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Abstract: The helicopter has grown in military stature for more than 40 years: its ascendancy has reformed the US Army. Unfortunately, the current army helicopter fleet consists predominantly of Vietnam-era aircraft approaching the end of their useful lives. We have captured complex procurement and modernization tasks in an optimization-based decision support system, christened PHOENIX, which recognizes yearly operating, maintenance, retirement, service-life extension, and new procurement costs while enforcing constraints on fleet age, technology mix, composition, and budgets over a multi-year planning horizon. The army has applied PHOENIX to helicopters with such success that it has already been adapted to tactical wheeled vehicles and is under consideration for further applications.

Keywords: Military—cost effectiveness, Programming—Integer applications, Finance—capital budgeting

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The helicopter has grown in military stature for more than 40 years: its ascendancy has reformed the US Army. Unfortunately, the current army helicopter fleet consists predominantly of Vietnam-era aircraft approaching the end of their useful lives. We have captured complex procurement and modernization tasks in an optimization-based decision support system, christened PHOENIX, which recognizes yearly operating, maintenance, retirement, service-life extension, and new procurement costs while enforcing constraints on fleet age, technology mix, composition, and budgets over a multi-year planning horizon. The army has applied PHOENIX to helicopters with such success that it has already been adapted to tactical wheeled vehicles and is under consideration for further applications.

Thus the whirlygig of time brings in his revenges.—William Shakespeare, *Twelfth Night*

Helicopters provide unique capabilities invaluable to the modern military. Their earliest use was quick, high-priority, short-haul transport of individuals between unimproved landing sites. Improvements in basic technology—principally stronger, lighter construction and more powerful turbine engines—have produced greater speed, greater load-carrying capacity, and greater survivability and reliability. These aircraft are now tailored to perform various specialized roles (called

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MILITARY—COST EFFECTIVENESS
PROGRAMMING—INTEGER APPLICATIONS
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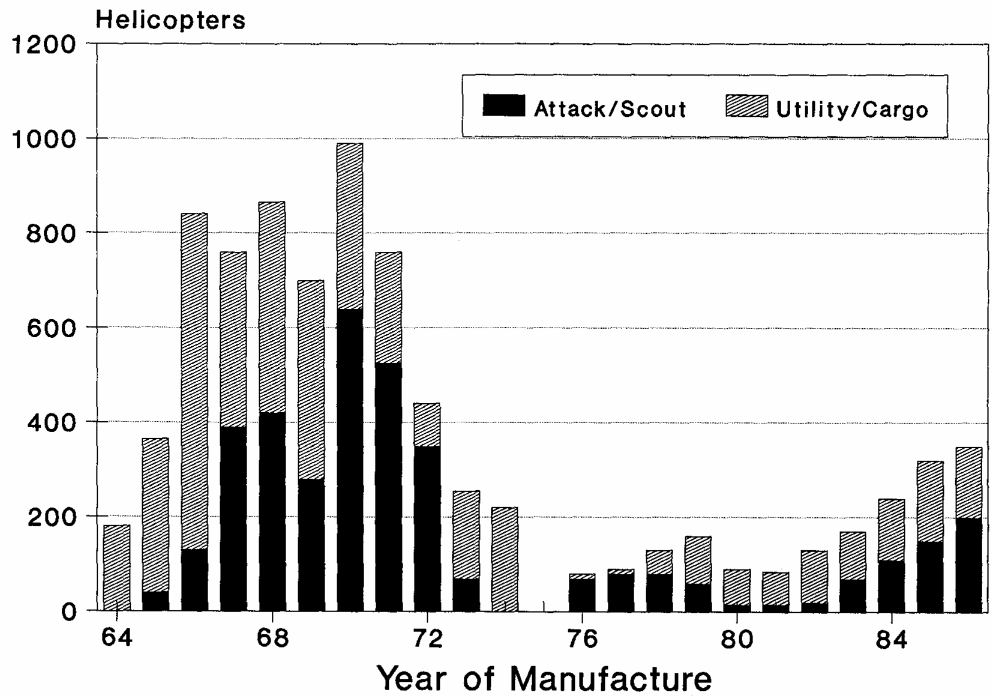

Figure 1: This summary of cohort inventories as of 1987 shows the preponderance of Vietnam-era helicopters.

craft histories. Figure 1 summarizes our initial helicopter inventory by cohort year of manufacture. Over years of service, each aircraft cohort suffers attrition through loss, conversion by SLEP, or by retirement.

Each production campaign is characterized by contiguous activity from a line's opening date to its closing date, and by minimum and maximum sustainable, economic, peace-time line capacities during production. For a particular aircraft model, the production line opening and closing dates may be restricted to certain years and require that prior production campaigns be ended before, or subsequent campaigns commenced after, its years of operation. For instance, the AH-64B Apache may not commence production until the AH-64A production ceases and may commence only in one of the years from 1990 through 1995 and may cease between 1992 and 2015 and must cease before production of the AH-64C begins, which will require a one-time conversion of tooling.

We assume that each aircraft must be retained for a minimum number of years but must be retired or SLEP-rebuilt no later than the date its maximum useful life expires. Actual life limits are expressed in flight hours, but we have assumed uniform annual flight program hours for each type of aircraft. Also, we have assumed that aircraft are purchased and supported individually, rather than in compatible unit sets. This assumption inflicts no harm because each cohort is produced contiguously, can be expected to require monotonically increasing maintenance costs with age, and thus is treated individually, much as a cohort-unit-set would be over time—a close approximation of reality.

Economic and political realities dictate that proposed annual constant-dollar budgets follow a regular pattern over time, regardless of myopic economies of scale in our program. Thus, a budget band of minimum and maximum future annual expenditures accounts for lags between payment for aircraft and their actual delivery. Unused budget monies are not carried forward to subsequent years.

A final, vital embellishment of our model is provision for violation of each of the foregoing requirements at a specified linear cost per unit of violation. Thus, each requirement is stated as an aspiration, or goal, which may not be achievable, but which can be approached with linear reward.

The mathematical formulation of PHOENIX is given in the appendix. In summary, we seek to minimize O & M costs subject to

(1) Minimum and maximum levels of operational aircraft by year and mission,
(2) A minimum fraction of high-technology aircraft by year and mission,
(3) A maximum average age by year and mission,
(4) Minimum and maximum expenditures by year,
(5) Certain production lines being open,
(6) Minimum and maximum production line capacities for open lines,
(7) Minimum and maximum production levels by year, for each possible line opening and closing year,
(8) The availability of suitable aircraft as raw material for SLEP conversions,
(9) The continuous operation of open production lines, and
(10) Over time, the aging, attrition, conversion by SLEP and retirement of aircraft cohorts.

Implementation and Computational Experience

This project began with rather urgent parallel efforts to develop data and a model. The direct impetus came from Major General Wilson A. Shoffner, Assistant Deputy Chief of Staff for Operations and Plans-Force Development, who was involved with certain strategic decisions to be made for fiscal 1989, just weeks from the start of the project.

Because of our 60-day sanction and because we were located on opposite sides of the continent, our efforts to develop the data and the model were not only parallel, they were highly independent. After one very long day of analysis, the model builders (primarily Brown and Wood with early help from Clemence) had a hand-written functional specification of the model and supporting data on a few sheets of paper (not much more detailed and a bit less accurate than the appendix). The model builders agreed to deliver a working prototype with extremely flexible capabilities aimed at capturing as much realism as possible at an annualized level of detail. The data development team (Clemence and Teufert) promised to mobilize whatever corporate wisdom was necessary to characterize the current fleet status, costs and the likely consequences of future procurement and manufacturing options. Over a dozen army analysts were involved in this process.

The model builders had the easier job. For the modeled entities (missions, production lines, aircraft, and so forth) they had to conjure up a great deal of detail in anticipation of actual data. The sheer numbers of entities would not be known, nor would their names, relationships, or data attributes, until the model was put into use. Naming conventions for model entities had to support many interactive model plays (data generation, solution, and interpretation) to be made over a short period of time; but, not knowing what kinds of issues would be of interest, and thus what model features to make especially user-friendly, the model builders had to invest much precious development time on extremely general model-manipulation functions that might never be used.

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Manufacturing activities presented classic concerns. Would a fixed and a unit production cost suffice to describe operations of a production line within prescribed volume limits? If not, should unit costs vary over time? When is a cost incurred, that is, when is a cost applied against a budget? Should candidate production campaigns using the same production facilities be modeled as a set of mutually exclusive entities with the same attributes or will these attributes, such as minimum and maximum production volumes, change with the length or starting year of a specific campaign? Some manufacturing activities compete for the same production lines, some must be scheduled in contiguous precedence, and some require availability of old aircraft as raw material. Hardest of all, how do you predict the types of questions that high-level managers will pose and how do you accommodate their guidance?

The model team was split into two parts: user interface and data validation, and model generation and report writing. The data for the MILP, for example, costs, bounds, and constraint coefficients, were to be generated directly into the working arrays of the solver. The generator was built assuming that the necessary data were scrubbed and available in specified arrays. The user interface was built to take raw data, in the form of a scenario script, check for consistency, and then present the data in the format required by the model generator. A rudimentary set of reports was designed and coded early in the process to help in debugging the model, while a final suite of reports was completed only after several weeks of experience with PHOENIX.

The model team decided to keep the user interface as simple as possible and to reuse proven software designs wherever feasible. An input script was built to allow for any combination of direct inputs from other computer programs and manual data entry via a standard full-screen editor. Input scripts accommodate arbitrarily detailed, free-format imbedded documentation which can be suppressed from reports when not needed. Scripts present data by type of entity in self-specified formats with scale factors for conversion from convenient user data units to common model units.

Each model entity is identified solely by its name. Names are accumulated in symbol tables as they are encountered, and references to undefined names are tolerated with mild rebuke: thus, removing an initial entity definition is sufficient to eliminate its influence throughout the script without further tedious deletions.

The goal was a script expressed in the user's terms which could be used to completely specify a model play. No model play would require any programming.

By previewing the data and by reviewing experience gained from solving many other models, the model builders realized that many plays of the model would generate constraints that could not be satisfied and that they must make provisions for this. Thus, the data script includes a linear cost to apply to each unit of constraint violation. We call these *elastic constraints* [Brown and Graves 1975], but other authors have suggested other names, for example, *goals* [Charnes and Cooper 1961].

Meanwhile, the data development team was much busier gathering expert opinions from throughout the army aviation community and evaluating their inputs. It analyzed existing and candidate helicopters and estimated and reestimated costs. Helicopters to employ new technology required much managerial input and artful data modeling. One entirely new program, the Light Helicopter Experimental (LHX), and options for AH-64 Apache upgrades posed vexing data development problems.

With the imposed deadline, data development had to be carried out in several parallel somewhat independent efforts. O & M costs, aviation overhead costs, fixed production costs, unit production costs, retirement costs, budget forecasts, production line data, aircraft inventory, force structure, and aviation policy were analyzed as functionally distinct areas. While the data were being collected and their form and nature became clearer, the team conducted concurrent analyses, and developed, documented, and standardized new data never before formally expressed by the army. For instance, diverse sources contributed a large amount of production line data which PHOENIX now expresses concisely. Annualized O & M costs derive from flying-hour cost data, flight program plans and modeling of maintenance costs.

Some seemingly important details proved inconsequential. For instance, aviation overhead costs such as air traffic control are not going to be influenced much by force modernization. These costs are treated as a constant component of the budget and otherwise ignored.

How do you predict the types of questions that high-level managers will pose?

Conversely, seemingly simple issues proved tricky. The data team discovered that fixed production costs are incurred well before a production line actually opens. (Unfortunately, the data team did not share this insight with the model team until later!)

Capital expenditures to open, operate, and close a production line are only roughly expressible as fixed and unit costs during the production campaign. Subtle learning effects, accounting and budgeting methods, collateral expenses for spares and support infrastructure, and so forth, all conspire to complicate the specification of realistic annualized fixed and variable (unit) costs. However, we are convinced that a combination of fixed and variable (unit) costs is absolutely necessary for a realistic model of this capital-intensive problem, and we have devoted much effort to deriving model costs that reflect, as accurately as possible, the true costs.

The Showdown

The model, by this time called PHOENIX, collided with its data on schedule, and with predictable consequences: when the model could be solved, it produced nonsensical answers. Pressing high-level demands for correct answers motivated the model and data teams in their energetic supplemental development and repair efforts.

New models, especially new optimization models, exhibit unpredictable behavior. Data errors and oversights are exploited perversely. Model assumptions are exercised to their extremes and weaknesses are inevitably revealed. Occasionally, bugs are discovered.

Review and revision soon produced trustworthy results but strange prescriptions. We diagnosed counter-intuitive behavior by enhancing report detail and by revising penalties governing constraint enforcement. Play was confidently begun in earnest.

Initial results were so compelling that high-level management posed questions leading to scores of scenario evaluations in just a few weeks. For instance, PHOENIX confirmed the necessity to reduce the size of the fleet. PHOENIX also revealed surprising advantages of a new LHX program over an extended effort to keep AH-64 Apache models current.

A great number of plays within scenarios was dictated in part by the nature of the model: a detailed optimization model was being used as an identity simulator to evaluate circumstances largely unforeseen by the model team. Promising scenarios mandated that we perform sensitivity analyses by further varying the data.

A typical 25-year scenario plans for 16 helicopter types and 300 potential campaigns for five production lines. The resulting MILP has about 4,000 constraints, 21,000 variables (of which 300 are binary with large fixed costs), and about 100,000 non-zero coefficients. Such problems are solved by the X-System [Brown and Graves 1975] on an IBM 3033AP using interactive VM/CMS in less than 3.5 megabytes. The typical scenario requires five to 10 minutes to find an optimal integer solution. (Subsequent work by Olson [1989] has reduced this time to about a minute.) The suite of reports developed to help in the detailed analysis of PHOENIX prescriptions includes

(1) Procurement schedule,
(2) Force composition by year,
(3) Force composition by cohort,
(4) Annual expenditures,
(5) Retirement schedule by model,
(6) Retirement schedule by cohort,
(7) Mission requirements,
(8) Average age,
(9) High-technology fraction, and
(10) Production line capacity utilization.

Once in a while, a scenario proves troublesome, requiring as much as 30 minutes to solve. These difficult scenarios are endowed with pathological model structure—nearly indistinguishable production alternatives—which give rise to numeric instability and a very long integer enumeration. We must frequently compare alternatives with small relative cost variations, say less than one percent. The solution effort required to obtain exact optimal solutions is justified by the scale factor of our objective function: billions of dollars.

The elastic constraints proved invaluable. On most plays, there are many violations. Given that linear penalties for constraint violations are meaningful in the context of the model, close scrutiny of such violations by optimal solutions was frequently rewarded with totally unforeseen insights. For instance, several of the best scenarios committed large, one-time budget overruns, balanced by large underruns. The reason proved clear enough: PHOENIX had to buy into a new production campaign in order to meet many other constraints but was unable to spend within a level budget band over the planning horizon.

Results

Many scenarios were evaluated leading to inescapable conclusions:

(a) Under projected budget limits, the size of the fleet must be slashed;

(b) Age-forced retirements create a large, near-term shortage in certain mission categories;

(c) New procurement and SLEP programs will require nonuniform funding levels over the planning horizon;

(d) Certain existing helicopters are not as cost-effective overall as thought; and

(e) Many alternatives show promise, but all require that we judiciously violate constraints derived from policy or resource guidelines. There is no perfect solution.

The practical impact of PHOENIX (and to some extent, the time-pressure under which PHOENIX was developed) may best be described by an anecdotal history.

On September 21, 1987, the *Washington Times* reported

> The army will go without new helicopters through much of the 1990s because of planning delays . . .

Chastened, the army encouraged us to begin the PHOENIX project during the 1987 year-end holidays. On January 13, 1988, the *Wall Street Journal* [Carrington 1988] reported that

> The army is all but certain to cancel plans for its new $50 billion LHX helicopter program.

The LHX had been considered the centerpiece of the army's aviation modernization program, and we were asked to see if the threatened cancellation should be challenged. We obtained initial results from the PHOENIX model in mid-February 1988. A press release on March 29, 1988 [Secretary of the Army 1988a] stated

> The army and the secretary of defense recently reached agreement on the objectives of the changed LHX program.

Extensive analysis using PHOENIX had proven the worth of the LHX even to its detractors. The same press release continues

> RDT & E funds for all army rotary wing aircraft programs will be managed in a consolidated fashion.

PHOENIX had impressed the brass sufficiently that they institutionalized the approach.

Airframe (Procurement Objective)	Budget Year												
	87	88	89	90	91	92	93	94	95	96	97	98	99–
AH-64 (863)	101	77	72	48	40	40	40	20					
OH-58D (477)	36	36	24	36	48	54	54	54	36				
UH-60A	82	72	71	61	61	72	72	72	72	72	72	72	606
MH-60K (2253)			1	11	11								
CH-47D	48	47	43	37	48	48							
MH-47E (472)		1	5	11									
SEMA (109)		3	10	20	14	21	27	14					
LHX (2096)									24	48	96	144	1784
Resources in Billions of Dollars			3.40	3.48	3.62	3.72	3.80	3.89	3.98	4.07	4.17	4.26	

Table 1: An army press release dated September 30, 1988 included Army Aviation Modernization Program (AAMP) production plans for significant, near-term planning epochs, along with budgetary assumptions.

A technical report of the PHOENIX effort was issued in August 1988 [Force Systems Directorate, 1988]. Finally, in a press release dated September 30, 1988, the secretary of the army [1988b] revealed a detailed fleet management plan for the Army Aviation Modernization Program (AAMP). The plan does preserve the future LHX program, and

> The funding provides for an efficient, cost-effective production rate of the UH-60A, CH-47D, and OH-58D aircraft in quantities required by the Army's force structure in meeting the requirements of the unified and specified commanders-in-chief, and to achieve an optimum program within the funding constraints.

(This is one of the few occasions where the term "optimum" has been suggested to the press in a technically correct manner.) Table 1, taken from this press release, gives detailed production plans for the Army's helicopter fleet and was extracted from a PHOENIX report.

After PHOENIX had been applied successfully to the army's helicopter fleet, it was adopted to plan the modernization of the army's fleet of over 335,000 tactical wheeled vehicles [HQ Department of the Army, 1989]. In an introduction to the report on this plan, the Army Chief of Staff states that

> The Army Tactical Wheeled Vehicle Modernization Plan is a roadmap that guides the Total Army to cost-effective development and acquisition of required tactical mobility assets.

Furthermore, he describes the criteria for modernization, which are virtually identical to those used in the helicopter plan:

> Establishment of key criteria for useful life, procurement objectives, service life extension programs, and retirement and washout. These criteria support decisions for vehicle improvement and replacement and assure needed warfighting capabilities now, and into the future.

The body of the report states (brackets ours)

Phoenix model [output] data served as the basis for determining which mix of vehicles minimized total TWV ownership costs to the army projected out to the year 2020.

Production of the LH (light helicopter) was authorized in April 1991 [Pasztor and Wartzman 1991].

Conclusions

Prior to PHOENIX, each force-planning scenario took about 14 man-days to work up manually. Given the complexity of most scenarios, integrated and consistent manual evaluations could not be guaranteed. Comparisons between such manual solutions were risky and in no sense were such solutions optimal.

Using PHOENIX, each scenario requires about half a day of data preparation. Optimal solutions from PHOENIX are trustworthy and easy to compare. PHOENIX has been designed to express scenarios in simple, universal terminology which is understood at all levels of review. Perhaps best of all, PHOENIX provides a "level playing field" for evaluating competing points of view at arm's length.

PHOENIX has its faults, too. We model helicopter wear as a function of calendar age, assuming a regular annual flight-hour program: actual flight hours are customarily used. Fixed and linear unit production costs are used to estimate actual costs: the efficiencies of lot sizes and learning effects may not be faithfully depicted. The length of planning horizon has been limited by foreseeable future procurement options: solutions are sensitive to this time limit. There are myriad procurement options: PHOENIX can be hard to solve if overwhelmed by too many nearly indistinguishable alternatives.

Choosing an objective function is difficult. For helicopters, PHOENIX minimizes O & M costs, relying on budget constraints to reconcile these costs with procurement and other costs. In the more recent army application to tactical wheeled vehicle modernization, PHOENIX minimizes the sum of procurement and ownership costs. In some models, it may be necessary to consider the personnel using and the personnel maintaining the weapons systems. In such a case, PHOENIX might better be used with a manpower objective function.

From the view of classical operations research, PHOENIX is tailored for long-range planning, at a high level of detail, of capital equipment procurement, use, repair, and retirement, where the fixed costs are large relative to other costs. Similar problems have been studied before, but principally in the private sector of our economy.

Our Department of Defense spent about a third of its budget on acquisitions of weapons systems in fiscal 1988—about \$84 billion. We think that the PHOENIX approach shows promise for other areas of military force planning.

Acknowledgments

Insight, Inc. of Alexandria, Virginia has provided the optimization software for PHOENIX. The Office of Naval Research has supported basic research in optimization by Brown and Wood and encourages its application to important problems.

APPENDIX: Mathematical Formulation

The PHOENIX model uses the following indices:

p = production line,
a = aircraft,
m = mission,
v = first year of a production campaign,
w = last year of a production campaign,

t = planning year, and
c = cohort year (year of manufacture)

The basic index sets are

A = aircraft type a,
C = aircraft cohorts c,
M = missions m,
T = planning (calendar) years t,
V = possible production campaign opening years,
W = possible production campaign closing years,
W_p = possible production campaign closing years for line p,
VW_p = possible pairs of opening and closing years (v, w), $v \in V$, $w \in W$, for line p,
VW_{pt} = the subset of VW_p such that $v \leq t \leq w$,
A'_a = aircraft a' which can be converted into aircraft a by SLEP,
A''_a = aircraft a'' which can be produced from aircraft a by SLEP,
A_p = aircraft a produced on production line p,
A_m = aircraft a which performs mission m,
C_a = cohorts c for aircraft a, and
PP = production line pairs (p, p') where line p precedes line p'.

The following data is needed to completely define derived index sets:

$\underline{\sigma}_a, \bar{\sigma}_a$ = minimum and maximum service life of aircraft a, and
l_a = lag in years between the year when aircraft a is paid for and the year it joins the operational fleet.

The final two index sets are

C_{ta} = cohort years c for aircraft a such that $\underline{\sigma}_a \leq t - c - l_a \leq \bar{\sigma}_a$, and
C'_{ta} = cohort years c for aircraft a such that $0 \leq t - c' - l_a \leq \bar{\sigma}_a$.

The set C_{ta} identifies cohorts of aircraft a which are eligible for retirement or SLEP in year t. The set C'_{ta} identifies cohorts which are part of the operational fleet in year t.

The remaining data for the model are

CP_a = the unit cost of producing aircraft a,
CR_a = the first-year cost of retiring aircraft a,
CO_{tac} = annual O&M cost for an aircraft a of age $t - c - l_a$ years,
CF_{ptvw} = the fixed costs paid in year t for line p as a result of starting a production campaign in year v and ending it in year w,
D_p = an indicator which is 1 if line p must be opened and 0 otherwise,
$\underline{PC}_p, \overline{PC}_p$ = minimum and maximum cumulative number of aircraft that must be produced during a production campaign on line p,
$\underline{N}_{ptvw}, \overline{N}_{ptvw}$ = minimum and maximum number of aircraft that can be produced on production line p in year t given that the production campaign begins in year v and ends in year w,
$\underline{B}_t, \overline{B}_t$ = minimum and maximum budget available in year t,
$CC_{a'a}$ = the per unit cost of converting aircraft a' into aircraft a by SLEP,
H_{ta} = an indicator that is 1 if aircraft a is classified as a high technology aircraft in year t and 0 otherwise,
$\overline{FA}_{tm}$ = maximum allowable average age of all operational aircraft in the fleet performing mission m in year t,
$\underline{FR}_{tm}, \overline{FR}_{tm}$ = minimum and maximum number of operational aircraft required in the fleet performing mission m in year t,
$\underline{FT}_{tm}$ = minimum fraction of aircraft performing mission m in year t that are required to be of high technology, and

α_a = annual survival rate (fraction) of aircraft a.

The decision variables of the model are

X_{tac} = inventory of operational aircraft of type a of cohort year c in year t (Note: X_{tat} is the number of aircraft a produced in year t),

R_{tac} = the number aircraft a in cohort c that are retired at the beginning of year t,

$S_{ta'ca}$ = the number aircraft a' in cohort c that are diverted by SLEP to produce aircraft a at the beginning of year t, and

O_{pvw} = an indicator variable which is 1 if production line p is opened at the beginning of year v and closed at the end of year w.

The model is a mixed integer linear program with standard constraints and elastic constraints. Elastic inequalities, denoted $\dot{\leq}$, can be violated at a linear cost per unit of violation:

Minimize

$$\sum_{t\in T}\sum_{a\in A}\sum_{c\in C'_{ta}} CO_{tac}X_{tac}$$

+ penalties for violating elastic constraints subject to

$$\underline{FR}_{tm} \,\dot{\leq}\, \sum_{a\in A_m}\sum_{c\in C'_{ta}} X_{tac} \,\dot{\leq}\, \overline{FR}_{tm} \qquad t\in T,\ m\in M \tag{1}$$

$$\sum_{a\in A_m}\sum_{c\in C'_{ta}} (H_{ta} - \underline{FT}_{tm})X_{tac} \,\dot{\geq}\, 0 \qquad t\in T,\ m\in M \tag{2}$$

$$\sum_{a\in A_m}\sum_{c\in C'_{ta}} ((t - c - l_a) - \overline{FA}_{tm})X_{tac} \,\dot{\leq}\, 0 \qquad t\in T,\ m\in M \tag{3}$$

$$\underline{B}_t \,\dot{\leq}\, \sum_{a\in A} CP_aX_{tat} + \sum_{a\in A}\sum_{a'\in A'_a, c\in C'_{ta'}} CC_{a'a}S_{ta'ca} + \sum_{a\in A}\sum_{c\in C'_{ta}} CO_{tac}X_{tac} + \sum_{p\in P}\sum_{(v,w)\in VW_{pt}} CF_{ptvw}O_{pvw} + \sum_{a\in A}\sum_{c\in C_{ta}} CR_aR_{tac} \,\dot{\leq}\, \bar{B}_t \qquad t\in T \tag{4}$$

$$D_p \leq \sum_{(v,w)\in VW_p} O_{pvw} \leq 1 \qquad p\in P \tag{5}$$

$$\sum_{(v,w)\in VW_p} \underline{PC}_pO_{pvw} \,\dot{\leq}\, \sum_{t\in T}\sum_{a\in A_p} X_{tat} \,\dot{\leq}\, \sum_{(v,w)\in VW_p} \overline{PC}_pO_{pvw} \qquad p\in P \tag{6}$$

$$\sum_{(v,w)\in VW_{pt}} \underline{N}_{ptvw}O_{pvw} \,\dot{\leq}\, \sum_{a\in A_p} X_{tat} \,\dot{\leq}\, \sum_{(v,w)\in VW_{pt}} \overline{N}_{ptvw}O_{pvw} \qquad t\in T\cap(\cup_{a\in A_p}C_a),\ p\in P \tag{7}$$

$$\sum_{a'\in A'_a}\sum_{c\in C_{ta'}} S_{ta'ca} - X_{tat} = 0 \qquad t\in T\cap C_a,\ a\in A \tag{8}$$

$$\sum_{v|(v,w)\in VW_p} O_{pvw} - \sum_{(v',w')\in\{VW_{p'}|v'=w+1\}} O_{p'v'w'} \geq 0 \qquad (p,p')\in PP,\ w\in W_p \tag{9}$$

$$-\alpha_aX_{t-1,ac} + X_{tac} + R_{tac} + \sum_{a''\in A''_a} S_{taca''} = 0 \qquad a\in A,\ c\in C,\ t\in\{T\,|\,0\leq t-c-l_a\leq \underline{\sigma}_a\} \tag{10}$$

$$X_{tac},\ R_{tac},\ S_{taca'} \geq 0$$

$$O_{ptvw} \in \{0, 1\}$$

Constraints (1) suggest that sufficient helicopters be available in each planning year to satisfy mission requirements. Constraints (2) suggest that each mission fleet contains at least a minimum fraction of high-technology aircraft in each planning year. Constraints (3) suggest that the average age of each mission fleet should not exceed a specified maximum age in each planning year. Constraints (4) suggest a minimum and maximum level of budget expenditure each year. These constraints contain aspects of the classic capital budgeting model [Lorie and Savage 1955]. Constraints (5) ensure that no more than one production campaign is initiated for each new aircraft design or SLEP. Constraints (6) suggest a limit on the total quantity produced on a production line. Constraints (7) suggest that annual aircraft production on open production facilities should fall within upper and lower economic limits during each year of the campaigns. Constraints (8) ensure that sufficient old aircraft are available for upgrade via SLEP. Constraints (9) enforce a contingent relationship between selected production lines: if production line p closes in year w, then production line p' must open in year $w + 1$ or not at all. These constraints, along with constraints (5), are examples of logical conditions placed on interdependent projects in capital budgeting models [Weingartner 1963]. Constraints (10) are balance equations between adjacent planning years for operational aircraft, aircraft designated for SLEP, and retiring aircraft. These constraints are modifications of standard production/inventory balance equations [for example, Arrow, Karlin, and Scarf 1958, pg. 25] where there is no outside demand but there is attrition from one time period to the next.

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