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Multi-criteria Approach in Configuration of Energy Efficient Sensor Networks

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Problem and Motivation: A designing of energy efficient wireless sensor networks is one of the most trendy research topics. In this paper the authors developed a new multi-criteria approach which allows filtering out non-efficient topologies during a phase of designing of a sensor network and also running a self-adjustment process during its functioning. It is shown that the problem of designing of an energy efficient wireless sensor network belongs to the class D of optimization combinatorial problems for which the equivalent recognition problems are NP-complete or open. The D-class (Peltsverger B., Khavronin O. (1999)) is such that according to a decomposition approach there exist particular criteria compatible with the main one, and the recognition task corresponding to a particular criterion always belongs to the P-class problem.

Decomposition approach allows constructing polynomial approximate algorithms for the D-class problems. Indeed, to determine a non-dominated point from a feasible set we must solve a recognition problem for every particular criterion. By finding a necessary number of non-dominated points we construct approximately Pareto's set where according to decomposition approach there is a solution of the initial problem (Statnikov, R., Bordetsky A., Statnikov A. (2004)). That approach is used to solve the designing of energy efficient wireless sensor network problem.

Background and Related Work: The problem of designing of an energy efficient wireless sensor network can be described by the following model. Given is a graph G = (V, E) with a singled out node i_0 (monitoring station); $\forall$ (i, j) $\in E$ given are $w_{ij} \geq 0$, power consumption for establishing connection between node i and node j; $v_{ij} \geq 0$, a unit data transmitting power along an arc (i,j). $\forall$ i $\in$ V \ {i_0} given is a size of the transmitting data $P_i \geq 0$.

Initial Problem: $$\sum_{(i,j)\in T}(w_{ij} + v_{ij} y_{ij}) \to \min,\ T \in \Omega, \quad (1.1)$$

where *T* – is a spanning tree of graph G, Ω is a set of the spanning trees of the graph *G*, y_{ij} is a data flow along an arc (i,j) in *T*.

The solution of the initial problem (IP) is a spanning tree T^*, which requires minimum power consumption. The NP-hardness of this problem is proved by reducing the minimal covering problem to it (Garey, M., Johnson, D. (1979)). It means that it is a difficult computational problem for large-scale networks such as wireless sensor networks with thousands sensors and communication links (Akyildiz et al., (2002)). Let's show that the problem of designing of an energy efficient wireless sensor network belongs to a class D and can be solved by use of the decomposition approach.

Approach and Uniqueness: Let's consider an initial NP-hard combinatorial optimization problem (IP)

$$F(x) \to \min,\ x \in X, \quad (2.1),$$

where *X* – is a finite set.

Along with the main criterion for the problem in question it is possible to introduce monotonically coordinated particular criteria $u_i(x) \to \min$, i = $\overline{1,m}$.

Particular criteria are monotonically coordinated if for any x', x'' $\in$ X, F(x') ≤ F(x'') follows $u_i(x') \leq u_i(x'')$, $\forall\ i \in \overline{1,m}$.

The problem

$$u_i(x) \to \min,\ x \in X,\ i \in \overline{1,m}. \quad (2.2)$$

is a particular problem (PP).

Definition. A class D is contained NP-hard problems (2.1) for which there exist monotonically coordinated particular criteria and particular problems (2.2) belong to a class P. In our case two particular criteria can be introduced:

1) reducing power consumption of establishing connection between nodes

PP$_1$: $u_1(T) = \sum_{(i,j)\in T} w_{ij} \longrightarrow \min, \quad T \in \Omega,$ (2.3)

a minimal spanning tree problem, belongs to the class *P*;

2) reducing power consumption of transmitting data

PP$_2$: $u_2(T) = \sum_{(i,j)\in T} v_{ij} y_{ij} \longrightarrow \min, \quad T \in \Omega,$ (2.4)

a shortest path tree problem, belongs to the class *P*.

The condition of coordination is satisfied, as $F(T) = u_1(T) + u_2(T)$.

According to decomposition approach the global solution of the problem (1.1) is reached on the effective solutions set of the problem

$$u = (u_1(T), u_2(T)) \rightarrow \min, T \in \Omega, \qquad (3.1)$$

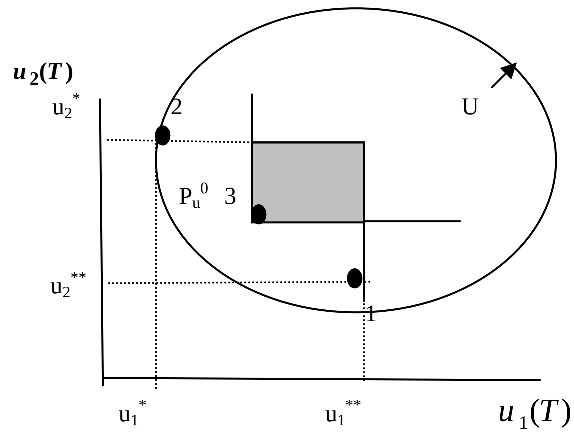
Let P_U, P_T – effective solutions (Pareto) sets in the space of criteria and the space of solutions correspondingly. So the problem (1.1) is reduced to the problem

$$F(T) \rightarrow \min, \; T \in P_T \subset \Omega.$$

It's evident that such an approach can be realized as the process of construction of the set P_U, (P_T) which is not labor consuming (see (2.3) and (2.4)). The Fig.1 illustrates a general scheme of designing of energy efficient sensor networks. By definition $T_i \in P_T$ if there exists no T_j such that $U_1(T_j) \leq U_1(T_i)$ and $U_2(T_j) \leq U_2(T_i)$ and one of the inequalities is strict. To each solution T_i of the problem (3.1) in the space of the criteria there is corresponded a rectangle D (T_i). On the Fig.1 is shown how the procedure can reduce a size of P_U^0 by cutting a rectangular which points are dominated by the point 3. The procedure allows finding lower and upper bounds of the function F (T) (energy consumption by the sensor network T): $F_0 \geq F_1 \geq F_2 \geq \ldots \geq F_* \geq F_{ideal} = F(u_1^*, u_2^{**})$.

Results and Contributions: A method of solving problem of designing an energy efficient wireless sensor network is based on the multi-criteria approach. As the initial criterion is the sum of the particular criteria the solution of the problem can be obtained when constructing a set of effective solutions P_T of the problem (3.1) by memorizing a record value F(T). The subdivision feasible set is based on the generator of spanning trees in order of weight increasing.

The generator is modified such as the spanning trees are generated in the order of weight increasing but in any previously determined step. This permits to make a uniform sample in a space of the particular criteria. This procedure can be also running during the time of functioning of sensor networks and used for self–configuration of the network by filtering out all topologies which do not belong to a Pareto set.



$u_1^* = u_1(T^*) = u_1(T) \longrightarrow \min, \; T \in \Omega, \quad u_2^* = u_2(T^*)$

$u_2^{**} = u_2(T^{**}) = u_2(T) \longrightarrow \min, \; T \in \Omega, \; u_1^{**} = u_1(T^{**})$

Fig. 1 Illustration of the General Scheme

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