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# Optimization of an internally finned rotating heat pipe.

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Optimization of an Internally
Finned Rotating Heat Pipe

by

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# ABSTRACT

A finite element formulation with linear triangular elements was used to solve the steady-state, two-dimensional conduction heat transfer equation in the condenser wall section of an internally finned rotating heat pipe. A FORTRAN program using this method was coupled with the ADS program for automated design of the internal heat pipe fin geometry to optimize heat transfer. An increase in surface area, which increases heat transfer, also increases the condensate level, which decreases heat transfer. The additional condensate level does not offset the advantage gained by the increased surface area. The investigation provided combinations of fin half angle, number of fins, and fin height for an optimum design. Water is used as the working fluid and the heat pipe is constructed from copper.

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# I. INTRODUCTION

## A. THE ROTATING HEAT PIPE

The rotating heat pipe is a closed container designed to transfer a large amount of heat in rotating machinery. Since the heat pipe operates on a closed two-phase cycle, the heat transfer capacity is greater than for solid conductors. Also, the thermal response time is less than with solid conductors. The three major elemental parts of the rotating heat pipe are: a cylindrical evaporator, a truncated cone condenser, and a fixed amount of working fluid as shown in figure 1.

An annulus is formed by the working fluid in the evaporator. This occurs at rotationary speeds above the critical speed. The addition of heat to the evaporator vaporizes the working fluid. A pressure differential between the evaporator and the condenser causes the vapor to flow towards the condenser. The vapor is transported to the condenser with its latent heat of vaporization. Condensation of the vapor on the inner wall is caused by external cooling. This condensation releases the latent heat of evaporation. This condensate is forced to flow back to the evaporator by a component, acting along the condenser wall, of the centrifugal force which is caused by the rotation of the heat pipe. As the condensate collects in the evaporator the cycle is repeated.

Since the evaporator and condenser portions of a heat pipe function independently, needing only common liquid and vapor streams, the area over which heat is introduced can differ in size and shape from the area

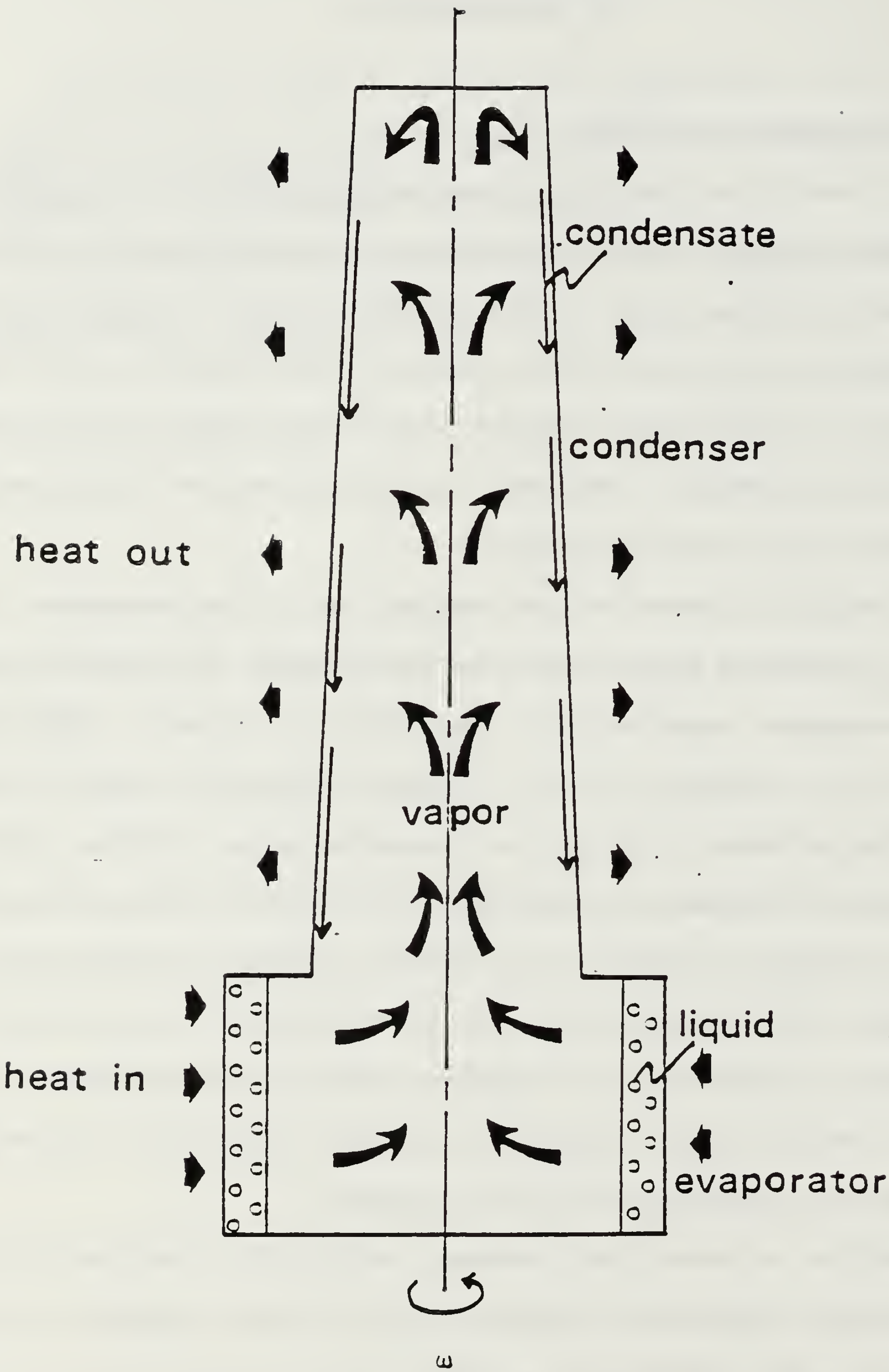



Figure 1. Schematic Drawing of a Rotating Heat Pipe

over which it is rejected, provided that the rate at which the liquid is vaporized does not exceed the rate at which it can be condensed. Therefore, high heat fluxes generated over relatively small areas can be dissipated over larger areas with reduced heat fluxes; allowing a cylindrical evaporator and a truncated cone condenser.

Capillary action acts to drive the condensate back to the evaporator in a conventional heat pipe. No limitation due to capillary action is encountered in a rotating heat pipe nor are external pumps or gravity depended on for the flow of the working fluid. Therefore, the rotating heat pipe can be used in any orientation [Ref. 1].

## B. OPERATING LIMITS OF A ROTATING HEAT PIPE

The first theoretical investigation of the rotating heat pipe conducted at the Naval Postgraduate School was performed by Ballback [Ref. 2] in 1969. Various fluid dynamic mechanisms limit the performance of a rotating heat pipe. Ballback [Ref. 2] studied these mechanisms and an estimation of the sonic limit, boiling limit, entrainment limit, and condensing limit of performance was made.

### 1. The Sonic Limit

The maximum flow of the vapor is set by the choked flow condition in the rotating heat pipe. This limiting vapor flow rate occurs when the heat flux is increased and limits the amount of energy the vapor can transport. The rotating heat pipe effectiveness is reduced due to this limitation. The limiting heat transfer rate due to this condition is:

$$Q_t = \rho_v U_v A h_{fg} \tag{1}$$

and the vapor velocity is considered to be sonic,

$$U_v = c = (g_o kRT)^{\frac{1}{2}} \tag{2}$$

where

$U_V$ = velocity of the vapor in ft/sec, and

A = cross sectional area for the vapor flow in ft

c = sonic velocity in ft/sec

$g_o$ = gravitational constant

k = ratio of specific heats

R = gas constant in ft-lbf/lbm R

T = absolute temperature in degrees Rankine

$\rho_V$ = density of vapor in lbm/ft$^3$

2. **The Boiling Limit**

The transition from nucleate to film boiling was hypothesized by Kutateladze [Ref. 3] to be a completely hydrodynamic process. He determined the following theoretical formula for predicting the burnout flux:

$$Q_t = K\sqrt{\rho_v}\, A_b h_{fg} \{\sigma g(\rho_f - \rho_v)\}^{\frac{1}{4}} \tag{3}$$

where

K = constant value

$A_b$ = heat transfer area in the boiler in ft$^2$

$h_{fg}$ = latent heat vaporization in Btu/lbm

$\sigma$ = surface tension in lbf/ft

g = acceleration of gravity in ft/hr$^2$

$\rho_f$ = density of fluid in lbm/ft$^3$

$\rho_v$ = density of vapor in lbm/ft$^3$.

A constant value for K in the range of 0.13 to 0.19 was suggested by the experimental data obtained by Kutateladze.

3. **The Entrainment Limit**

The flooding constraint in a wickless heat pipe was examined by Sakhuja [Ref. 4]. The correlation he developed is:

$$Q_t = \frac{A_x C^2 h_{fg}\sqrt{gD(\rho_f-\rho_v)\rho_v}}{\{1+(\rho_v/\rho_f)^{\frac{1}{4}}\}^2} \tag{4}$$

where

$Q_t$ = heat transfer rate in Btu/hr

$A_x$ = flow rate in ft$^2$

C = dimensionless constant, 0.725 for tube with sharp edged flange

$h_{fg}$ = latent heat of vaporization in Btu/lbm

g = acceleration due to gravity in ft/hr$^2$

D = inside diameter of heat pipe in ft

$\rho_f$ = density of the fluid in lbm/ft$^3$

$\rho_v$ = density of the vapor in lbm/ft$^3$.

4. **The Condensing Limit**

The condenser section of a rotating heat pipe was modeled as a truncated cone by Ballback [Ref. 2]. Using this model, the condensation limitation for a rotating heat pipe was determined by Ballback [Ref. 2]. He developed the following condensation limit:

$$Q_t = \pi\left\{\frac{2}{3}\frac{K_f\rho_f\omega^2 h_{fg}(T_s-T_\omega)^3}{\mu_f\sin^2\phi}\right\}^{1/4}\left[(R_o+L\sin\phi)^{8/3}-R_o^{8/3}\right]^{3/4} \quad (5)$$

where

$Q_t$ = total heat transfer rate in Btu/hr

$k_f$ = thermal conductivity of the condensate film in Btu/hr-ft-F

$\rho_f$ = density of fluid in lbm/ft$^3$

$\omega$ = angular velocity in 1/hr

$h_{fg}$ = latent heat of vaporization in Btu/lbm

$T_s$ = saturation temperature in F

$T_w$ = inside wall temperature in F

$\mu_f$ = viscosity of fluid in lbm/ft-hr

$\phi$ = half cone angle in degrees

$R_o$ = minimum wall radius in ft

L = length along the wall of the condenser in feet.

The geometry and speed of the rotating heat pipe, and the physical properties of the working fluid comprise the condensing limit equation.

For a rotating heat pipe with the physical characteristics as shown in Table I, the amount of heat that can be transferred from the rotating heat pipe is limited by the condensing limit. However, the limitations imposed by the sonic limit, boiling limit, and entrainment limit may become important as the heat pipe geometry and operating conditions vary.

TABLE I. ROTATING HEAT PIPE SPECIFICATIONS

| | |
|---|---|
| Length | 9.000 inches |
| Minimum Diameter | 1.55 inches |
| Wall Thickness | 0.03125 inches |
| Internal Half Angle | 1.000 degree |
| Rotating Speed | 3600 RPM |

To enhance the heat transfer capacity of the rotating heat pipe, internally finned condensers have been used to raise the condensing limit line. Thinner films occur near the ridges of the fins while thicker films occur in the troughs. The thinner film on the ridges provides a lower thermal resistance to heat flow, while the thicker film in the trough provides a higher resistance. A compromise between the improvement on the ridges and the degradation in the troughs is necessary for an overall heat transfer improvement [Ref. 5].

## C. ANALYSIS OF THE INTERNALLY FINNED ROTATING HEAT PIPE

Schafer [Ref. 6] developed an analytical model for a heat pipe with a triangular fin profile (figure 2). This model was developed in order to raise the condensing limit by the addition of internal fins. An assumption of one-dimensional heat conduction through the wall and fin was made for Schafer's model.

A two-dimensional heat conduction model using a Finite Element Method was developed for this same case by Corley [Ref. 7]. A parabolic temperature distribution along the fin surface was assumed by Corley [Ref. 7]. A significant improvement in the predicted heat transfer performance was indicated by his results. By using the two-dimensional model an increase of approximately 75 percent in the heat transfer performance was seen over the results from the use of the one dimensional model. However, Corley [Ref. 7] noted that at the fin apex an error as great as 50 percent was possible which could result in the total heat transfer being in error as much as 15 percent.

A modification was made to Corley's computer program by Tantrakul [Ref. 8]. In order to minimize the heat transfer error at the apex of the fin Tantrakul increased the number of finite elements used. His results with this modification converged with the results of Corley.

Purnomo developed a linear triangular finite element model (figure 3) used in a two-dimensional Finite Element Method solution. Purnomo's [Ref. 1] Finite Element Method program also worked and converged. To maximize the heat transfer from the rotating heat pipe the condenser geometry was varied. Using Purnomo's code parametric studies were conducted. However, the best geometry was not indicated in these studies. Purnomo's code was written to perform one analysis at a time. Davis [Ref. 9] modified Purnomo's code to allow for numerous analysis to be made using the optimization code COPES/CONMIN. Davis' Finite Element Method code incorporating the optimization worked and converged, resulting in an optimum design for an internally finned rotating heat pipe.

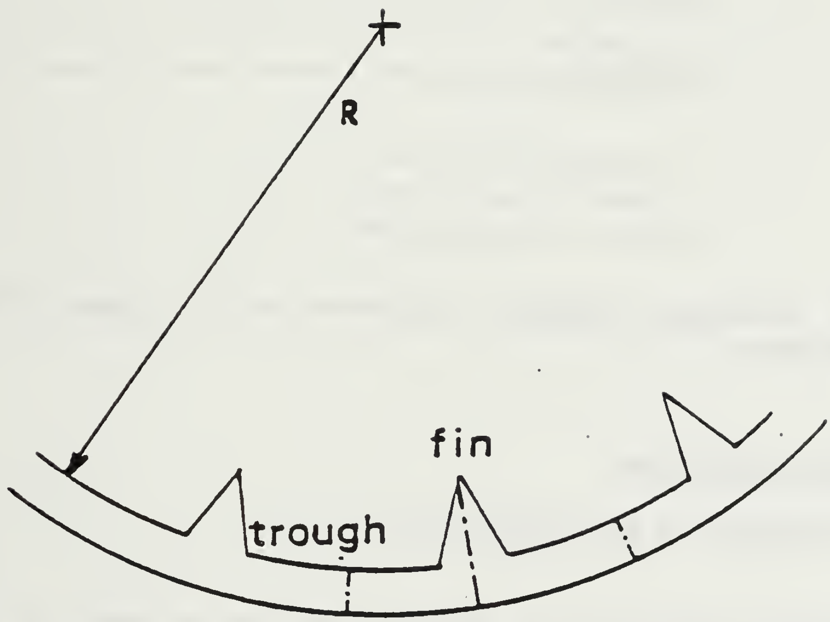
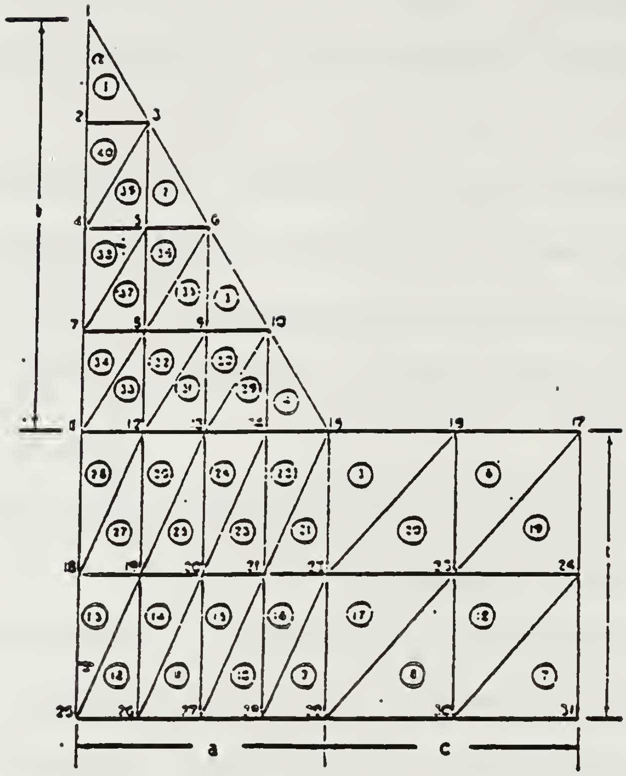



Figure 2. Internally Finned Condenser Geometry, Showing Fins, Troughs and Lines of Symmetry.




t = thickness

b = height of the fin

ø = condenser half angle

a = half of the fin width

c= half of the trough width

Figure 3. Condenser Geometry Considered with 40 Linear Triangular Finite Elements.

## D. THESIS OBJECTIVES

The objectives of this thesis were:

1. To modify Davis' [Ref. 9] computer program so that it is compatible with the ADS (Automated Design Synthesis) program [Ref. 10] and can be used for analysis and automated design of rotating heat pipes.

2. To use the resulting program to obtain an optimum design for an internally finned rotating heat pipe to obtain experimental data to compare with the analytical results.

3. To use the resulting program to obtain numerical results in place of data obtained from costly experimental operations.

# II. NUMERICAL OPTIMIZATION

## A. BACKGROUND

The parameter that is minimized or maximized during the design process is called the design objective. The design objective is minimized or maximized by changing the design variables within the design constraint limitations. This process is called numerical optimization. An assortment of physical, aesthetic, economic and, on occasion, political limitations must be met by the design constraints for the design to be acceptable. For the optimization process to work, the design criteria must be described in numerical terms. This is not always easy.

A computer program can be written to perform tedious and repetitive calculations necessary to optimize the problem once it is stated in numerical terms. For this reason, computer analysis is commonplace in most engineering organizations. For example, in heat transfer design the configuration, materials, and method of heat removal may be defined and a finite element analysis computer code is used to calculate temperatures, heat transfer rates, and other response quantities of interest. If any of these parameters are not within prescribed bounds, the engineer may change the method of cooling or other defined quantity and rerun the program. The engineer makes the actual design decisions, the computer code only provides the analysis of a proposed design. This is the commonly used approach which is called computer-aided design.

Analysis codes are commonly used for tradeoff studies. For example, an analysis code might be run on the distance a truck can go on a tank of fuel. For different loads, different distances are calculated which can be used in a range-payload study.

Fully automated design is the logical next step to computer-aided design. The computer makes the actual design decisions or trade-off studies based on input criteria in fully automated design. Minimal information is requested from the operator during the actual design process. Numerical optimization offers numerous improvements over the traditional approach to design. These improvements include: time reduction in design decision making; a rational, directed design procedure; and the procedure is unbiased by intuition or experience. The probability of obtaining a non-traditional solution is thereby improved. Engineering intuition and experience are still necessary to decide if the design obtained is an improvement and feasible.

## B. AUTOMATED DESIGN SYNTHESIS (ADS)

Vanderplaats [Ref. 10] developed a general purpose numerical optimization program containing a variety of algorithms, ADS. ADS is a FORTRAN program that optimizes a numerically defined objective function subject to a set of constraint limits. The solution of the problem is separated into three levels:

1. Strategy - Optimization strategy such as Augmented Lagrange Multiplier method or Sequential Linear Programming.
2. Optimizer - Actual algorithm to perform the optimization.
3. One-Dimensional Search - Line search routine used by optimizer.

Flexibility to solve a wide variety of engineering design problems is given by the combinations of nine strategies, five optimizers, and eight one-dimensional search options. The following definitions are necessary to discuss the use of ADS:

1. Design Variables - Those parameters which the optimization program is permitted to change within allowed bounds in order to improve the design. Design variables appear only on the right hand side of an equation and are continuous.

2. Design Constraints - An inequality constraint requires that some function of the design variable(s) remain less than a specified value. Design constraints may be linear or nonlinear, implicit or explicit, but they must be continuous functions of the design variable.

3. Objective Function - The parameter which is going to be minimized or maximized during the optimization process. The objective function may be linear or nonlinear, implicit or explicit, and must be a continuous function of the design variables. The objective function usually appear on the left side of an equation.

## C. PROGRAMMING GUIDELINES

Any computer code developed for engineering analysis should be written in such a way that it is easily coupled to a general purpose optimization program such as ADS. Therefore, a general programming practice is outlined here which in no way inhibits the use of the computer program in its traditional role as an analytical tool, but allows for simple adaption to ADS.

ADS is called by a user-supplied calling program. ADS does not call any user-supplied subroutines. Instead, ADS returns control to the calling program when function or gradient information is needed. The required information is evaluated and ADS is called again. This provides considerable flexibility in program organization and restart capabilities. Various internal parameters are defined on the first call to ADS which work well for the "average" optimization task. However, it is often desirable to change these in order to gain maximum utility of the program. Figure 4 is the program flow diagram for the case where the user wishes to over-ride one or more internal parameters, such as scaling, convergence criteria, or maximum number of iterations.

After initialization of basic parameters and arrays, the information parameter, INFO, is set to -2. ADS is then called to initialize all internal parameters and allocate storage space for internal arrays. Control is then returned to the user, at which point these parameters, for example convergence criteria, can be overridden if desired. At this point, INFO will have a value of 1 and the user must evaluate the objective function, OBJ, and constraint functions. ADS is called again and the optimization proceeds. Since, in this case, the gradient calculation control, IGRAD, has a value of zero, all gradient information is calculated by finite difference within ADS. When INFO has a value of zero, optimization is complete.

BEGIN

DIMENSION ARRAYS

DEFINE BASIC VARIABLES

IGRAD=0 (USE FINITE DIFFERENCE GRADIENTS)

INFO = -2

CALL ADS (INFO...)

IF INFO = 0, EXIT. ERROR WAS DETECTED

ELSE

OVER-RIDE DEFAULT PARAMETERS IN ARRAYS WK AND IWK IF DESIRED

CALL ADS (INFO...)

NO | YES

INFO = 0

EVALUATE OBJECTIVE AND CONSTRAINTS | EXIT OPTIMIZATION IS COMPLETE

Figure 4. Program Flow Logic: Over-Ride Default Parameters, Finite Difference Gradients [Ref. 10].

# III. FINITE ELEMENT SOLUTION

## A. REVIEW OF THE PREVIOUS ANALYSIS

As stated previously, the heat transfer solution for a one-dimensional model of an internally finned rotating heat pipe was studied by Schafer [Ref. 6]. The two-dimensional model was studied by Corley [Ref. 7]. The same assumptions and boundary conditions, similar to those used in the Nusslet analysis of film condensation on a flat wall, and based upon the analysis of Ballback [Ref. 2] were used for both. The more important of these assumptions are:

1. steady state operation,
2. film condensation, as opposed to dropwise condensation,
3. laminar flow of the condensate film along both the fin and the trough,
4. static balance of forces within the condensate,
5. one-dimensional conduction heat transfer through the film thickness (no convective heat transfer in the condensate film),
6. no liquid-vapor interfacial shear forces,
7. no condensate subcooling,
8. zero heat flux boundary conditions on both sides of the wall section (symmetry conditions), as shown in figure 5,
9. saturation temperature at the fin apex,
10. zero film thickness at the fin apex, and
11. negligible curvature of the condenser wall.

Figure 3 shows the linear triangular finite element model developed by Purnomo [Ref. 1] for use in obtaining a two-dimensional Finite Element solution.

The assumption that was used by Corley [Ref. 7] that the fin apex was at the saturation temperature of the working fluid was modified by Purnomo [Ref. 1]. The value of the temperature at the apex of the fin was allowed to float and a parabolic temperature distribution was assumed along the fin surface.

Purnomo's problem statement for the formulation of the Finite Element Method as shown in figure 5 is:

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{6}$$

with the following boundary conditions:

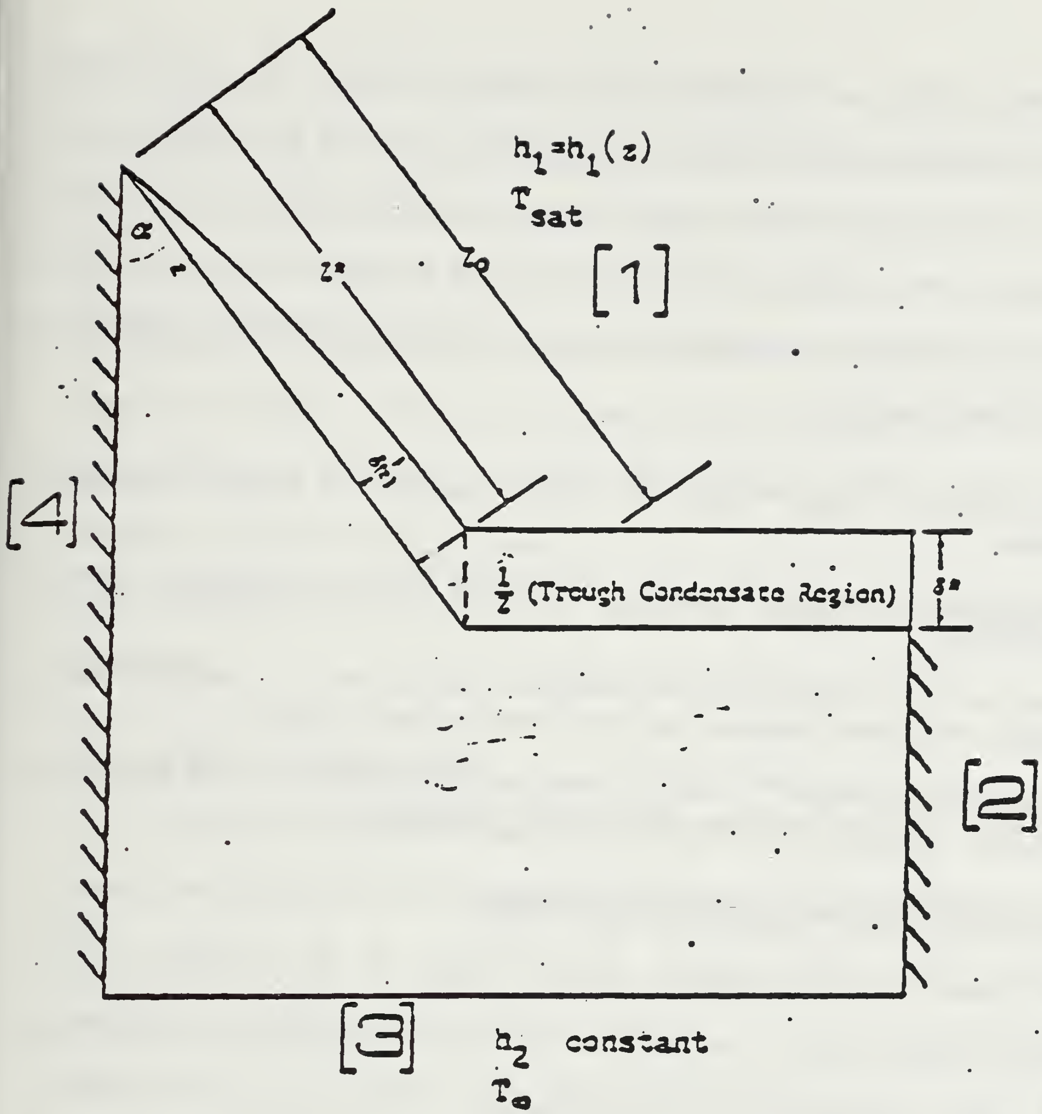
1. along boundary 1, $-K\ \partial T/\partial n = h_1(T - Tsat)$

2. along boundary 3, $-K\ \partial T/\partial n = h_2(T - T\infty)$

3. along boundaries 2 and 4, $\dfrac{\partial T}{\partial n} = 0$

A detailed description of the numerical formulation is presented in his thesis.

Davis used Constrained Function Minimization (CONMIN) as an optimization program. CONMIN is a FORTRAN program in subroutine form.




$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

b.c.

a) $-k \frac{\partial T}{\partial n} = h_1 (T - T_{sat})$ Along Boundary [1]

b) $-k \frac{\partial T}{\partial n} = h_2 (T - T_\infty)$ Along Boundary [3]

c) $\frac{\partial T}{\partial n} = 0$ Along Boundaries [2] and [4]

Figure 5. Differential Equation and Boundary Conditions Considered in the Analysis of Purnomo [Ref. 1].

Vanderplaats [Ref. 11] developed the Control Program for Engineering Synthesis (COPES) as a main program to simplify the use of CONMIN. Davis' computer program was written in subroutine form with SUBROUTINE ANALIZ (ICALC) as the main routine. The name ANALIZ is compatible with the COPES program and ICALC is a calculation control. Subroutine ANALIZ calls other subroutines as needed:

1. the routine "COORD" used to define positions of system coordinate points
2. the routine "FORMAF" used to formulate the Finite Element Method equations,
3. the routine "BANDEC" as an equation solver for a symmetric matrix which has been transformed into banded form, and
4. the routine "DPLORT" used to compute the roots of a real polynomial using a Newton-Raphson derivative technique.

## B. THE COMPUTER PROGRAM DEVELOPMENT

The basis for the present analysis code is Davis' [Ref. 9] two-dimensional finite element program. Davis' code was checked for validity as the first task undertaken in the development of this thesis. An error was discovered in calculating the fin condensate film thickness (AZS). In the initial calculation of HDEN only, the cubed term was merely multiplied by three. The correct form of the equation is shown below:

$$HDEN = -a_1z^3/3-b_1z^2/2+z(T_{sat}-T_1)$$

The effect of this error was minimal since subsequent equations were correct, a 0.00016% difference in the condensate level was noted.

The next task undertaken was to adapt the analysis code to permit automated design analysis using ADS. Many modifications were made, some of which are mentioned here. The program was rewritten to include a main program from which ADS is called and subroutines to perform various mathematical functions. Subroutine ANALIZ was deleted since the double precision version of ADS (DADS) was used. Modifications were also made to generalize the code and minimize the changes needed when varying the number of finite elements used.

A listing of the revised computer program is included as the Appendix.

## C. DESIGN OPTIMIZATION

There are thirteen parameters that can be used as design variables. There are geometric or functional parameters of the rotating heat pipe or the properties of the working fluid or environment. The possible design variables, possible constraint functions, and the objective function are listed below in Fortran. This code can pursue a wide variety of design problems.

The addition of fins by the designer increases the surface area which increases the heat transfer rate through the condenser wall. However, the addition of fins decreases the cross-sectional area through each fin for conduction and decreases the trough width which increases the condensate thickness in the trough. The increased condensate thickness decreases the heat transfer and if increased to the point of covering the fins it could dramatically reduce the heat transfer through the fin.

TABLE II. DESIGN VARIABLES

| DESIGN VARIABLES |
| --- |
| BFIN (fin height)<br>CANGL (cone half angle)<br>CLI (condenser length<br>FANGL (fin half angle)<br>HINF (external convective heat transfer coefficient)<br>NFIN (number of fins)<br>R2I (intermediate radius)<br>RBASEI (inside radius of condenser base)<br>RMP (rotational speed of the heat pipe)<br>THICKI (condenser wall thickness)<br>TINF (external temperature)<br>TSS (saturation temperature)<br>TZ (nodal point temperature) |
| **CONSTRAINT FUNCTIONS** |
| BOA (ratio of fin height over fin width)<br>ZOA (ratio of trough width over fin width)<br>DEL(NI) (condensate thickness) |
| **OBJECTIVE FUNCTION** |
| OBJ |

The purpose of the design study was to determine the fin height, number of fins, and fin half angle which would maximize the heat transfer rate. It was then decided that the design variables would be BFIN, FANGL, and NFIN. The number of fins was chosen vice the ratio of trough width to fin width (ZOA) as was previously used. This decision was made since it is easier to think in terms of the number of fins vice a ratio. Other potential design variables were held constant. The objective function to be maximized was OBJ=QT+QTF, the heat transfer through the fin plus the heat transfer through the trough.

The code was run with the three design variables using the internal scalar default parameters in ADS. The objective function was calculated using the input values of the design variables. The ADS program then changed the first design variable keeping the other two design variables at the input values. The calculations were made aging yielding a different value for the objective function. The new value would then be compared to the previous value of the objective function. If the difference in the objective function was not greater then the internal scalar default value, the first design variable would be returned to its original value and the process repeated with the second design variable. If the difference in the objective function was still not greater then the internal scalar default value, the second design variable would be returned to its initial value and the process repeated with the third design variable.

Each time the program was run, the optimization code would choose BFIN as the design variable to change first as it had the greater influence on the objective function. The remaining two variables would then either be kept constant or the number of fins would be maximized and the fin half angle minimized. Consistent results were not obtained with this method.

To improve the results, the internal scalar parameters were modified. These modifications included the constraint tolerances, the absolute and relative convergence criteria, the absolute and relative change in the design variables, the absolute and relative change in the objective function, the minimum absolute value of the finite difference step when calculation gradients, and the initial relative move limit. Better results were obtained as seen in the higher value for the objective function. However, the results were still not consistent depending on the initial values used for the three design variables. At this point, it was decided to concentrate on one design variable and on the basis of the previous calculations the design variable chosen was BFIN.

The external surface temperature was set equal to the working fluid saturation temperature and the theoretical upper limit on the heat transfer was calculated for comparison. An assumption is made that there is no thermal resistance across the condensate or the condenser wall. The upper limit of the heat transfer rate was predicted to be 69,492 BTU/HR using the following formula:

$$Q_{max} - 2\pi \bar{r} 1(T_{wall} - T_{\infty}) \tag{7}$$

where

h = outside convective heat transfer coefficient (5000 BTU/HR$\cdot$FT$^2\cdot$F)

$\bar{r}$ = average outside radius of condenser wall (0.07373 FT)

l = condenser length (0.75 FT)

$T_{wall}$ = temperature of the outside wall (100°F)

$T_{\infty}$ = ambient temperature (60°F)

Based on engineering judgement certain constraints were placed on the design. These constraints were applied to the number of fins (not to exceed 400) and the minimum fin half angle (not to be less than 10 degrees). The values were based on structural and manufacturing considerations.

# IV. RESULTS

## A. INTRODUCTION

The purpose of the design optimization was to maximize the heat transfer rate. This was accomplished by using the computer code in conjunction with ADS. Numerical results are discussed below.

## B. CONSTRAINED OPTIMIZATION

In the design problem undertaken to determine the optimum internal geometry for the maximum heat transfer, numerous runs were made for a condenser made of copper. This material has a thermal conductivity of 230 BTU/HR'FT'F. The working fluid was water.

Since the fin height (BFIN) was the design variable, the initial runs investigated whether there was an optimum fin height. Initially the fin half angle was held constant at 10 degrees and the number of fins was varied from 150 to 400. In each case, for the optimum design, the fin height was maximized and the trough was eliminated, as seen in figures 6 and 7.

The number of fins was then held constant and the fin half angle was varied from 10 to 25 degrees with the fin height remaining the design variable. Once again, the greatest heat transfer rate was achieved with the highest fin height for each number of fins (figure 8.)

As seen in figure 9, the highest heat transfer rate achieved was for 400 fins with a 10 degree fin half angle and a fin height of 0.0345 inches.

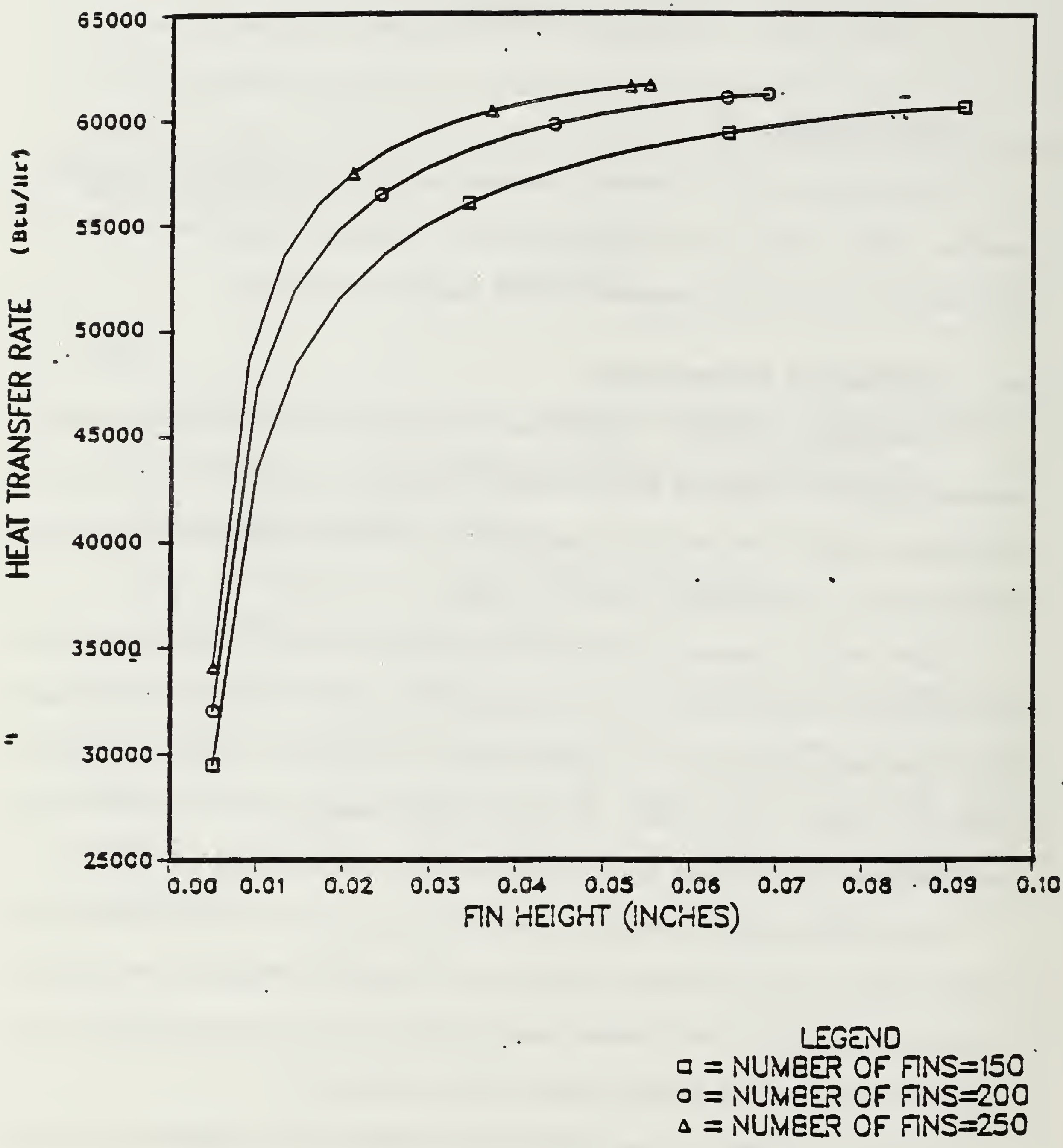



Figure 6. Heat Transfer Rate vs. Fin Height.

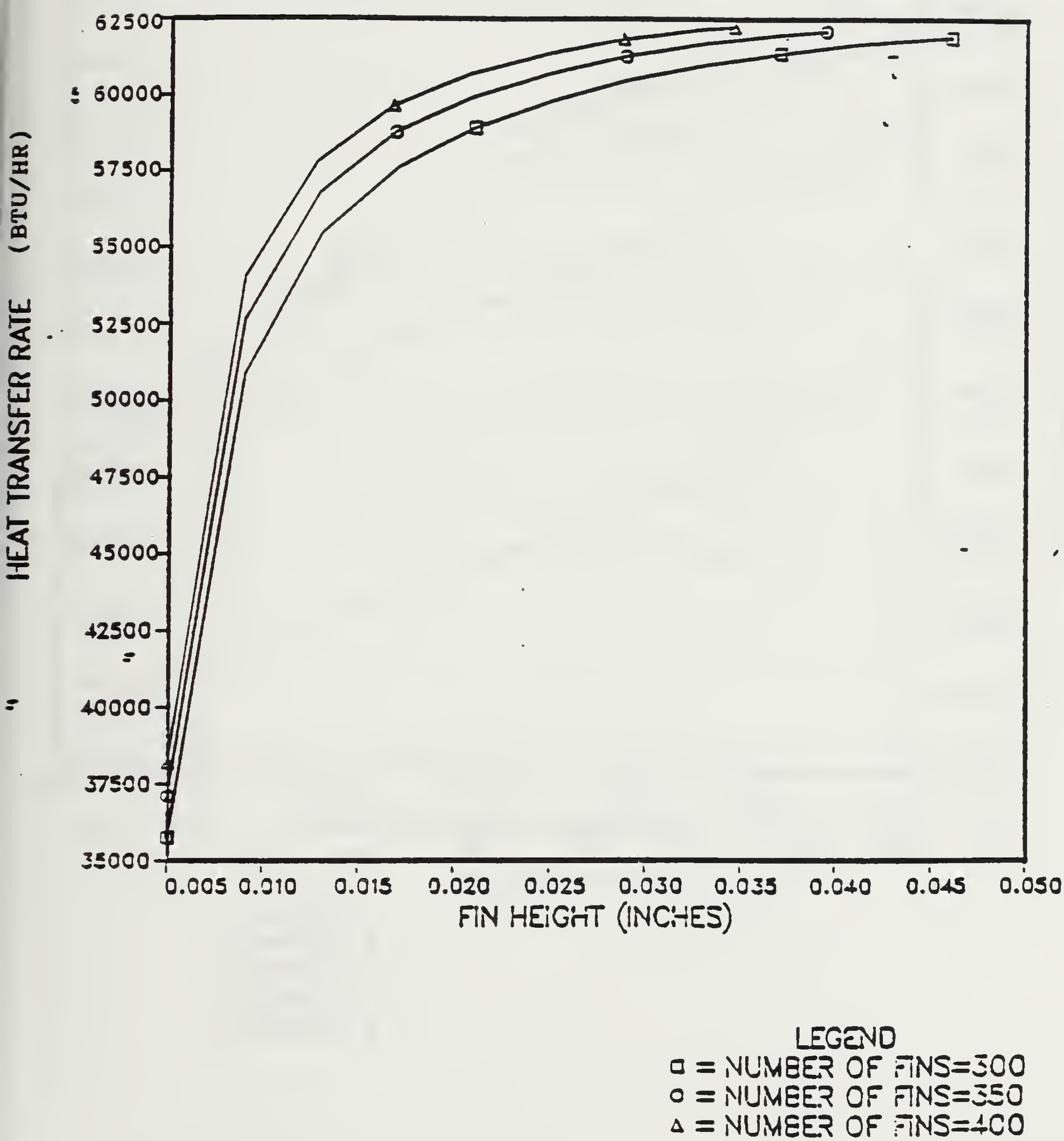



Figure 7. Heat Transfer Rate vs. Fin Height.

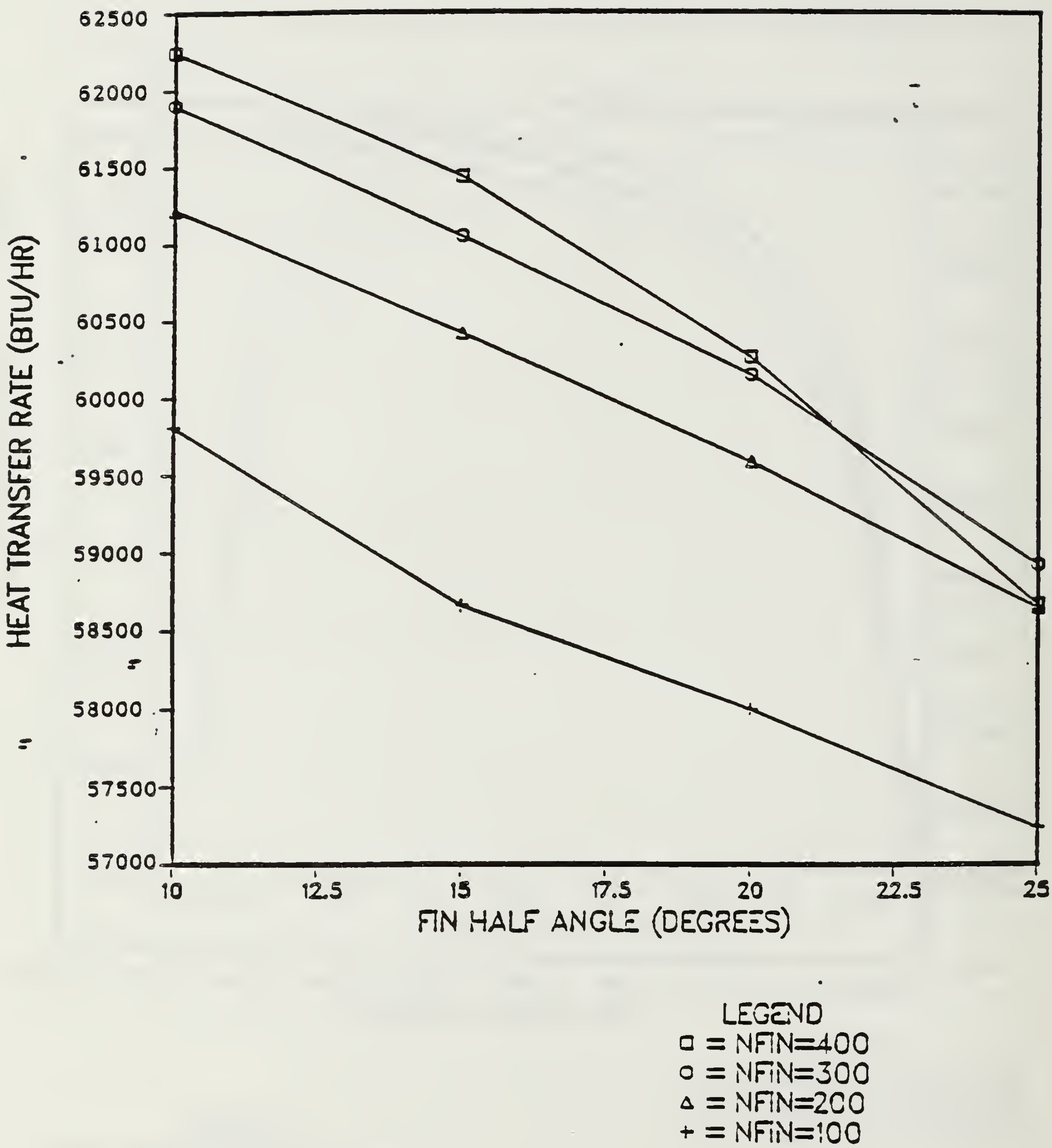



Figure 8. Heat Transfer Rate vs. Fin Half Angle (Optimum Fin Height).

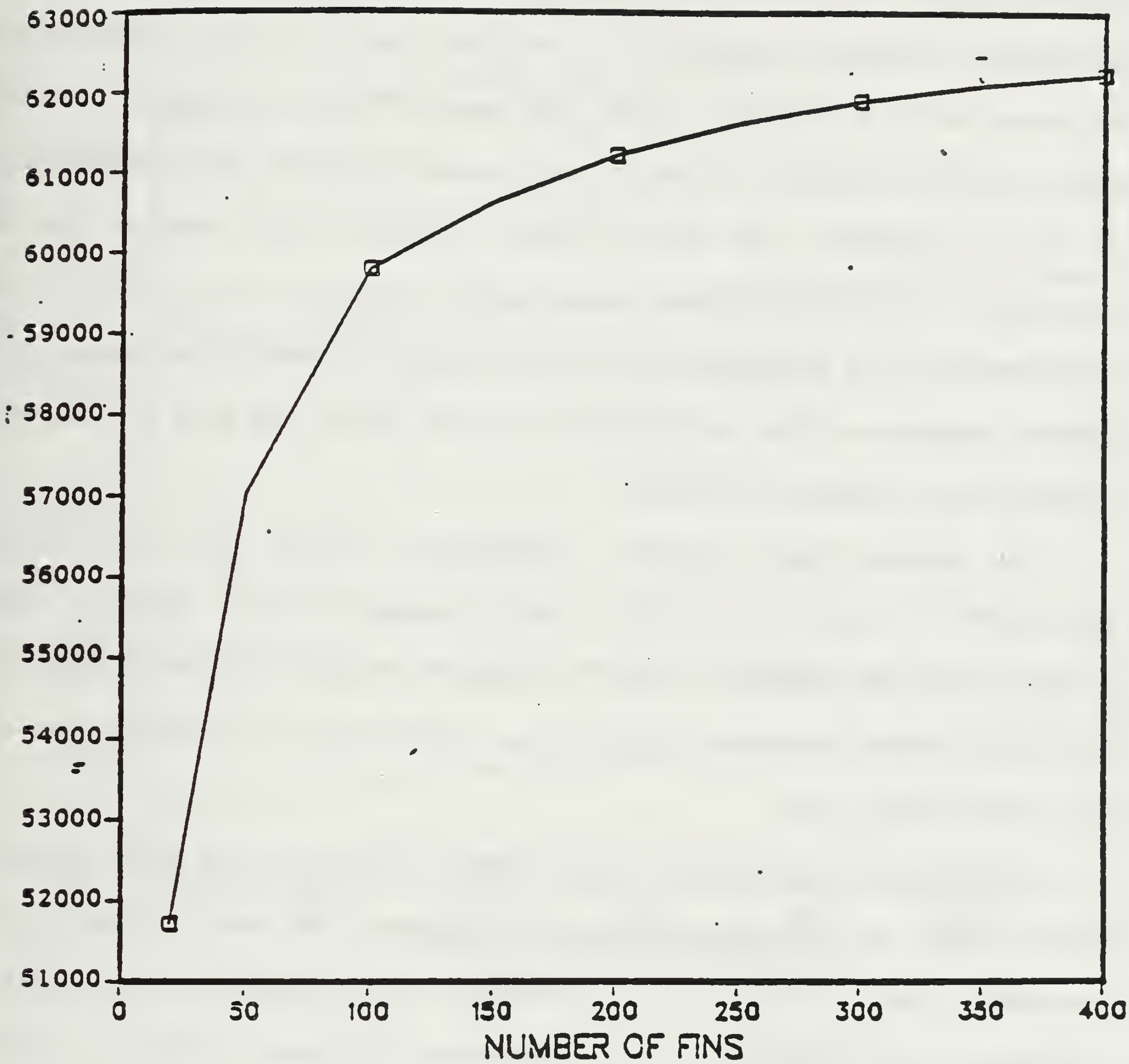



Figure 9. Heat Transfer Rate vs. Number of Fins (Optimum Design).

Figure 10 shows a plot of heat transfer rate versus fin half angle for a condenser with between 100 and 400 fins. The heat transfer rate, as a function of fin half angle for a constant fin height, increases as the fin half angle increases. Davis [Ref. 9] concluded that the heat transfer rate increased with an increase in fin half angle. This is correct if the fin height is kept constant. As the fin half angle increases, the surface area of the fin increases. The added surface area also has a thinner film of condensate on it which offers lower thermal resistance. The trough area decreases and the condensate film in the trough thickens, increasing the thermal resistance. This degradation does not offset the gain in the heat transfer rate caused by the fin.

For external heat transfer coefficients varying from 1000-50,000 BTU/HR•FT$^2$•F, the same optimum design geometry for a maximum heat transfer rate was obtained, which is stated in table III below. Figure 11 shows the strong influence the external heat transfer coefficient has on the heat transfer rate.

In figure 12, the effect of the rotating speed on the heat transfer rate is seen. As the rotational speed increases, the heat transfer rate increases, this is caused by an increase in the element heat transfer coefficient and a decrease in the condensate thickness on the fin which lowers the thermal resistance.

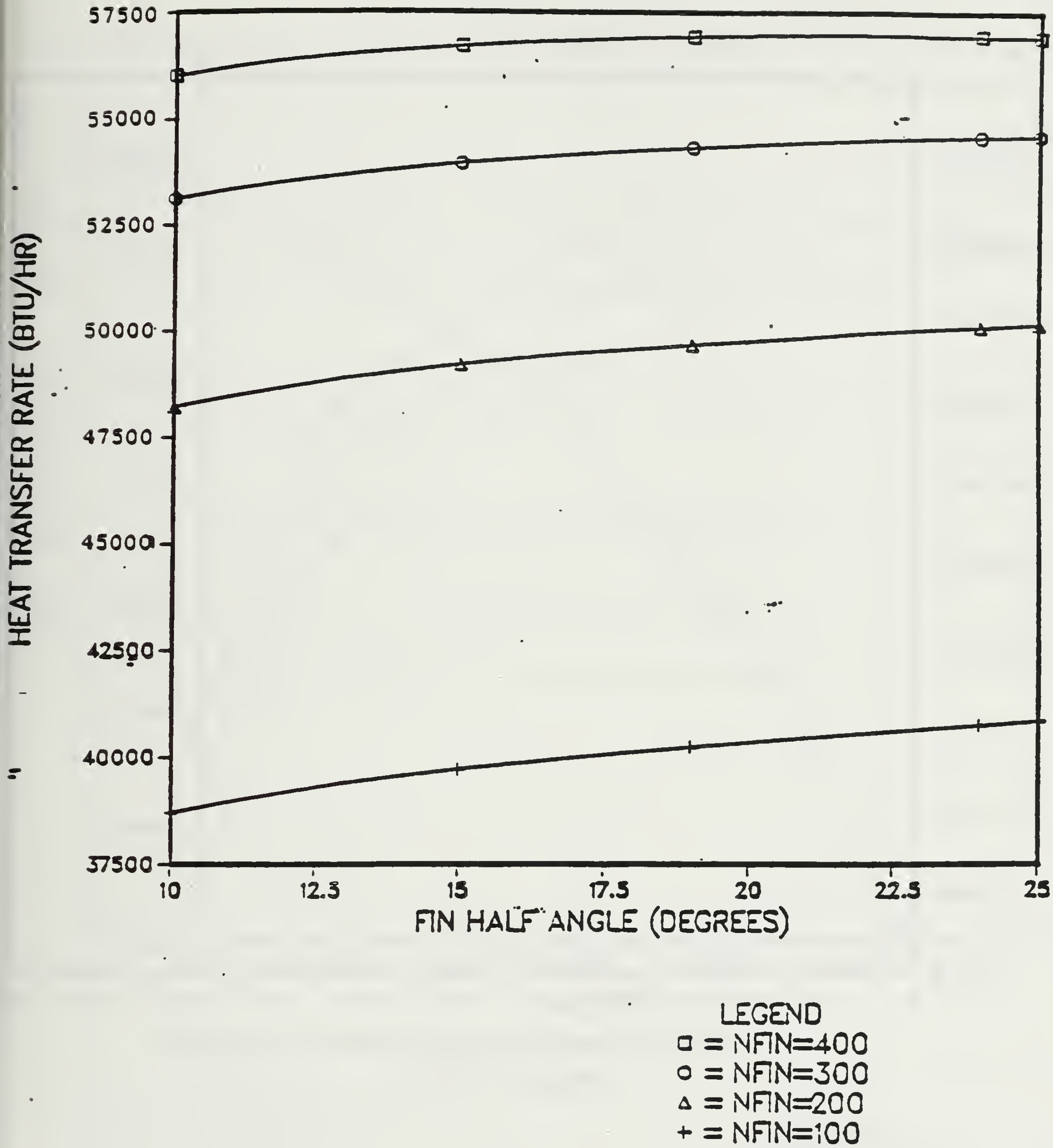



Figure 10. Heat Transfer Rate vs. Fin Half Angle (Constant Fin Height).

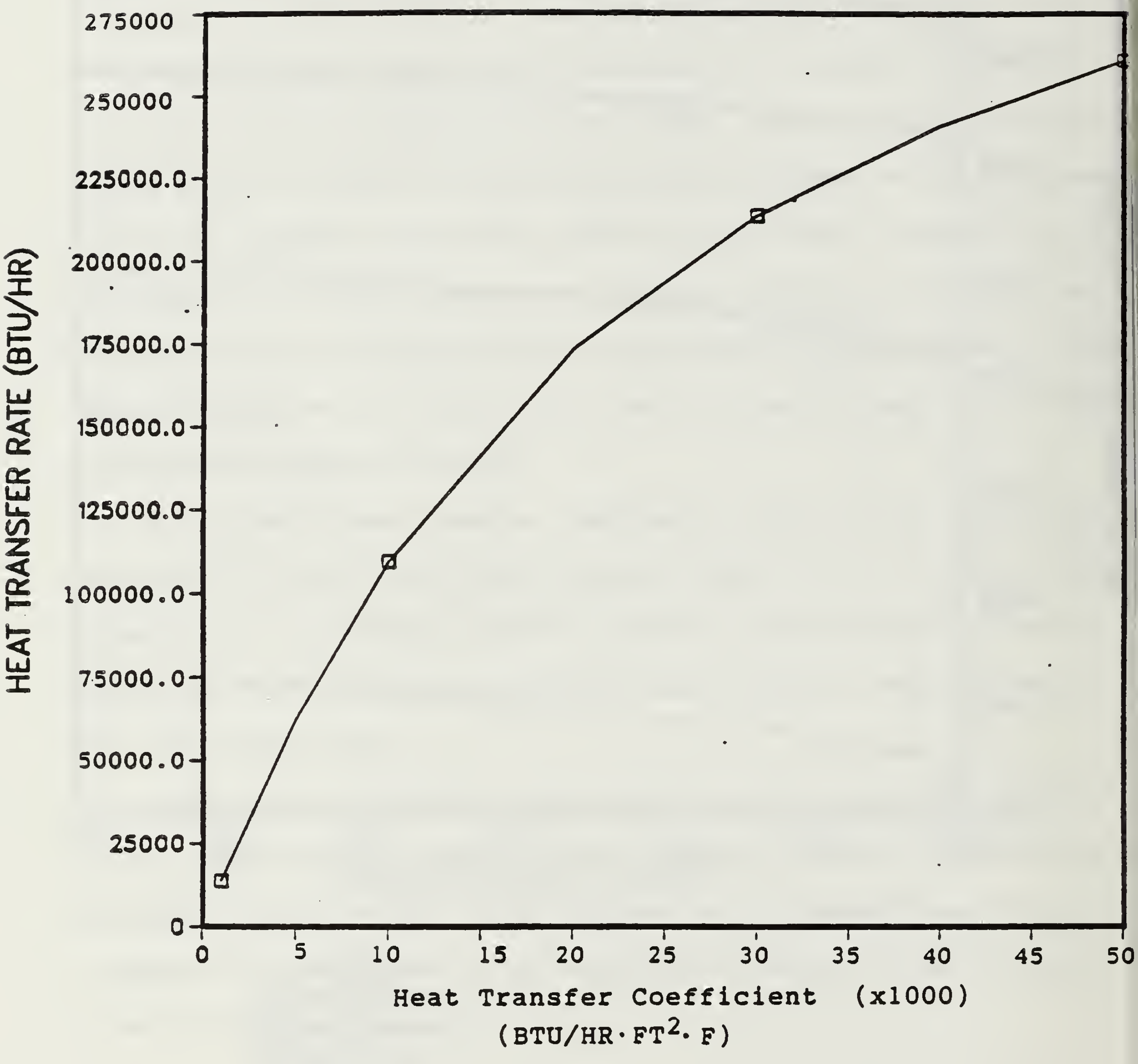



Figure 11. Heat Transfer Rate vs. Heat Transfer Coefficient.

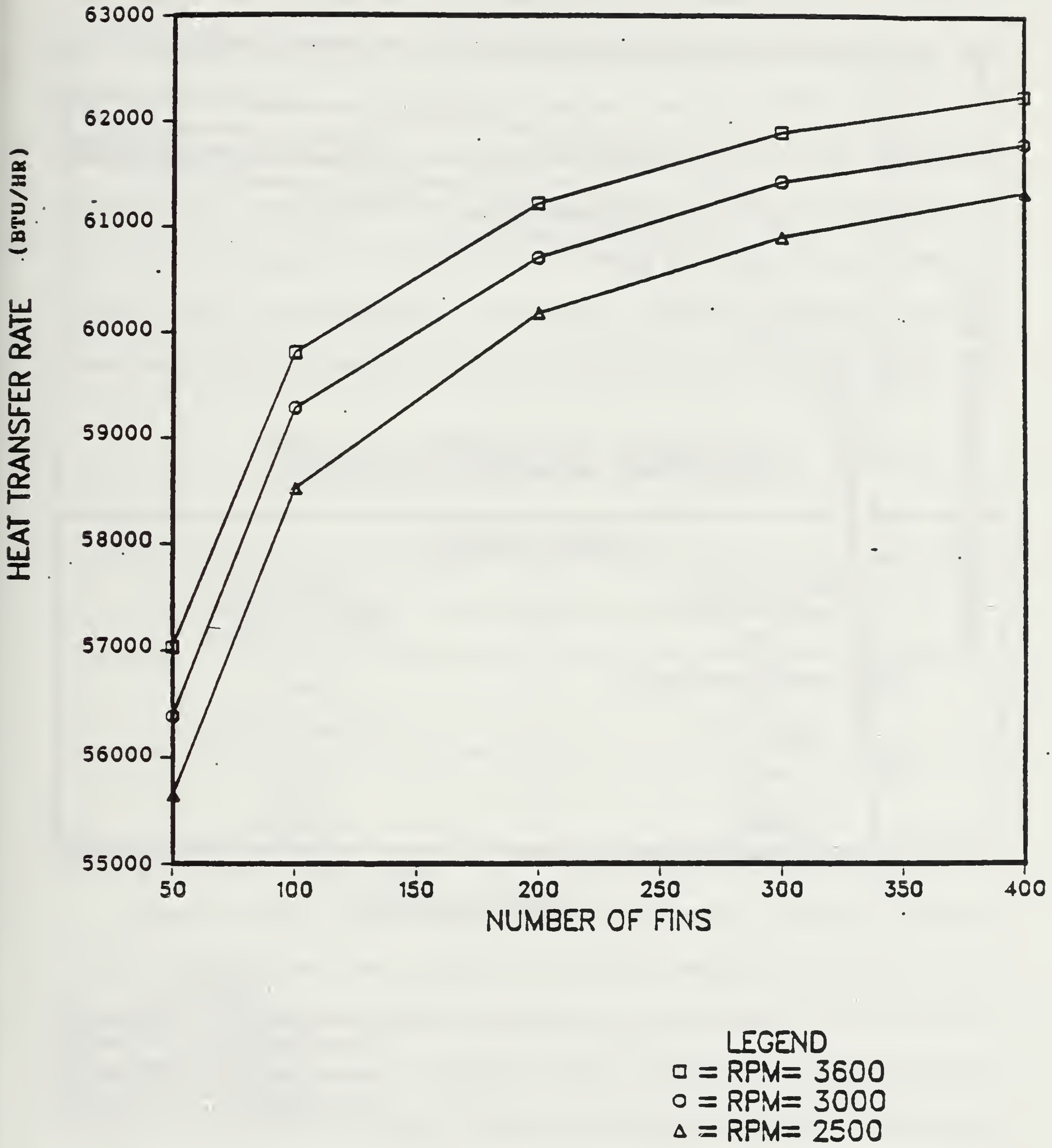



Figure 12. Heat Transfer Rate vs. Number of Fins (RPM Variation).

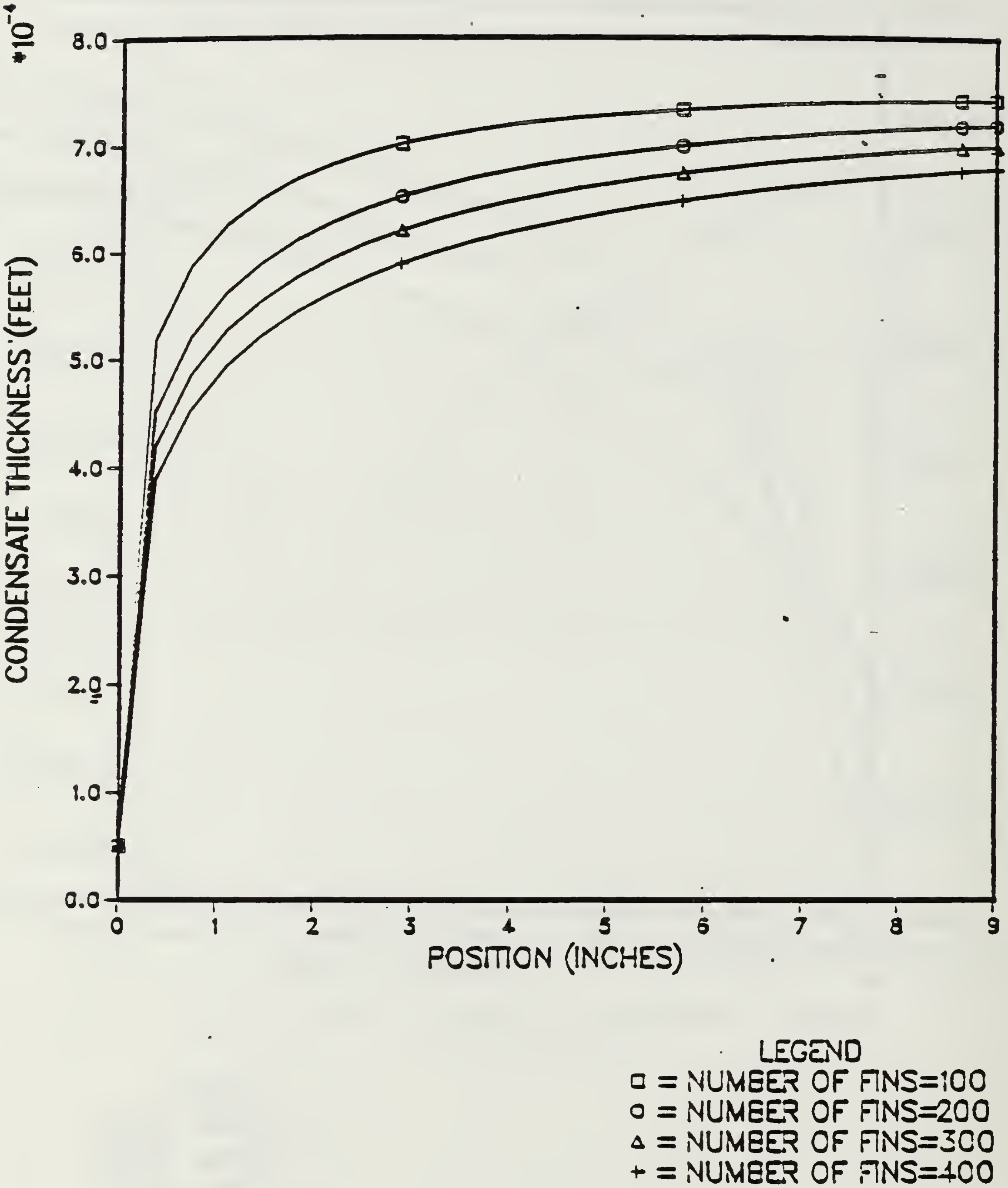



Figure 13. Condensate Level vs. Position (100-400 Fins).

When the number of fins was increased from 100 to 400, maintaining the same fin half angle, the condensate level decreased with the increased heat transfer rate (figure 13). This occurred because the thinner film over the fins decreased the resistance across the film which raised the temperatures along the fin, which in combination with the lower height fins increased the temperature on the outside of the pipe. This increase in temperature brings the operation closer to the condensing limit. When the fin half angle was increased for a specified number and height of fins, the condensate level decreased due to the increased trough width (figure 14).

**TABLE III. OPTIMIZATION RESULTS FOR VARIOUS HEAT TRANSFER COEFFICIENTS**

| OPTIMUM DESIGN | |
|---|---|
| Fin Height<br>Fin Half Angle<br>Number of Fins | 0.0345 inches<br>10.0 degrees<br>400 |
| $h_{external}$<br>(BTU/HR•FT$^2$•F) | HEAT TRANSFER RATE<br>(BTU/HR) |
| 1000<br>5000<br>50000 | 13765<br>62252<br>261003 |

Figures 15 and 16 show the effect the increase of surface area has on the heat transfer rate. In figure 15, the fin height for 400 fins with a 10 degree half angle is plotted against the heat transfer rate. As the fin height is increased up to a maximum value of 0.0345 inches the resulting design is a sawtooth. Figure 16 shows the effect adding more fins has on the heat transfer rate for a constant height fin. The greatest increase in the heat transfer rate is seen from the addition of 100 fins from a smooth tube.

In figure 17, the ratio of the actual surface area over the surface area for a smooth tube is plotted versus the number of fins. The fin height and the fin half angle are both held constant. As expected, the surface area ratio increases in a linear manner as the number of fins increases. The ratio of the heat transfer rate over the heat transfer rate for a smooth tube is plotted for two different heat transfer coefficients, h=1000 BTU/HR•FT$^2$•F and 5000 BTU/HR•FT$^2$•F. An increase in the number of fins results in not only an increase in the area ratio but also an increase in the heat transfer ratio. The increase in the heat transfer ratio is greatest when going from a smooth tube to a tube with 100 fins. The heat transfer ratio increase is greater for the heat transfer coefficient equal to 5000 BTU/HR•FT$^2$•F. This is because the heat transfer coefficient has a direct effect on the heat transfer rate, that is,

$$Q = hA(T-T_{wall})$$

Both curves approach an asymptotic value. However, the curve with the lower heat transfer coefficient approaches this asymptotic value with a fewer number of fins. Additionally, in view of the relatively small increase in the heat transfer ratio by the addition of fins for the lower heat transfer coefficient, consideration should be given to the cost of manufacturing the fins versus the benefit derived by their addition.

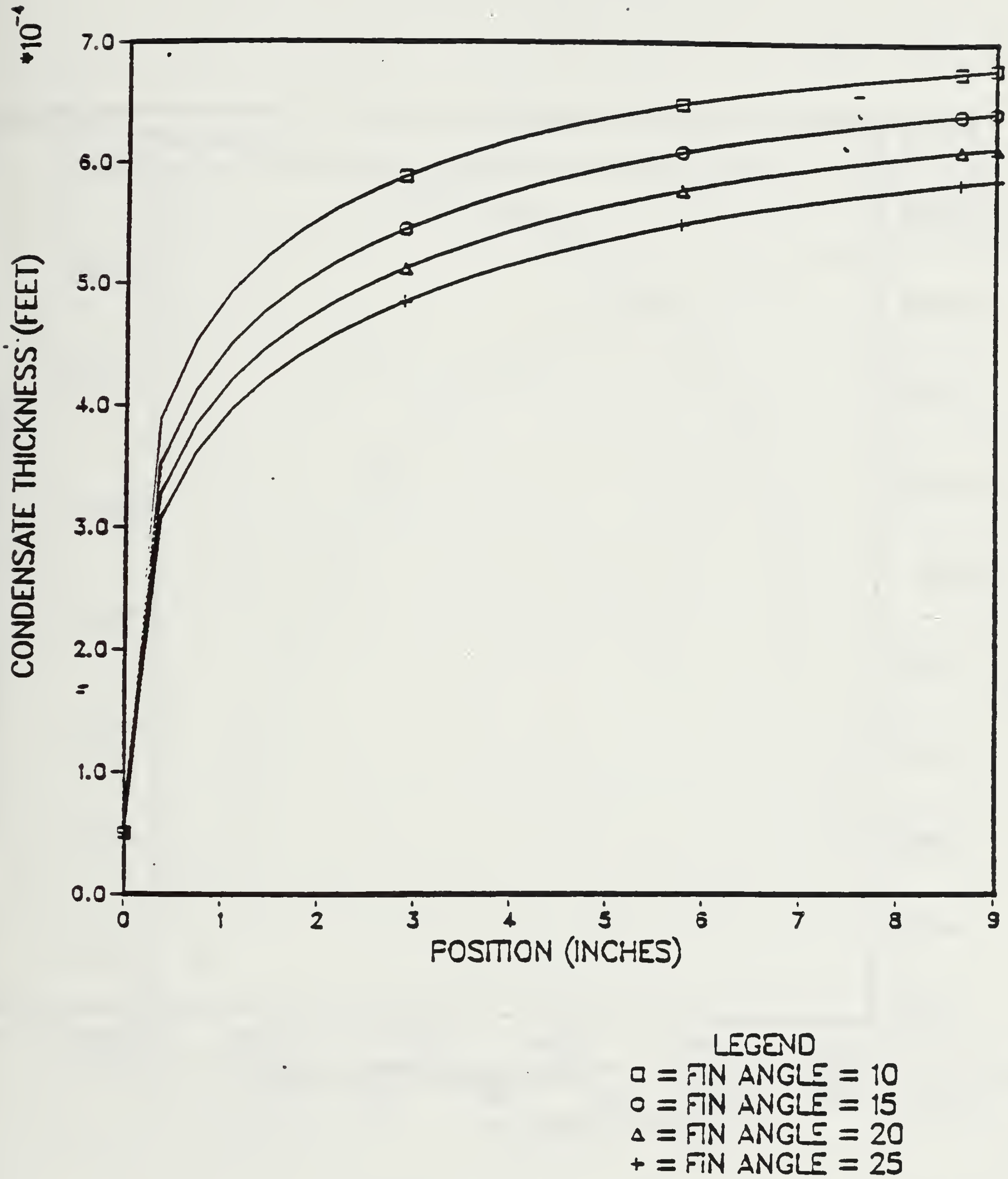



Figure 14. Condensate Level vs. Position (400 Fins, 10-25 Degree Fin Half Angle).

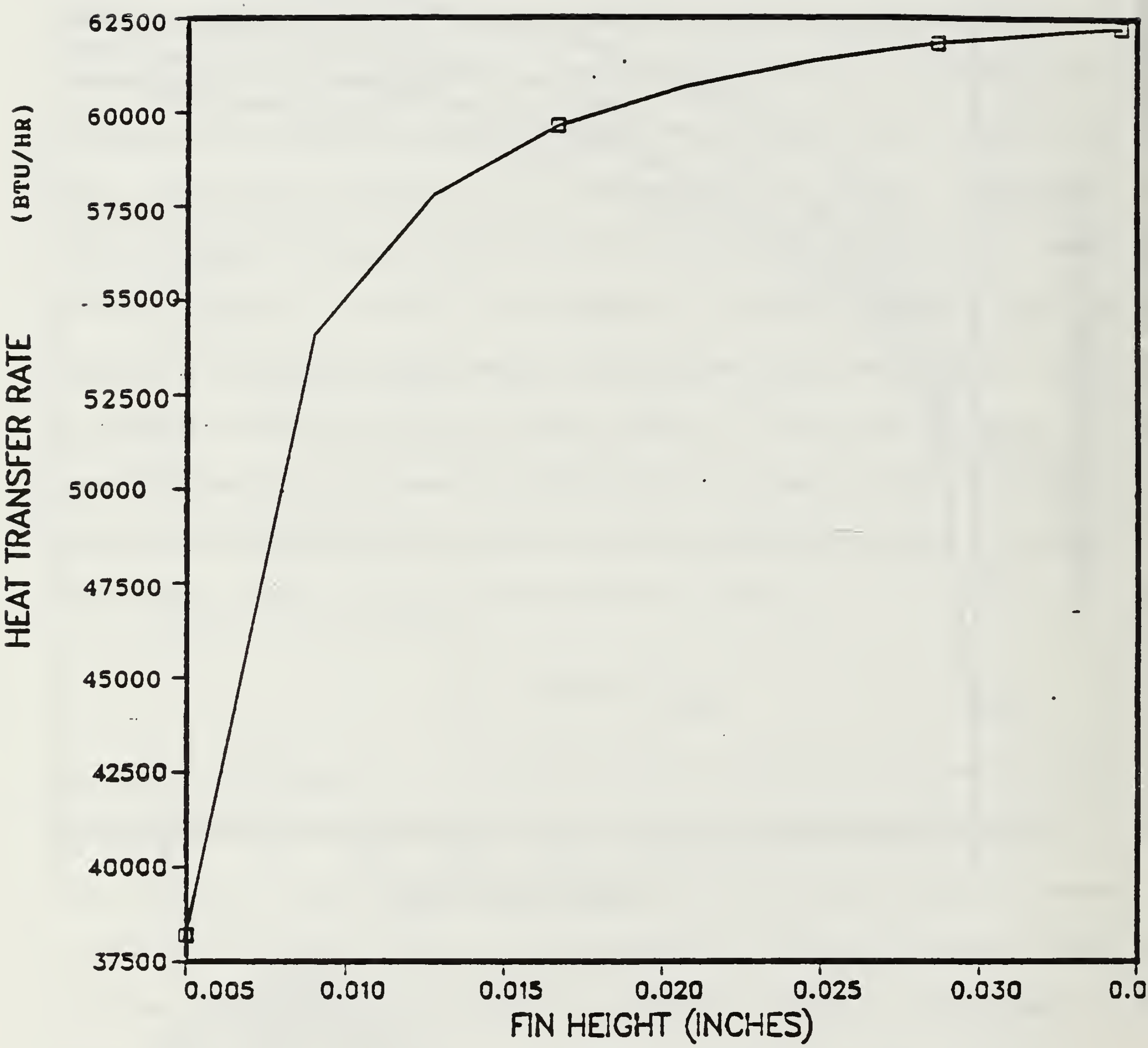



Figure 15. Heat Transfer Rate vs. Fin Height (400 Fins).

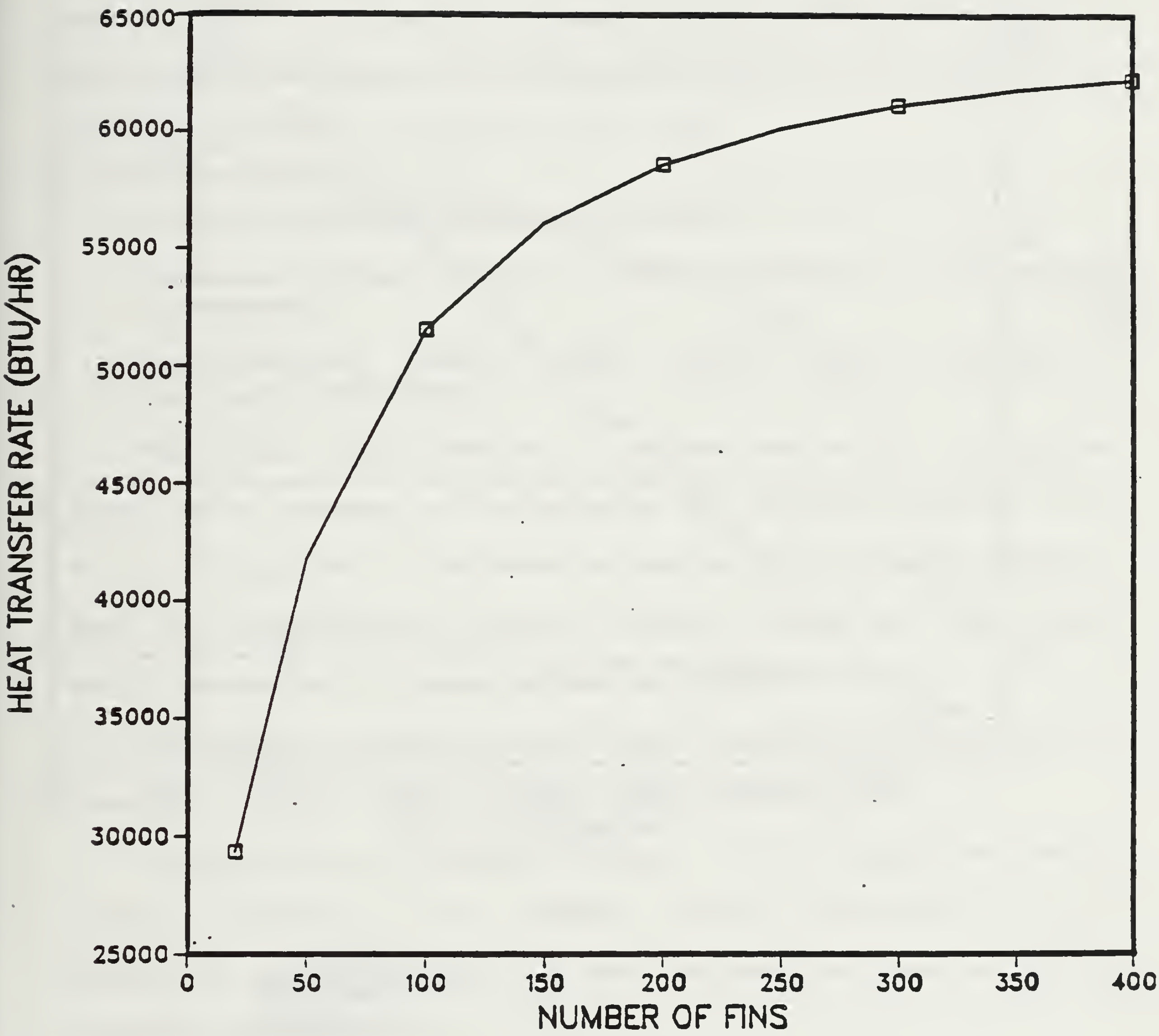



Figure 16. Heat Transfer Rate vs. Number of Fins (Constant Fin Height).

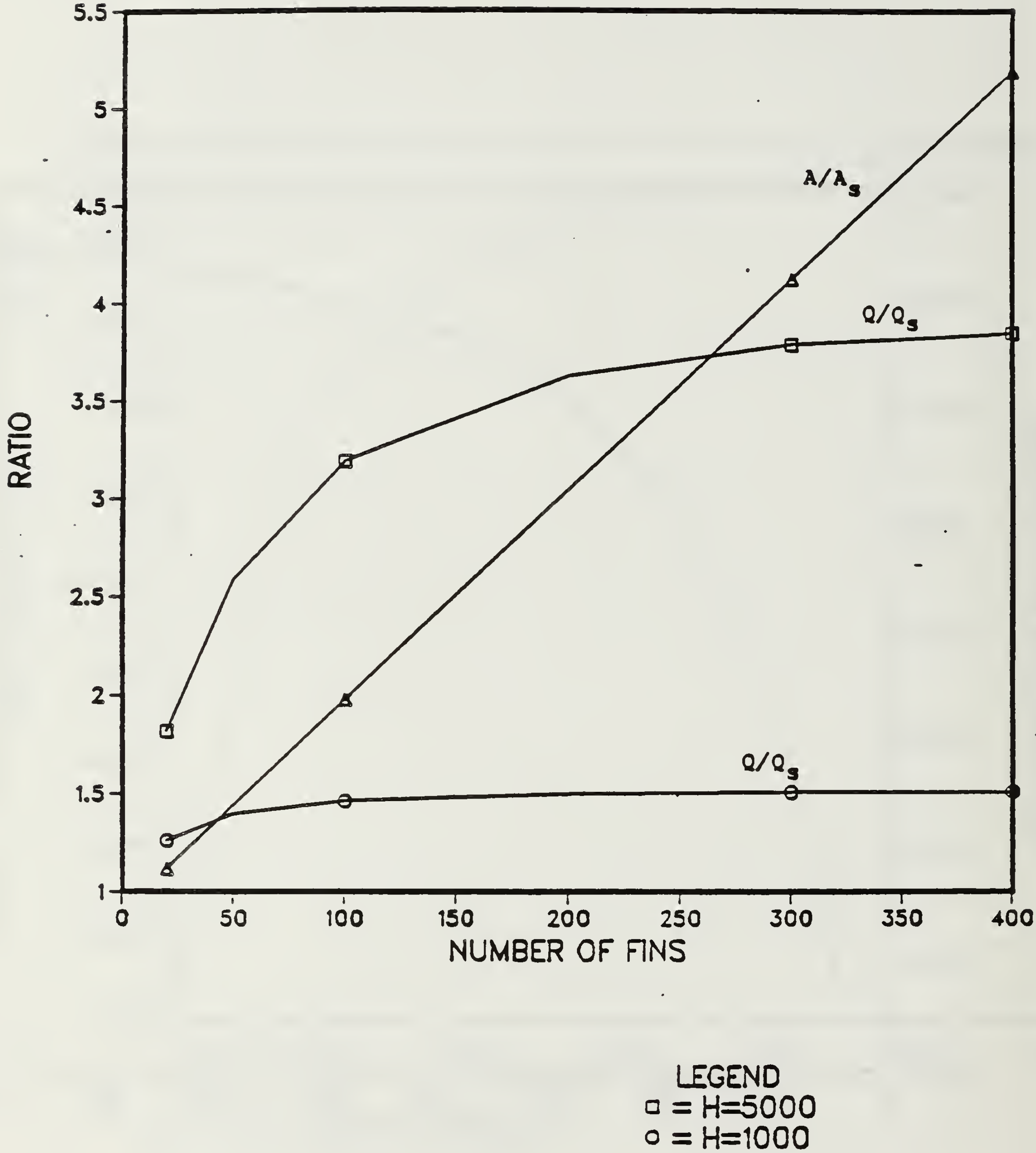



Figure 17. Heat Transfer and Area Ratios vs. Number of Fins.

## C. AUTOMATED DESIGN SYNTHESIS (ADS)

This optimization project was done on the IBM mainframe using the ADS optimization program. As stated previously, ADS is a general purpose numerical optimization program with a variety of algorithms that can be used to tailor the solution. The solution is separated into three levels in ADS: strategy, optimizer, and one-dimensional search. For this problem the following combination of algorithms were used:

1. Strategy: Sequential Linear Programming
2. Optimizer: Modified Method of Feasible Directions for constrained minimization
3. One-Dimensional Search: Golden Section Method followed by polynomial interpolation

The strategy used linearizes a nonlinear problem by a first order Taylor series expansion of the objective and constraint functions. The solution to this linear approximation is obtained. The problem is linearized again about this point and the new problem is solved with the process being repeated until a precise solution is achieved.

The optimizer chosen is used to find a search direction which will minimize the objective function while maintaining feasibility.

The combination of strategy, optimizer, and one-dimensional search chosen is not the only one available, nor is it necessarily the most efficient. It did yield results that were maximized and were within the constraint tolerances.

The ADS optimization program is complicated by the numerous internal parameters which must be changed to obtain an optimal design.

Complications arise when there is a vast difference in the scales of the design variables. The design variables themselves must be scaled which is further complicated when the variable itself covers a wide range. ADS also, in certain cases, allows for constraints to be violated. In some instances this might be acceptable but not in this case.

ADS does not have a scoping mechanism, that is the ability to decrease the rate of change of the design variable, and therefore to obtain a precise answer the internal parameters must be changed repeatedly. ADS also does not recognize integers, all numbers are real therefore, depending on the answer given, it may be necessary to round up or down.

# V. CONCLUSIONS

1. For an independent increase in fin half angle, rotational speed or number of fins an increase is seen in the heat transfer rate. As the parameters increase, the heat transfer rate levels off at the theoretical maximum heat transfer rate for the heat pipe. A decrease in the fin half angle with a corresponding increase in the fin height increases the heat transfer rate. If the fin half angle is decreased while the fin height is kept constant, then the heat transfer rate decreases.

2. Maximum heat transfer occurs for the same fin geometry regardless of the external heat transfer coefficient. For a specific condenser radius, as many fins as possible should be machined with a minimum fin half angle at the maximum fin height.

3. The computer code can be used for single analysis or the automated design of an internally finned rotating heat pipe.

4. The benefit of adding fins is dependent on the external heat transfer coefficient. Consideration of the cost of manufacturing the fins versus the increase in the heat transfer rate should be made.

# VI. RECOMMENDATIONS

1. Analyze different shaped fins including rectangular and curved.

2. Modify the code to allow for silmutaneous variations of more than one variable.

3. Use different working fluids and heat pipe materials to see if a different internal geometry occurs for the maximum heat transfer rate.

4. Modify the code to use the DOT optimization program vice ADS.

## APPENDIX: PROGRAM LISTING

```
   PROGRAM OLSON
*********************************************************
*                                                       *
*  ANALYSIS OF ROTATING HEAT PIPE , USING TRIANGULAR    *
*                ELEMENT MODEL                          *
*  COMPILED BY MAJOR IGNATIUS.S.PURNOMO IN JUNE 1978    *
*                                                       *
*      MODIFIED TO PERMIT NUMERICAL OPTIMIZATION        *
*             USING COPES/CONMIN                        *
*    BY LCDR WILLIAM A. DAVIS, JR. IN SEPTEMBER 1980    *
*    MODIFIED TO PERMIT USE OF THE ADS OPTIMIZATION     *
*       ROUTINE BY LT. G.L. OLSON  IN JUNE 1992         *
*                              *
*                                                       *
*********************************************************

   CHARACTER*20 NAME

   COMMON/ADS/DF(21),G(10),IDG(100),IGRAD,INFO,IOPT,IONED,IPRINT,
  :ISTRAT,IWK(2000),IZ(30),OBJ,S(2),VLB(2),VUB(2),W(21,30),WK(5000),
  :NCOLA,NCON,NDV,NGT,NRA,NRIWK,NRWK

   COMMON/OLLIE/A(200,50),AMTOT(200),APS,B(3),BFIN,BOA,BVIN,C(3),
  :CANGL,CF(200),CK,CLI,COF(5),CW(200),DEL(200),DMDOT(200),EA(3,3),
  :EPS(200),EZERO,F(200,1),FANGL,FNOBJ(100),H(200),HINF,HZ(200),
  :QB(200),QINC(200),QTINC(200),QTOT,QTOTAL(100),R(200),RB(200),
  :RBASEI,R2I,RHOF(200),ROOTI(4),RPM,ROOTR(4),T(200),TALFA,TB(200),
  :TCC(200),TE(200),THICK,THICKI,TIB(200),TINF,TS(200),TSAT,TSS,
  :TT(200),TZ,UF(200),X(200),XCOF(5),XPLOT(200),Y(200),Z(200),ZOA,
  :DOBF,DOTH,ICOR(200,3),IFF,JINT,JLC,JTC,KFF(50),KFIN(50),KT,NBAN,
  :NEL,NFIN,NSNP

   GUIDE TO FORTRAN VARIABLE NAMES

 AFOVAS  FIN AREA/SMOOTH AREA
 ALFA    FIN HALF ANGLE (RADIANS)
 BFIN    HEIGHT OF FIN (INCHES)
 BOA     TANGENT OF THE FIN HALF ANGLE
 BVIN    HEIGHT OF FIN (FEET)
 CALFA   COSINE OF ALFA
 CANGL   CONE HALF ANGLE (DEGREES)
 CBASE   INSIDE CIRCUMFERENCE OF CONDENSER (FEET)
 CEXIT   INSIDE CIRCUMFERENCE AT CONDENSER EXIT (FEET)
 CF      THERMAL CONDUCTIVITY OF CONDENSATE FILM (BTU/HR FT F)
 CL      CONDENSER LENGTH (FEET)
 CLI     CONDENSER LENGTH (INCHES)
 CPHI    COSINE OF PHI
 CRIT    CONVERGENCE CRITERION
```

```
C    DEL      FILM THICKNESS
C    DF       GRADIENT OF OBJECTIVE
C    DIV      FLOATING POINT VALUE OF NDIV
C    DMTOT    CONDENSATE MASS FLOW RATE
C    EPS      TROUGH WIDTH INCLUDING INCREMENTAL CHANGE
C    EPSEX    TROUGH WIDTH AT CONDENSER EXIT
C    EPSO     TROUGH WIDTH AT START OF CONDENSER
C    EZERO    FIN BASE WIDTH
C    F        FORCE VECTOR OF SYSTEM
C    FANGL    FIN HALF ANGLE (DEGREES)
C    G        CONSTRAINT VALUES ASSOCIATED WITH CURRENT DESIGN
C    H        CONVECTIVE HEAT TRANSFER COEFFICIENT (BTU/HR FT2 F)
C    HFG      LATENT HEAT OF VAPORIZATION (BTU/LBM)
C    IDG      CONSTRAINT TYPE IDENTIFIER
C    IEL      THE ELEMENT NUMBER
C    IFF      NO. OF ROWS MINUS ONE OF THE UPPER TRIANGULAR FIN
C    IFIN     EQUALS 0 FOR COPPER, AND 1 FOR STAINLESS STEEL
C    IFLUID   EQUALS 0 FOR WATER, AND 1 FOR FREON
C    IGRAD    GRADIENT CALCULATION IDENTIFIER
C    INFO     CONTROL PARAMETER
C    IONED    ONE DIMENSIONAL SEARCH IDENTIFIER
C    IOPT     OPTIMIZER IDENTIFIER
C    IPRINT   A FOUR DIGIT PRINT CONTROL
C    ISTRAT   OPTIMIZATION STRATEGY IDENTIFIER
C    JINT     NO. OF COLUMNS PLUS ONE BELOW TRIANGULAR FIN
C    JLC      NUMBER OF SYSTEM NODAL POINT LOCATED AT
C             THE CENTER OF SYSTEM COORDINATE
C    JTC      NUMBER OF SYSTEM NODAL POINT LOCATED AT
C             THE JUNCTION OF THE SYMMETRY BOUNDARY AND
C             THE LINE OF INTERSECTION BETWEEN THE FIN
C             AND THE CONDENSER WALL
C    KFF      NUMBER OF SYSTEM NODAL POINTS LOCATED ALONG
C             THE FIN CONVECTIVE BOUNDARY
C    KFIN     NUMBER OF SYSTEM NODAL POINTS LOCATED ON THE
C             SYSMMETRIC BOUNDARY OF TRIANGULAR FIN SECTION
C             NOT COUNTING POINTS AT BASE AND APEX
C    KT       NUMBER OF COLUMNS WITHIN THE TROUGH WALL SECTION
C    NBAN     SYSTEM BAND WIDTH
C    NBOTF    LAST ELEMENT AT BOTTOM SIDE
C    NBOTI    FIRST ELEMENT AT BOTTOM SIDE
C    NCON     NUMBER OF CONSTRAINTS
C    NDOBF    NUMBER OF ROWS WITHIN THE FIN
C    NDOTH    NUMBER OF ROWS WITHIN THE TROUGH
C    NDIV     NUMBER OF INCREMENT
C    NDV      NUMBER OF DESIGN VARIABLES
C    NEFB     ELEMENT NUMBER AT BASE OF FIN
C    NEL      NUMBER OF ELEMENTS
C    NEST     ELEMENT NUMBER AT END OF TROUGH
C    NRA      NUMBER OF ROWS IN ARRAY A
C    NRWK     DIMENSIONAL SIZE OF WK
C    NSNP     NUMBER OF SYSTEM NODAL POINTS
C    OBJ      VALUE OF THE OBJECTIVE FUNCTION ASSOCIATED WITH X
C    OMEGA    ROTATIONAL SPEED OF HEAT PIPE (RAD/HR)
C    PHI      CONE HALF ANGLE (RADIANS)
C    PI       PI
C    R2I      DISTANCE FROM CENTERLINE OF THE HEAT PIPE TO HALF THE F
C             HEIGHT
C    RBASE    INSIDE RADIUS OF CONDENSER BASE (FEET)
C    RBASEI   INSIDE RADIUS OF CONDENSER BASE (INCHES)
C    REXIT    INSIDE RADIUS OF CONDENSER EXIT (FEET)
```

```
RPM      REVOLUTIONS PER MINUTE
S        VECTOR OF DESIGN VARIABLES
SALFA    SINE OF ALFA
SPHI     SINE OF PHI
SURFAR   SURFACE AREA
THICK    CONDENSER WALL THICKNESS (FEET)
THICKI   CONDENSER WALL THICKNESS (INCHES)
TPHI     TANGENT OF PHI
TZ       AMBIENT TEMPERATURE
UF       VISCOSITY
VLB      LOWER BOUNDS ON THE DESIGN VARIABLES
VUB      UPPER BOUNDS ON THE DESIGN VARIABLES
WK       REAL WORK ARRAY
ZFIN     NUMBER OF FINS
ZOA      RATIO OF TROUGH WIDTH TO FIN BASE WIDTH
ZSTAR    SURFACE LENGTH OF THE FIN MINUS THE SURFACE LENGTH
         COVERED BY THE CONDENSATE IN THE TROUGH
ZZERO    SURFACE LENGTH OF FIN


  PRINT*, 'INPUT FILE NAME'
  READ*, NAME
  OPEN(10,FILE=NAME)
  OPEN(15,FILE='/HTPIPE OUTPUT')
  OPEN(14,FILE='/DUMP OUTPUT')
  OPEN(13,FILE='/GRAPH OUTPUT')


  THE FOLLOWING READS INPUT DATA, PERFORMS HEAT
  TRANSFER ANALYSIS, AND PRINTS RESULTS.



  *****   INPUT MODE   *****


         ELEMENT CONNECTIVITIES

  READ (10,420) NEL,NSNP,NBAN,IFLUID,IFIN
  WRITE (15,430) NEL,NSNP,NBAN
  WRITE (15,435) IFLUID,IFIN
  WRITE (15,436)
  WRITE (15,437)
  READ (10,440) (ICL,(ICOR(IEL,I),I=1,3),IEL=1,NEL)
  WRITE (15,450)

         THE CONDENSER GEOMETRY

  READ (10,460) CLI,CANGL,RBASEI,R2I,THICKI,BFIN,TZ
  WRITE (15,470) CLI,CANGL,RBASEI,R2I,THICKI,BFIN,TZ
  READ (10,480) NDIV,NEST,NEFB,NBOTI,NBOTF
  WRITE (15,490) NDIV,NEST,NEFB,NBOTI,NBOTF

         DATA FOR RUNNING

  READ (10,500) RPM,TSS,TINF,HINF
```

```
      WRITE (15,510) RPM,TSS,TINF,HINF
*
C            THE CONVERGENCE CRITERIAN
*
      READ (10,520) CRIT
      WRITE (15,530) CRIT
*
C            INTERNAL FIN GEOMETRY
*
      READ (10,540) FANGL,NFIN
      WRITE (15,550) FANGL,NFIN
      WRITE(*,*) FANGL,NFIN
      READ (10,560) IFF
      WRITE (15,570) IFF
      READ (10,580) (KFIN(I),KFF(I),I=1,IFF)
      READ (10,590) NDOBF,NDOTH,JTC,JLC,JINT,KT
      NHB=NEFB/2
      NBF=NBOTF+1
      DOBF = FLOAT(NDOBF)
      DOTH = FLOAT(NDOTH)
      WRITE (15,600) ICOR(NBOTI,2),ICOR(NEFB,1),ICOR(NEST,1),
     1ICOR(NBOTF,1)
*
*     SET CONSTRAINTS
      NRA=21
      NCOLA=30
      NRWK= 5000
      NRIWK=2000
      NDV= 1
      NCON= 2
      IGRAD= 0
*
*     INITIAL DESIGN
      S(1)= BFIN
*
*     BOUNDS
      VLB(1)= 0.0000001
      VUB(1)= 0.75
*
C     IDENTIFY CONSTRAINTS
C     NONLINEAR CONSTRAINT
      IDG(1)=1
C     LINEAR CONSTRAINT
      IDG(2)=2
*
      PRINT*, 'INPUT THE VALUES FOR ISTRAT,IOPT,IONED AND IPRINT'
      READ*, ISTRAT,IOPT,IONED,IPRINT
*
*
C     INITIALIZE COUNTER
      NO=0.0
C     CHANGE THE INTERNAL PARAMETERS
      INFO=-2
      CALL DADS(INFO,ISTRAT,IOPT,IONED,IPRINT,IGRAD,NDV,NCON,S,VLB,
     :VUB,OBJ,G,IDG,NGT,IZ,DF,W,NRA,NCOLA,WK,NRWK,IWK,NRIWK)
      IWK(2)=0
      IWK(3)=200
      IWK(5)=4
      IWK(7)=500
      WK(3)=-0.5
```

```
      WK(6)=-0.01
      WK(8)=0.05
      WK(9)=0.50
      WK(10)=0.05
      WK(11)=0.005
      WK(13)=0.001
      WK(14)=0.0001
      WK(21)=0.004
      WK(22)=0.002
      WK(26)=0.004
      WK(37)=0.0000000001

10    CALL DADS(INFO,ISTRAT,IOPT,IONED,IPRINT,IGRAD,NDV,NCON,S,VLB,
     :VUB,OBJ,G,IDG,NGT,IZ,DF,W,NRA,NCOLA,WK,NRWK,IWK,NRIWK)
      IF (INFO.EQ.0) GO TO 360

      *****    EXECUTION MODE    *****

      NO=NO+1

      CONVERT UNITS OF ALL DIMENSIONAL PARAMETERS
      TO FEET.  CONVERT UNITS OF ANGLES TO RADIANS.
      R2I=RBASEI-(S(1)/2)
      CL=CLI/12.0
      R2=R2I/12.0
      RBASE=RBASEI/12.0
      BVIN=S(1)/12.0
      DIV=FLOAT(NDIV)
      PI=3.14159265358979
      PHI=2.0*CANGL*PI/360.0
      SPHI=SIN(PHI)
      CPHI=COS(PHI)
      TPHI=TAN(PHI)
      DELX=CL/DIV
      CBASE=2.0*PI*RBASE
      REXIT=RBASE+CL*TPHI
      CEXIT=2.0*PI*REXIT
      THICK=THICKI/12.0
      ALFA=FANGL*2.0*PI/360.0
      SALFA=SIN(ALFA)
      CALFA=COS(ALFA)
      TALFA=TAN(ALFA)
      EZERO=2.0*(S(1)/12.0)*TALFA
      ZOA=((CBASE-(EZERO*NFIN))/NFIN)/EZERO

             BOUNDARY CONDITIONS AND TEMPERATURE ESTIMATES
             ALONG THE FIN BOUNDARY

      DO 20 NTINF=NBOTI,NBOTF
 20   TS(NTINF)=TINF
      DO 30 NNT=NBF,NEL
      TS(NNT)=0.0
 30   H(NNT)=0.0
      DO 40 IGT=1,NEST
      IE=ICOR(IGT,2)
 40   T(IE)=TZ
      IG=ICOR(NEST,1)
      T(IG)=TZ
      OMEGA IS IN RADIANS/HOUR
      OMEGA=RPM*2.0*PI*60.0
```

```
      DO 50 KL=NBOTI,NBOTF
   50 H(KL)=HINF
      HIFN=HINF
      TSAT=TSS
      EPSO=ZOA*EZERO
      BOA=TALFA
      SURFAR=NFIN*(2.0*(S(1)/(12*CALFA))+EPSO)
      EPSEX=(CEXIT-(NFIN*EZERO))/NFIN
      BETA=(EPSEX-EPSO)/DIV
      ZZERO=(S(1)/12)/CALFA
      AFOVAS=(ZOA+(1./SALFA))/(1.+ZOA)
      ZA=0.0
      DO 60 NSAT=1,NEST
   60 TS(NSAT)=TSAT
      TSOLID=(TSAT+TINF)/2.0
C       TEMPORARY CHANGE - TFILM
      ASMOOTH=0.0
      ACASE=0.0
      QT=0.0
      OBJ=0.0
      QT1=0.0
      QTF=0.0
      QTRF=0.0
      QTOT=0.0
      DMTOT=0.0
      NK=NDIV+1
      DO 350 NI=1,NK
C     R IS THE INCREMENTAL CHANGE IN THE RADIUS OF THE CONDENSER
      R(NI)=R2+NI*DELX*SPHI
      RB(NI)=RBASE+NI*DELX*SPHI
C     EPS IS THE INCREMENTAL CHANGE IN THE TROUGH WIDTH
      EPS(NI)=EPSO+NI*BETA
      APS=EPS(NI)
C
C            NODAL POINT COORDINATES
C
      CALL COORD
   65  Z(1)=ZA
      DO 70 IZEL=1,NEFB
      NA=ICOR(IZEL,1)
      NB=ICOR(IZEL,2)
      XE=X(NA)-X(NB)
      YE=Y(NA)-Y(NB)
      ELZ=SQRT(XE**2+YE**2)
   70 Z(IZEL+1)=Z(IZEL)+ELZ
      XZB=X(ICOR(NHB,1))-X(ICOR(1,2))
      YZB=Y(ICOR(NHB,1))-Y(ICOR(1,2))
      ZB=SQRT(XZB**2+YZB**2)
      ZC=ZZERO
      IM=1
C
C       PARABOLIC TEMPERATURE DISTRIBUTION ALONG THE FIN
C       BOUNDARY,USING LAGRANGE INTERPOLATION
C
   80 TP1=T(ICOR(1,2))
      TP2=T(ICOR(NHB,1))
      TP3=T(ICOR(NEFB,1))
      AP1=TP1/(ZB*ZC)
      AP2=TP2/(ZB*(ZB-ZC))
      AP3=TP3/(ZC*(ZC-ZB))
```

```
      BP1=-(ZB+ZC)*AP1
      BP2=-ZC*AP2
      BP3=-ZB*AP3
      A1=AP1+AP2+AP3
      B1=BP1+BP2+BP3
      TC=0.0
      DO 90 NY=1,NEST
   90 TC=TC+T(ICOR(NY,2))
      AY=FLOAT(NY+1)
      TF=(TC+T(ICOR(NY,1))+AY*TS(NY))/(2.0*AY)

              SOLID-FLUID PROPERTIES

     WATER PROPERTIES

      IF(IFLUID.EQ.1) GO TO 91
      HFG=1093.88-0.5703*TS(1)+0.00012819*(TS(1)**2)
     1-0.0000008824*(TS(1)**3)
      RHOF(NI)=62.774-0.00255698*TF-0.000053572*TF**2
      CF(NI)=0.3034+0.000738927*TF-0.00000147321*TF**2
      UF(NI)=0.001397-0.000014669*TF+0.0000000631253*TF**2-0.00000
     100000976569*TF**3

     FREON PROPERTIES

   91 IF(IFLUID.EQ.0) GO TO 92
      HFG=69.5459-0.0156011*TS(1)-0.000455294*(TS(1)**2)+0.00000104144*(
     1TS(1)**3)
      RHOF(NI)=102.059-0.025364*TF-0.000502649*(TF**2)+0.00000135407*(TF
     1**3)
      CF(NI)=0.0594858-0.000429765*TF+0.00000348218*TF**2-0.000000010416
     18*TF**3
      UF(NI)=0.00078-0.00000525*TF+0.0000000125*TF**2
   92 UF(NI)=3600*UF(NI)
      IF(IFIN.EQ.1) GO TO 93
      CW(NI)=231.7772-0.02222*TSOLID
   93 IF(IFIN.EQ.0) GO TO 94
      CW(NI)=8.776+0.00265*TSOLID
   94 CK=CW(NI)
      CONST=RHOF(NI)**2*OMEGA**2*HFG*CPHI*CALFA*R(NI)

      INITIAL FILM THICKNESS
      IF (NI.GT.1) GO TO 100
      DEL(1)=1.107*(((TSAT-TINF)*CF(NI)/(UF(NI)*HFG))**.25)*((UF(NI)/(
     1RHOF(NI)*OMEGA))**0.5)
  100 CONTINUE

              AVERAGE ELEMENT CONVECTIVE COEFFICIENT ALONG
              THE FIN BOUNDARY

      ZSTAR=ZZERO-DEL(NI)/CALFA
      AZZ=DEL(NI)/SALFA
      ZZ=ZSTAR
      HDEN=((-A1*ZZ**3)/3.0-(B1*ZZ**2)/2.0)+ZZ*(TSAT-T(1))
      AZS=ABS(4*CF(NI)*UF(NI)*HDEN/CONST)**0.25
      HAC=0.0
      DO 190 IEL=1,NEFB
      AZ=Z(IEL)
      BZ=Z(IEL+1)
      IF (ZSTAR.LE.BZ) GO TO 110
```

```
      GO TO 120
  110 IF (HAC.NE.0.0) GO TO 180
      BZ=ZSTAR
  120 IF (IEL.NE.1) GO TO 130
      AK=(BZ-AZ)/5.0
      ZZ=AK
      GO TO 140
  130 AK=(BZ-AZ)/4.0
      ZZ=AZ
  140 ZEL=4*AK
      DO 150 NH=1,5
      HDEN=(-1.0*(A1*ZZ**3/3.0+B1*ZZ**2/2.0))+ZZ*(TSAT-T(1))
      HZ(NH)=ABS(CF(NI)**3*CONST/(4*UF(NI)*HDEN))**0.25
  150 ZZ=ZZ+AK
C     AVERAGE H USING SIMPSONS RULE
      CONH=AK*(HZ(1)+4*HZ(2)+2*HZ(3)+4*HZ(4)+HZ(5))/(3*ZEL)
      IF (ZSTAR.EQ.BZ) GO TO 160
      H(IEL)=CONH
      GO TO 190
  160 AZ=ZSTAR
      HAZ=CONH*(AZ-Z(IEL))
      DELA=AZS
  170 BZ=Z(IEL+1)
      DELB=(BZ-ZSTAR)*AZZ/(ZZERO-ZSTAR)
      DELZ=(DELA+DELB)/2.0
      HAC=(BZ-AZ)*CF(NI)/DELZ
      H(IEL)=(HAZ+HAC)/(BZ-Z(IEL))
      GO TO 190
  180 AZ=Z(IEL)
      DELA=DELB
      HAZ=0.0
      GO TO 170
  190 CONTINUE
      NETI=NEFB+1
      DO 200 IEL=NETI,NEST
  200 H(IEL)=CF(NI)/DEL(NI)
C
C              ENTRY INTO THE FINITE ELEMENT SOLUTION
C
      CALL FORMAF
      CALL BANDEC (NSNP,NBAN,1)
C
C              THE TEMPERATURE DISTRIBUTION
C
      DO 210 NT=1,NSNP
  210 T(NT)=F(NT,1)
      TIB(NI)=T(ICOR(NBOTI,2))
      TT(NI)=T(ICOR(NEFB,1))
      TE(NI)=T(ICOR(NEST,1))
      TB(NI)=T(ICOR(NBOTF,1))
      TTS=0.0
      DO 220 NS=1,NSNP
  220 TTS=TTS+T(NS)
      PN=FLOAT(NS)
      TSOLID=TTS/PN
C
C              Q AT THE BOTTOM SIDE
C
      QBI=0.0
      DO 230 IBEL=NBOTI,NBOTF
```

```
      NKA=ICOR(IBEL,1)
      NKB=ICOR(IBEL,2)
      XB=X(NKA)-X(NKB)
      YB=Y(NKA)-Y(NKB)
      ELB=SQRT(XB**2+YB**2)
  230 QBI=QBI+(T(NKA)+T(NKB)-2*TS(IBEL))*ELB*H(IBEL)/2.0
      QB(NI)=QBI*DELX

              ITERATION UNTIL CONVERGENCE CRITERIA IS MET

      IF (IM.EQ.1) GO TO 240
      QJ=QBI
      GO TO 250
  240 QI=QBI
      IM=2
      GO TO 80
  250 AQ=ABS(QJ-QI)/QJ
      IF (AQ.LE.CRIT) GO TO 260
      QI=QJ
      GO TO 80
  260 DMDOT(NI)=2.*QBI*DELX/HFG
      DMTOT=DMTOT+DMDOT(NI)
      C1=RHOF(NI)**2*OMEGA**2*R(NI)*SPHI/(3*UF(NI))
      XCOF(1)=-DMTOT
      XCOF(2)=0.0
      XCOF(3)=0.0
      XCOF(4)=C1*EPS(NI)
      XCOF(5)=C1*TALFA
      M=4
      CALL DPOLRT (M,IER)
      IF (ROOTR(1).GT.0.0) GO TO 270
      IF (ROOTR(2).GT.0.0) GO TO 280
      IF (ROOTR(3).GT.0.0) GO TO 290
      IF (ROOTR(4).GT.0.0) GO TO 300

              THE CONDENSATE THICKNESS

  270 DEL(NI+1)=ROOTR(1)
      GO TO 310
  280 DEL(NI+1)=ROOTR(2)
      GO TO 310
  290 DEL(NI+1)=ROOTR(3)
      GO TO 310
  300 DEL(NI+1)=ROOTR(4)
  310 QEL=0.0
      IF (NI.NE.1) GO TO 320

              Q FROM THE TOP SIDE

      Q THROUGH FIN

      QEL=0.0
  320 DO 330 IQEL=1,NEFB
      KA=ICOR(IQEL,1)
      KB=ICOR(IQEL,2)
      XQEL=X(KA)-X(KB)
      YQEL=Y(KB)-Y(KA)
      ELM=SQRT(XQEL**2+YQEL**2)
      QEL=QEL+(2*TS(IQEL)-T(KA)-T(KB))*ELM*H(IQEL)/2.0
  330 CONTINUE
```

```
      QINC(NI)=QEL*DELX
      AMTOT(NI)=DMTOT
      QET=QEL*DELX*NFIN*2
      QT=QT+QET
      QA=QBI*DELX*NFIN*2
C
C     Q THROUGH TROUGH
C
      QTRF=0.0
      DO 340 IQEL=NEFB+1,NEST
      KA=ICOR(IQEL,1)
      KB=ICOR(IQEL,2)
      XQEL=X(KA)-X(KB)
      YQEL=Y(KB)-Y(KA)
      ELM=SQRT(XQEL**2+YQEL**2)
      QTRF=QTRF+(2*TS(IQEL)-T(KA)-T(KB))*ELM*H(IQEL)/2.0
  340 CONTINUE
      QTINC(NI)=QTRF*DELX
      QTOTAL(NI)=QINC(NI)+QTINC(NI)
      QTRFT=QTRF*DELX*NFIN*2.
      QTF=QTF+QTRFT
      QTOT=QTOT+QA
      ASMOOTH=2*PI*RB(NI)*DELX+ASMOOTH
      ACASE=ACASE+NFIN*(((2*S(1))/(12*CALFA))+EPSO)*DELX
  350 CONTINUE
C     EVALUATE OBJECTIVE FUNCTIONS AND CONSTRAINTS
      OBJ=-(QTOT)
      WRITE(15,*) 'OBJECTIVE FUNCTION=', OBJ,'BFIN=', S(1)
C     FIRST CONSTRAINT IS TO ENSURE THE RATIO ZOA IS NOT NEGATIVE
      G(1)=-(((CBASE-(2*NFIN*S(1)*TALFA/12))/NFIN)/(S(1)*TALFA/6))
      G(1)=1000.0*G(1)
C     THE SECOND CONSTRAINT IS TO ENSURE THE CONDENSATE LEVEL IS NO
C     GREATER THAN THE FIN HEIGHT
      G(2)=-((S(1)/12.0)-DEL(NI))
      XPLOT(NO)=S(1)
      FNOBJ(NO)=-OBJ
      ARATIO=ACASE/ASMOOTH
      WRITE(13,*) XPLOT(NO),FNOBJ(NO)
      WRITE(14,*) ARATIO, FNOBJ(NO)
      GO TO 10
  360 CONTINUE
C
      BFIN=S(1)
C     *****   OUTPUT MODE   *****
      WRITE (15,630)
      DO 370 NR=1,NK
  370 WRITE (15,640) NR,QINC(NR),QTINC(NR),QTOTAL(NR)
      WRITE (15,650) QT,QTF
      WRITE (15,660) CLI,CANGL,RBASEI,R2I,THICKI,BFIN,RPM,TSS,TINF,
     1RIT,FANGL,ZOA,IFF
      WRITE (15,661) AFOVAS
      WRITE (15,670) BOA,ZOA,NFIN,BVIN,SURFAR
      WRITE (15,680)
      DO 380 NP=1,NSNP
      TCC(NP)=.5555555*(T(NP)-32)
  380 WRITE (15,690) NP,X(NP),Y(NP),T(NP),TCC(NP)
      WRITE (15,700)
      DO 390 KKL=1,NBOTF
      NKX=ICOR(KKL,1)
      NKY=ICOR(KKL,2)
```

```
      XP=X(NKX)-X(NKY)
      YP=Y(NKX)-Y(NKY)
      EXY=SQRT(XP**2+YP**2)
      QEP=ABS((T(NKX)+T(NKY)-2*TS(KKL))*EXY*H(KKL)/2.0)
      QEP=QEP*DELX
  390 WRITE (15,710) KKL,H(KKL),EXY,QEP
      WRITE (15,720) CRIT
      WRITE (15,730) HFG,NFIN,H(NBOTF),TSAT,RPM,QTOT,QT,FANGL
      WRITE(15,734)
      WRITE(15,735) ROOTR(1),ROOTI(1)
      WRITE(15,735) ROOTR(2),ROOTI(2)
      WRITE(15,735) ROOTR(3),ROOTI(3)
      WRITE(15,735) ROOTR(4),ROOTI(4)
      WRITE (15,740)
      DO 400 NR=1,NK
  400 WRITE (15,750) NR,DEL(NR),QB(NR),AMTOT(NR),TIB(NR),TT(NR),TE(NR),
     1TB(NR)
      WRITE (15,760)
      DO 410 NG=1,NDIV,2
  410 WRITE (15,770) NG,CW(NG),CF(NG),RHOF(NG),UF(NG),EPS(NG),R(NG),
     1QINC(NG)
      RETURN

  412 FORMAT (8X,E12.5,8X,E12.5)
  420 FORMAT (5I5)
  430 FORMAT (/2X,15HNO.OF.ELEMENTS=,I5,10X,18HNO.OF.SYSTEM N.P.=,
     1I5,/,2X,13HNO.OF BANDED=,I5)
  435 FORMAT (/2X,'IFLUID=',I5,10X,'IFIN=',I5)
  436 FORMAT (2X,'IFLUID = 0 FOR WATER, AND 1 FOR FREON')
  437 FORMAT (2X,'IFIN = 0 FOR COPPER, AND 1 FOR STAINLESS STEEL')
  440 FORMAT (4I5)
  450 FORMAT (/2X,7HELEMENT,10X,3HNP1,14X,3HNP2,15X,3HNP3)
  460 FORMAT (7G10.5)
  470 FORMAT (4X,5HCLI= ,E12.5,/,4X,7HCANGL= ,E12.5,/,4X,8HRBASEI.=,E12.
     15,/,4X,5HR2I= ,E12.5,/,4X,8HTHICKI= ,E12.5,/,4X,6HBFIN= ,E12.5,/,4
     2X,4HTZ= ,E12.5)
  480 FORMAT (5I5)
  490 FORMAT (4X,6HNDIV= ,I10,/,4X,6HNEST= ,I10,/,4X,6HNEFB= ,I10,/,4X,7
     1HNBOTI= ,I10,/,4X,7HNBOTF= ,I10)
  500 FORMAT (4F10.2)
  510 FORMAT (4X,5HRPM= ,E12.5,/,4X,5HTSS= ,E12.5,/,4X,6HTINF= ,E12.5,/,
     14X,6HHINF= ,E12.5)
  520 FORMAT (G10.9)
  530 FORMAT (4X,6HCRIT= ,E12.5)
  540 FORMAT (2G10.5)
  550 FORMAT (4X,7HFANGL= ,E12.5,/,4X,6HNFIN= ,I5)
  560 FORMAT (I5)
  570 FORMAT (4X,5HIFF= ,I10)
  580 FORMAT (16I5)
  590 FORMAT (6I5)
  600 FORMAT (///5X,4HTIB=,I5,10X,3HTT=,I5,/,5X,3HTE=,I5,/
     1,6X,3HTB=,I5)
  610 FORMAT (//10X,17HCRASH,CRASH,CRASH)
  620 FORMAT (//5X,4(E12.7,3X))
  630 FORMAT (2X,7HSTATION,2X,4HQFIN,17X,7HQTROUGH,15X,6HQTOTAL)
  640 FORMAT (4X,I5,E12.5,10X,E12.5,10X,E12.5)
  650 FORMAT (//,4X,11HQFIN TOTAL=,E12.5,10X,15HQTROUGH TOTAL= ,E12.5)
  660 FORMAT (/////,4X,5HCLI= ,E12.5,5X,7HCANGL= ,E12.5,/,4X,8HRBASEI= ,
     1E12.5,2X,5HR2I= ,E12.5,/,4X,8HTHICKI= ,E12.5,2X,6HBFIN= ,E12.5,/,4
     2X,5HRPM= ,E12.5,5X,5HTSS= ,E12.5,/,4X,6HTINF= ,E12.5,4X,6HHINF= ,E
```

```
   312.5,/,4X,6HCRIT= ,E12.5,4X,7HFANGL= ,E12.5,/,4X,5HZOA= ,E12
   45HIFF= ,I10)
661 FORMAT(4X,'FIN AREA/SMOOTH AREA=',E12.5)
670 FORMAT (1H1,//2X,4HBOA=,G12.5,5X,4HZOA=,G12.5,5X,5HNFIN=,I5,
   1/,5HBVIN=,G12.5,5X,13HSURFACE AREA=,G12.5)
680 FORMAT (//5X,2HNP,6X,1HX,12X,1HY,12X,1HT,12X,2HTC)
690 FORMAT (/2X,I3,3X,4(F10.6,3X))
700 FORMAT (//2X,2HEL,8X,1HH,11X,9HEL-LENGTH,15X,4HQ-EL)
710 FORMAT (/2X,I2,3X,E12.5,3X,E12.5,10X,E12.5)
720 FORMAT (/2X,22HCONVERGENCE CRITERIAN=,E15.8)
730 FORMAT (1H ,//,5X,4HHFG=,E12.5,/,5X,11HNO.OF FINS=,I5,/,5X,
   16HH-OUT=,E12.5,/,5X,5HTSAT=,E12.5,/,5X,4HRPM=,E12.5,/,5X,6HQ
   2E12.5,/,5X,6HQFIN =,E12.5,/,5X,11HHALF-ANGLE=,F8.3)
734 FORMAT(/5X,'ROOTS:',5X,'REAL PARTS',15X,'IMAGINARY PARTS')
735 FORMAT(15X,E12.5,15X,E12.5)
740 FORMAT (1H0,6X,1HJ,4X,14HFILM THICKNESS,8X,2HQB,10X,8HMASS-T
   1/,4X,3HTIB,8X,2HTT,10X,2HTE,8X,2HTB)
750 FORMAT (1H ,4X,I4,4X,F12.10,4X,F10.4,6X,F9.5,6X,F5.1,/,
   16X,F5.1,6X,F5.1,6X,F5.1)
760 FORMAT (1H0,6X,1HJ,6X,6HK-WALL,4X,6HK-FILM,3X,7HDENSITY,4X,9
   1FILM,/,6X,7HEPSILON,5X,6HRADIUS,5X,4HQINC)
770 FORMAT (1H ,4X,I4,4X,F7.3,4X,F6.4,4X,F6.3,4X,F9.7,
   1/,4X,F9.7,4X,F7.5,5X,F5.1,1X,F7.3)
    END
*
*
*     THIS SUBROUTINE DEFINES THE POSITIONS OF SYSTEM COORDINATE P
*
      SUBROUTINE COORD
*
      COMMON/ADS/DF(21),G(10),IDG(100),IGRAD,INFO,IOPT,IONED,IPRIN
     :ISTRAT,IWK(2000),IZ(30),OBJ,S(2),VLB(2),VUB(2),W(21,30),WK(5
     :NCOLA,NCON,NDV,NGT,NRA,NRIWK,NRWK
*
      COMMON/OLLIE/A(200,50),AMTOT(200),APS,B(3),BFIN,BOA,BVIN,C(3
     :CANGL,CF(200),CK,CLI,COF(5),CW(200),DEL(200),DMDOT(200),EA(3
     :EPS(200),EZERO,F(200,1),FANGL,FNOBJ(100),H(200),HINF,HZ(200)
     :QB(200),QINC(200),QTINC(200),QTOT,QTOTAL(100),R(200),RB(200)
     :RBASEI,R2I,RHOF(200),ROOTI(4),RPM,ROOTR(4),T(200),TALFA,TB(2
     :TCC(200),TE(200),THICK,THICKI,TIB(200),TINF,TS(200),TSAT,TSS
     :TT(200),TZ,UF(200),X(200),XCOF(5),XPLOT(200),Y(200),Z(200),Z
     :DOBF,DOTH,ICOR(200,3),IFF,JINT,JLC,JTC,KFF(50),KFIN(50),KT,N
     :NEL,NFIN,NSNP
*
*
C     DELH IS THE STANDARD DIVISION OF FIN HEIGHT
      DELH=S(1)/(12*DOBF)
      X(1)=0.0
      Y(1)=THICK+(S(1)/12)
      N=1
      DO 20 I=1,IFF
      ICA=KFIN(I)
      ICB=KFF(I)
      CBA=FLOAT(ICB-ICA)
      AN=0.0
      DO 10 II=ICA,ICB
      X(II)=X(1)+N*AN*DELH*TALFA/CBA
      Y(II)=Y(1)-N*DELH
   10 AN=AN+1.0
   20 N=N+1
```

```
      AN=0.0
      ICD=ICB-ICA+1
      DO 50 J=JTC,JLC,JINT
      X(J)=X(1)
      Y(J)=(1.0-AN/DOTH)*THICK
      DO 30 JJ=1,ICD
      X(J+JJ)=X(J)+JJ*EZERO/(2*(CBA+1.0))
   30 Y(J+JJ)=Y(J)
      JJ=ICD
      DO 40 K=1,KT
      X(J+JJ+K)=X(J+JJ)+K*APS/(2.0*KT)
   40 Y(J+JJ+K)=Y(J)
   50 AN=AN+1.0
      RETURN
      END

      THIS SUBROUTINUE IS USED TO FORMULATE THE FEM EQUATIONS

      SUBROUTINE FORMAF

      COMMON/ADS/DF(21),G(10),IDG(100),IGRAD,INFO,IOPT,IONED,IPRINT,
     :ISTRAT,IWK(2000),IZ(30),OBJ,S(2),VLB(2),VUB(2),W(21,30),WK(5000),
     :NCOLA,NCON,NDV,NGT,NRA,NRIWK,NRWK

      COMMON/OLLIE/A(200,50),AMTOT(200),APS,B(3),BFIN,BOA,BVIN,C(3),
     :CANGL,CF(200),CK,CLI,COF(5),CW(200),DEL(200),DMDOT(200),EA(3,3),
     :EPS(200),EZERO,F(200,1),FANGL,FNOBJ(100),H(200),HINF,HZ(200),
     :QB(200),QINC(200),QTINC(200),QTOT,QTOTAL(100),R(200),RB(200),
     :RBASEI,R2I,RHOF(200),ROOTI(4),RPM,ROOTR(4),T(200),TALFA,TB(200),
     :TCC(200),TE(200),THICK,THICKI,TIB(200),TINF,TS(200),TSAT,TSS,
     :TT(200),TZ,UF(200),X(200),XCOF(5),XPLOT(200),Y(200),Z(200),ZOA,
     :DOBF,DOTH,ICOR(200,3),IFF,JINT,JLC,JTC,KFF(50),KFIN(50),KT,NBAN,
     :NEL,NFIN,NSNP


      DO 20 N=1,NSNP
      F(N,1)=0.0
      DO 10 MA=1,NBAN
   10 A(N,MA)=0.0
   20 CONTINUE
      DO 70 IEL=1,NEL
      IA=ICOR(IEL,1)
      IB=ICOR(IEL,2)
      IC=ICOR(IEL,3)
      B(1)=Y(IB)-Y(IC)
      B(2)=Y(IC)-Y(IA)
      B(3)=Y(IA)-Y(IB)
      C(1)=X(IC)-X(IB)
      C(2)=X(IA)-X(IC)
      C(3)=X(IB)-X(IA)
      LENGTH BETWEEN ELEMENT NODES 1 AND 2
      EL=SQRT(C(3)**2+B(3)**2)
      AREA OF TRIANGULAR ELEMENT
      AS=ABS((B(1)*C(2)-B(2)*C(1))/2.0)
      HC=H(IEL)/CK
      DO 60 J=1,3
      JJ=ICOR(IEL,J)
      DO 50 K=1,3
      KK=ICOR(IEL,K)
```

```
C     FORMING THE A MATRIX
      EA(J,K)=(B(J)*B(K)+C(J)*C(K))/(4*AS)
      IF (HC.EQ.0.0) GO TO 40
      HEL=HC*EL/6.0
      IF (J.EQ.3) GO TO 40
      IF (K.EQ.3) GO TO 40
      IF (J.EQ.K) GO TO 30
      EA(J,K)=EA(J,K)+HEL
      GO TO 40
   30 EA(J,K)=EA(J,K)+2*HEL
   40 IF (KK.LT.JJ) GO TO 50
      NW=KK-JJ+1
      A(JJ,NW)=A(JJ,NW)+EA(J,K)
   50 CONTINUE
   60 CONTINUE
C     FORMING THE F MATRIX
      FE=HC*TS(IEL)*EL/2.0
      F(IA,1)=F(IA,1)+FE
      F(IB,1)=F(IB,1)+FE
   70 CONTINUE
      RETURN
      END
*
*
*     EQUATION SOLVER OF A SYMMETRIC MATRIX THAT HAS BEEN TRANS-
*     FORMED INTO BANDED FORM.
*
      SUBROUTINE BANDEC (NEQ,MAXB,NVEC)
*
      COMMON/ADS/DF(21),G(10),IDG(100),IGRAD,INFO,IOPT,IONED,IPRIN
     :ISTRAT,IWK(2000),IZ(30),OBJ,S(2),VLB(2),VUB(2),W(21,30),WK(5
     :NCOLA,NCON,NDV,NGT,NRA,NRIWK,NRWK
*
      COMMON/OLLIE/A(200,50),AMTOT(200),APS,B(3),BFIN,BOA,BVIN,C(3
     :CANGL,CF(200),CK,CLI,COF(5),CW(200),DEL(200),DMDOT(200),EA(3
     :EPS(200),EZERO,F(200,1),FANGL,FNOBJ(100),H(200),HINF,HZ(200)
     :QB(200),QINC(200),QTINC(200),QTOT,QTOTAL(100),R(200),RB(200)
     :RBASEI,R2I,RHOF(200),ROOTI(4),RPM,ROOTR(4),T(200),TALFA,TB(2
     :TCC(200),TE(200),THICK,THICKI,TIB(200),TINF,TS(200),TSAT,TSS
     :TT(200),TZ,UF(200),X(200),XCOF(5),XPLOT(200),Y(200),Z(200),Z
     :DOBF,DOTH,ICOR(200,3),IFF,JINT,JLC,JTC,KFF(50),KFIN(50),KT,N
     :NEL,NFIN,NSNP
*
*
      LOOP=NEQ-1
      DO 20 I=1,LOOP
      MB=I+1
      NB=MIN0(I+MAXB-1,NEQ)
      DO 20 J=MB,NB
      L=J+2-MB
      D=A(I,L)/A(I,1)
      DO 10 MM=1,NVEC
   10 F(J,MM)=F(J,MM)-D*F(I,MM)
      MM=MIN0(MAXB-L+1,NEQ-J+1)
      DO 20 K=1,MM
      NN=L+K-1
   20 A(J,K)=A(J,K)-D*A(I,NN)
      DO 30 I=1,NVEC
   30 F(NEQ,I)=F(NEQ,I)/A(NEQ,1)
      DO 50 I=2,NEQ
```

```
      J=NEQ-I+1
      K=MIN0(NEQ-J+1,MAXB)
      DO 50 MM=1,NVEC
      DO 40 L=2,K
      MB=J+L-1
   40 F(J,MM)=F(J,MM)-A(J,L)*F(MB,MM)
   50 F(J,MM)=F(J,MM)/A(J,1)
      RETURN
      END


      SUBROUTINE DPOLRT COMPUTES THE ROOTS OF A REAL
      POLYNOMIAL USING A NEWTON-RAPHSON ITERATIVE
      TECHNIQUE.


      SUBROUTINE DPOLRT (M,IER)

      COMMON/ADS/DF(21),G(10),IDG(100),IGRAD,INFO,IOPT,IONED,IPRINT,
     :ISTRAT,IWK(2000),IZ(30),OBJ,S(2),VLB(2),VUB(2),W(21,30),WK(5000),
     :NCOLA,NCON,NDV,NGT,NRA,NRIWK,NRWK

      COMMON/OLLIE/A(200,50),AMTOT(200),APS,B(3),BFIN,BOA,BVIN,C(3),
     :CANGL,CF(200),CK,CLI,COF(5),CW(200),DEL(200),DMDOT(200),EA(3,3),
     :EPS(200),EZERO,F(200,1),FANGL,FNOBJ(100),H(200),HINF,HZ(200),
     :QB(200),QINC(200),QTINC(200),QTOT,QTOTAL(100),R(200),RB(200),
     :RBASEI,R2I,RHOF(200),ROOTI(4),RPM,ROOTR(4),T(200),TALFA,TB(200),
     :TCC(200),TE(200),THICK,THICKI,TIB(200),TINF,TS(200),TSAT,TSS,
     :TT(200),TZ,UF(200),X(200),XCOF(5),XPLOT(200),Y(200),Z(200),ZOA,
     :DOBF,DOTH,ICOR(200,3),IFF,JINT,JLC,JTC,KFF(50),KFIN(50),KT,NBAN,
     :NEL,NFIN,NSNP


      IFIT=0
      N=M
      IER=0
      IF (XCOF(N+1)) 10,40,10
   10 IF (N) 20,20,60

         SET ERROR CODE TO 1

   20 IER=1
   30 RETURN

         SET ERROR CODE TO 4

   40 IER=4
      GO TO 30

         SET ERROR CODE TO 2

   50 IER=2
      GO TO 30
   60 IF (N-36) 70,70,50
   70 NX=N
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 80 L=1,KJ1
      MT=KJ1-L+1
```

```
   80 COF(MT)=XCOF(L)
C
C        SET INITIAL VALUES
C
   90 XO=.00500101
      YO=0.01000101
C
C        ZERO INITIAL VALUE COUNTER
C
      IN=0
  100 XX=XO
C
C        INCREMENT INITIAL VALUES AND COUNTER
C
      XO=-10.0*YO
      YO=-10.0*XX
C
C        SET X AND Y TO CURRENT VALUE
C
      XX=XO
      YY=YO
      IN=IN+1
      GO TO 120
  110 IFIT=1
      XPR=XX
      YPR=YY
C
C        EVALUATE POLYNOMIAL AND DERIVATIVES
C
  120 ICT=0
  130 UX=0.0
      UY=0.0
      V=0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF (U) 140,270,140
  140 DO 150 I=1,N
      L=N-I+1
      XT2=XX*XT-YY*YT
      YT2=XX*YT+YY*XT
      U=U+COF(L)*XT2
      V=V+COF(L)*YT2
      FI=I
      UX=UX+FI*XT*COF(L)
      UY=UY-FI*YT*COF(L)
      XT=XT2
  150 YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF (SUMSQ) 160,230,160
  160 DX=(V*UY-U*UX)/SUMSQ
      XX=XX+DX
      DY=-(U*UY+V*UX)/SUMSQ
      YY=YY+DY
      IF (ABS(DY)+ABS(DX)-1.0E-05) 210,170,170
C
C        STEP ITERATION COUNTER
C
  170 ICT=ICT+1
      IF (ICT-500) 130,180,180
```

```
180 IF (IFIT) 210,190,210
190 IF (IN-5) 100,200,200
C
C       SET ERROR CODE TO 3
C
200 IER=3
    GO TO 30
210 DO 220 L=1,NXX
    MT=KJ1-L+1
    TEMP=XCOF(MT)
    XCOF(MT)=COF(L)
220 COF(L)=TEMP
    ITEMP=N
    N=NX
    NX=ITEMP
    IF (IFIT) 250,110,250
230 IF (IFIT) 240,100,240
240 XX=XPR
    YY=YPR
250 IFIT=0
    IF (ABS(YY/XX)-1.0E-04) 280,260,260
260 ALPHA=XX+XX
    SUMSQ=XX*XX+YY*YY
    N=N-2
    GO TO 290
270 XX=0.0
    NX=NX-1
    NXX=NXX-1
280 YY=0.0
    SUMSQ=0.0
    ALPHA=XX
    N=N-1
290 COF(2)=COF(2)+ALPHA*COF(1)
    DO 300 L=2,N
300 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
310 ROOTI(N2)=YY
    ROOTR(N2)=XX
    N2=N2+1
    IF (SUMSQ) 320,330,320
320 YY=-YY
    SUMSQ=0.0
    GO TO 310
330 IF (N) 30,30,90
    END
```

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