton, R., Bielicki, J., 2009. A scalable infrastructure model for carbon capture and storage: SimCCS. Energy Policy 37 (3), 1052–1060.

Middleton, R., Kuby, M., Wei, R., Keating, G., Pawar, R., 2012. A dynamic model for optimally phasing in CO_2 capture and storage infrastructure. Environmental Modelling & Software.

Pacala, S., Socolow, R., 2004. Stabilization wedges: solving the climate problem for the next 50 years with current technologies. Science 305 (5686), 968–972.

Rubin, E. S., Chen, C., Rao, A. B., 2007. Cost and performance of fossil fuel power plants with CO_2 capture and storage. Energy Policy 35 (9), 4444–4454.

Sumner, J., Bird, L., Dobos, H., 2011. Carbon taxes: a review of experience and policy design considerations. Climate Policy 11 (2), 922–943.